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Richard Arnott
Petia Petrova

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ABSTRACT

Consider an atomistic developer who decides when and at what density to develop his land, under a property tax system characterized by three time-invariant tax rates: the tax rate on pre-development land value, the tax rate on post-development residual site value, and the tax rate on structure. Arnott (2002) identified the subset of property value tax systems which are neutral. This paper investigates the relative efficiency of four idealized, non-neutral property value tax systems (Canadian property tax system, simple property tax system, residual site value tax system, and differentiated property tax system) under the assumption of a constant rental growth rate.

Richard Arnott
Department of Economics
Boston College
Chestnut Hill, MA 02467
and NBER
Tel: 617-551-3674
Fax: 617-552-1887
richard.arnott@bc.edu

Petia Petrova
Boston College

1 Introduction[1]

Through the centuries land and real property taxation have taken many forms. Land has been taxed on the basis of area, foot frontage, and agricultural rent generated. And real property has been taxed on the basis of the number of windows, chimneys, or balconies, property rents, and estimated property values, among other things. A central tradeoff in the choice of how to tax land and structures is between efficiency and ease of tax collection. Foot frontage is easy to measure but taxing foot frontage gives rise to long, narrow lots; taxing the number of windows is simple but leads to windows being bricked up in structures built before the tax was imposed, and to structures built subsequently having a small number of large windows; and so on.

In the Anglo-Saxon countries at least, the dominant debate today vis-à-vis land and real property taxation concerns the choice between land/site value taxation, property value taxation, or some hybrid. The defining difference between these taxes concerns the tax base of *developed* properties: under property taxation, the assessed market value of the developed property is taxed; under site value taxation, the tax base subsequent to development is the imputed value of the land; under differentiated property taxation, the imputed values of land and structure are both taxed but at different rates. The same tradeoff occurs. Site value taxation in its purest form is non-distortionary, but subsequent to development measuring such site value is fraught with difficulty. At the other extreme, property taxation is relatively easy to apply since property values can be estimated quite accurately from market transactions, but is distortionary — encouraging inefficiently low capital intensity in construction. Confronted with this tradeoff different jurisdictions have made different choices. Property taxation is the norm in North America; in mainland China, site value taxation is employed; while Australia and New Zealand (and Pittsburgh) have chosen differentiated property taxation.

An essential element of the debate entails quantifying the deadweight loss associated with the various forms of property taxation.[2] The traditional analysis due to Cannan (1899, reprinted 1959) and Marshall (1961) has two distinctive features. First, it is partial equilibrium, analyzing the effects of taxing a single property in isolation. Land is treated as being completely inelastic in supply, so its taxation is non-distortionary; a building

[1]The authors would like to thank Will Strange for proposing the topic of the paper, seminar participants at the University of California, Berkeley, the European University Institute, GREQAM, Queen's University and Syracuse University for helpful comments; Almos Tassonyi of the Department of Finance, Government of Ontario, Canada for calculating Toronto's effective property tax rates for 1985; Ian Mead for bringing data on effective property tax rates in the U.S. to our attention; Mark Liu for preparation of Figures 5 and 7; and Shihe Fu for research assistance. The first draft of the paper was written while Arnott was visiting the Fisher Center at the University of California, Berkeley, the second while he was visiting GREQAM. He would like to thank both for their hospitality. We would like to dedicate the paper in memory of Louis-André Gérard-Varet for his tireless efforts in encouraging the application of public economic theory to policy issues in French local public finance.

[2]This paper follows the literature in using "property taxation" in two senses: as a generic term for the wide variety of systems of taxing land and buildings, and for the specific form of taxation in which a developed property is taxed on the basis of its market value. Hopefully, the usage will be apparent from the context.

is treated as perfectly-elastically-supplied, so its taxation is distortionary. The second distinctive feature of the Marshallian analysis is that it treats the taxes as falling on rents rather than — as is actually the case — on values. The deadweight loss generated by the structure component of the property tax can then be portrayed diagrammatically as a conventional deadweight loss triangle.

One line of subsequent work (most notably, Mieszkowski (1972)) has analyzed property taxation from the perspective of static, general equilibrium theory à la Harberger. In the basic variant of the model, the structure component of the property tax is viewed as a tax on capital in the building sector. A more sophisticated variant recognizes that different jurisdictions tax property at different rates. The *average* rate of the structure component of the property tax is viewed as a tax on capital in the building sector, and jurisdiction-specific deviations in the tax rate from the average rate as generating excise tax effects.[3] This branch of the literature continues in the Marshallian tradition by treating the taxes as falling on rents rather than on values.

Another line of subsequent work retains Marshall's partial equilibrium perspective but employs a dynamic analysis — specifically capital asset pricing theory — and treats the taxes as falling on values rather than rents. Shoup (1970) investigated the effect of property taxation on a developer's choice of *when* to construct a fixed project on a vacant lot, taking the time path of rents as given. Arnott and Lewis (1979) extended Shoup's analysis to allow for variable building density. Capozza and Li (1994) subsequently investigated how these results are modified by uncertainty, with rents being generated by an exogenous stochastic process. A closely related group of papers has focused on the neutrality of site value taxation. A neutral tax does not alter the developer's choice of timing or density, and therefore generates no deadweight loss. The first three papers (Skouras (1978), Bentick (1979), and Mills (1981)) came to the unorthodox conclusion that site value taxation is distortionary. It was subsequently shown that this result hinges on how post-development site value is defined. Once an immobile and durable building is constructed on a site, the market provides a valuation for the property (site and building together) but not separate valuations for the site and the building. Thus, post-development site value must be *imputed*. Skouras, Bentick, and Mills all defined post-development site value as property value minus structure value, which is now termed *residual* site value, and hence showed that residual site value taxation is distortionary, discouraging density. Tideman (1982), following Vickrey (1970), demonstrated that the orthodox conclusion that site value taxation is neutral is restored if post-development site value is instead defined as "what the market value of the land would be if there were no building on the site (though in fact there is)"; this alternative definition is termed *raw* site value. The policy debate these papers spawned (e.g., Netzer (1998), Tideman (undated), and Mills (1998)) has concentrated on the practicability of employing either definition; in particular, how might post-development residual site value and raw site value be estimated in practice, and how accurate would such estimates likely be? The majority view is that residual site value could be more easily and accurately estimated than raw site value. Less accurate

[3]Hamilton (1975) introduces zoning into this model, and lays out a set of conditions under which the property tax becomes a non-distortionary benefits tax.

estimation would not only result in a tax system that was perceived as more capricious and hence less fair, but would also likely lead to more corruption in assessment and more wasteful litigation in assessment appeals. Thus, the central tradeoff is between the greater efficiency of the raw site value tax and the lower administrative costs (broadly speaking) of the residual site value tax. And the magnitude of the deadweight loss associated with residual site value taxation is an essential element of the debate.

This paper contributes to this strand of the literature by investigating the *deadweight loss* associated with alternative property tax systems, treating the taxes as taxes on values rather than rents and in a partial equilibrium context. More precisely, it considers the subset of property tax systems that can be characterized by three time-invariant tax rates — one on pre-development land value, a second on post-development structure value, and a third on post-development residual site value. And for this subset of property tax systems, it relates the deadweight loss from taxation applied to the single property to the three tax rates, as well as to the time path of rents, the form of the structure production function, and the interest rate. Particularly neat results are obtained for the "Canadian" property tax system, which exempts land prior to development from taxation and then taxes *property* value subsequent to development (hence taxing post-development residual site value and structure value at the same rate, τ). Under the simplifying assumptions that agricultural rent is zero and that floor rent grows at a constant rate η, it is shown that the present value of property tax revenues is maximized by setting the post-development property tax rate equal to the growth rate of floor rent; a higher property tax rate puts taxation on "the wrong side of the Laffer curve". The marginal deadweight loss associated with the revenue-maximizing tax rate is, of course, infinity. At lower tax rates the marginal deadweight loss is shown to be $\tau/(\eta - \tau)$; if therefore floor rent grows at two percent, the application of a one percent property tax (i.e., one percent of property value) generates a 100% marginal deadweight loss — the marginal dollar of tax revenue collected has a social cost of two dollars. In 1985, the average effective property tax rate in the City of Toronto for residential (six storeys or less) housing was 1.1% and for multi-family residential (more than six storeys) 4.2%[4] , while the annual growth rate in real apartment rents in the Toronto CMA between 1979 and 1989 was less than one percent.[5] These observations suggest that the Toronto property tax has been highly distortionary. Similar results are obtained for many other jurisdictions.[6]

Such back-of-the-envelope calculations are, of course, subject to numerous qualifications, but do indicate — as its critics have argued — that some forms of property taxation may be very inefficient, and accordingly that switching to less distortionary but administratively more costly forms of property taxation merits serious policy consideration.

[4]Source: Calculated from Ontario Ministry of Municipal Affairs and Housing, Municipal Analysis and Retrieval System (MARS) Database.

[5]Source: CMHC Rental Market Surveys. Over the period the rental growth rates on bachelor, one-bedroom, two-bedroom, and three-bedroom apartments were .84%, .90%, 1.00% and .93%.

[6]Fisher and Peters (1998), Table 4.3, provides data on the lowest and highest effective property tax rates in 1992 for a sample of cities within a state, for a selection of states. Indiana's figures are 3.72%, 4.39%; Iowa's 4.23%, 4.68%; and Minnesota's 4.81%, 5.30%. Thus, the effective property tax rate for Toronto is by no means an outlier.

Section 2.1 introduces the basic model in the absence of taxation. The rest of Section 2 introduces property taxation, and derives the general formulae for the present values of tax revenue collected and of deadweight loss. Section 3 applies the results to four broad classes of property tax systems, employing numerical examples. Section 4 presents qualifications, discusses possible extensions, and concludes.

2 Theory

Since the theory is developed at length in a companion paper (Arnott [2002]), its presentation here will be compact.

2.1 The model in the absence of taxation

The model is essentially that presented in Arnott and Lewis [1979]. An atomistic landowner owns a unit area of undeveloped land. He must decide when to develop the land and at what density (floor-area ratio). Once built, the structure is immutable; no depreciation occurs and no redevelopment is possible. He makes his decisions under perfect foresight. To simplify even further, it is assumed that the interest rate, the price per unit of capital, and the structure production function are invariant over time, and also that land prior to development generates no rent. The following notation is employed:

t	time ($t = 0$ today)
T	development time
K	capital-land ratio
$Q(K)$	structure production function ($Q' > 0$, $Q'' < 0$)
$r(t)$	rent per unit floor area at time t — floor or structure rent
i	interest rate
p	price per unit of structure capital

The structure production function indicates how many units of structure are produced when K units of capital are applied to the unit area of land. For concreteness, one may think of Q as the number of units of rentable floor area per unit area of land (floor-area ratio), or less precisely but more intuitively as the number of storeys in the building (which assumes an exogenous coverage ratio).

The developer's problem in the absence of taxation is[7]

$$\max_{T,K} \Pi(T,K) = \int_T^\infty r(t)Q(K)e^{-it}dt - pKe^{-iT}. \tag{1}$$

The first-order conditions are

$$T \ : \ \left(-r(T)Q(K) + ipK\right)e^{-iT} = 0 \tag{2}$$

$$K \ : \ \left(\int_T^\infty r(t)Q'(K)e^{-i(t-T)}dt - p\right)e^{-iT} = 0. \tag{3}$$

[7]We assume throughout that exogenous variables and parameter values are such that asset values are finite.

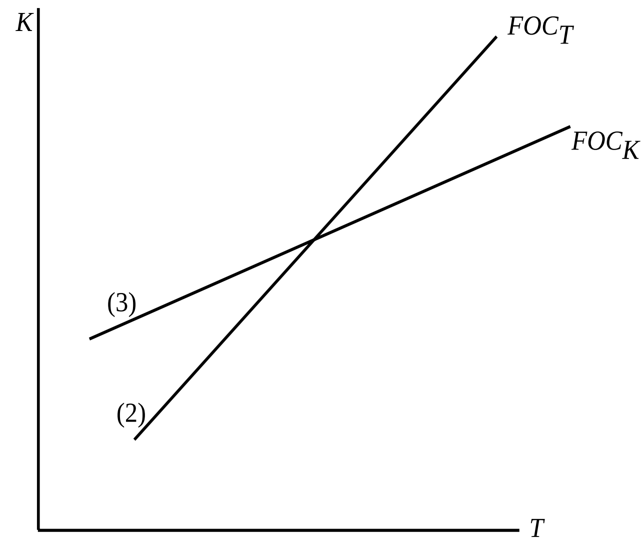

Figure 1: First-order conditions without taxation

Eq.(2) states that, K fixed, development time should be such that the marginal benefit from postponing construction one period (the one-period opportunity cost of construction funds) equal the marginal cost (the rent forgone). Eq.(3) states that, T fixed, capital should be added to the land up to the point where the increase in rental revenue due to an extra unit of capital, discounted to development time, equal the unit price of capital.

Figure 1 portrays the first-order conditions in T-K space. Both first-order conditions are positively-sloped, and the second-order conditions for an interior maximum require that rents be growing at development time and that the first-order condition for T be steeper than that for K (which, with a constant rate of rental growth, is equivalent to the requirement that the elasticity of substitution between land and capital in the production of structure be less than one). Multiple local interior maxima are possible and might indeed occur due to cyclical fluctuations, but to simplify the analysis we assume that there is a unique interior maximum, which is the global maximum.

