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AN EXPLORATION OF THE EFFECTS OF PESSIMISM AND DOUBT ON ASSET RETURNS

Andrew B. Abel

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ABSTRACT

The subjective distribution of growth rates of aggregate consumption is characterized by pessimism if it is first-order stochastically dominated by the objective distribution. Uniform pessimism is a leftward translation of the objective distribution of the logarithm of the growth rate. The subjective distribution is characterized by doubt if it is mean-preserving spread of the objective distribution. Pessimism and doubt both reduce the riskfree rate and thus can help resolve the riskfree rate puzzle. Uniform pessimism and doubt both increase the average equity premium and thus can help resolve the equity premium puzzle.

Andrew B. Abel
The Wharton School
University of Pennsylvania
2315 Steinberg-Dietrich Hall
Philadelphia, PA 19104
and NBER
abel@wharton.upenn.edu

The hypothesis that consumers have rational expectations about relevant economic variables is a maintained assumption in the vast majority of asset pricing models. This assumption is attractive because it seems optimal for self-interested consumers to use some resources to forecast variables of interest. This assumption is also attractive because it avoids the myriad modeling choices that arise once the discipline of rational expectations is removed. Nevertheless, the assumption of rational expectations is simply an assumption—an assumption that could turn out not to be true.

Recent work by Cecchetti, Lam and Mark (2000) has analyzed an asset pricing model with various departures from rational expectations. A major finding of that paper is that a model with conventional preferences in which consumers exhibit pessimistic beliefs can better match sample moments of asset returns than can a rational expectations model. I will use a simpler asset pricing model here to explore how two particular departures from rationality—pessimism and doubt—affect the means of asset returns. I define the subjective distribution of growth rates of aggregate consumption per capita to be characterized by pessimism if it is first-order stochastically dominated by the objective distribution.[1] The subjective distribution of growth rates is characterized by doubt if it is a mean-preserving spread of the objective distribution. I describe the analysis as exploratory in nature because I simply take pessimism and doubt as given, without modeling the source of departures from complete rationality of expectations. I focus on the implications of pessimism and doubt for asset returns, and explore whether pessimism and doubt can help resolve some asset pricing puzzles.

[1] This definition of pessimism is more restrictive and more precise than the use of the term "pessimism" in Cecchetti, Lam, and Mark (2000).

In section I, I develop a simple model of asset pricing in which consumers' subjective distribution of the growth rate of aggregate consumption differs from the objective distribution. I use this model in section II to show that pessimism reduces the riskfree rate. I define uniform pessimism as a leftward translation of the objective distribution of the logarithm of the growth rate of aggregate consumption, and then I show that uniform pessimism increases the average equity premium. In section III I focus on the effects of doubt on asset returns. I show that doubt reduces the riskfree rate and increases the average equity premium.

Pessimism and doubt can help resolve important asset pricing puzzles. As already mentioned, both pessimism and doubt reduce the riskfree rate, and thus they can help resolve the riskfree rate puzzle discussed by Weil (1989). Uniform pessimism and doubt both increase the average equity premium and thus can help resolve the equity premium puzzle of Mehra and Prescott (1985). To analyze the joint quantitative impact of pessimism and doubt, I assume in section IV that aggregate consumption growth is lognormal, and I calculate the riskfree rate and average equity premium under pessimism and doubt.

In section V, I calculate the values of the preference parameters that allow a calibrated version of the model with lognormality to match the historical average values of the riskfree rate and the equity premium. In the case of rational expectations, the value of the coefficient of relative risk aversion that allows the model to match the historical average equity premium is much higher than is conventionally deemed to be plausible. This high value of the coefficient of relative risk aversion is a reflection of the equity premium puzzle. I go on to show that with pessimism and doubt a calibrated version of the model can match the historical average equity premium with a much lower coefficient of relative risk aversion that is in the conventional range.

Having demonstrated the potential impact of pessimism and doubt on asset returns, I speculate about the potential sources and interpretations of pessimism and doubt in section VI. I intend this section to be an admission that I have not satisfactorily explained the sources of pessimism and doubt. I also use this section as an opportunity to speculate about possible avenues to explore. I present concluding remarks in section VII.

I. Asset Pricing when Consumers' Subjective Distribution Differs from the Objective Distribution

Consider a Lucas (1978) fruit-tree economy in which the only source of output is a large number of identical infinitely-lived fruit trees. Let $Y_t > 0$ be the amount of fruit produced by a representative tree in period t. This fruit is completely perishable after one period and cannot be used to increase the number of fruit trees. The only use for the fruit is consumption, and in equilibrium all fruit is consumed in the period in which it is produced.

The economy is populated by a large number of identical infinitely-lived consumers. Without loss of generality, normalize the number of consumers to equal the number of trees, which implies that Y_t is the amount of output per consumer or, equivalently, the dividend per consumer. All consumers have identical utility functions, identical initial asset holdings (equal to one fruit tree), and identical subjective distributions about future growth rates of Y_t. Thus, I can focus on the behavior of a representative consumer.

Let c_t be the consumption in period t chosen by the representative consumer, and let C_t be the aggregate consumption per consumer in period t. In equilibrium, $c_t = C_t = Y_t$, where $c_t = C_t$ reflects the assumption that all consumers are identical, and $C_t = Y_t$ reflects the fact that all output is consumed in the period in which it is produced. Therefore, individual consumption, c_t, and aggregate consumption per consumer, C_t, inherit the stochastic properties of the exogenous dividend process. Define $X_t > 0$ as the gross growth rate of dividends so that in equilibrium

$$X_t \equiv \frac{Y_t}{Y_{t-1}} = \frac{C_t}{C_{t-1}} = \frac{c_t}{c_{t-1}} . \tag{1}$$

Assume that X_t is i.i.d. over time which implies that ln Y_t , ln C_t, and ln c_t evolve as random walks, possibly with drift. Assume that all consumers know that X_t is i.i.d. over time. Let $F(X_t)$ be the true, or objective, distribution of X_t and let $F^*(X_t)$ be the subjective distribution that consumers think governs X_t. At this point I will not restrict the distributions $F(X_t)$ and $F^*(X_t)$ except to assume that they are non-degenerate distributions with finite moments.

