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**ABSTRACT**

This paper provides new perspective on the poor performance of exchange rate models by focusing on the information structure of FX trading. I present a new theoretical model of FX trading that emphasizes the role of incomplete and heterogeneous information. The model shows how an equilibrium distribution of FX transaction prices and orders can arise at each point in time from the optimal trading decisions of dealers. This result motivates an empirical investigation of how the equilibrium distribution of FX prices behaves using a new data set that details trading activity in the FX market. This analysis produces two striking results: (i) Much of the observed short-term volatility in exchange rates comes from sampling the heterogeneous trading decisions of dealers in an equilibrium distribution that, under normal market conditions, changes comparatively slowly. (ii) In contrast to the assumptions of traditional macro models, public news is rarely the predominant source of exchange rate movements over *any* horizon.

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# 1 Introduction

The origins of nominal exchange rate dynamics remain elusive. In particular, there is no widely accepted explanation for the sizable short and medium-term movements in the dollar during the floating-rate period. More generally, theoretical models relating exchange rates to macroeconomic fundamentals are still outperformed by simple time series models in forecasting spot rates over short and medium-term horizons (Frankel and Rose 1995).

This paper provides new perspective on the poor performance of exchange rate models by focusing on the information structure of trading between FX dealers in the spot market. I develop a new theoretical model of FX trading that emphasizes the role of incomplete and heterogeneous information in dealers' trading decisions. The model shows how an equilibrium distribution of FX transaction prices and orders can arise at each point in time from the optimal trading decisions of dealers. This result directs attention away from the traditional view that there is a single equilibrium value for the exchange rate. Instead, it motivates an empirical investigation of how the equilibrium distribution of FX transaction prices is determined. The paper undertakes this investigation with the aid of a new data set that details trading activity in the FX market. This analysis reveals a striking new perspective on the source of exchange rate movements over all horizons. In particular, I find that much of the short-term volatility in exchange rates comes from sampling the equilibrium distribution of transaction prices that, under normal market conditions, changes comparatively slowly. I also find that public news is rarely the predominant source of long term exchange rate movements, a result that contrasts with the assumptions of traditional macro models.

The theoretical model of FX trading has its antecedents in the simultaneous trade model of Lyons (1997) and is designed to capture the key features of the actual market. It focuses on trading between dealers that accounts for approximately 75% of total trading in major currency markets. The model is populated by a large number of dealers that trade directly with each other and with non-bank customers. Dealers receive private information from two sources. The first comes from outside the market in the form of customer orders. FX dealers cite customer-dealer transactions as an important source of information (Lyons 2000). The second source of private information comes from direct, interdealer transactions. As in the actual market, the details of each direct transaction (e.g., the bid and ask quotes, the amount and direction of trade) are only observed by the counterparties. This means that any information conveyed by a transaction diffuses slowly across the market. Markets with this information structure are said to "lack transparency".

The lack of transparency in both direct interdealer and customer-dealer transactions is a key feature of the model and differentiates it from other multiple dealer models (see, for example, Perraudin and Vitale 1995, Lyons 1997, and Evans and Lyons 1999). In those models, each dealer quotes a publicly observed price at which she will trade any amount with any number of other dealers. In equilibrium, this leads dealers to quote a common price to avoid being arbitraged. In this model, by contrast, a dealer can only quote a price (good for any amount) to one other dealer at a point in time. This restriction, and the lack of transparency, make it possible for different dealers to quote different prices without opening themselves to arbitrage. The task of the model is to show how heterogeneity in the customer orders received by different dealers, combined with the lack of transparency, leads to an equilibrium distribution of FX prices at which direct interdealer trading takes place.

Another key feature of the model is that it considers trading over a large (but finite) number of trading periods. This means that the duration of a trading period can be viewed as being as short as the few seconds it takes to complete a transaction. By contrast, existing multiple dealer models split the trading day into a few periods (e.g. Evans and Lyons 1999). These models are better suited for examining the cumulative effects of trading on exchange rate dynamics than the origins of high frequency intraday dynamics that I shall study. Of course, the introduction of a large number of trading periods is not without cost. In particular, it makes the optimization problem facing dealers much more complex. One "resolution" to this problem is to assume that dealers make rule-of-thumb trading decisions as in Chakrabarti (2000). My approach is to cast the model in an overlapping generations structure. Within this structure, the optimization problem facing dealers becomes tractable and an analytic solution for market equilibrium can be found. The use of an overlapping generations structure is new to the literature on FX trading.

Section 2 of the paper presents the model and examines how an equilibrium distribution of FX transaction prices arises. The model characterizes the dynamics of the transaction price distribution in terms of two exogenous shocks: customer-order shocks and common knowledge (CK) news shocks. The former drive the customer-orders received by dealers and represent the effects of portfolio shifts or changing liquidity demands by agents outside the FX market. CK news is characterized by the simultaneous arrival of new information to all market participants and their homogeneous interpretation of its implications for equilibrium prices. Clearly, CK news shocks are akin to the public news shocks found in macro exchange rate models. I use the different terminology to stress the fact that the news must be heard simultaneously and must be homogeneously interpreted.[1]

The model also characterizes the behavior of equilibrium interdealer order flow. This variable measures the direction of trade between dealers and is a proximate determinant of equilibrium prices in many trading models. It plays an important role in my analysis because the theoretical model makes predictions about the relationship between the distribution of prices and order flow that can be exploited empirically. In particular, the model shows that equilibrium order flow is (i) unaffected by CK news shocks, and (ii) lags behind prices in responding to customer order shocks. While result (i) comes from many trading models, and follows naturally from the definition of CK shocks, result (ii) is new to the theoretical literature.[2] It implies that market-wide price movements are useful in predicting future order flow. This seems counter-intuitive from an efficient market's perspective: Can't dealers exploit the information in prices about future orders to make larger trading profits? However in this model, the market's lack of transparency makes it impossible for dealers to learn about market-wide prices quickly enough to exploit their predictive ability for future order flow. Thus, the lack of transparency plays a central role in the model. In fact, one can demonstrate that the equilibrium breaks down if transparency in the market rises above a certain level.

With these results, I develop an empirical model that can be used to examine the origins of exchange rate dynamics. The empirical model decomposes observed changes in transaction prices into three components: a CK news component, an order flow component, and a sampling component. The former incorporates the traditional macro view of exchange rate dynamics in which all innovations in spot rates are attributable to the arrival of public news in a CK framework. The second component identifies the effects of customer order shocks, the external source of private information to dealers. This component identifies the degree to which asymmetric information affects equilibrium transaction prices. The sampling component arises from the fact that there is an equilibrium transaction price distribution at each point in time. The dispersion in this distribution reflects the heterogeneous trading decisions dealers make in a market that lacks transparency.

The empirical analysis begins in Section 3 with a description of the data. The data set details

[1] An example of the distinction between CK and public news can be found in Brennan and Cao (1997). They study a model where investors hold different priors about the value of assets. When public news arrives, better-informed investors change their valuation of the asset by more than the less well informed investors.

[2] This lead-lag relationship between prices and order flow does not appear in the Portfolio Shifts Model of Evans and Lyons (1999). In that model cumulative order flow during the trading day is a proximate determinant of the price at which customer/dealer trades are conducted at the end of the day. Thus, causality runs from order flow to prices at a daily frequency. The model presented here focuses on intraday trading, so result (ii) refers to the high frequency relationship between order flow and prices.

trading activity in the spot FX market over a four-month period, May 1 to August 1996. These data are unique in that they provide information on trading between FX dealers around the world.[3] In particular, they allow us to track the pattern of trade and FX prices in the direct interdealer market on a transaction-by-transaction basis. As such, the data series constitute sequences of irregularly spaced observations on a continuous trading process. This makes standard time series methods based on regularly space observations inapplicable. Section 3 describes how GMM estimation methods can be adapted to deal with the irregular spacing problem. Estimates are obtained from the data series sampled over a fixed 5-minute observation window, allowing for the fact that the window may correspond to varying spans of "market time": the time scale at which market processes evolve at a constant rate.

Section 4 reports the empirical results based trading in the DM/$, the most heavily traded currency pair. These data reveal that there is considerable variation in the intensity at which trading takes place between dealers. Some of the variation has a well-defined intraday pattern that appears consistent with dealers based in different geographical locations entering and leaving the market. However, actual trading intensity can differ greatly from the intraday pattern on any particular day. Transaction prices and order flow also display some interesting statistical characteristics. Price changes observed over a 5-minute interval appear to be non-normally distributed and display a significant degree of negative serial correlation. Order flow, by contrast, is positively autocorrelated and highly persistent. Although the serial correlation in price changes accords with the predictions of the trading model, the persistence in order flow does not.[4] In fact, order flow's persistence emerges as a puzzling feature of FX trading.

The paper's central empirical results come from the empirical model estimates and may be summarized as follows:

- The origins of exchange rate movements vary considerably according to the state-of-the market, measured by transaction intensity.

[3] They differ from the FX quotes shown on the screens of specialist information providers, such as Reuters, Telerate, Minex and Quotron. These quotes represent indicative prices rather than the firm prices at which a dealer will enter into a transaction. Their relation to the transaction price data used here is discussed in Evans (1997). There exists a large literature studying the quote data because this was the only source of market-wide information on FX trading. A partial list of papers includes; Baillie and Bollerslev (1991), Bollerslev and Domowitz (1993) and Bollerslev and Melvin (1994), Dacorogna, et al.(1993), Engle, Ito, and Lin, (1990), Goodhart and Giugale (1993), and Guillaume, et al (1994a).

[4] The source of the negative serial correlation in the trading model does not arise from "noise" or bid-ask bounce; reasons that are often suggested for the presence of negative serial correlation in indicative FX quote changes (see, for example, Zhou 1996). Nor, as I shall show, does the presence of serial correlation imply the existence of unexploited profitable trading opportunities.

- Under normal market conditions, the CK news accounts for approximately 15 per cent of the variance in short-term price changes. As trading intensity increases, the contribution of CK news rises to a peak of approximately 40 per cent.

- The sampling component accounts for approximately 80 per cent of the variance in short-term price changes under normal market conditions. When trading intensity is very high, the sampling component's contribution falls to 17 per cent.

- Long-term price movements originate from both CK news and customer order shocks.[5] When the transaction intensity is very low, more than 90 per cent of the variance of permanent shocks to the price level are attributable to CK news shocks. When the intensity is high, the CK contribution falls below 20 per cent. In these market states, approximately 80 per cent of the variance in permanent price shocks comes from customer orders.

- Customer order shocks affect transaction prices at least 20 minutes before interdealer order flow. Their peak effect on changing the price distribution occurs approximately 15 minutes after the shock and lasts for approximately 30 minutes. The strength of these effects increases with transaction intensity. When intensity is high, customer order shocks account for more than 50 per cent of the variance in price changes over a 30 minute to 2 hour horizon.

- Customer orders have both temporary and permanent effects on the level of prices in all market states.

These results provide new perspective on exchange rate dynamics in two respects. First, they provide strong empirical support for the idea that equilibrium in the FX market is described by a distribution of prices rather than a particular price level. The existence of this distribution is key to understanding the short-term dynamics of exchange rates because so much of the variance in observed price changes is attributable to the sampling component. Second, they challenge the traditional macro view that emphasizes the role of public news as the primary source of exchange rate movements. CK news shocks are rarely the predominant source of exchange rate changes over both long or short horizons. Moreover, the contribution of customer order shocks to permanent price movements points to a source of exchange rate dynamics, over macro relevant horizons, that has been overlooked by traditional models. The concluding section of the paper discusses how these observations may lead to the more empirically successful macro models of exchange rates.

[5]By definition, long-term price movements are not affected by the samplingly component.

The remainder of the paper is organized as follows. Section 2 presents the theoretical trading model, analyses the equilibrium behavior of transaction prices and order flow, and derives an empirical model that can investigate the source of exchange rate dynamics. Section 3 describes the data set and the econometric techniques used to study the data. Section 4 presents the empirical results. Section 6 concludes.

# 2 A Model of Direct Interdealer Trading

This section presents a simple model of direct interdealer trading in the FX market. Interdealer trading accounts for about 75% of total trading in major spot markets (the remaining 25% is between dealers and non-bank customers) and breaks into two transaction types, direct and brokered. Direct trading between dealers accounts for about 50% of interdealer trade and brokered trading accounts for about 50%.

Direct interdealer trades result from bilateral conversations between dealers typically over a sophisticated E-mail system (see below). A conversation is initiated when a dealer calls another dealer on the system to request a quote. Users of the system are expected to provide a fast two-way quote with a tight spread, which is in turn dealt or declined quickly (i.e., within seconds). Thus, trade follows the so-called dealer protocol where quotes precede orders. The system allows a large number of dealer pairs to hold conversations at the same time so many transactions can take place simultaneously. Importantly, details of each conversation, such as the quotes and the decision of the initiating dealer, are only known to the counterparties. They are never transmitted via the system to other dealers in the market.

In the model below, trading takes place simultaneously between pairs of dealers according to the dealer protocol. The model also incorporates the information structure of direct dealing: Details of each transaction are private information to the counterparties and only disseminate more widely as further trading takes place. The model also conforms to the actual market in that each dealer receives customer orders. These orders are a source of private information to dealers on the state of the market and play an important role in the dynamics of transaction prices.

## 2.1 Structure

Consider a market in which two assets are traded, one riskless, and one with a stochastic payoff representing FX. The market comprises a continuum of dealers $d \in [0, 1)$ who are split equally into two groups, $\{A : a \in [0,1/2)\}$ and $\{B : b \in [1/2,1)\}$. Dealers trade with the public and among themselves to maximize expected utility defined over future wealth subject to an inventory constraint that limits the number of periods they can hold a long or short FX position.[6] Let $I_t^d$ denote the inventory of dealer $d$ at the beginning of period $t$. Under the constraint, $I_t^d$ must equal zero at the end of each trading cycle that comprises two periods.

The structure of the model is most easily described in terms of a dealer's actions over the trading cycle. Consider dealer $a \in A$ beginning her trading cycle at the start of period $t$ with wealth $w_t^a$ and no inventory, $I_t^a = 0$. At the beginning of the period, the dealer receives an order from a customer for $c_t^a$ units of FX. This customer order is to be filled at the market price; that is, the same price as trades between dealers during period $t$. A positive value for $c_t^a$ represents the net purchase of FX by the public. The customer order is only known to the recipient, dealer $a$. It is not observed by other dealers in group $A$, or by any dealers in group $B$.

After receiving the customer order, dealer $a$ must choose the price, $p_t^a$, at which she will fill the customer order and trade with other dealers. Once this price is set, an order from dealer $b \in B$ arrives for $x_t^b$ units of FX. Dealer $b$ is matched with dealer $a$ according to an exogenous matching mechanism. A positive (negative) value for $x_t^b$ denotes that dealer $b$ wishes to purchase (sell) FX. Dealer $a$ fills the customer and dealer orders at the end of the period. Her wealth and inventory at the beginning of period $t+1$ are therefore $w_{t+1}^a = (c_t^a + x_t^b)p_t^a + w_t^a$ and $I_{t+1}^a = -(c_t^a + x_t^b)$.

At the beginning of period $t+1$, dealer $a$ has the opportunity to initiate trade with the public at price $s_{t+1}$. This price is good for any amount the dealer wishes to trade and is observed by all dealers. Dealer $a$ now chooses the fraction of her inventory to trade with the public knowing that any remaining inventory must be traded with a dealer in group $B$ later in the period in order to meet the inventory constraint of $I_{t+2}^a = 0$. Let $\lambda_{t+1}^a$ denote the chosen fraction of the inventory retained for trade with other dealers. Dealer $a$ therefore sells $(1-\lambda_{t+1}^a)I_{t+1}^a$ units of FX to the public for price $s_{t+1}$. Next, dealer $a$ is randomly matched with a dealer from group $B$ who has set a price of $p_{t+1}^b$ at which he is willing to trade any amount. To meet her inventory constraint, $a$ must sell $\lambda_{t+1}^a I_{t+1}^a$ units of FX to dealer $b \in B$. With these transactions complete, dealer $a's$ wealth at the beginning of period $t+2$ is $w_{t+2}^a = \lambda_{t+1}^a I_{t+1}^a p_{t+1}^b + (1-\lambda_{t+1}^a)I_{t+1}^a s_{t+1} + w_{t+1}^a$ and her inventory is $I_{t+2}^a = 0$, satisfying the constraint.

Figure 1 depicts the sequence of events across the market. The upper and lower panels of the

[6] Actual dealers face inventory constraints that limit the size and length of time they can hold FX positions. The constraint imposed on dealers in the model is more stringent but greatly simplifies the analysis.

figure show the decisions and actions of a typical dealer in groups $A$ and $B$ respectively. Here we can see how the trading cycles of dealers overlap in the typical manner of OLG models. During any period dealers from one group are setting transaction prices while dealers from the other are deciding how to manage their inventory. Notice that all interdealer transactions take place between dealers from different groups via an exogenous matching mechanism. This stops dealers from sharing inventory risk by trading within a group. Dealers are also prohibited from initiating trades with the public during the first period of their trading cycle.

Figure 1 also shows how information arrives to each dealer. At the beginning of each period all dealers learn about the outside price, $s_t$. At the start of their trading cycle, dealers also receive customer orders, $c_t^d$. These orders are only observed by the recipient and so constitute a source of private information. Dealers also receive information from trading. Trading takes place simultaneously between pairs of dealers so that no dealer knows market-wide prices or orders as he or she trades. In the actual FX market, there is no mechanism reporting past market-wide prices or orders, so dealers must infer these aggregates from their interactions with other individual dealers. To keep the model tractable, I assume that market-wide price and order flow information becomes public with a one period delay[7]: After trading has ending in period $t$, all dealers observe the average transaction price and dealer order flow from interdealer trading in period $t-1$, denoted by $p_{t-1}$ and $x_{t-1}$ respectively. Thus, dealers make trading decisions before they obtain any market-wide information on interdealer trading in the previous period.

Equilibrium transaction prices and dealer orders are determined endogenously by dealers in response to exogenous customer orders, $c_t^d$, and outside prices, $s_t$. The customer order received by each dealer is comprised of common and idiosyncratic components:

$$\begin{aligned} c_t^d &= c_t + u_t^d, \\ c_t &= v_t + \alpha v_{t-1}, \end{aligned} \tag{1}$$

where $u_t^i$ and $v_t$ are i.i.d. normal random variables with zero means and variances $\Sigma_u$ and $\Sigma_v$. The

[7]Ideally, the rate at which market-wide price and order flow information became public would be determined endogenously. In this case, trading between dealers via brokers would undoubtedly play a role because information on brokered trades is disseminated across the whole market (see Lyons 2000). Presumably, brokered trading speeds up the market-wide dissemination of information on direct interdealer trading. Hellwig (1981) provides the first example of a rational expectations trading model where the transmission of market prices is delayed.

outside price is assumed to follow a random walk

$$s_t = s_{t-1} + \varepsilon_t, \tag{2}$$

where $\varepsilon_t$ is an i.i.d. normal variable with mean zero and variance $\Sigma_s$.

Several comments concerning the specification of these processes are in order. Recall that the duration of a trading period in this model is very short: it is only as long as is necessary to conduct the transactions depicted in Figure 1. Thus, (1) and (2) characterize the behavior of $c_t^d$ and $s_t$ over very short time periods. With this perspective, the random walk specification implies that high frequency FX returns are unpredictable given their own history (i.e., $E[(s_{t+1} - s_t)/s_t | s_t, s_{t-1}, ...] = 0$). This implication of (2) makes outside prices semi-strong form efficient because their history is known to all dealers and members of the public. The random walk characterization is also consistent with existing empirical evidence on high frequency FX returns.[8] The short duration of a trading period also motivates the presence of serial correlation in $c_t^d$. The idea is that members of the public place FX orders with dealers to facilitate real transactions elsewhere in the economy. If these transactions take longer than FX transactions, the customer orders received by dealers will generally be serially correlated.[9] I assume that the serial correlation takes the form of an MA(1) process for tractability.

Equations (1) and (2) also embody an important assumptions about the public's price elasticity of demand for FX. Recall that $s_t$ represents the price at which dealers can trade any amount of FX with the public at the start of the second period in their trading cycle. Thus, $\Delta s_t = \varepsilon_t$ can be viewed as the change in the public's inverse demand function. If the demand for FX were less than infinitely elastic, $\varepsilon_t$ would generally be correlated with innovations in the public's holdings of FX. In this model, these holdings vary as a result of the trades initiated by dealers. Thus, in so far as dealers' trading decisions are affected by customer orders, $\varepsilon_t$ should be correlated with the lagged values of $v_t$. Equations (1) and (2) imply that this correlation is zero because $v_t$ and $\varepsilon_t$ are independent serially uncorrelated shocks. Thus the specification for customer orders and outside

[8]While there is some evidence of weak negative first-order serial correlation in the high frequency returns constructed from indicative quotes, these are typically attributed to bid-ask bounce or "noise" (e.g., Zhou 1996, and Andersen and Bollerslev 1998), factors that are absent from the theoretical model.

[9]As an example of how serial correlation in customer orders can arise, suppose agents I and II sign a contract at the start of period $t$ which specifies that I will pay II $z$ units of FX in exchange for a good. Agent I places the order for $c_t^a = z$ with dealer $a \in A$ and receives the FX within period $t$. The trade of goods and FX between I and II then takes place. The earliest time at which II can sell the FX is in period $t+1$ by placing a customer order of $c_{t+1}^b = -z$ for some dealer $b \in B$.

prices implicitly assumes that the public's demand for FX is infinitely elastic.[10] This assumption aids in the analysis that follows for two reasons. First, it greatly simplifies the optimal trading problem facing dealers. Specifically, it makes the speculative positions taken by dealers during the second period of their trading cycles independent of inventory holdings. The implications of relaxing this restrictions are discussed below. Second, it means that $\varepsilon_t$ shocks represent CK news about the value of FX: all dealers observe and interpret the implication of $\varepsilon_t$ shocks for equilibrium transaction prices in the same way. This would not be true if $\varepsilon_t$ were correlated with $v_{t-1}$. In this case, the pricing implications of $\varepsilon_t$ would be interpreted differently across dealers because they hold difference priors about $v_{t-1}$. Differentiating between the effects of CK news and other shocks would then be much harder.