We shall have occasion to use several different asset values. To simplify notation, we write these values on the assumption that development takes place at the profit-maximizing development time and density; for example, we write $P(t)$ for post-development *property value* instead of $P(t; T^*, K^*)$.

Post-development property value is

$$P(t) = \int_t^\infty r(u)Q(K)e^{-i(u-t)}du, \qquad t > T. \tag{4}$$

Some property tax systems (e.g., Australia, New Zealand and Pittsburgh) tax post-development structure value and post-development site value at different rates. Because of the spatial fixity of structures, land and structure value are not separately observable after development. Thus, post-development structure and site values are *imputed* values. The literature on the neutrality of land value taxation has analyzed two different concepts for post-development land or site value. The first, *residual site value*, denoted by $S(t)$, equals property value minus (depreciated — though here no depreciation is assumed) structure value:

$$S(t) = P(t) - pK, \qquad t > T. \tag{5}$$

The second *raw* site value, denoted by $\Sigma(t)$, is what the land would be worth at time t were it vacant, even though in fact it is developed:

$$\Sigma(t) = \max_{\widehat{T}(t),\widehat{K}(t)} \int_{\widehat{T}(t)}^{\infty} r(u)Q\left(\widehat{K}(t)\right)e^{-i(u-t)}du - p\widehat{K}(t)e^{-i(\widehat{T}(t)-t)}, \qquad t > T. \tag{6}$$

where $\widehat{T}(t)$ is the profit-maximizing development time[8] conditional on the land being undeveloped at time t, and $\widehat{K}(t)$ the corresponding profit-maximizing capital-land ratio. Since no depreciation is assumed, post-development *structure value* is simply pK. Pre-development *land value* is

$$V(t) = \int_{T}^{\infty} r(u)Q(K)e^{-i(u-t)}du - pKe^{-i(T-t)}, \qquad t < T. \tag{7}$$

Because development occurs at the profit-maximizing time and density,

$$V(T^-) = S(T^+) = \Sigma(T^+). \tag{8}$$

2.2 The model with taxation

In what follows we ignore the taxation of raw site value since in our opinion the difficulty of estimating it would render its taxation impractical. We should, however, note that a hypothetical property tax system which taxes pre-development land value and (post-development) raw site value at the same rate, and exempts (post-development) structure value from taxation, is neutral — does not affect the developer's choice of development time and density, and hence entails no deadweight loss. Since the developer's tax liability over time is independent of her choices, raw site value taxation is lump sum and so does not affect her decisions.

We restrict our analysis to the class of property tax systems characterized by three time-invariant tax rates: the tax rate on pre-development land value, τ_V; the tax rate on post-development residual site value, τ_S; and the tax rate on post-development structure

[8] By definition, $\widehat{T}(t) \geq t$. With smoothly growing rents, if the land remains undeveloped after the profit maximizing development time has passed ($t > T$), conditional on the land being undeveloped at time t it is profit-maximizing to develop it right away, in which case $\widehat{T}(t) = t$. It is possible, however, that the housing market is in a slump at time t so that even though the profit-maximizing development time has passed, conditional on the land being undeveloped at time t it is profit maximizing to develop later, in which case $\widehat{T}(t) > t$.

value, τ_K . We shall examine four different types of property tax systems. Under a *s̲imple* property tax system, $\tau_V = \tau_S = \tau_K \equiv \tau_s$; the tax rates on pre-development land value and post-development residual site value and structure value are all the same. Under a *C̲anadian* property tax system,$\tau_V = 0$, $\tau_S = \tau_K \equiv \tau_c$; property is untaxed prior to development[9], while after development property value is taxed, which is equivalent to taxing post-development residual site value and structure value at the same rate. Under a *r̲esidual* site value tax system $\tau_V = \tau_S \equiv \tau_r$, with $\tau_K = 0$; pre-development land value and post-development residual site value are taxed at the same rate while post-development structure value is exempt from taxation. Finally, a *differentiated* property tax system is like the common property tax system, except that the common tax rate on pre-development land value and post-development residual site value is higher than that on post-development structure value: $\tau_S = \tau_V \equiv \tau_L > \tau_K > 0$.

The rest of this section treats the general case. In the next section, we shall explore the properties of the above four different types of property tax systems using numerical examples.

We first derive the asset valuation formulae and the corresponding first-order conditions. Post-development residual site value is

$$\begin{aligned} S(t) &= \int_t^\infty r(u)Q(K)e^{-i(u-t)}du - pK - \\ &\quad -\tau_S \int_t^\infty S(u)e^{-i(u-t)}du - \tau_K \int_t^\infty pKe^{-i(u-t)}du \\ &= \int_t^\infty \left(r(u)Q(K) - ipK - \tau_S S(u) - \tau_K pK\right) e^{-i(u-t)}du. \end{aligned} \tag{9a}$$

Differentiation with respect to t yields

$$\dot{S} = -rQ + (i + \tau_K)pK + (i + \tau_S)S,$$

which has the solution

$$S(t) = \int_t^\infty \left(r(u)Q(K) - (i + \tau_K)pK\right) e^{-(i+\tau_S)(u-t)}du. \tag{9b}$$

Thus, the tax on structure value increases the cost of capital by the tax rate on structure value, while the tax on residual site value increases the post-development discount rate by the tax rate on residual site value. Pre-development land value equals

$$V(t) = \max_{T,K} S(T)e^{-i(T-t)} - \tau_V \int_t^T V(u)e^{-i(u-t)}du, \tag{9}$$

which, employing the procedure above and using (8), yields

[9] Under the property tax systems in most Canadian provinces, pre-development agricultural land is taxed on the basis of what it would be worth if it were held in agricultural use forever. Under our assumption that land prior to development generates no rent, this corresponds to a zero tax rate on pre-development land value.

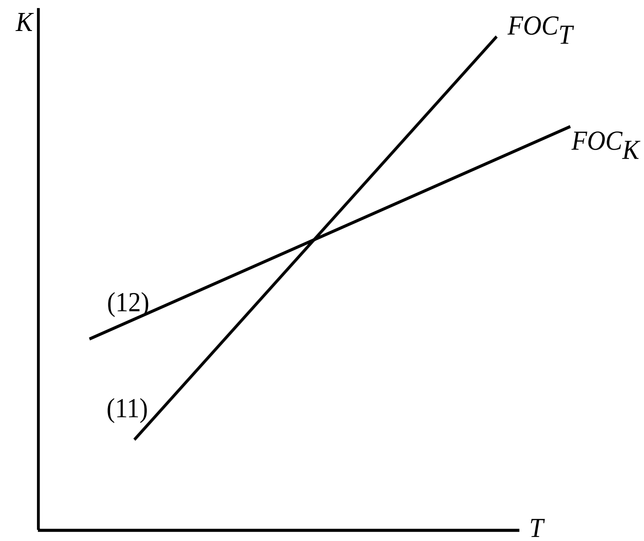

Figure 2: First-order conditions with taxation

$$V(t) = \max_{T,K} \left[\int_T^{\infty} (r(u)Q(K) - (i + \tau_K)pK) \, e^{-(i+\tau_S)(u-T)} du \right] e^{-(i+\tau_V)(T-t)}. \quad (10)$$

Hence, the pre-development land value tax increases the pre-development discount rate by the tax rate on pre-development land value.

The developer chooses T and K so as to maximize (10). The first-order conditions are

$$T \; : \; [-r(T)Q(K) + (i + \tau_K)pK + (\tau_S - \tau_V) \, V(T)] \, e^{-(i+\tau_V)(T-t)} = 0 \quad (11)$$

$$K \; : \; \left[\int_T^{\infty} \left(r(u)Q'(K) - (i + \tau_K)p \right) e^{-(i+\tau_S)(u-T)} du \right] e^{-(i+\tau_V)(T-t)} = 0. \quad (12)$$

Eq.(11) states that optimal development time occurs when the marginal benefit from postponing development one period equals the marginal cost. The marginal benefit equals the savings from postponing construction cost one period, which equals construction costs times the user cost of capital, $i + \tau_K$, plus the savings in site/land value tax payments, $(\tau_S - \tau_V) \, V(T)$. The marginal cost equals the rent forgone. Eq.(12) states that capital should be added to the site up to the point where the discounted rent attributable to the last unit of capital equals the discounted value of the user cost, with the discount rate equal to the interest rate plus the tax rate on post-development residual site value.

The effects of each of the three tax rates on profit-maximizing development time and density can be derived with the use of Figure 2 which is the same as Figure 1 except for the presence of property taxation. We assume that rents are "on average" rising over times. Under this assumption, the geometric implications of the second-order conditions are qualitatively the same as for Figure 1. Consider first τ_V. From (11), $\left.\frac{\partial K}{\partial \tau_V}\right|_{(11)} = \frac{V(T)}{-r(T)Q'(K) + (i+\tau_K)\,p}$; from (12), the assumption that rents are on average rising over time implies that the denominator is positive. A rise in the pre-development land value tax rate therefore causes (11) to shift up. From (12), $\left.\frac{\partial K}{\partial \tau_V}\right|_{(12)} = 0$. Thus, as intuition suggests, a rise in the pre-development land value tax rate causes earlier development at lower density. These and the other comparative static results with respect to T and K are recorded in Table 1.

Table 1

Comparative static effects of tax rates on development time and density

	τ_V	τ_S	τ_K
T	-	?*	?*
K	-	?*	?*

Note:*Building earlier at higher density can be ruled out.

Since the inefficiency due to a property tax derives from "how far from neutral" the property tax system is, it is of interest to characterize neutral property tax systems. For most of the paper, we shall focus on the special but central case where floor rent grows at a constant rate. For this case, Arnott (2002, Prop. 2) gives the result that: When floor rent grows at a constant rate η, a neutral property tax system has the properties that

$$\tau_K = \tau_S \left(-\frac{\eta}{i + \tau_S - \eta} \right) \qquad \tau_V = 0, \tag{13}$$

and provides two different intuitive explanations. The first is casual, the second exact. A residual site value tax system ($\tau_S = \tau_V > 0, \tau_K = 0$) has no effect on the development timing condition (see (11)), but by increasing the discount rate causes the development density condition to shift down, resulting in earlier development at lower density. Take this as the starting point and consider how τ_V, τ_S and τ_K should be modified to restore neutrality. First, capital should be subsidized to offset the depressing effect of residual site value taxation on development density. But from (11), the subsidization of capital advances development by reducing the marginal benefit from postponing development. The development timing condition, which was undistorted with residual site value taxation, becomes distorted, leading to excessively early development. This can be corrected by setting the pre-development land value tax rate below the post-development site value tax rate. This intuition suggests that a neutral property tax system has $\tau_S > \tau_V$ and $\tau_K < 0$.[10] The precise intuition is that the tax system described in (13) is equivalent to a

[10]This intuition is correct for the special case of a constant growth rate of rents, but not generally. See

site rent tax system with a time–invariant tax rate (post-development site rent is defined as rent net of amortized construction costs) which is neutral. Thus, how distortionary a property tax system is depends on how far it deviates from site rent taxation at a constant rate.

2.3 Deadweight loss

Define $Y(t)$ to be the discounted (and brought forward) social surplus from the site, evaluated at time t. This equals the discounted social benefit from the site, which equals the discounted revenue it generates, minus discounted construction costs. Thus,

$$Y(t) = \int_T^\infty r(u)Q(K)e^{-i(u-t)}du - pKe^{-i(T-t)}. \tag{14}$$

Letting ^{b} denote the pre-tax situation, ^{a} the after-tax situation, and $\mathcal{D}(t)$ the deadweight loss from the site evaluated at time t:

$$\mathcal{D}(t) = Y^b(t) - Y^a(t). \tag{15}$$

The social surplus from the site accrues to landowners, in the form of land value less brought-forward tax payments (before development) or property value less brought-forward construction costs less brought-forward past tax payments (after development), $\mathcal{L}(t)$, and to the government, in the form of discounted (and brought-forward) tax revenues, $\mathcal{R}(t)$. Hence,

$$Y^b(t) = \mathcal{L}^b(t) \qquad Y^a(t) = \mathcal{L}^a(t) + \mathcal{R}(t) \tag{16a,b}$$

$$\mathcal{D}(t) = \big(\mathcal{L}^b(t) - \mathcal{L}^a(t)\big) - \mathcal{R}(t). \tag{16c}$$

Eq. (16c) indicates that deadweight loss may be calculated as the loss in landowner surplus minus tax revenue.

With a tax on pre-development land value, the value of tax revenue collected depends on when the tax was first imposed. Let I ($\leq T$ and possibly < 0) represent this date, and $\mathcal{R}^-(t)$ denote the value of tax revenue collected prior to development. Then

$$\begin{aligned}
\mathcal{R}^-(t) &= \left(\tau_V \int_I^T V(u)e^{-i(u-t)}du\right)^a \\
&= \left(\tau_V e^{it} \int_I^T V(T)e^{-(i+\tau_V)(T-u)}e^{-iu}du\right)^a \\
&\qquad\qquad \left(\text{using } \overset{\bullet}{V} = (i+\tau_V)V \text{ from (10)}\right) \\
&= \left(\tau_V e^{it}V(T)e^{-(i+\tau_V)T} \int_I^T e^{\tau_V u}du\right)^a \\
&= \left(V(T)e^{-i(T-t)}(1-e^{-\tau_V(T-I)})\right)^a.
\end{aligned} \tag{17a}$$

Appendix 2 to Arnott [2002].

