NBER WORKING PAPER SERIES

THE EFFECTS OF INVESTING SOCIAL SECURITY FUNDS IN THE STOCK MARKET WHEN FIXED COSTS PREVENT SOME HOUSEHOLDS FROM HOLDING STOCKS

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Working Paper 7739
http://www.nber.org/papers/w7739

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 2000

This paper previously circulated under the title "The Aggregate Effects of Including Equities in the Social Security Trust Fund." I thank Bill Dupor, Janice Eberly, Doug Elmendorf, Martin Feldstein, Robert Hall, Adriano Rampini, Matthew Shapiro, Nick Souleles, Stephen Zeldes, an anonymous referee and the participants in the 1998 NBER Summer Institute Conference on Risk and Distribution Issues in Social Security Reform, the EFG Program Meeting of the NBER, the International Workshop on Reforming Social Security, and seminars at the University of Chicago, Northwestern University, Princeton University, and the University of Virginia for helpful comments. The views expressed herein are those of the author and not necessarily those of the National Bureau of Economic Research.

The Effects of Investing Social Security Funds in the Stock Market
When Fixed Costs Prevent Some Households from Holding Stocks
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NBER Working Paper No. 7739
June 2000
JEL No. H55

ABSTRACT

With fixed costs of participating in the stock market, consumers with high income will participate in the stock market, but consumers with lower income will not participate. If a fully-funded defined-contribution social security system tries to exploit the equity premium by selling a dollar of bonds per capita and buying a dollar of equity per capita, consumers who save but do not participate in the stock market will increase their consumption, thereby reducing saving and capital accumulation. Calibration of a general equilibrium model indicates that this policy could reduce the aggregate capital stock substantially, by about 50 cents per capita.

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The Social Security system in the United States currently runs a surplus of $100 billion per year, which is used to buy bonds that are held in the Social Security Trust Fund. The value of this trust fund is projected to grow to more than $2.3 trillion 1999 dollars in the year 2015. However, as the baby boom generation retires and collects social security benefits, the trust fund will be drawn down until it is exhausted in 2034.[1]

Various sorts of policies could delay, perhaps indefinitely, the date at which the trust fund is exhausted. Social security taxes could be increased by increasing payroll tax rates or increasing the retirement age, and future benefits could be reduced explicitly or more subtly by changing the calculation of the CPI, taxing future benefits more heavily, increasing the retirement age, or adopting means testing. However, policies to increase social security taxes are opposed by workers, and policies to reduce social security benefits are opposed by social security beneficiaries, both current and future. A third type of policy that neither directly increases taxes nor reduces benefits is to try to earn a higher rate of return on the Social Security Trust Fund by taking advantage of the equity premium on stocks relative to bonds. Historically, the rate of return on stocks has exceeded the rate of return on short-term riskless bills by an average of about 6% per year. The prospect of earning a higher rate of return on even a fraction of the huge Social Security Trust Fund has generated a variety of proposals to invest some of the trust fund in equity.

Some economists have argued that investing part of the Social Security Trust Fund in equity is simply a rearrangement of paper assets without any real allocational

[1] These figures, which are from the intermediate projection in Table III.B2 of the 1999 Annual Report of the Board of Trustees of the Federal Old-Age and Survivors Insurance and Disability Insurance Trust Funds, refer to the combined OASI and DI Trust Funds.

effects, and they have described such a policy as a "shell game." The shell game argument is similar to the Ricardian equivalence proposition in public finance and macroeconomics and the Modigliani-Miller theorem in corporate finance. The argument is that private investors will react to any rearrangement of the social security system's portfolio in a way that completely neutralizes the effect of the portfolio change. For example, if the social security system sells a dollar of bonds and purchases a dollar of equity, private investors would buy a dollar of bonds and sell a dollar of equity. There would be no effect on asset prices or on the integrated portfolio of the private sector and the social security system. Like the Ricardian Equivalence proposition and the Modigliani-Miller theorem, the shell game argument is based on a set of assumptions about optimizing behavior and perfect markets. The shell game argument assumes that all consumers participate in the markets for bonds and equity, and their holdings of these assets are at interior optima. However, the argument breaks down, and changing the portfolio of the social security system is not a shell game if, as is indeed true, some consumers save, but do not participate in the stock market.

The 6% average annual equity premium and widespread non-participation in the stock market are two aspects of the equity premium puzzle that was first exposited by Mehra and Prescott (1985). Mehra and Prescott showed that a prototypical asset-pricing model could not explain an equity premium larger than 35 basis points per year using conventional values for preference parameters. They coined the term "equity premium puzzle" to describe the fact that the historical average equity premium greatly exceeds the equity premium that can be explained by conventional models. Though much of the subsequent study of the equity premium puzzle has focused on this high equity premium, another aspect of the puzzle, highlighted by Mankiw and Zeldes (1991), is that a majority of consumers do not own any stocks, despite the large average reward to owning stocks.

Although the 6% average equity premium is the aspect of the equity premium puzzle that has attracted the attention of policy makers trying to preserve the solvency of the social security system, the widespread non-participation in the stock market is the important aspect for addressing the shell game issue. In this paper, I develop and analyze a general equilibrium overlapping-generations model with fixed costs of participating in the stock market and with a constraint that prevents young consumers from borrowing to finance consumption. In addition, I introduce intra-cohort heterogeneity in earnings. The fixed costs, together with the heterogeneity, generate an equilibrium in which high-income consumers find it beneficial to pay the fixed cost and to participate in the stock market, while middle-income and low-income consumers do not participate in the stock market. Young middle-income consumers save some of their current income and hold riskless bonds. Young low-income consumers are prevented from borrowing by a binding borrowing constraint and consume all of their available resources.

Because middle-income and low-income consumers do not directly own equity, they cannot offset the change in the portfolio of the social security system when it sells bonds and buys equity. Thus, the standard argument underlying the shell game result does not hold in this model. For the cases analyzed in Sections V and VI of this paper, this change in the portfolio of the social security system reduces the nation's capital stock, despite the fact that the social security system increases (from an initial value of zero) its holding of capital. The reduction in the aggregate capital stock is the result of an income effect on middle-income consumers. These consumers benefit from an increase in the expected present value of lifetime resources when the social security system holds some equity instead of lower-yielding bonds. In response to this increase in lifetime resources, these consumers increase their consumption, thereby reducing national saving and national capital accumulation. Low-income consumers, however, do not have the resources to increase their consumption when they are young, and thus their consumption when young is unchanged by the change in the portfolio of the social security system.

It is interesting to compare these results to those by Diamond and Geanakoplos (1999), hereafter D-G, who present a model in which the aggregate capital stock *increases* when the social security trust fund sells bonds in exchange for risky capital. The D-G model has two classes of consumers, corresponding to the low-income and high-income consumers in the model I present here. By ignoring consumers who save but do not participate in the stock market (middle-income consumers in the current model), D-G eliminate the income effect that drives the results in the current model. Instead, they focus on the interest rate effects on aggregate saving. As in the current paper, D-G find that the change in the portfolio of the social security system increases the equilibrium riskless interest rate. In the current paper, with logarithmic preferences, the interest rate has no effect on aggregate saving, but D-G use a class of preferences for which an increase in the interest rate increases saving. Since the D-G model does not have middle-income consumers, through whom the income effect operates, the interest rate effect on aggregate saving determines the effect on the aggregate capital stock.[2]

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[2] Though the D-G model incorporates an interest rate effect on saving, the model in the current paper has several advantages relative to the D-G model. The model in the current paper has three classes of consumers, which include the two classes in D-G, and the empirically important class of consumers who save but do not own stock. Membership in these classes is determined endogenously in the current model, but is exogenous in D-G. All consumers participate in the social security system in the current model, but

The presentation of the model begins in Section I with a description of the production function and resulting factor prices. The production function is consistent with endogenous growth and has the implication that the rate of return to risky capital, which is traded in the stock market, is random and independent of the amount of risky capital. The decisions of individual consumers are examined in Section II. Consumers choose how much to consume when young and they decide how to allocate their portfolios to stocks and bonds. The portfolio decision includes a decision about whether to incur the fixed cost necessary to participate in the stock market. After deriving the demands for bonds and stocks by individuals in Section II, I discuss equilibrium in these markets in Section III.

I begin analyzing the effect of having the social security system sell bonds and buy equity in Section IV. In order to focus sharply on the question of the effect of the social security system's portfolio, I analyze a fully-funded defined-contribution social security system that initially holds only bonds. I present an expression in Section IV for the effect on the aggregate capital stock of a small change in the social security system's portfolio. This expression is the sum of two effects. The first effect reduces the aggregate capital stock because of the income effect on middle-income consumers described above. The second effect reflects the behavior of "switchers" who would hold stock in the initial situation, but who would not participate in the stock market if the social security system holds some stock in its portfolio. With the formulation of fixed

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only the class of non-saving workers participate in social security in D-G. All consumers choose consumption based on explicit optimization in the current model, but in D-G, workers are assumed to be non-savers. Specifically, D-G assumes that workers set consumption when young equal to after-tax wage income when young. The model in the current paper is easily calibrated, but the D-G model needs additional assumptions to make it amenable to calibration.

costs in this model, switchers respond to the avoidance of fixed costs by increasing their saving. Beginning in Section V, I ignore switchers and show that the change in the portfolio of the social security system reduces the aggregate capital stock, increases the riskless interest rate, and increases the maximized expected utility of all young consumers. However, the increase in the utility of young consumers, who are the current workers, is achieved at least partly at the expense of future workers and taxpayers. In Section VI, I present numerical estimates of the reduction in the aggregate capital stock, and show that this reduction can be substantial. I present concluding remarks in Section VII.