## 2.2 Equilibrium

Equilibrium in this model is a described by a sequence of transaction price and order flow distributions consistent with market clearing and the rational trading decisions of dealers. The requirements of market clearing are very simple. In the first period of the trading cycle, dealers must fill customer and dealer orders at a single price. In the second period, the dealer must trade with her assigned counterparty at the price set so as to finish the trading cycle with no FX inventory. Hence, the order *from* dealer $d$ at the end of her trading cycle is

$$x_t^d = -\lambda_t^d I_t^d = \lambda_t^d (c_{t-1}^d + x_{t-1}^*), \tag{3}$$

where $x_{t-1}^*$ represents the dealer order she received one period earlier. (Hereafter, "*" denotes variables chosen by another dealer.) This condition must hold for every dealer in the market. It implies that the dealer orders received during the first period of a trading cycle match the inventory another dealer wishes to trade in the second period of their trading cycle.

Trading decisions are made to maximize expected utility defined over wealth at the end of trading in period $T_d$ for $d = \{a, b\}$, where $I_{T_d}^d = 0$. Formally, for dealer $d$ starting her trading cycle

[10]This contrasts with Evans and Lyons (1999) where the public's overnight demand of FX is less than infinitely elastic. The model present in that paper focuses on how the cumulative effects of FX trading affect the daily dynamics of exchange rates and so complements this analysis of intraday dynamics.

in period $t$, prices and orders are chosen as

$$p_t^d = \arg\max E\left[\mathcal{U}(w_{T_d}^d)|\Omega_t^d\right], \tag{4}$$

$$\lambda_{t+1}^d = \arg\max E\left[\mathcal{U}(w_{T_d}^d)|\Omega_{t+1}^d\right], \tag{5}$$

subject to

$$w_{t+2}^d = w_t^d + (c_t^d + x_t^*)(p_t^d - s_{t+1}) - \lambda_{t+1}^d(c_t^d + x_t^*)(p_{t+1}^{d*} - s_{t+1}), \tag{6}$$

where the dealer's information sets are

$$\Omega_t^d = \{s_t, p_{t-2,}x_{t-2}, c_t^d, p_{t-1}^*\} \cup \Omega_{t-1}^d, \tag{7}$$

$$\Omega_{t+1}^d = \{s_{t+1}, p_{t-1}, x_{t-1}, x_t^*\} \cup \Omega_t^d. \tag{8}$$

The terms on the right of (7) and (8) show the new information received between the beginning of one period and the next. Public information arrives in the form of outside prices and the average of past prices and order flow. Dealers also receive private information between the start of periods $t-1$ and $t$ in the form of $c_t^d$ and $p_{t-1}^*$, the customer order and the price they were quoted during trading in $t-1$. Between the start of $t$ and $t+1$ dealers receive private information in the form of $x_t^*$, the dealer order received in period $t$.

To find the equilibrium, I first posit a form for the dynamics of the equilibrium distribution of prices and dealer orders. Using these processes, I then solve the dealer's optimization problem to find how optimal prices and orders are set over the trading cycle. Finally, I check that the posited dynamics for prices and orders are consistent with the solution for the optimization problems facing all dealers. Appendix A provides a detailed derivation of the equilibrium. The results are presented below.

**Proposition:** *If dealers hold rational expectations about the equilibrium process for prices and orders, and expected utility is defined as*

$$E\left[\mathcal{U}(w_{T_d}^d)|\Omega_t^d\right] = E\left[w_{T_d}^d|\Omega_t^d\right] - \frac{\theta}{2}Var\left(w_{T_d}^d|\Omega_t^d\right),$$

*the solutions to the optimization problems in (4) - (8) are*

$$p_t^d - E[s_{t+1}|\Omega_t^d] = \frac{(c_t^d + E[x_t^*|\Omega_t^d])}{\theta Var(x_t^*|\Omega_t^d)} - \varphi \frac{(E[p_{t+1}^*|\Omega_t^d] - s_t)}{Var(x_t^*|\Omega_t^d)} \tag{9}$$

$$\lambda_{t+1}^d = -\frac{(E[p_{t+1}^*|\Omega_{t+1}^d] - s_{t+1})}{\theta Var(p_{t+1}^*|\Omega_{t+1}^d)(c_t^d + x_t^*)} \tag{10}$$

*for $d = \{a, b\}$, $a \in A$, and $b \in B$. The coefficient $\varphi$, and the conditional variances $Var(x_t^*|\Omega_t^d)$* and *$Var(p_{t+1}^*|\Omega_{t+1}^d)$ are constants that depend upon the parameters governing equilibrium price and order flow distributions. Typical elements of these distributions can be written as*

$$p_t^d = s_t + \eta_1 c_t^d + \eta_2 \tilde{v}_{t-1}^d \tag{11}$$

$$x_t^d = \eta_3(v_{t-1} + u_{t-1}^d) \tag{12}$$

*where $\eta_1$, $\eta_2$ and $\eta_3$ are all functions of the structural parameters, $\theta$, $\alpha$, $\Sigma_v, \Sigma_u$ and $\Sigma_s$. $\tilde{v}_{t-1}^d$ is dealer $d'$s estimate of $v_{t-1}$ at the end of her trading cycle in $t-1$. In equilibrium, this estimate is equal to $v_{t-1} + e_{t-1}^d$, where $e_{t-1}^d \sim i.i.d.N(0, \Sigma_e)$ with $\Sigma_e = \Sigma_u/(1 - (\eta_2/\eta_1)^2)$.*

The equilibrium displays several noteworthy features. First, (11) and (12) describe elements in the cross-sectional distribution of prices and orders. Customer orders are the sole source of this heterogeneity. Differences in the customer orders received at the start of the trading cycle directly contribute to the dispersion of prices (via $c_t^d$ in eqn. 11) and also to the dispersion of dealer orders chosen next period (via $u_{t-1}^d$ in eqn. 12). Differences in customer orders also lead dealers to have different estimates of $v_{t-1}$, denoted by $\tilde{v}_{t-1}^d$, which also contribute to the dispersion of prices. Appendix A shows that the heterogeneity in prices and orders disappears if there are no idiosyncratic shocks to customer orders.

The second feature to note is that changes in outside prices have a one-to-one effect on equilibrium prices but no impact on order flow. The reason is that changes in $s_t$ represent CK news about the value of FX: Recall that $s_t$ follows a random walk with i.i.d. innovations, so $E[s_{t+1}|\Omega_t^d] = s_t$ for all dealers $d$. Innovations in $s_t$ therefore lead all dealers to revise their forecast for outside prices in the same way. Since prices are set as a markup over $E[s_{t+1}|\Omega_t^d]$ that depends on expected dealer orders (see eqn. 9), and dealer orders depend on the expected markup, $E[p_{t+1}^*|\Omega_{t+1}^d] - s_{t+1}$ (see eqns. 3 and 10), in the rational expectations equilibrium no dealer setting prices expects a change in dealer orders, and no dealer choosing their order expects a change in the markup. Consequently, prices move one-for-one with outside prices and interdealer order flow is unaffected.

Notice also that customer order shocks affect both equilibrium prices and order flow. Equilibrium markups, $p_t^d - s_t$, follow an MA(1) process while order flow depends on customer order shocks lagged one period. The intuition behind these results is most easily understood by considering how a customer order affects the optimal trading decisions of an individual dealer. Customer orders are received at the beginning of the trading cycle and contain three components, a common shock, $v_t$, an idiosyncratic shock $u_t^d$, and a lagged common shock, $v_{t-1}$. Dealers need to estimate $v_t$ and $v_{t-1}$ in order to optimally set prices because their forecasts of $x_t^*$ and $p_{t+1}^*$ depend on these estimates. Appendix A shows that these estimates are formed by combining the information in customer orders and past prices:

$$E\left[v_{t+i}|\Omega_t^d\right] = \begin{cases} 0 & i > 0 \\ \phi(c_t^d - \alpha\delta_e\tilde{v}_{t-1}^d) & i = 0 \\ \delta_e\tilde{v}_{t-1}^d + \alpha\phi(c_t^d - \alpha\delta_e\tilde{v}_{t-1}^d) & i = -1 \end{cases}, \tag{13}$$

where

$$\tilde{v}_{t-1}^d \equiv \frac{1}{\eta_1}\left(p_{t-1}^* - s_{t-1}\right) - \frac{(\alpha+\eta_2/\eta_1)}{\eta_1}\left(p_{t-2} - s_{t-2}\right) + \frac{(\alpha+\eta_2/\eta_1)^2}{\eta_3}x_{t-2}, \tag{14}$$

with $\phi \equiv \Sigma_v(1-\delta_e)/\left(\Sigma_u + (1+\alpha^2(1-\delta_e))\Sigma_v\right)$ and $\delta_e \equiv \Sigma_v/(\Sigma_v + \Sigma_e)$. Dealer estimates contain two idiosyncratic components: $p_{t-1}^*$, the price the dealer was quoted in period $t-1$ trading, and $c_t^d$, the customer order received before period $t$ trading. Equation (13) shows how a customer order changes the estimates of $v_{t-1}$ and $v_t$. Because dealers are paired through an independent random matching process, dealer $d$'s estimate of $u_{t-1}^*$ (the idiosyncratic customer shock received by $d$'s period $t$ counterpart) is zero, so the rationally expected order flow is $E[x_t^*|\Omega_t^d] = \eta_3 E[v_{t-1}|\Omega_t^d]$ from (12). Similarly, (11) implies that $E[p_{t+1}^*|\Omega_t^d] - s_t = \eta_1 E[c_{t+1}^*|\Omega_t^d] + \eta_2 E[\tilde{v}_t^*|\Omega_t^d]$, which in turn is equal to $(\eta_1\alpha+\eta_2)E[v_t|\Omega_t^d]$ because $E[e_t^*|\Omega_t^d] = 0$ through random matching. Thus, customer orders affect prices directly via the first term in (9) and indirectly through revisions in expectations, with a net effect measured by the $\eta_1$coefficient.

In the second period of the trading cycle the dealer must decide how much of her inventory to retain for trade with other dealers. As (10) shows, this decision depends on her expectations concerning the quote she will receive. In equilibrium these expectations are given by

$$E[p^*_{t+1}|\Omega^d_{t+1}] = s_{t+1} + \eta_1 E\left[\left(v_{t+1} + u^*_{t+1}\right)|\Omega^d_{t+1}\right] + (\eta_1\alpha + \eta_2)E\left[v_t|\Omega^d_{t+1}\right] + \eta_2 E[e^*_t|\Omega^d_{t+1}]. \quad (15)$$

As above, random matching ensures that $E\left[u^*_{t+1}|\Omega^d_{t+1}\right] = E[e^*_t|\Omega^d_{t+1}] = 0$, so expectations depend on $E\left[v_{t+1}|\Omega^d_{t+1}\right]$, $E\left[v_t|\Omega^d_{t+1}\right]$ and $s_{t+1} \in \Omega^d_{t+1}$.Recall from (8) that the dealer only learns the values of $\{s_{t+1}, p_{t-1}, x_{t-1}, x^*_t\}$ between the start of periods $t$ and $t+1$. On the basis of this information, Appendix A shows that the dealer's best estimates of $v_t$ are given by

$$E\left[v_{t+i}|\Omega^d_{t+1}\right] = \begin{cases} 0 & i > 0 \\ \delta_u\left(c^d_t - \frac{\alpha}{\eta_1}\left(p_{t-1} - s_{t-1}\right) + \frac{\alpha(\alpha+\eta_2/\eta_1)}{\eta_3}x_{t-1}\right) & i = 0 \end{cases}. \quad (16)$$

Equation (16) shows that a customer order received in period $t$ affects the dealer's estimate of $E\left[v_t|\Omega^d_{t+1}\right]$ but not $E\left[v_{t+1}|\Omega^d_{t+1}\right]$. Thus, customer orders received at the start of the trading cycle affect dealer orders next period because they contain information that dealers find useful in forecasting quotes.

Customer orders also have a lagged effect on prices. Because the price set by dealer $d$ reflects in part the customer order she received in period $t$, the information received by her trading counterpart in $t$ is also affected by that order. Specifically, combining (11) and (14) gives

$$\tilde{v}^*_t \equiv \frac{1}{\eta_1}\left(\eta_1 c^d_t + \eta_2\tilde{v}^d_{t-1}\right) - \frac{(\alpha+\eta_2/\eta_1)}{\eta_1}\left(p_{t-1} - s_{t-1}\right) + \frac{(\alpha+\eta_2/\eta_1)^2}{\eta_3}x_{t-1}$$

which is a component of $E[v_{t+i}|\Omega^*_{t+1}]$. As we saw above, these estimates are used by dealers to set prices optimally. Hence, a customer order received by one dealer will affect the price set by another the next period because transaction prices convey information that is useful in making subsequent trading decisions.

The origins of the price and order flow dynamics now become clear. Persistence in the $c^d_t$ process means that the customer orders received by dealers in one group are correlated with the customer orders received by dealers in the other. Since trading always takes place between traders from different groups, this correlation means that customer orders affect dealers' expectations about the dealer orders or quotes they will received while trading. As a consequence, customer orders affect dealer orders and transaction prices.

One particularly important feature of the model is that dealers only observe market-wide transaction prices and order flow with a delay. After trading has ended in period $t$, all dealers observe the average transaction price and dealer order flow from interdealer trading in period $t-1$. This lack of transparency means that dealers must make trading decisions before there is precise information about the common shock to customer orders last period. To appreciate how this lack of information affects trading behavior, it is useful to consider what would happen if all dealers learned the value of $v_{t-1}$ immediately after period $t-1$ trading ended so that $v_{t-1} \in \Omega_t^d$. In this case, (15) simplifies to $E[p_{t+1}^*|\Omega_{t+1}^d] = s_{t+1} + (\eta_1\alpha + \eta_2)v_t$ so dealers hold the same expectations about future quotes. These expectations, in turn, imply that dealer orders are $x_{t+1}^d = -\psi(\eta_1\alpha + \eta_2)v_t$ from (3) and (10). Since these orders are completely predictable to dealers setting prices in $t+1$, $Var(x_{t+1}^*|\Omega_{t+1}^d) = 0$ and the optimal markup shown in (9) becomes infinite. Thus, increasing the degree of transparency leads to a breakdown in the equilibrium.

One further perspective one the equilibrium comes from considering the origins of dealer order flow. In this model dealer orders represent the unwinding of the speculative position taken midway through the trading cycle. At this point, dealers observe outside prices and decide on the fraction of their inventory to retain for trade with other dealers. Recall that for a dealer starting a trading cycle in period $t$, her inventory at the start of $t+1$ is $I_{t+1}^d = -(c_t^d + x_t^*)$. Thus, a positive value of $\lambda_{t+1}^d(c_t^d + x_t^*)$ represents a short position established at the start of $t+1$ that must be unwound by purchasing $x_{t+1}^d = \lambda_{t+1}^d(c_t^d + x_t^*)$ from another dealer later in the period. Equation (12) shows that the size of these positions depend on the difference between the outside price and the price they expect to be quoted by another dealer, and the variance of that quote. The optimal position does not depend on the inventory the dealer started the period with because the public's demand for any unwanted inventory is infinitely elastic. Thus, customer orders have no direct effect on subsequent order flow through an inventory-control channel.[11] Instead, they affect dealer order flow because they change expectations and induce dealers to take speculative positions that ultimately are unwound through dealer orders. Specifically, suppose a positive $v_t$ shock raises customer orders received by group $A$ dealers in period $t$. If $\alpha$ is positive (negative), these dealers will revise their forecasts of $c_{t+1}^*$ upward (downward), and also their forecast of the quote they will receive from a group $B$ dealer in $t+1$ trading. Consequently, group $A$ dealers will establish longer (shorter) speculative positions at the start of $t+1$ that in turn lead to lower (higher) average dealer order flow, $x_{t+1}$.

[11] If the public's demand was less than perfectly elastic (see Evans and Lyons 1999, for example), optimal speculative positions would depend in part on the dealer's inventory. Although this modification would make the model very much harder to solve, it would introduce an operative inventory control channel that in principle could lead to serially correlated order flow.

### 2.3 Empirical Implications

Equations (11) and (12) describe elements in the equilibrium cross-sectional distributions of prices and orders at a point in time. Suppose we observe the average order $x_t$, and an randomly drawn price $p_t^o$, from these distributions at time $t$. Since $p_t^o = p_t^d$ for some dealer $d$, equations (1), (2) and (11) imply that we can write the observed price as

$$p_t^o = p_t + \omega_t^o, \tag{17}$$

where $\omega_t^o = \eta_1 u_t^d + \eta_2 e_{t-1}^d$ is an idiosyncratic shock. $p_t$ is the average price that follows

$$\begin{aligned} \Delta p_t &= \eta_1 v_t + (\eta_2 - (1-\alpha)\eta_1)v_{t-1} - (\eta_1\alpha + \eta_2)v_{t-2} + \varepsilon_t \\ &= B(L)v_t + \varepsilon_t, \end{aligned} \tag{18}$$

where $B(L)$ represents a polynomial in the lag operator, $L$. The size of the $\omega_t^o$ shock depends on the identity of the dealer whose price we observe. If the observed price is drawn randomly from the cross-sectional distribution of prices every period, $\omega_t^o$ will be serially independent. It will also be independent from leads and lags of $\Delta p_t$.

Combining (17) and (18) gives the period-by-period change in the observed price as

$$\Delta p_t^o = \varepsilon_t + B(L)v_t + \omega_t^o - \omega_{t-1}^o. \tag{19}$$

This equation decomposes observed price changes into three components: the CK news component $\varepsilon_t$, the order flow component $B(L)v_t$, and the sampling component $\omega_t^o - \omega_{t-1}^o$. The first component incorporates the traditional macro view of exchange rate determination in which all innovations in spot rates are attributable to the arrival of public news. More precisely, this view assumes that: (i) all information relevant for exchange rate determination is CK, and (ii) the mapping from information to equilibrium prices is also CK. $\varepsilon_t$ shocks play the role of CK news in this model because all dealers learn the value of $\varepsilon_t$ simultaneously and hold the same view about the mapping from $\varepsilon_t$ to the equilibrium transaction price distribution. The second and third components come from the trading-theoretic perspective of the model. The second captures the role of customer orders as a source of asymmetric information to dealers while the third arises from the existence of an equilibrium distribution of transaction prices at a point in time.

While the three components are mutually independent, they cannot be separately identified without some further information. This is provided via the trading model in the form of average dealer order flow. In particular, (12) implies that average dealer order flow follows,

$$x_t = \eta_3 v_{t-1} = C(L)v_t, \tag{20}$$

so we can rewrite (19) as

$$\Delta p_t^o = D(L)x_t + \varepsilon_t + \omega_t^o - \omega_{t-1}^o \tag{21}$$

where $D(L) = B(L)C(L)^{-1}$.

Equation (21) takes the form of regression with an MA(1) error-structure that can be used to estimate the different price change components. For example, in the case of the model presented above, (21) becomes

$$\Delta p_t^o = a_1 x_{t+1} + a_2 x_t + a_3 x_{t-1} + \varepsilon_t + \omega_t^o - \omega_{t-1}^o. \tag{22}$$

where $a_1 = \eta_1/\eta_3, a_2 = (\eta_2 + (\alpha - 1)\eta_1)/\eta_3$, and $a_3 = -(\eta_2 + \alpha\eta_1)/\eta_3$. Note that the right hand side of this equation contains both a lead and lag of order flow. The leading term arises because $v_t$ shocks affect prices contemporaneously but order flow with a one period lag. Hence, the value of $a_1$ captures the contemporaneous effect of $v_t$ on prices.

Although the structure of the trading model honors the main features of direct interdealer trading, it limits the actions of dealers in ways that the market does not. In particular, the model imposes a rigid structure to the timing of events and assumes the presences of an exogenous matching mechanism that has no exact market counterpart: Actual dealers can choose to initiate conversations with any other dealer at any time they are not responding to a quote request from another dealer. In recognition of these limitations, I will not attempt structural estimation of the trading model with (22).

I will focus instead on (21). This equation allows us to study the origins of exchange rate movements beyond the confines of the specific trading model. In particular, decompositions of exchange rate changes using (21) rest on three identification assumptions: (i) the orthogonality of $\omega_t^o$ to all leads and lags of $\Delta p_t$, (ii) the dependence of $x_t$ on only $v_t$ shocks, and (iii) the absence of serial correlation in $\varepsilon_t$. Assumption (i) will hold provided that the observed price is an independent random drawing from the distribution of prices each period. This is a reasonable assumption in a market where there are a large number of dealers who can execute transactions at any time. Assumption (ii) rules out the possibility that common knowledge shocks affect order flow. This assumption holds true in a wide class of rational trading models and has been used elsewhere in empirical research (see, for example, Hasbrouck 1991 and Payne 1999). Assumption (iii) implies that all CK shocks have permanent affects on the spot exchange rate. This assumption is a little stronger because, as macro models show, it is possible for public news to have a temporary effect on spot rates. However, given that the estimated speed of mean reversion in spot rates following such shocks is typically measured in weeks, months or longer, and we will be considering observations over five minute intervals, this assumption is also rather weak. Notice that none of these assumptions restrict the form of the polynomial $D(L)$, which captures the dynamics of prices and order flow via $B(L)$ and $C(L)$. The model presented above places restrictions on $B(L)$ and $C(L)$ that would almost surely be different in richer theoretical settings. Thus, in the empirical analysis that follows, I will estimate (21) with quite general specifications for $D(L)$. The parameter estimates thus obtained will then be used to decompose price changes into their various components.

# 3 Empirical Analysis

## 3.1 The Data

The analysis below utilizes new data on trading activity in the DM/$ spot FX market over a four-month period, May 1 to August 31, 1996. The data set contains time-stamped tic-by-tic data on actual transactions taking place through the Reuters Dealing 2000-1 system via an electronic feed that was customized for the purpose by Reuters. This is the most widely used electronic dealing system. According to Reuters, over 90% of the world's direct interdealer transactions took place through the system.

Trades on the D2000-1 system take the form of electronic bilateral conversations. The conversation is initiated when a dealer calls another dealer on the system to request a quote. Users of the system are expected to provide a fast two-way quote with a tight spread, which is in turn dealt or declined quickly (i.e., within seconds). To settle disputes, Reuters keeps a temporary record of all the conversations on the system. This record is the source of the transactions data. Every time an electronic conversation on D2000-1 results in a trade, the Reuters feed provides a time-stamped record of the transactions price, a bought or sold indicator, and a measure of cumulative trading volume.

