NBER WORKING PAPER SERIES

THE ADVANTAGE OF TRANSPARENT INSTRUMENTS OF MONETARY POLICY

Andrew Atkeson
Patrick Kehoe

Working Paper 8681
http://www.nber.org/papers/w8681

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 2001

Atkeson and Kehoe thank the National Science Foundation for financial support. The views expressed herein are those of the authors and not necessarily those of the National Bureau of Economic Research, the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

The Advantage of Transparent Instruments of Monetary Policy
Andrew Atkeson and Patrick Kehoe
NBER Working Paper No. 8681
December 2001
JEL No. E5, E52, E61, F33, F41

ABSTRACT

Is the exchange rate or the money growth rate the better instrument of monetary policy? A common argument is that the exchange rate has a natural advantage because it is more transparent: it is easier for the public to monitor than the money growth rate. We formalize this argument in a simple model in which the government chooses which instrument it will use to target inflation. We find that when the government cannot commit to its policies, the greater transparency of the exchange rate makes it easier to provide the government with incentives to pursue good policies. Hence, transparency gives the exchange rate a natural advantage over the money growth rate as the monetary policy instrument.

Andrew Atkeson
Department of Economics
UCLA
Bunche Hall 9381
Box 951477
Los Angeles, CA 90095-1477
and NBER
andy@atkeson.net

Patrick Kehoe
Research Department
Federal Reserve Bank of Minneapolis
90 Hennepin Avenue
Minneapolis, MN 55480-0291
and NBER
pkehoe@res.mpls.frb.fed.us

> By the simple virtue of being a price rather than a quantity, the exchange rate provides a much clearer signal to the public of the government's intentions and actual actions than a money supply target. Thus, if the public's inflationary expectations are influenced to a large extent by the ability to easily track and continuously monitor the nominal anchor, the exchange rate has a natural advantage (Calvo and Végh 1999, p. 1589).

> True, the exchange rate has some special properties. In particular, it is easily observable, so the private sector can directly monitor any broken promises by the central bank. But we know of no convincing argument that turns these properties into an explanation for why it would be a more efficient method to achieve credibility to target the exchange rate rather than, say, the money growth rate (Persson and Tabellini 1994, p. 17).

A classic question in international economics is whether the exchange rate or the money growth rate is the better instrument of monetary policy. A common answer, illustrated by the quotation from Calvo and Végh (1999) above, is that the exchange rate has a natural advantage over the money growth rate as an instrument of monetary policy because the exchange rate is easier for the public to observe; hence, it is more transparent. Skeptics of this view agree that the exchange rate is easier for the public to monitor. However, as Persson and Tabellini (1994) point out in the other quotation above, no clear theoretical argument has been made that explains why the transparency of the exchange rate gives it a natural advantage as a monetary policy instrument. We provide such a theoretical argument here.

We formalize this argument by building on the analyses of Canzoneri (1985) and

Zarazaga (1993) and using a simple model of sustainable monetary policy similar to that of Kydland and Prescott (1977) and Barro and Gordon (1983). In our model, each period, the government chooses one of two regimes for monetary policy: one in which the exchange rate is the instrument or one in which the money growth rate is the instrument. Under the *exchange rate regime*, the government picks the rate of depreciation of the exchange rate with some foreign country. By choosing this exchange rate, the government sets the mean inflation rate, and realized domestic inflation varies with shocks to the inflation rate in the foreign country. Under the *money regime*, the government picks a money growth rate, thus setting the mean inflation rate, and realized inflation varies with domestic inflation shocks. Hence, under both regimes, the government sets the mean inflation rate, and realized inflation varies with exogenous shocks.

The key difference between the two regimes is the degree to which the public can observe the monetary policy instrument. The exchange rate regime is *transparent* in that agents can directly observe the exchange rate. The money regime is *opaque* in that agents cannot directly observe money growth rates; rather, agents observe inflation, which serves as a noisy signal of money growth. In all other respects, the two regimes are symmetric. Note that in both regimes, the government is targeting inflation; it is just using different instruments to implement its target.

Exchange rate regimes gain an advantage from their transparency only because this characteristic helps mitigate credibility problems that arise when the government cannot commit to its monetary policies. To emphasize this point, we consider the government's choice of monetary policy instrument when the government can and cannot commit to its policies. First we consider an environment in which the government can commit and, hence, has no credibility problems. We demonstrate that here, even though exchange rates are easier to monitor, exchange rate regimes have no natural advantage: an exchange rate regime is preferred to a money regime if and only if the volatility of foreign inflation shocks is smaller than that of domestic inflation shocks.

We then consider an environment in which the government has credibility problems because it cannot commit to its policies. Here, regardless of the regime for monetary policy, the government has an incentive in the short run to surprise the public with higher than expected inflation in order to decrease unemployment. In equilibrium, this short-run incentive is balanced against the costs that arise when agents adjust their expectations of future policies—and, hence, their future actions—when they perceive that the government has deviated from its policies. We evaluate the two regimes by comparing their best equilibria.

In the environment without commitment, the exchange rate regime has a natural advantage because of its transparency; when the volatilities of foreign and domestic shocks are equal, the exchange rate regime is strictly preferred. Under the exchange rate regime, agents can directly observe any deviation by the government from its expected policy action, and so they can respond precisely whenever a deviation occurs. In contrast, under the money regime, agents cannot directly observe deviations by the government. Instead, agents respond to inflation, which is a noisy signal of the government's choice of money growth rate; thus, their response to a deviation by the government must necessarily be less precise. This inability of agents to precisely tailor their behavior in response to deviations by the government makes deterring the government from surprise inflation harder in the money regime. Hence, the exchange rate regime has a natural advantage.

The result that the exchange rate regime has a natural advantage when the government cannot commit is easiest to show under the assumptions that inflation is the only signal of money growth and that money growth is never observable. We show that we can relax both of these assumptions and still obtain our result.

We also characterize the outcomes that occur in the best exchange rate regime and the best money regime in the two environments, with and without commitment. When the government can commit to its policies, the two regimes' outcomes are symmetric. In both regimes, the government sets its instrument to the same constant level every period. This constant level determines the mean inflation rate, and realized inflation varies randomly around this constant mean.

In contrast, when the government cannot commit to its policies, the optimal outcomes under the two regimes are not symmetric. In the best equilibrium under an exchange rate regime, the government chooses a low rate of depreciation of the exchange rate and achieves a low average inflation rate. It maintains this low depreciation rate, and thus this low average inflation rate, in every period, regardless of the realization of inflation. Observed inflation fluctuates randomly around this constant mean.

The equilibrium outcome under the best money regime looks quite different. Under a money regime, agents cannot tell whether high realized inflation is the result of the government's choice of a high money growth rate or is simply the result of a large domestic inflation shock. Because of this lack of transparency, the optimal outcome necessarily oscillates at random between two extreme phases, with low and high average inflation. This random oscillation along the equilibrium path in the best money regime is analogous to the outcomes obtained by Green and Porter (1984) and Abreu, Pearce, and Stacchetti (1986) in their analyses of equilibrium price wars among oligopolists.

Among the literature on monetary policy, our analysis is most closely related to the seminal contribution of Canzoneri (1985), who was the first to use the logic of Green and Porter (1984) to explain periodic bouts of high inflation. (See also Zarazaga 1995.) There is also some work in this literature on the issue of transparency in monetary policy. Cukierman and Meltzer (1986) and Faust and Svensson (2001, forthcoming) explore linear signaling outcomes in models with unobserved types.

Among the international economics literature, the most closely related work is by Canavan and Tommasi (1997) and Herrendorf (1999), who use two-period signaling games to argue that governments can signal their preferences for low inflation by choosing an exchange rate rather than a money growth rate. For related work in a domestic context, see the analysis of Backus and Driffill (1985).

Here we have used a simple reduced-form model of money. Chang (1998), Albanesi, Chari, and Christiano (2001), and Phelan and Stacchetti (forthcoming) have used the recursive methods of Abreu, Pearce, and Stacchetti (1990) to analyze some general equilibrium macroeconomic models with perfect monitoring.

1. Two Monetary Policy Instruments

Here we present a model of monetary policy in which, each period, the government selects either an exchange rate regime, in which it uses the exchange rate as its policy instrument, or a money regime, in which it uses the money growth rate as its policy instrument.

In the model, time is discrete, and time periods are denoted $t = 0, 1, 2, \ldots$. The economy consists of a government, which dislikes both unemployment and inflation, and a continuum of agents who each choose the rate of change of their individual nominal wages.