I. Production and Factor Prices

The economy is populated by overlapping generations of consumers who live for two periods. Consumers are endowed with $n > 0$ effective units of labor when they are young, and they supply this labor inelastically. Old consumers do not work. There is intra-generational heterogeneity in the amount of effective units of labor per young consumer. Let $G(x)$ be the measure of workers with $n \leq x$. Suppose that $G(x)$ is differentiable for $x > 0$, and adopt the normalizations $G(\infty) = 1$ and

$$N \equiv \int_0^\infty n dG(n) = 1 \tag{1}$$

where N is the aggregate effective labor input.

The production function for an individual firm is taken from the endogenous growth literature[3]

$$Y_i = AK_i^{\alpha}\left(KN_i\right)^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{2}$$

where Y_i is the output of firm i, K_i is the capital stock of firm i, N_i is labor used by firm i, K is the aggregate capital stock, and $A > 0$ is a random variable. The private marginal product of capital accruing to firm i is

$$\frac{\partial Y_i}{\partial K_i} = \alpha A\left(\frac{K_i}{N_i}\right)^{\alpha-1} K^{1-\alpha} \tag{3}$$

and the marginal product of labor is

$$\frac{\partial Y_i}{\partial N_i} = \left(1-\alpha\right)A\left(\frac{K_i}{N_i}\right)^{\alpha} K^{1-\alpha}\,. \tag{4}$$

In equilibrium, the capital-labor ratio K_i/N_i is identical for all firms and is equal to the aggregate capital-labor ratio, K/N. Since $N = 1$,

$$\frac{K_i}{N_i} = K\,. \tag{5}$$

The gross rate of return on capital, R, is the private marginal product of capital, so equations (3) and (5) imply

$$R = \alpha A. \tag{6}$$

[3] See, for example, Barro and Sala-i-Martin (1995, p. 150).

Note that the rate of return R is random, positive, and independent of the capital stock.

The wage rate, w, equals the marginal product of labor, so equations (4) and (5) imply

$$w = (1-\alpha)AK. \tag{7}$$

II. The Decisions of Individual Consumers

There is a continuum of young consumers indexed by n, the endowment of effective units of labor. I will show in this section that the optimal consumption and portfolio decisions of a young consumer assign that consumer to one of three groups. Young consumers who save some of their disposable income and participate in the stock market are assigned to group H. Young consumers who save some of their disposable income but do not participate in the stock market are assigned to group M, and young consumers who consume all of their disposable income and do not participate in the stock market are assigned to group L. I will show that group H consists of high-income consumers, group M consists of middle-income consumers, and group L consists of low-income consumers.

A young consumer endowed with n effective units of labor earns labor income of wn where w is the wage rate in equation (7). Each young consumer pays a tax T. Old consumers do not earn labor income, but receive benefits from a fully-funded defined-contribution social security system. Specifically, the social security system invests $\theta_b \geq 0$ in riskless bonds and $\theta_k \geq 0$ in risky capital on behalf of each young consumer. When these consumers are old, they receive social security benefits of $\theta_b r + \theta_k R$ where $r \geq 0$ is the (gross) rate of return on riskless bonds and $R > 0$ is the (gross) rate of return on risky capital in equation (6).

Each consumer must decide how much to save when young and how to allocate this saving between riskless bonds and risky capital, subject to a borrowing constraint that prevents young consumers from borrowing to finance consumption. There are no transactions costs associated with investing in riskless bonds, but there is a fixed cost of participating in the stock market, where the claims to risky capital are traded. I will model this fixed cost to include both a resource cost and a utility cost, but either type of cost alone is sufficient for the results in this paper. The resource cost is a fixed amount $F_1 \geq 0$ that must be paid when young in order to participate in the stock market. The utility cost will be described below when the utility function is introduced.

Having described all of the opportunities facing a consumer, I will present the budget constraint more formally. After consuming an amount c, a young consumer has private saving of $s \equiv wn - T - \delta F_1 - c$, where δ is a dummy variable that equals one if the consumer participates in the stock market, and equals 0 otherwise. Part of this saving is used to purchase risky capital, k, in the stock market and the remainder, $s - k$, is used to purchase riskless bonds. Let $b \equiv s - k + \theta_b$ denote the amount of riskless bonds held, both directly and indirectly through the social security system, by a young consumer, and let $h \equiv k + \theta_k$ be the amount of risky capital held, both directly and indirectly. Define $a \equiv b + h$ as the total value of the assets held directly and indirectly. Using the definition of s yields

$$a \equiv b + h = \Omega - \delta F_1 - c, \qquad (8a)$$

where $$\Omega \equiv wn - T + \theta_b + \theta_k \tag{8b}$$

is the value of wealth owned, both directly and indirectly, by the consumer when young.[4]

An old consumer consumes all available resources, which consist of *br*, the principal and interest on both direct and indirect holdings of riskless bonds, and *hR*, the gross value of risky capital held both directly and indirectly. Letting *x* denote consumption when old yields

$$x = br + hR. \tag{9}$$

The utility function is[5]

$$U = \ln(c - \delta F_2) + \beta E\{\ln x\}, \qquad \beta > 0 \tag{10}$$

where $F_2 \geq 0$ is a utility cost of participating in the stock market. The consumer can avoid this cost by not participating in the stock market, thereby setting $\delta = 0$. As mentioned above, either F_1 or F_2 can be zero, but $F_1 + F_2 > 0$.

[4] The borrowing constraint prevents consumption when young from exceeding disposable income, $wn - T$. The present value of the claim on future social security benefits, $\theta_b + \theta_k$, is part of total wealth, Ω, but is not part of disposable income, $wn - T$.

[5] The quantitative results in Table 2 below would be unchanged if the utility function in equation (10) were replaced by the more general utility function $U = \ln(c - \delta F_2) + \frac{\beta}{1-\phi} \ln E\{x^{1-\phi}\}$, which, like the utility function in equation (10) has an intertemporal elasticity of substitution equal to one. However, the more general utility function presented here has a constant coefficient of relative risk aversion equal to ϕ, whereas the utility function in equation (10) has a coefficient of relative risk aversion equal to one. With the more general utility function, the marginal propensity to consume out of wealth (when the borrowing constraint is not binding) equals $1/(1+\beta)$ when $\theta_k = 0$, just as for the utility function in equation (10).

II.A. Savers Who Participate in the Stock Market

A consumer who participates in the stock market chooses current consumption c, bonds b, and risky capital h, to maximize the utility function in equation (10) subject to the constraints in equations (8) and (9).[6] Equivalently, the consumer chooses a, the total amount of assets held directly and indirectly, and $\gamma \equiv h/a$, the share of the total assets held in risky capital. Recognizing that $b = (1-\gamma)a$ and $h = \gamma a$, substitute equations (8) and (9) into the utility function in equation (10), with $\delta = 1$, to obtain

$$\max_{a,\gamma} \quad \ln(\Omega - F_1 - F_2 - a) + \beta \ln a + \beta\psi(\gamma, r) \tag{11a}$$

where

$$\psi(\gamma, r) \equiv E\{\ln[(1-\gamma)r + \gamma R]\}. \tag{11b}$$

The optimal value of a is determined by differentiating the maximand in equation (11a) with respect to a, setting the derivative equal to zero, and solving to obtain

$$a = \frac{\beta}{1+\beta}(\Omega - F_1 - F_2). \tag{12}$$

Equation (12) shows that the optimal size of the portfolio, $a \equiv b+h$, consisting of both direct and indirect holdings, is independent of the rates of return on the riskless and risky

[6] I have assumed that the social security parameters θ_b and θ_k and the fixed costs F_1 and F_2 are such that the borrowing constraint is not binding for consumers who choose to participate in the stock market. See footnote 16.

assets, and is proportional to wealth, Ω, adjusted for the fixed costs of participating in the stock market. The fixed cost F_1 is a resource cost and is directly subtracted from first-period resources, thereby reducing the optimal size of the portfolio. The fixed cost F_2 is a utility cost and hence does not directly reduce resources. However, an increase in F_2 reduces the optimal size of the portfolio because an increase in F_2 increases the marginal utility of consumption when young, and therefore shifts consumption away from old age toward youth.[7]

Now consider the optimal allocation of the portfolio. The first-order condition for γ is obtained by differentiating the maximand in equation (11a) with respect to γ, setting the derivative equal to zero, and solving to obtain

$$Z(\gamma, r) \equiv E\left\{\frac{R-r}{(1-\gamma)r+\gamma R}\right\} = 0. \tag{13}$$

Equation (13) implicitly defines the optimal equity share $\gamma(r)$ as a function of the riskless interest rate r, given the distribution of the risky rate of return on capital R. It is straightforward to show that[8] $\gamma'(r) < 0$ and that[9] $\gamma(r)$ has the same sign as the (*ex ante*)

[7] However, if fixed costs involve a reduction in utility when old as well as a reduction in utility when young, they could increase the optimal size of the portfolio. For example, if the utility function is $U = \ln(c-\delta F_2) + \beta E\{\ln(x-\delta F_3)\}$, the decision problem of a participant in the stock market can be written as $\max_{b,h} \ln(\Omega - b - h - F_1 - F_2) + \beta E\{\ln(br + hR - F_3)\}$. The first-order conditions with respect to b and h are $\frac{1}{\Omega-b-h-F_1-F_2} = \beta E\left\{\frac{r}{br+hR-F_3}\right\}$ and $\frac{1}{\Omega-b-h-F_1-F_2} = \beta E\left\{\frac{R}{br+hR-F_3}\right\}$, which can be combined to yield $\frac{b+h-F_3/r}{\Omega-b-h-F_1-F_2} = \beta$, which implies $a \equiv b+h = \frac{\beta}{1+\beta}(\Omega - F_1 - F_2 + F_3/(\beta r))$. If $F_1 + F_2 - F_3/(\beta r) > 0$, fixed costs reduce optimal total assets. However, if $F_1 + F_2 - F_3/(\beta r) < 0$, fixed costs increase optimal total assets. The optimal total amount of assets is unaffected by the fixed costs in the special case with (1) no resource costs of participation ($F_1 = 0$); (2) $F_2 = F_3$; and (3) $\beta r = 1$.