A representative consumer chooses individual consumption, c_t, and holdings of shares in fruit trees to maximize

$$E_t^*\left\{\frac{1}{1-\alpha}\sum_{j=0}^{\infty}\beta^j c_{t+j}^{1-\alpha}\right\} \tag{2}$$

where $\beta > 0$ is the time preference discount factor, $\alpha > 0$ is the coefficient of relative risk aversion, and the operator $E_t^*\{\ \}$ denotes the subjective expectation conditional on information available at time t using the subjective distribution $F^*(X)$. The utility function in equation (2) is a standard utility function widely used in the asset pricing literature except that the expectation is evaluated using the subjective distribution $F^*(X)$ rather than the objective distribution $F(X)$ as would be the case if consumers had rational expectations.

In equilibrium, the marginal rate of substitution between c_t and c_{t+1} for the utility function in equation (2) is

$$M_{t+1} \equiv \beta X_{t+1}^{-\alpha} . \tag{3}$$

It is well known that in the absence of any transaction costs or other capital market imperfections, the conditional expectation of the product of M_{t+1} and R_{t+1} equals one, where R_{t+1} is the gross rate of return on any asset. When the subjective distribution of X_{t+1} differs from the objective distribution of X_{t+1}, it is the subjective distribution that is used in calculating the conditional expectation. Thus,[2]

$$E_t^*\{M_{t+1}R_{t+1}\} = 1 . \tag{4}$$

The pricing condition in equation (4) holds for any asset. I will apply this condition first to the riskfree rate and then to the rate of return on equity.

Let R^f_{t+1} be the gross rate of return on a one-period riskfree asset held from period t to period t+1. Substituting R^f_{t+1} for R_{t+1} in equation (4), using the expression for the

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[2] The representative consumer's Euler equation is $E_t^*\left\{\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha} R_{t+1}\right\} = 1$. Substituting equation (1) into this Euler equation and using equation (3) yields equation (4).

marginal rate of substitution in equation (3), and recognizing that R^f_{t+1} is known at time t yields[3]

$$R^f = \frac{1}{\beta E^*\left\{X^{-\alpha}\right\}}. \tag{5}$$

The riskfree rate in equation (5) is the riskfree rate that prevails when consumers have the subjective distribution $F^*(X_t)$. If consumers have rational expectations, the riskfree rate is obtained from equation (5) by replacing the subjective expectation $E^*\{X^{-\alpha}\}$ by the objective expectation $E\{X^{-\alpha}\}$. Calibrations of equation (5) under rational expectations typically use historical data on X_t to estimate the moments of the distribution of X_t. When these calibrations use conventional values of the preference parameters α and β, the resulting riskfree rate is much higher than its historical average of one or two percent per year in the United States. The typical finding that the calibrated value of the riskfree rate is much higher than the observed average value has been dubbed the "riskfree rate puzzle" by Weil (1989). Later in this paper I will examine the extent to which pessimism and doubt can help resolve the riskfree rate puzzle.

Equity is a claim on the future stream of dividends accruing to a fruit tree. Thus, P_t, the ex-dividend price of equity in period t, is the price of a claim on all dividends accruing to a fruit tree from time t+1 onward. The gross rate of return on equity between periods t and t+1 is

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[3] Because X_t is i.i.d. over time, the conditional expectation $E_t^*\{X_{t+1}^{-\alpha}\}$ equals the unconditional expectation $E^*\{X_{t+1}^{-\alpha}\}$. The time subscript t+1 is omitted from $X^{-\alpha}$ without loss of generality or clarity here and elsewhere in the paper.

$$R_{t+1}^{e} = \frac{P_{t+1} + Y_{t+1}}{P_t}. \tag{6}$$

Because the growth rate of consumption and dividends, X_t, is i.i.d., and because the utility function is isoelastic, the equilibrium ex-dividend price of equity is proportional to the current dividend. Specifically, as can be easily verified,

$$P_t = \omega Y_t. \tag{7}$$

Substituting equation (7) into equation (6) and recalling the definition of the gross growth rate of dividends in equation (1) yields

$$R_{t+1}^{e} = \frac{\omega + 1}{\omega} X_{t+1}. \tag{8}$$

The value of the price-dividend ratio ω can be determined by substituting equation (8) into equation (4) and using the expression for the marginal rate of substitution in equation (3) to obtain

$$\frac{\omega + 1}{\omega} = \frac{1}{\beta E^{*}\left\{X^{1-\alpha}\right\}}. \tag{9}$$

Finally, substitute equation (9) into equation (8) to obtain the equilibrium rate of return on equity

$$R_{t+1}^{e} = \frac{X_{t+1}}{\beta E^{*}\left\{X^{1-\alpha}\right\}}. \tag{10}$$

Equation (10) indicates that the equilibrium rate of return on equity is proportional to the gross rate of dividend growth.

