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ABSTRACT

The classical model of balance of payments crises implicitly assumes that the central bank sits passively as international reserves dwindle. In practice, however, central banks typically defend pegs aggressively by raising short-term interest rates. This paper analyzes the feasibility and optimality of raising interest rates to delay a potential BOP crisis. Interest rate policy works through two distinct channels. By raising demand for domestic, interest-bearing liquid assets, higher interest rates tend to delay the crisis. Higher interest rates, however, increase public debt service and imply higher future inflation, which tends to bring forward the crisis. We show that, under certain conditions, it is feasible to delay the crisis, but raising interest rates beyond a certain point may actually hasten the crisis. A similar non-monotonic relationship emerges between welfare and the increase in interest rates. It is thus optimal to engage in some active interest rate defense but only up to a certain point. In fact, there is a whole range of interest rate increases for which it is feasible to delay the crisis but not optimal to do so.

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1 Introduction

In the aftermath of the currency crises in Europe (1992), Mexico (1994), Asia (1997), Russia (1998) and, more recently, Brazil (January 1999), there has been renewed interest in further understanding the mechanics of balance of payments (BOP) crises. For the better part of the last two decades, the profession's view of BOP crises has essentially followed the scenario envisaged by Paul Krugman in his 1979 seminal paper. Under a fixed exchange rate regime, any attempt to maintain a fixed exchange rate for the domestic currency while simultaneously expanding domestic credit to finance a fiscal deficit introduces a fundamental inconsistency into the system. Since real money demand is given, any attempt to monetize a given fiscal deficit under a fixed exchange rate regime will lead to a continuous loss of international reserves. If reserves cannot fall below a certain threshold, then countries pursuing such policies will inevitably be forced to abandon the peg at some point. Krugman (1979) showed how, at the time of the crisis, a speculative attack will deplete the central bank's reserves. This is a remarkable feature since the attack occurs in a context in which the public has rational expectations and can thus perfectly anticipate all future events.[1]

Another notable — but seldom discussed — feature of Krugman-type models is that the central bank is a rather defenseless agent. It sits passively as it watches the public hurrying to the central bank window and taking home its international reserves. In this paradigm, the central bank is – like Gulliver in his unfortunate encounter with the Lilliputians – a sleeping giant caught by surprise and unable to react. In reality, of course, central banks are hardly sleeping giants. Quite to the contrary, they typically fight long and hard before giving up a peg.

The key weapon in the central banks' arsenal is the use of short-term interest rates. When reserves begin to fall, central banks raise interest rates to often very high levels to induce investors to hold on to domestic currency denominated assets rather than switching to foreign currency denominated assets (see, for example, IMF (1997)). During the ERM crisis, for example, Sweden's monetary authority pushed overnight interest rates to around 500 percent per year. A more recent example is Brazil (Figure 1, Panel A).[2] In October 1997, and in the midst of the Asian turmoil, Brazil's central bank

[1] Apart from Krugman (1979), a few other notable papers which develop this line of thinking include Calvo (1987), Flood and Garber (1984), and Obstfeld (1984, 1986).

[2] Figure 1, Panel A depicts a short-term market rate (SELIC). This market rate essentially moved within a band defined by two interest rates set by the central bank (TBC and TBAN).

moved forcefully to double already high interest rates to defend the peg. Interest rates were also raised sharply in the aftermath of the Russian crisis. In practice, therefore, rather than sitting still and let events unfold, central banks are actively engaged in defending the peg by raising key short-term interest rates.

Naturally, even when it succeeds in defending a peg – at least temporarily – a high interest rate policy is not without costs. The main costs – as reflected in countless policy discussions – fall into three areas. First, high interest rates increase the debt service of public debt, thus raising the fiscal deficit. This issue was particularly evident in the case of Brazil. As Brazil pushed interest rates to close to 50 percent in the fall of 1997 to defend the peg (instituted in July 1994 in the context of the Real stabilization plan), the operational deficit jumped from less than 3 percent of GDP in September 1997 to 8.4 percent in November 1998, mainly due to the rapid build-up of interest payments on public debt (Figure 1, Panel B). Second, higher interest rates are likely to lead to an economic contraction, which could worsen even further the fiscal situation by reducing tax revenues. Third, higher interest rates may aggravate the financial situation of an already weak or poorly regulated banking sector.[3]

Policy disagreements over the virtues of raising interest rates to defend a peg ultimately come down to an assessment of the benefits (currency stability) versus the costs (typically some combination of the three factors just mentioned). On the one hand, higher interest rates to defend and/or strengthen the currency have been a typical component of IMF programs. According to the IMF, it is essential to fend off attacks on the currency with a "timely and forceful tightening of interest rates".[4] On the other

[3] During 1994, Mexican policymakers were reluctant to raise interest rates to defend the peg because of the perceived fragility of the banking system (see Sachs, Tornell, and Velasco (1996)).