[8] Applying the implicit function theorem to equation (13) along with

equity premium $E\{R\text{-}r\}$. In the general equilibrium in this model, $\gamma(r)$ is positive so the equity premium is positive. Henceforth, I will confine attention to cases with a positive equity premium.

Let $b_H(\Omega, r, \theta_k)$ and $h_H(\Omega, r, \theta_k)$ be the optimal values of b and h for a consumer who participates in the stock market. (The subscript "H" indicates that the consumer is a high-income consumer.) Equation (12) and the definition of $\gamma(r)$ imply

$$b_H(\Omega, r, \theta_k) = [1-\gamma(r)]\frac{\beta}{1+\beta}(\Omega - F_1 - F_2) \tag{14}$$

and

$$h_H(\Omega, r, \theta_k) = \gamma(r)\frac{\beta}{1+\beta}(\Omega - F_1 - F_2). \tag{15}$$

Let $V_H(\Omega, r, \theta_k)$ denote the maximized value of the utility function for a consumer who participates in the stock market. Substituting the optimal values of b and h from equations (14) and (15) into equation (11a) and simplifying yields

$$V_H(\Omega, r, \theta_k) = \beta \ln\beta - (1+\beta)\ln(1+\beta) + (1+\beta)\ln(\Omega - F_1 - F_2) + \beta\psi(\gamma, r). \tag{16}$$

Inspection of equation (16) yields

$$\frac{\partial Z(\gamma, r)}{\partial \gamma} = -E\left\{\frac{(R-r)^2}{[(1-\gamma)r + \gamma R]^2}\right\} < 0 \tag{*}$$

and $$\left.\frac{\partial Z(\gamma, r)}{\partial r}\right|_{Z(\gamma, r)=0} = -E\left\{\frac{R}{[(1-\gamma)r + \gamma R]^2}\right\} < 0 \tag{**}$$

proves that $\gamma'(r) < 0$.

[9] Since $Z(\gamma, r)$ is strictly decreasing in γ (see equation (*) in footnote 8) and $Z(0,r) = E\{R\text{-}r\}/r$ has the same sign as the equity premium, the optimal value of γ, which satisfies equation (13), has the same sign as the equity premium.

$$\frac{\partial V_H(\Omega, r, \theta_k)}{\partial \Omega} = \frac{1+\beta}{\Omega - F_1 - F_2} > 0 \tag{17a}$$

$$\frac{\partial V_H(\Omega, r, \theta_k)}{\partial r} = \beta \frac{\partial \psi(\gamma, r)}{\partial r} = \beta(1-\gamma) E\left\{\frac{1}{(1-\gamma)r + \gamma R}\right\} \tag{17b}$$

and

$$\frac{\partial V_H(\Omega, r, \theta_k)}{\partial \theta_k} = 0. \tag{17c}$$

Equation (17a) shows that an increase in lifetime resources Ω increases the maximized value of utility. Whether an increase in the riskless rate r increases or decreases maximized utility depends on whether the consumer would choose a long or a short position in bonds. As shown in equation (17b), a consumer who would choose a long position in bonds ($1-\gamma > 0$), and thus would be a recipient of interest, benefits from an increase in r; a consumer who would choose a short position in bonds ($1-\gamma < 0$),[10] and thus would be a payer of interest, suffers a loss in utility when the interest rate on bonds increases. Equation (17c) shows that a change in θ_k that leaves Ω unchanged has no effect on the maximized value of utility. For instance, an increase in θ_k accompanied by an equal decrease in θ_b leaves Ω unchanged and, according to equation (17c), has no effect on the maximized value of utility.

[10] The borrowing constraint does not allow the consumer to borrow to finance consumption, but does permit borrowing to finance the holding of capital, so $\gamma(r)$ could exceed one.

II.B. Savers Who Do Not Participate in the Stock Market

For a consumer who does not participate in the stock market, the direct holding of risky capital, k, is zero, and thus the total (indirect) holding of risky capital is $h = \theta_k$. By not participating in the stock market, this consumer avoids the resource cost F_1 and the utility cost F_2. In this subsection, I focus on consumers who do not participate in the stock market, but who choose to consume less than their disposable income, $wn - T$. Such consumers choose b to maximize the utility function in equation (10) with $\delta = 0$ and subject to the constraints in equations (8) and (9) with h set equal to the parameter θ_k. The decision problem can be written as

$$\max_b \ \ln(\Omega - \theta_k - b) + \beta E\{\ln(br + \theta_k R)\}. \tag{18}$$

Let $b_M(\Omega, r, \theta_k)$ denote the optimal value of b for a saver who does not participate in the stock market. (The subscript "M" indicates a middle-income consumer.) In Appendix A, I show that

$$b_M(\Omega, r, 0) = \frac{\beta}{1+\beta}\Omega \tag{19}$$

and that

$$\left.\frac{\partial b_M(\Omega, r, \theta_k)}{\partial \theta_k}\right|_{\theta_k = 0} = -\left(1 + \frac{1}{1+\beta}\frac{E\{R\} - r}{r}\right) = -\frac{E\left\{\frac{R}{r}\right\} + \beta}{1+\beta} < -1 \tag{20}$$

where the inequality in equation (20) follows from the fact that the equity premium, $E\{R\}-r$, is positive.

The optimal holding of bonds by a saver who does not participate in the stock market is particularly simple in the case in which the social security system does not hold any equity. Formally, this case is represented by $\theta_k = 0$, and equation (19) shows that in this case, the optimal holding of bonds is proportional to wealth, Ω, and is independent of the riskless interest rate, r. Thus, a one-dollar increase in wealth, Ω, increases optimal bond holding by $\beta/(1+\beta)$.

To interpret the partial derivative in equation (20) it is important to observe that wealth, Ω, is held fixed when θ_k is changed. Suppose, for the moment that the expected rate of return on risky capital, $E\{R\}$, equals the riskless rate r. In this case, equation (20) indicates that a one-dollar increase in θ_k, accompanied by a one-dollar decrease in θ_b to keep Ω unchanged, reduces the optimal bond holding by one dollar so that the total size of the portfolio is unchanged. Now suppose that the expected rate of return on risky capital exceeds the riskless rate. In this case, a one-dollar increase in θ_k accompanied by a one-dollar decrease in θ_b has a positive income effect that stimulates consumption and reduces the size of the consumer's portfolio. More precisely, this change in θ_k and θ_b effectively increases the consumer's expected second-period resources by the equity premium, $E\{R\}- r$, which has a present value of $[E\{R\} - r]/r$. Since the marginal propensity to consume when young is $1/(1+\beta)$, consumption increases by $\Delta c_M \equiv [1/(1+\beta)][E\{R\} - r]/r$, and the total portfolio falls by Δc_M. Since the consumer's (indirect) holding of equity increases by one dollar, the fall in the consumer's holding of bonds is 1 + $\Delta c_M = 1 + [1/(1+\beta)][E\{R\} - r]/r$, as shown in equation (20).

Let $V_M(\Omega, r, \theta_k)$ denote the maximized value of the utility function for a saver who does not participate in the stock market. Substituting the optimal value of b from equation (19) into equation (18) and simplifying yields

$$V_M(\Omega, r, 0) = \beta \ln \beta - (1+\beta)\ln(1+\beta) + (1+\beta)\ln \Omega + \beta \ln r \tag{21}$$

Inspection of equation (21) yields

$$\left.\frac{\partial V_M(\Omega, r, \theta_k)}{\partial \Omega}\right|_{\theta_k=0} = \frac{1+\beta}{\Omega} > 0 \tag{22a}$$

and

$$\left.\frac{\partial V_M(\Omega, r, \theta_k)}{\partial r}\right|_{\theta_k=0} = \frac{\beta}{r} > 0\ . \tag{22b}$$

An increase in wealth, Ω, increases the maximized value of utility as shown in equation (22a). Because a saver who does not hold any risky capital, either directly or indirectly, has a long position in bonds, an increase in the riskless interest rate r increases the maximized value of utility as shown in equation (22b).