Let $\mathcal{R}^+(t)$ denote the value of the revenue collected after development. Then

$$
\begin{aligned}
\mathcal{R}^+(t) &= \left(\tau_K \int_T^\infty pKe^{-i(u-t)}du + \tau_S \int_T^\infty S(u)e^{-i(u-t)}du\right)^a \\
&= \left(\left[\int_T^\infty r(u)Q(K)e^{-i(u-T)}du - pK - S(T)\right] e^{-i(T-t)}\right)^a \text{(using (9a))} \\
&= Y^a(t) - \left(V(T)e^{-i(T-t)}\right)^a \quad \text{(using (14) and } V(T) = S(T)\text{)}. \qquad \text{(17b)}
\end{aligned}
$$

Also,

$$
\begin{aligned}
\mathcal{R}(t) &= \mathcal{R}^-(t) + \mathcal{R}^+(t) \\
&= Y^a(t) - \left(V(T)e^{-i(T-t)}e^{-\tau_V(T-I)}\right)^a. \qquad \text{(17c)}
\end{aligned}
$$

The landowner's pre- and post-tax present-value surpluses are[11]

$$
\mathcal{L}^b(t) = \left(V(T)e^{-i(T-t)}\right)^b \qquad \text{(18a)}
$$

and

$$
\begin{aligned}
\mathcal{L}^a(t) &= \left(V(T)e^{-i(T-t)}\right)^a - \mathcal{R}^-(t) \\
&= \left(V(T)e^{-i(T-t)}e^{-\tau_V(T-I)}\right)^a \qquad \text{(using (17a))}. \qquad \text{(18b)}
\end{aligned}
$$

In evaluating property tax systems, we assume that there is currently ($t = 0$) no property tax system in place and a choice is to be made concerning what property tax system to apply from today forward. This choice would be uninteresting if the site is already developed since property taxation would then have no real effects. Thus, *we examine the effects of a property tax system applied to an undeveloped site from today on.*[12] This conceptual excercise has two important implications. First, if the profit-maximizing development time computed per (11) and (12) is positive, then the pre-development land value tax is first applied today, i.e. $I = 0$. Second, if the profit-maximizing development time computed per (11) and (12) is negative, then (11) is replaced by the condition that development will occur at the most profitable time *from today forward.*

[11]Suppose $I = -\infty$, $\tau_V T$ is finite and $\tau_V I = -\infty$. Then, from (17c) and (18b), $\mathcal{R}(t) = Y^a(t)$ and $\mathcal{L}^a(t) = 0$. If, furthermore, τ_V, τ_S, and τ_K are all small, then the government expropriates the entire surplus from the site with no distortion. The first result states that if the government initially imposes the pre-development land tax infinitely far into the past at a rate such that $\tau_V I = -\infty$, it expropriates the entire surplus from the site. The second result indicates that this expropriation can be achieved with essentially no distortion if additionally the tax rates are sufficiently small. The practical relevance of this neutrality result is open to question!

[12]There is a problem with our conceptual exercise. Consider two property tax systems A and B. Choosing between the two tax systems today, A is preferred to B. However, if the choice were to be made ten years from now, B might be preferred to A. Such time inconsistency could arise if, for example, both tax systems yielded the same profit-maximizing development time and density, but tax system A generated more discounted revenue than B from $t = 0$ on while tax system B generated more discounted revenue from $t = 10$ on.

The equations derived earlier in this subsection were general. We now particularize them to the conceptual excercise we are performing. Let $\breve{T}$ denote profit-maximizing development time computed per (11) and (12), and $\tilde{T} \geq 0$ denote the profit-maximizing development time from today forward.

If $\breve{T} > 0$, the following system of equations applies: (14), (15), (16a,b,c) with $t = 0$, and

$$\mathcal{R}^{-}(0) = \left(V(\breve{T})e^{-i\breve{T}}\left(1 - e^{-\tau_V \breve{T}}\right)\right)^a = \left(V(0)\left(e^{\tau_V \breve{T}} - 1\right)\right)^a \tag{17a$'$}$$

$$\mathcal{R}^{+}(0) = Y^a(0) - \left(V(\breve{T})e^{-i\breve{T}}\right)^a \tag{17b$'$}$$

$$\mathcal{R}(0) = Y^a(0) - \left(V(\breve{T})e^{-(i+\tau_V)\breve{T}}\right)^a = Y^a(0) - (V(0))^a \tag{17c$'$}$$

$$\mathcal{L}^b(0) = \left(V(\breve{T})e^{-i\breve{T}}\right)^b = V^b(0) \tag{18a$'$}$$

$$\mathcal{L}^a(0) = \left(V(\breve{T})e^{-(i+\tau_V)\breve{T}}\right)^a = V^a(0) \tag{18b$'$}$$

If $\breve{T} < 0$, the following system of equations applies: (15), (16a,b,c,) with $t = 0$, and

$$Y(0) = \int_{\tilde{T}}^{\infty} r(u)\, Q(K)\, e^{-iu} du - pKe^{-i\tilde{T}} \tag{14$''$}$$

$$\mathcal{R}^{-}(0) = \left(V(\tilde{T})e^{-i\tilde{T}}\left(1 - e^{-\tau_V \tilde{T}}\right)\right)^a = \left(V(0)\left(e^{\tau_V \tilde{T}} - 1\right)\right)^a \tag{17a$''$}$$

$$\mathcal{R}^{+}(0) = Y^a(0) - \left(V(\tilde{T})e^{-i\tilde{T}}\right)^a \tag{17b$''$}$$

$$\mathcal{R}(0) = Y^a(0) - \left(V(\tilde{T})e^{-(i+\tau_V)\tilde{T}}\right)^a = Y^a(0) - (V(0))^a \tag{17c$''$}$$

$$\mathcal{L}^b(0) = \left(V(\tilde{T})e^{-i\tilde{T}}\right)^b = V^b(0) \tag{18a$''$}$$

$$\mathcal{L}^a(0) = \left(V(\tilde{T})e^{-(i+\tau_V)\tilde{T}}\right)^a = V^a(0) \tag{18b$''$}$$

In all the examples we shall consider, when $\breve{T} < 0$, $\tilde{T} = 0$.

Since by assumption no tax revenue is raised before $t = 0$, whether $\breve{T} < 0$ or $\breve{T} > 0$, landowner surplus at $t = 0$ equals land value at $t = 0 : \mathcal{L}(0) = V(0)$. Thus, from (16c)

$$\mathcal{D}(0) = V^b(0) - V^a(0) - \mathcal{R}(0)$$

or

$$V^b(0) = V^a(0) + \mathcal{R}(0) + \mathcal{D}(0), \tag{19}$$

which states that *today*, immediately after the imposition of any form of property taxation, land value plus discounted tax revenue plus discounted deadweight loss equals land value prior to taxation.[13]

The analysis obtains the deadweight loss from applying alternative property tax systems to a particular site in isolation. Extending the analysis to determine the efficiency of alternative tax systems applied to an entire jurisdiction requires a fuller model which accounts for the heterogeneity of sites as well as (unless the jurisdiction is completely open) for the endogeneity of rents. Thus, the reader should be cautious not to over-interpret the paper's very partial equilibrium analysis.

3 Four Property Tax Systems

Throughout this section we shall assume that structure rents grow at a constant rate η over time, which considerably simplifies the analysis. To simplify the algebra, we ignore the complications which arise when $\check{T} < 0$; we do, however, take account of these complications in our numerical examples. To further simplify notation, we omit the superscript ^{a} on after-tax variables, when there is no danger of ambiguity.

With this assumption, from (10):

$$V(t) = \max_{T,K} \left[\frac{r(T)Q(K)}{i+\tau_S-\eta} - \frac{i+\tau_K}{i+\tau_S} pK \right] e^{-(i+\tau_V)(T-t)}. \tag{20}$$

The corresponding first-order conditions are

$$T \;:\; \left[-\frac{i+\tau_V-\eta}{i+\tau_S-\eta} r(T)Q(K) + \frac{(i+\tau_V)(i+\tau_K)}{(i+\tau_S)} pK \right] e^{-(i+\tau_V)(T-t)} = 0 \tag{21}$$

$$K \;:\; \left[\frac{r(T)Q'(K)}{i+\tau_S-\eta} - \frac{i+\tau_K}{i+\tau_S} p \right] e^{-(i+\tau_V)(T-t)} = 0. \tag{22}$$

Dividing the two first-order conditions yields

$$\frac{Q(K)}{Q'(K)K} = \frac{i+\tau_V}{i+\tau_V-\eta}. \tag{23}$$

By the second-order conditions, the elasticity of substitution of Q is less than one. Since $Q/Q'K$ is therefore increasing in K, (23) implies that *profit-maximizing structural density decreases with τ_V and is independent of τ_S and τ_K*. Thus, we may write $K = K(\tau_V)$ with $K' < 0$. Letting $r_0 \equiv r(0)$, (21) and (23) imply

$$T = \frac{1}{\eta} \ln \left[\frac{(i+\tau_K)(i+\tau_S-\eta)p}{(i+\tau_S) r_0 Q'(K(\tau_V))} \right], \tag{24}$$

[13]The general relationship is $\mathcal{L}^b(t) = \mathcal{L}^a(t) + \mathcal{R}(t) + \mathcal{D}(t)$. Also in general, $V^b(t) = \mathcal{L}^b(t)$, so that $V^b(t) = \mathcal{L}^a(t) + \mathcal{R}(t) + \mathcal{D}(t)$. Thus (19) applies when $V^a(0) = \mathcal{L}^a(0)$, which holds since no tax revenue is collected prior to $T = 0$.

which indicates that profit-maximizing development time increases with τ_K and τ_S and decreases with τ_V. The second-order conditions, $\eta > 0$, $Q'' < 0$, and $\sigma < 1$, guarantee that there is a unique maximum, which is interior. These results are of sufficient importance that we record them in:

Proposition 1 *If the rate of rental growth is constant and if the second-order conditions of the developer's profit-maximization problem are satisfied, then: i) development density is decreasing in τ_V and independent of τ_S and τ_K; ii) development time increases with τ_S and τ_K and and decreases with τ_V.*

Following the discussion of the previous section, we shall take $I = 0$. Then from (17c), (14), and (20):

$$\mathcal{R}(0) = \left[r(T)Q(K) \left(\frac{1}{i-\eta} - \frac{e^{-\tau_V T}}{i+\tau_S-\eta} \right) - pK \left(1 - \frac{i+\tau_K}{i+\tau_S} e^{-\tau_V T} \right) \right] e^{-iT}. \tag{25}$$

Finally, from (14) and (15):

$$\mathcal{D}(0) = \left[\left(\frac{r(T)Q(K)}{i-\eta} - pK \right) e^{-iT} \right]^b - \left(\frac{r(T)Q(K)}{i-\eta} - pK \right) e^{-iT}. \tag{26}$$

We now consider four idealized property value tax systems. We start with what we have referred to as the Canadian property tax system, since it is the neatest and easiest to analyze.

3.1 Canadian property tax system ($\tau_V = 0$, $\tau_S = \tau_K \equiv \tau_c > 0$)

Many Canadian provinces and some U.S. states tax agricultural land on the basis of what it would be worth if it were held in agriculture forever (Youngman and Malme (1993)). Since pre-development land rent in our model is zero, application of such a tax system would result in no tax liability prior to development, which corresponds to $\tau_V = 0$. It can be seen from (21) and (22) that this property tax system causes both the timing and density first-order conditions to shift down, and from (23) in such a way that development occurs at the same density as in the absence of taxation but at a later date — as displayed in Figure 3.[14] That density is unaffected by the property tax system considerably simplifies the calculations. From (24):

$$T^a = T^b + \frac{1}{\eta} \ln \left(\frac{i+\tau_c-\eta}{i-\eta} \right). \tag{27}$$

From (25), the discounted revenue raised from the tax is

$$\mathcal{R}(0) = \frac{r(T)Q(K)\tau_c e^{-iT}}{(i-\eta)(i+\tau_c-\eta)}. \tag{28}$$

[14] This result was first demonstrated in Arnott and Lewis (1979).