The timing of actions within each period is as follows. At the beginning of a period, the government chooses a regime for monetary policy, namely, whether it will use the exchange rate or the money growth rate as its policy instrument in the current period. If it chooses the exchange rate regime, the government opens a trading desk at which it trades domestic and foreign currency. If it chooses the money regime, the government does not open this desk. The presence or absence of the trading desk is thus an observable indicator of the current regime. After the government's choice of regime, agents choose their nominal wages. Finally, depending on the regime, the government chooses either the rate of depreciation of the exchange rate or the money growth rate. The government is free to switch regimes at the beginning of each period.

For convenience, we will describe the economy for a given period t starting at the end of the period and working backward to the beginning. At the end of the period, the government chooses the rate of depreciation of the exchange rate or the money growth rate. The government takes as given the average rate of wage inflation x set by agents earlier in the period. Unemployment is equal to a constant U plus the gap between average wage inflation x and realized inflation π. The government's per period payoff for a given value of x and a realization of π is

$$(1) \qquad r(x,\pi) = -\frac{1}{2}\left[(U + x - \pi)^2 + \pi^2\right].$$

Under the two regimes, realized inflation is a function of monetary policy as follows. Under the exchange rate regime, the government chooses a rate of change in the exchange rate denoted $e_t = s_t - s_{t-1}$, where s_t is the level of the exchange rate. For simplicity, however, we refer to e_t as the *exchange rate*. Inflation in the home country is given by

$$(2) \qquad \pi = e + \pi^*$$

where π^* is inflation in the foreign country, which has a normal distribution with mean 0 and variance $\sigma^2_{\pi^*}$. Thus, by choosing an exchange rate, the government sets the mean inflation rate to be e, while the variance of domestic inflation is determined by shocks in the foreign country which are outside the domestic government's control. Foreign inflation π^* is observed only after the exchange rate is chosen. We let $g(\pi|e)$ denote the density of realized domestic inflation given the choice of exchange rate e.

Under the money regime, the government chooses a money growth rate μ. Given μ, realized inflation π is given by

$$(3) \qquad \pi = \mu + \varepsilon$$

where ε represents domestic inflation shocks which are normally distributed with mean 0 and variance σ^2_π. Thus, by choosing the money growth rate, the government sets the mean inflation rate to be μ, and the variance of domestic inflation is determined by domestic shocks outside of the government's control. We interpret the imperfect connection between money growth and inflation as arising from some combination of the government's imperfect control over actual (as opposed to desired) money growth and a noisy relation between money growth and inflation. We let $f(\pi|\mu)$ denote the density of realized inflation given the choice of money growth rate μ and let σ^2_π denote the variance of domestic inflation shocks.

To model the idea that exchange rates are more easily monitored than money growth rates, we assume that under both regimes, agents can see the exchange rate e and the inflation rate π but not the money growth rate μ. Thus, under an exchange rate regime, agents directly see the actions of the government, while under a money regime they do not. In the money regime, inflation serves as a noisy signal of the government's actions.

Under both regimes, equations (2) and (3) both hold. In the exchange rate regime, e is the choice variable and the money growth rate μ is endogenously determined, while in the money regime, μ is the choice variable and the exchange rate e is endogenously determined. In these regimes, the government's choice of either e or μ determines the mean inflation rate. The only difference in the regimes, besides observability of the instruments, is the variance of the resulting inflation. In this sense, in both regimes, the government is targeting inflation.

The government's expected per period payoff under an exchange rate e is

$$S(x,e) = \int r(x,\pi) g(\pi|e)\, d\pi$$

and under a money growth rate μ is

$$R(x,\mu) = \int r(x,\pi) f(\pi|\mu)\, d\pi.$$

With our functional forms, these become

$$S(x,e) = -\frac{1}{2}\left[(U+x-e)^2 + e^2\right] - \sigma_{\pi^*}^2 \tag{4}$$

$$R(x,\mu) = -\frac{1}{2}\left[(U+x-\mu)^2 + \mu^2\right] - \sigma_{\pi}^2. \tag{5}$$

Notice that the government's payoffs in the two regimes are symmetric with respect to the policy variables e and μ. In particular, the functions S and R differ only with respect to the uncontrollable variances $\sigma_{\pi^*}^2$ and σ_{π}^2, which are constants. We ensure that the government's payoffs are bounded by assuming that the policies e and μ are bounded above and below by some arbitrarily large constants.

In the middle of each period, each agent chooses the change in the agent's own wage rate $z_t = w_t - w_{t-1}$. For simplicity, we refer to z_t as *individual wages.* We let x_t denote the average change in the wage rate in period t, which, again for simplicity, we refer to as *average wages.* An agent's payoff for a given value of z and a realization of π is

$$(6) \qquad r^A(z, \pi) = -\frac{1}{2}\left[(z-\pi)^2 + \pi^2\right].$$

Each agent can choose z differently depending on whether the regime is an exchange rate regime or a money regime. We denote these choices z_e and z_μ. An agent's expected per period payoff under an exchange rate regime with exchange rate e is

$$(7) \qquad S^A(z_e, e) = \int r^A(z_e, \pi) g(\pi|e)\, d\pi = -\frac{1}{2}\left[(z_e - e)^2 + e^2\right] - \sigma_{\pi^*}^2$$

while this agent's expected per period payoff under a money regime with money growth rate μ is

$$(8) \qquad R^A(z_\mu, \mu) = \int r^A(z_\mu, \pi) f(\pi|\mu)\, d\pi = -\frac{1}{2}\left[(z_\mu - \mu)^2 + \mu^2\right] - \sigma_\pi^2.$$

Notice that under either regime, agents aim to choose wages equal to mean inflation, either e or μ, depending on the regime.

Notice also that the objective function of these agents differs from that of the government. In our simple reduced-form model, this difference generates the conflict of interests between the government and the agents that leads to a time consistency problem. We think of this setup as a reduced-form way of capturing the tension that occurs in a general equilibrium model in which the government and the agents have the same objectives but there are distortions in the economy that lead to a time consistency problem. (See Chari, Kehoe, and Prescott 1989 for a more complete discussion.)

The payoff for the government is the discounted value of its expected per period payoffs

$$(9) \qquad (1-\beta)\sum_{t=0}^{\infty}\beta^t[(1-i_t)S(x_{et},e_t)+i_tR(x_{\mu t},\mu_t)]$$

where $1 \geq \beta > 0$ is the discount factor and i_t is a variable that indicates the regime chosen in period t, where $i_t = 0$ for the exchange rate regime and $i_t = 1$ for the money regime. Here x_{et} denotes the average wages chosen in period t if an exchange rate regime is chosen and $x_{\mu t}$ denotes the average wages chosen in period t if a money regime is chosen. The discounted payoffs for the agents are written similarly.

2. With Commitment

We first suppose that the government can commit to a monetary policy once and for all in period 0. We show that when the government can commit to its policies, a classic result holds: an exchange rate regime is preferred to a money regime if and only if the volatility of foreign inflation shocks is lower than that of domestic inflation shocks. Thus, for this environment, the exchange rate has no natural advantage over the money growth rate as an instrument of monetary policy even though the exchange rate is more easily monitored.

In this environment with commitment, at the beginning of period 0, the government chooses the sequence $\{i_t, e_t, \mu_t\}_{t=0}^{\infty}$ indicating the regime it will follow and the exchange rate or money growth rate it will implement under that regime in each period. After this, in each period t, agents choose wages z_{et} or $z_{\mu t}$, depending on the regime. Given (7) and (8), it is clearly optimal for agents to choose $z_{et} = e_t$ and $z_{\mu t} = \mu_t$; hence, in equilibrium average wages satisfy

$$(10) \qquad x_{et} = e_t \text{ and } x_{\mu t} = \mu_t.$$

Here the optimal policies and allocations solve the Ramsey problem of choosing sequences $\{i_t, e_t, \mu_t, x_{et}, x_{\mu t}\}_{t=0}^{\infty}$ to maximize the government's discounted payoff (9) subject to the equilibrium condition on agents' average wages (10). This problem reduces to a sequence of static problems of choosing e and μ to solve $\max_e S(e, e)$ and $\max_\mu R(\mu, \mu)$ and then choosing the regime that leads to the higher payoff. Since the government's payoffs are symmetric with respect to the policy variables, the optimal exchange rate and money growth rate are identical (both 0), and the government simply picks the regime with the lower variance of inflation. We denote this maximum payoff as v^R and refer to it as the *Ramsey payoff.* We summarize this result as

PROPOSITION 1. NO NATURAL ADVANTAGE WITH COMMITMENT. *When the government can commit to its monetary policies, an exchange rate regime is preferred to a money regime if and only if* $\sigma^2_{\pi^*} \leq \sigma^2_{\pi}$.