To determine the effect on utility of an increase in θ_k from an initial value of zero, differentiate equation (18) with respect to θ_k and evaluate the resulting derivative at $b = b_M(\Omega, r, 0)$ and $\theta_k = 0$ to obtain

$$\left.\frac{\partial V_M(\Omega, r, \theta_k)}{\partial \theta_k}\right|_{\theta_k=0} = \frac{1+\beta}{\Omega}\left[E\left\{\frac{R}{r}\right\} - 1\right] > 0. \tag{23}$$

A one-dollar increase in θ_k, accompanied by a one-dollar decrease in θ_b, which keeps Ω constant, increases the expected present value of lifetime resources by $[E\{R\} - r]/r$ dollars. Multiplying this increase in expected present value by the effect of Ω on V_M shown in equation (22a) yields the effect on the maximized value of utility shown in equation (23).

II.C. The Participation Decision

For young consumers who save some of their disposable income, an important decision is whether to participate in the stock market. If the benefit of participating in the stock market outweighs the fixed cost of participation, then the consumer will participate. Otherwise the consumer will not directly hold any risky capital. To make this decision, a saver must compare $V_H(\Omega, r, \theta_k)$, which is the maximized value of utility if the consumer participates in the stock market, with $V_M(\Omega, r, \theta_k)$, which is the maximized value of utility if the consumer does not participate in the stock market. Define

$$D(\Omega, r, \theta_k) \equiv V_H(\Omega, r, \theta_k) - V_M(\Omega, r, \theta_k). \tag{24}$$

If $D(\Omega, r, \theta_k) > 0$, the consumer will choose to participate in the stock market, and if $D(\Omega, r, \theta_k) < 0$ the consumer will choose not to participate in the stock market. If $D(\Omega, r, \theta_k) = 0$, the consumer will be indifferent about participating in this market.

Define the critical value $\hat{\Omega}(r,\theta_k)$ as the value of Ω, given[11] r and θ_k, for which the consumer is indifferent about participating in the stock market. Formally, $\hat{\Omega}(r,\theta_k)$ satisfies $D\left(\hat{\Omega}(r,\theta_k), r, \theta_k\right) = 0$. To compute the critical value $\hat{\Omega}$ in the case with $\theta_k = 0$, substitute equations (16) and (21) into the definition of $D(\Omega, r, \theta_k)$ in equation (24), use the definition of $\psi(\gamma, r)$ in equation (11b), and evaluate the resulting expression at $\theta_k = 0$ to obtain

$$D(\Omega, r, 0) = (1+\beta)\ln\left(1 - \frac{F_1 + F_2}{\Omega}\right) + \beta\Psi(r) \tag{25a}$$

where[12]

$$\Psi(r) \equiv \max_\gamma E\left\{\ln\left[1 - \gamma + \gamma\frac{R}{r}\right]\right\} > 0. \tag{25b}$$

Setting the right hand side of equation (25a) equal to zero and solving for Ω, yields the critical value of Ω as

$$\hat{\Omega}(r,0) = \frac{F_1 + F_2}{1 - \exp\left[-\frac{\beta}{1+\beta}\Psi(r)\right]} > F_1 + F_2 > 0. \tag{26}$$

For consumers with $\Omega < \hat{\Omega}(r,0)$, $D(\Omega, r, 0) < 0$ and it is optimal not to participate in the stock market. For consumers with $\Omega > \hat{\Omega}(r,0)$, $D(\Omega, r, 0) > 0$ and it is optimal to

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[11] The distribution of the risky rate of return R is also given, though this distribution is not reflected in the notation.

[12] $\Psi(r)$ can be written as $\max_\gamma \psi^*(\gamma, r)$ where $\psi^*(\gamma, r) \equiv E\{\ln[1 - \gamma + \gamma R/r]\}$. Observe that $\psi^*(0, r) = 0$ so that $\Psi(r) \geq 0$. Recall from footnote 9 that if the ex ante equity premium is positive, the optimal value of γ is positive, which implies $\Psi(r) > 0$.

participate in the stock market. Put simply, high-income consumers will buy risky capital and middle-income and low-income consumers will not.

An increase in the riskless interest rate r reduces the benefit of investing in the stock market relative to holding only riskless bonds, and thus increases the critical value $\hat{\Omega}(r,0)$.[13] An increase in θ_k provides some of the benefit of investing in the stock market and thus reduces the gain to participating directly in the stock market relative to holding only riskless bonds in the directly-held portfolio. Therefore, an increase in θ_k increases the critical value $\hat{\Omega}$.[14]

The only source of intra-generational heterogeneity in this model, and thus the only factor that causes Ω to differ among consumers in a given generation, is variation in endowments of effective units of labor, n. Since Ω is a linear function of n, there is a unique critical value of n, call it $\hat{n}$, that separates young participants in the stock market and young non-participants. It follows from the definition of Ω in equation (8b) that

$$\hat{n} = \frac{1}{w}\left(\hat{\Omega} + T - \theta_b - \theta_k\right). \tag{27}$$

Young consumers with $n > \hat{n}$ will participate in the market for risky capital, and young consumers with $n < \hat{n}$ will not participate in this market.

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[13] If the ex ante equity premium $E\{R\} - r$ is positive, the optimal value of γ in the definition of $\Psi(r)$ is positive, and hence $\Psi(r)$ is a decreasing function of the riskless interest rate r. Therefore, equation (26) implies that $\hat{\Omega}(r,0)$ is an increasing function of r.

[14] Formally, $\left.\frac{\partial D(\Omega,r,\theta_k)}{\partial \theta_k}\right|_{\theta_k=0} = \left.\frac{\partial V_H(\Omega,r,\theta_k)}{\partial \theta_k}\right|_{\theta_k=0} - \left.\frac{\partial V_M(\Omega,r,\theta_k)}{\partial \theta_k}\right|_{\theta_k=0} = -\left.\frac{\partial V_M(\Omega,r,\theta_k)}{\partial \theta_k}\right|_{\theta_k=0} < 0$, where the first equality follows from the definition of $D(\Omega, r, \theta_k)$ in equation (24), the second equality follows from equation (17c), and the inequality follows from equation (23). Thus, an increase in θ_k must be offset by an increase in Ω to maintain the right hand side of equation (24) equal to zero.

Consumers for whom $\Omega = \hat{\Omega}$ (equivalently, $n = \hat{n}$) are indifferent about participating in the stock market. I will refer to these consumers as "switchers" because a small change in r or θ_k will change $\hat{\Omega}$, and these consumers may switch their participation status. Confine attention to the case in which $\theta_k = 0$ and define $\Delta_{\hat{b}} \equiv b_M(\hat{\Omega}, r, 0) - b_H(\hat{\Omega}, r, 0)$ as the difference in the holdings (direct and indirect) of bonds and $\Delta_{\hat{k}} \equiv -h_H(\hat{\Omega}, r, 0)$ as the difference in the holdings of risky capital between those consumers who do not participate in the stock market and those who participate in the stock market. It follows from equations (19), (14), and (15) that

$$\Delta_{\hat{b}} = \frac{\beta}{1+\beta}\left[\gamma(r)\hat{\Omega} + (1-\gamma(r))(F_1 + F_2)\right] > 0 \tag{28}$$

and

$$\Delta_{\hat{k}} = -\gamma(r)\frac{\beta}{1+\beta}\left(\hat{\Omega} - F_1 - F_2\right) < 0. \tag{29}$$

A switcher who chooses not to participate in the stock market holds more bonds and less risky capital than a switcher who chooses to participate in the stock market.

The effect of the participation decision on the size of the total portfolio of a switcher is calculated by adding equations (28) and (29) to obtain

$$\Delta_{\hat{b}} + \Delta_{\hat{k}} = \frac{\beta}{1+\beta}(F_1 + F_2) > 0. \tag{30}$$

As shown in equation (30), a switcher who chooses not to participate in the stock market has a larger total portfolio in the first period than a switcher who chooses to participate in the stock market. By not participating in the stock market, the switcher avoids the fixed cost $F_1 + F_2$ and therefore increases the total value of the first-period portfolio by $[\beta/(1+\beta)](F_1+F_2)$. Of course, in terms of utility, the avoidance of the fixed cost F_1+F_2, which increases the size of the portfolio, is offset by the fact that the rate of return on the portfolio is lower when the switcher has no direct holding of risky capital than when the switcher participates directly in the stock market.

II.D. Young Consumers Who Do Not Save[15]

Young consumers are allowed to borrow to purchase risky capital but are not allowed to borrow to finance consumption in excess of disposable income. Formally, this borrowing constraint is

$$c \leq wn - T. \tag{31}$$

Suppose that $\theta_k = 0$ and consider a young consumer with $\Omega < \hat{\Omega}$, which implies that the consumer does not participate in the stock market. In the absence of the borrowing constraint in equation (31), this consumer would consume $\frac{1}{1+\beta}\Omega$. This amount of consumption exceeds the amount of consumption permitted by the borrowing constraint in equation (31) if $\Omega > (1+\beta)(wn - T)$, or equivalently if

[15] I thank Martin Feldstein for suggesting consideration of young consumers who face binding borrowing constraints that prevent their consumption from increasing in response to a change in the portfolio of the social security system.

$$\Omega < \Omega_0 \equiv \frac{1+\beta}{\beta}\theta_b. \tag{32}$$

Henceforth, I will assume that the level of wealth at which the borrowing constraint just binds, Ω_0, is less than the level of wealth at which savers are indifferent about participating in the stock market, $\hat{\Omega}$.[16] Thus, Ω_0 and $\hat{\Omega}$ define three regions in the wealth distribution. Low-income consumers have Ω smaller than Ω_0. These consumers consume their disposable income $wn - T$ when they are young and do not participate in the stock market. Middle-income consumers have Ω greater than Ω_0, so that the borrowing constraint is not binding, but smaller than $\hat{\Omega}$, so that they do not participate in the stock market. High-income consumers have Ω greater than $\hat{\Omega}$. They save some of their disposable and participate in the stock market.

