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DISINFLATION AND FISCAL REFORM: A NEOCLASSICAL PERSPECTIVE

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ABSTRACT

During the last two decades, many Latin American countries engaged in disinflation programs based on both exchange rate management and fiscal reforms. However, in most instances, part of the fiscal reform was delayed or not implemented completely, so the fiscal deficit increased and the program had to be abandoned. The aftermath of these programs is not encouraging, since most of these policies turned out to be failures, lowering reserves and causing higher inflation rates. Given this record, it is worth asking why governments start a disinflation program even though the fiscal equilibrium is not guaranteed. In this paper we show that, if the reform process is uncertain and inflation has welfare costs, the optimal exchange rate policy implies the initiation of a disinflation program at the announcement of the fiscal reform. Additionally, we show that even if there exists a possibility of a balance of payments crisis, it is still optimal to initiate a disinflation program. This means that, in this set up, avoiding the crisis with probability one is suboptimal. Finally, we show that it is optimal to engage in a sequence of stabilization programs until one of them is successful.

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1. Introduction

During the last two decades, many Latin American countries engaged in disinflation programs based on both exchange rate management and fiscal reforms. However, in most instances, part of the fiscal reform was delayed or not implemented completely, so the fiscal deficit increased and the program had to be abandoned. The aftermath of these programs is not encouraging: since most of these policies turned out to be failures, lowering reserves and causing higher inflation rates. Given this record, it is worth asking why governments start a disinflation program even though the fiscal equilibrium is not guaranteed. A more sensible strategy would be to stabilize the fiscal accounts first, and then reduce inflation.

The literature has explained this behavior based on four alternative theories: the Olivera-Tanzi effect, optimal tax composition, exchange rate management as a disciplinary device, and political economy issues. First, if the economy is working in the wrong side of the Laffer curve, there exists another equilibrium with lower inflation. The idea is that the lag that exists between the realization of income and the time income tax is paid reduces real revenue. A stabilization moves the economy to the left hand side of the Laffer curve and no fiscal effort is required. This is the Olivera-Tanzi effect (see Olivera (1967) and Tanzi (1978)). Second, the disinflation program might be the result of an optimal tax choice problem. For example, consider an economy that has a high inflation tax and a low income tax. Moving toward the optimal tax portfolio implies a reduction in inflation and an increase in income tax. This kind of tax recomposition are common in the Latin American experience, and are an important component of their reform processes. Third, the disinflation program can be thought as a commitment or disciplinary device to encourage fiscal responsibility. If there is a conflict between the central bank and the government, and the central bank is the stronger one, then the monetary authority initiates a managed exchange rate to force the fiscal authority to reduce expenditure.[1] Fourth, there are political economy models that concentrates on the choice of the exchange rate regime. For example, Tornell and Velasco (1995) analyze a political economy model that explains when a fixed exchange rate is more likely to be adopted.

These theories capture important aspects of the disinflation programs in Latin America. They fail, however, to explain several of the issues in those processes. The first two theories cannot justify why disinflation programs usually end with a balance of payments crisis. Both predict that no need for extra financing is required during the disinflation. The third explanation does not seem to capture the institutional arrangements that prevail in Latin America; central bank independence is a relatively new concept for the continent, and, in general, we observe that the monetary authority abandons the policy, and not the converse. Finally, the fourth hypothesis depends on political economy institutions that are not necessarily

[1] The European disinflation experiences of the 80's can be classified as examples of the use of monetary policy as a commitment device.

common across the region. These are undoubtedly important components of the story, however, here we abstract from them and emphasize an alternative explanation.

In this paper[2], we present a simple model that accounts for the behavior of the government based on three assumptions: the process of reform is uncertain, inflation has welfare costs, and disinflations are costly. The model has three main implications: First, the optimal exchange rate policy implies the initiation of a disinflation program at the announcement of a fiscal reform. Second, even if there exists a possibility of a balance of payments crisis, it is still optimal to initiate the disinflation program. Third, it is optimal to engage in a sequence of stabilization programs until one of them is successful, or until a balance of payments crisis occurs[3].

The intuition is that the announcement of a fiscal reform conveys good news in the future in the form of lower expected fiscal deficits. Seigniorage has welfare costs, therefore it is optimal for the Central Bank, to smooth the inflationary tax. Hence, a disinflation program is initiated at the announcement of the reform and it is financed with reserves. If the reform never takes place and the disinflation program has to be abandoned, the ex-post inflation rate is higher than the one that existed before the program was initiated; it looks as if the government made a mistake when they implemented the stabilization program in the first place.[4]

The paper is organized as follows: In section two, we summarize some of the Latin American stabilization experiences. In section three, we present the basic setup. In section four, we solve the model when there is no constraint on the level of reserves and prove that the optimal policy indeed implies the initiation of a disinflation program at the announcement of a fiscal reform. In section five, we extend the model to include the possibility of a balance of payments crisis. We show that even though there exists the possibility of a balance of payments crisis, still it is optimal to initiate the disinflation. In section six, we allow the government to re-initiate a new fiscal reform after the previous one had failed. The model implies a sequence of stabilization programs, each one leading to a higher inflation rate. Section seven concludes and offers recommendations for future research.

[2]This paper is closely related to the literature studying disinflation programs under exchange rate managements. See Calvo (1986, 1987) and Calvo & Drazen (1995). See also Rodriguez (1982) and Calvo & Vegh (1993) for the boom-recesion cycle, and Calvo & Vegh (1993), Velasco (1993), Agenor & Montiel (1996) for the interest rates movements.

[3]It is important to point out that here we are concerned with the timing between the disinflation and the fiscal reform. As we discuss below, the model only captures a small part of the disinflation (around 25%). In an earlier version of the paper, the model included sticky prices and it was able to account for a sizeable fraction of the exchange rate peg.

[4]The paper is closely related to Calvo & Drazen (1995). They also study the impact of uncertain policies on the path of the economy. In their case, they concentrate on the existence of market imperfections and its interaction with the uncertain duration of the policies.

2. Latin American Experiences.

In this section we study some Latin American stabilization programs. We are interested in characterizing the typical stabilization experience; both from the fiscal and the inflation point of view. Thus, this is suggestive evidence of the patterns we are interested in explaining later in the paper.

In tables 2.1 and 2.2, we classify the stabilization programs for eight countries in the last 30 years, according to two criteria (Following Tornell & Velasco (1995): whether it was successful or not in permanently reducing inflation, and whether the fiscal deficit was reduced before, during or never after the program was initiated. This list is not exhaustive, although suggestive. We define that an stabilization is unsuccessful when inflation increases above the initial level or when another stabilization program is initiated. Second, we decide that there was a fiscal effort if the fiscal deficit changed by more than five percent of GDP or there is a fiscal surplus.[5]

	Before	During	Never
Successful	Chile 78	Argentina 91 Bolivia 85 Peru 90	
Not Successful	Mexico 87 Uruguay 79	Argentina 79 Argentina 85 Brazil 90 Chile 75 Mexico 82 Uruguay 74	Brazil 86 Venezuela 89 Venezuela 94

Table 2.1: Latin American Stabilizations: When expenditure was Reduced?

Two main points can be extracted from these tables: First, notice that there are few successful cases. Second, notice that there are few cases where the fiscal deficit was reduced before the stabilization program. More importantly, these two cases (Chile 78 and Uruguay 79) were preceded by another stabilization program (Chile 75 and Uruguay 74). Note that not all unsuccessful programs were abandoned because a fiscal disequilibrium occurred. For example, in Chile 78 the program was abandoned because inflation was too inertial and the program was ineffective in reducing inflation, and not because there was an increase in the fiscal deficit.

The typical Latin American country, then, starts a disinflation program when there is a problem of high inflation and fiscal deficit. To take care of the inflation problem a nominal anchor is implemented, while to take care of the fiscal problem, a fiscal reform is initiated. However, the fiscal reform takes time and the deficit or the expenditure are not reduced at the speed the government thought. In the end, most

[5] In other words, in Bolivia (1985) the fiscal deficit was reduced from 30 percent of GDP to 5 percent. Even though the country still had a considerably large fiscal deficit we assume that enough efforts were made. On the other hand, if the fiscal deficit decreases from 6 percent to 5 percent we decide that no effort was made.

	Before	During	Never
Successful	Chile 78	Argentina 91 Bolivia 85 Peru 90	
Not Successful	Uruguay 79	Brazil 90 Chile 75 Mexico 82 Mexico 87 Uruguay 74 Venezuela 89 Venezuela 94	Argentina 79 Argentina 85 Brazil 86

Table 2.2: Latin American Stabilizations: When the fiscal deficit was reduced?

of the time the monetary policy is abandoned and the economy returns to a higher level of inflation. In this situation, a new stabilization program is announced and the cycle starts all over again. The model presented in this paper formalizes this intuition.

3. Basic Model.

As was mentioned before, the three main ingredients of the model are the following: the reform process is uncertain, inflation has welfare costs, and a disinflation program is costly. Some coments about each of these components have to be made.

We are interested in the permanent fiscal reforms such as privatization, social security reforms, labor market liberalization, reduction in the size of the government, new tax laws, etc. In general, these reforms imply a permanent change in the fiscal deficit process that affects consumer's choices. However, they require negotiations with congress, unions, and industries, and the experience of several Latin American countries has shown that their implementation is difficult, time-consuming, and sometimes unsuccessful.[6] In practice, governments are able to reduce expenditure and fiscal deficit in many different ways. There are short run measures that are relatively easy to implement, such as elimination of subsidies, reduction in public investment, delay in the increase of public sector wages, etc. Some of these measures, however, are not sustainable and in a model of perfect foresight agents, ineffective. In other words, the present value of the deficit does not change and therefore, there is no effect on consumer's decisions. In this paper, we concentrate on the long run permanent measures.