3. Without Commitment

Now we suppose that the government cannot commit to its policies. Instead, in each period, it chooses a regime and then, after agents set their wages, the government chooses the level of its monetary policy instrument. We show that when the government cannot commit to its policies, the exchange rate regime has a natural advantage over the money regime because of the exchange rate's transparency.

In this environment without commitment, both the government and agents choose their actions as functions of the observed history of aggregate variables: the choice of regime, the exchange rate, and inflation. In period t, this history is given by $h_t = (i_0, e_0, \pi_0; \ldots; i_{t-1}, e_{t-1}, \pi_{t-1})$. A strategy for the government is a sequence of functions $\sigma^G = \{i_t(h_t), e_t(h_t), \mu_t(h_t)\}$ that map histories into the choice of regime i_t and corresponding exchange rates e_t or money growth rates μ_t. A strategy for agents is a sequence of functions $\sigma^A = \{z_{et}(h_t), z_{\mu t}(h_t)\}_{t=0}^{\infty}$ that map histories into actions z_t, where $z_{et}(h_t)$ is only relevant if $i_t(h_t) = 0$ and $z_{\mu t}(h_t)$ is only relevant if $i_t(h_t) = 1$. We also define a sequence of functions $\sigma^X = \{x_{et}(h_t), x_{\mu t}(h_t)\}_{t=0}^{\infty}$ that record the average wages chosen by agents after each history. Let $\sigma = (\sigma^G, \sigma^A, \sigma^X)$ denote the strategies of the government, the strategies of the agents, and the average wages. Notice that in the histories we need not record the history of average wages since a deviation by any one agent cannot affect this average. (See, for example, Chari and Kehoe 1990 for details.)

A *perfect equilibrium* in this environment is a collection of strategies σ such that (i) after every history h_t, the agents' strategy σ^A is optimal given the government's strategy σ^G and the average of other agents' wages σ^X; (ii) after every history h_t, the government's strategy σ^G is optimal given the average of agents' wages σ^X; and (iii) after every history h_t, σ^A and σ^X agree.

Let V denote the set of perfect equilibrium payoffs. In what follows, it will prove convenient to allow public randomization to guarantee that this set V is convex and thus equal to an interval $[v^w, v^b]$, where v^w is the lowest, or the *worst, equilibrium payoff* and v^b is the highest, or the *best, equilibrium payoff.* This public randomization is accomplished by adding to the model a random variable θ_t that the government and the agents observe at the beginning of each period. We modify the histories h_t to include the realizations of this variable from period 0 through period t.

Clearly, given agents' payoffs (7) and (8), after any history h_t, the agents' best response to the government strategy σ^G is to choose wages $z_{et}(h_t) = e_t(h_t)$ or $z_{\mu t}(h_t) = \mu_t(h_t)$, depending on the regime. Thus, in any perfect equilibrium, average wages must satisfy

$$x_{et}(h_t) = e_t(h_t) \text{ and } x_{\mu t}(h_t) = \mu_t(h_t).$$

That is, in equilibrium, wage inflation must equal expected inflation.

We formulate the incentive constraint of the government recursively, by drawing on the work of Abreu, Pearce, and Stacchetti (1986, 1990). Their basic idea as follows. In a repeated game, a strategy is a prescription for current actions and all future actions following every possible history. When evaluating the government's current payoffs and current incentive constraints, however, we need not specify the whole sequence of future actions for the government and agents that follow every possible current action that the government might take. Rather, we need specify only how the government's payoff from the next period on, namely, its *continuation value*, will vary as the government's current action varies. In a perfect equilibrium, these continuation values are also equilibrium payoffs for the repeated game starting from next period on. This simple observation forms the basis for a recursive approach to describing the incentive compatibility constraints for the government and to finding the set of equilibrium payoffs.

Consider a period in which the government has chosen an exchange rate regime and agents have chosen wages x_e. We formulate the current incentive constraint on the government's choice of exchange rate e recursively as follows. What matters to the government in choosing the exchange rate is how its current period payoff and its continuation value vary with its action e. Its current period payoff is $(1-\beta)S(x_e, e)$, where the agents' choice of x_e is taken as given. Agents observe the government's choice of exchange rate e; therefore, their future choices of wages, and thus the future payoffs of the government, can vary directly with e. Rather than describe the entire sequence of future actions taken by agents and the government, contingent on the government's current choice of e, we simply describe the government's continuation value from those actions as some function $w(e)$. Since, in a perfect equilibrium, the strategies that the government and agents follow from next period on must also be perfect equilibrium strategies of the repeated game starting from that period, the government's continuation values $w(e)$ must lie in the set V of perfect equilibrium payoffs for the government.

Given any such continuation value function $w(e) \in V$, we say that an *exchange rate e is incentive compatible in the current period* if

$$(11) \quad (1-\beta)S(x_e, e) + \beta w(e) \geq (1-\beta)S(x_e, e') + \beta w(e')$$

for all e'. This incentive constraint simply requires that the government get a higher discounted sum of current and future payoffs from choosing e than from choosing any other e'. It is a standard result that such a recursive incentive constraint is necessary and sufficient for full incentive compatibility.

Consider next a period in which the government has chosen a money regime and agents have chosen wages x_μ. We can formulate the current incentive constraint on the government's choice of money growth rate μ recursively as well. This constraint is different from the constraint (11) above because here agents do not observe the government's action, the money growth rate μ. Instead, they observe only inflation $\pi = \mu + \varepsilon$, which is a noisy signal of μ. Hence, the government's continuation value cannot vary with μ directly; it can vary only with π. (It should be clear that it is feasible but redundant to also condition this value on the endogenously determined exchange rate $e = \pi - \pi^*$.) Thus, we write the continuation value function for the government when it has chosen a money regime as $w(\pi)$. These continuation values $w(\pi)$ must also lie in the set V of the government's perfect equilibrium payoffs.

Given any such continuation value function $w(\pi) \in V$, we say that a *money growth rate μ is incentive compatible in the current period* if

$$(12) \quad (1-\beta)R(x_\mu, \mu) + \beta \int w(\pi) f(\pi|\mu)\, d\pi \geq (1-\beta)R(x_\mu, \mu') + \beta \int w(\pi) f(\pi|\mu')\, d\pi$$

for any possible μ'. This incentive constraint simply requires that the government get a higher discounted payoff from choosing μ than from choosing any other μ'. Notice that here the government's continuation values vary with μ only to the extent that changes in the money growth rate μ shift the distribution of inflation $f(\pi|\mu)$.

Notice also that the set of equilibrium payoffs V is independent of which regime is used in the current period. This is because we have assumed that the government can switch regimes at the beginning of any period; hence, the game from the next period on is independent of the regime used in the current period. Note as well that the set V in which the government's continuation values must lie is unknown. One can solve for this set recursively. To show that an exchange rate regime has a natural advantage, however, we do not need to solve for V. Instead, we treat this set $V = [v^w, v^b]$ as a parameter and show that an exchange rate regime has a natural advantage from transparency for any nondegenerate set V of equilibrium payoffs.

We now compare the exchange rate regime to the money regime. We first compute the highest payoff that can be achieved if the exchange rate regime is used in the current period. This payoff is the highest perfect equilibrium payoff for the government given that it uses an exchange rate regime in the current period and is free to switch regimes in each future period. We then compute the corresponding highest payoff for the government given that it uses a money regime in the current period and is free to switch regimes in each future period. We compare these payoffs to characterize when the exchange rate regime is preferred to the money regime.

Given a set $V = [v^w, v^b]$ of perfect equilibrium payoffs, the *best payoff for the government under an exchange rate regime* is the solution to the following problem: choose current actions x_e and e and a continuation value function $w(e) \in V$ to maximize

$$(1-\beta)S(x_e, e) + \beta w(e)$$

subject to the incentive constraints $x_e = e$ and (11). Notice that the left side of the incentive constraint (11) is the payoff to be maximized, so setting $w(e) = v^b$ is clearly optimal. To relax the incentive constraint (11) as much as possible, it is optimal to minimize the right side of this constraint by setting $w(e') = v^w$. Using this argument and substituting out $x_e = e$, we can write the problem as

$$(13) \quad \max_{e} \; (1-\beta)S(e, e) + \beta v^b$$

subject to

$$(14) \quad (1-\beta)\left[S(e, e') - S(e, e)\right] \leq \beta(v^b - v^w)$$

for all e'.