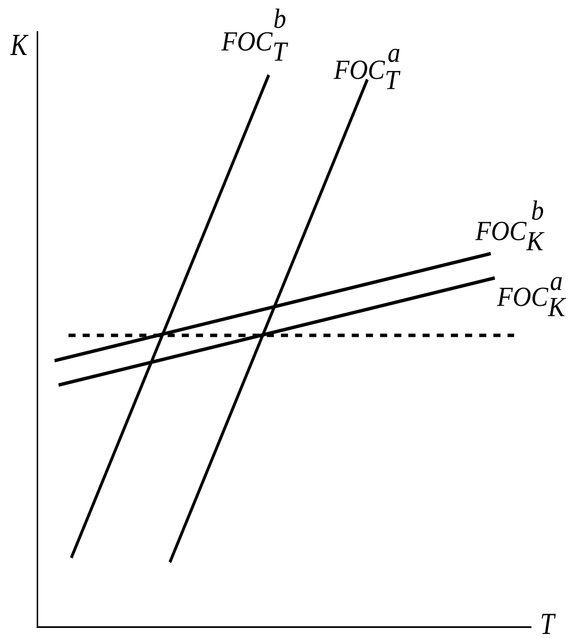

Figure 3: The Canadian property tax system

From (20) and (21):

$$V^b(0) = \left[\left(\frac{r(T)Q(K)}{i-\eta} - pK\right)e^{-iT}\right]^b = \left[\frac{\eta r(T)Q(K)}{i\,(i-\eta)}e^{-iT}\right]^b. \tag{29}$$

Substituting (29) and (27) into (28) yields

$$\mathcal{R}(0) = V^b(0)\left(\frac{i\tau_c}{\eta\,(i-\eta)}\right)\left(\frac{i-\eta}{i+\tau_c-\eta}\right)^{\frac{i}{\eta}}. \tag{30}$$

From (20):

$$\begin{aligned} V^a(0) &= \left(\frac{r(T)Q(K)}{i+\tau_c-\eta} - pK\right)e^{-iT} = \left(\frac{r(T)Q(K)}{i+\tau_c-\eta}\right)\left(\frac{\eta}{i}\right)e^{-iT} \text{ (using (21))} \\ &= V^b(0)\left(\frac{i-\eta}{i+\tau_c-\eta}\right)^{\frac{i}{\eta}} \text{ (using (27) and (29))}. \end{aligned} \tag{31}$$

Then from (19):

$$\mathcal{D}(0) = V^b(0)\left(1 - \left(\frac{\eta i - \eta^2 + i\tau_c}{\eta(i-\eta)}\right)\left(\frac{i-\eta}{i+\tau_c-\eta}\right)^{\frac{i}{\eta}}\right). \tag{32}$$

Two results are of sufficient interest that we record them as:

Proposition 2 *Under the Canadian property tax system:a) The revenue-maximizing tax rate is $\tau_c = \eta$. b) The marginal deadweight loss (MDWL) is $\dfrac{\tau_c}{\eta - \tau_c}$.*

Proof:
a) Follows directly from (30).
b) $MDWL = \dfrac{\partial \mathcal{D}(0)/\partial \tau_c}{\partial \mathcal{R}(0)/\partial \tau_c}$. The result then follows directly from (30) and (32).■

The results are so simple that there should be a simple explanation of them; so far, however, an incisive explanation has eluded us.[15]

Part a) of Prop. 2 is interesting since it suggests that jurisdictions in which the rental growth rate is less than the property tax rate may be on the wrong side of the Laffer curve. Part b) suggests that the Canadian property tax system can be highly distortionary at even modest tax rates.

We now present a numerical example. We choose units so that $K^b = Q^b = 1$, assume that $i = .03$ and $\eta = .02$, and choose $r_0 = .024731$ and $p = 2.2408$ so that development in the no-tax situation occurs at $T^b = 50$ and $V^b(0) = 1$.

Table 2
Numerical example with Canadian property tax

τ	K	T	$V(0) = \mathcal{L}(0)$	$\mathcal{R}^+(0) = \mathcal{R}(0)$	$\mathcal{D}(0)$	$MDWL$
0	1	50	1	0	0	0
.01	1	84.7	.354	.530	.116	1.0
.02	1	104.9	.192	.577	.230	∞
.03	1	119.3	.125	.563	.312	-3.0

Notes: i) recall that under our assumption that no tax revenue is collected prior to $t = 0$, $V(0) = \mathcal{L}(0)$; ii) recall (19), that the imposition of taxation at $t = 0$ does not change $V(0) + \mathcal{R}(0) + \mathcal{D}(0)$ and; iii) a marginal deadweight loss of -3.0 implies that as the tax rate is raised on the wrong side of the Laffer curve, a dollar reduction in revenue is associated with an increase in deadweight loss of 3.0.

Because profit-maximizing structural density is independent of the property tax rate, the above results hold independent of the form of the structure production function. The results, displayed in Table 2, indicate that the effects of the Canadian property tax are substantial. With the chosen parameters, a two-percent tax rate, for example, causes land value to fall to only 19% of pre-tax value, generates a deadweight loss of 23% of pre-tax value, and causes development of the land to be postponed 55 years!

If the tax is imposed at a location for which r_0 is lower, with the exogenous functions and other exogenous parameters held fixed, the only change is that everything occurs later. If, for example, $r_0' = \dfrac{r_0}{2}$, everything occurs $\dfrac{1}{\eta} \ln\left(\dfrac{r_0}{r_0'}\right) = 34.7$ years later (see (24)).

[15] A somewhat mechanical explanation is provided in Appendix 2.

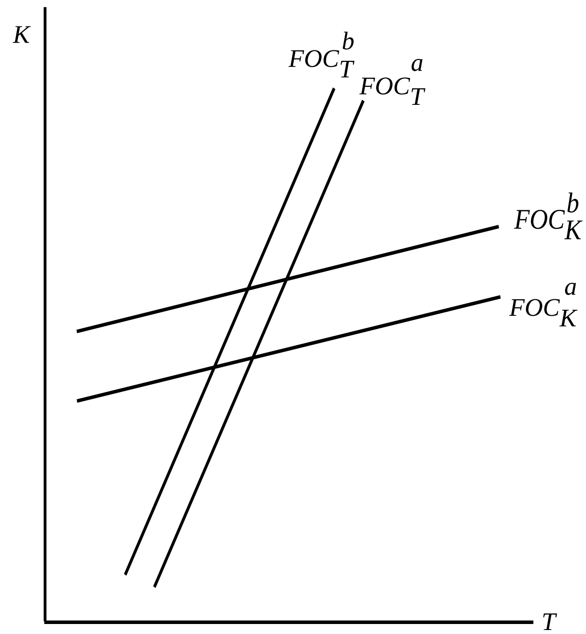

Figure 4: The simple property tax system

It bears repeating that the above results describe the effects of imposing the property tax on a single parcel of land. The general equilibrium effects, which could be examined in a growing, fully-closed monocentric city model, would be considerably more complicated.

3.2 Simple property tax system $(\tau_V = \tau_K = \tau_S \equiv \tau_s > 0)$

We now consider a simple tax system in which the tax rates applied to pre-development land value and post-development property value are the same. This is the tax system most Americans would identify as "the property tax".

As displayed in Figure 4, the simple property tax system causes both the timing and density first-order conditions to shift to the right (see (21) and (22)) such that: i) development density falls; and ii) development time may either be postponed or brought forward, depending on parameter values and the form of the structure production function.

From (25) and (21):

$$\mathcal{R}(0) = r(T)Q(K)e^{-iT}\left(\frac{\eta + \tau_s}{(i-\eta)(i+\tau_s)} - \frac{\eta e^{-\tau_s T}}{(i+\tau_s-\eta)(i+\tau_s)}\right). \tag{33}$$

From (20) and (21):

$$V^a(0) = \frac{\eta r(T)Q(K)}{(i+\tau_s)(i+\tau_s-\eta)}e^{-(i+\tau_s)T}. \tag{34}$$

And then, from (29), (32) and (33):

$$\begin{aligned}\mathcal{D}(0) &= V^b(0) - (V^a(0) + \mathcal{R}(0)) \\ &= \left[\frac{\eta r(T)Q(K)}{i(i-\eta)}e^{-iT}\right]^b - \frac{(\eta+\tau_s)r(T)Q(K)}{(i+\tau_s)(i-\eta)}e^{-iT}. \qquad (35)\end{aligned}$$

Unfortunately, the results for this type of property tax system are not as neat as those for the Canadian property tax system because the tax alters structural density, which complicates the algebra. We can, however, obtain some analytical results for the special case of CES production functions. We assume that

$$Q(K) = c_0(1 + c_1 K^\rho)^{\frac{1}{\rho}},$$

where $\sigma = \dfrac{1}{1-\rho}$ in the elasticity of substitution between land and capital in the structure production function. Then from (23):

$$K = \left(\frac{\eta c_1}{i+\tau_s-\eta}\right)^{\frac{\sigma}{1-\sigma}} \text{ and } Q(K) = c_0\left(\frac{\eta}{i+\tau_s}\right)^{\frac{\sigma}{1-\sigma}}. \qquad (36a,b)$$

From (21):

$$T = \frac{1}{\eta}\left(\ln\left(\frac{p(i+\tau_s)}{c_0 r_0}\right) + \frac{\sigma}{1-\sigma}\ln\left(\frac{c_1(i+\tau_s)}{i+\tau_s-\eta}\right)\right), \qquad (37)$$

from which it follows that development is postponed or brought forward according to whether $\ln\left(\dfrac{i+\tau_s}{i}\right) + \dfrac{\sigma}{1-\sigma}\ln\left(\left(\dfrac{(i+\tau_s)}{i}\right)\left(\dfrac{i-\eta}{i+\tau_s-\eta}\right)\right)$ is greater or less than zero, or since $1-\sigma > 0$ according to whether $\ln\left(\dfrac{i+\tau_s}{i}\right)$ is greater or less than $\sigma \ln\left(\dfrac{i+\tau_s-\eta}{i-\eta}\right)$. And from (21) and (36a):

$$r(T)Q(K) = (i+\tau_s)\,p\left(\frac{\eta c_1}{i+\tau_s-\eta}\right)^{\frac{\sigma}{1-\sigma}}. \qquad (38)$$

Inserting (38) into (33) – (35) yields

$$\mathcal{R}(0) = p\left(\frac{\eta c_1}{i+\tau_s-\eta}\right)^{\frac{\sigma}{1-\sigma}} e^{-iT}\left(\frac{\eta+\tau_s}{i-\eta} - \frac{\eta e^{-\tau_s T}}{i+\tau_s-\eta}\right) \qquad (33')$$

$$V^a(0) = \frac{\eta p}{(i+\tau_s-\eta)}\left(\frac{\eta c_1}{i+\tau_s-\eta}\right)^{\frac{\sigma}{1-\sigma}} e^{-(i+\tau_s)T} \qquad (34')$$

and

$$\mathcal{D}(0) = \left[\frac{\eta r(T)Q(K)}{i\,(i-\eta)}e^{-iT}\right]^b - \frac{(\eta+\tau_s)\,p}{i-\eta}\left(\frac{\eta c_1}{i+\tau_s-\eta}\right)^{\frac{\sigma}{1-\sigma}} e^{-iT}. \qquad (35')$$

We shall now use these formulae in a numerical example. We employ the same parameters as in the numerical example of the previous subsection, and choose c_0 and c_1 such that $K = Q = 1$ in the absence of taxation. We shall consider three values for the elasticity of substitution in the structure production function: $\sigma = .25, .5, .75$. In the absence of taxation the site is developed at $t = 50$ and has $V^b = 1$. The results are displayed in Table 3.

Table 3

Numerical example with simple property tax

τ	K	Q	T	$V(0) = \mathcal{L}(0)$	$\mathcal{R}^-(0)$	$\mathcal{R}^+(0)$	$\mathcal{R}(0)$	$\mathcal{D}(0)$	$MDWL$
0	1	1	50	1	0	0	0	0	0.00
.01	.794	.909	57.6	.177	.138	.632	.770	.052	0.39
.02	.693	.843	65.7	.039	.105	.721	.826	.135	−3.41
.03	.630	.794	73.1	.009	.070	.709	.780	.212	−1.22
(a) $\sigma = .25,\ c_0 = 1.14471$									
.01	.500	.750	44.1	.192	.106	.597	.703	.105	1.30
.02	.300	.600	46.2	.050	.075	.624	.699	.252	−1.65
.03	.250	.500	50.0	.014	.048	.563	.611	.375	−1.13
(b) $\sigma = .50,\ c_0 = 1.5$									
.01	.125	.422	3.56	.243	.009	.503	.512	.245	−4.99
.02*	.037	.281	0	.101	0	.463	.463	.436	−2.07
.03*	.016	.208	0	.051	0	.386	.386	.563	−1.41
(c) $\sigma = .75,\ c_0 = 3.375$									
Parameter values in all panels : $i = .03$, $\eta = .02$, $p = 2.2408$, $r_0 = .024731$, $c_1 = .5$									

Notes: 1. *The formulae presented in this subsection are for the case $\breve{T} > 0$. If $\breve{T} < 0$ according to (37), development occurs today. The calculations are modified accordingly. (Eq. (21) does not apply and T is set to 0. K is then determined from (22) alone, from which Q follows. $V^a(0)$ is then determined from (20), $\mathcal{D}(0)$ from (26), and $\mathcal{R}(0)$ as a residual per (19) from (25). $\mathcal{R}(0) = \dfrac{r_0 Q(K)\tau_s}{(i-\eta)(i+\tau_s-\eta)}$,where $Q(K) = c_0\left(1 + c_1 K^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$ and $K^{\frac{1-\sigma}{\sigma}} + c_1 = \left(\dfrac{c_0 c_1 r_0}{p(i+\tau_s-\eta)}\right)^{1-\sigma}$).

2. $\mathcal{R}^-(0)$ is calculated from (17a$'$), and $\mathcal{R}^+(0)$ as $\mathcal{R}^+(0) = \mathcal{R}(0) - \mathcal{R}^-(0)$.

3. The revenue-maximizing tax rate and maximum revenue are: $\sigma = .25$, $\tau_s = .0176$, $\mathcal{R}(0) = .8286$; $\sigma = .5$, $\tau_s = .0140$, $\mathcal{R}(0) = .725$; $\sigma = .75$, $\tau_s = .00850$, $\mathcal{R}(0) = .517$.