In a very long time series of observations, the sample average rate of return on equity equals the objective expectation of the return in equation (10).[4] Using the objective distribution $F(X)$ to calculate this expectation yields

$$E\left\{R^{e}\right\}=\frac{E\left\{X\right\}}{\beta E^{*}\left\{X^{1-\alpha}\right\}}. \tag{11}$$

The ex post equity premium is the amount by which the realized rate of return on equity exceeds the riskfree rate. In ratio form, the equity premium is

$$\frac{R_{t+1}^{e}}{R^{f}}=\frac{X_{t+1}E^{*}\left\{X^{-\alpha}\right\}}{E^{*}\left\{X^{1-\alpha}\right\}} \tag{12}$$

and the objective expectation of the equity premium is

$$\frac{E\left\{R^{e}\right\}}{R^{f}}=\frac{E\left\{X\right\}E^{*}\left\{X^{-\alpha}\right\}}{E^{*}\left\{X^{1-\alpha}\right\}}. \tag{13}$$

The average equity premium that prevails under rational expectations is obtained from equation (13) by replacing the subjective expectations $E^{*}\left\{X^{-\alpha}\right\}$ and $E^{*}\left\{X^{1-\alpha}\right\}$ by the respective objective expectations $E\left\{X^{-\alpha}\right\}$ and $E\left\{X^{1-\alpha}\right\}$. Calibrations of the average equity premium under rational expectations using conventional values of the

[4] In a very long time series of observations, consumers might learn the objective distribution, thereby making the subjective distribution identical to the objective distribution. In this paper, I ignore learning, which is consistent with the viewpoint of robust control, which I discuss in section VI. Anderson, Hansen, and Sargent (2000, p. 4) state: "The perspective of a robust decision maker differs substantially from that of one who learns. In our dynamic settings, the robust decision maker accepts model misspecification as a permanent state of affairs, and devotes his thoughts to designing robust controls, rather than to using data to improve his model specification over time. The robust decision maker turns his back on learning."

preference parameters α and β typically yield an equity premium that is much smaller than the historical average equity premium of about 6% per year in the United States.

II. The Effect of Pessimism on Asset Returns

I will use the term *pessimism* to mean that consumers' subjective distribution $F^*(X)$ is first-order stochastically dominated by the objective distribution $F(X)$. In this section I will show that pessimism can help resolve the both the riskfree rate and equity premium puzzles.

First consider the effect of pessimism on the riskfree rate. Since $X^{-\alpha}$ is a strictly decreasing function of X, pessimism implies that $E^*\{X^{-\alpha}\}$ is larger than $E\{X^{-\alpha}\}$. Thus, it follows immediately from equation (5) that pessimism reduces the riskfree rate relative to the riskfree rate that would prevail under rational expectations.

The role of pessimism in reducing the riskfree rate has a simple interpretation. If consumers are pessimistic about the growth rate of dividends, then, relative to the case of rational expectations, they will attempt to reduce current consumption and increase current saving. The attempt to increase current saving puts downward pressure on the real interest rate. Equivalently, a low expected growth rate of consumption can be consistent with optimality only if the real interest rate is low.

Because pessimism reduces the riskfree rate, it can help explain the riskfree rate puzzle. To analyze the magnitude of the impact of pessimism on the riskfree rate I will introduce the notion of *uniform pessimism*, which I define as a leftward translation of the objective distribution of $x \equiv \ln X$. Specifically, the subjective distribution $F^*(X)$ is characterized by uniform pessimism if

$$F^*(X) = F(Xe^{\Delta}) \tag{14}$$

for $\Delta > 0$. It is straightforward to show that under uniform pessimism

$$E^*\left\{X^a\right\} = e^{-a\Delta} E\left\{X^a\right\} \tag{15}$$

for an arbitrary constant a.

I will use a circumflex to denote a rate of return that would prevail under rational expectations. For instance, $\hat{R}^f$ is the gross riskfree rate under rational expectations. Letting $\hat{r}^f \equiv \ln \hat{R}^f$ denote the net riskfree rate under rational expectations and r_P^f denote the net riskfree rate, $\ln R^f$, under uniform pessimism, it follows from equations (5) and (15) that

$$r_P^f = \hat{r}^f - \alpha\Delta . \tag{16}$$

Equation (16) indicates that uniform pessimism reduces the riskfree rate by $\alpha\Delta$.

Now consider the effect of uniform pessimism on the average equity premium. Define $\hat{\varepsilon} \equiv \ln\left(E\left\{\hat{R}^e\right\} / \hat{R}^f\right)$ as the (logarithm of the) objective expectation of the equity premium that would prevail under rational expectations and define ε_P as the logarithm of the objective expectation of the equity premium, $\ln\,[E\{R^e\}/R^f]$, under pessimism. It follows from equations (13) and (15) that

$$\varepsilon_P = \hat{\varepsilon} + \Delta , \qquad \text{under uniform pessimism} \tag{17}$$

According to equation (17), uniform pessimism increases the average equity premium by Δ. To understand this effect, it is helpful to examine the *subjective* expectation of the equity premium, $E^*\{R^e\}/R^f$. It follows from equations (12) and (15) that

$$\frac{E^*\{R^e\}}{R^f} = \frac{E\{\hat{R}^e\}}{\hat{R}^f}, \qquad \text{under uniform pessimism.} \tag{18}$$

Equation (18) shows that the equity premium (subjectively) expected by consumers with uniform pessimism is identical to the equity premium expected by consumers in a rational expectations equilibrium. Thus, uniform pessimism has no effect on consumers' subjective expectation of the equity premium. The reason that pessimism increases the objective expectation of the equity premium is not that pessimistic consumers require a higher equity premium. Instead, they require the same equity premium as under rational expectations, but their pessimism leads them to underestimate the mean growth rate of dividends and hence the average rate of return on equity. Thus, the objective expectation of the equity premium is greater than consumers expect and hence is greater than the expectation of the equity premium under rational expectations. Indeed, because consumers underestimate the average net growth rate $x_t \equiv \ln X_t$ by Δ, they underestimate the net expected return to equity by Δ. Thus, pessimism increases the objective expectation of the equity premium by Δ.