The critical value that separates consumers for whom the borrowing constraint binds from consumers for whom the constraint does not bind can be expressed in terms of the endowment of labor. Substituting $\Omega_0 \equiv \frac{1+\beta}{\beta}\theta_b$ into the definition of wealth in equation (8b), and recalling that $\theta_k = 0$, yields $n_0 = \frac{1}{w}\left(T + \frac{1}{\beta}\theta_b\right)$. Young consumers with $n < n_0$ will consume all of their disposable income, and young consumers with $n > n_0$ will save some of their disposable income.

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[16] Equations (26) and (32) imply that for $\theta_k = 0$ the condition that $\Omega_0 < \hat{\Omega}$ is satisfied if and only if

$$F_1 + F_2 > \left[1 - \exp\left(-\frac{\beta}{1+\beta}\Psi(r)\right)\right]\frac{1+\beta}{\beta}\theta_b. \tag{*}$$

With the more general preferences introduced in footnote 5, the benefit of optimally allocating a portfolio to stocks and bonds, $\Psi(r)$, is $\max_\gamma \frac{\beta}{1-\phi}\ln E\left\{\left(1-\gamma+\gamma\frac{R}{r}\right)^{1-\phi}\right\}$. Allowing the coefficient of relative risk

III. Capital Market Equilibrium

In this section I examine equilibrium in the markets for riskless bonds and risky capital. Let $B^D(r, \theta_k, \theta_b, T, AK)$ be the aggregate demand for bonds, which can be written as

$$\begin{aligned} B^D(r,\theta_k,\theta_b,T,AK) \equiv & \int_{\hat{n}}^{\infty} b_H\big((1-\alpha)AKn - T + \theta_b + \theta_k, r, \theta_k\big)dG(n) \\ & + \int_{n_0}^{\hat{n}} b_M\big((1-\alpha)AKn - T + \theta_b + \theta_k, r, \theta_k\big)dG(n) \, . \\ & + \theta_b G(n_0) \end{aligned} \tag{33}$$

The first integral on the right hand side of equation (33) is the demand for bonds by high-income consumers, and the second integral on the right hand side is the demand for bonds by middle-income consumers. The final term on the right hand side, $\theta_b G(n_0)$, is the amount of bonds held indirectly by low-income consumers, who do not directly hold any bonds.

Riskless bonds are issued by the government. In the context of the United States, riskless bonds are issued by the Treasury and are held by individuals and by the Social Security Trust Fund. Let B denote the aggregate quantity of riskless bonds issued by the Treasury.[17] In principle, private consumers, or groups of private consumers could issue bonds, but since these privately issued bonds would be assets of some private agents and

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aversion ϕ to exceed one can reduce $\Psi(r)$ relative to its value in equation (25b), and thus can reduce the size of the fixed cost $F_1 + F_2$ needed to make condition (*) hold.

[17] Assume that $B \leq \frac{\beta}{1+\beta}\int_0^{\infty}(wn - T + \theta_b + \theta_k)dG(n)$. The right hand side of this condition is the aggregate demand for bonds that would arise if no consumers participated in the stock market. The aggregate demand for bonds cannot exceed this value, and thus equilibrium requires that the supply of bonds be no larger than this amount.

liabilities of other private agents, the aggregate net supply of bonds would continue to be equal to B. Equilibrium in the bond market is represented by

$$B = B^{D}\left(r, \theta_k, \theta_b, T, AK\right). \tag{34}$$

Let $K^D(r, \theta_k, \theta_b, T, AK)$ be the aggregate demand for risky capital, which can be written as

$$K^{D}\left(r, \theta_k, \theta_b, T, AK\right) \equiv \theta_k G(\hat{n}) + \int_{\hat{n}}^{\infty} h_H\left((1-\alpha)AKn - T + \theta_b + \theta_k, r, \theta_k\right) dG(n). \tag{35}$$

Low-income and middle-income young consumers hold capital only indirectly, through the social security system, and their aggregate holding of risky capital is shown by the first term on the right hand side of equation (35). The second term on the right hand side of this equation is the demand for risky capital by high-income consumers.

Let K' be the aggregate amount of risky capital held at the end of the current period. It is distinct from K, which is the aggregate amount of capital held at the beginning of the current period (the end of the previous period) and used in production during the current period. Equilibrium in the market for risky capital is represented by

$$K' = K^{D}\left(r, \theta_k, \theta_b, T, AK\right). \tag{36}$$

IV. The Effect of Investing a Small Amount of Social Security in the Stock Market

Now consider the effects of changing the portfolio of the social security system. Specifically, assume that initially the social security system holds only riskless bonds so that $\theta_k = 0$. Then suppose that in the current period, the social security system sells a small amount of bonds per capita and uses the proceeds of this sale to purchase risky capital. Formally, $d\theta_b = - d\theta_k < 0$ so that the total value of assets in the social security system is unchanged. In addition, suppose that the Treasury does not change the current amount of bonds, B, outstanding at the end of the current period. The analysis in this paper will focus on the effects of this policy during the current period.[18]

Because the change in the portfolio of the social security system leaves the value of $\theta_b + \theta_k$ unchanged, the value of wealth, Ω, is unchanged for each young consumer in the current period. This invariance of Ω to the composition of the portfolio of the social security system implies that if all consumers participate in all markets, including the stock market, they will not change their total holdings (direct plus indirect) of either bonds or stocks. Specifically, when the social security system sells a dollar of bonds and purchases a dollar of risky capital on behalf of each young consumer, each young consumer would offset this asset swap by purchasing an additional dollar of bonds and selling one dollar of risky capital. In this case, investing social security funds in the stock market is merely a "shell game" with no effects on the real allocation of consumption.

[18] A fully dynamic analysis that examines future periods as well as the current period would have to take account of the Treasury's budget constraint $B_{t+1} = r_t B_t + G_t - T_t$ where B_t is the aggregate value of bonds outstanding at the beginning of period t, r_t is the gross riskless interest rate on assets held from period t-1 to period t, G_t is aggregate government purchases by the Treasury during period t, and T_t is total tax collections during period t. None of the variables r_t, B_t, B_{t+1}, G_t and T_t is affected by the change in policy in period t. However, the riskless interest rate r_{t+1} will change as a result of this policy, which will require *future* changes in government purchases, taxes, or the path of bonds outstanding. These changes do not affect the equilibrium in the current period (period t), which is the focus of this analysis. Because Diamond and Geanakoplos (1999) analyze future periods as well as the period in which the portfolio of the social security system is changed, they must take a stand on how the government reacts to a change in the interest rate on its bonds. They assume that an increase in the interest rate leads the government to increase taxes on old consumers to maintain the level of government bonds unchanged.

The shell game result depends on the assumption that all consumers participate in the stock market and hence can offset the effects of the change in the portfolio of the social security system. But most consumers hold no stock in their portfolios,[19] and hence the assumptions underlying the shell game result are violated. The present model takes account of non-stockholders and the consequent departure from the shell game result. In Appendix B, I show that

$$\left.\frac{dK^D(r,\theta_k,\theta_b,T,AK)}{d\theta_k}\right|_{\substack{\theta_k=0\\ d\theta_b=-d\theta_k}} = -\frac{1}{1+\beta}\frac{E\{R\}-r}{r}\left[G(\hat{n})-G(n_0)\right]+\left(\Delta_{\hat{b}}+\Delta_{\hat{k}}\right)\frac{d\hat{n}}{d\theta_k}dG(\hat{n}). \tag{37}$$

The effect on the capital stock in equation (37) is the sum of two terms. The first term represents the reduction in aggregate saving resulting from the increased consumption of middle-income consumers, who do not participate in the stock market. This increased consumption arises because a one-dollar shift from bonds to risky capital increases expected second-period income by the equity premium $E\{R\} - r$, and the present value of this increase is $[E\{R\} - r]/r$. The marginal propensity to consume when young is $1/(1+\beta)$, so the increase in the present value of expected income increases the consumption of each middle-income consumer by $\Delta c_M \equiv [1/(1+\beta)]\,[E\{R\} - r]/r$.[20] The aggregate effect, represented by the first of the two terms in equation (37), is obtained by multiplying Δc_M by the measure of middle-income consumers, $G(\hat{n})-G(n_0)$.

[19] See footnote 25.

[20] Low-income consumers also enjoy an increase in expected second-period income but are prevented from increasing their current consumption by a binding borrowing constraint.