With a low elasticity of substitution for the structure production function, the developer responds to the tax by building somewhat later at slightly lower density. Compare

Table 2 with Table 3A. The relative efficiency of the Canadian tax system and the simple tax system with $\sigma = .25$ can be gauged by comparing the deadweight losses for a given amount of revenue collected, or vice versa. Recall that with a constant rate of rental growth, the tax rates for a neutral property tax system are given in (13). It is not obvious which of the Canadian or simple tax systems deviates more from the neutral tax system. But for the parameter values of the numerical example, with $\sigma = .25$ the simple tax system with is the more efficient. For intermediate values of the elasticity of substitution, the developer responds to the tax principally by building at moderately lower density, and for high values by building considerably earlier at considerably lower density. The tax system becomes more distortionary the higher the elasticity of substitution, and for $\sigma = .75$ the Canadian tax system is more efficient than the simple tax system. Based on empirical studies which estimate the elasticity of substitution between land and capital in the structure production function (reviewed in McDonald(1981)), the current wisdom is that σ lies between .6 and .7; and in this range the efficiency of the two tax systems is similar.

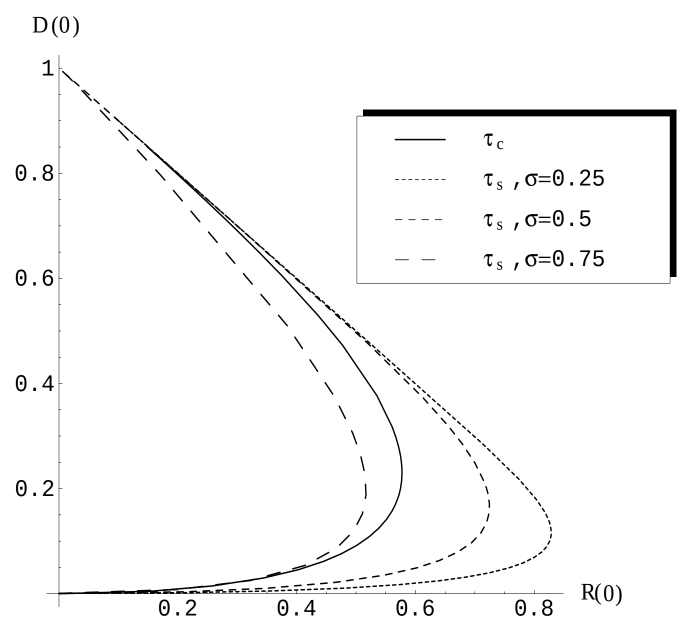

Figure 5

Figure 5 plots $\mathcal{D}(0)$ versus $\mathcal{R}(0)$ for the Canadian property tax system and the simple property tax system with $\sigma = .25, .5$, and $.75$. As noted above, the simple property tax system is more efficient the lower the elasticity of substitution between land and capital in the structure production function. Furthermore, the Canadian property tax system is less efficient than the simple property tax system with $\sigma = .25$ and $\sigma = .5$ but more efficient with $\sigma = .75$. Since empirical evidence suggests that σ lies in the range $(.5, .75)$, for the parameters of the example at least the Canadian and simple property tax systems are comparable in efficiency.

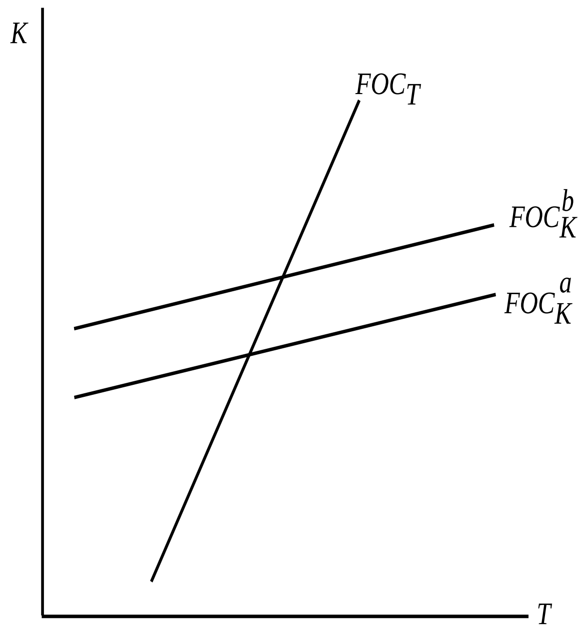

Figure 6: Residual site value taxation

3.3 Residual site value tax system ($\tau_K = 0,\ \tau_V = \tau_S \equiv \tau_r$)

Residual site value taxation is employed in China (Wong(1999)) and in some Australian states (Youngman and Malme (1993)). Residual site value taxation leaves unchanged the position of the timing first-order condition and causes the density first-order condition to shift down — as displayed in Figure 6. As a result, as the tax rate rises development occurs earlier and at a lower density.

From (25) and (21):

$$\mathcal{R}(0) = r(T)Q(K)e^{-iT}\left(\frac{\eta}{(i-\eta)\,i} - \frac{\eta e^{-\tau_r T}}{(i+\tau_r)\,(i+\tau_r-\eta)}\right). \tag{39}$$

From (20) and (21):

$$V^a(0) = \frac{\eta r(T)Q(K)}{(i+\tau_r)\,(i+\tau_r-\eta)}e^{-(i+\tau_r)T}. \tag{40}$$

And from (29), (39), and (40):

$$\begin{aligned}\mathcal{D}(0) &= V^b(0) - (V^a(0) + \mathcal{R}(0)) \\ &= \left[\frac{\eta r(T)Q(K)}{i\,(i-\eta)}e^{-iT}\right]^b - \frac{\eta r(T)Q(K)}{i\,(i-\eta)}e^{-iT}.\end{aligned} \tag{41}$$

As with the simple property tax system, we assume that the structure production function is CES. Then from (23):

$$K=\left(\frac{\eta c_1}{i+\tau_r-\eta}\right)^{\frac{\sigma}{1-\sigma}} \quad \text{and} \quad Q(K)=c_0\left(\frac{\eta}{i+\tau_r}\right)^{\frac{\sigma}{1-\sigma}}, \tag{42a,b}$$

which are the same expressions as for the simple property tax, but with τ_r replacing τ_s. From (21):

$$T=\frac{1}{\eta}\left(\ln\left(\frac{ip}{c_0 r_0}\right)+\frac{\sigma}{1-\sigma}\ln\left(\frac{c_1\left(i+\tau_r\right)}{i+\tau_r-\eta}\right)\right) \tag{43}$$

and

$$r(T)Q(K)=ip\left(\frac{\eta c_1}{i+\tau_r-\eta}\right)^{\frac{\sigma}{1-\sigma}}. \tag{44}$$

Then:

$$\mathcal{R}(0)=ip\left(\frac{\eta c_1}{i+\tau_r-\eta}\right)^{\frac{\sigma}{1-\sigma}} e^{-iT}\left(\frac{\eta}{(i-\eta)\,i}-\frac{\eta e^{-\tau_r T}}{\left(i+\tau_r\right)\left(i+\tau_r-\eta\right)}\right) \tag{45}$$

$$V^a(0)=\frac{\eta ip}{\left(i+\tau_r\right)\left(i+\tau_r-\eta\right)}\left(\frac{\eta c_1}{i+\tau_r-\eta}\right)^{\frac{\sigma}{1-\sigma}} e^{-(i+\tau_r)T} \tag{46}$$

and

$$\mathcal{D}(0)=\left[\frac{\eta r(T)Q(K)}{i\,(i-\eta)}e^{-iT}\right]^b-\frac{\eta p}{i-\eta}\left(\frac{\eta c_1}{i+\tau_r-\eta}\right)^{\frac{\sigma}{1-\sigma}} e^{-iT}. \tag{47}$$

We now turn to the numerical example. The parameters are exactly the same as for corresponding case for the simple property tax system example. The results are displayed in Table 4.

Comparing this table with Table 3, it is evident, for the example considered at least, that residual site value taxation is more efficient than simple property taxation. The only difference between the two tax systems is that residual site value taxation exempts structures. The supplementary taxation of structures under the simple property tax system is so distortionary that in a number of the cases treated — those with higher rates of taxation and higher substitution elasticities — setting the tax *rates* equal, the revenue raised from residual site value taxation is higher than under simple property taxation. Put alternatively, holding the tax rate constant, revenue would be increased by exempting structures from taxation.

3.4 Differentiated property tax system ($\tau_V=\tau_S\equiv\tau_L>0, \tau_K$ unrestricted)

There is potentially a considerable variety of hybrid property tax systems. One is the neutral property tax system discussed in Arnott (2000). Here we discuss the *differentiated*

Table 4

Numerical example with residual site value taxation.

τ	K	Q	T	$V(0) = \mathcal{L}(0)$	$\mathcal{R}^-(0)$	$\mathcal{R}^+(0)$	$\mathcal{R}(0)$	$\mathcal{D}(0)$	$MDWL$
0	1	1	50	1	0	0	0	0	0.00
.01	.794	.909	43.2	.237	.128	.608	.736	.028	0.17
.02	.693	.843	40.2	.0833	.103	.744	.847	.070	1.13
.03	.630	.794	38.4	.0351	.076	.780	.856	.109	−4.14
(a) $\sigma = .25$									
.01	.500	750	29.7	.256	.089	.574	.663	.081	0.75
.02	.300	.600	20.6	.107	.054	.644	.698	.195	−3.77
.03	.250	.500	15.3	.0588	.033	.619	.651	.293	−1.56
(b) $\sigma = .5$									
.01*	.172	.492	0	.320	.000	.512	.512	.168	0.42
.02*	.132	.433	0	.180	.000	.596	.596	.216	1.27
.03*	.112	.399	0	.121	.000	.615	.615	.265	3.64
(c) $\sigma = .75$									

Notes: *1. The formulae presented in this subsection are for the case $\breve{T} > 0$. If $\breve{T} < 0$ according to (43), development occurs today. The calculations are modified accordingly.

2. The revenue-maximizing tax rate and maximum revenue are: $\sigma = .25$, $\tau_r = .0267$, $\mathcal{R}(0) = .8574$; $\sigma = .5$, $\tau_r = .0167$, $\mathcal{R}(0) = .7035$; $\sigma = .75$, $\tau_r = .0415$, $\mathcal{R}(0) = .619$.

property tax system, employed in Australia, New Zealand, and Pittsburgh, under which pre-development land value and post-development residual site value are taxed at the same positive rate, and structure value at a different rate, where τ_L is the tax rate on "land value". The standard rationale for this type of system goes as follows: Site value taxation is non-distortionary but does not raise sufficient revenue to finance the level of public services demanded in a modern economy, and so is supplemented with distortionary structure value taxation but at a lower tax rate. Since these jurisdictions measure post-development site value as residual site value, and since residual site value taxation is distortionary, the standard rationale is flawed.[16] It is nonetheless of interest to enquire: Within this class of property tax systems, what ratio of $\frac{\tau_K}{\tau_L}$ is the most efficient and what

[16] There is a more basic logical flaw in the standard rationale for a differentiated property tax system. A pure land value tax (tax on market land value prior to development and on raw site value after development), set at an infinite rate, extracts a site's entire discounted surplus. No *anticipated* property tax system can raise more than this amount. To do so would require a negative landowner discounted surplus, which is not possible with anticipation. Consider the effects of broadening the tax base to include structures. At low tax rates, the addition of structures to the tax base does result in higher revenue. But above a certain tax rate, the revenue generated from structures is more than offset by the decline in revenue generated from the land deriving from the distortion caused by the taxation of structures.

does this ratio depend on?

The analysis is complicated by the presence of two regimes: $\breve{T} > 0$ and $\breve{T} < 0$. The way we shall proceed is to: first, examine the systems of equations that applies with $\breve{T} > 0$ without reference to the constraint that this system of equations applies when and only when $\breve{T} > 0$; second, examine the systems of equations that applies with $\breve{T} < 0$ (and hence $T = 0$) without reference to the constraint that this system of equations applies when and only when $\breve{T} < 0$; and finally put the pieces together.

3.4.1 $\breve{T} > 0$

From (23), application of this differentiated property tax system results in construction at lower than the efficient density. And since this tax system has the simple property tax system and residual site value tax systems as special cases, it is clear from previous results that its application may cause development to be either postponed or brought forward. We continue to assume that structure production exhibits constant elasticity of substitution. Thus, from (23):

$$K = \left(\frac{\eta c_1}{i + \tau_L - \eta}\right)^{\frac{\sigma}{1-\sigma}} \qquad\qquad Q(K) = c_0 \left(\frac{\eta}{i + \tau_L}\right)^{\frac{\sigma}{1-\sigma}}. \tag{48a,b}$$

And then from (21):

$$T = \frac{1}{\eta}\left(\ln\left(\frac{p(i + \tau_K)}{c_0 r_0}\right) + \frac{\sigma}{1-\sigma}\ln\left(\frac{c_1\,(i + \tau_L)}{i + \tau_L - \eta}\right)\right). \tag{49}$$

From (25) and (21):

$$\mathcal{R}(0) = p\left(\frac{\eta c_1}{i + \tau_L - \eta}\right)^{\frac{\sigma}{1-\sigma}} e^{-iT}\left(\frac{\eta + \tau_K}{i - \eta} - \frac{e^{-\tau_L T}(i + \tau_K)\eta}{(i + \tau_L)\,(i + \tau_L - \eta)}\right) \tag{50}$$

and

$$\mathcal{D}(0) = Y^b(0) - \left(\frac{\eta + \tau_K}{i - \eta}\right) p \left(\frac{\eta c_1}{i + \tau_L - \eta}\right)^{\frac{\sigma}{1-\sigma}} e^{-iT}. \tag{51}$$

Define the *efficiency locus* to be the locus of (τ_K, τ_L) that raise a given amount of revenue with minimum deadweight loss. Analytical characterization of the efficiency locus is algebraically messy.[17] Consequently, we examine diagrammatically the efficiency locus for the three examples considered in Tables 3 and 4.