III. The Effects of Doubt on Asset Returns

In this section I analyze the effects of doubt on asset returns. I will use the term doubt to mean that consumers' subjective distribution $F^*(X)$ is a mean-preserving spread of the objective distribution $F(X)$. To determine the effect of doubt on the riskfree rate, observe that $X^{-\alpha}$ is a strictly convex function of X because $\alpha > 0$. Hence, a mean-preserving spread on the distribution of X increases the expected value of $X^{-\alpha}$. Therefore, if the subjective distribution $F^*(X)$ is characterized by doubt, $E^*\{X^{-\alpha}\}$ is larger than $E\{X^{-\alpha}\}$, and it follows immediately from equation (5) that doubt reduces the riskfree rate relative to the riskfree rate that would prevail under rational expectations.

There is a simple economic explanation for the effect of doubt on the riskfree rate. An increase in doubt increases consumers' perceived risk. This increase in perceived risk leads consumers to seek safety in the riskfree asset, which drives up the price of the riskfree asset, or equivalently, drives down the riskfree rate.

To measure the effect of doubt on the expected equity premium, define

$$\theta \equiv \ln\left(1 + \frac{Var^*\{X\}}{\left[E^*\{X\}\right]^2}\right) - \ln\left(1 + \frac{Var\{X\}}{\left[E\{X\}\right]^2}\right), \tag{19}$$

where $E^*\{X\}$ and $Var^*\{X\}$ are the mean and variance, respectively, of $F^*(X)$, and $E\{X\}$ and $Var\{X\}$ are the mean and variance, respectively, of $F(X)$. The parameter θ is approximately equal to the difference in the squared coefficient of variation of X when comparing $F(X)$ and $F^*(X)$. I show in the Appendix that under doubt, with $E^*\{X\} = E\{X\}$ and $Var^*\{X\} > Var\{X\}$,

$$\ln E^*\left\{X^a\right\} - \ln E\left\{X^a\right\} \cong \frac{1}{2} a(a-1)\theta > 0. \tag{20}$$

Define ε_D as the logarithm of the objective expectation of the equity premium, ln $[E\{R^e\}/R^f]$, under doubt. It follows from equations (13) and (20) that under doubt

$$\varepsilon_D = \hat{\varepsilon} + \alpha\theta . \tag{21}$$

Equation (21) shows that doubt causes the expected equity premium to exceed the expected equity premium under rational expectations, $\hat{\varepsilon}$, by $\alpha\theta$.

To understand the effect of doubt in equation (21), it is helpful to examine the subjective expectation of the equity premium, $E^*\{R^e\}/R^f$. It follows from equations (12) and (20) that

$$\ln\left[E^*\left\{R^e\right\}/R^f\right] = \hat{\varepsilon} + \alpha\theta , \qquad \text{under doubt.} \tag{22}$$

According to equation (22), consumers require a higher (subjective) expected equity premium under doubt than under rational expectations because they perceive a higher level of risk associated with equity. The increase in the required equity premium, $\alpha\theta$, is proportional to the coefficient of relative risk aversion α and to the perceived increase in risk θ.

IV. Lognormality

I have shown that uniform pessimism and doubt both reduce the riskfree rate and increase the average equity premium. These results do not depend on the specification of the distribution $F(X)$. To illustrate the quantitative impacts of uniform pessimism and doubt on these rates of return, and to examine the role of preference parameters, I will assume that $F(X)$ and $F^*(X)$ are lognormal. The assumption that $x_t \equiv \ln X_t$ is normal is commonly adopted in studies of asset pricing.

Suppose that the objective distribution of ln X_t is $N(\mu, \sigma^2)$ and that the subjective distribution of ln X_t is $N(\mu^*, \sigma^{*2})$ where $\mu^* \equiv \mu - \Delta - ½\,\theta$, $\sigma^{*2} \equiv \sigma^2 + \theta$, $\Delta \geq 0$, and $\theta \geq 0$. The assumption of lognormality implies that for an arbitrary constant a

$$E\{X^a\} = \exp\left[a\mu + \frac{1}{2}a^2\sigma^2\right] \tag{23a}$$

and

$$E^*\{X^a\} = E\{X^a\}\exp\left[-a\Delta + \frac{1}{2}a(a-1)\theta\right]. \tag{23b}$$

Using equations (5) and (23a) yields the net riskfree rate under rational expectations

$$\hat{r}^f = -\ln\beta + \alpha\mu - \frac{1}{2}\alpha^2\sigma^2. \tag{24}$$

Using equations (5) and (23b), the riskfree rate under pessimism and doubt is

$$r^f = \hat{r}^f - \alpha\Delta - \frac{1}{2}\alpha(1+\alpha)\theta. \tag{25}$$

Equation (25) reiterates the findings from sections II and III that both pessimism ($\Delta > 0$) and doubt ($\theta > 0$) reduce the riskfree rate relative to the riskfree rate under rational expectations. Under lognormality, the effects of pessimism and doubt are additive.