Given a set $V = [v^w, v^b]$ of perfect equilibrium payoffs, the *best payoff for the government under a money regime* is the solution to the following problem: choose current actions x_μ and μ and a continuation value function $w(\pi) \in V$ to maximize

$$(15) \quad (1-\beta)R(x_\mu, \mu) + \beta \int w(\pi) f(\pi|\mu)\, d\pi$$

subject to the incentive constraints $x_\mu = \mu$ and (12). Substituting $x_\mu = \mu$ and rearranging the incentive constraint, we can write this problem as

$$(16) \quad \max_{\mu}(1-\beta)R(\mu,\mu) + \beta \int w(\pi) f(\pi|\mu)\, d\pi$$

subject to

$$(17) \quad (1-\beta)\left[R(\mu,\mu') - R(\mu,\mu)\right] \leq \beta \int w(\pi)\left[f(\pi|\mu) - f(\pi|\mu')\right] d\pi.$$

We use the following result to compare the two regimes in Proposition 2.

LEMMA 1. V NONDEGENERATE. If the variance of foreign inflation shocks is less than or equal to that of domestic inflation shocks, then the set $V = [v^w, v^b]$ has $v^b > v^w$ and v^b is greater than the payoff from the static Nash outcome repeated in every period.

The proof is in Appendix A. We then have

PROPOSITION 2. A NATURAL ADVANTAGE WITHOUT COMMITMENT. *When the government cannot commit to its monetary policies, an exchange rate regime is preferred to a money regime even if the variances of foreign and domestic inflation shocks are the same.*

Proof. When $\sigma^2_{\pi^*} = \sigma^2_{\pi}$, the current period payoffs are the same, in that $S(\mu,\mu') = R(\mu,\mu')$. Clearly, then, the exchange rate regime is weakly preferred to the money regime. Now we show that the exchange rate regime is strictly preferred. The continuation value for the government is lower under the money regime than under the exchange rate regime, since $w(\pi) \leq v^b$ implies that

$$(18) \quad \int w(\pi) f(\pi|\mu)\, d\pi \leq v^b.$$

Suppose first that $w(\pi)$ is such that (18) is an equality. Then $w(\pi) = v^b$ (almost everywhere), the continuation payoff of the government is independent of the government's current action, and the only incentive compatible actions under a money regime are the static Nash actions. From Lemma 1, we know that the government can achieve a payoff that is strictly higher than that of static Nash with an exchange rate regime. Hence, if (18) is an equality, then an exchange rate regime is strictly preferred to a money regime.

Next, suppose that $w(\pi)$ is such that (18) is a strict inequality. Note that the incentive constraint is tighter under a money regime than under an exchange rate regime, since

$$\int w(\pi)\left[f(\pi|\mu) - f(\pi|\mu')\right] d\pi < v^b - v^w.$$

As a result, here also the best payoff the government can achieve under an exchange rate regime is strictly higher than the best payoff it can achieve under a money regime. Q.E.D.

We illustrate the results of Propositions 1 and 2 in Figure 1. There we show how the optimal regime varies with the relative volatility of the domestic and foreign inflation shocks. When the government can commit to its policies, the exchange rate regime is preferred if and only if the variance of foreign inflation shocks, $\sigma^2_{\pi^*}$, is lower than the variance of domestic shocks, σ^2_{π}. This is the region labeled A in the figure. When the government cannot commit to its policies, the exchange rate regime is preferred even if the variances of the shocks are equal. Thus, the region for which the exchange rate regime is preferred expands to include the region labeled B as well as A.

4. Relaxing Some Assumptions

In modeling the idea that exchange rates are easier to monitor than money growth rates, we have made the simple but extreme assumptions that inflation is the only signal of the money growth rate and that money growth rates are never observed. Here we show that we can relax these assumptions and still derive our main result that when the government cannot commit to its policies, exchange rate regimes have a natural advantage because of the transparency of their monetary policy instrument.

Suppose first that, in addition to inflation, agents observe another noisy signal of money growth denoted by η. In an environment in which the government has imperfect control over money growth, we might interpret this signal η as the realized money growth rate. Let $f(\pi, \eta|\mu)$ be the density of inflation π and the noisy signal η given the money growth rate μ. Here the government's continuation value can vary only with π and η and can be written as $w(\pi, \eta)$. The government's incentive constraint now becomes

$$(1-\beta)R(x_\mu, \mu) + \beta \int \int w(\pi, \eta) f(\pi, \eta|\mu)\, d\pi d\eta \geq$$

$$(1-\beta)R(x_\mu, \mu') + \beta \int \int w(\pi) f(\pi, \eta|\mu')\, d\pi d\eta$$

for any possible μ'. Proving the analogue of Proposition 2 in this environment is straightforward.

Suppose next that while inflation is the only signal of the money growth rate that agents can observe in the current period, the money growth rate is perfectly observable with a lag, which for simplicity we take to be one period. Specifically, assume that the money growth rate μ_{t-1} is observed after agents set their wages in period t. Here, the history on which agents condition their actions is

$$h_t = (i_0, e_0, \pi_0; i_1, e_1, \pi_1, \mu_0; \ldots; i_{t-1}, e_{t-1}, \pi_{t-1}, \mu_{t-2})$$

and the history for the government is

$$H_t = (i_0, e_0, \pi_0, \mu_0; i_1, e_1, \pi_1, \mu_0; \ldots; i_{t-1}, e_{t-1}, \pi_{t-1}, \mu_{t-1}).$$

The strategies for the agents and the government are defined as functions of these histories in the standard way.

The intuition for why exchange rates have a natural advantage in this environment is clear. Under the money regime, any deviation in period t is not directly observed in that period. Thus, in period $t+1$, agents can react only to a noisy signal of that action. Of course, by period $t+2$, agents have observed the government's action in period t, and agents at that time can precisely react to any deviation in period t. This lag in the ability to precisely react leads to a tighter incentive constraint under the money regime and thus gives the exchange rate regime its advantage.

The technical difficulty in proving this result is that this economy does not lend itself to recursive analysis as easily as our original economy does. In our original economy, the governments and the agents have the same information at the beginning of each period; hence, the economy starting from any period t looks identical to the economy starting in period 0. In this new economy with information lags, however, the government and the agents do not have the same information at the beginning of each period. In particular, in the continuation of this economy starting from period t, the government has private information, namely, its actions in period $t-1$, that the agents do not have. In the economy starting in period 0, however, the government has no private information. Hence, when there are lags, the economy starting in period t does not look identical to the economy starting in period 0, and the recursive analysis of equilibrium payoffs that we used in proving Proposition 2 does not apply.

In Appendix B, we prove the following proposition without using recursive methods. Let $V = \left[v^w, v^b\right]$ be the set of equilibrium payoffs for the government of the economy starting in period 0. Assume that

$$(19) \quad v^b < v^R$$

so that the best equilibrium payoff to the economy with no commitment is strictly less than the Ramsey payoff.

Proposition 3. *Under* (19), *if the variance of the domestic and foreign inflation shocks are equal, then an exchange rate regime is strictly preferred to a money regime.*

The idea of the proof of this proposition is to show that the tighter incentive constraint under the money regime makes the best equilibrium under a money regime have a lower value. The role of our assumption in (19) is to ensure that the incentive constraint in both regimes strictly binds. This assumption will hold whenever the government discounts the future by a sufficient amount. If the government discounts the future sufficiently little, then the incentive constraint in both regimes is slack, both regimes attain the Ramsey payoff, and the best money regime is tied with the best exchange rate regime.

5. The Best Equilibria Without Commitment

So far, we have compared the best payoffs the government can achieve under exchange rate and money regimes. Here we characterize the equilibrium outcomes that produce these best payoffs. We begin by describing these optimal outcomes under the two regimes. Then we present a formal characterization of the outcomes.

A. Optimal Outcomes

When the exchange rate regime is the preferred regime, the equilibrium outcome is simple. In each period, the government chooses an exchange rate regime and sets the exchange rate equal to the best exchange rate policy e^b. If the government deviates from this policy, then the government and agents revert to the actions that implement the worst equilibrium payoff v^w. These actions may correspond to either an exchange rate regime or a money regime, depending on the variances of the shocks. In equilibrium, of course, there are no deviations; hence, the exchange rate is set to e^b in every period, and inflation randomly fluctuates around this mean level e^b. This result follows immediately from the solution to (13).