Figure 7A displays iso-revenue contours, iso-deadweight-loss contours, and the corresponding efficiency locus for $\sigma = .25$. For all levels of revenue indicated, $\tau_L > \tau_K$. The revenue-maximizing tax rates are $\tau_K = .0039$ and $\tau_L = .0251$, and the maximum revenue is .862.

[17] It can however be shown that structure value should be taxed not subsidized — see Appendix 3.

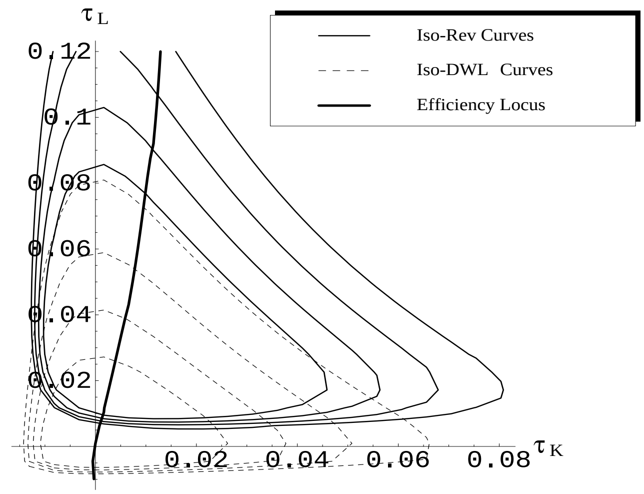

Figure 7A: Efficient land/site and structure value tax rates: σ=.25

Notes: What is labelled the "efficiency locus" is in fact the locus of tangency points of iso-revenue and iso-deadweight loss contours. The relevant portion of this locus runs from the origin to the maximum revenue point.

The simple property tax system is the same as the differentiated property system with the constraint imposed that $\tau_K = \tau_L$. Thus, comparison of the differentiated and simple property tax systems indicates the gains that can be achieved by taxing post-development residual site value and structure value at different rates. With the corresponding simple property tax system (from Table 3), the revenue-maximizing tax rate is .0176 and the maximum revenue .829. Plotting $\mathcal{D}(0)$ against $\mathcal{R}(0)$ for the differentiated property tax system and comparing the locus with the corresponding locus in Figure 5 for the simple property tax system would indicate the proportional efficiency gain achievable at different revenue levels from employing differentiated rather than simple property taxation.

The residual site value tax system is the same as the differentiated property tax system, with the constraint imposed that $\tau_K = 0$. Thus, comparison of the differentiated and residual site value tax systems indicates the gain that can be achieved over residual site value taxation by taxing structure value. The maximum revenue that can be achieved under residual site value taxation is .857. Thus, for this example, the extra revenue that can be generated under the differentiated property tax system from being able to tax structure value in addition to land value is quite modest.

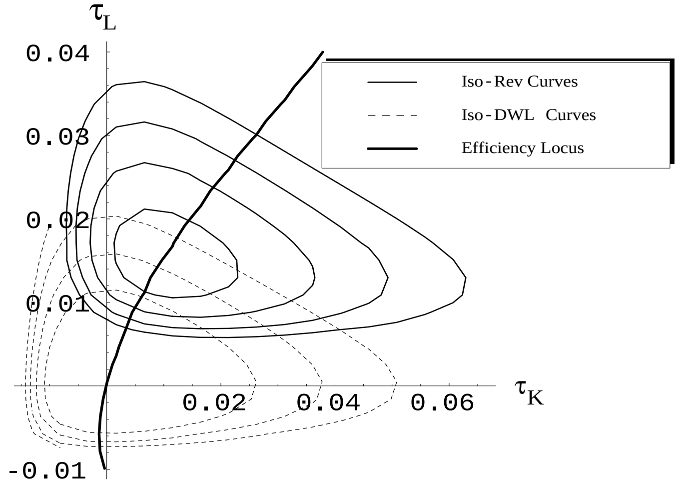

Figure 7B: Efficient land and structure value tax rates: σ=.5

Notes: What is labelled the "efficiency locus" is in fact the locus of tangency points of iso-revenue and iso-deadweight loss contours. The relevant portion of this locus runs from the origin to the maximum revenue point.

Figure 7B is the same as Figure 7A but is for $\sigma = .5$ (and the corresponding parameters from Table 3). The revenue-maximizing tax rates are $\tau_K = .0099$ and $\tau_L = .0150$, and the maximum revenue is .728. Observe that $\frac{\tau_K}{\tau_L}$ is higher at the revenue maximum with $\sigma = .5$ than with $\sigma = .25$. For $\sigma = .5$, the revenue-maximizing tax rate under the simple property tax system is .0140 and the maximum revenue is .725. The improvement in efficiency from being able to tax post-development site value and structure value at different rates is small. Under the residual site value tax system, the revenue-maximizing tax rate is .0167 and the maximum revenue is .703.

Figure 7C is the same as Figures 7A and 7B but is for $\sigma = .75$. The revenue-maximizing tax rates are $\tau_K = .0072$ and $\tau_L = .0229$, and the maximum revenue is .540. Under the simple property tax system, the revenue-maximizing tax rate is .0122 and the maximum revenue is .499 , while under the residual site value tax system the corresponding figures are .008 and .460.

These examples illustrate that, for $\breve{T} > 0$, residual site value taxation is more efficient than simple property taxation for low levels of σ, and less efficient for higher levels, which is explained below. In all cases, the relative efficiency gains achieved from employing differentiated property taxation (with τ_K and τ_L set at optimal levels) rather than simple or residual site value taxation appear quite modest; the robustness of the result is worth examining. Finally, for the simple, residual, and differentiated property tax systems, the deadweight loss due to property taxation appears to be more sensitive to the elasticity of substitution between land and structures in the structure production function than to the form of property taxation employed, which points to the importance of obtaining more accurate estimates of this elasticity.

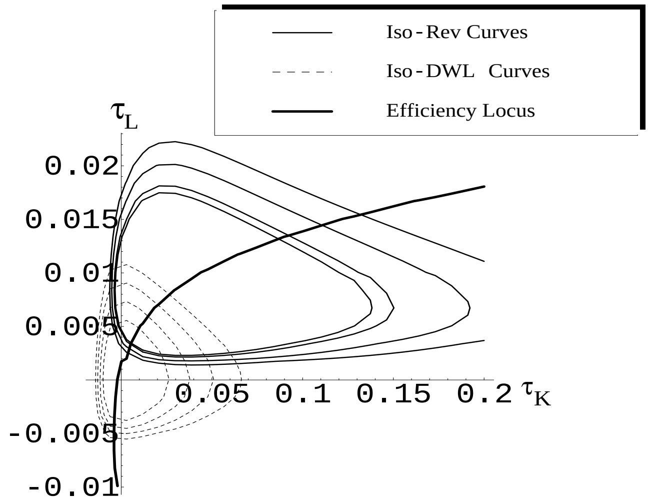

Figure 7C: Efficient land and structure value tax rates: σ=.75

Notes: What is labelled the "efficiency locus" is in fact the locus of tangency points of iso-revenue and iso-deadweight loss contours. The relevant portion of this locus runs from the origin to the maximum revenue point.

A differentiated Canadian property tax system under which $\tau_V = 0$ and $\tau_S > \tau_K$ is possible. Indeed, the neutral property tax system identified in (13) is an example, under which full surplus extraction with no deadweight loss is achievable. Recall that the maximum revenue with an undifferentiated Canadian property tax system (from Table 2, with $\tau = .02$) is .577. Thus, for the Canadian property tax system differentiation of the post-development tax rates generates significant efficiency gains.

3.4.2 $\breve{T} < 0$

In this case, T is set to 0. K is then determined from (22) alone, from which Q follows. $V^a(0)$ is then determined from (20), $\mathcal{D}(0)$ from (26) and $\mathcal{R}(0)$ as a residual per (19) from (25). Since there are now two tax rates and only the capital first-order condition, in the light of the argument presented in Arnott(2002), it should not be surprising that by setting $\tau_K = \dfrac{-\eta\tau_L}{i + \tau_L - \eta}$, *conditional on* $T = 0$ revenue can be raised without distortion, with the amount rising with τ_L until with $\tau_L = \infty$ full surplus extraction is achieved.

3.4.3 Putting the pieces together

Turn to Fig. 7A, which applies to the case $\breve{T} > 0$. This case applies when (τ_K, τ_L) are such that T in (49) is greater than or equal to zero. Since T in (49) is increasing in τ_K and decreasing in τ_L, and since $T = 50$ with $(\tau_K, \tau_L) = (0, 0)$, $\breve{T} = 0$ is a positively-sloped locus lying above the origin. Below the locus, the equations for $\breve{T} > 0$ apply, and above the locus those for $\breve{T} < 0$ apply. Below the $\breve{T} = 0$ locus, the efficiency locus is positively-sloped; above the locus, it is given by $\tau_K = \dfrac{-\eta\tau_L}{i + \tau_L - \eta}$. Thus there is a discontinuity in the efficiency locus as it crosses the $\breve{T} = 0$ locus; below the $\breve{T} = 0$ locus, the efficiency locus takes into account that a rise in tax revenue affects two margins, timing and density, while above the $\breve{T} = 0$ locus, the efficiency locus takes into account that there is only the density margin.

Now consider determination of the revenue-maximizing tax rate. There are two local revenue maxima. The first is at $(\tau_K, \tau_L) = (-\eta, \infty)$; here, timing is distorted with $T = 0$ instead of $T = 50$, but density is efficient conditional on timing. The second is the revenue-maximizing (τ_K, τ_L) below or on the $\breve{T} = 0$ locus. If it is on the locus, it is dominated by $(\tau_K, \tau_L) = (-\eta, \infty)$ since that pair of tax rates is efficient conditional on $T = 0$. If it is below the locus, there are two local maxima which must be compared; the former entails distortion on only the timing margin, the latter less distortion on the timing margin but more on the density margin.

The numerical results are displayed in Table 5.

Table 5

Numerical example with differentiated property value taxation.
Revenue-maximizing tax rates.

	Local maximum 1 $\breve{T} < 0$					*Local maximum 2* $\breve{T} > 0$					
σ	τ_L	τ_K	$\mathcal{R}(0)$	K	Q	τ_L	τ_K	$\mathcal{R}(0)$	K	Q	T
.25	∞	$-.02$	.554	.593	.761	.0251	.0039	.862*	.658	.817	45.39
.50	∞	$-.02$	.753*	.410	.676	.0150	.0099	.728	.399	.666	38.69
.75	∞	$-.02$	.892*	.298	.632	.0092	.0229	.540	.188	.513	28.04

Notes: 1. Parameter values are the same as for the corresponding case of σ in Tables 3 and 4.

2. * indicates the global-maximizing set of tax rates and revenue.

A pattern is evident. For $\breve{T} > 0$, as the elasticity of substitution increases, the maximum surplus extraction and $\frac{\tau_L}{\tau_K}$ at the revenue maximum fall. To understand these results, consider first the extreme case of $\sigma = 0$. Density is unaffected by taxation, and efficient timing can be maintained by setting $\tau_K = 0$ (see (49)), with full surplus extraction being achieved by setting $\tau_L = \infty$. In this extreme case, therefore, residual site value taxation is non-distortionary, while simple property taxation is distortionary. As the elasticity of substitution increases, both timing and density, and hence deadweight loss, become relatively more sensitive to τ_L than to τ_K (see (48), (49) and (51)) implying that simple property taxation increases in efficiency relative to residual site value tax system. For $\breve{T} < 0$ and $T = 0$, in contrast, as the elasticity of substitution increases the maximum surplus extraction rises and the revenue-maximizing tax rates remain unchanged. To understand the former result, consider first the limiting case as σ approaches 1. The timing and density first-order conditions then almost *coincide,* implying that almost the same surplus can be achieved at $T = 0$ and an appropriately reduced density as at the no-tax optimum, and the appropriately reduced density is achieved with full surplus extraction with $\tau_K = -0.02$ and $\tau_L = \infty$. At the other extreme with $\sigma = 0$, construction occurs at the same density as at the no-tax optimum but fifty years earlier, which entails considerable distortion.

4 Concluding Comments

This paper examined the deadweight loss due to the property tax when the property tax is realistically modeled as a tax on value rather than as a tax on rent. The analysis was partial equilibrium, examining the effects of the property tax when it is applied to a single, small parcel of land, and paying no attention to the disposition of the tax revenue raised. To keep the algebra manageable, a number of simplifying assumptions were made. Most notably, the extreme case of infinitely durable structures was considered. The landowner decides when to build a structure on his vacant land and at what density, and subsequently that structure remains on the site forever with no depreciation. Thus, the deadweight loss due to the property tax derives from its changing the timing and density of construction.