[4] On IMF policies, it is worth quoting from Stan Fischer's (IMF's First Deputy Managing Director) remarks on recent IMF-supported programs in Asia (see Fischer (1998)): "Are the programs too tough? In weighing this question, it is important to recall that when they approached the IMF, the reserves of Thailand and Korea were perilously low, and the Indonesian rupiah was excessively depreciated. Thus, the first order of business was, and still is, to restore confidence in the currency. To achieve this, countries have to make it more attractive to hold domestic currency, which, in turn, requires increasing interest rates temporarily, even if higher interest costs complicate the situation of weak banks and corporations. This is a key lesson of the tequila crisis in Latin America 1994-95, as well as from the more recent experience of Brazil, the Czech Republic, Hong Kong and Russia, all of which have fended off attacks on their currencies in recent months with a timely and forceful tightening of interest rates along with other supporting policy measures. Once confidence is restored, interest rates can return to more normal levels."

hand, critics like Jeff Sachs contend that the costs of raising interest rates are typically too high. On Brazil, Sachs has argued (*Financial Times*, January 22, 1999) that "at that point [when the Asian crises hit], an urgent re-assessment of monetary exchange rate policy was due. And yet the IMF defended the Brazilian decision in October 1997 to put up interest rates to 50 percent per year precisely in order to hold the currency. This decision was fateful. It cemented the end of Brazilian economic growth, and built in a fiscal time bomb. When the misguided defence of the currency began, the deficit was about 4 percent of GDP. A fiscal adjustment, supposedly of 2 percent of GDP was announced, and praised by the IMF. But instead of reducing the deficit to 2 percent of GDP, the 1998 budget deficit in fact jumped to 8 percent of GDP, in large part the result of the self-induced economic slowdown (which reduced tax collection) and the rapid build-up of interest payments on public debt."[5]

For all its practical importance, the effectiveness and desirability of using high interest rates to defend an exchange rate peg has received scant attention in the theoretical literature. This surprising neglect may be partly explained by the difficulties that the profession has faced when it comes to thinking about interest rate policy (i.e., about central banks using interest rates as policy instruments). As is well-known from Sargent and Wallace (1975), controlling – or targeting – nominal interest rates in standard monetary models leads to price level indeterminacy. Far from "solving" the problem, sticky prices only lead to a higher-order indeterminacy (Calvo (1983)). While the literature has come up with different ways of dealing with this problem, it is fair to say that the profession lacks a widely-accepted framework to think about interest rates as policy instruments.[6] Still, as we await a wider consensus on this issue, it would seem important to have a simple analytical framework to think about the use of higher interest rates to fight speculative attacks against the domestic currency. We could then pose the following questions: given a peg which is not consistent with fundamentals, can higher interest rates delay a balance of payment crisis? If so, by how long? Is it optimal to delay a crisis?

The main purpose of this paper is to develop such a framework and provide some answers to the questions just raised. To this end, we incorporate interest rate policy in an otherwise standard, optimizing, small open economy prone to Krugman-type crises. Following Calvo and Végh (1995,

[5] For a detailed critique of IMF policies in Asia, see Radelet and Sachs (1998).

[6] See, for instance, Auernheimer and Contreras (1993), Calvo and Végh (1995, 1996), McCallum (1981), Reinhart (1992), and Woodford (1995)).

1996), we sidestep all indeterminacy problems by thinking of interest rate policy as the central bank's ability to set the interest rate on an interest-bearing liability (say, a liquid bond). If this liquid bond is an imperfect substitute for cash in the public's liquid portfolio, then the central bank can use this interest rate as an *additional* policy instrument. In other words, the central bank can – in addition to setting the exchange rate – set this interest rate and thus affect the total demand for liquid assets. Raising the interest rate on this liquid bond will, other things being equal, reduce the opportunity cost of holding it and thus increase money demand (defined as the demand for cash and the liquid bond). In this framework, therefore, interest rate policy amounts to paying interest on part of the money supply.[7] This seems like a natural way to think about interest rate policy in the context of currency crises since, in practice, the rationale for raising interest rates is precisely to increase the attractiveness of domestic currency denominated assets.