The equilibrium outcome under the best money regime looks quite different. Under this regime, the government starts by setting the money growth rate equal to some low growth rate μ^b and continues to do that as long as low inflation is realized. Specifically, the government sets the money growth rate to μ^b as long as the domestic inflation shock ε is small enough so that $\mu^b + \varepsilon \leq \pi^b$, where π^b is the relatively low cutoff level of inflation used in the best money regime. In equilibrium, eventually a large enough domestic inflation shock must occur so that the realized inflation exceeds π^b. After such a shock, the government and agents revert to the actions that implement the worst equilibrium payoff v^w. Thus, under the money regime, the actions that implement the worst equilibrium payoffs are eventually observed. We prove this result later in Proposition 4.

The worst equilibrium payoff v^w can occur under either an exchange rate regime or a money regime, depending on the variances of domestic and foreign inflation shocks. This worst equilibrium payoff is the larger of two payoffs: the worst payoff under an exchange rate regime v_e^w and the worst payoff under a money regime v_μ^w. That is, $v^w = \max\{v_e^w, v_\mu^w\}$. The worst equilibrium payoff is the larger of these two payoffs because, at the beginning of each period, the government can choose which regime it prefers.

It turns out that when the variances are such that a money regime implements the best payoff, that regime also implements the worst payoff. In this worst regime, the government starts by setting the money growth rate equal to some high growth rate μ^w and continues to do that as long as the domestic inflation shock ε is small enough so that $\mu^w + \varepsilon \leq \pi^w$, where π^w is the relatively high cutoff level of inflation used in the worst money regime. When a sufficiently large domestic inflation shock occurs so that realized inflation exceeds π^w, the government and agents revert to the actions that implement the best equilibrium payoff. In this sense, in the worst money regime, extremely high inflation must be realized before average inflation can fall. We prove this result later in Proposition 5.

In Figure 2 we illustrate a typical path of money growth and inflation outcomes observed in the best equilibrium over time when the money regime is used in both the best and worst equilibria. In period 0, agents choose low wages $x_\mu = \mu^b$, the government chooses a low money growth rate μ^b, and realized inflation is this low money growth rate plus the domestic inflation shock $\pi_0 = \mu^b + \varepsilon_0$. In the figure, we assume that realized inflation π_0 is less than the critical value π^b. Hence, in period 1, agents again choose wages $x_\mu = \mu^b$, the government again chooses a low money growth rate μ^b, and realized inflation is $\pi_1 = \mu^b + \varepsilon_1$. The outcome continues in this fashion, with agents choosing low wages and the government choosing a low money growth rate, until the domestic inflation shock is large enough so that realized inflation exceeds the critical value π^b. In the figure, this occurs in period 4. In period 5, agents choose high wages $x_\mu = \mu^w$, the government chooses high money growth rate μ^w, and realized inflation is $\pi_5 = \mu^w + \varepsilon_5$. This pattern continues until the domestic inflation shock is high enough so that realized inflation exceeds the high critical value π^w. In the figure, this occurs in period 7. In period 8, the outcome reverts back to the pattern of agents choosing low wages and the government choosing a low money growth rate. After that, the outcome cycles stochastically between these two phases, depending on the realizations of the domestic inflation shocks.

We use an argument similar to that in Proposition 2 to characterize the regions of the parameter space in which the exchange rate regime and the money regime are used in the best and worst equilibrium outcomes. When the variances of domestic and foreign inflation shocks are the same, the worst payoff under an exchange rate regime is lower than that under a money regime; that is, $v_e^w < v_\mu^w$. This is because here the current period payoff functions R and S are the same and the incentive constraint is looser under an exchange rate regime than under a money regime. Hence, when these variances are the same, the worst equilibrium payoff $v^w = \max\{v_e^w, v_\mu^w\}$ is equal to that under a money regime. Clearly, increasing the variance of foreign inflation shocks above that of the domestic shocks reduces v_e^w and leaves v_μ^w unchanged. Hence, $v^w = v_\mu^w$ when the variance of foreign inflation shocks exceeds that of domestic inflation shocks.

In Figure 3, we combine this result with that in Proposition 2 to characterize which regimes are used in the best and worst outcomes in each part of the parameter space. If the variance of foreign shocks is sufficiently high relative to that of domestic shocks, as in region C of the figure, then the government follows a money regime in both the best and the worst equilibria. If the variance of foreign shocks is sufficiently low relative to that of domestic shocks, as in region E, then the government follows an exchange rate regime in both the best and the worst equilibria. When the variances of the two inflation shocks are similar, as in region D, then the government uses an exchange rate regime in the best equilibrium and a money regime in the worst equilibrium. In regions D and E, the observed best outcome is an exchange rate regime with a constant e in every period. The observed best outcome in region C stochastically cycles between the best money regime and the worst money regime as discussed above.

B. Formal Description

We begin our formal characterization of these outcomes with the best money regime. The recursive representation of this regime is the solution to problem (15). To solve this problem, we first replace the incentive constraint (12) with the first-order condition associated with maximizing the left side of this incentive constraint with respect to μ. The resulting constraint is

$$(20) \quad (1-\beta)R_{\mu}(x_{\mu},\mu)+\beta\int w(\pi)f_{\mu}\left(\pi|\mu\right)\,d\pi=0$$

where $R_{\mu}(x,\mu) = \partial R(x,\mu)/\partial\mu$ and $f_{\mu}(\pi|\mu) = \partial f(\pi|\mu)/\partial\mu$. This first-order condition is necessary and sufficient to ensure that (12) holds when the function defined by the left side of (12) is concave in μ. In Proposition 4 below, we simply assume that this approach is valid and characterize the resulting $w(\pi)$. In Lemma 2, we show that, given the resulting form of $w(\pi)$, the left side of (12) is concave in μ when the variance of domestic inflation shocks is sufficiently large.

Under the assumption that our first-order condition approach is valid, in problem (15) we can replace the government's incentive constraint (12) with constraint (20). In any solution to this problem, the continuation values necessarily have a *bang-bang* form:

$$(21) \quad w^{b}(\pi)=\left\{\begin{array}{c} v^{b}\ \mathit{if}\ \ \pi\leq\pi^{b} \\ v^{w}\ \mathit{if}\ \ \pi>\pi^{b} \end{array}\right\}.$$

That is, there is a cutoff inflation level π^b such that the optimal continuation value function $w^b(\pi)$ is set to the best payoff v^b if the realized inflation rate is less than π^b and to the worst payoff v^w if the realized inflation rate is greater than π^b.

Part of the rationale for the optimal continuation value taking the form (21) is intuitive. Since higher money growth rates make higher inflation more likely, in order to discourage the government from choosing a high money growth rate, the continuation value function must specify a low continuation payoff for the government when realized inflation is high. Slightly less intuitive is that the best continuation value function must assign only the best and worst possible equilibrium payoffs. Mechanically, this occurs because both the payoffs and the incentive constraint are linear in the continuation values. We demonstrate this formally in Proposition 4.

PROPOSITION 4. *Under the assumption that the first-order condition approach is valid, the optimal continuation value function has the form of* (21).

Proof. With λ as the multiplier on the government's incentive constraint (20), the term in the Lagrangian that involves $w(\pi)$ is

$$\beta \int w(\pi) \left[1 + \lambda \frac{f_\mu(\pi|\mu)}{f(\pi|\mu)}\right] f(\pi|\mu)\, d\pi.$$

Notice that this term is linear in each value of $w(\pi)$, so that it is optimal to set

$$w^b(\pi) = \left\{ \begin{array}{l} v^b \; if \; \left[1 + \lambda \frac{f_\mu(\pi|\mu)}{f(\pi|\mu)}\right] > 0 \\ v^w \; if \; \left[1 + \lambda \frac{f_\mu(\pi|\mu)}{f(\pi|\mu)}\right] < 0 \end{array} \right\}.$$

These first-order conditions imply that the optimal continuation values are always extreme, that is, either v^b or v^w. The only issue is, for what values of π are the payoffs v^b and v^w assigned? To determine these values, we start by observing that with our assumption of normality, $f_\mu(\pi|\mu) = f(\pi|\mu)(\pi - \mu)/\sigma_\pi$, so that our densities satisfy the monotone likelihood ratio property; that is, the ratio

$$\frac{f_\mu(\pi|\mu)}{f(\pi|\mu)} = (\pi - \mu)/\sigma_\pi$$

is increasing in π. Thus, $w^b(\pi)$ is increasing in π if $\lambda > 0$ and decreasing in π if $\lambda < 0$.