In the model, we assume that inflation is the only available tax, and that it generates welfare costs. First, the assumption that inflation is the only available tax is capturing the fact that in Latin America,

[6] Alesina and Drazen (1991) provide a theoretical explanation of why these negotiations might require time. Also, see Alesina and Perotti (1996).

seigniorage has been an important share of the government's revenue, especially before the reform. Moreover, it also reflects that during the 70's and 80's inflation was used as the marginal instrument to raise revenue; the tax system was rigid and the only two sources the government had to finance a shock were reserves and seigniorage.[7] The second assumption is that inflation has welfare costs. In the literature, there are several papers that discuss the nature and measures of these costs.[8] In this paper, we simplify and capture them with a concave utility function and a cash in advance constraint. This particular formulation has the advantage that it can be interpreted as a tax smoothing problem, where inflation is a distortionary tax. Barro (1979) showed that when taxes are distortionary, the optimal policy is to spread the tax burden across time: *tax smoothing*[9]. In our case, the tax smoothing result implies inflation smoothing. Note that the smoothing motive justifies the implementation of the disinflation program. In other words, the announcement of the fiscal reform implies that future welfare costs might be smaller. If the cost function is convex, then consumers want to transfer part of the future benefits to today, which requires reducing current inflation.[10]

Finally, a disinflation program is costly because it deprives the government of a source of revenue. The loss in reserves today leads to a higher level of inflation in the future, as the government seeks to recover revenue. This is the Sargent and Wallace (1980) effect, which in our case, appears as a reduction in reserves, rather than as an increase in debt.[11]

As it should be clear by now, in the model the tax smoothing motive drives the timing of the disinflation, while the Sargent and Wallace effect generates the costs of the program.

Two additional remarks: First, failed stabilization programs are far more costly than just the Sargent and Wallace effect. In practice, the failure to implement a disinflation program is costly, not only in loss of reserves, but in several and probably more important ways, such as recessions, loss in credibility in future programs, etc. These costs could be included in the model. However, it would complicate the analysis

[7] The assumption that inflation is the only tax eliminates issues of optimal composition of taxes, and the Olivera-Tanzi effect. In the literature on optimal inflation tax see Phelps (1973) for the first contribution. Additionally, Fischer (1983) studies optimal inflation tax in the context of different exchange rate regimes, Vegh (1989a) studies it in the context of currency substitution, and Aizenman (1987), De Gregorio (1993), and Vegh (1989b) study it in the context of different degrees of efficiency in the tax system. For the Olivera Tanzi effect see the seminal contributions by Olivera (1967) and Tanzi (1978). The exclusion of these issues simplifies the analysis; however, it is important to mention that if those aspects are introduced in the model, the results hold.

[8] Several authors had measured the welfare costs of inflation. The literature started with Bailey (1956), Fischer (1981) and Lucas (1981) where they argue that the welfare cost of moderate inflations is low. Recent constributions include Colley and Hansen (1989, 1991), İmrohoroğlu (1992), İmrohoroğlu and Prescott (1991), and Jones and Manuelli (1993). In general, the literature agrees on important welfare costs at high inflation rates, but moderate to small effects are found at low inflation.

[9] See Barro (1988), Calvo and Guidotti (1992), Ball and Mankiw (1994), Mankiw (1984) and Saint-Paul (1994).

[10] It is important to mention that the assumption that the welfare costs are convex can be relaxed. It can be shown that if the concavity of seignorage is larger than the concavity of the welfare costs of inflation, then the optimal strategy still is to smooth inflation.

[11] See also Liviatan (1984, 1986) and van Wijnbergen (1988)

without improving the intuition. Second, the amount of disinflation predicted by the model comes from the tax smoothing motive, and therefore, is relatively small in comparison with the data. The Latin American experience on average implies a reduction in the exchange rate depreciation from 200 percent to almost zero. The tax smoothing (at best) would be able to account for one quarter of that. This caveat, however, can be solved if sticky prices or inflation inertia (*à la* Calvo) is introduced in the model.[12] Moreover, issues of credibility, transparency of policy, and or political economy will contribute to explain the size of the disinflation. In this paper, however, we are more concerned with the timing of the disinflation, rather than its magnitude. The inclusion of sticky prices (for example) complicates the analysis but does not change the date at which the disinflation program is initiated. In that model, only the "intensity" of the disinflation program is changed.

3.1. Environment and Consumers

Consider a small open economy where there is a single tradable good and where PPP holds. Assume there is perfect capital mobility and zero foreign inflation. All bonds are indexed, thus the domestic nominal inflation rate is equal to the rate of depreciation, and the domestic interest rate is equal to the depreciation rate plus the foreign real interest rate (assumed to be constant).[13] There are three agents: an infinitely lived representative consumer, the government and the central bank.

Consumers choose their consumption path and portfolio holdings taking as given the exchange rate policy. Formally, the consumer's problem is,

$$\max_{\{c_t\}} \quad E\int_0^\infty \ln c_t e^{-\rho t} dt \tag{3.1}$$

$$\begin{aligned} s.t. & \\ \dot{a}_t &= \rho a_t + y - c_t - i_t m_t \\ c_t &\leq \frac{1}{\alpha} m_t \\ \lim_{t\to\infty} a_t e^{-\rho t} &= 0 \end{aligned}$$

where c_t is consumption, y is output (assumed to be constant), a_t are the asset holdings denominated in tradables, m_t denotes money balances in terms of tradables, ρ is the discount rate (assumed to be constant), and i_t is the nominal domestic interest rate. The first equation is the consumer's objective function. The second one is the budget constraint in terms of tradables, where the interest rate has been

[12] See Agenor and Montiel (1996) for an open economy model with cash in advance contraints and sticky prices.

[13] We assume that there is no growth in the world economy and that it is in steady state, thus the international real interest rate is equal to the discount rate.

already substituted by the international interest rate. The third one is the cash in advance constraint. And the fourth one is the transversality condition on consumer's assets.

There are four technical assumptions used in the model that simplify the analysis. First, we assume that consumers do not derive utility from government expenditure. Second, we assume that output is exogenously given. Relaxing these two assumptions does not change the results. If output depends on the level of expenditure or consumers derive utility from public expenditure, this makes the expenditure reduction less desirable. However, if reducing expenditure is welfare improving, then there is a reduction in tax requirements in the future and the results still hold.

Third, we adopt a cash in advance formulation. An equivalent formulation is one where money enters in the utility function. The same general results hold with the exception that the path of money holdings might be different. Cash in advance assumes that money and consumption are complements, and money in the utility function relaxes this assumption. We choose a cash in advance formulation because it captures the distortionary inflation tax in a simpler way. See Appendix E for the solution of the money in the utility case.

And fourth, we assume log utility. The choice of log utility simplifies the consumer's solution making current consumption independent of the future interest rate path. A different utility function implies that current consumption is a function of the future path of interest rates. Thus, some intertemporal substitution is made by the consumers at the announcement of the reform. However, full smoothing is only achieved if there is tax smoothing (this result comes from Barro (1979)). So, still it is the case that the optimal strategy involves inflation smoothing.

The solution for the consumer's problem is,

$$c_t = \frac{y + \rho a_0}{1 + \alpha i_t} \tag{3.2}$$

$$m_t = \alpha \frac{y + \rho a_0}{1 + \alpha i_t} \tag{3.3}$$

3.2. Government

The government finances an exogenous expenditure on tradables by inflationary tax and interest earnings on reserves. We assumed that the government expenditure has no impact on output or the consumer's utility; it is wasteful expenditure. At time zero, the government announces an uncertain fiscal reform, in the sense that it is not sure when it can be implemented or if it will ever be. We assume that all agents have the same prior about the probability of success of such reform.

Assume that the expenditure's process is described by,

$$g_t = \begin{cases} g_h & t < \tau \\ \\ g_h \quad wp \quad 1-q & t \geq \tau \\ g_l \quad wp \quad q & \end{cases} \tag{3.4}$$

where q, τ and $g_h > g_l$ are exogenously given. Define the expenditure improvement as $\Delta g = g_h - g_l$. Define the bad state of the world as the state in which expenditure is not reduced, and the good state of the world as the state in which expenditure is permanently reduced.

There are three technical remarks about these stochastic process: First, the timing of the adjustment is known, but not its outcome. In section 6, we show that the results still hold if this assumption is relaxed. Second, the drift of the process is negative, thus there is a true process of reform in place. In the appendix, a more general process is analyzed and the relative importance of the drift is studied. Third, the expenditure process is exogenous. The question we are addressing is why countries peg their exchange rates, conditional on having a fiscal reform in place. Thus, the exogeneity of the process can be interpreted as the existence of conflicts between monetary and fiscal policy, and that the fiscal authority is the stronger one. Hence, the expenditure process can be considered as exogenous by the central bank.

The government's budget constraint is given by,

$$\dot{B}_t = e_t g_t - \Omega_t + i_t B_t \tag{3.5}$$

where B_t denotes the government debt held by the central bank and Ω_t represents the central bank's profits, discussed below. We assume that the government's debt is in nominal terms but indexed. This eliminates the incentives for discrete devaluations or surprise inflations to reduce its real value.

In the present model, the government has been oversimplified; it has no choices to make. It follows a very simple rule. It maintains a high expenditure and at time τ, if lucky, it can reduce it. In further research, the endogeneity of both the expenditure and the reform process should be introduced to study the political economy aspects of stabilization programs.

3.3. Central Bank

The central bank decides the path of exchange rate depreciations that maximize consumer's utility, taking as given the government's expenditure path and the consumer's reaction function. This is a benevolent Central Bank in the sense that its objective function is exactly the same as that of the consumers. Obviously different results would be obtained if the Central Bank has a different objective. However, in this paper we want to analyze what is the optimal policy, from the consumer's perspective, conditional on a fiscal reform. As will become clear later, even in this restrictive environment we can justify the initiation of a disinflation program even though the fiscal accounts are not yet in equilibrium.[14] This is indeed the most important contribution of this paper. The rest of the sections show how robust it is.

The central bank's balance sheet and flow profits in nominal terms are given by:

$$\begin{aligned} M_t &= e_t r_t + B_t \\ \Omega_t &= i_t B_t + \left(i_t^* + \hat{e}_t\right) e_t r_t \end{aligned} \tag{3.6}$$

where M_t represents the nominal money holdings, r_t is total reserves in foreign currency, i_t^* is the foreign nominal interest rate, and $\hat{e}_t$ denotes the exchange rate depreciation. The first equation is the central bank's balance sheet. The second equation is the central bank's profits which consist of nominal interest earnings on government's debt, foreign interest earnings on reserves, and the capital gains on reserves due to a depreciation.