Several features of the data are particularly noteworthy. First, they provide transaction information for the whole interbank market over the full 24-hour trading day. This contrasts with earlier transaction data sets covering single dealers over some fraction of the trading day (e.g. Lyons 1995, Yao 1997a, and 1997b, and Bjonnes and Rime 1998). The data set makes it possible, for the first time, to analyze trading patterns and prices at the level of "the market." The only other multiple-dealer data set in the literature covers brokered interdealer transactions (the electronic system examined by Goodhart, Ito and Payne 1996, and Payne 1999). The system they examine, however, accounts for only a small fraction of daily trading volume.[12]

Second, these market-wide transactions data are not observed by individual FX dealers on the system as they trade. Though dealers have access to their own transaction records, they do not have access to others' transactions on the system. The transactions data therefore represents a history of market activity that market participants could only infer indirectly. This feature has important implications for interpreting the results reported below.

Third, the data cover a relatively long time span (four months) in comparison with other micro data sets. This span provides as truly vast number of minute-by-minute observations on trading activity across a wide variety of "market states". In the analysis that follows, I utilize that data collected between 00:00:01 BST on Monday to 24:60:60 BST on Friday. (All time is measured relative to British Summer Time (BST) which corresponds to GMT plus one hour). This time interval appears to span the week of trading in the DM/$ fairly well. Although the D2000-1 system runs 24 hours a day, 7 days a week, it rarely recorded trades outside this interval. Excluding weekends and a feed interruption caused by a power failure, there are 79 full trading days in the sample.

The analysis below concentrates on (i) transaction prices, (ii) interdealer order flow, and (iii) trade intensity. Transactions take two forms in the data. If a dealer initiating a conversation ends up buying dollars, the transaction price is equal to the ask quote in DMs per dollar offered by the other dealer. I refer to this as the DM purchase price for dollars, $p_t^+$. If the dealer initiating a conversation ends up selling dollars, the transaction price will be equal to the bid quote given by the other dealer. I refer to this as the DM sale price for dollars, $p_t^-$. Thus the designation of a transaction price as a purchase or sale price depends on who initiates the transaction. Buyer-initiated trades take place at the purchase price while seller-initiated trades take place at the sale

[12]There is also evidence that dealers attach more informational importance to direct interdealer order flow than to brokered interdealer order flow, (see Bjonnes and Rime 1998).

price. Interdealer order flow, $x_t$, is defined by the difference between the number of buyer-initiated orders and seller-initiated orders per period. Trade intensity, $n_t$, is defined as the number of trades per period.

### 3.2 Econometrics

The spot FX market is open continuously in the sense that dealers can trade with one another via the D2000-1 system 24 hours a day, 7 days a week. The system also allows conversations between many dealer pairs to take place at the same time. Thus, it is possible for multiple trades to be concluded, and recorded, at the same instant. The resulting data set constitutes a sequence of irregularly spaced observations on a continuous trading process. At some points in the sample, the gaps between successive trades span many minutes, while at others several trades appear with the same second-by-second time stamp. I will not attempt to directly model these irregular timing patterns in the analysis below. Instead I will utilize prices, order flow and trade intensity measured relative to a fixed 5 minute observation interval. Hence, $p_t^+$ and $p_t^-$ are respectively the last dollar purchase and sale price recorder during interval $t$; $x_t$ is the difference between the number of buyer-initiated and seller-initiated trades during interval $t$; and $n_t$ is the number of transactions during interval $t$.

One drawback to adopting a fixed observation interval is that there are periods of the day when no transactions take place during the interval. I designate the price and order flow observations from these periods as "missing". All the statistics and estimates reported below are calculated without the use of these observations. For example, in computing the first order autocorrelation coefficient in the $\Delta p_t^+$ series, I only use consecutive observations on $\Delta p_t^+$ and $\Delta p_{t-1}^+$ for which none of the values for $p_t^+$ were "missing". More generally, I employ the GMM estimation procedure described below.

All the statistics and empirical models considered below can be written in the state-space form:

$$\begin{aligned} \xi_t &= A\xi_{t-1} + \zeta_t, \\ y_t &= C\xi_t, \end{aligned} \tag{23}$$

where $\xi_t$ is a $q$-dimensioned state vector, and $y_t$ is a $r$-dimensioned vector of observed variables. $\zeta_t$ is a $q$-dimensioned vector of shocks with zero means that are uncorrelated with $\xi_{t-1}$, serially uncorrelated and have covariance matrix $\Omega$. Although the form of $A$, $C$ and $\Omega$ vary according to the particular application, in all cases the eigenvalues of $A$ lie inside the unit circle so that $\xi_t$ and $y_t$ follow stationary processes. Hence, (23) implies that the unconditional means of $\xi_t$ and $y_t$ are respectively equal to a $q$ and $r$-dimensioned vector of zeros. (23) also implies that the covariance of the states, $\Gamma(k) \equiv Cov(\xi_t, \xi'_{t-k})$, is computed as $\Gamma(k) = A\Gamma(k-1)$ with $\Gamma(0) = vec^{-1}\left[(I - A \otimes A)^{-1} vec(\Omega)\right]$. The covariance of the observed variables is therefore given by

$$Cov(y_t y'_{t-k}) \equiv \gamma(k) = C\Gamma(k)C'. \tag{24}$$

In some applications, I also make us of a $j$-dimensioned vector of instruments, $z_t$, with the property $cov(y_t, z'_{t-i}) = \mathbf{0}$ for $i \geq 0$.

Let $\theta$ represent the vector of parameters to be estimated. As in the standard GMM case, I consider orthogonality conditions of the form

$$E\left[m_t(k;\theta)\right] = 0 \tag{25}$$

where

$$m_t(k;\theta) = \mathcal{D}(k)\begin{bmatrix} vec(y_t z'_{t-k}) \\ vec(y_t y'_{t-k} - \gamma(k;\theta)) \end{bmatrix}$$

for $k = 0, 1, ..K$. $\mathcal{D}(k)$ is a vector of ones and zeros that selects the moments to be included in $m_t(k;\theta)$. (25) gives a maximum of $rj + r^2$ independent conditions when $k > 0$ and $rj + r(r+1)/2$ conditions when $k = 0$.

To compute the GMM estimates, let $m_t(\theta) = [m_t(0;\theta), m_t(1;\theta).....m_t(K;\theta)]'$ be vector of selected moment conditions. While all the elements of $m_t(\theta)$ can be computed for any period $t$, if a particular element involves a value for $y_t$ or $y_{t-k}$ designated as a "missing" observation, the result is also designated "missing". This holds true irrespective of the value of $\theta$ so the set of "missing" elements in $m_t(\theta)$ will not vary with $\theta$ for a particular $t$. Let $\Lambda = \{t_1^*, .t_2^*...t_T^*\}$ denote the set of observations for which none of the elements in $m_t(.)$ are "missing". The estimates of $\theta$ are found by minimizing

$$Q(\theta) = m_{T^*}(\theta)' W^{-1} m_{T^*}(\theta) \tag{26}$$

where

$$m_{T^*}(\theta) = \frac{1}{T^*} \sum\nolimits_{\Lambda} m_t(\theta),$$

with $T^*$ equal to the number of observations in $\Lambda$. I follow the standard practice of first setting the weighting matrix $W$ equal to the identity to obtain consistent estimates of $\theta$. These estimates, $\tilde{\theta}$, are then used to compute a consistent estimate of the optimal weighting matrix. The form of this weighting matrix varies across applications according to whether elements of $m_t(\theta)$ are serially correlated under the null hypothesis of a correctly specified model. The most general weighting matrix I consider follows the form proposed by Newey and West (1987):

$$\tilde{W} = \Gamma_{0,T^*} + \sum_{v=1}^{\kappa} \left\{ 1 - \left[ \frac{v}{\kappa+1} \right] \right\} \left( \Gamma_{v,T^*} + \Gamma'_{v,T^*} \right),$$

where

$$\Gamma_{v,T^*} = \frac{1}{T^*} \sum\nolimits_{\Lambda} m_t(\tilde{\theta}) m_{t-v}(\tilde{\theta})'.$$

In applications where all the elements of $m_t(\theta)$ are serially uncorrelated, $\kappa$ is set to zero so that $\tilde{W} = \Gamma_{0,T^*}$. The GMM estimates, $\hat{\theta}$, are found by minimizing (26) with $W = \tilde{W}$. The asymptotic covariance matrix of the resulting estimates is $\hat{V} = [\hat{G}\tilde{W}^{-1}\hat{G}']^{-1}$where $\hat{G} = \partial m_{T^*}(\hat{\theta})/\partial\theta'$.

Several facets of this estimation technique may be illustrated by considering its application to the regression in (22). In this case, $y_t = \Delta p_t^o - a_1 x_{t+1} - a_2 x_t - a_3 x_{t-1}, \xi_t' = [\varepsilon_t, \omega_t^o, \omega_{t-1}^o]$, and $\zeta_t' = [\varepsilon_t, \omega_t^o, 0]$ with

$$C = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \qquad \text{and} \qquad \Omega = \begin{bmatrix} \Sigma_\varepsilon & 0 & 0 \\ 0 & \Sigma_\omega & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The GMM estimates of $\theta = [a_1, a_2, a_3, \Sigma_\varepsilon, \Sigma_\omega]$ can be found with instruments $z_t = [1, x_{t+1}, x_t, x_{t-1}]$ and the moments

$$m_t(\theta) = \begin{bmatrix} y_t & y_t x_{t+1} & y_t x_t & y_t x_{t-1} & y_t^2 - \gamma(0;\theta) & y_t y_{t-1} - \gamma(1;\theta) & ..... & y_t y_{t-k} - \gamma(k;\theta) \end{bmatrix}.$$

It is clear in this application that the GMM technique does not necessarily provide the most efficient parameter estimates. For example, if $\varepsilon_t$ and $\omega_t^o$ were normally distributed and the time series for $y_t$ contained no missing observations, $\theta$ would be most efficiently estimated by maximum likelihood with the aid of the Kalman Filter. However, balanced against this, the GMM technique offers two important advantages. First, it does not require any distributional assumptions regarding the error processes. Although the trading model implies that $\varepsilon_t$ and $\omega_t^o$ are normally distributed, this arises from modeling assumptions made for analytic convenience rather than any deep economic reason. In reality, the distribution of $\varepsilon_t$ and $\omega_t^o$ could be far from normal. The second advantage stems from the presence of missing observations. Although the Kalman Filtering algorithm can be extended to deal with one or two missing observations (see Harvey 1989), dealing with many adds considerable to the complexity of the filter and makes estimation very computational burdensome. By contrast, the GMM technique can deal with many missing observations very easily. Moreover, because the data set spans four months, the adoption of the five minute observation interval provides us with a truly vast number of (non-missing) observations. As a consequence, the GMM estimates of $\theta$ are extremely precise judged by the metric of their asymptotic distribution.

# 4 Results

## 4.1 Sample Statistics

Although the D2000-1 system runs 24 hours a day, the vast majority interdealer transactions in the DM/$ are concentrated during the European trading. This institutional feature gives rise to recurrent intraday patterns in the data. Exemplifying this phenomena, Engle, Ito and Lin (1990), Baillie and Bollerslev (1991), Bollerslev and Domowitz (1993), Goodhart and Giugale (1993), Payne (1997) and Andersen and Bollerslev (1998) have all studied the intraday patterns in the volatility of indicative quotes.

There is also a pronounced intradaily pattern in transactions. Figure 2 plots the average trade intensity during each five minute observation interval calculated over the 79 trading days in the sample. As the figure shows, average trading activity follows a three humped pattern. The first hump occurs between 01:00 and 05:00 with peak intensity of approximately 5 trades per minute.[13]

[13]Trading in the DM/$ is comparatively light during this period because only Asian based dealers are typically in the market. It is also possible that the Reuter's Dealing system is used much less by Asian based dealers that their counterparts in Europe and the US.

The second hump begins around 07:00, peaks with approximately 25 trades per minute at 09:30, and drops to 15 trades per minute by about 12:30. Trading during this period is dominated by dealers situated in Europe. The third hump begins at 12:30, and rises quickly to a peak of approximately 30 trades per minute at 14:30. This increase coincides with U.S. based dealers entering the market. Activity remains around 25 trades per minute until approximately 16:00, when European based dealers typically leave the market. Thereafter, there is a gradual decline in activity with fewer than 5 trades per minute taking place after 19:00.

It is important to stress that Figure 2 plots average trade intensity over the sample of 79 trading days. Actual intensity can vary considerably from day to day. For example, the most active period of trading in the sample occurs between 09:00 and 10:00 when more that 200 transactions per minute take place, 8 times the average for that time of day. More generally, the difference between the actual and average intensity during each five minute interval has a sample standard deviation of 58, so that trading intensity on any particular day could differ significantly from the pattern displayed in Figure 2.

Further evidence on the variability of trading activity is provided by Table 1. The upper panel of this table reports estimates of a 6-state first-order Markov process for trade intensity. The states are defined as $\{j : n \in [S_j, S_{j+1})\}$ for $j = 1, ..6$ where $S_j$ is the lower bound for state $j$ reported in the table. Thus, an observation of $n_t = 10$, would represent state $j = 3$ of the Markov process. The body of the table reports the matrix of estimated transition probabilities, with entry $i, j$ denoting the probability of transition from state $i$ to state $j$. Two features of these estimates are noteworthy. First, the probability of remaining in the same state, reported on the leading diagonal, is less than 75 per cent in every case. Second, the probability of leaving the current state is highest at intermediate levels of trading activity. These features are more pronounced in the lower panel of the table which reports estimates of a 6-state Markov process for the "deseasonalize" trading rates. In this case, states are defined as $\{j : n - \bar{n} \in [S_j, S_{j+1})\}$ where $\bar{n}$ denotes the average trade intensity for the interval from which $n$ is observed. These estimates indicate that unusually high or low trade intensities are more likely to persist than intensities that are closer to the norm for that particular time of day. They also show that trade intensities can vary over the complete range of states from period to period. In contrast to the upper panel, all the off-diagonal transition probabilities are non-zero.

These results show that there are considerable variations in trade intensity over the sample period. Some of these variations can be attributed to a fairly well-defined intraday pattern that appears consistent with dealers in different locations entering and leaving the market. However, on any particular day actual trade intensity can vary considerable from this norm. This raises the possibility that the 5 minute observation interval spans varying periods of "market time": the time scale at which market process evolve at a constant rate. If this time scale is grounded in the rate at which information becomes available to dealers, and the arrival of private information is an important source of interdealer trade, then the variable trading intensities we observe may in part result from the speeding up and slowing down of market time. This phenomena is referred to as time deformation (Stock 1988) and I shall examine its possible implications below.

Sample statistics for the change in purchase price, $\Delta p_t^+$, and order flow, $x_t$ are reported in Table 2.[14] From the statistics in the right hand columns of the upper panel, the unconditional distributions for both variables appear non-normal. The distribution for transaction price changes is skewed to the left and is highly leptokuritic. The distribution for order flow is also fat-tailed but skewed to the right. The lower panel reports estimated autocorrelation coefficients together with the p-values for the null hypothesis of a zero coefficient. In the case of purchase prices, these estimates indicate the presence of a MA(1) process for $\Delta p_t^+$ : there is a significant negative coefficient at lag one, while all coefficients at higher lags are insignificantly different from zero. By contrast, order flow displays positive autocorrelation that is statistically significant at the 1% level on lags one through six.

Further evidence on the dynamics of price changes and order flow is provided by Table 3. Here I report GMM estimates for various ARMA specifications using the variance and the first twelve autocorrelations as moments (see Appendix B). The number of overidentifying restrictions for each set of estimates are reported in the right hand column together with the Hansen (1982) J-statistic and its associated p-value.

The upper panel of the table reports estimates of ARMA models for the change in purchase price. Consistent with the statistics in Table 2, there is strong evidence of a moving average component in this series: the moving average coefficient(s) are highly significant in all but one model. Although the MA(1) model appears well-specified when judged by the J-statistic, the autoregressive coefficients also appear to be significant in the ARMA(1,1) and ARMA(2,2) models. This is an interesting finding because it provides some preliminary evidence that order flow contributes to price changes. Recall from (19) that observed price changes can be written as $\Delta p_t^o = \varepsilon_t + B(L)v_t + \omega_t^o - \omega_{t-1}^o$, where $\varepsilon_t$, $v_t$ and $\omega_t$ are serially uncorrelated. According to this equation, price changes should follow an

[14]The statistics for the difference in sales prices, $\Delta p_t^-$, are almost identical to those for $\Delta p_t^+$ and so are not reported to conserve space. All statistics on $\Delta p_t^+$ are calculated without "missing" observations. Appendix B provides details on all the empirical results presented below.

MA(1) process if $B(L)v_t = 0$. However, the estimates in Table 3 suggest that price changes follow a higher order process. According to (19), this must originate from the order flow component, $B(L)v_t$.

ARMA model estimates for order flow are reported in the lower panel of Table 3. These estimates indicate that order flow follows either an ARMA(1,2), ARMA(2,1) or ARMA(2,2) process with a high degree of persistence. Judged by the J-statistics, lower order models do not appear to capture the high degree of persistence in the process implied by the autocorrelations reported in Table 2. For example, the estimates of the ARMA(1,2) model imply a root for the order flow process of 0.84.

The persistence in the order flow is somewhat puzzling. To see why, consider the implications of these results for the behavior of inventories and customer orders. Market clearing ensures that $\Delta I_{t+1} = -(c_t + x_t)$ where $I_t$ denotes the average level of dealer inventories at the start of period $t$. Now suppose that dealers' trading decisions are in part motivate by the desire to control inventories. One minimal implication of an operative inventory control channel is that $Cov(\Delta I_{t+1}, \Delta I_{t-i}) < 0$ for $i > 0$, because dealers adjust their quotes so inventories do not stray too far from their (constant) desired level.[15] When this condition holds, market clearing implies that

$$Cov(c_t, x_{t-i}) + Cov(x_t, c_{t-i}) + Cov(c_t, c_{t-i}) + Cov(x_t, x_{t-i}) = Cov(\Delta I_{t+1}, \Delta I_{t-i}) < 0. \qquad (27)$$

The order flow results in Tables 2 and 3 could be squared with the presence of an operative inventory control channel if the sum of first three terms is negative. Let us therefore consider these terms in turn. Interdealer order flow cannot be observed by non-bank customers so there is little reason to think that the first term is negative. The second term depends on the extent to which inventory imbalances resulting from customer orders are passed on from one dealer to another; a phenomena termed "hot potato" trading by Lyons (1997). If "hot potato" trading is prevalent, interdealer order flow will be positively correlated with lagged customer orders so $Cov(x_t, c_{t-i}) > 0$. Together these observations suggest an upper bound for $Cov(c_t, c_{t-i})$ of $-Cov(x_t, x_{t-i})$ in the presence of an inventory control channel. Since $Cov(x_t, x_{t-i})$ appears significantly positive for a number of lags $i$, the bound implies that customer orders are negatively autocorrelated over similar horizons. Although customer orders are taken as exogenous in the trading model, in a more general setting they would be determined endogenously by the behavior of the non-bank public. Whether

[15] See Hasbrouck (1998) for a discussion of the canonical inventory control model where this condition holds. Empirical evidence on the importance of the inventory control in dealer decision-making is provided in Lyons (1995). He estimates a half life for unwanted inventory of approximately ten minutes for the dealer in his study.

such negative autocorrelations could arise endogenously remains to be seen.

Equation (27) provides a further perspective on the behavior of order flow. Suppose that customer orders are serially uncorrelated. If $Cov(x_t, c_{t-i}) \geq 0$ and $Cov(c_t, x_{t-i}) = 0$ for the reasons cited above, then $Cov(x_t, x_{t-i})$ is a lower bound on $Cov(\Delta I_{t+1}, \Delta I_{t-i})$. From this perspective, the order flow estimates in Tables 2 and 3 suggest that average inventory changes are positively correlated: an equally puzzling result that is inconsistent with the micro evidence on dealer inventories in Lyons (1995).

A key assumption in these arguments is that the dynamics of order flow hitting individual dealers (i.e., $x_t^*$) is well-represented by the dynamics of the aggregate measure, $x_t$. If there is a significant degree of heterogeneity in dynamics of individual the order flows, the estimates in Tables 2 and 3 will not be informative about the behavior of $x_t^*$. And, as a result, the puzzles alluded to above become more apparent than real. If this is indeed the case, the order flow results simply serve to reinforce my emphasis on the important role of heterogeneity.

### 4.2 Structural Models

I now turn to examine the origins of exchange rate movements. Table 4 reports GMM estimates of the regression in (21) for ten different specifications for $D(L)$, the polynomial on order flow. The dependent variable in all cases is the change in purchase price, $\Delta p_t^+$. (Results using the change in sale price, $\Delta p_t^-$, are nearly identical.) The table reports the coefficient estimates on each of the order flow terms, together with their standard errors which are corrected for conditional heteroskedasticity and an MA(1) error process. Inspection of these estimates reveals that the coefficients on $x_{t-2}$, $x_{t-3}$, $x_{t+5}$ and $x_{t+6}$ are (individually) insignificantly different from zero at the 5 per cent level whenever they are included in a specification. This evidence suggests that $D(L)$ is well-characterized by a $6'th$ order polynomial (i.e., one that includes terms in $L^{-4}, L^{-3}, L^{-2}, L^{-1}, L^0$, and $L$). As row VII of the table shows, all the coefficient estimates in this specification are statistically significant. The importance of the leading order flow terms is further emphasized by the results in the last row. Here the table reports estimates for the case where $D(L)$ includes only $L^0$, $L$, $L^2$ and $L^3$ terms. The $R^2$ statistic from this specification is 0.005, much lower than the $R^2$ statistics for the other specifications. Thus, order flow contributes most to the predicable variation in price changes through the leading terms.

The right hand columns of the table report the estimated sum of the order flow coefficients together with some regression diagonstics. In every case, the sum of the coefficients is positive and statistically significant. Since the dependent variable is the change in price, these estimates imply that order flow variations have a permanent effect on the price level. This is a surprising finding when judged against the background of the trading model. There customer order shocks only exerted a temporary affect on prices so that $D(1) = 0$. The estimates in Table 4 strongly reject this restriction. The trading model makes more accurate predictions about the structure of the regression residuals. The right hand column of the table reports $l-$ statistics (Cumby and Huizinga 1993) for the null hypothesis that the regression residuals follow an MA(1) process. For the preferred specification in row VII, the $l-$statistic is significant at the 9 per cent level.