We will show that $\lambda < 0$ and $w^b(\pi)$ is decreasing in π as follows. First, note that at the optimum, $R_\mu(x^b, \mu^b) \geq 0$. This follows since the optimum must weakly improve on the static Nash payoff and thus must have a money growth rate less than or equal to the static Nash level. That is, $x^b = \mu^b \leq U$. Since $R_\mu(x, \mu) = U + x - 2\mu$, $R_\mu(x^b, \mu^b) \geq 0$. Next, since $R_\mu(x^b, \mu^b) \geq 0$, the incentive constraint (20) implies that

$$\int w^b(\pi) f_\mu(\pi|\mu)\, d\pi \leq 0. \tag{22}$$

Since inflation is normally distributed with mean μ, increasing μ increases the distribution of inflation in the sense of first-order stochastic dominance. Thus, increasing μ increases $\int w^b(\pi) f(\pi|\mu)\, d\pi$ when $w^b(\pi)$ is increasing and decreases this integral when $w^b(\pi)$ is decreasing. Thus, to satisfy (22), $w^b(\pi)$ must be decreasing. Q.E.D.

In Lemma 2, proved in Appendix A, we justify our use of the first-order approach. We let ϕ and Φ denote the density and cumulative distribution functions of a standard normal, respectively.

Lemma 2. First-order approach valid. Given that $w^b(\pi)$ has the bang-bang form (21) and is decreasing, if $\sigma_\pi^2 > \frac{\beta}{1-\beta}(v^b - v^w)\phi(1)/2$, then the incentive constraint (12) is satisfied if and only if the first-order condition (20) holds.

To complete our characterization of the outcome under the best money regime, we must also characterize the outcome under the worst money regime. In the worst money regime, continuation values $w^w(\pi)$ are assigned to give the government the incentive to choose a higher money growth rate than it would choose in the static Nash outcome. This entails giving the government high continuation values when high inflation is realized and low continuation values when low inflation is realized. Thus, when the equilibrium reverts to the worst money regime, the government chooses a high money growth rate and keeps choosing this high rate until a sufficiently high level of inflation is realized. This result is proved in the next proposition.

As before, under the assumption that the first-order condition approach is valid, we can write the problem of finding the worst payoff under a money regime as

$$\min_{\mu,x,w(\pi)} (1-\beta)R(x,\mu) + \beta \int w(\pi) f(\pi,\mu)\, d\pi \tag{23}$$

subject to the constraints $x = \mu$ and (20).

PROPOSITION 5. *Under the assumption that the first-order approach is valid, the optimal continuation value function for the worst equilibrium in the money regime has the form*

$$w^w(\pi) = \left\{ \begin{array}{c} v^w \; if \; \pi \leq \pi^w \\ \\ v^b \; if \; \pi > \pi^w \end{array} \right\} \tag{24}$$

for some cutoff inflation rate π^w.

Proof. The proof is similar to that of Proposition 4. Specifically, the first-order condition of the problem (23) with respect to $w(\pi)$ implies that $w^w(\pi)$ has a bang-bang form around some cutoff π^w. To see that $w^w(\pi)$ must be increasing, note that at the optimum, $R_\mu(x^w, \mu^w) \leq 0$, so that the current period payoff for the government is decreased when the government deviates to a higher money growth rate. Accordingly, the incentive constraint (20) implies that

$$\int w^w(\pi) f_\mu(\pi|\mu)\, d\pi \geq 0$$

which gives the result that $w^w(\pi)$ is increasing. Q.E.D.

6. Conclusion

Here we have considered the advantage of transparency in a model in which the exchange rate is observable and the money growth rate is only observable with noise, at least contemporaneously. In the best equilibrium of an exchange rate regime, the rate of depreciation of the exchange rate is constant. This occurs because our simple model abstracts from all shocks that would lead the optimal mean inflation rate to vary over time. As such, our model does not provide a rationale for fixing exchange rates; rather, it provides a rationale for using the exchange rate rather than the money growth rate as the instrument of monetary policy.

We have shown here that a certain price, the exchange rate, has an advantage over a certain quantity, the money growth rate, as a monetary policy instrument. This basic idea, that prices have an advantage over quantities as policy instruments, might also be applied to a comparison of interest rates and any other quantity instrument that is more difficult to monitor.

Appendix A: Proofs of Lemmas 1 and 2

Proof of Lemma 1.

Here we show that the set of equilibrium payoffs contains a point with strictly higher value than that of the static Nash outcome. In any period, in an exchange rate regime, the government's static best response to average wages x_e is to choose e to maximize $S(x_e, e)$. This best response is given by $B(x_e) = (U + x_e)/2$. Likewise, under a money regime, the government's static best response to wages x_μ is $B(x_\mu) = (U + x_\mu)/2$, and the static Nash outcomes are $e = x_e = U$ and $\mu = x_\mu = U$. Repeating the static Nash outcomes in every period, regardless of the history, is a perfect equilibrium that leads to a payoff for the government of

$$v^N = \max[S(U,U), R(U,U)].$$

Thus, $v^N \in V$.

We now construct a higher equilibrium payoff using trigger strategies. Let $\hat{e}$ be some exchange rate that is strictly lower than the static Nash exchange rate U, and let $\hat{v} = S(\hat{e}, \hat{e})$ be the government's payoff when $x_e = \hat{e}$ and this $\hat{e}$ is played in every period. The trigger strategies specify the following. Begin with the government choosing an exchange rate regime, agents setting $x_e = \hat{e}$, and the government choosing $\hat{e}$. Continue with these actions in every period until the government deviates in the choice of regime or in choosing $e \neq \hat{e}$. After any such deviation, both the government and the agents revert to the static Nash outcome forever. These strategies constitute an equilibrium if the government has no incentive to deviate in that

$$(1-\beta)\,[S(\hat{e}, B(\hat{e})) - S(\hat{e}, \hat{e})] \leq \beta(\hat{v} - v^N) \tag{25}$$

holds. It is easy to show with our functional forms that (25) is satisfied for $\hat{e} = U - \varepsilon$ for some sufficiently small ε. Thus, $\hat{v}$ and v^N are equilibrium payoffs that satisfy $v^b \geq \hat{v} > v^N \geq v^w$. Q.E.D.

Proof of Lemma 2.

Here we show that the solution to the problem with incentive constraint (12) is satisfied if and only if the first-order condition (20) holds when $\sigma_\pi^2 > \frac{\beta}{1-\beta}(\bar{w} - \underline{w})\frac{\phi(1)}{2}$.

Using (21), we can write the constraint (12) as

$$(26) \quad \mu \in \arg\max_{\mu}(1-\beta)R(x,\mu) + \beta\left\{\bar{w}\Phi\left(\frac{\pi^h - \mu}{\sigma_\pi}\right) + \underline{w}\left[1 - \Phi\left(\frac{\pi^h - \mu}{\sigma_\pi}\right)\right]\right\}.$$

Since $F(\pi^h, \mu) = \Phi((\pi^h - \mu)/\sigma_\pi)$, we can write the first- and second-order conditions of the maximization problem (26) as

$$(27) \quad (1-\beta)R_\mu(x^h, \mu) - \beta\left(\frac{\bar{w} - \underline{w}}{\sigma_\pi}\right)\phi\left(\frac{\pi^h - \mu}{\sigma_\pi}\right) = 0$$

and for all μ

$$(28) \quad (1-\beta)R_{\mu\mu}(x^h, \mu) - \beta(\bar{w} - \underline{w})\left(\frac{\pi^h - \mu}{\sigma_\pi^2}\right)\phi\left(\frac{\pi^h - \mu}{\sigma_\pi}\right) \leq 0$$

which can be written as

$$(29) \quad -2(1-\beta) - \beta\frac{(\bar{w} - \underline{w})}{\sigma_\pi^2}\phi(z)z \leq 0$$

for all $z \in [-\infty, \infty]$. The expression $\phi(z)z$ in (29) is minimized at $z = -1$. Since $\phi(-1) = \phi(1)$, the inequality $\sigma_\pi^2 > \frac{\beta}{1-\beta}(\bar{w} - \underline{w})\frac{\phi(1)}{2}$ guarantees that the second-order condition holds globally, and thus, (20) is both necessary and sufficient for (12). Q.E.D.

Appendix B: Proof of Proposition 3

We prove Proposition 3 in three steps.