The second term on the right hand side of equation (37) is the increase in aggregate saving that arises as switchers endogenously change their status from group *H*, which holds risky capital, to group *M*, which does not participate in the stock market. The saving of each switcher increases by $\Delta_{\hat{b}} + \Delta_{\hat{k}}$, and the measure of switchers is

$$\frac{d\hat{n}}{d\theta_k} dG(\hat{n}).$$

V. Ignoring the Effects of Switchers

For the remainder of this paper, I will ignore the behavior of switchers. One rationale for ignoring switchers is that I am analyzing a change in the portfolio of the social security system after the current generation of workers has decided whether to participate in the stock market for risky capital, but before they make a final choice about how much to consume when young. In this scenario, consumers are allowed to react to the change in the portfolio of the social security system. Consumers who had been indifferent about participating in the stock market, but who had nonetheless chosen to participate, have already incurred the fixed cost of participation. Since this cost is already sunk, these consumers will not switch to non-stockholder status. Simply put, consumers with $n = \hat{n}$ will not switch away from participating in the stock market.[21]

A second rationale for ignoring switchers is that the direction in which their saving responds to the change in the portfolio of the social security system is not robust to the specification of fixed costs. With the formulation of fixed costs in this paper,

[21] The direction of the switch is important for the sunk cost argument here. In this case, the change in the portfolio of the social security system increases the riskless interest rate *r*, which increases $\hat{\Omega}$. In addition,

switchers who choose not to hold stocks have higher saving than switchers who choose to hold stocks, but this result could be reversed by a different formulation of fixed costs.[22]

V.A. The Effect on Portfolios of Individual Consumers

Ignoring the behavior of switchers, the effect on the aggregate capital stock of the change in the portfolio of the social security system is given by the first of the two terms in equation (37). Table 1 illustrates this result by focusing on the portfolios of individual consumers. First examine the row for low-income consumers. Young low-income consumers do not hold any assets directly. All of their assets are held indirectly through the social security system, so the change in the portfolio of the social security system increases capital by one dollar and reduces bonds by one dollar for each young low-income consumer. Now examine the row for middle-income consumers. As discussed below equation (20), a one-dollar increase in θ_k accompanied by a one-dollar decrease in θ_b causes middle-income consumers to increase their consumption by $\Delta c_M \equiv [1/(1+\beta)][E\{R\}-r]/r$ dollars, and therefore to reduce their total assets by Δc_M dollars. Because each middle-income consumer increases the holding of risky capital by one dollar while reducing total assets by Δc_M dollars, each middle-income consumer's holding

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the increase in θ_k increases $\hat{\Omega}$. Since $\hat{\Omega}$ increases, potential switchers would switch from participating to not participating in the stock market, but they would not avoid the fixed cost already incurred.

[22] Suppose that in addition to the fixed costs F_1 and F_2, participation in the stock market involves a utility cost when old. For example, if the utility function is $U = \ln(c - \delta F_2) + \beta E\{\ln(x - \delta F_3)\}$, then, as shown in footnote 7, $a \equiv b + h = \frac{\beta}{1+\beta}(\Omega - F_1 - F_2 + F_3/(\beta r))$. Recall from equation (19) that a middle-income consumer's total assets equal $\frac{\beta}{1+\beta}\Omega$. If $F_1 + F_2 - F_3/(\beta r) > 0$, as in the model in the text, the total assets of a switcher who participates in the stock market will be less than those of a switcher who does not participate in the stock market. However, if $F_1 + F_2 - F_3/(\beta r) < 0$, the total assets of a switcher who participates in the stock market will be greater than those of a switcher who does not participate in the stock market.

of bonds is reduced by $1 + \Delta c_M$ dollars, as shown in the final column of the middle-income consumer's row.

Table 1 Effect of $\Delta\theta_k = -\Delta\theta_b = 1$, if $dG(\hat{n}) = 0$			
Type of consumer [measure]	Change in:		
	Total Assets (per capita)	Capital (per capita)	Bonds (per capita)
Low-Income $[G(n_0)]$	0	1	-1
Middle-Income $[G(\hat{n}) - G(n_0)]$	$-\Delta c_M \equiv -\frac{1}{1+\beta}\frac{E\{R\}-r}{r}$	1	$-(1+\Delta c_M)$
High-Income $[1 - G(\hat{n})]$	0	$-\frac{G(\hat{n}) + \Delta c_M[G(\hat{n}) - G(n_0)]}{1 - G(\hat{n})}$	$\frac{G(\hat{n}) + \Delta c_M[G(\hat{n}) - G(n_0)]}{1 - G(\hat{n})}$
Aggregate	$-\Delta c_M[G(\hat{n}) - G(n_0)]$	$-\Delta c_M[G(\hat{n}) - G(n_0)]$	0

Since the aggregate supply of bonds is held fixed, equilibrium in the bond market requires that high-income consumers increase their holdings of bonds by the amount shown in the final column of Table 1, so that the change in the aggregate holding of bonds, shown in the lower right corner of Table 1, is zero. High-income consumers are induced to increase their holdings of bonds by an increase in the riskless interest rate r, which causes them to shift some of their assets from risky capital to riskless bonds.[23] Since the total value of assets held by each high-income consumer is unchanged, the

[23] The effect on the riskless interest rate can be calculated by setting $dG(\hat{n}) = 0$ in equation (B.6) to obtain $\frac{dr}{d\theta_k} = -\frac{B_\theta}{B_r} > 0$ where $B_\theta < 0$ is defined in equation (B.4) and $B_r > 0$ is defined in equation (B.3).

increase in bond holdings in the final column is offset by an equal decrease in the holding of risky capital, as shown in Table 1.

The effect on the aggregate holding of risky capital is obtained by weighting the effects on low-income, middle-income, and high-income consumers by their respective measures. As shown in the final row of Table 1, the aggregate effect on the amount of risky capital is $-\Delta c_M\left[G(\hat{n})-G(n_0)\right]$, which equals the first of the two terms on the right hand side of equation (37).

V.B. The Effects on Utility

The replacement of a small amount of bonds by an equal value of risky capital in the portfolio of the social security system changes the utility of middle-income consumers by dV_M, where

$$dV_M = \left(\left. \frac{\partial V_M(\Omega, r, \theta_k)}{\partial \theta_k} \right|_{\theta_k=0} \right) d\theta_k + \left(\left. \frac{\partial V_M(\Omega, r, \theta_k)}{\partial r} \right|_{\theta_k=0} \right) dr > 0 \tag{38}$$

and the sign is obtained using equations (23) and (22b) along with $d\theta_k > 0$ and $dr > 0$. Middle-income consumers benefit in two ways from the change in the portfolio of the social security system. First, they benefit from holding risky capital, which has a higher expected rate of return than bonds. In addition, because they are recipients of interest income, they benefit from the increase in the riskless interest rate. Similarly, low-income consumers benefit from holding risky capital, and because they are recipients of interest on their indirect holdings of bonds in the social security system, they benefit from the increase in the riskless interest rate.

The effect on the utility of high-income consumers is

$$dV_H = \frac{\partial V_H}{\partial r} dr = \beta(1-\gamma) E\left\{\frac{1}{(1-\gamma)r + \gamma R}\right\} dr \tag{39}$$

where the second equality uses equation (17b). High-income consumers, who hold capital directly, are not directly affected by the change in θ_k because they can offset this change in the social security system's portfolio by changing their own directly-held portfolios. However, the change in the riskless interest rate affects the maximized utility of high-income consumers. Since $dr > 0$, the sign of the effect in equation (39) is the same as the sign of $1 - \gamma$, which is the share of bonds in the portfolios of high-income consumers. In principle, if the aggregate supply of bonds, B, is sufficiently small, the value of $1 - \gamma$ could be negative, but the empirically relevant case is one in which high-income consumers have positive holdings of bonds so that $1 - \gamma$ is positive. In this case, high-income consumers benefit from the increase in the riskless interest rate.

I have shown that if high-income consumers have long positions in bonds, the change in the portfolio of the social security system increases the utility of young consumers in all three income classes. Old consumers are unaffected by this change. So does the change in the portfolio of the social security system represent a free lunch, making some consumers better off without harming others? The answer is no. The next generation will work with a smaller capital stock and thus will have smaller wage income than it would have had otherwise. In addition, the increase in the riskless interest rate is costly to the issuer of bonds. Since the government is the issuer of bonds, the increase in the riskless interest rate is costly to the government, which means that it is costly to future taxpayers.[24]

VI. An Estimate of the Effect on the Aggregate Capital Stock

The effect of a change in the portfolio of the social security system on the aggregate capital stock, ignoring switchers, is given by the first of the two terms on the right hand side of equation (37)

$$\left.\frac{dK^D(r,\theta_k,\theta_b,T,AK)}{d\theta_k}\right|_{\substack{\theta_k=0\\ d\theta_b=-d\theta_k}} = -\frac{1}{1+\beta}\frac{E\{R\}-r}{r}\left[G(\hat{n})-G(n_0)\right], \text{ ignoring switchers.} \quad (40)$$

This effect is the product of three term terms. The first term, $1/(1+\beta)$, is the marginal propensity to consume when young. I will assume that one period in the model corresponds to 25 years and that the annual rate of time preference is 1%. Under these assumptions, $\beta = (1.01)^{-25} = 0.7798$, and the marginal propensity to consume is $1/(1+\beta) = 0.5619$.

The second term is the present value of the equity premium, $[E\{R\}-r]/r$. Cecchetti, Lam, and Mark (1993, Table 2, p. 35) report that over the period 1892-1987, the average value of the equity premium, $E\{R\} - r$, is 0.0663 and the average (gross) riskless rate is $r = 1.0119$, so the average rate of return on equity, $E\{R\}$, is 1.0782. These

[24] There is an element of a free lunch here because the social security system can purchase risky capital on behalf of consumers without incurring the costs F_1 or F_2. However, it remains true that future generations of workers and taxpayers are harmed by the reduction in their wages and by the consequences of the increased cost of debt service.

average rates of return are expressed on an annual basis. Treating a period in the model as 25 years implies $[E\{R\}-r]/r = [1.0782^{25} - 1.0119^{25}]/1.0119^{25} = 3.8870$.