While the results in Table 4 provide strong evidence on the price-impact of order flow, more can be learnt about the origins of price movements by utilizing both the purchase and sale prices, $p_t^+$ and $p_t^-$. Specifically, consider $p_t^+$ and $p_t^-$ to represent random drawings from the respective distributions of purchase and sales prices so that $p_t^+ = p_t + \omega_t^+$ and $p_t^- = p_t + \omega_t^-$ where $p_t$ is the average price level. Combining these expressions with (18) and (20) gives

$$\begin{bmatrix} \Delta p_t^+ \\ \Delta p_t^- \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} D(L)x_t + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \varepsilon_t + \begin{bmatrix} \omega_t^+ - \omega_{t-1}^+ \\ \omega_t^- - \omega_{t-1}^- \end{bmatrix}. \tag{28}$$

When observed prices are drawn independently from their respective distributions, $\omega_t^-$ and $\omega_t^+$ will be serially uncorrelated and independently distributed. They will also be independent from the CK news shock $\varepsilon_t$, and from leads and lags of order flow.

Equation (28) provides us with a generalizing of (21) that utilizes both purchase and sales price changes to estimate the dynamics of the average price change $\Delta p_t$, which is represented by the first two terms on the right hand side. The third term is the sampling component of observed price changes that arises from the presence of the transaction price distribution. To estimate the model, I draw on the results in Table 4 by assuming that $D(L)$ is a 6'th order polynomial containing terms in $L^{-4}$ to $L$. I also assume that $\omega_t^+$ and $\omega_t^-$ have the same variance, $\Sigma_\omega$, and constant means.[16] The GMM estimates are obtained using instruments $z_t = [x_{t+4}, ....x_{t-1}]$ and moments derived from the covariances of observed price changes.

The GMM estimates of the Bivariate model in (28) are reported in Table 5. The order flow

[16] The latter assumption implies that there is no period-by-period change in the difference between the average purchase and sales price, $\bar{p}_t^+ - \bar{p}_t^-$. This is not a completely innocuous assumption because dealers can change the spread between their bid and ask quotes, which in turn could alter $\bar{p}_t^+ - \bar{p}_t^-$. However, spreads in the interdealer market are extremely small (for example, Lyons 2000 reports a median spread of DM 0.0003/$ for the dealer he studied), so keeping the means of $\omega_t^+$ and $\omega_t^-$ constant is not unreasonable.

coefficient estimates are generally very similar to those reported in row VII of Table 4, and are all highly statistically significant. The sum of the estimated coefficients is approximately 0.24 with a standard error of 0.02. This estimate is also similar to the one in Table 4 and implies a strong rejection of the null hypothesis that customer order shocks only have temporary affects on prices. The estimates of $\Sigma_{\varepsilon}^{1/2}$ and $\Sigma_{\omega}^{1./2}$, the standard deviations of CK news and the idiosyncratic shocks are rather similar. Thus, the cross-sectional dispersion of transaction prices appears to be of the same order of magnitude as the variance of CK shocks.

These estimates provide empirical support for the existence of an equilibrium distribution of transaction prices at a point in time. While the trading model shows how an equilibrium distribution can arise from heterogeneity and a lack of transparency, are other interpretations possible? In particular, couldn't the estimates simply imply the presence of some sort of measurement error? In one sense, the answer is clearly no. The data on $p_t^+$ and $p_t^-$ come from the Reuters trading system and constitute part of the audit trail. There is not doubt that transactions took place at exactly the prices recorded in the $p_t^+$ and $p_t^-$ series. Of course, it is possible that dealers make typing errors or irrational decisions when using the trading system. $p_t^+$ and $p_t^-$ will then contain measurement errors in the sense that they do not correspond to equilibrium transaction prices in any theoretical model that excludes these possibilities. But even in this case, the assumptions of the Bivariate model are valid so long as the $\omega_t^o$ shocks are serially uncorrelated. Thus, while it may be possible to attribute some of the price distribution implied by the estimates to economic factors ignored by the trading model, it cannot be even partially attributed to pure measurement error in the data.

The estimates reported in Table 5 are based on the dynamics of price changes and order flow measured over a fixed 5-minute observation window. As I noted above, it is quite possible that this fixed interval corresponds to varying spans of "market time". Consequently, the estimates may be affected by the presence of time deformation. To examine this possibility, I use trade intensity as a state variable to test for the presence of state-dependency in the dynamics of prices changes and order flow. If market time is in part determined by the rate at which dealers receive new information from outside the market, and arrival of non-common knowledge (NCK) information is an important source of interdealer trade, the large variations in trade intensity we observe may be signaling the presence of time deformation.

The upper panel of Table 6 reports the results of the state-dependency tests for the dynamics of price changes and order flow. In the former case, I estimate models for price changes that include the distributed lag of order flow $D(L)x_t$, together with the interaction terms $D(L)x_t n_t$, $D(L)x_t n_t^2$

and $D(L)x_t n_t^3$. I then test the null hypothesis that the estimated coefficients on all the included interactions terms are zero using a Wald test corrected for the presence of heteroskedasticity and an MA(1) residual error structure. As the table shows, these test statistics strongly reject the null. The Wald tests for state-dependency in the order flow dynamics show much weaker evidence against the null; none are significant at the 5 per cent significance level. These tests are based on AR(6) models for order flow (i.e., $D(L) = d_1 L + ... + d_6 L^6$), but the results are robust to the use of higher-order models.

The lower panel of Table 6 reports tests for heteroskedasticity in the variances of $\varepsilon_t$, $\omega_t^+$ and $\omega_t^-$. The center three columns report Glesjer (1969) tests for heteroskedasticity using combinations of $n_t, n_t^2$ and $n_t^3$. As the table shows, there is very strong evidence against the null of homoskedasticity in all three cases. The right hand column reports LM statistics for first order ARCH (Engle 1982) for each of the three shocks. These statistics also imply a rejection of the homoskedastic null at very high significance levels. The table also reports tests for heteroskedasticity in the variance of the innovations in order flow. The tests use the innovations from the estimates of the ARMA(2,2) order flow model in Table 3. All four test statistics are highly significant.

Overall, the results in Table 6 strongly indicate that the dynamics of price changes vary significantly with the state of the market as measured by trading intensity. While this does not constitute direct evidence of time deformation, it is certainly consistent with the idea that speed of market processes varies through time. The results also point to the need to incorporate state-dependency into the Bivariate model. To this end, I consider the following extension:

$$\begin{bmatrix} \Delta p_t^+ \\ \Delta p_t^- \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} D(L, n_t)x_t + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \varepsilon_t + \begin{bmatrix} \omega_t^+ - \omega_{t-1}^+ \\ \omega_t^- - \omega_{t-1}^- \end{bmatrix}, \tag{29}$$

where $D(L, n_t)$ denotes a state-dependent 6'th order polynomial

$$D(L, n) = d_1(n)L^{-4} + d_2(n)L^{-3} + ..... + d_5(n) + d_6(n)L,$$

with state-dependent coefficients $d_i(.)$. As above, $\varepsilon_t$, $\omega_t^+$and $w_t^-$ are mutually independent and serially uncorrelated shocks, but their variances are now state-dependent: $Var(\varepsilon_t) = \Sigma_\varepsilon(n)$ and

$Var(\omega_t^+) = Var(\omega_t^-) = \Sigma_\omega(n)$. I model state-dependency in the coefficients and variances as

$$\begin{aligned} d_i(n) &= d_i(0)\exp(-n/\gamma) + d_i(\infty)(1-\exp(-n/\gamma)), \\ \Sigma_i(n) &= \Sigma_i(0)\exp(-n/\gamma) + \Sigma_i(\infty)(1-\exp(-n/\gamma)), \end{aligned}$$

where $d_i(0)$, $d_i(\infty)$, $\Sigma_i(0)$ and $\Sigma_i(\infty)$ are all parameters to be estimated. These functional forms bound the coefficients between $d_i(0)$ and $d_i(\infty)$, and the variances between $\Sigma_i(0)$ and $\Sigma_i(\infty)$ as the transaction rate varies between 0 and $\infty$. For the sake of parsimony, the positive scaling parameter $\gamma$, is common to all functions.

Table 7 reports the GMM estimates of the state-dependency model in (29) with $\gamma$ set equal to 100. Attempts to estimate the model with $\gamma$ unrestricted gave estimates close to 100 but the standard errors on all the other parameters were much larger than the values reported in the table. (Re-estimating the model with $\gamma$ set to different values of 90 and 110 had negligible effects on the estimated parameters.) The upper portion of the table reports the estimated bounds in the order flow polynomial, $D(L, n)$. When compared to the estimates in Table 5, we see that the estimates of $d_i(0)$ are generally smaller in absolute value than their state-independent counterparts, while the estimates of $d_i(\infty)$ are generally larger. The estimated range for the individual $d_i(.)$s appears quite large. A Wald test for the null hypothesis of $d_i(0) = d_i(\infty)$ for $i = \{1, 2, ...6\}$, reported in the lower panel, is highly significant. This test statistic supports the presence of state-dependency in the price change dynamics. The $d_i(.)$ estimates also imply significant state dependence in $D(1, n)$, which measures the long run impact of order flow on the price level. The estimated lower and upper limits are -0.15 and 1.32 with standard errors of 0.03 and 0.10 as $n$ ranges from zero to infinity.

The center of the table reports estimates for the variance parameters. In models where all four parameters were left unrestricted, the estimates of $\Sigma_\varepsilon(0)$ and $\Sigma_\omega(\infty)$ were very close to zero (i.e. $< 0.0001$), so the table reports estimates where these parameters are restricted to zero. With these restrictions, $Var(\varepsilon_t) = \Sigma_\varepsilon(\infty)(1-\exp(-n/100))$ and $Var(\omega_t^+) = Var(\omega_t^-) = \Sigma_\omega(0)\exp(-n/100)$. The estimated value for $\Sigma_\omega(0)$ implies that the standard deviation for the idiosyncratic shocks slowly falls from 0.047 to approximately 0.006 as $n$ varies from 0 to 200. The estimate for $\Sigma_\varepsilon(\infty)$ implies that the standard deviations of CK shocks is smaller than $Var(\omega_t)^{1/2}$ for $n$ less than 35. As the transactions rate rise beyond 35 the standard deviation increases slowly from 0.042.towards 0.090.

The lower portion of the table reports the results of various diagnostic tests. The Hansen

(1982) $J$ -test for the over-identifying restrictions of the model has a p-value of 0.96. The table also reports the results of LM-type tests for misspecification in the estimated $D(L,n)$, $\Sigma_\varepsilon(n)$ and $\Sigma_\omega(n)$ functions. None of these statistics are statistically significant. This suggests that the model did manage to incorporate most of the state-dependency in price dynamics. The model is less successful in accounting for all the heteroskedasticity exhibited by price changes. The bottom of the table reports autocorrelations for the standardized estimates of the shocks. If the model had completely captured the heteroskedasticity in price changes, these autocorrelations should all be close to zero. As the table shows, this is not the case. In particular, there appears to be significant first-order serial correlation in the estimates of $\varepsilon_t^2/\Sigma_\varepsilon(n_t)$.

Overall, the results in Table 7 show that the state-dependent dynamics of price changes are reasonably well-characterized by the model specified in (29). While the model does not identify *all* the sources of heteroskedasticity in prices, it does appear to capture the role played by changing trade intensity.

### 4.3 Implications

The results above allow us to examine the origins of price changes in several different ways. In particular, we can (i) study the dynamic response of prices to CK and NCK shocks, (ii) decompose the variance of observed price changes into different theoretical components, and (iii) examine the sources of seasonality in price heteroskedasticity.

The results in Table 7 provide us with estimates of the observed price process:

$$\Delta p_t^o = D(L,n_t)x_t + \varepsilon_t + \omega_t^o - \omega_{t-1}^o. \tag{19}$$

Recall from Section 2.3 that equilibrium order flow can be represented as $x_t = C(L)v_t$, where $v_t$ is a common component of customer orders. Although this process could also be state-dependent with the coefficients in $C(L)$ functions of the transaction rate, the empirical evidence in Table 6 suggests that order flow follows a reasonably stable process. Thus, as an empirical matter, we can substitute $C(L)v_t$ for $x_t$ in (19) to give

$$\Delta p_t^o = B(L,n_t)v_t + \varepsilon_t + \omega_t^o - \omega_{t-1}^o, \tag{30}$$

where $B(L,n) = D(L,n)C(L)$.

Equation (30) provides us with the means to empirically examine the origins of price changes.

In particular, the state-dependent polynomial $B(L, n)$ identifies the impulse response of prices to a one standard deviation order flow shock while trade intensity remains at $n$. To calculate these impulse responses, I first use the estimated ARMA models for order flow in Table 3 to calculate the coefficients in $C(L)$. I then combine these values with the estimates of $D(L, n)$ from Table 7 to compute $B(L, n)$. Panels A, B and C of Figure 3 report the results for $n = 5, 20$ and 40. As there is little to choose between the ARMA(2,1), ARMA(1,2) and ARMA (2,2) estimates for order flow, the figure plots the impulse responses implied by each specification. For completeness, panel D plots the impulse responses implied by the estimates of $D(L)$ in Table 5 (i.e., without state-dependency).

Figure 3 displays three noteworthy features. First the impulse responses appear fairly robust to the choice of ARMA specification used in calculating $C(L)$. Second, the dynamic response of prices seems to vary considerably with trade intensity. When the state of the market is characterized by low trade intensity, order flow shocks have comparatively small effects on prices. Prices rise for the first three periods following the shock and then fall back towards their original level. In fact the total effect on the price level (i.e., $B(1, 5)$) is less than 0.002 in all three cases. Thus, order flow shocks have small and transitory effects on prices when transaction rates are low. Panels B and C show very different responses. Here prices rise for approximately 7 periods (35 minutes) following the shock. The cumulative effect on the price level is approximately 0.40 when $n = 20$, and 0.63 when $n = 40$. Thus, order flow shocks have much larger and long-lasting price effects when trade intensity is high.

The third noteworthy feature concerns the time of the peak response. In all cases order flow shocks have their largest (positive) effect on price changes during the 3rd. period, 15 minutes after the shock. To understand how such a delay might arise, recall from the trading model that the customer order received by one dealer affected the price quoted to a counterparty, who in turn used the information in setting prices next period. In a model with a richer timing structure that more closely resembles trading in the interdealer market, it may take some time before the transmission of information through transaction prices affects the price-setting decisions of a significant number of dealers. Consequently, the delayed peaked response may well reflect the importance of transaction prices as a medium for information flows between dealers and the relatively slow speed at which such information diffuses across the whole market.

We can also use equation (30) to decompose the variance of observe price changes into different theoretical components. In particular, consider the $k$-period price change, $\Delta^k p_t^o \equiv \Sigma_{i=0}^{k-1} \Delta p_{t+i}^o$.

Substituting for $\Delta p_t^o$ with (30), gives

$$\Delta^k p_t^o = \omega_t^o - \omega_{t-k}^o + \sum_{i=0}^{k-1} \varepsilon_{t-i} + B(L,k,n_t)v_t, \tag{31}$$

where $B(L,k,n_t) = \sum_{i=0}^{k-1} B(L, L^i n_t)L^i$. Since the $v_t$, $\varepsilon_t$ and $\omega_t^o$ shocks are mutually independent and serially uncorrelated, we can use (31) to write the variance of price changes as

$$Var(\Delta^k p_t^o) = \Sigma_\omega(n_t) + \Sigma_\omega(n_{t-k}) + \sum_{i=0}^{k-1} \Sigma_\varepsilon(n_{t-i}) + Var\left(B(L,k,n_t)v_t\right). \tag{32}$$

The first two terms identify the contribution of the idiosyncratic sampling shocks to the variance. These terms can be calculated from the estimates of the $\Sigma_\omega(.)$ function in Table 7. The contribution of CK news, given by the third term, can be similarly calculated from the estimates of $\Sigma_\varepsilon(.)$. The forth term identifies the contribution of the order flow shocks (from the common component of customer orders). This term can be further decomposed as

$$Var\left(B^*(L,k,n_t)v_t\right) + Var\left(B(1,k,n_t)v_t\right) + 2B^*(0,k,n_t)B(1,k,n_t)Var(v_t),$$

where $B^*(L,k,n_t) \equiv B(L,k,n_t) - B(1,k,n_t)$ identifies the transitory effect on prices over $k$-periods of a one standard deviation order flow shock. The first term in the expression above gives us the contribution of such shocks to the variance of $k$-period price changes. The permanent contribution of order flow shocks is shown by the second term. The last term identifies twice the covariance between $B^*(L,k,n_t)v_t$ and $B(1,k,n_t)v_t$.

To compute these components we need estimates of $B(L,k,n_t)$ and the variance of the $v_t$ shocks. In light of the state-dependency results reported in Table 6, it is clearly inappropriate to assume that $v_t$ is homoskedastic. I therefore re-estimated the ARMA(2,2) model for order flow allowing for a state-dependence variance, $\Sigma_v(n) = \Sigma_v(0)\exp(-n/\gamma) + \Sigma_v(\infty)(1-\exp(-n/\gamma))$. With $\gamma$ set equal to 100 (as above), the GMM estimates of the AR and MA coefficients are almost identical to those reported in Table 3 while the estimates of $\Sigma_v(0)$ and $\Sigma_v(\infty)$ are 0.001 and 0.022 respectively. For the sake of comparison, I also computed variance decompositions assuming that $v_t$ was homoskedastic with $\Sigma_v$ set equal to the variance of the innovations from the ARMA(2,2) model. These results are reported in Appendix C.

Table 8 reports variance decompositions based on (32) for various horizons $k$ and trade intensities $n$. The upper panel reports the fraction of the price change variance attributable to sampling,

$R_\omega(k,n) \equiv 2\Sigma_\omega(n)/Var(\Delta^k p_t^o)$. According to these estimates, most of the short-term variability in prices is attributable to sampling unless trade intensity is very high. For example, $R_\omega(1,10)$ is approximately 84 per cent. Although $R_\omega(k,n)$ falls at all intensities as the horizon increases, sampling continues to contribute more than 17 per cent at the 2 hour horizon for intensities of 10 or less. These results indicate that, under most conditions, a majority of the high frequency variations in observed transaction prices result from the presence of significant dispersion in the equilibrium price distribution.

The middle panel of Table 8 reports the contribution of the order flow shocks to the variance of price changes: $R_v(k,n) \equiv Var\left(B(L,k,n_t)v_t\right)/Var(\Delta^k p_t^o)$. These estimates show that order flow shocks contribute more to price volatility in states of the market with higher trade intensities. Order flow shocks also make a larger contribution to the price variance as the horizon rises. Across all intensities, order flow contributes approximately 40 per cent of the variance in prices at the two hour horizon. The contribution of order flow shocks are even higher in states where trade intensities are at least 40.

These findings are inconsistent with the traditional macro view that stresses the importance of CK news. According to this view, the estimates of both $R_\omega(k,n)$ and $R_v(k,n)$ should be close to zero. Of course, the macro view may still be accurate at much longer horizons. To address this possibility, the right hand column of the center panel reports the contribution of the order flow shocks to the variance of the permanent innovations in prices. According to (30), observed prices can be written as $p_t^o = \bar{p}_t + I(0)$ terms, where $\Delta\bar{p}_t = \varepsilon_t + B(1,n)v_t$. Hence, the relative contribution of order flow can be calculated as $R_v(\infty,n) \equiv B(1,n)^2\Sigma_v/Var(\Delta\bar{p}_t)$. The right hand column of the table shows that the estimates of $R_v(\infty,n)$ follow a "U-shaped" pattern, with a minimum value of zero when $n = 5$. At high trade intensities, the estimates of $R_v(\infty,n)$ are more than 80 per cent. These estimates are consistent with regression finding in Evans and Lyons (1999) that order flow accounts for more than 60 per cent of daily price changes. They also stand in sharp contrast to the traditional macro view concerning the origins of exchange rate movements.

Further information on the importance of order flow shocks comes from the lower panel of Table 8. Here I report the contribution of order flow shocks that temporary affect the price level, $R_{v^*}(k,n) \equiv Var\left(B^*(L,k,n_t)v_t\right)/Var(\Delta^k p_t^o)$. Once again, the estimates of $R_{v^*}(k,n)$ increase as $n$ rises. The largest estimates of over 22 per cent are found at high trade intensities with the 30 minute horizon. Across all intensities, the temporary order flow component contributes approximately 12 per cent to the variance of the prices changes over 30 minutes. This pattern is consistent with the delayed peak in the impulse responses plotted in Figure 3.

Finally, we can use the variance components to examine sources of seasonal heteroskedasticity. Recall from Figure 2 that there are pronounced intraday patterns in trade intensities. Figure 4 combines these patterns with the estimates of $R_\omega(k,n)$ and $R_v(k,n)$ to show how different shocks contribute to the variance of price changes over a typical 24 hour period. Three features of these plots stand out. First, sampling variability is a very significant source of price variance outside of European trading hours. For example, panel A shows that the estimates of $R_\omega(1,n)$ are more than 80 per cent before 7:00 hrs. and after 17:00 hrs.. Although sampling contributes less to the price variance over longer horizons, $R_\omega(k,n)$ remains above 30 per cent even at the two hour horizon. Second, although sampling contributes less to the variance of price changes during European Trading, it continues to be a very important source at the 5 minute horizon; the estimates of $R_\omega(1,n)$ remain above 60 per cent. Third, order flow shocks contribute most to price variance during European trading. The peak values for $R_v(k,n)$ range from 10 per cent at the 5 minute horizon to approximately 50 per cent at two hours. When combined with the values for $R_\omega(k,n)$, these estimates imply a peak contribution for CK news to the price variance, defined as $R_\varepsilon(k,n) \equiv 1 - R_\omega(k,n) - R_v(k,n)$, of approximately 30 per cent at the 5 minute horizon, and 50 per cent at 2 hours.

## 5 Conclusion

The primary aim of this paper has been to provide a new perspective on the source of exchange rate dynamics. The perspective comes from considering how trading in the FX market actually takes place. To this end, I presented a theoretical model of FX trading that emphasized how the lack of transparency in dealer-customer and dealer-dealer transactions can lead to an equilibrium distribution of transaction prices rather than a single price level. I then showed how the predictions of this model could be used to develop an empirical framework for studying exchange rate dynamics. Applying this framework to transactions data for the DM/$, several striking results emerge: First, there is strong evidence supporting the presence of equilibrium price distribution. Second, CK news shocks are rarely the predominant source of exchange rate changes over both long and short horizons. Third, NCK news shocks are an empirically important source of long-term exchange-rate dynamics.