The first step is to show that in any period, the government's continuation payoff can never be lower than the worst equilibrium payoff v^w starting from period 0. This follows because each period, the government always has the option of not conditioning its action on what it has done in the past. Therefore, the government cannot be held to a payoff lower than the worst it can attain in period 0 when there is no past on which to condition its actions. Thus, the continuation value for the government following a money regime, when the government has private information, can never be worse than that following an exchange rate regime, when it does not.

The second step in the proof of Proposition 3 is

LEMMA 3. If the variances of foreign and domestic inflation shocks are the same, then for any equilibrium strategy profile σ that starts in the first period with a money regime, there exists an alternative equilibrium strategy profile $\tilde{\sigma}$ that starts in the first period with an exchange rate regime and implements the same payoff for the government.

Proof. The current period payoff functions $R(x_\mu, \mu)$ and $S(x_e, e)$ are identical, and the distributions of realized inflation π are also identical, whenever $e = \mu$ and $x_e = x_\mu$. The alternative equilibrium strategy profile $\tilde{\sigma}$ is constructed by in the first period having the government choose an exchange rate regime, agents set wages $x_e = x_\mu$, and the government choose $e_0 = \mu_0$ and, in subsequent periods having the government's and the agents' actions vary with realized inflation π_0 in the same way as these actions varied with π_0 under the original strategy profile σ.

After a deviation in which the government chooses a money regime in the first period or in which it chooses an exchange rate regime but sets $e_0 \neq \mu_0$, let the continuation of $\tilde{\sigma}$ implement the worst possible equilibrium payoff; namely, let it be a strategy profile that delivers the payoff v^w for the government. From the first step above, we know that this punishment for deviations is (weakly) more severe than any punishment for deviations specified in σ, and hence, $\tilde{\sigma}$ must also be incentive compatible. The establishes the second step. Q.E.D.

The third step completes the proof of Proposition 3 as follows. Let σ be an equilibrium strategy profile in which the government chooses a money regime in period 0. By Lemma 3, we know that we can construct an alternative equilibrium in which the government chooses an exchange rate regime and attains the same value. We now show that by choosing an exchange rate regime, we can relax the period 0 incentive constraint and thus attain a strictly higher payoff for the government. This will complete the proof since, under the assumption that attaining the Ramsey payoff is infeasible, the incentive constraint strictly binds in any money regime.

Given any value of realized inflation in period 0 and any money growth rate in period 0, the government's continuation value from period 1 on lies between v^w and v^b. Since agents' actions in period 1 cannot be contingent on μ_0, but rather must depend on realized inflation, the cost to the government of a deviation from period 0 in terms of the change in the expected continuation value must be strictly less than $v^b - v^w$. In contrast, under an exchange rate regime, there is an equilibrium in which the cost to the government after a deviation is equal to $v^b - v^w$. Hence, the incentive constraint is strictly looser when an exchange rate regime is chosen in period 0 than when a money regime is chosen. Thus, the best equilibrium payoff is strictly higher under an exchange rate regime than under a money regime. Q.E.D.

References

Abreu, Dilip. 1986. Extremal Equilibria of Oligopolistic Supergames. *Journal of Economic Theory* 39(1, June): 191–225.

Abreu, Dilip; Pearce, David; and Stacchetti, Ennio. 1986. Optimal Cartel Equilibria with Imperfect Monitoring. *Journal of Economic Theory* 39(1): 251–69.

__________. 1990. Toward a Theory of Discounted Repeated Games with Imperfect Monitoring. *Econometrica* 58(5, September): 1041–63.

Albanesi, Stefania; Chari, V.V.; and Christiano, Lawrence J. 2001. How Severe is the Time Consistency Problem in Monetary Policy? Working Paper 8139. National Bureau of Economic Research.

Backus, David, and Driffill, John. 1985. Inflation and Reputation. *American Economic Review* 75(3, June): 530–38.

Barro, Robert J., and Gordon, David B. 1983. Rules, Discretion and Reputation in a Model of Monetary Policy. *Journal of Monetary Economics* 12(1, July): 101–21.

Calvo, Guillermo A., and Végh, Carlos. 1999. Inflation Stabilization and BOP Crises in Developing Countries. In *Handbook of Macroeconomics,* Vol. 1C, ed. John B. Taylor and Michael Woodford, pp. 1531–1614. Amsterdam: North-Holland.

Canavan, Chris, and Tommasi, Mariano. 1997. On the Credibility of Alternative Exchange Rate Regimes. *Journal of Development Economics* 54(1, October): 101–22.

Canzoneri, Matthew B. 1985. Monetary Policy Games and the Role of Private Information. *American Economic Review* 75(5, December): 1056–70.

Chang, Roberto. 1998. Credible Monetary Policy in an Infinite Horizon Model: Recursive

Approaches. *Journal of Economic Theory* 81(2, August): 431–61.

Chari, V. V., and Kehoe, Patrick J. 1990. Sustainable Plans. *Journal of Political Economy* 98(4, August): 783–802.

Chari, V. V.; Kehoe, Patrick J.; and Prescott, Edward C. 1989. Time Consistency and Policy. In *Modern Business Cycle Theory*, ed. Robert J. Barro, pp. 265–305. Cambridge, Mass.: Harvard University Press.

Cukierman, Alex, and Meltzer, Allan H. 1986. A Theory of Ambiguity, Credibility, and Inflation under Discretion and Asymmetric Information. *Econometrica* 54(5, September): 1099–128.

Faust, Jon, and Svensson, Lars E. O. 2001. Transparency and Credibility: Monetary Policy with Unobservable Goals. *International Economic Review* 42(2, May): 369–97.

__________. 1999. Forthcoming. The Equilibrium Degree of Transparency and Control in Monetary Policy. *Journal of Money, Credit, and Banking.*

Green, Edward J., and Porter, Robert H. 1984. Noncooperative Collusion under Imperfect Price Information. *Econometrica* 52(1, January): 87–100.

Herrendorf, Berthold. 1999. Transparency, Reputation, and Credibility under Floating and Pegged Exchange Rates. *Journal of International Economics* 49(1, October): 31–50.

Kydland, Finn, and Prescott, Edward C. 1977. Rules Rather than Discretion: The Inconsistency of Optimal Plans. *Journal of Political Economy* 85(3, June): 473–91.

Persson, Torsten, and Tabellini, Guido. 1994. Introduction. In *Monetary and Fiscal Policy,* Vol. 1, *Credibility*, ed. Torsten Persson and Guido Tabellini, pp. 1–31. Cambridge, Mass.: MIT Press.

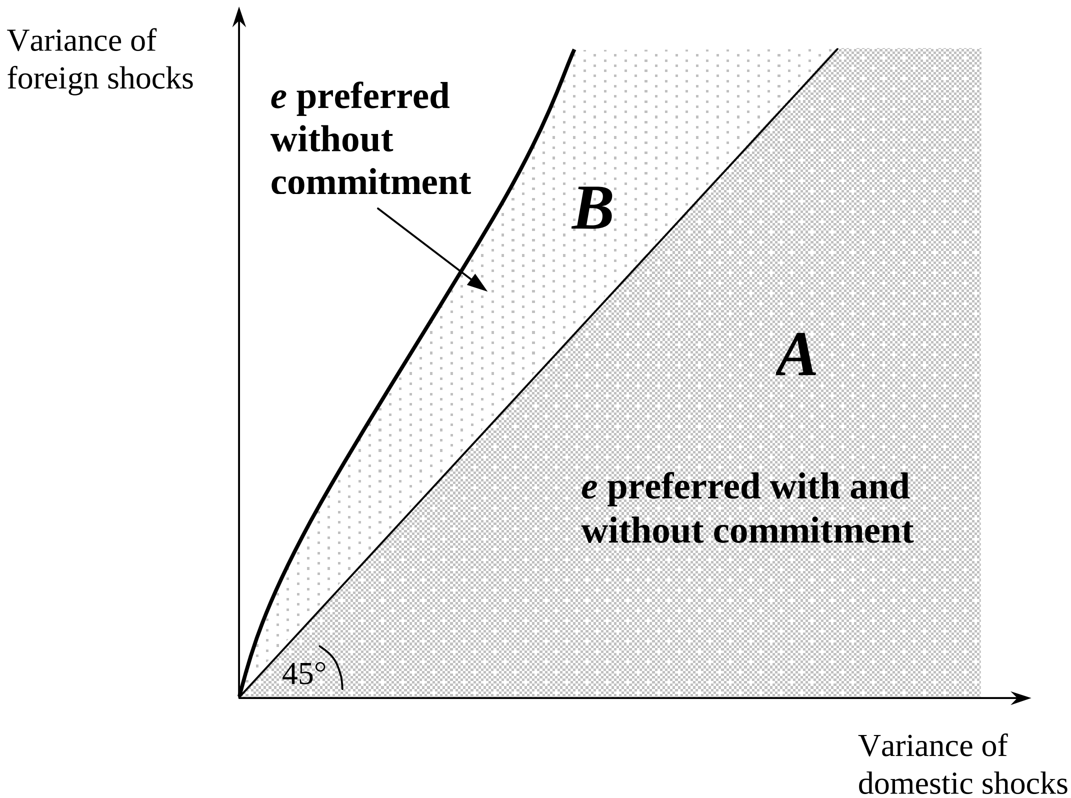
Phelan, Christopher, and Stacchetti, Ennio. Forthcoming. Sequential Equilibria in a Ramsey

Tax Model. *Econometrica*.