One implication of perfect capital mobility, the indexed government debt and the PPP assumptions is that choosing the exchange rate depreciation is the same as choosing the inflation rate or the nominal interest rate.[15] Given this equivalence we assume that the central bank chooses the nominal interest rate. Formally, the problem is,

$$\max_{\{i_t\}} \quad E\int_0^\infty \ln\left(\frac{y+\rho a_0}{1+\alpha i_t}\right) e^{-\rho t} dt \tag{3.7}$$

$$\begin{aligned} s.t. & \\ \dot{b}_t &= \rho b_t + g_t - i_t m_t \\ \lim_{t\to\infty} b_t e^{-\rho t} &= 0 \\ r_t &\geq \bar{r} \end{aligned}$$

The first constraint is the government's budget constraint in real terms. This is obtained by substituting (3.6) into (3.5), and rewriting it in terms of tradables. Again, the real interest rate has been substituted out by the international interest rate. The second constraint is the transversality condition on the government's debt. The third constraint is an international liquidity constraint reflected in a minimum level of reserves.

[14] This approach to optimal monetary policy is now standard in the literature. See Lahiri and Vegh (2000).

[15] Additionally, these assumptions imply that government foreign debt and reserves are perfect substitutes, so, a constraint on the level of reserves is equivalent to a constraint on the level of debt.

4. Disinflation Program: Optimal Exchange Rate Policy

The main question of the paper is why do governments initiate a disinflation program even though the fiscal equilibrium is not guaranteed.[16] This section shows that in our framework that behavior is, indeed, optimal.

We have simplified the model in several dimensions. First, there are no political economy issues: There is a representative consumer (thus no distributional problems) and all agents maximize the same utility function. Second, there are no Olivera-Tanzi effects and there is no choice between inflation and other taxes, which eliminates these motives as possible sources of the disinflation program. Finally, the central bank is weaker than the fiscal authority, so no disciplinary arguments apply. This is reflected in the exogeneity of g_t. In summary, under these assumptions, the explanations given in the literature would imply that a flexible exchange regime is the optimal policy. If a disinflation is started, it is due to the tax smoothing motive.

In this section, to isolate the adoption of the program, we solve the simple case when there are no reserves constraints. The main result is that the optimal exchange rate path is a managed exchange rate regime with a depreciation rate lower than the one implied by flexible exchange rate.[17] In later sections we generalize the model and show that the result is still robust to most of them.

The central bank's problem is to choose the path of nominal interest rates that solves (3.7) when $\bar{r} \rightarrow -\infty$. First, we solve the model for a flexible exchange rate as a benchmark. Second, we solve for the optimal exchange rate policy.

4.1. Flexible exchange rate.

We define the flexible exchange rate as the one that implies a constant level of reserves; thus the government's debt is also constant. Imposing $\dot{b}_t = 0$ on the government's budget constraint we obtain,

$$\rho b_0 + g_t = i_t m_t$$

This equation implies that the seigniorage has to be equal to the total government expenditures every period. Given the money demand, equation (3.2), we can solve for the interest rate, which implicitly solve

[16] An alternative ways of posing the same question is: why for some period of time the fiscal and monetary policy seems to be inconsistent?

[17] A formal solution is shown in appendix A.

for the exchange rate depreciation.

$$\begin{aligned} \frac{1}{1+\alpha i_t} &= 1-\frac{g_t+\rho b_0}{y+\rho a_0} \\ \hat{e}_t &= i_t-\rho \end{aligned} \tag{4.1}$$

Denote i_h^f (i_l^f) as the interest rate in the flexible regime consistent with a high (low) level of expenditure.

4.2. Optimal interest rate path

Lets show that the optimal exchange rate path before τ, is a managed exchange rate with a depreciation rate smaller than the one implied by flexible exchange rate, and that after τ, the optimal regime is a flexible exchange rate. The problem is solved by backward induction.

We know that, after τ, expenditure is constant in each of the states of the world. By tax smoothing, the optimal regime is one that implies a constant inflationary tax. The only constant rate of depreciation consistent with the government's transversality condition is a flexible exchange rate. Denote the government debt at τ as b_τ. Substituting in equation (4.1) we obtain the interest rate in each state of the world.

$$\frac{1}{1+\alpha i_h^1} = 1-\frac{g_h+\rho b_\tau}{y+\rho a_0} \tag{4.2}$$

$$\frac{1}{1+\alpha i_l^1} = 1-\frac{g_l+\rho b_\tau}{y+\rho a_0} \tag{4.3}$$

where i_h^1 is the interest rate consistent with the higher level of expenditure and i_l^1 is the one consistent with the lower level of expenditure.

The second step is to solve for the interest rate before τ. Writing the Hamiltonian and optimizing we obtain that the interest rate is constant prior to τ and that it satisfies the following constraint:

$$i^1 = (1-q)\, i_h^1 + q i_l^1 \tag{4.4}$$

where i^1 is the interest rate between $[0, \tau]$. Equation (4.4) comes from equating expected marginal utilities of consumption before and after τ. Finally, we use the law of motion of debt to compute its value at time τ, given i^1.

$$b_\tau = b_0 + \frac{e^{\rho\tau}-1}{\rho}\left[g_h+\rho b_0-(y+\rho a_0)\left[1-\frac{1}{1+\alpha i^1}\right]\right] \tag{4.5}$$

Equations (4.2), (4.3), (4.4), and (4.5) constitute a system of four equations with four unknowns. The solution for the interest rate, debt, reserves and consumption are shown in figure 4.1.

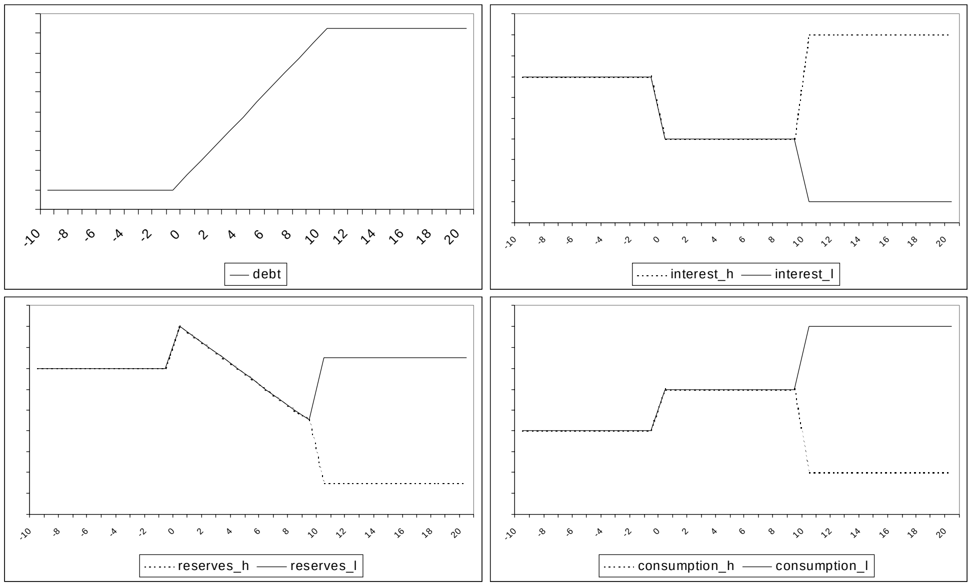

Figure 4.1: Solution to the unconstrained economy.

Proposition 4.1. *Along the optimal path, the exchange rate depreciation between* $[0, \tau]$ *is smaller than the one implied by flexible exchange rate. Moreover, foreign debt is increasing or, equivalently, reserves are falling.*

Proof. The proof is by contradiction. Suppose the proposition is false, assume that $i^1 \geq i_h^f$. Substituting in the intertemporal budget constraint of the government, we obtain $\dot{b}_t < 0$. This is because the larger interest rate implies a larger seigniorage. Thus, at time τ, the total debt is smaller than the initial debt b_0. Then, according to equation (4.1), (4.2), and (4.3), $i_h^1 < i_h^f$ and $i_l^1 < i_l^f$. However, using equation (4.4) the interest rate i^1 is a weighted average of the interest rates after τ. In particular, it has to be always smaller than i_h^1, which is smaller than i_h^f. But this is a contradiction. ■

The proposition states that a disinflation program is initiate even though expenditure has not been adjusted. The disinflation causes an increase in debt due to the reduction in seigniorage. If the fiscal adjustment fails, so the bad state of the world is realized, the new equilibrium depreciation rate is higher than the one that would prevail if a flexible exchange rate were adopted in the first place. Ex-post, it looks as if the country made a mistake initiating the stabilization program.

The intuition of the result is the following. The announcement of the fiscal reform conveys good news in terms of future expected reductions in expenditure; the expected equivalent annuity of expenditure falls. By the intertemporal budget constraint of the government the expected equivalent annuity of taxation should fall too. Because inflation generates welfare costs the optimal path of inflation tax is to have a constant expected rate of inflation. Thus a disinflation is initiated.[18]

Finally, note that there is no guarantee that reserves are positive in the bad state of the world. If τ or the expected expenditure improvement are large enough, reserves can be negative, especially when the fiscal adjustment does not take place. [19] We return to this point in the next section.

4.3. Discussion

There are three caveats of the model worth to be mentioned. First, the optimal exchange rate regime is a managed exchange rate and not a fixed exchange rate. Several countries, however, had fixed their exchange rates as the nominal anchor. Other reasons as visibility, credibility, or political economy have to be introduced to explain the adoption of a pure fixed exchange regime.

Second, the path of the optimal exchange rate predicted by the model does not fully characterize the observed behavior in two important dimensions. First, the managed exchange rate predicted by the model is a small fraction of the observed pegs. The average reduction in exchange rate depreciation is from 200 percent to almost zero. The model can explain (at most) 50 percent. Second, the model implies that if a reform is successful, there should be a further discrete jump in the depreciation rate. In the data, excluding some particular cases (Chile and Mexico) there is almost never a reduction in the depreciation rate after the reform is approved. The two facts can be accounted for if inflation inertia (*à la* Calvo) is introduced in the model. The intuition is that the Central Bank reduces the depreciation rate below the one predicted by fully prices in order to achieve the desired reduction in seigniorage. The Central Bank equates the benefit of reducing the inflation rate faster with the cost of distorting the nominal interest rate. This helps explain the size of the disinflation.