The first of these findings is key to understanding the short-term dynamics of exchange rates. Unless trading between dealers is extremely active, the dispersion in the equilibrium distribution is large enough to account for most of the observed variance in high frequency price changes. This finding puts a new perspective on the high frequency volatility of exchange rates. It implies that much of the volatility we observe comes from sampling the heterogeneous trading decisions of dealers in an equilibrium distribution that, under normal market conditions, changes comparatively slowly.

The second finding speaks more directly to assumptions that lie at the heart of the traditional macro view of exchange rate dynamics. Recall that this view assumes (i) all information relevant for exchange rate determination is CK, and (ii) the mapping from information to equilibrium prices is also CK. CK news shocks meet both of these requirements but account for only 56 per cent of the persistent movements in exchange rates across all market states, and 20 per cent when trading activity is high. Thus, a key implication of my results is that models based on assumptions (i) and (ii) are simply too restrictive to account for all the persistent exchange rate movements we observe. In this sense, my findings do direct attention away from the common knowledge framework modeled in traditional macro exchange rate models.

Where should our attention be redirected? The third finding points towards NCK news shocks. The trading model identified customer orders as one source of such shocks, but this is not a wholly satisfactory answer: Customer orders are surely determined endogenously by agents outside the market. Although general equilibrium models have yet to be developed identifying the source of customer orders, there are suggestions in the literature about their origins. For example, Evans and Lyons (1999) point out that orders can embody NCK information about valuation numerators (i.e. future interest differentials) and denominators (i.e., anything that affects discount rates). In the former case, customer orders could reflect changes in the expected future path of interest rates. Moreover, there is mounting microeconomic evidence that private information plays an important role in FX trading (Lyons 1995, Yao 1997a, Bjonnes and Rime 1998, Cheung and Wong 1998, Ito Lyons and Melvin, 1988, Covrig and Melvin 1998, and Payne 1999). Customer orders need not be the only source of NCK news. Public news announcements, for example, may represent NCK news to dealers because they hold differing views about its implications for equilibrium prices. If this is the case, prices and interdealer order flow will be affected by these differing views until a consensus arises; a implication consistent with the variance decompositions presented above.

In sum, my results direct attention away from exchange rate models with a common-knowledge environment dominated by a small number of macro variables and towards models with richer informational structures.

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# A Trading Model Solution

This appendix derives the equilibrium solution to the trading model presented in Section 2. I start with four lemmas.

**Lemma 1** *When prices and order flow follow (11) and (12), the information sets of dealer $d$ starting her trading cycle in period $t$, can be characterized by*

$$\Omega_t^d = \{q_t, c_t^d, v_{t-2}, \tilde{v}_{t-1}^d\} \cup \Omega_{t-1}^d, \tag{A.1}$$

$$\Omega_{t+1}^d = \{q_{t+1}, v_{t-1}, u_{t-1}^*\} \cup \Omega_t^d, \tag{A.2}$$

*where $\tilde{v}_{t-1}^d = v_{t-1} + e_{t-1}^d$, with $e_{t-1}^d \sim N(0, \Sigma_e)$ and $\Sigma_e = \Sigma_u/(1-(\eta_2/\eta_1)^2)$.*

**Proof** At the end of period $t-1$ trading dealers learn the values of $p_{t-2}$ and $x_{t-2}$.Combining (11) and (12) with (1) and averaging across dealers (with the average values of $u_t^d$ and $e_t^d$ set equal to zero), gives

$$p_{t-2} = q_{t-2} + \eta_1 v_{t-2} + (\eta_1\alpha + \eta_2)v_{t-3}, \tag{A.3}$$

$$x_{t-2} = \eta_3 v_{t-3}. \tag{A.4}$$

Since $q_{t-2}$ is announced publicly at the start of $t-2$, these equations imply that

$$v_{t-2} = \frac{1}{\eta_1}(p_{t-2} - q_{t-2}) - \frac{(\eta_1\alpha + \eta_2)}{\eta_3\eta_1} x_{t-2} \in \Omega_t. \tag{A.5}$$

Dealers also receive private information in the form of dealer orders $x_t^*$ and transactions prices $p_{t-1}^*$. Specifically, equation (12) implies that

$$x_t^* - \eta_3 v_{t-1} = \eta_3 u_{t-1}^*.$$

By the start of period $t+1$, all the terms on the left are known so the dealer can precisely infer the value of $u_{t-1}^*$ as shown in (A.2).

Finally, combine (11) and (1) to write transaction prices as

$$p_{t-1}^* = q_{t-1} + \eta_1(u_{t-1}^* + v_{t-1}) + \eta_1\alpha v_{t-2} + \eta_2\tilde{v}_{t-2}^*.$$

Define $\tilde{v}^d_{t-1} \equiv \frac{1}{\eta_1}\left[p^*_{t-1} - q_{t-1} - (\eta_1\alpha + \eta_2)v_{t-2}\right]$ as the signal dealer $d$ extracts from $p^*_{t-1}$ given $q_{t-1}$ and $v_{t-2} \in \Omega^d_t$. Substituting for $p^*_{t-1}$ in this definition gives

$$\tilde{v}^d_{t-1} = v_{t-1} + \frac{\eta_2}{\eta_1}(\tilde{v}^*_{t-2} - v_{t-2}) + u^*_{t-1}.$$

Thus, $\tilde{v}^d_{t-1}$ is equal to the true value of $v_{t-1}$ plus an estimation error that depends on the idiosyncratic shock to the customer order received by dealer $*$ ($d's$ counterparty in $t-1$), and their estimation error, $\tilde{v}^*_{t-2} - v_{t-2}$. Solving this equation recursively implies that $\tilde{v}^d_{t-1} - v_{t-1}$ is a infinite weighted average of the past $u^d_t$ shocks received by a sequence of dealers that have traded with each other. Since $u^d_t \sim i.i.d.N(0, \Sigma_u)$ for all $d$, and $|\eta_2| < |\eta_1|$ (see below), $e^d_{t-1} \equiv \tilde{v}^d_{t-1} - v_{t-1}$ $\sim i.i.d.N(0, \Sigma_e)$ with $\Sigma_e = \Sigma_u/(1 - (\eta_2/\eta_1)^2)$.

**Lemma 2** *If dealer $d$ starts her trading cycle in period $t$, with information sets described by (A.1) and (A.2),*

$$E\left[v_{t+i}|\Omega^d_t\right] = \begin{cases} 0 & i > 0 \\ \phi(c^d_t - \alpha\delta_e\tilde{v}^d_{t-1}) & i = 0 \\ \delta_e\tilde{v}^d_{t-1} + \alpha\phi(c^d_t - \alpha\delta_e\tilde{v}^d_{t-1}) & i = -1 \end{cases}, \tag{A.6}$$

*where $\delta_e \equiv \Sigma_v/(\Sigma_v+\Sigma_e)$ and $\phi \equiv \Sigma_v(1-\delta_e)/\left(\Sigma_u + (1+\alpha^2(1-\delta_e))\Sigma_v\right)$. At the beginning of period $t+1$,*

$$E\left[v_{t+i}|\Omega^d_{t+1}\right] = \begin{cases} 0 & i > 0 \\ \delta_u(c^d_t - \alpha v_{t-1}) & i = 0 \end{cases} \tag{A.7}$$

*where $\delta_u \equiv \frac{\Sigma_v}{\Sigma_v+\Sigma_u}$*

**Proof** Dealer estimates of $v_t$ are found by applying the following well-known property of bivariate normal distributions: If $x \sim N(\mu, \Omega)$ where $x = [x_1, x_2]'$, $\mu = [\mu_1, \mu_2]'$ and $\Omega = [\Omega_{ij}]$, then $x_1|x_2 \sim N(\mu_{1|2}, \Omega_{1|2})$ where $\mu_{1|2} = \mu_1 + (\Omega_{12}/\Omega_{22})(x_2 - \mu_2)$ and $\Omega_{1|2} = \Omega_{11} - \Omega_{12}(\Omega_{22})^{-1}\Omega_{12}$.

For dealer $d$ starting her trading cycle in period $t$, Lemma 1 implies that $E[v_{t-1}|\Omega^d_t] = E[v_{t-1}|\tilde{v}^d_{t-1}, c^d_t]$, $E\left[v_t|\Omega^d_t\right] = E[v_t|\tilde{v}^d_{t-1}, c^d_t]$, and $\tilde{v}^d_{t-1} \sim N(0, \Sigma_e + \Sigma_v)$. Applying the result above with $x_1 = v_{t-1}$ and $x_2 = \tilde{v}^d_{t-1}$, gives $v_{t-1}|\tilde{v}^d_{t-1} \sim N(\delta_e\tilde{v}^d_{t-1}, \Sigma_v(1-\delta_e))$, where $\delta_e \equiv \Sigma_v/(\Sigma_v + \Sigma_e)$. Combining this with (1) implies that $c^d_t|\tilde{v}^d_{t-1} \sim N(\alpha\delta_e\tilde{v}^d_{t-1}, \Sigma_u + \Sigma_v[1 + \alpha^2(1-\delta_e)])$.

Next, I apply the result above with $x_1 = v_{t-1}|\tilde{v}^d_{t-1}$ and $x_2 = c^d_t|\tilde{v}^d_{t-1}$ to get $E[v_{t-1}|\Omega^d_t] = \delta_e\tilde{v}^d_{t-1} + \alpha\phi(c^d_t - \alpha\delta_e\tilde{v}^d_{t-1})$. Similarly, if $x_1 = v_t|\tilde{v}^d_{t-1}$ and $x_2 = c^d_t|\tilde{v}^d_{t-1}$, I obtain $E\left[v_t|\Omega^d_t\right] = \phi(c^d_t - \alpha\delta_e\tilde{v}^d_{t-1})$.

When dealer $d$ reaches the second period of her trading cycle in $t+1$, Lemma 1 implies that $E[v_t|\Omega^d_{t+1}] = E[v_t|c^d_t, v_{t-1}]$ and $E[v_{t+1}|\Omega^d_{t+1}] = E[v_{t+1}|c^d_t, v_{t-1}] = 0$. Let $x_1 = v_t|v_{t-1} \sim N(0, \Sigma_v)$ and $x_2 = c^d_t|v_{t-1} \sim N(\alpha v_{t-1}, \Sigma_v + \Sigma_u)$. Applying the result above, gives $E[v_t|\Omega^d_{t+1}] = \delta_u(c^d_t - \alpha v_{t-1})$ with $\delta_u = \Sigma_v/(\Sigma_v + \Sigma_u)$. Equations (13), (14) and (16) follow from Lemma 2, (A.5) and the definition of $\tilde{v}^d_t$.

**Lemma 3** *For a dealer starting her trading cycle in period $t$, with expected utility defined in the Proposition, the solution to the optimization problems in (4) - (8) is*

$$p^d_t - E[q_{t+1}|\Omega^d_t] = \frac{(c^d_t + E[x^*_t|\Omega^d_t])}{\theta Var(x^*_t|\Omega^d_t)} + \frac{Cov(x^d_{t+1}(p^*_{t+1} - q_{t+1}) - \Delta^\tau w_T, x^*_t|\Omega^d_t)}{Var(x^*_t|\Omega^d_t)}, \tag{A.8}$$

*and*

$$x^d_{t+1} = \lambda^d_{t+1}(c^d_t + x^*_t) = \frac{(q_{t+1} - E[p^*_{t+1}|\Omega^d_{t+1}])}{\theta Var(p^*_{t+1}|\Omega^d_{t+1})} + \frac{Cov\left(\Delta^\tau w^d_T, p^*_{t+1}|\Omega^d_{t+1}\right)}{Var(p^*_{t+1}|\Omega^d_{t+1})}, \tag{A.9}$$

*where $\Delta^\tau w^d_T \equiv w^d_T - w^d_{t+2}$ is the change in wealth from $t+2$ until the end of trading in period $T$ (denoted $T_d$ in the text).*

**Proof** Let $E^d_t$, $Var^d_t(.)$ and $Cov^d_t(.)$ respectively denote $E[.|\Omega^d_t], Var(.|\Omega^d_t)$ and $Cov(.|\Omega^d_t)$. First iterate the budget constraint in (6) forward to give

$$w^d_T = \Delta^\tau w^d_T + w^d_t + (c^d_t + x^*_t)(p^d_t - q_{t+1}) - \lambda^d_{t+1}(c^d_t + x^*_t)(p^*_{t+1} - q_{t+1}),$$

where $\tau = T - (t+2)$. Now consider a dealer starting her trading cycle in period $t$. At the beginning of period $t+1$, the dealer chooses $\lambda^d_{t+1}$ to maximize $E^d_{t+1}\mathcal{U}(w^d_{T_d}) = E^d_{t+1}(w^d_{T_d}) - \frac{\theta^2}{2}Var^d_{t+1}(w^d_{T_d})$ where

$$\begin{aligned}
E^d_{t+1}(w^d_T) &= E^d_{t+1}\Delta^\tau w^d_T + w^d_t + (c^d_t + x^*_t)(p^d_t - q_{t+1}) - \lambda^d_{t+1}(c^d_t + x^*_t)(E^d_{t+1}p^*_{t+1} - q_{t+1}), \\
Var^d_{t+1}(w^d_T) &= (\lambda^d_{t+1})^2(c^d_t + x^*_t)^2 Var^d_{t+1}(p^*_{t+1}) + Var^d_{t+1}(\Delta^\tau w^d_T) \\
&\quad -\lambda^d_{t+1}2(c^d_t + x^*_t)Cov^d_{t+1}\left(\Delta^\tau w^d_T, p^*_{t+1}\right).
\end{aligned}$$

Making these substitutions and differentiating $E^d_{t+1}\mathcal{U}(w^d_T)$ with respect to $\lambda^d_{t+1}$ gives (A.9).

In period $t$, the dealer chooses $p_t^d$ to maximize $E_t^d\mathcal{U}(w_T^d) = E_t^d w_T^d - \frac{\theta}{2}Var_t^d(w_T^d)$. The mean and variance are calculated from the budget constraint with $x_{t+1}^d = \lambda_{t+1}^d(c_t^d + x_t^*)$ using the fact that $q_{t+1}$ is independent of $x_t^*$ :

$$\begin{aligned} E_t^d(w_T^d) &= E_t^d\left[\Delta^\tau w_T^d - x_{t+1}^d(p_{t+1}^* - q_{t+1})\right] + w_t^d + (c_t^d + E_t^d x_t^*)(p_t^d - E_t^d q_{t+1}), \\ Var_t^d(w_T^d) &= Var_t^d(\Delta^\tau w_T^d - x_{t+1}^d(p_{t+1}^* - q_{t+1})) + Var_t^d(x_t^*)[(p_t^d - E_t^d q_{t+1})^2 + \Sigma_s] \\ &\quad + (c_t^d + E_t^d x_t^*)^2\Sigma_q - 2(p_t^d - E_t^d q_{t+1})Cov_t^d(\Delta^\tau w_T^d - x_{t+1}^d(p_{t+1}^* - q_{t+1}), x_t^*). \end{aligned}$$

Making these substitutions and differentiating $E_t^d\mathcal{U}(w_T^d)$ with respect to $p_t^d$ gives (A.8).

**Lemma 4** *If $y_1, y_2$ and $y_3$ are jointly normally distributed random variables,*

$$Cov(y_1y_2, y_3) = Ey_1Cov(y_2, y_3) + Ey_2Cov(y_1, y_3).$$

**Proof** From the identity, $y_1y_2 = Ey_1Ey_2 + Ey_1\breve{y}_2 + Ey_2\breve{y}_1 + \breve{y}_1\breve{y}_2$ where $\breve{y}_i \equiv y_i - Ey_i$, we can write

$$Cov(y_1y_2, y_3) = Ey_1Cov(y_2, y_3) + Ey_2Cov(y_1, y_3) + Cov\left(\breve{y}_1\breve{y}_2, y_3\right).$$

To show that the last term in these expression equals zero, project $\breve{y}_1$ and $\breve{y}_2$ on $\breve{y}_3$ to get $\breve{y}_1 = a_1\breve{y}_3 + e_1$ and $\breve{y}_2 = a_2\breve{y}_3 + e_2$ where $e_i$ is uncorrelated with $\breve{y}_3$ and $Ee_i = 0$. The projection errors are also independent from $\breve{y}_3$ because the $\breve{y}_i$'s are normally distributed. Hence,

$$\begin{aligned} Cov\left(\breve{y}_1\breve{y}_2, y_3\right) &= E\left[\breve{y}_1\breve{y}_2\breve{y}_3\right] \\ &= a_1a_2E\left[\breve{y}_3^3\right] + E\left[a_1e_2 + a_2e_1\right]E\left[\breve{y}_3^2\right] + E_t\left[e_1e_2\right]E[\breve{y}_3]. \end{aligned}$$

Since $\breve{y}_3$ is normally distributed with $E\breve{y}_3 = 0$, and $Ee_i = 0$, all the terms in the second line equal zero.

**Proof of Proposition in Section 2** The proof proceeds in two steps. Step 1 is to show that

$$Cov(\Delta^\tau w_T, x_t^*|\Omega_t^d) = 0, \tag{A.10}$$

$$Cov\left(\Delta^\tau w_T^d, p_{t+1}^*|\Omega_{t+1}^d\right) = 0, \tag{A.11}$$

$$Cov(x_{t+1}^d(p_{t+1}^* - q_{t+1}), x_t^*|\Omega_t^d) = \varphi(q_t - E_t^d p_{t+1}^*). \tag{A.12}$$

so that (A.8) and (A.9) in Lemma 3, can be written as

$$x_{t+1}^d = \psi(q_{t+1} - E_{t+1}^d p_{t+1}^*) \tag{A.13}$$
$$p_t^d - E_t^d q_{t+1} = \beta(c_t^d + E_t^d x_t^*) + \beta\theta\varphi(q_t - E_t^d p_{t+1}^*) \tag{A.14}$$

where $\psi \equiv (\theta Var_{t+1}^d(p_{t+1}^*))^{-1}$, $\beta = (\theta Var_t^d(x_t^*))^{-1}$ and $\varphi$ is a constant determined below. These equations have the same form as (9) and (10) in the text. Step 2 shows that (11) and (12) are consistent with (A.13) and (A.14) when dealers hold rational expectations.

*Step 1:* To derive the results in (A.10) - (A.12), I first use the identity $\Delta^\tau w_T^d = \sum_{i=1}^{\tau/2} \Delta^2 w_{t+2(i+1)}^d$, to write

$$Cov_t^d\left(\Delta^\tau w_T^d, x_t^*\right) = \sum\nolimits_{i=1}^{\tau/2} Cov_t^d(\Delta^2 w_{t+2(i+1)}^d, x_t^*), \tag{A.15}$$
$$Cov_{t+1}^d\left(\Delta^\tau w_T^d, p_{t+1}^*\right) = \sum\nolimits_{i=1}^{\tau/2} Cov_{t+1}^d(\Delta^2 w_{t+2(i+1)}^d, p_{t+1}^*). \tag{A.16}$$

The dealer's budget constraint implies that

$$\Delta^2 w_{t+2(i+1)}^d = (c_{t+2i}^d + x_{t+2i}^*)(p_{t+2i}^d - q_{t+2i+1}) - x_{t+2i+1}^d(p_{t+2i+1}^* - q_{t+2i+1}). \tag{A.17}$$

so covariance terms on the right of equations (A.15) and (A.16) can be written as

$$\begin{aligned} Cov_t^d(\Delta^2 w_{t+2(i+1)}^d, x_t^*) &= Cov_t^d\left((c_{t+2i}^d + x_{t+2i}^*)(p_{t+2i}^d - q_{t+2i+1}), x_t^*\right) \\ &\quad - Cov_t^d\left(x_{t+2i+1}^d(p_{t+2i+1}^* - q_{t+2i+1}), x_t^*\right), \end{aligned} \tag{A.18}$$
$$\begin{aligned} Cov_{t+1}^d(\Delta^2 w_{t+2(i+1)}^d, p_{t+1}^*) &= Cov_{t+1}^d\left((c_{t+2i}^d + x_{t+2i}^*)(p_{t+2i}^d - q_{t+2i+1}), p_{t+1}^*\right) \\ &\quad - Cov_{t+1}^d\left(x_{t+2i+1}(p_{t+2i+1}^* - q_{t+2i+1}), p_{t+1}^*\right). \end{aligned} \tag{A.19}$$

Equations (11) and (12) imply that all the terms in the the covariance functions are normally distributed so each covariance can be evaluated using Lemma 4. Thus in the case of (A.18)

$$\begin{aligned}
& Cov_t^d\left((c_{t+2i}^d + x_{t+2i}^*)(p_{t+2i}^d - q_{t+2i+1}), x_t^*\right) \\
= \; & E_t^d\left[(c_{t+2i}^d + x_{t+2i}^*)\right] Cov_t^d\left((p_{t+2i}^d - q_{t+2i+1}), x_t^*\right) \\
& + E_t^d\left[(p_{t+2i}^d - q_{t+2i+1})\right] Cov_t^d\left((c_{t+2i}^d + x_{t+2i}^*), x_t^*\right) \\
& Cov_t^d\left(x_{t+2i+1}^d(p_{t+2i+1}^* - q_{t+2i+1}), x_t^*\right) \\
= \; & E_t^d\left[x_{t+2i+1}^d\right] Cov_t^d\left((p_{t+2i+1}^* - q_{t+2i+1}), x_t^*\right) \\
& + E_t^d\left[(p_{t+2i+1}^* - q_{t+2i+1})\right] Cov_t^d\left(x_{t+2i+1}^d, x_t^*\right)
\end{aligned} \tag{A.20}$$

Using (11), (12) and the results in lemma 2, it is straightforward to check that all the expectations in (A.20) equal zero for $i \geq 1$, and hence, by (A.15), (A.18) and (A.20), $Cov_t^d\left(\Delta^\tau w_T^d, x_t^*\right) = 0$ as shown in (A.10).