The property tax was modeled as a triple of time-invariant tax rates, the first applied to pre-development land value, the second to post-development (residual) site value (defined as property value minus construction costs), and the third to structure value (measured by construction costs). A number of variants of the property tax were examined: i) the "Canadian" property tax system under which property value is taxed after development, with pre-development land value exempt from taxation; ii) the simple property tax system under which pre-development land value and post-development property value are taxed at the same rate; iii) the residual site value tax system under which pre-development land value and post-development residual site value are taxed at the same rate, with structure value exempt; and iv) the differentiated property tax system, which is like the residual site value tax system except that structure value is taxed but at a different rate. The paper focused on the special case where the rate of rental growth is constant.

All four tax systems are distortionary (Arnott (2002) derives the property tax system that is neutral in the context of the model). The Canadian property tax system causes later development at unchanged density; the simple and differentiated property tax systems have an ambiguous effect on development time but unambiguously discourage density; and the residual site value tax system results in earlier development at lower density. For the Canadian property tax system, the revenue-maximizing tax rate equals the growth rate of rents and the marginal deadweight loss equals the tax rate divided by the growth rate of rents less the tax rate. The corresponding results for the other property tax systems are not as neat, depending *inter alia* on the current level of rents and the elasticity of substitution between land and capital in the structure production function. For each of the property tax systems considered, for a set of plausible parameter values we computed deadweight loss as a function of the tax rate. Two results were particularly striking. First, in all the numerical examples, the revenue-maximizing tax rate was lower than the actual effective property tax rate employed in many jurisdictions, suggesting that some jurisdictions may be "on the wrong side of the Laffer curve" with respect to property taxation. Second, in all the examples except for the Canadian property tax system, deadweight loss was strongly positively related to the elasticity of substitution between land and capital in the production of structures, and the revenue-maximizing tax rate strongly negatively related to it, which points to the importance for policy analysis of precise estimation of this elasticity.

In the course of our analysis, we encountered a general conceptual issue of how to compare the efficiency of two inter–temporal tax systems. Though doing so entails time inconsistency, we compared their efficiency from today forward.

In the paper, we provided a reasonably thorough analysis of the efficiency effects of a variety of idealized property tax systems, but in the context of a specific, partial equilibrium model. It remains to be seen how robust our results are. There are several important issues for future research:

1. We modeled the property tax as applying to a single, atomistic property. How should the model be generalized to a metropolitan area or to an entire country, and how will this generalization affect the results?[18, 19]

2. We assumed that structures are completely immalleable. Does introducing depreciation and property rehabilitation and redevelopment significantly alter the results?

3. We made a number of simplifying assumptions: no technical change, a constant

[18]An appealing way to address this question would be to analyze the effects of alternative property value tax systems in the context of a model of growing, monocentric city with completely durable housing model in which developers have perfect foresight (Fujita (1976), Arnott (1980), and Wheaton (1982)). Our analysis generalizes straightforwardly to an open city since equilibrium housing rents are then unaffected by property taxation. In a closed city, however, housing rents are affected by property taxation, which considerably complicates the analysis.

In a growing monocentric city with completely durable housing different locations have different development times, which complicates the comparison of property tax systems to be implemented today. For properties that have already been developed, the (unanticipated) imposition of a property tax from today forward entails no distortion; for properties that will be developed far off into the future, the bulk of the present value of revenue collected from the tax will be collected prior to development. Thus, a tax system that is relatively efficient at some locations may be relatively inefficient at others, and the overall efficiency of a tax system entails averaging over locations.

[19]Think of the standard partial equilibrium tax analysis. The deadweight loss for a given tax rate or a given amount of tax revenue is higher, the larger are the demand and supply elasticities. Looking at a single property for which rent is fixed is analagous to assuming a perfectly elastic demand curve. Thus, it is natural to conjecture that the deadweight loss from a given property tax system, whether for a particular property tax rate or for a given amount of tax revenue collected, is lower the less elastic is demand. Accordingly, looking at a single property with exogenous rent tends to overstate the deadweight loss due to property taxation.

This intuition can be formalized by considering a growing closed (exogenous population) monocentric city in which demand is such that housing unit size is fixed *and* the elasticity of substitution between land and capital in the production of the floor area is zero. At a given point in time, housing unit size, the floor-area ratio of housing, and population are all independent of the property tax system in place; so too therefore is the residential area of the city and the boundary of urban development. In this special case, therefore, all property tax systems have no effect on the density and timing of development, and hence generate no deadweight loss. If the elasticity of substitution in housing production is non-zero, however, property taxation generates deadweight loss by inducing inefficient factor proportions—structural density.

This line of reasonong leads to the conjecture that the deadweight loss due to a property tax system is higher the greater the elasticity of demand for housing and the greater is the elasticity of substitution between capital and land in the production of housing. The difficulty in formalizing the conjecture is in deciding what to hold fixed while these elasticities are being varied.

interest rate, no change in construction costs, zero pre-development land rent, and a constant growth rate of floor rent. How does relaxing these assumptions affect our results qualitatively and quantitatively?

4. Our analysis ignored uncertainty, which is surely important in property development. Since Capozza and Li (1994) introduced stochasticity into the Arnott-Lewis model, one obvious approach is to extend their analysis to treat a variety of property value tax systems.

5. We assumed that the developer has complete discretion concerning when and at what density to build. But in practice his choice is constrained by a variety of development controls and zoning regulations.[20] If they are so strict that they result in the same development time and density with and without a particular property tax system, the property tax system is neutral.

6. In an earlier version of the paper, we contrasted our analysis of the property tax as a tax on value with the conventional analysis which treats the property tax as a tax on rent, especially with respect to measurement of deadweight loss. This topic merits consideration.

7. It is shown in Arnott (2002) that a tax on "net site rent" (equal to zero prior to development and $r(t)Q(K) - ipK$ after development) is neutral. Net site rent taxation is presumably not employed because net site rent is unobservable or observable only at prohibitive cost. Our analysis treated observability only implicitly. Perhaps a more explicit treatment would be fruitful.

In the introduction, we emphasized that the choice of property tax system entails a tradeoff between conventional deadweight loss and administrative costs broadly speaking. At one extreme is a classical land value tax system, which is non–distortionary but very costly to administer. Our paper focused on quantifying deadweight loss. As important but more difficult will be to quantify the other side of the tradeoff.

[20] In analyzing the efficiency of alternative property tax systems with development controls and zoning regulations, it is important to recognize that they are imposed to deal with perceived market failure.

Appendix 1

(Not for publication)

Relative Efficiency of the Canadian and Simple Property Tax Systems

We start by recording the formulae for revenue raised from the Canadian property tax system, and the deadweight loss:

$$\mathcal{R}(0) = V^b(0)\left(\frac{i\tau_c}{\eta(i-\eta)}\right)\left(\frac{i-\eta}{i+\tau_c-\eta}\right)^{\frac{i}{\eta}} \tag{30}$$

$$\mathcal{D}(0) = V^b(0)\left(1-\left(\frac{\eta i-\eta^2+i\tau_c}{\eta(i-\eta)}\right)\right)\left(\frac{i-\eta}{i+\tau_c-\eta}\right)^{\frac{i}{\eta}}. \tag{32}$$

To obtain the corresponding formulae for the simple tax system will require some additional algebra. Rewrite (33) and (35), using (29):

$$\mathcal{R}(0) = V^b(0)\frac{i\left(r(T)Q(K)e^{-iT}\right)^a}{\eta\left(r(T)Q(K)e^{-iT}\right)^b}\left(\frac{\eta+\tau_s}{i+\tau_s}-\frac{e^{-\tau_s T^a}(i-\eta)}{(i+\tau_s)(i+\tau_s-\eta)}\right) \tag{A1.1}$$

$$\mathcal{D}(0) = V^b(0)\left(1-\frac{i\left(r(T)Q(K)e^{-iT}\right)^a}{\eta\left(r(T)Q(K)e^{-iT}\right)^b}\left(\frac{\eta+\tau_s}{i+\tau_s}\right)\right). \tag{A1.2}$$

From (36b) and (37):

$$\frac{\left(r(T)Q(K)e^{-iT}\right)^a}{\left(r(T)Q(K)e^{-iT}\right)^b} = \left(\frac{i}{i+\tau_s}\right)^{\frac{\sigma}{1-\sigma}} e^{-(i-\eta)(T^a-T^b)} \tag{A1.3}$$

with

$$\begin{aligned} T^a - T^b &= \frac{1}{\eta}\left(\ln\left(\frac{i+\tau_s}{i}\right)+\frac{\sigma}{1-\sigma}\ln\left(\left(\frac{i+\tau_s}{i}\right)\left(\frac{i-\eta}{i+\tau_s-\eta}\right)\right)\right) \\ &= \frac{1}{\eta(1-\sigma)}\left(\ln\left(\frac{i+\tau_s}{i}\right)+\sigma\ln\left(\frac{i-\eta}{i+\tau_s-\eta}\right)\right) \end{aligned} \tag{A1.4}$$

so that

$$e^{-(i-\eta)(T^a-T^b)} = \left(\frac{i}{i+\tau_s}\right)^{\frac{i-\eta}{\eta(1-\sigma)}}\left(\frac{i+\tau_s-\eta}{i-\eta}\right)^{\frac{\sigma(i-\eta)}{\eta(1-\sigma)}}. \tag{A1.5}$$

Substituting (A1.5) into (A1.3) yields

$$\frac{\left(r(T)Q(K)e^{-iT}\right)^a}{\left(r(T)Q(K)e^{-iT}\right)^b} = \left(\frac{i}{i+\tau_s}\right)^{\frac{\eta\sigma+i-\eta}{\eta(1-\sigma)}}\left(\frac{i+\tau_s-\eta}{i-\eta}\right)^{\frac{\sigma(i-\eta)}{\eta(1-\sigma)}}. \tag{A1.6}$$

And substituting (A1.6) and (37) into (A1.1) and (A1.2) yields

$$\mathcal{D}(0) = V^b(0)\left(1 - \left(\frac{\eta+\tau_s}{\eta}\right)\left(\frac{i}{i+\tau_s}\right)^{\frac{i}{\eta(1-\sigma)}}\left(\frac{i+\tau_s-\eta}{i-\eta}\right)^{\frac{\sigma(i-\eta)}{\eta(1-\sigma)}}\right). \tag{A1.7}$$

$$\begin{aligned}\mathcal{R}(0) &= V^b(0)\left(\frac{i}{i+\tau_s}\right)^{\frac{i}{\eta(1-\sigma)}}\left(\frac{i+\tau_s-\eta}{i-\eta}\right)^{\frac{\sigma(i-\eta)}{\eta(1-\sigma)}} \\ &\quad\times\left(\frac{\eta+\tau_s}{\eta} - \left(\frac{i-\eta}{\eta(i+\tau_s-\eta)}\right)\left(\frac{c_0 r_0}{p\,(i+\tau_s)}\right)^{\frac{\tau_s}{\eta}}\left(\frac{i+\tau_s-\eta}{c_1\,(i+\tau_s)}\right)^{\frac{\sigma\tau_s}{\eta(1-\sigma)}}\right).\end{aligned} \tag{A1.8}$$

The relative efficiency of the two tax systems for a given set of parameter values can be calculated as follows. Set τ_s and from (A1.7) and (A1.8) calculate the corresponding deadweight loss and tax revenue. From (30) calculate the τ_c which raises the same amount of tax revenue and from (32) calculate the corresponding deadweight loss. Then compare the deadweight losses.

Appendix 2

(For publication)

Some Isomorphisms

Some insights can be gained by viewing our model as a static model, with prices and quantities in present value terms. One unit of structure constructed today generates present discounted revenue of $\int_0^\infty r(t)e^{-it}dt = \dfrac{r_0}{i-\eta} = \overline{\rho}$. Interpret this as the exogenous consumer price of structure. Accordingly, the quantity of structure produced on a unit of area of land — in present value terms — is $Q(K)e^{-(i-\eta)T}$, and the quantity of capital employed — in present value terms — is Ke^{-iT}. Defining $D \equiv e^{-iT}$, the developer's net revenue or profit function in the absence of taxation is

$$\Pi(K, D) = \overline{\rho} Q(K) D^{\frac{i-\eta}{i}} - pKD. \tag{A2.1}$$

The corresponding first-order conditions for profit-maximization are

$$D \quad : \quad \overline{\rho} Q(K) \left(\frac{i-\eta}{i} \right) D^{-\frac{\eta}{i}} - pK = 0 \tag{A2.2a}$$

$$K \quad : \quad \overline{\rho} Q^{'}(K) D^{\frac{i-\eta}{i}} - pD = 0, \tag{A2.2b}$$

which have straightforward interpretations.

With taxation, from (20) the developer's profit function is

$$\Pi(K, D) = \overline{\rho} Q(K) D^{\frac{i+\tau_V-\eta}{i}} \left(\frac{i-\eta}{i+\tau_S-\eta} \right) - \frac{i+\tau_K}{i+\tau_S} pKD^{\frac{i+\tau_V}{i}}. \tag{A2.3}$$

The isomorphisms arise when $\tau_V = 0$. For this special case,

$$\Pi(K, D) = \overline{\rho} Q(K) D^{\frac{i-\eta}{i}} \left(\frac{i-\eta}{i+\tau_S-\eta} \right) - \frac{i+\tau_K}{i+\tau_S} pKD. \tag{A2.4}$$

Comparing (A2.1) and (A2.4), we obtain the following isomorphisms.