In the same model, when the reform is successful (so the fiscal deficit is permanently reduced) the degree of inertia of the inflation process is reduced[20]. In this circumstance, there are two forces of opposite direction that determine the optimal exchange rate policy. On the one hand, the realization of the reform conveys good news about the future, and a smaller depreciation rate is desired. This is the direct implication

[18] The model has additional implications that are well in line with the existing literature on exchange rate based stabilization programs.First, on impact, reserves go up and decrease thereafter. The reduction in the nominal interest rate implies an increase in demand for real balances, which is reflected in an increase in reserves on the implementation of the disinflation. Second, there is a consumption boom at the announcement of the reform. Third, the trade balance and the current account deteriorate. See Calvo (1986 and 1987), Calvo and Vegh(1993), Agenor and Montiel (1986), Rodriguez (1982).

[19] The comparative statics is analyzed in the appendix. An increase in q unambiguously increases debt at τ, and reduces current inflation. An increase in τ increases debt at time τ, and increases current inflation.

[20] This occurs in the Calvo model both through the expectations of future inflation rates and the reduction of excess domestic demand.

of the model developed in the previous section. On the other hand, the reduction in inflation inertia implies that a less aggressive peg is required in order to achieve the same disinflation. Depending on the degree of inertia the second effect can dominate and therefore, no changes in the depreciation rate are going to be observed after the reform is implemented.

The third caveat of the model is that the fiscal deficit should increases on impact. The reduction in inflation immediately reduce government revenue. In several Latin American experiences this is not the case; the fiscal deficit usually falls on the announcement of the reform. We know, however, that unsustainable short term measures can be, and had been implemented to reduce these deficits. In our model, consumers have perfect foresight and only permanent changes affect consumption. Consumers only care about the equivalent annuity of government expenditure (or fiscal deficit). To reconcile our implication with the data it is important to look at the counter part of the fiscal deficit, which is the path of debt: the model, indeed implies that debt should be accumulated through out the disinflation program. We know that in most of the unsuccessful cases (the only exception is Mexico 87) the government debt is increased through out the program.[21] In other words, the equivalent annuity of the fiscal deficit increased during the years of the disinflation. An alternative view to the increase in government debt is to observe a decrease in total domestic holdings of foreign assets. In the model, the counter part of the fiscal deficit is the deterioration of the current account. This is a standard fact observed in exchange rate based stabilization programs.

In summary, when the fiscal reform is uncertain, the announcement of it induces the Central Bank to implement a disinflation program. The extent of the peg depends not only in the characteristics of the reform process, but also on the degree of price inertia. During the disinflation, the economy experiences a consumption boom, a deterioration of the current account, and an increase in government debt. If the reform does not take place, inflation increases above the original level.

5. Balance of payments crisis: The Latin American case.

One of the most distinct features of the Latin American disinflation programs is how they end: with a Balance of Payments crisis. As the experience has shown, these crises are costly to the economies in terms of output losses, unemployment, and recession. In this context, we could ask whether the conclusions from the previous section would hold in the presence of the possibility of a balance of payments crises. In fact, a strategy that implies zero probability of facing a crisis is one in which the disinflation program is started *after* the fiscal reform has been implemented.

[21] For example, Argentina today has 5 times the domestic debt it had in 1991.

In this section, we explore the optimal exchange rate policy when the country faces a constraint in the level or reserves (or equivalently in its level of debt). In this section we build heavily on the previous literature on balance of payments crises.[22] It has, however, the additional implication that a disinflation program is initiated even though it implies a positive probability of facing a crises.

In the previous section, we argued that there are parameters that under the optimal policy imply negative reserves. In these cases, if there exists a constraint on the level of reserves (assume that for simplicity it is zero), the central bank is unable to implement the optimal unconstrained strategy. The solution to the constrained optimization problem is then a corner solution.[23] The optimal polity implies that the central bank sets the interest rate to the minimum one that guarantees that at time τ, in any event of the world, reserves are greater or equal than the minimum. In other words, the central bank sets the interest rate such that in the bad state of the world reserves are zero.[24] Notice that the balance of payments crisis occurs *á la* Krugman (1979) with the twist that here the timing is given and not the fiscal deficit. In Krugman's model, the engine of the crisis is an exogenous fiscal deficit. Thus, the timing is determined by the necessity to finance the deficit with reserves. In our case, the expenditure process is exogenous, but not the fiscal deficit. The timing of the crisis is given by the realization of not implementing the reform, and the inflation tax revenue (or equivalently the fiscal deficit) adjusts to make the crisis rational at τ. In other words, the inflation tax is such that there is a fiscal deficit financed by reserves that makes optimal a speculative attack at τ.

Assume the constraint is hit, we know that after τ reserves are zero in the bad state; therefore by the balance sheet of the central bank, domestic debt and money holdings are equal.

$$b_\tau = m_\tau \;\Rightarrow\; b_\tau = \alpha \frac{y + \rho a_0}{1 + \alpha i_h^c}$$

where i_h^c stands for the interest rate when the level of expenditure is high and there is a constraint on the level of reserves. The interest rate after τ also has to satisfy the transversality condition on the government debt, so, it is determined by equation (4.1). Solving for the maximum level of debt,

$$\bar{b}_\tau = \frac{\alpha}{1 + \alpha\rho} \left(y + \rho a_0 - g_h\right) \tag{5.1}$$

The interest rate prior to τ has to be consistent with a debt accumulation such that debt is equal to equation (5.1) at time τ. Using the equation for debt accumulation we solve for the interest rate (i^c) prior

[22] See Krugman (1979), Flood and Garber (1986), Calvo (1986 and 1987).

[23] The formal solution is in appendix B.

[24] When expenditure is not adjusted interest rates increase and reserves fall. Thus, if the constraint is binding it has to be binding in the bad state of the world.

to τ.

$$\frac{1}{1+\alpha i^c} = \frac{1}{1+\alpha i^f} + \frac{\alpha\rho}{(1+\alpha\rho)\left(e^{\rho\tau}-1\right)}\left[1 - \frac{g_h + \rho b_0\left(1+\frac{1}{\alpha\rho}\right)}{y+\rho a_0}\right] \tag{5.2}$$

Proposition 5.1. *The optimal path implies that a disinflation program is initiated at the announcement of the fiscal reform. Most importantly, there is a positive probability of a balance of payments crisis.*

Proof. i^1 implies an accumulation of debt that generates negative reserves, and we constructed i^c to have a lower rate of debt accumulation. Thus, $i^c > i^1$ by construction. To show that $i^f > i^c$ we follow the same proof by contradiction of proposition (4.1), or by inspection of equation (5.2).

Finally, the interest rate is computed such that the reserves reach their minimum in the case of not adjusting the expenditure. This means, that there is a balance of payments crisis at τ that occurs with probability equal to the probability that the bad state of the world is realized. In other words, when it is known that the fiscal reform has failed there is a speculative attack. ■

Note that the proposition implies that a government initiates a disinflation program even though there is a risk of a balance of payments crisis.[25] The intuition is that the announcement of the fiscal reform conveys good news in the future and the government wants to transfer part of those future benefits to today in the form of higher real balances. The extent in which this transfer can be made is limited by the debt constraint. Therefore there is no full smoothing of consumption and money holdings. It is always optimal, however, to transfer some of those benefits to today.

Two remarks about the cost of a balance of payments crises: First, in this model, the only cost of the balance of payments crisis is the elimination of reserves and the lack of foreign credit; this is the Sargent & Wallace effect. Balance of payments crises, however, are likely to be more costly than this. Considering additional costs does not change the qualitative implications of the model.

In particular, the proposition (almost) continues to be true if additional costs have to be paid after the crisis occurs. The intuition is that the crisis is avoided with probability one if the interest rate implemented is an ε larger than i^c. Thus, in the case in which the costs of the balance of payments crisis are paid after the speculative attack, the model predicts the same timing for the initiation of the disinflation program, and a similar depreciation rate.

Second, a more realistic cost of the balance of payment would assume that the interest rate faced by the government is a decreasing (convex and differentiable) function of the level of reserves. The disinflation

[25] This result is robust to alternative formulations of preferences and expenditure processes.

will be initiated at the announcement of the reform, but the size of the disinflation would be smaller. There is a marginal benefit of reserves on top of its financing role that limits the extent of the depreciation.

In summary, we show that the nature of the solution does not change if a maximum level of debt exists. Still it is the case that the government initiates a disinflation program when a fiscal reform is announced. The important result is that the optimal policy implies that there exists a positive probability of a balance of payments crisis. Notice that this does not mean that a balance of payments crisis is optimal. Rather it means that it is optimal to initiate a disinflation program even if there exists a probability of a balance of payments crisis.

6. Sequence of Stabilization Programs.

In figure 6.1, the Brazilian monthly inflation rate in the late 80's is plotted. The shaded area represents periods where disinflation programs where in place.[26] There are two facts that we can extract from this figure. First, notice that there is a sequence of unsuccessful stabilization programs. Second, that every time the program fails, the inflation rate is higher than the inflation before the program was initiated.

This experience is not exclusively Brazilian. For example, Venezuela since 1983 had implemented five stabilization programs, and Argentina did the same in the 70's.

In this section, we show that this pattern is the optimal policy. We show that the central bank implements a sequence of stabilization programs, even though each time it is harder to reduce inflation and it is more costly if the program fails.

To capture this dynamics we change our basic framework and assume that the government is continuously trying to reduce expenditure: every time a fiscal reform fails, the government announces a new one. This behavior should raise naturally from the assumption that expenditure is wasteful. A very simple way of modelling this is to assume that expenditure follows a Poisson process, which implies that there is a

[26] In the particular case of Brazil, several of those cases involve price controls.