In the case of (A.19),

$$\begin{aligned}
& Cov_{t+1}^d\left((c_{t+2i}^d + x_{t+2i}^*)(p_{t+2i}^d - q_{t+2i+1}), p_{t+1}^*\right) \\
= \; & E_{t+1}^d\left[(c_{t+2i}^d + x_{t+2i}^*)\right] Cov_{t+1}^d\left((p_{t+2i}^d - q_{t+2i+1}), p_{t+1}^*\right) \\
& + E_{t+1}^d\left[(p_{t+2i}^d - q_{t+2i+1})\right] Cov_{t+1}^d\left((c_{t+2i}^d + x_{t+2i}^*), p_{t+1}^*\right), \\
& Cov_{t+1}^d\left(x_{t+2i+1}^d(p_{t+2i+1}^* - q_{t+2i+1}), p_{t+1}^*\right) \\
= \; & E_{t+1}^d\left[x_{t+2i+1}^d\right] Cov_{t+1}^d\left((p_{t+2i+1}^* - q_{t+2i+1}), p_{t+1}^*\right) \\
& + E_{t+1}^d\left[(p_{t+2i+1}^* - q_{t+2i+1})\right] Cov_{t+1}^d\left(x_{t+2i+1}^d, p_{t+1}^*\right).
\end{aligned} \tag{A.21}$$

As above, it is straightforward to check that all the expectations in (A.21) are equal to zero for $i \geq 1$. Hence, (A.16), (A.19) and (A.21) together imply that $Cov_{t+1}^d\left(\Delta^\tau w_T^d, p_{t+1}^*\right) = 0$ as shown in (A.11). (A.9) of Lemma 3 and (A.11) imply (A.13).

To derive (A.12), I use Lemma 4 in conjunction with (A.13) to write

$$\begin{aligned} Cov_t^d\left(x_{t+1}^d(p_{t+1}^* - q_{t+1}), x_t^*\right) &= \psi(q_t - E_t^d p_{t+1}^*) Cov_t^d\left((p_{t+1}^* - q_{t+1}), x_t^*\right) \\ &\quad + \psi(E_t^d p_{t+1}^* - q_t) Cov_t^d\left((q_{t+1} - E_{t+1}^d p_{t+1}^*), x_t^*\right). \end{aligned}$$

With $q_{t+1}$ following an independent random walk, and $Cov_t^d\left(p_{t+1}^*, x_t^*\right) = Cov_t^d\left(E_{t+1}^d p_{t+1}^*, x_t^*\right)$ because $x_t^* \in \Omega_{t+1}^d$, this expression simplifies to

$$Cov_t^d\left(x_{t+1}^d(p_{t+1}^* - q_{t+1}), x_t^*\right) = 2\psi(q_t - E_t^d p_{t+1}^*) Cov_t^d\left(E_{t+1}^d p_{t+1}^*, x_t^*\right). \tag{A.22}$$

Equation (11) and Lemma 2 imply that $E_{t+1}^d p_{t+1}^* = q_{t+1} + (\alpha\eta_1 + \eta_2)\delta_u(v_t + u_t^d)$, so

$$Cov_t^d\left(E_{t+1}^d p_{t+1}^*, x_t^*\right) = (\alpha\eta_1 + \eta_2)\eta_3 Cov_t^d(v_t, v_{t-1}). \tag{A.23}$$

Lemma 2 also implies that

$$\begin{aligned} v_{t-1} - E_t^d v_{t-1} &= (1-\phi)\alpha v_{t-1} - \alpha\phi v_t - \alpha\phi u_t^d - \delta_e(1-\alpha^2\phi)\tilde{v}_{t-1}^d \\ v_t - E_t^d v_t &= (1-\phi)v_t - \phi u_t^d - \phi\alpha v_{t-1} - \phi\alpha\delta_e\tilde{v}_{t-1}^d \end{aligned}$$

so

$$\begin{aligned} E_t^d\left[(v_{t-1} - E_t^d v_{t-1})\left(v_t - E_t^d v_t\right)\right] &= \phi\alpha\left[(1-\phi)(1-\alpha(1+\delta_e)) + (\delta_e+1)(1-\alpha^2\phi)\delta_e\right]\Sigma_v \\ &\quad + \alpha\phi^2\Sigma_u + \delta_e\alpha\phi(1-\alpha^2\phi)\Sigma_e. \end{aligned} \tag{A.24}$$

Combining (A.22) -(A.24) gives.(A.12) where

$$\varphi = 2\psi(\alpha\eta_1 + \eta_2)\eta_3\left\{\phi\alpha\left[(1-\phi)(1-\alpha(1+\delta_e)) + (\delta_e+1)(1-\alpha^2\phi)\delta_e\right]\Sigma_v + \alpha\phi^2\Sigma_u + \delta_e\alpha\phi(1-\alpha^2\phi)\Sigma_e\right\}.$$

*Step 2:* I proceed using the method of undetermined coefficients with dealer $d$'s expectations be represented by

$$E_t^d x_t^* = \gamma_1 c_t^d + \gamma_2 v_{t-2} + \gamma_3 \tilde{v}_{t-1}^d, \tag{A.25}$$
$$E_{t+1}^d p_{t+1}^* = q_{t+1} + \pi_1 c_t^d + \pi_2 v_{t-1}, \tag{A.26}$$

for $\pi_i$ and $\gamma_i$ determined below.

Consider the orders from dealer $b$ in period $t$ : Lagging (A.13) one period with $d = b$, and substituting for $E_t^b p_t^a$ with (A.26), gives

$$x_t^b = \psi q_t - \psi \left[ q_t + \pi_1 c_{t-1}^b + \pi_2 v_{t-2} \right].$$

Next, we take expectations conditioned on $\Omega_t^d$ (dealer $a's$ time $t$ information) to obtain,

$$E_t^a x_t^b = -\psi\pi_1 E_t^a v_{t-1} - \psi(\pi_2 + \alpha\pi_1) E_t^a v_{t-2} - \psi\pi_1 E_t^a u_{t-1}^b.$$

Since $t$ is the first period of dealer $a'$ trading cycle, Lemma 2 implies that $E_t^a v_{t-1} = \delta_e \tilde{v}_{t-1}^a + \alpha\phi(c_t^a - \alpha\delta_e \tilde{v}_{t-1}^a)$, .random matching ensures that $E_t^a u_{t-1}^b = 0$, and Lemma 1 implies that $E_t^a v_{t-2} = v_{t-2}$. Combining these results with the equation above, and simplifying, gives

$$E_t^a x_t^b = -\psi\alpha\phi\pi_1 c_t^a - \psi\left(\pi_1\alpha + \pi_2\right) v_{t-2} - \psi\delta_e\pi_1(1 - \alpha^2\phi)\tilde{v}_{t-1}^a. \tag{A.27}$$

Next, consider the pricing decision made by group $B$ dealers at the beginning of their trading cycle in period $t+1$. According to (A.14) these prices are given by

$$p_{t+1}^b = q_{t+1} + \beta c_{t+1}^b + \beta E_{t+1}^b x_{t+1}^a + \beta\theta\varphi(q_{t+1} - E_{t+1}^b p_{t+2}^a). \tag{A.28}$$

Leading (A.26) one period forward, and taking expectations conditioned on $\Omega_{t+1}^b$, gives

$$E_{t+1}^b p_{t+2}^a = q_{t+1} + \pi_1 c_{t+1}^b + \pi_2 E_{t+1}^b v_t. \tag{A.29}$$

Since period $t+1$ is the first period of dealer $b$'s trading cycle, $E_{t+1}^b v_{t+1} = \phi(c_{t+1}^b - \alpha\delta_e \tilde{v}_t^b)$ and $E_{t+1}^b v_t = \delta_e \tilde{v}_t^b + \alpha\phi(c_{t+1}^b - \alpha\delta_e \tilde{v}_t^b)$ from (A.6) of Lemma 2. Combining these results with (A.25), (A.28) and (A.29), gives

$$p_{t+1}^{b} = q_{t+1} + \beta\left[1+\gamma_1 - \theta\varphi\pi_1 - \theta\varphi\pi_2\alpha\phi\right]c_{t+1}^{b} + \beta\gamma_2 v_{t-1} + \beta\left[\gamma_3 - \theta\varphi\pi_2\delta_e(1-\alpha^2\phi)\right]\tilde{v}_t^{b}.$$

Next, take expectations conditioned on $\Omega_{t+1}^{a}$ to give

$$\begin{aligned} E_{t+1}^{a}p_{t+1}^{b} &= q_{t+1} + \beta\left[1+\gamma_1 - \theta\varphi\pi_1 - \theta\varphi\pi_2\alpha\phi\right]E_{t+1}^{a}c_{t+1}^{b} + \beta\gamma_2 v_{t-1} \\ &\quad +\beta\left[\gamma_3 - \theta\varphi\pi_2\delta_e(1-\alpha^2\phi)\right]E_{t+1}^{a}\tilde{v}_t^{b}. \end{aligned}$$

Period $t+1$ is the second period of dealer $a'$s trading cycle so $E_{t+1}^{a}c_{t+1}^{b} = \alpha E_{t+1}^{a}v_t = \alpha\delta_u(c_t^{a} - \alpha v_{t-1})$ and $E_{t+1}^{a}\tilde{v}_t^{b} = E_{t+1}^{a}v_t = \delta_u(c_t^{a} - \alpha v_{t-1})$ from Lemmas 1 and 2. Substituting in these expectations into the expression above gives

$$\begin{aligned} E_{t+1}^{a}p_{t+1}^{b} &= q_{t+1} + \beta\left[1+\gamma_1 - \theta\varphi\pi_1 - \theta\varphi\pi_2\alpha\phi\right]\alpha\delta_u(c_t^{a} - \alpha v_{t-1}) + \beta\gamma_2 v_{t-1} \\ &\quad +\beta\left[\gamma_3 - \theta\varphi\pi_2\delta_e(1-\alpha^2\phi)\right]\delta_u(c_t^{a} - \alpha v_{t-1}). \end{aligned} \tag{A.30}$$

which is in the form of (A.26) above.

Equating coefficients in (A.25) and (A.26) with (A.27) and (A.30) respectively, gives the following set of equations:

$$\begin{aligned} &\pi_1 = \beta\left[1+\gamma_1 - \theta\varphi\pi_1 - \theta\varphi\pi_2\alpha\phi\right]\alpha\delta_u + \beta\left[\gamma_3 - \theta\varphi\pi_2\delta_e(1-\alpha^2\phi)\right]\delta_u \\ &\pi_2 = \beta\gamma_2 - \pi_1\alpha, \qquad \gamma_1 = -\psi\alpha\phi\pi_1, \\ &\gamma_2 = -\psi\left(\pi_1\alpha + \pi_2\right) \quad \gamma_3 = -\psi\delta_e\pi_1(1-\alpha^2\phi). \end{aligned} \tag{A.31}$$

This establishes that dealer's expectations take the form of (A.25) and (A.26).

To establish that equilibrium prices and orders follow (11) and (12), first note that the equations for $\pi_2$ and $\gamma_2$ in (A.31) imply that $\gamma_2 = 0$, and $\pi_2 = -\alpha\pi_1$. Using these results with (A.25), (A.26) (A.29) and lemma 2 to substitute for the expectations in (A.13) and (A.14):

$$\begin{aligned} p_t^{d} &= q_t + \beta\left(1 - \left(\theta\varphi(1-\alpha^2\phi) + \psi\phi\alpha\right)\alpha\delta_u\beta\Xi\right)c_t^{d} \\ &\quad -\beta^2\delta_u\alpha\Xi\delta_e\left(1-\alpha^2\phi\right)\left(\psi - \alpha\theta\varphi\right)\tilde{v}_{t-1}^{d} \end{aligned} \tag{A.32}$$

$$x_t^{d} = -\psi\beta\delta_u\alpha\Xi(c_{t-1}^{d} - \alpha v_{t-2}) \tag{A.33}$$

(A.32) and (A.33) take the same form as the posited process for equilibrium prices and orders in (9) and (10), with

$$\begin{aligned}
&\eta_1 = \beta\left(1 - \left(\theta\varphi(1-\alpha^2\phi) + \psi\phi\alpha\right)\alpha\delta_u\beta\Xi\right), \quad \eta_3 = -\psi\beta\delta_u\alpha\Xi, \\
&\eta_2 = -\beta^2\delta_u\alpha\Xi\delta_e\left(1-\alpha^2\phi\right)(\psi - \alpha\theta\varphi), \\
&\Xi^{-1} = 1 + \beta\alpha\delta_u(\psi\alpha\phi + \theta\varphi(1-\alpha^2\phi)) + \beta\delta_e(1-\alpha^2\phi)(\psi - \alpha\theta\varphi)\delta_u
\end{aligned} \tag{A.34}$$

Together with (1), these equations imply that

$$\begin{aligned}
Var_t^d(x_t^*) &= {\eta_3}^2\left[\Sigma_u + \Sigma_v\left(1-\delta_e)(1-\alpha^2\phi\right)\right], \\
Var_{t+1}^d(p_{t+1}^*) &= \Sigma_q + \eta_1^2(\Sigma_v + \Sigma_u) + \eta_2^2\Sigma_e + (\eta_1\alpha + \eta_2)^2\Sigma_v(1-\delta_u).
\end{aligned}$$

When there are no idiosyncratic shocks to customer orders (i.e., $\Sigma_u = 0$), $\Sigma_e = 0$, $\delta_u = \delta_e = 1$ $\phi = 0$ and $\varphi = 0$. Substituting these restrictions into the expressions for $\eta_i$ implies that $p_t^d = q_t + \beta(v_t + \frac{\alpha}{1+\beta\psi}v_{t-1})$ and $x_t^d = -\frac{\beta\psi\alpha}{1+\beta\psi}v_{t-1}$ so the heterogeneity in prices and orders disappears.

Finally, we need to establish that $|\eta_2/\eta_1| < 1$ in order for $\Sigma_e$ to be finite. I argue by contradiction. Suppose $|\eta_2/\eta_1| \geq 1$, so that $\delta_e = 0$. Then $\eta_2 = 0$ by (A.34) above. Hence, $\eta_1$ must also be equal to zero. According to (A.34), this requires that either (i) $\beta \equiv (\theta Var_t^d(x_t^*))^{-1} = 0$ or (ii) $1 = \left(\theta\varphi(1-\alpha^2\phi) + \psi\phi\alpha\right)\alpha\delta_u\beta\Xi$.Condition (i) cannot hold because $Var_t^d(x_t^*)$ is finite even when $\Sigma_e$ is not because $|\eta_3| < \infty$ when $\delta_e = 0$. All that remains is condition (ii). Substituting for $\Xi$ with $\delta_e = 0$, it is easy to check that (ii) cannot hold.

# B Estimation Details

The appendix describers how the GMM technique is applied to produce the estimation results reported in the tables.

## B.1 Autocorrelations

To estimate $Corr(w_t, w_{t-i})$, let $y_t = w_t - \alpha_0 - \alpha_1 w_{t-i}$ and $z_t' = [1, w_t]$. The GMM estimates of $\alpha_0$ and $\alpha_1$ are then computed using $m_t(\theta) = y_t z_{t-i}'$ with weighting matrix $\Gamma_{0T^*}$. Table 2 reports the estimate of $\alpha_1$, together with the p-value for the Wald test of the null hypothesis of $\alpha_1 = 0$.

## B.2 ARMA Models

To illustrate how the ARMA model estimates in Table 3 are calculated, consider the ARMA(2,2) case:

$$w_t = \alpha_1 w_{t-1} + \alpha_2 w_{t-2} + \nu_t + \beta_1 \nu_{t-1} + \beta_2 v_{t-2}. \tag{B.1}$$

The first step is to write the model in the state space form of (23):

$$\begin{bmatrix} w_t \\ w_{t-1} \\ \nu_t \\ \nu_{t-1} \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & \theta_1 & \theta_2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w_{t-1} \\ w_{t-2} \\ \nu_{t-1} \\ \nu_{t-2} \end{bmatrix} + \begin{bmatrix} \nu_t \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$y_t = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_t \\ w_{t-1} \\ \nu_t \\ \nu_{t-1} \end{bmatrix}, \qquad \Omega = \begin{bmatrix} \Sigma_\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The estimates are then computed with $m_t(k;\theta) = y_t y_{t-k} - \gamma(k;\theta)$, for $k = 0, 1, ..12$, where $\gamma(k;\theta)$ $\equiv Cov(y_t, y_{t-k})$ is calculated from the state space form. The weighting matrix is $\Gamma_{0T^*}$.

## B.3 Decomposition Regressions

Table 4 reports GMM estimates of

$$\Delta p_t^+ = \sum\nolimits_{i=\underline{k}}^{i=\overline{k}} \alpha_i x_{t-i} + \nu_t,$$

where $\nu_t$ follows an MA(1) process that is independent from all leads and lags of $x_t$. Let $y_t = \Delta p_t^+ - \sum_{i=\underline{k}}^{i=\overline{k}} \alpha_i x_{t-i}$, and $z_t' = [x_{t-\underline{k}} ......x_{t-\overline{k}}]$. The GMM estimates of the $\alpha_i' s$, are computed using $m_t(\theta) = y_t z_t'$ and the Newey-West weighting matrix with $\kappa = 1$, *i.e.*, $\Gamma_{0T^*} + \frac{1}{2}(\Gamma_{1T^*} + \Gamma_{1T^*}')$. Notice that this choice allows for both heteroskedasticity and an MA(1) process in $\nu_t$.

## B.4 Bivariate Model

The Bivariate model in (28) can be written in the state space form of (23) as

$$\begin{bmatrix} \omega_t^+ \\ \omega_t^- \\ \omega_{t-1}^+ \\ \omega_{t-1}^- \\ \varepsilon_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_{t-1}^+ \\ \omega_{t-1}^- \\ \omega_{t-2}^+ \\ \omega_{t-2}^- \\ \varepsilon_{t-1} \end{bmatrix} + \begin{bmatrix} \omega_t^+ \\ \omega_t^- \\ 0 \\ 0 \\ \varepsilon_t \end{bmatrix}, \qquad \Omega = \begin{bmatrix} \Sigma_\omega & 0 & 0 & 0 & 0 \\ 0 & \Sigma_\omega & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Sigma_\varepsilon \end{bmatrix},$$

$$y_t \equiv \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} \equiv \begin{bmatrix} \Delta p_t^+ - \sum_{i=1}^{i=-4} \alpha_i x_{t-i} \\ \Delta p_t^- - \sum_{i=1}^{i=-4} \alpha_i x_{t-i} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \omega_t^+ \\ \omega_t^- \\ \omega_{t-1}^+ \\ \omega_{t-1}^- \\ \varepsilon_t \end{bmatrix}.$$

To obtain the GMM estimates, I use instruments, $z_t' = [x_{t-1}......x_{t+4}]$, and moments

$$\begin{aligned} m_t(0;\theta) &= \begin{bmatrix} vec(y_t z_t') \\ vec(y_t y_t') - \gamma(0;\theta) \end{bmatrix}, \\ m_t(k;\theta) &= vec(y_t y_{t-k}') - \gamma(k;\theta) \qquad k = 1, 2, \end{aligned}$$

with the Newey-West ($\kappa = 1$) weighting matrix, $\Gamma_{0T^*} + \frac{1}{2}(\Gamma_{1T^*} + \Gamma_{1T^*}')$.

## B.5 Tests for State-Dependency

The upper panel of Table 6 reports Wald tests for non-linearity in models of the form:

$$\Delta p_t^+ = D_0(L)x_{t-i} + D_1(L)n_t x_t + D_2(L)n_t^2 x_t + D_3(L)n_t^3 x_t + \nu_t,$$

where $\nu_t$ follows an MA(1) process that is independent from all leads and lags of $x_t$ and $D_i(L) = d_1^i L + d_2^i L^0 + d_3^i L^{-1} + ... d_6^i L^{-4}$. Each cell reports the Wald statistic and p-value for the null hypothesis of $d_i^j = 0$ for $i = \{1, 2..6\}$ with $j = 1$ (left hand column), $j = 1$ and 2 (center column), and $j = 1, 2,$ and 3 (right hand column). In each case, the $d_i^j$ coefficients are estimated by GMM for the specification including $D_0(L)x_t$ and the regressors listed at the head of each column along the lines described in B.4.

The lower panel of Table 6 reports tests for heteroskedasticity in the shocks to the Bivariate Model and ARMA(2,2) order flow model. Estimates of the shocks from the Bivariate Model are obtained as:

$$\begin{aligned}
\left(\widetilde{\widetilde{\varepsilon}}_t\right)^2 &= \hat{y}_{1t}\hat{y}_{2t} = \left(\hat{\varepsilon}_t + \hat{\omega}_t^+ - \hat{\omega}_{t-1}^+\right)\left(\hat{\varepsilon}_t + \hat{\omega}_t^- - \hat{\omega}_{t-1}^-\right) = \left(\hat{\varepsilon}_t\right)^2 + \zeta_t^{\varepsilon}, \\
\left(\widetilde{\widetilde{\omega}}_t^+\right)^2 &= -\hat{y}_{1t+1}\hat{y}_{1t} = -\left(\hat{\varepsilon}_{t+1} + \hat{\omega}_{t+1}^+ - \hat{\omega}_t^+\right)\left(\hat{\varepsilon}_t + \hat{\omega}_t^+ - \hat{\omega}_{t-1}^+\right) = \left(\hat{\omega}_t^+\right)^2 + \zeta_t^+, \\
\left(\widetilde{\widetilde{\omega}}_t^-\right)^2 &= -\hat{y}_{2t+1}\hat{y}_{2t} = -\left(\hat{\varepsilon}_{t+1} + \hat{\omega}_{t+1}^- - \hat{\omega}_t^-\right)\left(\hat{\varepsilon}_t + \hat{\omega}_t^- - \hat{\omega}_{t-1}^-\right) = \left(\hat{\omega}_t^-\right)^2 + \zeta_t^-,
\end{aligned} \tag{B.2}$$

where "hats" denote the GMM estimates. The estimated innovations to the ARMA(2,2) order flow model are found from (B.1) as

$$\left(\widetilde{\widetilde{v}}_t\right)^2 = \left(w_{t+2} - \hat{\alpha}_1 w_{t+1} - \hat{\alpha}_2 w_t\right)\left(w_t - \hat{\alpha}_1 w_{t-1} + \hat{\alpha}_2 w_{t-2}\right)/\hat{\beta}_2 = \left(\hat{v}_t\right)^2 + \zeta_t^v.$$

Under the null of a correctly specified model, all the error terms, $\zeta_t^i$, have mean zero and are serially uncorrelated. To implement the Glesjer (1969) tests, I estimate $\varpi_t^2 = \alpha_0 + \underline{n}_t'\alpha + \xi_t$ for each shock $\varpi_t$, where the vector $\underline{n}_t$ includes the terms listed at the head of each column. The GMM estimates of $\alpha_0$ and $\alpha$ are then computed using $m_t(\theta) = y_t z_t'$ where $y_t = \varpi_t^2 - \alpha_0 - \underline{n}_t'\alpha$ and $z_t' = [1, \underline{n}_t']$ with weighting matrix $\Gamma_{0T^*}$. The table reports the Wald test for $\alpha = 0$ based on these estimates.