Zarazaga, Carlos E.J.M. 1995. Hyperinflations and Moral Hazard in the Appropriation of Seigniorage: An Empirical Implementation with a Calibration Approach. Research Department Working Paper 95-17. Federal Reserve Bank of Philadelphia.

Figure 1

Parameter regions for which an exchange rate regime is preferred to a money regime with and without commitment*



*With commitment, exchange rate regimes are preferred in region *A*, where the variance of domestic inflation shocks is greater than the variance of foreign inflation shocks. With no commitment, exchange rate regimes have an additional advantage; they are preferred in both region *A* and region *B*.

Figure 2

Outcomes with money regime in the best and worst equilibria

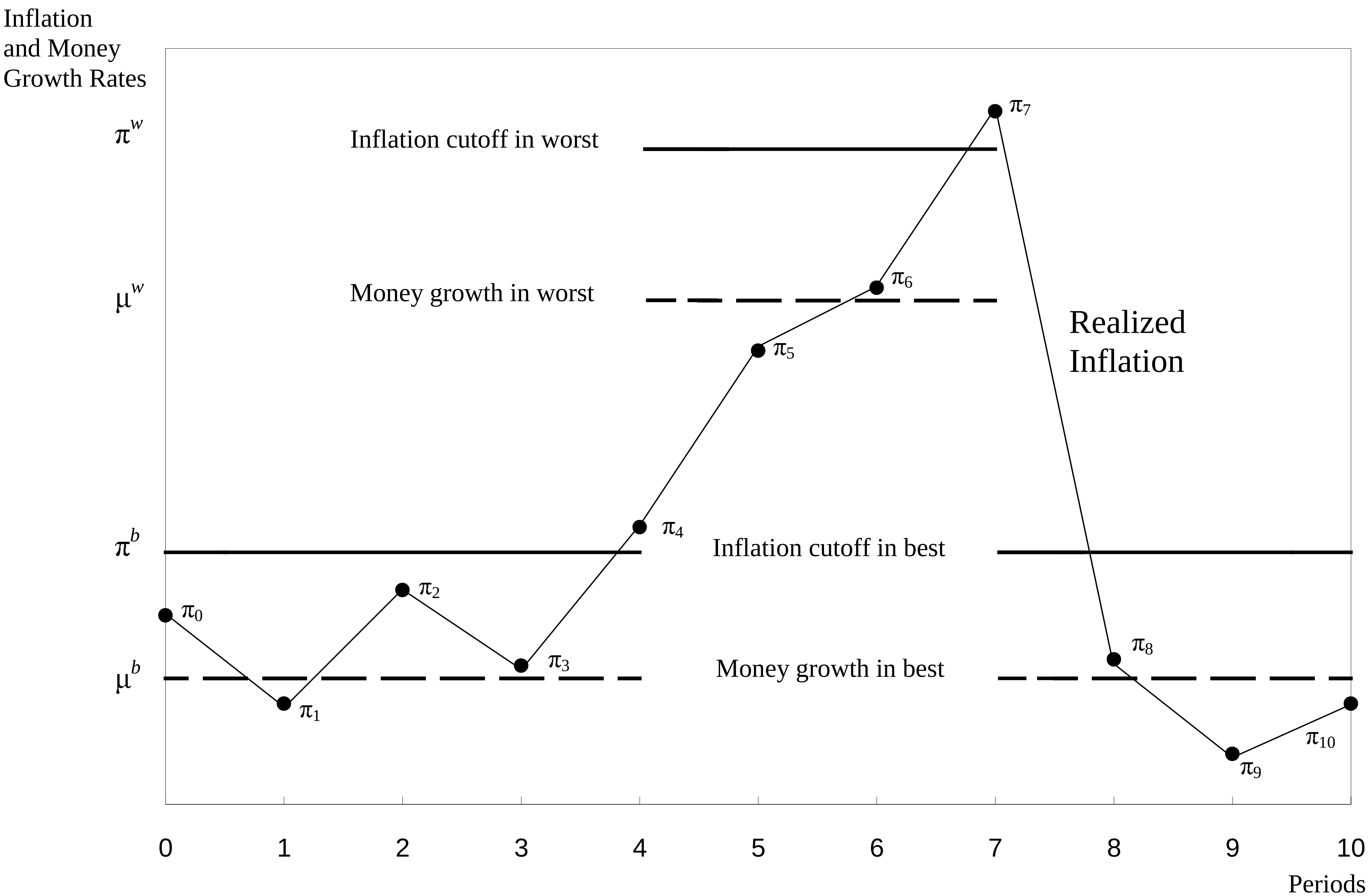
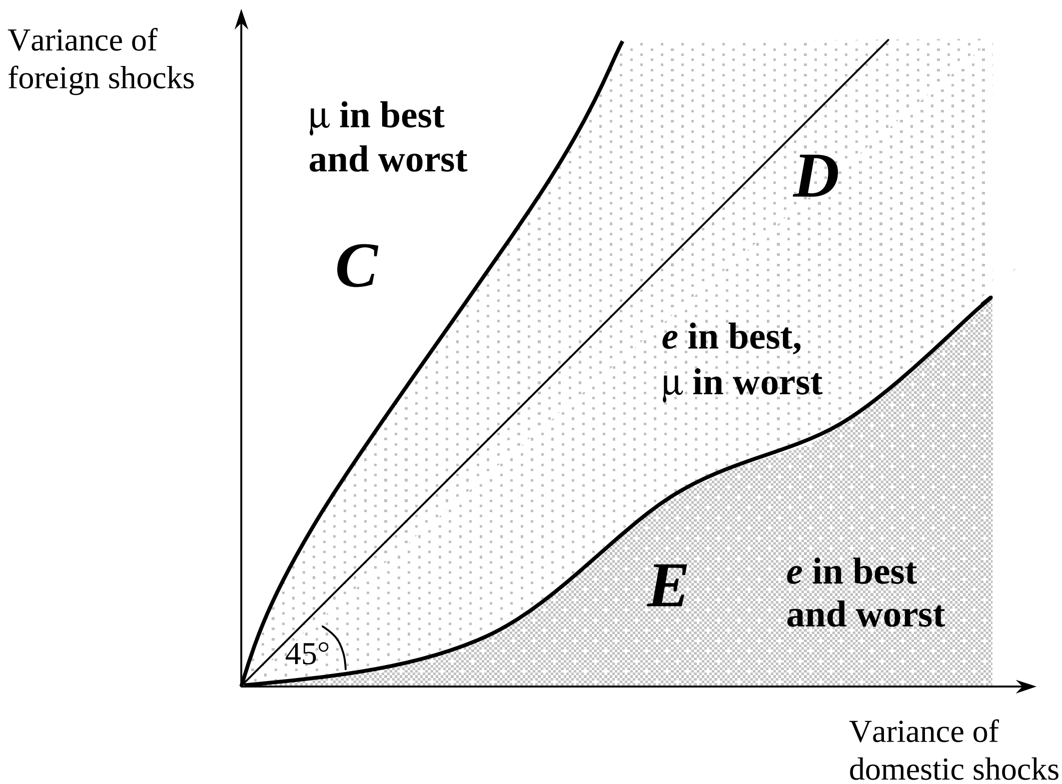


Figure 3

Regimes in the best and worst equilibrium outcomes



*In region *C*, the money regime is followed in the best and the worst equilibria. In region *D*, the exchange rate regime is followed in the best equilibria and the money regime is followed in the worst equilibria. In region *E*, the exchange rate regime is followed both in the best and in the worst equilibria.