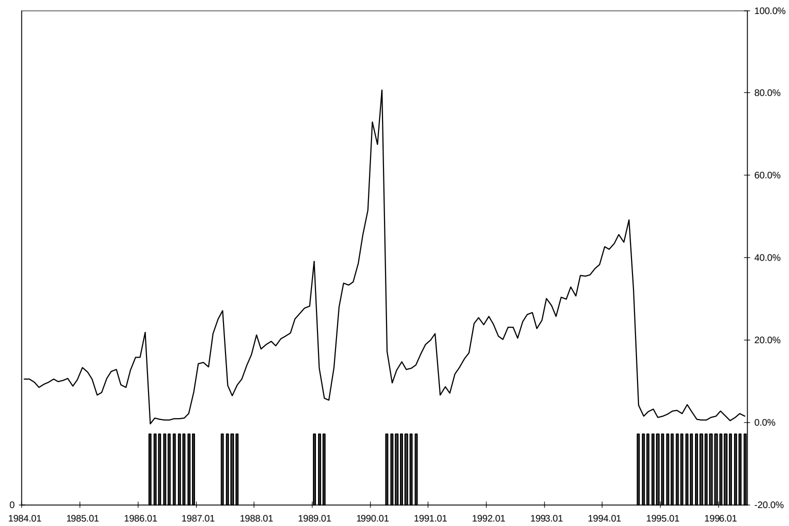

Figure 6.1: Stabilization programs and monthly inflation: Brazil.

fiscal reform at every point in time with probability qdt of being successful.[27]

$$g_{t+dt} = \begin{cases} g_h \quad w/p \quad 1-qdt \quad \text{if } g_t = g_h \\ g_l \quad w/p \quad qdt \\ \\ g_l \quad w/p \quad 1 \quad \text{if } g_t = g_l \end{cases}$$

Define the high state when there is a high level of expenditure, and the low state when there is a low level of expenditure. The rest of the section is organized as follows: First, we solve the problem assuming no debt constraints. We show that the optimal policy implies a continuum of disinflation programs (thus a sequence of them). Second, we introduce a reserve constraint and show that, even though there exists the possibility of a balance of payments crisis, the optimal strategy is to implement a sequence of disinflation programs until one is successful. Finally, we introduce the possibility of foreign help (in the form of IMF and World Bank loans) and show that this implies that after a loan is made, the government implements a disinflation program until the balance of payments reappears.

[27] There are two ways in which this process can be interpreted. First, at every time t the government announces a fiscal reform for time $t+dt$. If it fails, then the government announces a new one. Second, there is only one permanent fiscal reform, but the government is uncertain about when it is going to succeed. Thus, this section might also be interpreted as a relaxation of the expenditure process assumed earlier where the timing is exogenous.

6.1. Unconstrained economy.

The first result is that the optimal policy is to implement a continuum of disinflation programs. This is shown by proving that when expenditure is high, the optimal nominal interest rate is always smaller than the one implied by flexible exchange rate. To solve the problem we define a value function in each of the states of the world.

$$\begin{aligned} \rho V^{l}\left(b_{t}\right) &= \max_{\theta_{t}}\left\{\ln\theta_{t}+\left[\rho b_{t}+g_{l}-\left(y+\rho a_{0}\right)\left(1-\theta_{t}\right)\right]V_{b}^{l}\right\} \\ \rho V^{h}\left(b_{t}\right) &= \max_{\theta_{t}}\left\{\ln\theta_{t}+\left[\rho b_{t}+g_{h}-\left(y+\rho a_{0}\right)\left(1-\theta_{t}\right)\right]V_{b}^{h}+q\left[V^{l}-V^{h}\right]\right\} \end{aligned} \tag{6.1}$$

where V^{l} is the value function when expenditure is low and V^{h} is the value function when expenditure is high.[28]

Proposition 6.1. *If expenditure is high, the optimal strategy involves a rate of depreciation smaller than the one implied by flexible exchange rate, inflation and government debt are increasing every unsuccessful fiscal reforms, and the optimal strategy approaches the flexible exchange rate at high levels of debt.*

If expenditure is low, the optimal strategy is either a flexible or a fixed exchange regime. This is because the optimal flexible regime is a constant exchange rate.

Proof. The proof is in appendix C see propositions C.1 and C.2. ∎

The proposition implies that the optimal strategy when expenditure is high, is a managed exchange rate regime. Additionally, it implies that the larger the level of debt, the smaller the disinflation effort. In other words, the difference in the nominal interest rate between the optimal and the implied by flexible exchange rate is a decreasing function of debt.

The differential equations implied by equation 6.1 do not have a close form solution, thus we solve them numerically. The solutions for the optimal policy when expenditure is high is shown in figure 6.2. Debt as a percentage of GDP is measured in the x-axis, i_l is the interest rate implied by flexible exchange rate when expenditure is low (the bottom schedule), i_f is the interest rate implied by flexible exchange rate when expenditure is high (the top schedule), and i_h is the solution of the differential equation when expenditure is high. The interest rate is increasing with debt and is always smaller than the interest rate implied by flexible exchange rate in the high state.

[28] A formal solution is in appendix C.

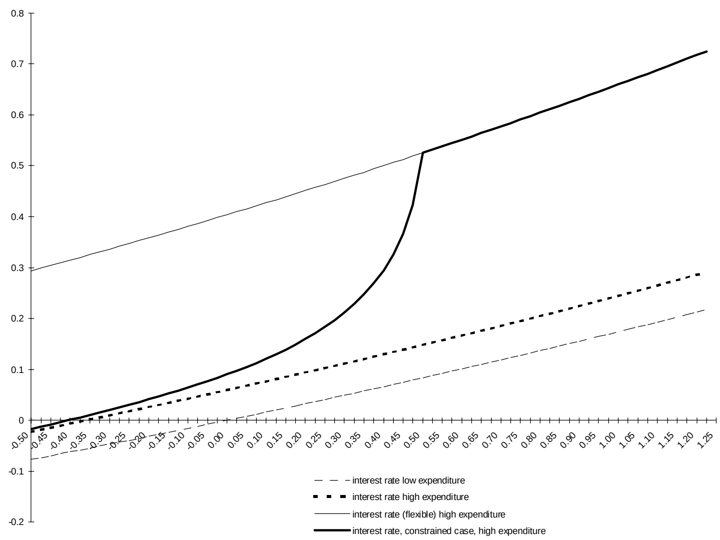

Figure 6.2: Solution to the poisson process.

6.2. Debt Constraint

We now introduce the possibility of a balance of payments crisis. Similarly as in the previous section, the maximum level of debt is given by equation (5.1). At this level of debt the optimal strategy is a flexible exchange rate regime; thus we use this constraint as a boundary condition for the differential equation. After substituting by the FOC, the differential equation is,

$$
\begin{aligned}
\left[\rho b_t + g_h - (y+\rho a_0)\left(1 - \frac{1}{1+\alpha i^h}\right)\right]\frac{\partial i^h}{\partial b_t} &= q\left[i^h - i^l\right] \\
i^h(\bar{b}) &= \frac{1}{\alpha}\left(\frac{g_h + \rho\bar{b}}{y+\rho a_0 - g_h - \rho\bar{b}}\right) \\
\bar{b} &= \frac{\alpha}{1+\alpha\rho}\left(y+\rho a_0 - g_h\right)
\end{aligned}
$$

The solution is shown in figure 6.2, where the interest rate of the constrained economy is computed. Note that for low levels of debt, the solutions for the constrained and unconstrained economy are similar. On the other hand, when debt is increasing the constrained economy approaches the flexible exchange rate faster than the unconstrained economy. Finally, when the maximum level of debt is reached, the regime changes to a flexible exchange rate in the constrained economy. In other words, when reserves are zero there are no possibilities of financing a reduction in inflation, other than implementing the fiscal reform.

6.3. Foreign Help

In this section, we show that if the country is close to hit the debt constraint a foreign loan is welfare improving and it implies an immediate adoption of a disinflation program. To clarify the intuition, assume that the economy has reached the maximum level of debt, so it has a flexible exchange rate. Lets interpret the debt level net of foreign help. This means that a loan from the IMF or the World Bank increases the debt capacity of the country. In terms of our model, the economy jumps to the left in figure 6.2. Therefore, a more aggressive disinflation program is initiated, real balances increase, and the consumer's utility goes up.

In summary, the results in this section are the following: First, the optimal strategy implies a sequence of disinflation programs even though there exists the possibility of a balance of payments crisis. Second, the inflation rate is increasing through the path every time the disinflation program fails. Third, the larger the debt, the smaller the disinflation effort. In other words, the "harder" the disinflation is.

7. Conclusions

Several Latin American countries have initiated stabilization programs based on fiscal reforms and on exchange rate managements. In most of these cases, the program was abandoned, and *ex-post*, it seemed as if it was a bad idea to initiate it in the first place. This paper has shown that if inflation has convex welfare costs and the fiscal reform is uncertain, it is possible to explain the government's behavior.

The paper shows why a government would implement a disinflation program even though the fiscal support has not come. The results show that the timing of the disinflation is unaffected even though there exists a possibility of a balance of payments crisis. Additionally, the analysis implies that countries will implement a sequence of disinflation programs until one of them is successful, or until a balance of payments crisis occurs. Each failed stabilization increases the inflation rate and makes the next disinflation program tougher to implement.

We show that these results are robust to several specifications of the expenditure process. Moreover, similar results can be obtained in more general models with alternative utility functions and specifications of money demand (for example, money in the utility function).

The two most important caveats of the model, however, are its inability to explain the size of the disinflation program and the oversimplification of the costs of failed stabilization programs. As was discussed in section 4.3, the first problem can be solved if inflation inertia is introduced in the model (Calvo (1983)). In that case, inflation is a smooth process and the exchange rate disinflation overshoots in order to help inflation to come down. Moreover, that model is also able to explain the process of inflation after the reform is approved. Even though such model could be more realistic, the inclusion of price stickiness complicates the analysis without providing additional intuition of the reasons that explain the initiation of the disinflation. In fact, the timing is unaffected by considering sticky prices.

The second problem of the model is that the only cost of the disinflation program is the Sargent & Wallace effect, which is likely to be small. In reality, disinflation programs are more costly than just its impact on future financing (especially when they fail). As was argued before, the model can be extended in this direction. The conjecture is that the qualitative results remain the same, and the timing of the disinflation is unaffected by these issues.

Finally, three important dimensions of stabilization programs have not been considered in the paper and should be the subject of future research: First, we have not considered political economy and credibility issues to explain the adoption of these reforms. These are important aspects and a more complete story should include them. The consideration of the political economy reasons and the tax smoothing motive in the adoption of the program are aspects that should be further investigated in order to improve our understanding of the disinflation experiences.

Second, almost all countries experienced an increase in real interest rate on the implementation of the program. In our model, the real interest rate is constant because we assumed that government debt is indexed. Credibility and imperfect capital mobility could explain the path of the real interest rate. First, if credibility is associated with the process of reform, then risk premium can explain the changes in the real interest rate (see Velasco (1993)). Second, if the capital account is closed, and the government is involved in a tight monetary policy, the interest rate on impact might increase (see Calvo, Leiderman and Reinhart (1993)).