The average equity premium under rational expectations is calculated using equations (13) and (23a) to obtain

$$\hat{\varepsilon} = \alpha\sigma^2. \tag{26}$$

The average equity premium under pessimism and doubt is calculated using equations (13) and (23b) to obtain

$$\varepsilon = \hat{\varepsilon} + \Delta + \alpha\theta \,. \tag{27}$$

As noted earlier, uniform pessimism ($\Delta > 0$) and doubt ($\theta > 0$) increase the average equity premium relative to its value under rational expectations. Equation (27) illustrates that in the case of lognormality the effects on the equity premium of uniform pessimism and doubt are additive.

Equations (25) and (27) show that pessimism and doubt—either separately or in combination—can help resolve the riskfree rate and equity premium puzzles. The riskfree puzzle is that rational expectations asset pricing models produce a riskfree rate that is too high, but equation (25) shows that pessimism and doubt reduce the riskfree rate. The equity premium puzzle is that rational expectations models produce an average equity premium that is too low, but equation (27) shows that both pessimism and doubt increase the average equity premium.

V. Choosing Preference Parameters Values to Match Sample Mean Returns

In this section I present an alternative view of the equity premium and riskfree rate puzzles by calculating the values of α and β that allow the model to match the historical average values of r^f and ε. I will maintain the assumption that $F(X)$ and $F^*(X)$ are lognormal and analyze the quantitative extent to which pessimism and doubt can help resolve the equity premium and riskfree rate puzzles.

Let a bar over a variable denote the average value of a variable in a historical sample of data. Thus, $\bar{r}^f$ is the sample average value of the net riskfree rate r^f, and $\bar{\varepsilon}$ is the sample average value of the equity premium ε. Mehra and Prescott (1985) report that for the period 1889-1978, $\bar{r}^f = 0.0080$ and $\bar{\varepsilon} = 0.0595$.[5] The challenge posed by Mehra and Prescott is to calibrate the parameters of consumption growth, μ and σ, to match U.S. data, and then to find plausible values of the preference parameters α and β for which the predicted value of the riskfree rate equals the sample value $\bar{r}^f$ and the predicted value of the average equity premium equals the sample value $\bar{\varepsilon}$.

It is straightforward to find the values of α and β that allow the model's predictions of r^f and ε to match the sample values $\bar{r}^f$ and $\bar{\varepsilon}$ respectively. Let α^* be the value of the coefficient of relative risk aversion for which the model's prediction of the average equity premium equals the sample value $\bar{\varepsilon}$ when consumers have the subjective distribution $F^*(X)$. To calculate α^*, set ε equal to $\bar{\varepsilon}$ in equation (27) and use equation (26) to obtain

$$\alpha^* = \frac{\bar{\varepsilon} - \Delta}{\sigma^2 + \theta} \quad . \tag{28}$$

Let β^* be the value of β for which the model's predictions of r^f and ε equal the respective sample values $\bar{r}^f$ and $\bar{\varepsilon}$. Setting $r^f = \bar{r}^f$ in equation (25) and using equation (24) yields

$$\beta^* = \exp\left[-\bar{r}^f + \alpha^*(\mu - \Delta) - \frac{1}{2}\alpha^{*2}\sigma^2 - \frac{1}{2}\alpha^*(1+\alpha^*)\theta\right]. \tag{29}$$

[5] Mehra and Prescott found the average riskfree rate was 0.0080 and the average rate of return on equity was 0.0698. Thus $\bar{r}^f = \ln(1.0080) = 0.0080$ and $\bar{\varepsilon} = \ln(1.0698/1.0080) = 0.0595$.

Let $\hat{\alpha}$ and $\hat{\beta}$ be the values of the preference parameters that allow the model's predictions under rational expectations to match the sample values $\bar{r}^f$ and $\bar{\varepsilon}$. The values of $\hat{\alpha}$ and $\hat{\beta}$ are obtained from equations (28) and (29) by setting $\Delta = \theta = 0$ to obtain

$$\hat{\alpha} = \frac{\bar{\varepsilon}}{\sigma^2} \tag{30}$$

and

$$\hat{\beta} = \exp\left[-\bar{r}^f + \hat{\alpha}\mu - \frac{1}{2}\hat{\alpha}^2\sigma^2\right]. \tag{31}$$

Mehra and Prescott (1985) report the moments of consumption growth for the period 1889-1978 to be $E\{X\} = 1.0183$ and $Var\{X\} = (0.0357)^2$. Since ln X is $N(\mu, \sigma^2)$, these values for the mean and variance of X imply that $\mu = 0.01752$ and $\sigma = 0.03505$.[6] Using these sample values along with $\bar{r}^f = 0.0080$ and $\bar{\varepsilon} = 0.0595$, equation (30) yields $\hat{\alpha} = 48.44$. Mehra and Prescott state that reasonable values for α are in the range from 0 to 10. Because $\hat{\alpha}$ is so much larger than 10, Mehra and Prescott argue that their model (which includes the assumption of rational expectations) fails to account for the sample average equity premium.

Despite the contention by Mehra and Prescott that the coefficient of relative risk aversion α must be smaller than 10, Kandel and Stambaugh (1991) argue forcefully that there is no evidence that would lead one to rule out values of α greater than 10. Using $\hat{\alpha} = 48.44$ from equation (30), equation (31) implies that the value of $\hat{\beta}$ is 0.549. This low

[6] Since ln X is $N(\mu, \sigma^2)$, $E\{X\} = \exp[\mu + (1/2)\sigma^2]$ and $Var\{X\}/[E\{X\}]^2 = \exp(\sigma^2) - 1$.

value of $\hat{\beta}$ is a consequence of the strong precautionary saving motive induced by the high value of $\hat{\alpha}$. A strong desire for precautionary saving puts downward pressure on the interest rate. In order to prevent the interest rate from being too low (or even negative) consumers must have a high rate of time preference, which is captured by a low value of β.[7]