## B.6 Bivariate Model with State-Dependence

The State-Dependent model in (29) can be written in the state space form of (23) as

$$\begin{bmatrix} \omega_t^+ \\ \omega_t^- \\ \omega_{t-1}^+ \\ \omega_{t-1}^- \\ \varepsilon_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_{t-1}^+ \\ \omega_{t-1}^- \\ \omega_{t-2}^+ \\ \omega_{t-2}^- \\ \varepsilon_{t-1} \end{bmatrix} + \begin{bmatrix} \omega_t^+ \\ \omega_t^- \\ 0 \\ 0 \\ \varepsilon_t \end{bmatrix},$$

$$\Omega(n_t) = \begin{bmatrix} \Sigma_\omega(n_t) & 0 & 0 & 0 & 0 \\ 0 & \Sigma_\omega(n_t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Sigma_\varepsilon(n_t) \end{bmatrix}.$$

$$y_t \equiv \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} \equiv \begin{bmatrix} \Delta p_t^+ - \sum_{i=1}^{i=-4} \alpha_i(n_t) x_{t-i} \\ \Delta p_t^- - \sum_{i=1}^{i=-4} \alpha_i(n_t) x_{t-i} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \omega_t^+ \\ \omega_t^- \\ \omega_{t-1}^+ \\ \omega_{t-1}^- \\ \varepsilon_t \end{bmatrix},$$

where

$$\begin{aligned} \Sigma_i(n) &= \Sigma_i(0)\exp(-n/100) + \Sigma_i(\infty)(1-\exp(-n/100)), \\ \alpha_i(n) &= \alpha_i(0)\exp(-n/100) + \alpha_i(\infty)(1-\exp(-n/100)). \end{aligned}$$

To obtain the GMM estimates, I use instruments,

$$z_t' = [x_{t-1}, ......x_{t+4}, x_{t-1}\exp(-n_t/100), ......x_{t+4}\exp(-n_t/100)],$$

and moments

$$\begin{aligned} m_t(0;\theta) &= \begin{bmatrix} vec(y_t z_t') \\ vec(y_t y_t') - \gamma_t(0;\theta) \end{bmatrix}, \\ m_t(k;\theta) &= vec(y_t y_{t-k}') - \gamma(k;\theta, n_t) \qquad k = 1, 2. \end{aligned}$$

where

$$\gamma_t(0;\theta) = \begin{bmatrix} \Sigma_\omega(n_t)+\Sigma_\omega(n_{t-1})+\Sigma_\varepsilon(n_t) & \Sigma_\varepsilon(n_t) \\ \Sigma_\varepsilon(n_t) & \Sigma_\omega(n_t)+\Sigma_\omega(n_{t-1})+\Sigma_\varepsilon(n_t) \end{bmatrix}$$

$$\gamma_t(1;\theta) = \begin{bmatrix} -\Sigma_\omega(n_{t-1}) & 0 \\ 0 & -\Sigma_\omega(n_{t-1}) \end{bmatrix}, \qquad \gamma_t(2;\theta) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

As in the Bivariate Model, I use the Newey-West ($\kappa = 1$) weighting matrix, $\Gamma_{0T^*} + \frac{1}{2}(\Gamma_{1T^*} + \Gamma'_{1T^*})$.

The Wald test for $D(L,0) = D(L,\infty)$ is computed as $\nabla\hat{\alpha}' \left(\hat{\Omega}_{\nabla\alpha}\right)^{-1} \nabla\hat{\alpha}$ where

$$\nabla\hat{\alpha} = [\hat{\alpha}_1(0) - \hat{\alpha}_1(\infty), \ldots\ldots, \hat{\alpha}_{-4}(0) - \hat{\alpha}_{-4}(\infty)]$$

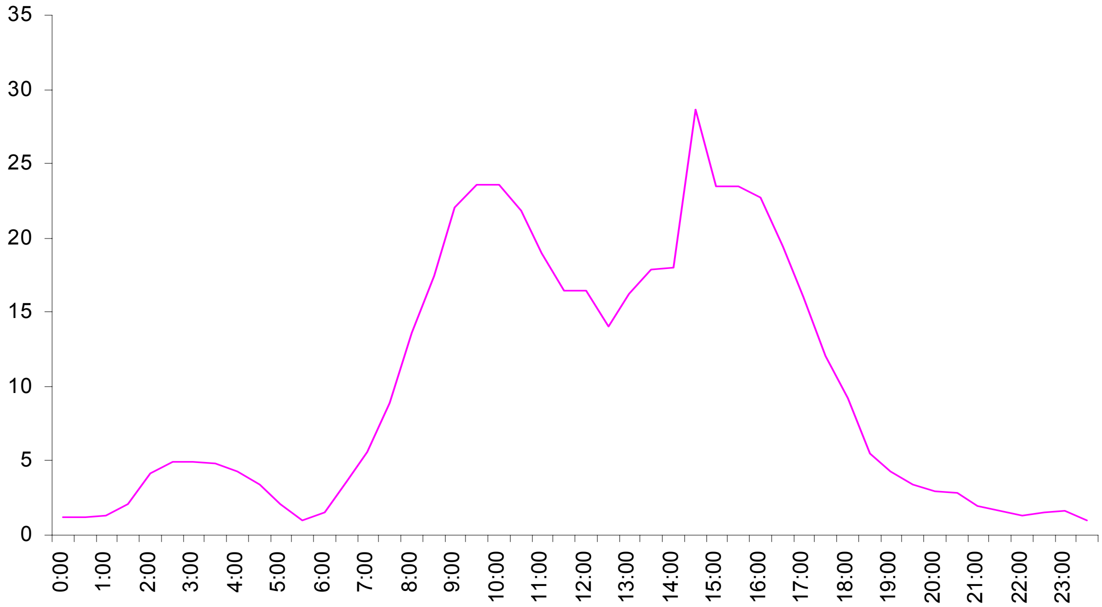
and $\hat{\Omega}_{\nabla\alpha}$ is the estimated asymptotic covariance matrix of $\nabla\alpha$. To test for misspecification in the $\alpha_i(n)$ and $\Sigma_i(n)$ functions, I use the GMM version of the LM test developed by Newey and West (1987). In the case of the $\alpha_i(n)$ functions, I consider alternative specifications of the form $\tilde{\alpha}_i(n) = \alpha_i(n) + \varphi_i n$. To test the null hypothesis that $\varphi_i = 0$ for all $i$, I use the two step procedure suggested by Greene (1997). First I compute the derivative for the GMM criterion function $Q(\theta)$ with $\tilde{\alpha}_i(n)$ replacing $\alpha_i(n)$ at the GMM estimates with $\varphi_i = 0$. I then calculate the Wald statistic for the null hypothesis that this vector of derivatives equals zero. In the case of the variance functions $\Sigma_\omega(n)$ and $\Sigma_\varepsilon(n)$ functions, the alternative specifications take the form of $\tilde{\Sigma}_\omega(n) = \Sigma_\omega(n) + \varphi n$ and $\tilde{\Sigma}_\varepsilon(n) = \Sigma_\varepsilon(n) + \varphi n$.

The lower panel of the table reports autocorrelations for the estimated shocks. The shocks are calculated from the GMM estimates as in (B.2), and standardized as $\tilde{\varpi}_t^2 = \varpi_t^2[\hat{\Sigma}_\varpi(n_t)]^{-1}$ where $\varpi_t$ denotes the shock in question. I then estimate $\tilde{\varpi}_t^2 = \alpha_0 + \alpha_i\tilde{\varpi}_{t-i}^2 + \xi_t$ by GMM using $m_t(\theta) = y_t z'_{t-i}$ where $y_t = \tilde{\varpi}_t^2 - \alpha_0 - \alpha_i\tilde{\varpi}_{t-i}^2$ and $z'_t = \left[1, \tilde{\varpi}_t^2\right]$ with weighting matrix $\Gamma_{0T^*}$. The table reports the estimate of $\alpha_i$ and GMM standard error allowing for heteroskedasticity. Under the null hypothesis that $Var(v_t) = \Sigma_v(n_t)$, $\alpha_i = 0$ for all lags $i$.

## Figure 1: Trading Cycle

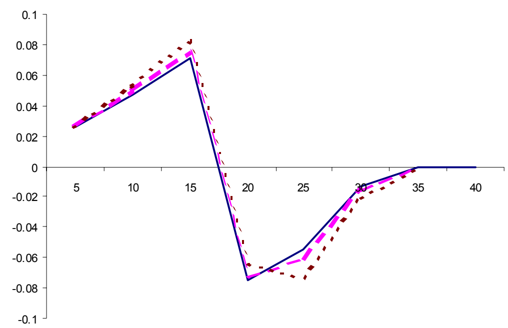
| | | | | | | |
|---|---|---|---|---|---|---|
| **Dealer** $a \in A$ | | | | | | |
| New Information | $q_{t-1}, x_{t-3}, p_{t-3}$ | $p^b_{t-1}$ | $q_t, x_{t-2}, p_{t-2}\ c^a_t$ | $x^b_t$ | $q_{t+1}, x_{t-1}, p_{t-1}$ | $p^b_{t+1}$ |
| Decisions | Choose inter dealer order flow $x^a_{t-1}$ | | Set price $p^a_t$ | | Choose inter dealer order flow $x^a_{t+1}$ | |
| Actions | Trade with public at price $q_{t-1}$ | Buy $x^a_{t-1}$ from dealer $b$ at $p^b_{t-1}$ | | Sell $x^b_t$ to dealer $b$ at $p^a_t$<br>Sell $c^a_t$ to customers at $p^a_t$ | Trade with customers at $q_{t+1}$ | Buy $x^a_{t+1}$ from dealer $b$ at $p^b_{t+1}$ |
| Trading Periods | *t-1* | | *t* | | *t*+1 | |
| **Dealer** $b \in B$ | | | | | | |
| New Information | $q_{t-1}, x_{t-3}, p_{t-3},\ c^b_{t-1}$ | $x^a_{t-1}$ | $q_t, x_{t-2}, p_{t-2}$ | $p^a_t$ | $q_{t+1}, x_{t-1}, p_{t-1},\ c^b_{t+1}$ | $x^a_{t+1}$ |
| Decisions | Set price $p^b_{t-1}$ | | Choose inter dealer order flow $x^b_t$ | | Set price $p^b_{t+1}$ | |
| Actions | | Sell $x^a_{t-1}$ to dealer $a$ at $p^b_{t-1}$<br>Sell $c^b_{t-1}$ to customers at $p^b_{t-1}$. | Trade with public at price $q_t$ | Buy $x^b_t$ from dealer $a$ at $p^a_t$ | | Sell $x^a_{t+1}$ to dealer $a$ at $p^b_{t+1}$<br>Sell $c^b_{t+1}$ to customers at $p^b_{t+1}$. |

## Figure 2: Average Trade Intensity

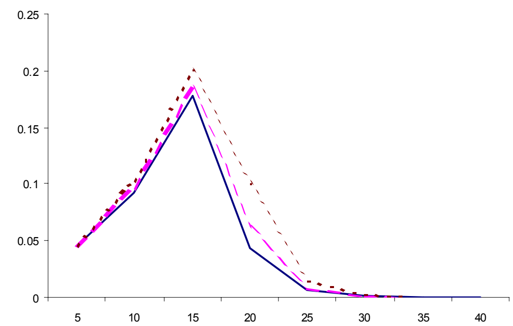



The figure shows the average number of direct-interdealer transactions per minute over the 79 trading days in the sample plotted over 24 hours.

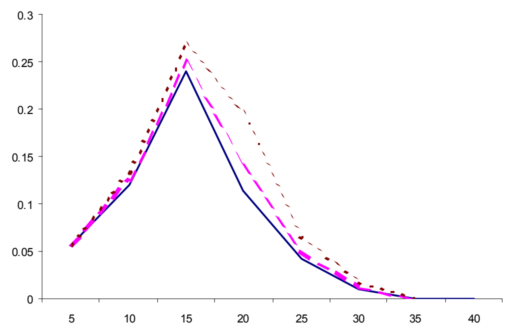
# Figure 3: Impulse Response Functions



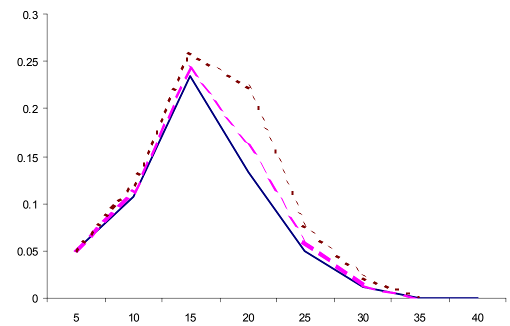

A: $n = 5$




B: $n = 20$



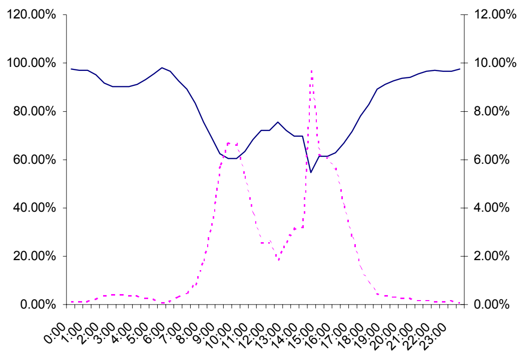

C: $n = 40$



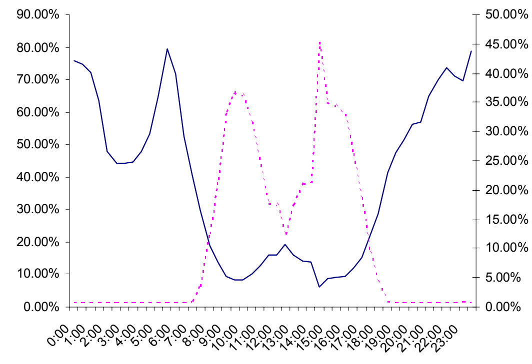

D: $n$ = all

Panels A, B, and C plot $B(L,n) = D(L,n)C(L)$ where $D(L,n)$ is the estimated state-dependent polynomial on order flow from Table 6 and $C(L)$ is the polynomial implied by the estimated ARMA model for order flow in Table 3. Panel D plots $B(L,n) = D(L)C(L)$ where $D(L)$ is estimated the polynomial on order flow from Table 5 (without state-dependence). The solid, dashed and dotted lines respectively show the impulse response based on ARMA(2,2), ARMA(2,1) and ARMA(1,2) order flow models.

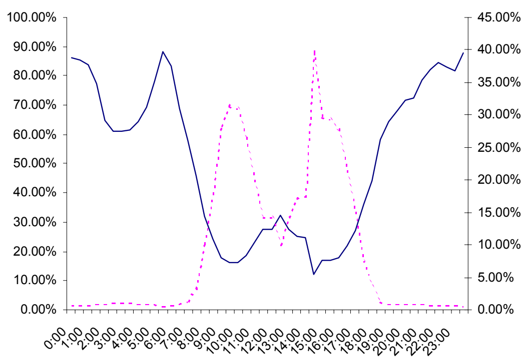
# Figure 4: Variance Decompositions Over a Typical Trading Day



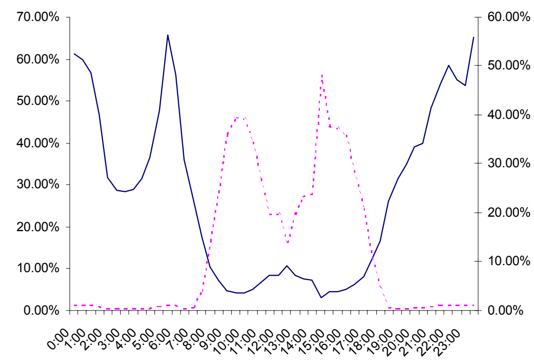

A: 5 minute Horizon ($k$-=1)




B: 30 Minute Horizon ($k$=6)




C: 60 Minute Horizon ($k$=12)




D: 120 Minute Horizon ($k$=24)

Solid lines plot $R_{\omega}(k,n_t)$ against the left hand axis. Dashed lines plot $R_{\upsilon}(k,n_t)$ against the right hand axis. $n_t$ is the average trade intensity rate over the sample during the each 5-minute interval.

## Table 1: Markov Models for Trade Intensities

| **I:With Seasonals** | Transition Probabilities | | | | | |
|---|---|---|---|---|---|---|
| | State | | | | | |
| State | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 0.743 | 0.188 | 0.028 | 0.001 | 0.000 | 0.001 |
| 2 | 0.215 | 0.568 | 0.209 | 0.008 | 0.001 | 0.000 |
| 3 | 0.040 | 0.237 | 0.574 | 0.301 | 0.087 | 0.022 |
| 4 | 0.001 | 0.006 | 0.160 | 0.496 | 0.453 | 0.201 |
| 5 | 0.000 | 0.001 | 0.019 | 0.126 | 0.274 | 0.245 |
| 6 | 0.000 | 0.000 | 0.010 | 0.068 | 0.186 | 0.532 |
| Ergodic Probabilities | 0.210 | 0.244 | 0.283 | 0.160 | 0.053 | 0.050 |
| Lower Bounds | 0 | 1 | 4 | 14 | 26 | 36 |

| **II:Without Seasonals** | Transition Probabilities | | | | | |
|---|---|---|---|---|---|---|
| | States | | | | | |
| State | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 0.725 | 0.110 | 0.068 | 0.108 | 0.105 | 0.036 |
| 2 | 0.108 | 0.497 | 0.266 | 0.182 | 0.081 | 0.034 |
| 3 | 0.066 | 0.265 | 0.466 | 0.278 | 0.129 | 0.047 |
| 4 | 0.060 | 0.104 | 0.167 | 0.297 | 0.288 | 0.164 |
| 5 | 0.021 | 0.016 | 0.020 | 0.086 | 0.232 | 0.221 |
| 6 | 0.019 | 0.007 | 0.013 | 0.049 | 0.165 | 0.498 |
| Ergodic Probabilities | 0.248 | 0.251 | 0.250 | 0.150 | 0.050 | 0.050 |
| Lower Bounds | -28.672 | -3.644 | -0.974 | 1.775 | 8.472 | 16.582 |

Notes: The tables report estimates of the transition probabilities for a first-order, 6-state Markov processes for trade intensities. Panel I reports estimates based on the raw intensities, while panel II shows estimates for the deseasonalized intensities, where the latter are computed as $n_t - \overline{n_t}$ with $\overline{n_t}$ denoting the sample average rate for observation period $t$ (plotted in Figure 2). The table reports the lower bounds that define the 6 states, and the estimated unconditional (ergodic) probability of each state occurring. The transition probabilities are estimated as the relative frequency that a particular transition occurred over the sample.

## Table 2: Sample Statistics

| | mean | max. | min. | Sdt. | skewness | kurtosis |
|---|---|---|---|---|---|---|
| $\Delta p_t^+$ | 0.000 | 0.500 | -0.790 | 0.076 | -0.194 | 7.291 |
| | | | | 0.008 | 0.020 | 0.039 |
| $x_t$ | 0.005 | 69.000 | -72.000 | 5.211 | 0.102 | 14.260 |
| | | | | 0.007 | 0.017 | 0.034 |

Autocorrelations (p-values)

| lag | 1 | 2 | 3 | 4 | 5 | 6 | 12 | 18 | 24 |
|---|---|---|---|---|---|---|---|---|---|
| $\Delta p_t^+$ | -0.319 | -0.014 | -0.005 | 0.002 | -0.006 | 0.004 | 0.005 | 0.020 | 0.001 |
| | (0.000) | (0.170) | (0.650) | (0.858) | (0.539) | (0.732) | (0.607) | (0.037) | (0.899) |
| $x_t$ | 0.232 | 0.105 | 0.092 | 0.077 | 0.060 | 0.058 | 0.025 | 0.027 | 0.005 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.007) | (0.005) | (0.578) |

Notes: $p_t^+$ is 100 times the last DM purchase price for dollars on the Reuter's D2000-1 system during observation interval $t$. $x_t$ is the difference between the number of buyer-initiated and seller-initiated trades during observation interval $t$. The autocorrelations are computed by GMM. The p-values are calculated from Wald tests of the null hypothesis of a zero correlation allowing for conditional heteroskedasticity (see Appendix B for details).

## Table 3: ARMA Models

| Coeffs. | $a_1$ | $a_2$ | $b_1$ | $b_2$ | $\sigma^2$ | J-stat. | p-value | Df. |
|---|---|---|---|---|---|---|---|---|
| $\Delta p_t^+$ | -0.2347 | | | | 0.0728 | 53.8420 | 0.0000 | 11 |
| | (0.0084) | | | | (0.0006) | | | |
| | | | 0.3079 | | 0.0719 | 4.9999 | 0.9312 | 11 |
| | | | (0.0122) | | (0.0007) | | | |
| | 0.0768 | | 0.3903 | | 0.0717 | 1.8596 | 0.9973 | 10 |
| | (0.0269) | | (0.0308) | | (0.0007) | | | |
| | 0.0809 | 0.0012 | 0.3945 | | 0.0717 | 1.8587 | 0.9935 | 9 |
| | (0.0874) | (0.0246) | (0.0902) | | (0.0007) | | | |
| | 0.0972 | | 0.4108 | -0.0063 | 0.0717 | 1.8587 | 0.9935 | 9 |
| | (0.4022) | | (0.4041) | (0.1244) | (0.0007) | | | |
| | -1.0129 | 0.0898 | -0.7000 | 0.4305 | 0.0717 | 1.1029 | 0.9975 | 8 |
| | (0.2873) | (0.0321) | (0.2882) | (0.1066) | (0.0007) | | | |
| $x_t$ | 0.2602 | | | | 5.7451 | 107.4869 | 0.0000 | 11 |
| | (0.0100) | | | | (0.0615) | | | |
| | | | -0.2241 | | 5.7527 | 155.4307 | 0.0000 | 11 |
| | | | (0.0128) | | (0.0612) | | | |
| | 0.7572 | | 0.5953 | | 5.8144 | 32.9568 | 0.0003 | 10 |
| | (0.0166) | | (0.0229) | | (0.0601) | | | |
| | 0.9851 | -0.1133 | 0.7755 | | 5.8769 | 8.9239 | 0.4443 | 9 |
| | (0.0348) | (0.0176) | (0.0300) | | (0.0604) | | | |
| | 0.8417 | | 0.6282 | 0.0959 | 5.8771 | 7.5449 | 0.5806 | 9 |
| | (0.0176) | | (0.0207) | (0.0141) | (0.0604) | | | |
| | 0.7663 | 0.0577 | 0.5531 | 0.1411 | 5.8762 | 7.3681 | 0.4975 | 8 |
| | (0.1356) | (0.1023) | (0.1348) | (0.0802) | (0.0605) | | | |

Notes: The table reports GMM estimates and standard errors for ARMA models of the form

$$z_t = a_1 z_{t-1} + a_2 z_{t-2} + w_t + b_1 w_{t-1} + b_2 w_{t-2},$$

where $Ew_t = 0$ and $E\omega_t^2 = \sigma^2$. The models are estimated from the mean, variance and first 12 autocorrelations of the data (see Appendix B). The right hand columns report the results of Hansen (1982) J-tests for each specification. The column headed Df. reports the degrees of freedom associated with each test. The variables are: $p_t^+$, 100 times the last DM purchase price for dollars on the Reuter's D2000-1 system during observation interval $t$; and $x_t$, the difference between the number of buyer-initiated and seller-initiated trades during observation interval $t$.