Third, in the model presented, the government and the supply side (the reform process) have been oversimplified. In future research, the process of the fiscal reform should be endogenized in order to understand its interaction with the disinflation, its timing and size.

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A. The basic model

In this section we solve the simple model presented in the text.

A.1. Consumer's problem

The consumer's problem is given by,

$$
\begin{aligned}
& \max_{\{c_t\}} \quad E\int_0^\infty \ln c_t \, e^{-\rho t}dt \\
s.t. & \\
\dot{a}_t &= \rho a_t + y - c_t - i_t m_t \\
c_t &\leq \frac{1}{\alpha} m_t \\
\lim_{t\to\infty} a_t e^{-\rho t} &= 0
\end{aligned}
$$

where the first equation is the objective function. The second equation is the intertemporal budget constraint in terms of tradables. The third equation is the cash in advance constraint. And the fourth equation is the transversality condition on consumer's assets. Solving for consumption, money holdings, and the multiplier,

$$
\begin{aligned}
c_t &= \frac{1}{\lambda_0}\frac{1}{1+\alpha i_t} \\
m_t &= \frac{1}{\lambda_0}\frac{\alpha}{1+\alpha i_t}
\end{aligned}
$$

Substituting in the intertemporal budget constraint, integrating and imposing the transversality condition we obtain,

$$
\frac{1}{\lambda_0} = y + \rho a_0
$$

Note that the consumer's consumption and money function does not depend on the future path of interest rates. This result comes from the log utility assumption.

A.2. Central Bank's problem

We derive the solution in two steps. First we solve the problem when there is no fiscal uncertainty and expenditure is constant. Second, we solve the problem with the stochastic process assumed in the text.

Solution without fiscal uncertainty. We show that the solution when there is no risk in government expenditure is a flexible exchange rate. The problem is the following,

$$
\begin{aligned}
& \max_{\{i_t\}} \quad E\int_0^\infty \ln \frac{1}{1+\alpha i_t} e^{-\rho t}dt \\
s.t. & \\
\dot{b}_t &= \rho b_t + g_t - (y + \rho a_0)\left(1 - \frac{1}{1+\alpha i_t}\right) \\
\lim_{t\to\infty} b_t e^{-\rho t} &= 0
\end{aligned}
$$

Define,

$$\theta_t = \frac{1}{1+\alpha i_t} \tag{A.1}$$

The first order conditions are,

$$\begin{aligned} \frac{1}{\theta_t} + (y+\rho a_0)\lambda_t &= 0 \\ \dot{\lambda}_t &= 0 \end{aligned}$$

Note that if $\dot{\lambda}_t = 0$, then $\dot{\theta}_t = 0$. This means that the multiplier is constant and that marginal consumption is constant. Which means that the optimal strategy for the government is to have a constant inflation; smooth the inflationary tax. To determine the level of the multiplier we substitute in the budget constraint and impose the transversality condition. This implies that,

$$\theta_i = 1 - \frac{g_i + \rho b_0}{y + \rho a_0}$$

This means that the solution is a constant depreciation rate equal to the flexible exchange rate.

Fiscal uncertainty. We solve the problem by backward induction. Given that we know that without fiscal uncertainty the solution is a constant interest rate, then after τ, there should be a constant inflation rate consistent with a flexible exchange rate given the level of debt at τ. Formally,

$$\begin{aligned} \theta_h^1 &= 1 - \frac{g_h + \rho b_\tau}{y + \rho a_0} \\ \theta_l^1 &= 1 - \frac{g_l + \rho b_\tau}{y + \rho a_0} \end{aligned}$$

where θ_h^1 is the inverse of the interest rate when the level of expenditure is high, and θ_l^1 is the inverse of the interest rate when the level of expenditure is low. Substituting in the utility function and using the definition of the debt,

$$\begin{aligned} &\max_{\{\theta_t\}} \left[\int_0^\tau \ln \theta_t e^{-\rho t} dt + \frac{1}{\rho} e^{-\rho t} \left(q \ln \theta_l^1 + (1-q) \ln \theta_h^1 \right) \right] \\ &s.t. \\ &\dot{b}_t = \rho b_t + g_i - (y + \rho a_0)\left(1 - \frac{1}{1+\alpha i_t}\right) \end{aligned}$$

writing the Hamiltonian and solving the first order conditions we find (as before) that the optimal interest rate has to be constant between $[0, \tau]$. Using the debt accumulation equation and substituting in the maximization problem, the first order condition implies (after some algebra),

$$\frac{1}{\theta^1} = \frac{q}{\theta_l^1} + \frac{1-q}{\theta_h^1}$$

This condition is saying that marginal utility of consumption before τ is equal to the expected marginal utility of consumption after τ. Which is the usual Euler condition on consumption. Substituting by the definitions of θ we obtain the equation (4.4) in the text for the interest rate.

To solve for the interest rate we have the following system of equations,

$$
\begin{aligned}
\frac{1}{1+\alpha i_h^1} &= 1-\frac{g_h+\rho b_\tau}{y+\rho a_0} \\
\frac{1}{1+\alpha i_l^1} &= 1-\frac{g_l+\rho b_\tau}{y+\rho a_0} \\
i^1 &= (1-q)\, i_h^1 + q i_l^1 \\
b_\tau &= b_0 + \frac{e^{\rho\tau}-1}{\rho}\left[g_h+\rho b_0-(y+\rho a_0)\left[1-\frac{1}{1+\alpha i^1}\right]\right]
\end{aligned}
$$

Solving implicitly for the debt level and the interest rate, it can be shown that i^1 is smaller that the pure flexible exchange rate. This means that a managed exchange rate has been initiated with the announcement of the fiscal reform. Second, it can be also shown that there is an increase in debt, meaning that the disinflation program is financed either by reserves or foreign debt.

B. Model with debt constraint.

In this section we solve the problem when there is a constraint in the level of reserves or, equivalently, a constraint in the level of debt. For simplicity assume that reserves have to be positive. The problem is solved in two steps: First, we define the level of debt constraint. Second, we solve the Kuhn-Tucker problem.

We know that when the constraint is hit the expenditure is high. At that moment, money demand and total debt are equal. Moreover, the interest rate has to be one in which there is no change in the level of debt, thus it is the flexible exchange rate. Formally,

$$
\begin{aligned}
b_\tau &= m_\tau \\
m_\tau &= \alpha\frac{y+\rho a_0}{1+\alpha i_\tau} \\
\frac{1}{1+\alpha i_\tau} &= 1-\frac{g_h+\rho b_\tau}{y+\rho a_0}
\end{aligned}
$$

This system of equations uniquely determines the debt at τ, which is consistent with hitting the constraint.

$$
b_\tau^c = \frac{\alpha}{1+\alpha\rho}\left(y+\rho a_0-g_h\right) \tag{B.1}
$$

Thus, if the constraint is binding, then the interest rate between $[0,\tau]$ has to be one such that the debt accumulated until time τ is equal to equation (B.1). Using the government's debt law of motion, we solve for the interest rate,

$$
\begin{aligned}
\theta^c &\equiv 1+\Psi_\tau\left[\alpha\rho-\frac{g_h\left((1+\alpha\rho)\, e^{\rho\tau}-1\right)+\rho b_0\,(1+\alpha\rho)\, e^{\rho\tau}}{y+\rho a_0}\right] \\
\Psi_\tau &\equiv \frac{1}{(1+\alpha\rho)\left(e^{\rho\tau}-1\right)}
\end{aligned} \tag{B.2}
$$

After some algebra, we obtain equation (5.2) in the text. Now, lets look at the central bank's problem,

$$\max_{\{\theta_t\}} \quad E\int_0^\infty \ln\theta_t e^{-\rho t}dt$$

$$\begin{aligned} s.t. & & \\ \dot{b}_t &=& \rho b_t + g_t - (y+\rho a_0)(1-\theta_t) \\ \lim_{t\to\infty} b_t e^{-\rho t} &=& 0 \\ \theta_t &\leq& \theta^c \text{ for all } t\epsilon[0,\tau] \end{aligned}$$

where the last constraint implies that the nominal interest has to be always larger or equal to i^c otherwise, the constraint on the level of reserves would be hit. Writing the Hamiltonian we solve the Kuhn-Tucker problem. If the constraint is not binding then the solution is the same as the previous section. If the constraint is binding, then the solution is,

$$\theta_t = \begin{cases} \theta^c & & \tau < 0 \\ \theta_h^c = 1 - \frac{g_h + \rho b_\tau^c}{y+\rho a_0} & g_t = g_h & \tau \geq 0 \\ \theta_l^c = 1 - \frac{g_l + \rho b_\tau^c}{y+\rho a_0} & g_t = g_l & \tau \geq 0 \end{cases}$$

where b_τ^c is given by equation (B.1) and θ^c is given by equation (B.2). Notice that by construction b_τ^c is smaller than the debt obtained in the optimal unconstrained strategy, thus $\theta_h^c > \theta_h^1$ and $\theta_l^c > \theta_l^1$, but it is positive which implies that there is a disinflation program before τ. In other words, even though there exists the possibility of a balance of payments crisis, still it is optimal to initiate a disinflation program financed with reserves.

C. Expenditure follows a Poisson process

In this section we assume that the expenditure follows a Poisson process. First we solve the problem without reserves constraints. Second, we show the solution when there are reserves constraints.