The third term is the fraction of consumers who save some of their disposable income but do not participate in the stock market, i.e., the fraction of consumers that are middle-income consumers. Ameriks and Zeldes (2000, Exhibit 9B) report that 36.8% of households held less than \$5000 of financial wealth and less than \$1000 of stock in any form in 1995. I will treat these households as low-income households and thus set $G(n_0) = 0.368$. They also report that 36.7% of households owned more than \$1000 of stock in 1995. I will treat these households as high-income households in the model, and thus set $G(\hat{n}) = 0.633$.[25] Therefore, the fraction of middle-income consumers, $G(\hat{n}) - G(n_0)$, is 0.265.[26]

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[25] Mankiw and Zeldes (1991, Table 1) report that in 1984, 72.4% of households did not own any stock and 4.5% of households held some stock but less than \$1000 of stock, so the comparable value of $G(\hat{n})$ in 1984 was 0.769. The decline in $G(\hat{n})$ from 1984 to 1995 reflects the increasing extent of stock ownership in the U.S. economy over that time.

[26] I have assumed that all young consumers have equal claims on future social security benefits, despite the fact that they have different pre-tax income, *wn*. An alternative specification is that a consumer with effective labor endowment n has a claim on future social security benefits with a present value of $\lambda(n)\theta$, where $\lambda'(n) \geq 0$ and $\int_0^\infty \lambda(n)dG(n) = 1$, so that the present value of the aggregate claim on future social security benefits is θ. In this case, the term $G(\hat{n}) - G(n_0)$ in equations (37) and (40) is replaced by $\int_{n_0}^{\hat{n}} \lambda(n)dG(n)$. In the case examined in the text, $\lambda(n) \equiv 1$, so that $\int_{n_0}^{\hat{n}} \lambda(n)dG(n) = G(\hat{n}) - G(n_0)$. However, in the case with $\lambda(n)$ increasing in n, $\int_{n_0}^{\hat{n}} \lambda(n)dG(n)$ can be greater than, equal to, or less than $G(\hat{n}) - G(n_0)$. Consider an example in which n is uniformly distributed on the interval [1, 4], and the present value of the claim on future social security benefits is proportional to wage income *wn*, so that $\lambda(n) = n/2.5$. Suppose that $n_0 = 2$ and $\hat{n} = 3$. In this case, $G(\hat{n}) - G(n_0) = 1/3$ and $\int_{n_0}^{\hat{n}} \lambda(n)dG(n) = 1/3$ so that the effect of the change in the portfolio of the social security system is the same whether the social security benefit is proportional to *wn* or independent of *wn*. If social security benefits and taxes are capped, then for some $a > 0$, $\lambda(n) = awn$ if $n \leq \bar{n}$ and $\lambda(n) = aw\bar{n}$ if $n \geq \bar{n}$. In this case, $\int_{n_0}^{\hat{n}} \lambda(n)dG(n)$ exceeds $G(\hat{n}) - G(n_0)$, if the distribution of n is uniform on the interval [1, 4], $n_0 = 2$, $\hat{n} = 3$, and $1 < \bar{n} < 4$. In this case, the effect on the aggregate capital stock is higher if social security benefits are proportional to *wn* (up

Multiplying the marginal propensity consume, 0.5619, by the equity premium, 3.8870, and by the fraction of middle-income consumers, 0.265, yields

$$\left.\frac{dK^D(r,\theta_k,\theta_b,T,AK)}{d\theta_k}\right|_{\substack{\theta_k=0 \\ d\theta_b=-d\theta_k}} = -0.5788, \qquad \text{ignoring switchers.} \tag{41}$$

If the social security system increases its holding of risky capital by one dollar per capita and reduces its holding of riskless bonds by one dollar per capita, the aggregate holding of risky capital will fall by more than 57 cents per capita. Because the magnitude of this effect is surprisingly large, I explore the sensitivity of this calculation to various factors. Table 2 reports the quantitative effects of various changes that are designed to reduce the magnitude of this effect. Column (1) contains the baseline calculation presented in equation (41). The effect on capital stock is reported in the final row. Reducing the rate of time preference to zero reduces the marginal propensity to consume, $1/(1+\beta)$, to 0.5 and reduces the magnitude of the effect on the capital stock to 0.52 (column (2)). Reducing the assumed number of years per period to 20 reduces both the marginal propensity to consume and the average equity premium per period, and thus reduces the magnitude of the effect on the capital stock to 0.37 (column (3)). Reducing the expected rate of return on risky capital, $E\{R\}$, by 1 percentage point per year reduces the magnitude of the effect on the capital stock to 0.43 (column (4)). Increasing the riskless interest rate by one percentage point per year reduces the magnitude of the effect to 0.42 (column (5)). Although each of the changes explored in columns (2) – (5) reduces the magnitude of the effect, in all cases the magnitude of the effect remains greater than

to the cap) than if they are independent of *wn*. Of course, the results of this example could be reversed for a

0.35. Putting together all of these changes reduces the magnitude of the effect to 0.19 as shown in column (6). Thus, even allowing for the possibility that the baseline calculation overstates the magnitude of the effect, the effect appears to be quite substantial.

Table 2
Effect of $d\theta_k = -d\theta_b > 0$ on Aggregate Capital Stock

	(1) baseline	(2) low time pref.	(3) low N	(4) low $E\{R\}$	(5) high r	(6) small effect
Rate of time pref. (annual)	0.01	0	0.01	0.01	0.01	0
Mean risky rate (annual)	1.0782	1.0782	1.0782	1.0682	1.0782	1.0682
Mean riskless rate (annual)	1.0119	1.0119	1.0119	1.0119	1.0219	1.0219
$G(\hat{n})$	0.633	0.633	0.633	0.633	0.633	0.633
$G(n_0)$	0.368	0.368	0.368	0.368	0.368	0.368
N = number of years per period	25	25	20	25	25	20
Propensity to consume, $1/(1+\beta)$ (per N-year period)	0.5619	0.5000	0.5496	0.5619	0.5619	0.5000
Equity premium, $[E\{R\} - r]/r$ (per N-year period)	3.8870	3.8870	2.5582	2.8715	2.8218	1.4260
Effect on capital stock	-0.58	-0.52	-0.37	-0.43	-0.42	-0.19

VII. Concluding Remarks

The general equilibrium model developed in this paper shows that if the social security system sells some bonds and purchases an equal amount of equity, the effect on the aggregate capital stock is the sum of two terms. The first term reflects the income effect of middle-income consumers who benefit from the increased rate of return on assets held in the portfolio of the social security system and have the resources to increase their consumption when they are young. In response to this increase in their effective wealth, middle-income consumers increase their consumption, which reduces national saving and capital accumulation. The second term reflects the behavior of switchers who

different distribution of n and different values of n_0 and $\hat{n}$.

would have been indifferent about participating directly in the stock market before the change in the social security system's portfolio. As a result of the social security system's holding of equity, these switchers would choose not to participate directly in the stock market and thus would avoid paying the fixed cost of participating in that market. With the formulation of fixed costs in this model, avoiding the fixed costs induces switchers to increase their saving, but this effect is not particularly robust. To the extent that switchers increase their saving, their response to the change in the social security system's portfolio works in the opposite direction of the response of middle-income consumers.

To study the quantitative impact on the aggregate capital stock of the change in the social security system's portfolio, I have chosen to ignore the behavior of switchers for two reasons: (1) if I consider a change in the social security system after the current generation of workers has already incurred the fixed cost of participating in the stock market, they will not switch; and (2) under a reasonable alternative formulation, switchers might reduce rather than increase their saving. Ignoring switchers, I have shown that if the social security system sells one dollar of bonds and buys one dollar of equity, the aggregate capital stock would fall by a little more than a half dollar. Of course, the estimated effect on the aggregate capital stock would be reduced by taking account of switchers, *if* they have not already incurred the fixed cost of stock market participation, and if fixed costs are as modeled in this paper. Future research is needed to explore the potential quantitative importance of switchers.

Though the income effect on middle-income consumers, which is the driving force behind the reduction in the aggregate capital stock, is very straightforward in this model, it is important to consider factors that mitigate, or even reverse, this effect. These factors could be usefully addressed in future research. In the current model, middle-income consumers would choose to hold a positive amount of risky capital if there were no fixed costs. Thus, when the trust fund of the social security system sells some bonds in exchange for risky capital, these middle-income consumers regard themselves as better off and increase their consumption when young. However, if middle-income consumers would choose to hold a negative amount of risky capital in the absence of fixed costs, then they would reduce their consumption and increase their saving when the social security trust fund sell bonds in exchange for risky capital. High risk aversion alone would not lead consumers to choose a negative amount of risky capital in this model, because the equity premium on risky capital is positive, and there are no other risks in the model. But if consumers face other risks, such as homeownership or ownership interests in small businesses, that are positively correlated with the return on risky capital, then high risk aversion may lead consumers to choose a negative amount of risky capital in their portfolios.[27]

The logarithmic utility function in equation (10) has both an intertemporal elasticity of substitution equal to one and a coefficient of relative risk aversion equal to one. The unitary intertemporal elasticity of substitution, together with the assumption that consumers do not work when old, implies that saving does not depend on the interest rate. However, to the extent that an increase in the interest rate leads to an increase in saving by consumers, the quantitative results in this paper will be mitigated or possibly even reversed. As discussed in the introduction, Diamond and Geanakoplos (1999) have

[27] A more general point is that the model does not contain a good reason for the institution of social security. Perhaps the inclusion of such a reason might alter the conclusions in this paper. However, the

analyzed a model that emphasizes the interest rate effect on saving, but their model has no middle-income consumers and thus ignores the income effect emphasized in this paper.