# Table 4: Decomposition Regressions

| | $D(L)$ Coefficients (x100) | | | | | | | | | | | Diagnostics | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $x_t$ | $x_{t-1}$ | $x_{t-2}$ | $x_{t-3}$ | $x_{t+1}$ | $x_{t+2}$ | $x_{t+3}$ | $x_{t+4}$ | $x_{t+5}$ | $x_{t+6}$ | $D(L)$ | $R^2$ | SEE | l-test |
| I | -0.134 | -0.018 | | | 0.200 | | | | | | 0.048 | 0.028 | 7.479 | 2.045 |
| | (0.012) | (0.011) | | | (0.016) | | | | | | (0.018) | | | (0.360) |
| II | -0.140 | -0.028 | 0.001 | | 0.167 | 0.157 | | | | | 0.156 | 0.044 | 7.364 | 2.219 |
| | (0.012) | (0.011) | (0.013) | | (0.016) | (0.012) | | | | | (0.021) | | | (0.330) |
| III | -0.145 | -0.031 | -0.002 | -0.001 | 0.165 | 0.140 | 0.085 | | | | 0.210 | 0.051 | 7.331 | 4.359 |
| | (0.012) | (0.011) | (0.014) | (0.011) | (0.016) | (0.013) | (0.012) | | | | (0.021) | | | (0.113) |
| IV | -0.146 | -0.031 | -0.004 | -0.002 | 0.162 | 0.139 | 0.072 | 0.052 | | | 0.242 | 0.053 | 7.307 | 6.046 |
| | (0.012) | (0.011) | (0.014) | (0.011) | (0.016) | (0.013) | (0.013) | (0.011) | | | (0.023) | | | (0.049) |
| V | -0.146 | -0.031 | -0.003 | -0.002 | 0.161 | 0.138 | 0.072 | 0.050 | 0.011 | | 0.249 | 0.054 | 7.299 | 5.287 |
| | (0.012) | (0.011) | (0.014) | (0.011) | (0.016) | (0.013) | (0.013) | (0.012) | (0.011) | | (0.023) | | | (0.071) |
| VI | -0.146 | -0.031 | -0.005 | 0.000 | 0.161 | 0.138 | 0.072 | 0.050 | 0.011 | 0.002 | 0.251 | 0.055 | 7.289 | 4.429 |
| | (0.012) | (0.011) | (0.014) | (0.011) | (0.016) | (0.013) | (0.013) | (0.012) | (0.011) | (0.010) | (0.024) | | | (0.109) |
| VII | -0.145 | -0.033 | | | 0.159 | 0.139 | 0.071 | 0.055 | | | 0.247 | 0.051 | 7.318 | 4.794 |
| | (0.012) | (0.011) | | | (0.016) | (0.013) | (0.012) | (0.011) | | | (0.021) | | | (0.091) |
| VIII | -0.146 | -0.033 | | | 0.159 | 0.138 | 0.071 | 0.052 | 0.012 | | 0.253 | 0.052 | 7.315 | 6.315 |
| | (0.012) | (0.011) | | | (0.016) | (0.013) | (0.012) | (0.012) | (0.011) | | (0.022) | | | (0.043) |
| IX | -0.145 | -0.033 | | | 0.159 | 0.138 | 0.071 | 0.051 | 0.012 | 0.003 | 0.256 | 0.053 | 7.297 | 6.046 |
| | (0.012) | (0.011) | | | (0.016) | (0.013) | (0.013) | (0.012) | (0.011) | (0.010) | (0.023) | | | (0.049) |
| X | -0.091 | -0.013 | 0.017 | 0.014 | | | | | | | -0.073 | 0.005 | 7.559 | 0.075 |
| | (0.013) | (0.012) | (0.014) | (0.011) | | | | | | | (0.017) | | | (0.963) |

Notes: The table reports GMM estimates of the coefficients in the polynomial, $D(L)$ for the regression:

$$\Delta p_t^+ = D(L)x_t + \varepsilon_t + \omega_t^+ - \omega_{t-1}^+,$$

where $p_t^+$ is 100 times the last DM purchase price for dollars on the Reuter's D2000-1 system during observation interval $t$; and $x_t$, the difference between the number of buyer-initiated and seller-initiated trades during observation interval $t$. The GMM estimates and standard errors allow for the presence of conditional heteroskedasticity and an MA(1) error structure (see Appendix B). The column headed $D(1)$ reports the sum of the estimated coefficients in $D(L)$ and its standard error. The right-hand column reports Cumby Huizinga $l$-test statistics for the null hypothesis that the errors follow an MA(1) process. The associated p-values are reported in parenthesis.

# Table 5: Estimates of Bivariate Model

| | | | | | | |
|---|---|---|---|---|---|---|
| | $x_{t+4}$ | $x_{t+3}$ | $x_{t+2}$ | $x_{t+1}$ | $x_t$ | $x_{t-1}$ |
| Coefficients in $D(L)$ | 0.0494 | 0.0700 | 0.1488 | 0.1613 | -0.1461 | -0.0374 |
| (x100) | (0.0103) | (0.0114) | (0.0112) | (0.0156) | (0.0107) | (0.0101) |
| | $(\Sigma_\varepsilon)^{1/2}$ | 0.0391 | (0.0011) | $(\Sigma_\omega)^{1/2}$ | 0.0433 | (0.0004) |
| | $D(1)$ | 0.2461 | (0.0219) | J-statistic | 7.6044 | (0.8684) |

Notes: The table reports GMM estimates of the Bivariate model:

$$\begin{bmatrix} \Delta p_t^+ \\ \Delta p_t^- \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} D(L)x_t + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \varepsilon_t + \begin{bmatrix} \omega_t^+ - \omega_t^+ \\ \omega_t^- - \omega_{t-1}^- \end{bmatrix}.$$

where $w_t^+$, $w_t^-$ and $\varepsilon_t$ are mutually independent and serially uncorrelated shocks with $E\omega_t^+ = \omega^+$, $E\omega_t^- = \omega^-$, $E\varepsilon_t = 0$ and $Var(\varepsilon_t) = \Sigma_\varepsilon$, $Var(\omega_t^+) = Var(\omega_t^-) = \Sigma_\omega$. $p_t^+$ and $p_t^-$ are respectively 100 times the last DM purchase and sales price for dollars on the Reuter's D2000-1 system during observation interval *t*. $x_t$ is the difference between the number of buyer-initiated and seller-initiated trades during observation interval *t*. Asymptotic standard errors corrected for heteroskedasticity and serial correlation are reported below the parameter estimates (see Appendix B). The table also reports Hansen's J-statistic with its associated p-value in parenthesis.

## Table 6: Tests For State-Dependency

| Variable | Non–linearity: $D(L)x_t n_t$ | $D(L)x_t n_t$ $D(L)x_t n_t^2$ | $D(L)x_t n_t$ $D(L)x_t n_t^2$ $D(L)x_t n_t^3$ |
|---|---|---|---|
| $\Delta p^+$ | 97.844<br>(<0.001) | 92.988<br>(<0.001) | 166.284<br>(<0.001) |
| $\Delta p^-$ | 167.426<br>(<0.001) | 252.563<br>(<0.001) | 296.390<br>(<0.001) |
| $x$ | 11.254<br>(0.081) | 16.868<br>(0.158) | 28.132<br>(0.060) |

| Shock | Heteroskedasticity: $n$ | $n$ $n^2$ | $n$ $n^2$ $n^3$ | ARCH |
|---|---|---|---|---|
| $\varepsilon_t$ | 545.127<br>(<0.001) | 609.454<br>(<0.001) | 662.451<br>(<0.001) | 123.525<br>(<0.001) |
| $\omega_t^+$ | 6.272<br>(0.013) | 6.471<br>(0.039) | 14.234<br>(0.003) | 17.981<br>(<0.001) |
| $\omega_t^-$ | 2.542<br>(0.117) | 5.017<br>(0.081) | 25.759<br>(<0.001) | 41.626<br>(<0.001) |
| $v_t$ | 4705.604<br>(<0.001) | 5152.894<br>(<0.001) | 5188.820<br>(<0.001) | 2097.581<br>(<0.001) |

Notes: See Table 5 for the definitions of $\Delta p^+, \Delta p^-$, and $x$. The upper panel reports Wald tests for the null hypothesis of zero coefficients on all the terms listed at the head of each column in models of the form

$$z_t = D(L)x_t + D(L)x_t n_t + D(L)x_t n_t^2 + D(L)x_t n_t^3 + w_t.$$

In each case, the model including $D(L)x_t$ and the listed terms was estimated by GMM allowing for heteroskedasticity and an MA(1) error structure. These estimates are then used to construct the Wald test. P-values are reported below each statistic. In cases where the change in price is the dependent variable, D(L) takes the form of the 6'th-order polynomial in the Bivariate model. For the case of order flow, D(L) = $d_1 L + d_2 L^2 + \ldots.. d_6 L^6$. The lower panel reports the results of tests for heteroskedasticity. The center three columns report Glesjer (1969) tests for heteroskedasticity in the variance of each shock using the variables listed at the head of each column. $v_t$ is the innovation to the ARMA(2,2) model for order flow estimated in Table 3 above, while $\varepsilon_t$, $\omega_t^+$, and $\omega_t^-$ are the shocks from the Bivariate model. The right hand column reports LM statistics for first-order ARCH. In all cases, p-values for the null hypothesis of homoskedasticity are shown in parenthesis.

## Table 7: Bivariate Model with State-Dependency

**I: Estimates**

| | $x_{t+4}$ | $x_{t+3}$ | $x_{t+2}$ | $x_{t+1}$ | $x_t$ | $x_{t-1}$ |
|---|---|---|---|---|---|---|
| Coefficients in $D(L,0)$ | 0.0195 | 0.0154 | 0.0075 | -0.0579 | -0.0963 | -0.0428 |
| (Std. Errs.) (x100) | (0.0169) | (0.0160) | (0.0167) | (0.0182) | (0.0171) | (0.0165) |
| Coefficients in $D(L,\infty)$ | 0.1575 | 0.2555 | 0.5384 | 0.6424 | -0.2697 | -0.0072 |
| (Std. Errs.) (x100) | (0.0659) | (0.0567) | (0.0563) | (0.0594) | (0.0426) | (0.0517) |
| | $\Sigma_\varepsilon(0)^{1/2}$ | $\Sigma_\varepsilon(\infty)^{1/2}$ | $\Sigma_\omega(0)^{1/2}$ | $\Sigma_\omega(\infty)^{1/2}$ | $D(1,0)$ | $D(1,\infty)$ |
| | 0.000 | 0.0900 | 0.0479 | 0.0000 | -0.1546 | 1.3169 |
| | (N/A) | (0.0005) | (0.0001) | (N/A) | (0.0300) | (0.1020) |

**II: Diagnostics**

| | Statistic | p-value |
|---|---|---|
| J-Test for Over identifying Restrictions | 9.315 | (0.968) |
| Wald Test for $D(L,0)=D(L,\infty)$ | 255.247 | (<0.001) |
| LM Test for misspecification in *D(L,n)* | 0.726 | (0.999) |
| LM Test for misspecification in $\Sigma_\varepsilon(n)$ | 0.128 | (0.721) |
| LM Test for misspecification in $\Sigma_\omega(n)$ | 0.094 | (0.760) |

| Residual | Residual Autocorrelations (Std. Errs) lag = 1 | 2 | 3 | 4 | 5 | 6 | 12 |
|---|---|---|---|---|---|---|---|
| $\varepsilon_t^2 / \Sigma_\varepsilon(n_t)$ | 0.1051 | 0.0087 | 0.0243 | 0.031 | 0.024 | 0.0312 | 0.0034 |
| | (0.0222) | (0.0120) | (0.0098) | (0.0148) | (0.0158) | (0.0191) | (0.0081) |
| $\left(\omega_t^+\right)^2 / \Sigma_\omega(n_t)$ | 0.0605 | 0.0152 | 0.0174 | -0.0085 | 0.0008 | 0.02 | -0.0023 |
| | (0.0233) | (0.0138) | (0.0171) | (0.0111) | (0.0095) | (0.0121) | (0.0094) |
| $\left(\omega_t^-\right)^2 / \Sigma_\omega(n_t)$ | 0.0543 | 0.0219 | 0.0103 | 0.0023 | 0.0035 | 0.0037 | 0.017 |
| | (0.0230) | (0.0097) | (0.0109) | (0.0103) | (0.0086) | (0.0096) | (0.0112) |

Notes: The model takes the form:

$$\begin{bmatrix}\Delta p_t^+ \\ \Delta p_t^-\end{bmatrix} = \begin{bmatrix}1 \\ 1\end{bmatrix} D(L,n_t)x_t + \begin{bmatrix}1 \\ 1\end{bmatrix}\varepsilon_t + \begin{bmatrix}\omega_t^+ - \omega_t^+ \\ \omega_t^- - \omega_{t-1}^-\end{bmatrix},$$

where $\omega_t^+$, $\omega_t^-$ and $\varepsilon_t$ are mutually independent and serially uncorrelated shocks with $E\omega_t^+ = \omega^+, E\omega_t^- = \omega^-, E\varepsilon_t = 0$. The variances are given by $Var(\varepsilon_t) = \Sigma_\varepsilon(n_t)$ $Var(\omega_t^+)$ $=Var(\omega_t^-)$ $=\Sigma_\omega(n_t)$ with $\Sigma_i(n) = \Sigma_i(0)\exp(-n/100) + \Sigma_i(\infty)(1-\exp(-n/100))$. The state-dependent polynomial is $D(L,n_t) = d_1(n_t)L^{-4} + d_2(n_t)L^{-3} + .... + d_6 L$. Variables are defined in Table 5. Panel I shows GMM estimates and asymptotic standard errors corrected for heteroskedasticity and serial correlation (see Appendix B). Panel II reports the Hansen J-statistic, a Wald statistic for the null that $D(L,0) = D(L,\infty)$, and LM-type statistics for misspecification state-dependent polynomial, and variances (see Appendix B). The lower portion of the table reports autocorrelations, and standard errors, in the estimated standardized shocks.

## Table 8: Variance Ratios

| $R_\omega(k,n) = Var(\omega_t^o - \omega_{t-k}^o) / Var(\Delta^k p_t^o)$ | | | | | |
|---|---|---|---|---|---|
| $n \setminus k$ | 5 | 30 | 60 | 120 | $\infty$ |
| 2 | 96.39% | 81.78% | 69.45% | 53.36% | 0.00% |
| 5 | 91.37% | 64.22% | 47.64% | 31.42% | 0.00% |
| 10 | 83.70% | 45.89% | 29.94% | 17.66% | 0.00% |
| 20 | 69.61% | 24.65% | 13.79% | 7.33% | 0.00% |
| 30 | 56.60% | 13.48% | 6.88% | 3.48% | 0.00% |
| 40 | 45.04% | 7.73% | 3.75% | 1.85% | 0.00% |
| 60 | 27.65% | 3.07% | 1.42% | 0.68% | 0.00% |
| 80 | 17.13% | 1.49% | 0.68% | 0.32% | 0.00% |
| all | 67.11% | 20.13% | 10.72% | 5.54% | 0.00% |
| $R_v(k,n) = Var(B(L,k,n)v_t) / Var(\Delta^k p_t^o)$ | | | | | |
| $n \setminus k$ | 5 | 30 | 60 | 120 | $\infty$ |
| 2 | 0.18% | 0.77% | 0.91% | 1.08% | 1.67% |
| 5 | 0.38% | 1.00% | 0.74% | 0.49% | 0.00% |
| 10 | 0.80% | 3.11% | 3.54% | 3.87% | 4.34% |
| 20 | 3.25% | 17.68% | 21.71% | 24.10% | 26.82% |
| 30 | 8.53% | 36.71% | 42.22% | 45.07% | 47.98% |
| 40 | 15.95% | 52.09% | 57.28% | 59.75% | 62.16% |
| 60 | 32.32% | 70.31% | 73.97% | 75.60% | 77.11% |
| 80 | 45.91% | 79.19% | 81.80% | 82.93% | 83.97% |
| all | 5.53% | 30.62% | 36.81% | 40.22% | 43.87% |
| $R_{v^*}(k,n) = Var(B^*(L,k,n)v_t) / Var(\Delta^k p_t^o)$ | | | | | |
| $n \setminus k$ | 5 | 30 | 60 | 120 | $\infty$ |
| 2 | 0.17% | 0.53% | 0.38% | 0.29% | 0.00% |
| 5 | 0.38% | 1.00% | 0.74% | 0.49% | 0.00% |
| 10 | 0.74% | 2.54% | 2.06% | 1.22% | 0.00% |
| 20 | 2.22% | 8.72% | 6.49% | 3.59% | 0.00% |
| 30 | 4.52% | 14.45% | 10.15% | 5.52% | 0.00% |
| 40 | 6.68% | 18.04% | 12.37% | 6.70% | 0.00% |
| 60 | 9.43% | 21.46% | 14.44% | 7.83% | 0.00% |
| 80 | 10.73% | 22.86% | 15.28% | 8.29% | 0.00% |
| all | 3.13% | 12.16% | 8.64% | 4.75% | 0.00% |

Notes: Variance decompositions derived from estimates of the State-Dependent Bivariate model in Table 7 and the ARMA(2,2) model for order flow with state-dependent heteroskedasticity. The column headings show the horizon $k$ measured in minutes. $R_\omega(k,n)$ and $R_v(k,n)$ respectively measure the contribution of sampling and order flow shocks to the variance of observed price changes. $R_{v^*}(k,n)$ measures the contribution of order flow shocks that only temporarily affect the price level.

# Appendix C: Variance Ratios

| $R_{\omega}(k,n)=Var(\omega_t^o-\omega_{t-k}^o)/Var(\Delta^k p_t^o)$ | | | | | |
|---|---|---|---|---|---|
| $n \setminus k$ | 5 | 30 | 60 | 120 | $\infty$ |
| 2 | 95.21% | 77.64% | 65.35% | 49.64% | 0.00% |
| 5 | 90.62% | 62.82% | 46.87% | 31.09% | 0.00% |
| 10 | 83.26% | 44.98% | 29.26% | 17.22% | 0.00% |
| 20 | 69.91% | 25.25% | 14.20% | 7.57% | 0.00% |
| 30 | 58.57% | 15.76% | 8.26% | 4.23% | 0.00% |
| 40 | 49.15% | 10.63% | 5.36% | 2.69% | 0.00% |
| 60 | 35.04% | 5.66% | 2.74% | 1.35% | 0.00% |
| 80 | 25.50% | 3.44% | 1.63% | 0.79% | 0.00% |
| all | 67.11% | 20.13% | 10.72% | 5.54% | 0.00% |

| $R_{v}(k,n)=Var(B(L,k,n)v_t)/Var(\Delta^k p_t^o)$ | | | | | |
|---|---|---|---|---|---|
| $n \setminus k$ | 5 | 30 | 60 | 120 | $\infty$ |
| 2 | 1.41% | 5.79% | 6.75% | 7.99% | 11.88% |
| 5 | 1.20% | 3.14% | 2.35% | 1.56% | 0.00% |
| 10 | 1.32% | 5.03% | 5.70% | 6.22% | 6.96% |
| 20 | 2.83% | 15.68% | 19.36% | 21.57% | 24.09% |
| 30 | 5.34% | 25.99% | 30.68% | 33.20% | 35.84% |
| 40 | 8.29% | 34.11% | 38.96% | 41.41% | 43.89% |
| 60 | 14.25% | 45.17% | 49.71% | 51.87% | 53.96% |
| 80 | 19.48% | 52.03% | 56.16% | 58.07% | 59.89% |
| all | 5.53% | 30.62% | 36.81% | 40.22% | 43.87% |

| $R_{v^*}(k,n)=Var(B^*(L,k,n)v_t)/Var(\Delta^k p_t^o)$ | | | | | |
|---|---|---|---|---|---|
| $n \setminus k$ | 5 | 30 | 60 | 120 | $\infty$ |
| 2 | 1.36% | 3.98% | 2.81% | 2.13% | 0.00% |
| 5 | 1.20% | 3.15% | 2.36% | 1.56% | 0.00% |
| 10 | 1.21% | 4.06% | 3.29% | 1.96% | 0.00% |
| 20 | 1.95% | 7.84% | 5.84% | 3.23% | 0.00% |
| 30 | 3.09% | 10.92% | 7.68% | 4.15% | 0.00% |
| 40 | 4.22% | 13.08% | 8.92% | 4.79% | 0.00% |
| 60 | 6.02% | 15.71% | 10.44% | 5.57% | 0.00% |
| 80 | 7.21% | 17.19% | 11.30% | 6.03% | 0.00% |
| all | 3.13% | 12.16% | 8.64% | 4.75% | 0.00% |

Notes: Variance decompositions derived from estimates of the State-Dependent Bivariate model in Table 7 and the ARMA(2,2) model for order flow in Table 3. (Unlike Table 8, this version assumes homoskedastic variance for order low shocks). The column headings show the horizon $k$ measured in minutes. $R_{\omega}(k,n)$ and $R_{v}(k,n)$ respectively measure the contribution of sampling and order flow shocks to the variance of observed price changes. $R_{v^*}(k,n)$ measures the contribution of order flow shocks that only temporarily affect the price level.