C.1. No reserves constraint

Assume that expenditure follows,

$$g_{t+dt} = \begin{cases} g_h & w/p & 1-qdt & \text{if } g_t = g_h \\ g_l & w/p & qdt & \\ & & & \\ g_l & w/p & 1 & \text{if } g_t = g_l \end{cases}$$

In this case, we have two value functions. One for the low level of expenditure and one for the high level of expenditure. The Bellman's equations are,

$$\begin{aligned} \rho V^l(b_t) &=& \max_{\theta_t}\left\{\ln\theta_t + [\rho b_t + g_l - (y+\rho a_0)(1-\theta_t)]\,V_b^l\right\} \\ \rho V^h(b_t) &=& \max_{\theta_t}\left\{\ln\theta_t + [\rho b_t + g_h - (y+\rho a_0)(1-\theta_t)]\,V_b^h + q\left[V^l - V^h\right]\right\} \end{aligned}$$

The solution for the first one is the following: The first order condition and the envelope theorem equations are,

$$\frac{1}{\theta_t^l} + (y + \rho a_0) V_b^l = 0 \tag{C.1}$$

$$\left[\rho b_t + g_l - (y + \rho a_0)\left(1 - \theta_t^l\right)\right] V_{bb}^l = 0$$

This means that the solution is the flexible exchange rate regime.

$$\theta^l(b_t) = 1 - \frac{g_l + \rho b_t}{y + \rho a_0} \tag{C.2}$$

Substituting in the Bellman equation it is possible to solve for the value function. Notice that the value function is twice differentiable, decreasing and concave, and the interest rate policy function is increasing and convex. Now, we solve the problem for the value function with high level of expenditure. The first order condition and the envelope theorem imply,

$$\frac{1}{\theta_t^h} + (y + \rho a_0) V_b^h = 0 \tag{C.3}$$

$$\left[\rho b_t + g_h - (y + \rho a_0)\left(1 - \theta_t^h\right)\right] V_{bb}^h = q\left[V_b^h - V_b^l\right] \tag{C.4}$$

Proposition C.1. $\forall b_t < \infty \Rightarrow \theta^h(b_t) < \theta^l(b_t)$

Proof. Lets first show that they can not be equal, and then show that $\theta^h(b_t)$ can not be larger than $\theta^l(b_t)$. Assume $\theta^h(b_t) = \theta^l(b_t)$. Then equations (C.1) and (C.3) imply that, $V_b^l = V_b^h$. Substituting in the right hand side of equation (C.4) we obtain,

$$\left[\rho b_t + g_h - (y + \rho a_0)\left(1 - \theta_t^h\right)\right] = 0$$

which implies that the solution for $\theta^h(b_t)$ is,

$$\theta^h(b_t) = 1 - \frac{g_h + \rho b_t}{y + \rho a_0} \neq \theta^l(b_t) \;\; \forall \; b_t < \infty$$

which is a contradiction for any finite level of debt.

Now assume $\theta^h(b_t) > \theta^l(b_t)$. In this case, equations (C.1) and (C.3) imply,

$$(y + \rho a_0) V_b^h = -\frac{1}{\theta_t^h} > -\frac{1}{\theta_t^l} = (y + \rho a_0) V_b^l \;\Rightarrow\; V_b^h > V_b^l$$

This implies that the right hand side of equation (C.4) is always positive. Given the properties of the value function we know that V_{bb}^h is negative. Thus this would imply that the term in the brackets is negative.

$$\left[\rho b_t + g_h - (y + \rho a_0)\left(1 - \theta^h(b_t)\right)\right] < 0$$

Solving for $\theta^h(b_t)$

$$\theta^h(b_t) < 1 - \frac{g_h + \rho b_t}{y + \rho a_0} < \theta^l(b_t)$$

which is a contradiction. Therefore, $\theta^h(b_t) < \theta^l(b_t)$ for any finite level of debt. ■

Note that this proposition implies also, that $V_b^h < V_b^l$. Thus the right hand side of equation (C.4) is negative. Now lets show that the optimal solution implies a disinflation program when expenditure is high.

Proposition C.2. $\forall b_t < \infty \Rightarrow \theta^h(b_t) > \theta^f(b_t)$

Proof. Remember that we define $\theta^f(b_t)$ as the solution to

$$\left[\rho b_t + g_h - (y + \rho a_0)\left(1 - \theta^f(b_t)\right)\right] = 0$$

Given the concavity of the value function and proposition (C.1) $\left(V_b^h < V_b^l\right)$ we know that,

$$\left[\rho b_t + g_h - (y + \rho a_0)\left(1 - \theta^h(b_t)\right)\right] > 0$$

Therefore, the optimal path implies a reduction in reserves (increasing debt) and $\theta^h(b_t) > \theta^f(b_t)$ for any finite level of debt. ■

Substituting the definitions of θ and the value functions, we obtain the following differential equation for the interest rate when expenditure is high.

$$\left[\rho b_t + g_h - (y + \rho a_0)\left(1 - \frac{1}{1 + \alpha i^h}\right)\right] \frac{\partial i^h}{\partial b_t} = q\left[i^h - i^l\right] \tag{C.5}$$

where $i^l(b_t)$ has a close form solution from equation (C.2). The boundary condition for the differential equation is,

$$\lim_{b \to -\infty} i^f = \lim_{b \to -\infty} i^h = \lim_{b \to -\infty} i^l = -\frac{1}{\alpha}$$

The solution is shown in figure 6.2. The schedule in the bottom is the interest rate when there is a low level of expenditure. The schedule on the top is the interest rate implied by a flexible exchange rate when expenditure is high. The schedule in the middle is the solution for the differential equation when expenditure is high.

C.2. Reserves constraint

In this section we assume that reserves have to be positive. This imposes a limit on the maximum level of debt. As we did in appendix B, the level of debt when the constraint is hit is,

$$b_t = \frac{\alpha}{1 + \alpha\rho}(y + \rho a_0 - g_h)$$

the solution for the constrained economy implies the same differential equation as before but with a different boundary condition. Formally,

$$\begin{aligned}
\left[\rho b_t + g_h - (y + \rho a_0)\left(1 - \frac{1}{1 + \alpha i^h}\right)\right] \frac{\partial i^h}{\partial b_t} &= q\left[i^h - i^l\right] \\
i^h(\bar{b}) &= \frac{1}{\alpha}\left(\frac{g_h + \rho\bar{b}}{y + \rho a_0 - g_h - \rho\bar{b}}\right) \\
\bar{b} &= \frac{\alpha}{1 + \alpha\rho}(y + \rho a_0 - g_h)
\end{aligned}$$

The properties of the solution are conserved. The numerical solutions is shown in figure 6.2. We compare the solution for the constrained and unconstrained economies. Notice that for low levels of debt the two solutions behave similarly. However, when the crisis is close the interest rate starts increasing faster in the constrained case.

D. Expenditure follows a Jump Diffusion Process

In the paper we solve the optimal interest rate when the process follows a particular case of a Poisson process. In this section we find the solution for a more general stochastic process. The process assumed in the text has two characteristics: First, its drift is negative. Second, the timing of the change is known. In this section we analyze under which conditions a disinflation program is initiated.

Assume that the expenditure follows,

$$dg_t = \mu_g dt + \sigma_g dz_t + \lambda_g dq$$

where dz_t is the standard Weiner process, and dq is a Poisson process that takes value 0 with probability $1 - qdt$, and value 1 with probability qdt. The Bellman's equation in continuous time is,

$$\rho V(b_t, g_t) = \max_{\theta_t} \{\ln \theta_t + EdV\}$$

Using Itô's lemma we can show that,

$$\rho V(b_t, g_t) = \max_{\theta_t} \left\{ \begin{array}{c} \ln \theta_t + [\rho b_t + g_t - (y + \rho a_0)(1 - \theta_t)] V_b(b_t, g_t) + \\ \mu_g V_g(b_t, g_t) + \frac{1}{2}\sigma_g^2 V_{gg}(b_t, g_t) \\ -q[V(b_t, g_t + \lambda_g) - V(b_t, g_t)] \end{array} \right\}$$

where V_i represents the partial derivative with respect to argument i. The first order condition implies,

$$\frac{1}{\theta_t} + (y + \rho a_0) V_b = 0$$

Notice that the value function has the following properties. First, it is continuous and twice differentiable. Second, it is decreasing with respect to debt and government expenditure. Notice that this implies that the policy function (θ_t) is non-increasing with debt and expenditure. Finally, V is concave with respect to its arguments.

The envelope theorem implies,

$$\begin{aligned} [\rho b_t + g_t - (y + \rho a_0)(1 - \theta_t)] V_{bb} &= -\mu_g V_{gb} - \frac{1}{2}\sigma_g^2 V_{ggb} \\ &\quad + q[V_b(\cdot, g_t + \lambda_g) - V_b] \end{aligned} \tag{D.1}$$

After some algebra the differential equation on the interest rate is,

$$\begin{aligned} & \left[-\frac{\partial i(b_t, g_t)}{\partial b_t}\right] \left[\rho b_t + g_t - (y + \rho a_0)\left(1 - \frac{1}{1 + \alpha i(b_t, g_t)}\right)\right] \\ = \; & \mu_g \frac{\partial i(b_t, g_t)}{\partial g_t} + \frac{1}{2}\sigma_g^2 \frac{\partial^2 i(b_t, g_t)}{\partial g_t^2} - q[i(b_t, g_t + \lambda_g) - i(b_t, g_t)] \end{aligned} \tag{D.2}$$

This is a delayed partial differential equation, and it does not have a close form solution. However, we are more interested in its characterization. In this case, it is important to understand when the interest rate implied by this equation is smaller than the interest rate from flexible exchange rate. In other words, what are the conditions on the stochastic process that generates a managed exchange rate with a loss in reserves.

Define i^f as the interest rate implied by flexible exchange rate as before. This interest rate requires

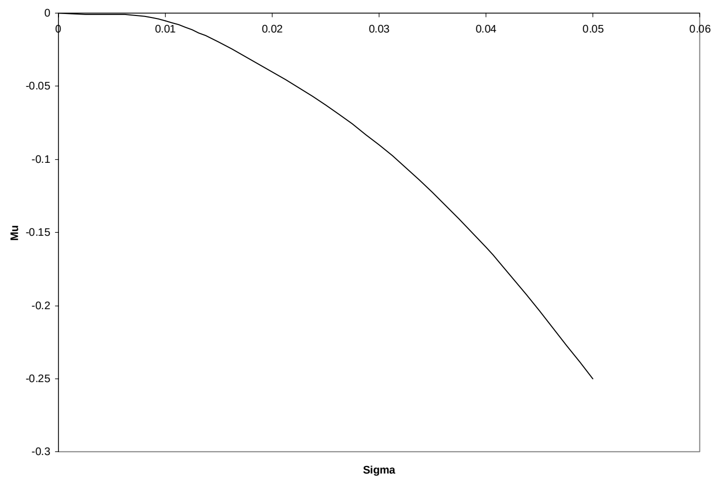

Figure D.1: Determination of the region where the optimal interest rate coincides with the flexible exchange rate.

the right hand side of equation D.2 to be equal to zero. The first term in the left hand side is always is negative, thus the sign of $\rho b_t + g_t - (y + \rho a_0)\left(1 - \frac{1}{1+\alpha i(b_t,g_t)}\right)$ is the opposite sign of the right hand side. Therefore, a disinflation program occurs if the right hand side is negative.

$$\mu_g \frac{\partial i_t}{\partial g_t} + \frac{1}{2}\sigma_g^2 \frac{\partial^2 i_t}{\partial g_t^2} - q\left[i_t\left(\cdot, g_t + \lambda_g\right) - i_t\right] < 0$$

Notice that this is an equilibrium condition, which makes it very difficult to characterize. Using numerical methods we can find the required drift to satisfy the condition, given certain level of uncertainty. We solve the problem for the case $q = 0$ and find the set of points $\left[\mu, \sigma^2\right]$ such that the solution for the differential equation implies a flexible exchange rate. The solution is shown in figure D.1.