1. A property tax system with $\tau_V = 0$ and $\tau_K = \dfrac{-\eta\tau_S}{i+\tau_S-\eta}$ is isomorphic to a profit tax at rate $\tau_\Pi = \dfrac{\tau_S}{i+\tau_S-\eta}$. This is the neutral tax system identified in Arnott (2002).

2. A property tax system with $\tau_V = 0$ and $\tau_S = 0$ is isomorphic to a tax on the input KD at rate $\tau_I = \dfrac{\tau_K}{i}$.

3. A property tax system with $\tau_V = 0$ and $\tau_K = \tau_S$ — a Canadian property tax system — is isomorphic to an *ad valorem* tax on the output $Q(K)D^{\frac{i-\eta}{i}}$ at rate $\tau_\Phi = \dfrac{\tau_S}{i-\eta}$. The *ad valorem* tax drives a wedge between the producer and consumer price. Since the consumer price is exogenous (and the demand function therefore perfectly elastic) $\rho(1+\tau_\Phi) = \overline{\rho}$, where ρ is the producer price. From (A2.4), $\rho = \overline{\rho}\left(\dfrac{i-\eta}{i+\tau_S-\eta}\right)$. Combining these two formulae establishes the result.

This result allows us to provide an explanation, albeit a rather mechanical one, for the results presented in Prop. 2. Define $q \equiv Q(K)D^{\frac{i-\eta}{i}}$. Then the tax revenue from the Canadian property tax system may be written as

$$\mathcal{R}(0) = \tau_\Phi \rho q(\rho) = (\overline{\rho} - \rho)q(\rho), \qquad \text{(using } \rho(1+\tau_\Phi) = \overline{\rho}\text{)}\,. \tag{A2.5}$$

where $q(\rho)$ is the supply curve. Maximizing $\mathcal{R}(0)$ with respect to ρ gives an expression for the tax-revenue-maximizing producer price:

$$\rho : -q(\rho) + (\overline{\rho} - \rho)q^{'}(\rho) = 0 \Rightarrow \frac{\overline{\rho} - \rho}{\rho} = \tau_\Phi = \frac{1}{\varepsilon_S}. \tag{A2.6}$$

All that remains is to solve the elasticity of supply:

$$\frac{dq}{d\rho} = q\left(\frac{Q^{'}(K)}{Q(K)}\frac{dK}{d\rho} + \left(\frac{i-\eta}{i}\right)\frac{1}{D}\frac{dD}{d\rho}\right). \tag{A2.7}$$

From (23) and (27):

$$\frac{dK}{d\tau_c} = 0 \qquad \frac{dT}{d\tau_c} = \frac{1}{\eta\,(i + \tau_c - \eta)}. \tag{A2.8}$$

Recalling that $\rho = \overline{\rho}\left(\dfrac{i-\eta}{i+\tau_S-\eta}\right)$, $\tau_c = \tau_S$, and $D = e^{-iT}$ gives

$$\frac{dK}{d\rho} = \frac{dK}{d\tau_c}\frac{d\tau_c}{d\rho} = 0 \text{ and } \frac{dD}{d\rho} = \frac{dD}{dT}\frac{dT}{d\tau_c}\frac{d\tau_c}{d\rho} = \frac{iD}{\eta\rho}. \tag{A2.9}$$

Combining (A2.7) and (A2.9) yields

$$\varepsilon_S = \frac{\rho}{q}\frac{dq}{d\rho} = \frac{i-\eta}{\eta}, \tag{A2.10}$$

from which it follows that the revenue-maximizing tax rates are

$$\tau_\Phi = \frac{\eta}{i-\eta} \quad \text{and} \quad \tau_c = \eta. \tag{A2.11}$$

Thus, the revenue-maximizing tax rate under the Canadian property tax system equals the growth rate of rents because:

i) the Canadian property tax system with an exogenous time path of rents is isomorphic to an output tax in a static model with perfectly elastic demand;

ii) the revenue-maximizing tax rate for an output tax in a static model with perfectly elastic demand equals the inverse of the elasticity of supply;

iii) the elasticity of supply in the static model isomorphic to the dynamic model of the Canadian property tax system is $\dfrac{i-\eta}{\eta}$; and

iv) an output tax rate of $\dfrac{i-\eta}{\eta}$ in the static model maps into a Canadian property tax rate of η.

Unfortunately, the pre-development land tax rate enters the developer's profit function, (A2.3), in a way that does not correspond to the way in which linear input, output or profit taxes enter the profit function in static models. As a result, there seems to be no simple isomorphisms when $\tau_V \neq 0$.

Appendix 3

(Not for publication)

With an Efficient Differentiated Property Tax System, when $\breve{T} > 0$, $\tau_K > 0$

We prove this by showing that for $\tau_K \ \epsilon \ (-i, 0]$, $\dfrac{d\mathcal{D}(0)}{d\tau_K} \leq 0$ while $\dfrac{d\mathcal{R}(0)}{d\tau_K} > 0$

- $\left.\dfrac{d\mathcal{D}(0)}{d\tau_K}\right|_{\tau_K \ \epsilon \ (-i,0]} \leq 0$

  From (51):

$$\frac{d\mathcal{D}(0)}{d\tau_K} = -\left(\frac{\eta+\tau_K}{i-\eta}\right) p \left(\frac{\eta c_1}{i+\tau_L-\eta}\right)^{\frac{\sigma}{1-\sigma}} \left(\frac{1}{\eta+\tau_K} - i\frac{dT}{d\tau_K}\right) e^{-iT}. \tag{A3.1}$$

From (49):

$$\frac{dT}{d\tau_K} = \frac{1}{\eta(i+\tau_K)}. \tag{A3.2}$$

Combining (A3.1) and (A3.2) yields

$$\begin{aligned} \frac{d\mathcal{D}(0)}{d\tau_K} &= -\left(\frac{\eta+\tau_K}{i-\eta}\right) p \left(\frac{\eta c_1}{i+\tau_L-\eta}\right)^{\frac{\sigma}{1-\sigma}} \left(\frac{-(i-\eta)\tau_K}{(\eta+\tau_K)\,\eta\,(i+\tau_K)}\right) e^{-iT} \\ &= p\left(\frac{\eta c_1}{i+\tau_L-\eta}\right)^{\frac{\sigma}{1-\sigma}} \frac{\tau_K}{\eta\,(i+\tau_K)}, \end{aligned} \tag{A3.3}$$

so that $\left.\dfrac{d\mathcal{D}(0)}{d\tau_K}\right|_{\tau_K \ \epsilon \ (-i,0]} \leq 0.$

- $\left.\dfrac{d\mathcal{R}(0)}{d\tau_K}\right|_{\tau_K \ \epsilon \ (-i,0]} > 0$

  From (50)

$$\begin{aligned} \frac{d\mathcal{R}(0)}{d\tau_K} &= p\left(\frac{\eta c_1}{i+\tau_L-\eta}\right)^{\frac{\sigma}{1-\sigma}} e^{-iT} \\ &\quad \times \left[\begin{array}{c} \left(\dfrac{1}{i-\eta} - \dfrac{\eta e^{-\tau_L T}}{(i+\tau_L)\,(i+\tau_L-\eta)}\right) \\ -i\dfrac{dT}{d\tau_K}\left(\dfrac{\eta+\tau_K}{i-\eta} - \dfrac{e^{-\tau_L T}\,(i+\tau_K)\,\eta}{(i+\tau_L)\,(i+\tau_L-\eta)}\right) \\ +\tau_L\dfrac{dT}{d\tau_K}\dfrac{e^{-\tau_L T}\,(i+\tau_K)\,\eta}{(i+\tau_L)\,(i+\tau_L-\eta)} \end{array} \right] \end{aligned} \tag{A3.4}$$

Substituting in (A3.2) gives

$$\begin{aligned}
\frac{d\mathcal{R}(0)}{d\tau_K} &= p\left(\frac{\eta c_1}{i+\tau_L-\eta}\right)^{\frac{\sigma}{1-\sigma}} e^{-iT} \\
&\times \left[\begin{array}{c} \frac{1}{i-\eta} - \frac{\eta e^{-\tau_L T}}{(i+\tau_L)(i+\tau_L-\eta)} - \frac{i(\eta+\tau_K)}{\eta(i-\eta)(i+\tau_K)} + \\ \frac{(i+\tau_L)e^{-\tau_L T}}{(i+\tau_L)(i+\tau_L-\eta)} \end{array}\right] \\
&= p\left(\frac{\eta c_1}{i+\tau_L-\eta}\right)^{\frac{\sigma}{1-\sigma}} e^{-iT}\left[-\frac{\tau_K}{\eta(i+\tau_K)} + \frac{e^{-\tau_L T}}{(i+\tau_L)}\right], \qquad \text{(A3.5)}
\end{aligned}$$

so that

$$\left.\frac{d\mathcal{R}(0)}{d\tau_K}\right|_{\tau_K \in (-i,0]} > 0.$$

Thus starting at (τ_K, τ_L) (with $\tau_L > -(i-\eta)$), increasing τ_K from a level in $(-i, 0]$ to 0^+ increases revenue and decreases deadweight loss. The model is not well-defined for $\tau_K < -i$ or $\tau_L < -(i-\eta)$.

References

[1] Arnott, R. (1980) "A Simple Urban Growth Model with Durable Housing", *Regional Science and Urban Economics*, Vol.10, pp. 53–76.

[2] Arnott, R. (2002) "Neutral Property Taxation" http://FMWWW.bc.edu/EC-V/Arnott.fac.html

[3] Arnott, R.J., and F.D. Lewis (1979) "The Transition of Land to Urban Use" *Journal of Political Economy*, Vol. 87, pp.161–169.

[4] Bentick, B.L. (1979) "The Impact of Taxation and Valuation Practices on Timing and Efficiency of Land Use" *Journal of Political Economy*, Vol.87, pp. 858–868.

[5] Capozza, D., and Y. Li (1994) "The Intensity and Timing of Investment: The Case of Land" *American Economic Review*, Vol. 84, pp. 889–904.

[6] Cannan, E. (1959) "Minutes of Royal Commission on Local Taxation, 1899" reprinted in Readings in the Economics of Taxation, R. Musgrave and C. Shoup, eds. (Homewood, IL:R.Irwin), 171–201.

[7] Fisher, S. and A.H. Peters (1998) *Industrial Incentives: Competition among American States and Cities* (Kalamazoo, MI: Upjohn).

[8] Fujita, M. (1976) "Spatial Patterns of Urban Growth: Optimum and Market" *Journal of Urban Economics*, Vol. 3, pp. 209–241.

[9] Hamilton, B.W. (1975) "Zoning and Property Taxation in a System of Local Governments" *Urban Studies*, Vol. 12, pp. 205–211.

[10] Ladd, H.F. (1997) "Theoretical Controversies: Land and Property Taxation" *Local Government Tax and Land Use Policies in the United States: Understanding the Links,* Ch.2 (Cheltenham, U.K.: Edward Elgar).

[11] Marshall, A. *Principles of Economics*, 9th ed., 1961, Appendix G (1961:MacMillan).

[12] McDonald, J. (1981) "Capital–Land Substitution in Urban Housing: A Survey of Empirical Estimates" *Journal of Urban Economics*, Vol.9, pp.190–221.

[13] Mieszkowski, P. (1972) "The Property Tax: An Excise Tax or a Profit Tax ?" *Journal of Public Economics*, Vol.1, pp. 73–96.

[14] Mills, D.E. (1981) "The Non–neutrality of Land Value Taxation" *National Tax Journal*, Vol. 34, pp. 125–129.

[15] Mills, E.S. (1998) "Is Land Taxation Practical?" *Illinois Real Estate Letter*, Fall, pp. 1– 5.

[16] Netzer, D. (1966) *Economics of the Property Tax* (The Brookings Institution).

[17] Netzer, D., Ed.(1998) *Land Value Taxation: Can It and Will It Work Today?* (Cambridge, MA: Lincoln Institute).

[18] Shoup, D. (1970) "The Optimal Timing of Urban Land Development" *Papers of the Regional Science Association*, Vol. 25, pp. 33–44.

[19] Skouras,A. (1978) "The Non–neutrality of Land Taxation" *Public Finance*, Vol.30, pp.113–134.

[20] Tideman, N.T. (1982) "A Tax on Land Is Neutral" *National Tax Journal*, Vol.35, pp. 109–111.

[21] Tideman, N. (undated) "Taxing Land is Better than Neutral: Land Taxes, Land Speculation and the Timing of Development" mimeo.

[22] Vickrey, W.S. (1970) "Defining Land Value for Taxation Purposes", in *The Assessment of Land Value* by D.M. Holland, ed. (Milwaukee, WI: University of Wisconsin Press).

[23] Wheaton, W. C. (1982) "Urban Residential Growth under Perfect Foresight" *Journal of Urban Economics*. Vol. 12, p 1–21.

[24] Wong, K.C. (1999) "The Evolution of Land Value Taxation in China" *Construction and Real Estate Review*, Vol.3, pp. 1–4.

[25] Youngman, J.M. and J.H. Malme (1993) *An International Survey of Taxes on Land and Buildings* (Deventer, The Netherlands: Kluwer).