Pessimism and doubt can help resolve the riskfree rate puzzle and the equity premium puzzle by allowing the model to match the sample moments $\overline{r}^f$ and $\overline{\varepsilon}$ using values of the preference parameters α and β that are in the range examined by Mehra and Prescott. Table 1 presents the values of α^* and β^* for various combinations of pessimism, measured by Δ, and doubt, measured by θ. Each row of Table 1 corresponds to a fixed level of doubt, which is given in the first column. The level of doubt in each row is represented by two (equivalent) values in the first column: the top element in each cell in the first column is the value of θ and the bottom element in each cell is the standard deviation of the subjective distribution in the absence of pessimism (i.e., when $\Delta = 0$). Each column of Table 1 (except the first column which contains θ and $S.D.^*\{X\}$ for $\Delta = 0$) corresponds to a fixed level of pessimism which is represented by the value of Δ at the top of the column. All of the cells contain two numbers representing preference parameters: the top number is α^* and the bottom number is β^*.

[7] Kandel and Stambaugh (1991) do not have such a low value of β because their model has a richer structure of serial correlation for X_t, which changes the implied moments of asset returns.

Table 1						
Values of α^* and β^* under Pessimism (Δ) and Doubt (θ)						
θ	Δ					
$S.D.^*\{X\}$ for $\Delta = 0$	0	0.005	0.01	0.02	0.03	0.055
0	48.44	44.37	40.30	32.16	24.02	3.66
0.0357	0.549	0.516	0.495	0.485	0.516	0.858
0.0010	26.70	24.46	22.21	17.73	13.24	2.02
0.0481	0.706	0.684	0.669	0.663	0.687	0.915
0.0020	18.43	16.88	15.33	12.24	9.14	1.39
0.0579	0.777	0.761	0.750	0.747	0.766	0.937
0.0040	11.38	10.42	9.47	7.55	5.64	0.86
0.0737	0.844	0.833	0.827	0.826	0.841	0.957
0.0060	8.23	7.54	6.85	5.46	4.08	0.62
0.0867	0.875	0.868	0.864	0.864	0.877	0.966
0.0100	5.30	4.85	4.41	3.52	2.63	0.40
0.1082	0.905	0.901	0.899	0.901	0.911	0.974
0.0150	3.67	3.36	3.05	2.43	1.82	0.28
0.1302	0.923	0.921	0.920	0.923	0.931	0.979
0.0163	3.39	3.11	2.82	2.25	1.68	0.26
0.1354	0.926	0.924	0.923	0.926	0.935	0.980

The case of rational expectations is shown in the first row of results ($\theta = 0$, $S.D.^*\{X\} = 0.0357$) and in the column headed $\Delta = 0$. In this case there is neither pessimism nor doubt. This cell shows that the model with rational expectations will match the riskfree rate and average equity premium with $\alpha = 48.44$ and $\beta = 0.549$, as discussed above. Moving to the right across the first row, pessimism, measured by Δ, increases. As Δ increases, α^* decreases linearly in Δ. The value of β^* first falls and then increases as Δ increases.[8] Starting from the rational expectations cell and moving down

[8] Substituting equation (28) into equation (29) and rearranging yields

this column, the degree of doubt, measured by θ, increases. As doubt increases, α^* decreases and β^* increases.[9] Thus, both pessimism and doubt, either individually or in combination, can reduce α^* and increase β^*, thereby moving both preference parameters toward the conventional values that were the focus of Mehra and Prescott.

One's view of the equity premium puzzle and the riskfree rate puzzle depends on the range of preference parameter values that one regards as plausible. Underlying the seminal discussion of the equity premium puzzle by Mehra and Prescott is the belief that the preference parameters α and β must satisfy $0 < \alpha \leq 10$ and $0 < \beta < 1$. As noted earlier, Kandel and Stambaugh (1991) argue that there is no compelling reason to restrict α to be less than ten. Nevertheless, much of the literature regards 10 as an upper bound for α. For comparability with this literature, I have shaded those cells in Table 1 for which $0 < \alpha^* \leq 10$ and $0 < \beta^* < 1$. To the extent that the equity premium puzzle represents a challenge to match $\bar{r}^f$ and $\bar{\varepsilon}$ with values of α and β deemed reasonable by Mehra and Prescott, the shaded cells represent combinations of pessimism and doubt that meet this challenge, though the levels of pessimism and doubt may be implausibly high in some cases.

$\ln \beta^* = -\bar{r}^f + \frac{\bar{\varepsilon} - \Delta}{\sigma^2 + \theta}\left(\mu - \frac{1}{2}(\bar{\varepsilon} + \Delta + \theta)\right)$ so that $\ln \beta^*$ is quadratic in the pessimism parameter Δ.

[9] Differentiating equation (28) with respect to θ yields $\frac{d\alpha^*}{d\theta} = \frac{-\alpha^*}{\sigma^2 + \theta} < 0$ if $\alpha^* > 0$.

Taking the logarithm of each side of equation (29), differentiating with respect to θ, and using equation (28) yields $\frac{d \ln \beta^*}{d\theta} = -\frac{1}{2}\alpha^*(1 + \alpha^*) + \left[\mu - \Delta - \alpha^*(\sigma^2 + \theta) - \frac{1}{2}\theta\right]\frac{d\alpha^*}{d\theta}$. Then using the expression for $\frac{d\alpha^*}{d\theta}$ yields $\frac{d \ln \beta^*}{d\theta} = \frac{\alpha^*}{\sigma^2 + \theta}\left[\left(\frac{1}{2}\bar{\varepsilon} - \mu - \frac{1}{2}\sigma^2\right) + \frac{1}{2}\Delta\right]$. In Table 1, $\bar{\varepsilon} = 0.0595$, $\mu = 0.0183$, and $\sigma = 0.0357$ so $\frac{1}{2}\bar{\varepsilon} - \mu - \frac{1}{2}\sigma^2 = 0.0108$. Therefore, $\frac{d \ln \beta^*}{d\theta} > 0$, if $\alpha^* > 0$ and $\Delta \geq 0$.