In this paper, I focused on the effect on aggregate capital accumulation, and for the purpose of calibration, I matched the equity premium in the model with the empirical average equity premium. I did not attempt to explain the equity premium in terms of fundamental preferences and technologies. As is well known from the literature on the equity premium puzzle, the unitary coefficient of relative risk aversion associated with the logarithmic preferences in this paper is too low to account for the observed equity premium. I have presented, in footnote 5, a utility function that retains the unitary intertemporal elasticity of substitution but has an arbitrary coefficient of relative risk aversion. The quantitative results presented in Table 2 continue to hold under this more general utility function. However, a higher coefficient of relative risk aversion will increase the equilibrium equity premium and can help reduce the size of the fixed cost necessary to prevent stock market participation by middle-income consumers.

There are various proposals to reform social security, and, no doubt, several new proposals will appear. I have chosen to focus not on a particular reform proposal, but instead to focus on a particular feature—investing part of the trust fund in risky capital—that is found in some proposals. Proposals that include risky investment by the trust fund include other aspects, such as guaranteed minimum benefits to retirees, that require further study. Nor have I addressed any corporate governance issues that might arise if the social security system owned stock in private enterprises. With these same caveats,

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goal of this paper is simply to take the existence of social security as given and to analyze a specific change the analysis of this paper can also be applied to proposals that include individual accounts without investment options for individuals.

To sharpen the analysis of the social security system's portfolio, I have confined attention to fully-funded, defined-contribution social security systems in which the shell game argument is transparent. However, the current social security system in the United States is closer to a pay-as-you-go, defined-benefit plan. In the fully-funded, defined-contribution system analyzed in this paper, the current workers bear the risk associated with next period's risky return to equity held by the social security system.[28] However, in a pay-as-you-go, defined-benefit system there is more scope for intergenerationally sharing the risk associated with the return to capital held by the social security system.

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in the way the trust fund is invested.

[28] In a defined-benefit system, the additional investment income obtained by investing some of the trust fund in higher-yielding risky capital rather than bonds accrues to young consumers in the form of lower payroll taxes and hence higher disposable income. Middle-income and high-income young consumers would save part of the additional disposable income and consume part of it, and low-income consumers would consume all of it. Of course, in the period in which the social security system begins to invest in risky capital, which is the focus of this paper, the effects would not yet be operative. For an analysis of risky trust fund investment in a defined-benefit system along a growth path in an overlapping generations model with a representative agent in each generation, see Abel (1999).

Appendix A
Optimal Bond Holdings by a Saver Who Does Not Participate in the Stock Market

The optimal value of b for a middle-income consumer, who saves but does not participate in the stock market, is determined by differentiating equation (18) with respect to b, and setting the derivative equal to zero to obtain

$$\frac{1}{\Omega-\theta_k-b}=\beta r E\left\{\frac{1}{br+\theta_k R}\right\}. \tag{A.1}$$

It is helpful to define the following function

$$f\left(b,\theta_k,\Omega,r\right)\equiv \beta r\left(\Omega-\theta_k-b\right)E\left\{\frac{1}{br+\theta_k R}\right\} \tag{A.2}$$

Define b^* as the value of b that satisfies $f(b^*, \theta_k, \Omega, r) = 1$, and note that b^* satisfies the first-order condition in equation (A.1). Differentiating $f(b, \theta_k, \Omega, r)$ with respect to b and θ_k, and evaluating the derivatives at $b = b^*$ yields

$$f_b\left(b^*,\theta_k,\Omega,r\right)=-\frac{1}{\Omega-\theta_k-b^*}\left[1+\beta E\left\{\left(\frac{\left(\Omega-\theta_k-b^*\right)r}{b^*r+\theta_k R}\right)^2\right\}\right]<0 \tag{A.3}$$

$$f_{\theta_k}\left(b^*,\theta_k,\Omega,r\right)=-\frac{1}{\Omega-\theta_k-b^*}\left[1+\beta r E\left\{\left(\frac{\Omega-\theta_k-b^*}{b^*r+\theta_k R}\right)^2 R\right\}\right]<0 \tag{A.4}$$

Note that $f(b^*, 0, \Omega, r) = \beta(\Omega-b^*)/b^*$ so

$$b_M\left(\Omega,r,0\right)=\frac{\beta}{1+\beta}\Omega, \tag{A.5}$$

which is equation (19). The derivatives in equations (A.3) – (A.4), evaluated at $\theta_k = 0$, are

$$f_b\left(b^*,0,\Omega,r\right)=-\frac{1+\beta}{b^*}<0 \tag{A.6}$$

$$f_{\theta_k}\left(b^*,0,\Omega,r\right)=-\frac{1}{b^*}\left[\beta+E\left\{\frac{R}{r}\right\}\right]<0\,. \tag{A.7}$$

Applying the implicit function theorem to $f(b^*, 0, \Omega, r) = 1$, and using equations (A.6) - (A.7) yields equation (20) in the text.

Appendix B
The Effect on the Aggregate Capital Stock of Investing a Small Amount of Social Security in the Stock Market

The aggregate demand for bonds is

$$B^D = \theta_b G(n_0) + \int_{n_0}^{\hat{n}} b_M(\Omega(n), r, \theta_k) dG(n) + \int_{\hat{n}}^{\infty} b_H(\Omega(n), r, \theta_k) dG(n) \quad \text{(B.1)}$$

where the first term is the demand for bonds by low-income consumers, the second term is the demand for bonds by middle-income consumers, and the third term is the demand for bonds by high-income consumers. Totally differentiate equation (B.1) with respect to B^D, r, θ_k, θ_b, n_0 and $\hat{n}$ using equations (14), (20), and the fact that $\frac{\partial b_M(\Omega, r, 0)}{\partial r} = 0$, set $d\theta_b = -d\theta_k$, recognize that $b_M(\Omega(n_0), r, 0) = \theta_b$, and evaluate the expression at $\theta_k = 0$ to obtain

$$dB^D \Big|_{\substack{\theta_k = 0 \\ d\theta_b = -d\theta_k}} = B_r dr + B_\theta d\theta_k + (d\hat{n}) dG(\hat{n}) \Delta_{\hat{b}} \quad \text{(B.2)}$$

where

$$B_r \equiv -\gamma'(r) \frac{\beta}{1+\beta} \int_{\hat{n}}^{\infty} (\Omega(n) - F_1 - F_2) dG(n) > 0 \quad \text{(B.3)}$$

$$B_\theta \equiv -G(n_0) - \frac{E\left\{\frac{R}{r}\right\} + \beta}{1+\beta} \left(G(\hat{n}) - G(n_0)\right) < 0 \quad \text{(B.4)}$$

$$\Delta_{\hat{b}} \equiv b_M(\hat{\Omega}, r, 0) - b_H(\hat{\Omega}, r, 0) > 0. \quad \text{(B.5)}$$

The supply of bonds, B, is fixed, and in equilibrium $B^D = B$. Therefore, $dB^D = 0$ so that equation (B.2) implies

$$(d\hat{n}) dG(\hat{n}) = -\frac{1}{\Delta_{\hat{b}}} \left[B_r dr + B_\theta d\theta_k \right]. \quad \text{(B.6)}$$

The aggregate demand for risky capital is

$$K^D = \theta_k G(\hat{n}) + \int_{\hat{n}}^{\infty} h_H(\Omega(n), r, \theta_k) dG(n). \quad \text{(B.7)}$$

Totally differentiate equation (B.7) with respect to K^D, r, θ_k, and $\hat{n}$ using equation (15), and evaluate the expression at $\theta_k = 0$ to obtain

$$dK^D\Big|_{\substack{\theta_k=0\\ d\theta_b=-d\theta_k}} = G(\hat{n})d\theta_k + \left[\gamma'(r)\frac{\beta}{1+\beta}\int_{\hat{n}}^{\infty}\left(\Omega(n)-F_1-F_2\right)dG(n)\right]dr + (d\hat{n})dG(\hat{n})\Delta_{\hat{k}} \tag{B.8}$$

where

$$\Delta_{\hat{k}} \equiv -h_H\left(\hat{\Omega},r,0\right) < 0 \tag{B.9}$$

Rewrite equation (B.8) using equations (B.4) and (B.3) to obtain

$$dK^D\Big|_{\substack{\theta_k=0\\ d\theta_b=-d\theta_k}} = -\frac{E\left\{\frac{R}{r}\right\}-1}{1+\beta}\left(G(\hat{n})-G(n_0)\right)d\theta_k - \left(B_r dr + B_\theta d\theta_k\right) + (d\hat{n})dG(\hat{n})\Delta_{\hat{k}} \tag{B.10}$$

Substitute equation (B.6) into equation (B.10) to obtain

$$dK^D\Big|_{\substack{\theta_k=0\\ d\theta_b=-d\theta_k}} = -\frac{E\left\{\frac{R}{r}\right\}-1}{1+\beta}\left(G(\hat{n})-G(n_0)\right)d\theta_k + \left(\Delta_{\hat{b}}+\Delta_{\hat{k}}\right)(d\hat{n})dG(\hat{n}), \tag{B.11}$$

which implies equation (37) in the text.

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