Note that a negative drift is necessary (but not sufficient). In other words, there has to be a reform in place otherwise there is no rationale for a disinflation program. The negative drift can be obtain either by the Brownian motion part, or the Poisson process.

D.1. Solution when there is a debt constraint.

In the same way we did for the Poisson case, the debt constraint adds a boundary constraint in the partial differential equation.

$$b_t \le \frac{\alpha}{1+\alpha\rho}\left(y + \rho a_0 - g_t\right)$$

Moreover, we know that at that level of debt the interest rate is given by the flexible exchange rate one. We use this as a boundary condition for the partial differential equation.

The solution for the partial differential equation are shown in figure D.2 and D.3. Figure D.2 is the solution for the unconstrained economy, and figure D.3 is the solution for the constrained economy. As before, the behavior is similar at low levels of debt.

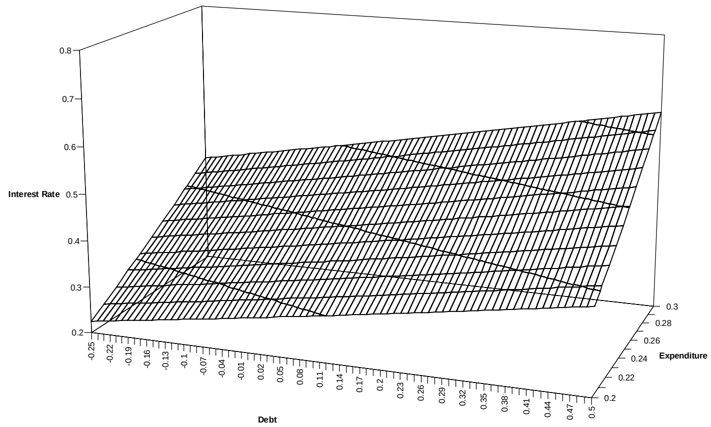

Figure D.2: Solution to the brownian motion case. Unconstrained economy.

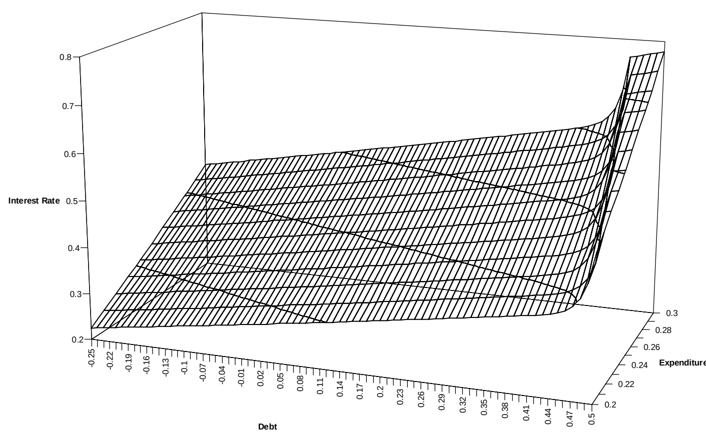

Figure D.3: Solution to the brownian motion case. Constrained economy.

E. Solution for the Money in the Utility model.

In this section we show that the main result of the paper can be obtained in a Money in the utility model. The two most important results in the model are: First, the tax smoothing result that implies that optimal interest rates are constant if government expenditure is constant. Second, that expected marginal utilities are equalized at the time of reform.

Assume the consumers derives utility from holding real balances and that its objective function is

$$\max_{\{c_t,m_t\}} \quad E\int_0^\infty U\left(c_t,m_t\right)\ e^{-\rho t}dt$$

$$\begin{aligned} s.t. & & \\ \dot{a}_t &=& \rho a_t + y - c_t - i_t m_t \\ \lim_{t\to\infty} a_t e^{-\rho t} &=& 0 \end{aligned}$$

The first order conditions are

$$U_c = \lambda_t \tag{E.1}$$
$$U_m = i_t\lambda_t \tag{E.2}$$
$$\dot{\lambda}_t = 0 \tag{E.3}$$
$$\dot{a}_t = \rho a_t + y - c_t - i_t m_t \tag{E.4}$$

This implies that consumption is constant and that the total inflationary taxes paid satisfy

$$\begin{aligned} U_c &=& \lambda_0 \\ c_0 &=& y + \rho a_0 - \rho\int_0^\infty i_t m_t\ e^{-\rho t}dt \end{aligned}$$

Now assume the expenditure is constant, and lets show that the optimal solution is to have consumption, monetary holdings, and interest rate constant. The Central Bank maximizes the same utility function as consumers, subject to its budget constraint and the solution of the consumer expressed by equations (E.1) to (E.4).

$$\max_{\{i_t\}} \quad E\int_0^\infty U\left(c_t,m_t\right)\ e^{-\rho t}dt$$

$$\begin{aligned} s.t. & & \\ \dot{b}_t &=& \rho b_t + g - i_t m_t \\ & & \text{Equations (E.1) to (E.4)} \\ \lim_{t\to\infty} b_t e^{-\rho t} &=& 0 \end{aligned}$$

The Hamiltonian is

$$H = U\left(c_t,m_t\right) + \mu_t\left(\rho b_t + g - i_t m_t\right)$$

where the FOC are

$$\begin{aligned} U_c \frac{\partial c_t}{\partial i_t} + U_m \frac{\partial m_t}{\partial i_t} &= \mu_t \left(m_t + i_t \frac{\partial m_t}{\partial i_t} \right) \\ \dot{\mu}_t &= 0 \\ \dot{b}_t &= \rho b_t + g - i_t m_t \end{aligned}$$

Using the government budget constraint we can show that the net present value of total expenditures has to be equal to the net present value of total taxes. Thus

$$0 = g + \rho b_0 - \rho \int_0^\infty i_t m_t \, e^{-\rho t} dt$$

Which implies that the total consumption is constant and equal to

$$c_0 = y + \rho a_0 - (g + \rho b_0) \tag{E.5}$$

Note that equation (E.5) implies that consumption is independent of the path of taxes. This is because in this model there is Ricardian Equivalence. If the taxes are reduced today, those will have to be recovered in the future. This implies that $\frac{\partial c_t}{\partial i_t} = 0$.

Substituting the solution of the consumer problem (E.2) in the FOC's of the Central Bank we obtain

$$\frac{i_t}{m_t} \frac{\partial m_t}{\partial i_t} = -\frac{\mu_0}{\mu_0 - \lambda_0}$$

or in other words,

$$m_t \frac{U_{mm}}{U_m} = -\left(1 - \frac{\lambda_0}{\mu_0}\right)$$

Therefore, if the demand for real balances is well behaved (monotonic) the optimal solution for the central bank is to have a constant elasticity of substitution on the money holdings. For example, in a CES this implies a unique money demand for each level of consumption. Given that the level of consumption is unique due to equation (E.5), this implies that the interest rate is constant too. This proves the first part of the results. If expenditure is constant, and the utility function is well behaved (decreasing and monotonic demand functions) the optimal monetqary policy is to set a constant tax.

From the budget constraint of the government debt it is easy to show that the solution implies that

$$i_t m_t = g + \rho b_0$$

The second result comes directly from the concavity of the utility function. The optimal monetary policy will equate the expected utility before and after the resolution of the uncertainty takes place. The reason is that otherwise there will be a jump in the exchange rate that would have been anticipated. In order to avoid it, the expected utility after τ and the marginal utility before τ are the same.

Given some assumptions on the utility function we can obtain that increases in expenditure need increases in interest rate to compensate the extra resources. This implies that after τ if the fiscal reform is successful there is a decrease in the interest rate. Thus, the interest rate before τ has to be a weighted average of the interest rate assuming there is a high or low expenditure. Because in our set up, the high expenditure after τ is the same as that one that exists at time $t = 0$, this implies that the interest rate between 0 and τ is smaller than the one that exists before $t = 0$ smaller than the one that exists after τ if the reform is unsuccessful, but bigger than the one that will prevail if the reform is successful.

E.1. Solution for a CES

In this section we specialize the previous results for the case of a CES. Assume that the instantaneous utility is

$$U\left(c_t, m_t\right) = \left(c_t^{\frac{\phi-1}{\phi}} + m_t^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}$$

for some $\phi > 1$. Then the FOC of the consumers imply that

$$\left(\frac{c_t}{m_t}\right)^{-\frac{1}{\phi}} = \frac{1}{i_t}$$

But consumption is given by equation (E.5)

$$c_t = y + \rho a_0 - (g + \rho b_0)$$

and given the budget constraint of the government we have that

$$\begin{aligned} g + \rho b_0 &= i_t m_t \\ &= m_t^{\frac{\phi-1}{\phi}} c_0^{\frac{1}{\phi}} \end{aligned}$$

which implies a unique solution for m_t

$$m_t = \frac{\left[g + \rho b_0\right]^{\frac{\phi}{\phi-1}}}{\left[y + \rho a_0 - (g + \rho b_0)\right]^{\frac{1}{\phi-1}}} \tag{E.6}$$

$$i_t = \left[\frac{y + \rho a_0 - (g + \rho b_0)}{g + \rho b_0}\right]^{\frac{1}{\phi-1}} \tag{E.7}$$

Note that in equation (E.6) and (E.7) an increase in government expenditure increases the interest rate (decrasing money holdings) if $\phi > 2$ which implies an elasticity of substitution between money and consumption smaller than $1/2$. Under that assumption the results from the paper are all replicated in this set up.