VI. The Sources and Sizes of Pessimism and Doubt: Some Speculative Thoughts

I have demonstrated that pessimism and doubt may help resolve the riskfree rate and equity premium puzzles, but this demonstration leads naturally to the next question: How much pessimism and doubt might characterize subjective distributions? It is difficult to answer this question without a more complete understanding of the sources of pessimism and doubt, and the persistence of these departures from rationality. It may be tempting to gauge the amount of pessimism or doubt by using sampling theory to estimate the standard errors of the estimates of the moments of growth rates, but this approach presumes that the source of pessimism and doubt is simply sampling error. At this point in the exploratory analysis, I would entertain a broader class of potential sources of departures from rationality.

A complete analysis of possible sources of pessimism and doubt is beyond the scope of this paper, but I will take this opportunity to illustrate quantitatively, without necessarily endorsing, one possible source. In the Lucas (1978) fruit-tree model used here, consumption, output, and dividends are identically equal and hence the growth rate X_t represents the growth rate of consumption, the growth rate of output, and the growth rate of dividends. The majority of the asset pricing literature calibrates X_t using consumption data. But what if consumers used dividend data to calibrate X_t ? Cecchetti, Lam, and Mark (1990, Table 1, p. 402) report moments of annual growth rates of consumption (1889-1985), output (1869-1985), and dividends (1871-1985), all in real per capita terms. The mean growth rate of dividends was only -0.38% per year compared with the mean consumption growth rate of 1.84% per year. Thus, if the sample mean growth rate of dividends were used as the subjective mean of X_t, the degree of pessimism,

Δ, would equal 0.0184 - (-0.0038) = 0.0222. The standard deviation of dividend growth was 0.1359 and the standard deviation of consumption growth was only 0.0379. Thus, if the standard deviation of dividend growth were used as the standard deviation of $F^*(X)$, while the mean growth rate of consumption, 1.84% per year, is used as the mean of $F^*(X)$,[10] then θ would equal $\ln(1 + (0.1359/1.0183)^2) - \ln(1 + (0.0379/1.0183)^2) = 0.0163$. Table 1 shows that if $\Delta = 0$ and $\theta = 0.0163$, then $\alpha^* = 3.39$ and $\beta^* = 0.926$, values that are well within the conventional range established by Mehra and Prescott.

Another interpretation of pessimism and doubt is based on the equivalence of risk-sensitive optimal control and robust control discussed by Hansen, Sargent, and Tallarini (1999) and references cited therein. Risk-sensitive optimal control introduces a single new parameter in the intertemporal utility function that can magnify the sensitivity of optimal behavior to risk even when the subjective distribution of growth rates is identical to the objective distribution. In contrast to risk-sensitive optimal control, which alters the intertemporal utility function without departing from rational expectations, robust control does not alter the usual intertemporal utility function, but allows the subjective distribution to differ from the objective distribution. In particular, under robust control, a consumer views her specification of the distribution function for the growth rate X as an approximation in the sense that she believes that there is a set of candidate distribution functions near the approximating model that might actually govern the growth rate.[11,12] Robust decision rules have favorable properties even if one of these

[10] This is a case of pure doubt, i.e., no pessimism, with the objective distribution represented by the historical distribution of consumption growth rates.

[11] The "risk-sensitivity" parameter can be viewed as describing the size of the set of distributions near the consumer's approximating model.

other distributions turns out to govern the data. The robust decision maker designs a decision rule by formulating a min-max game in which nature chooses a distribution from this set to minimize the consumer's utility. By playing a best response against that minimizing distribution, it turns out that the consumer's decision rule works well enough over the set of distributions near the approximating model. Thus, the consumer acts "as if" she is pessimistic in order to achieve robustness. One might interpret the subjective distribution $F^*(X)$ in the current paper as the most unfavorable distribution of growth rates X in a set of candidate distributions for a consumer solving a robust control problem. Though the framework in this paper is not identical to that in Hansen, Sargent, and Tallarini (1999), one might be able to establish a similar equivalence between the decision problem in this paper and a form of risk-sensitive control, but at this point, such an equivalence is merely speculation.

A completely different interpretation of the distributions $F(X)$ and $F^*(X)$ is based on the Peso problem, in which investors take account of the possibility of unlikely large events that do not occur during the sample period studied. In the framework of the Peso problem, one might interpret $F^*(X)$, which governs consumers' decisions, as the true distribution of growth rates. The distribution $F^*(X)$ may assign a small positive probability to a very adverse realization of the growth rate, i.e., to a very low realization of X.[13] In the Peso problem framework, the distribution $F(X)$ would be the empirical distribution of growth rates observed during a sample period. If the very adverse

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[12] Anderson, Hansen, and Sargent (2000) and Maenhout (1999) have applied robust control to study asset pricing.

[13] A small probability of large adverse events does not fit the lognormal example in sections IV and V, but the more general framework in the preceding sections can accommodate a small probability of large unlikely events.

realization of the growth rate happened not to occur during the sample period, then $F(X)$ would differ from $F^*(X)$. Pessimism and doubt that characterize $F^*(X)$ reflect unfavorable events that have not been observed during the sample period. Under this interpretation, the average equity premium reported in equation (13) is the average value of the equilibrium equity premium in samples in which the distribution of growth rates X is identical to the empirical distribution observed during the sample period. It is an empirical question whether a Peso problem can account for enough pessimism or doubt to have a substantial impact on observed equilibrium rates of return.[14]

VII. Conclusion

I have used a very simple asset pricing model to explore the implications of pessimism and doubt for the average returns that would be observed in large samples. Within the context of this model, both pessimism and doubt, either individually or together, can help resolve the riskfree rate puzzle by reducing the equilibrium riskfree rate. Under pessimism, consumers underestimate the mean growth rate of aggregate consumption and thus try to reduce current consumption and increase current saving. The attempt to increase current saving drives down the interest rate. Doubt reduces the interest rate by increasing the perceived risk associated with equity, thereby driving consumers to seek safety in the riskfree asset and driving down the interest rate on this asset.

[14] Reitz (1988) tries to resolve the equity premium puzzle by arguing that consumers are concerned about the small possibility of a very large drop in consumption. Though he does not mention the Peso problem by name, he states its essence: "To the extent that equity returns have been high with no crashes, equity owners have been compensated for crashes that happened not to occur" (p. 118). While Reitz claims that the size of unlikely (and as yet unobserved) crashes needed to resolve the equity premium puzzle is plausible, Mehra and Prescott (1988) argue that these crashes are implausibly large. Even if the Peso problem cannot plausibly

Uniform pessimism and doubt tend to increase the average equity premium and thus may help to resolve the equity premium puzzle. Under uniform pessimism, consumers require a (subjective) expected equity premium that is equal to the average equity premium under rational expectations. Though consumers think that the average equity premium will be relatively small, as it is under rational expectations, the average equity premium turns out larger than consumers expect precisely because consumers' expectations of consumption growth, and hence equity returns, are biased downward under pessimism. Doubt increases the average equity premium through a different channel: under doubt, consumers perceive a higher degree of risk associated with equity and thus require a higher expected equity premium.

The effects of pessimism and doubt on asset returns are encouraging because each of these features, represented by a single parameter, can individually move both the riskfree rate and the average equity premium in the "right" directions. In the context of the illustrative example in which consumers use historical dividend growth to estimate the standard deviation of X_t, doubt alone could resolve the equity premium puzzle without appeal to any pessimism at all. Specifically, using only doubt, the model could match the sample mean riskfree rate and sample mean equity premium with preference parameters well within the range of values deemed permissible by Mehra and Prescott.

I have emphasized that the analysis here is exploratory. I used a very simple asset pricing model for a brief foray into the territory of nonrational expectations to see whether departures from rationality might have interesting asset pricing implications in general, and whether they could help resolve some asset pricing anomalies in particular.

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resolve the equity premium puzzle by itself, it may have a substantial quantitative impact on asset returns and

The results of this foray are encouraging. It appears that pessimism, and perhaps especially doubt, may have quantitatively important effects on the moments of asset returns. Given the potential quantitative importance of these departures from rationality, the next challenge is to explain why pessimism and doubt may occur. The results of this paper suggest that understanding such departures from rationality may greatly extend our understanding of asset returns.

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could be one of several factors contributing to a resolution.

Appendix: Effect of Doubt on $E\{X^a\}$

Suppose that $F^*(X)$ is characterized by doubt and let μ_X be the common mean of $F(X)$ and $F^*(X)$. Define $g(X) \equiv X^a$. The second-order Taylor series expansion of $g(X)$ around μ_X is

$$g(X) \cong \mu_X{}^a + a\mu_X{}^{a-1}(X-\mu_X) + \frac{1}{2}a(a-1)\mu_X{}^{a-2}(X-\mu_X)^2. \qquad \text{(A1)}$$

Take the expectation of both sides of equation (A1) using $F(X)$ to obtain

$$E\{X^a\} = E\{g(X)\} \cong \left[1 + \frac{1}{2}a(a-1)\frac{\sigma_X{}^2}{\mu_X{}^2}\right]\mu_X{}^a \qquad \text{(A2)}$$

where σ_X is the standard deviation of $F(X)$. Take the expectation of both sides of equation (A1) using $F^*(X)$ to obtain

$$E^*\{X^a\} = E^*\{g(X)\} \cong \left[1 + \frac{1}{2}a(a-1)\frac{\sigma_D^2}{\mu_X{}^2}\right]\mu_X{}^a \qquad \text{(A3)}$$

where $\sigma_D > \sigma$ is the standard deviation of $F^*(X)$, which is characterized by doubt. Take logarithms of both sides of equations (A2) and (A3) and use ln (1+z) ≅ z to obtain

$$\ln E^*\{X^a\} - \ln E\{X^a\} \cong \frac{1}{2}a(a-1)\left(\frac{\sigma_D^2}{\mu_X{}^2} - \frac{\sigma_X{}^2}{\mu_X{}^2}\right). \qquad \text{(A4)}$$

Observe that $\theta \equiv \ln\left(1 + \frac{\sigma_D^2}{\mu_X{}^2}\right) - \ln\left(1 + \frac{\sigma_X^2}{\mu_X{}^2}\right) > 0$ and use ln (1+z) ≅ z to obtain

$\frac{\sigma_D^2}{\mu_X{}^2} - \frac{\sigma_X{}^2}{\mu_X{}^2} \cong \theta$, so that equation (A4) can be rewritten as

$$\ln E^*\{X^a\} - \ln E\{X^a\} \cong \frac{1}{2}a(a-1)\theta > 0. \qquad \text{(A5)}$$

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