In our framework, higher interest rates work through two main channels: a money demand effect and a fiscal effect. The money demand effect refers to the fact that higher interest rates on domestic liquid bonds lead, other things being equal, to a higher demand for liquid bonds and, hence, for domestic assets. This mechanism enables the monetary authority to postpone the crisis. There is, however, no such a thing as a free lunch. Higher interest rates on liquid public debt imply a higher debt service burden, which will need to be financed by higher inflation in the future (the fiscal effect). The expectation of higher inflation in the future tends to bring forward the crisis by reducing the amount of liquid domestic assets that the public wishes to hold once the crisis occurs. The fiscal consequences of higher interest rates will thus tend to offset the beneficial effect of higher interest rates in delaying the crisis.[8]

The first question that we ask concerns the *feasibility* of defending a peg: can higher interest rates postpone a crisis? And, if so, for how long? We

[7] Of course, in practice, this link may be more indirect. For instance, if commercial banks hold a large fraction of their portfolio as interest-bearing liabilities of the government – as is often the case in emerging markets – then the interest rate paid by banks on their liabilities will be heavily affected by the interest rate they receive on government bonds.

[8] In this paper, we abstract from the other two potential costs of higher interest rates on output and the banking system mentioned above. Obviously, this is not to say that we believe that these other two effects are unimportant. As a methodological question, however, we believe that as a first pass it is necessary to tackle one cost at a time to understand the fundamental issues involved. In a companion paper (Lahiri and Végh (1999)), we abstract from the fiscal effect and focus on the output costs of higher interest rate policy in a model in which firms need to resort to bank credit to operate.

show that, under the most plausible range for money demand elasticities, raising interest rates will indeed postpone the crisis. As interest rates become higher, however, the fiscal effect mentioned above will at some point begin to dominate and further raising interest rates beyond a certain point will actually hasten the crisis. There is thus a *non-monotonic* relationship between the increase in interest rates and the time of the crisis. In other words, there is a certain increase in interest rates that will maximize the delay of the crisis. Hence, our analysis shows that the central bank can in principle buy precious time by raising interest rates which, in practice, would give the fiscal authority the chance to put its house in order. Raising interest rates too much, however, would be self-defeating.

Naturally, the ability to delay a costly crisis by raising interest rates does not necessarily mean that it is optimal to do so. Our second question is thus: is it optimal (i.e., welfare maximizing) to raise interest rates to defend a peg and delay the crisis? The answer is "yes, but only up to a point." We first show that it is optimal for the monetary authority to engage in *some* active interest rate defense, as opposed to sitting still and doing nothing (as implied by the Krugman case). In other words, starting from an equilibrium in which the central bank is totally passive, raising interest rates is welfare improving. However, the relationship between the increase in interest rates and welfare may be non-monotonic. Hence, raising interest rates beyond a certain point will begin to reduce welfare. In fact, we show that there is a whole range of interest rate increases for which it is *feasible* to further delay the crisis but *not optimal* to do so. From a policy point of view, our results thus suggest that it may be optimal for central banks to engage in *some* interest rate defense. However, raising interest rates too much may be counterproductive due to the resulting future inflationary effects.

Our work is part of an incipient theoretical and empirical literature on the effectiveness of higher interest rates in defending a peg (or, under floating rates, strengthening the currency). On the theoretical side, Drazen (1999a,b) addresses this same issue – interest rate defense of a peg – in a "second-generation", Barro-Gordon type model. His main focus is on the signalling effect of higher interest rates. His analysis thus complements ours (a "first-generation" model), since we totally abstract from such issues. In Drazen's framework, the central bank's ability to fend off speculative attacks by raising interest rates is complex because, depending on the information that speculators have, higher interest rates may signal either that a government is less able or more able to defend an exchange rate peg. Hence, Drazen's work reinforces the message that follows from our work (but, of course, for completely different reasons) of a non-linear relationship between higher interest rates and a successful defense of a peg. More recently, Flood and Jeanne (2000) show how to incorporate active interest rate defense in the traditional, first-generation, reduced-form Krugman-Flood-Garber model of BOP crises. In their framework, raising interest rates (on a non-liquid bond) before the crisis is never effective and, in fact, always brings the crisis forward as it worsens the fiscal situation. The key difference with our analysis is that, in their model, the bonds whose interest rate is increased provide no liquidity. Hence, higher interest rates do not lead to a higher real money demand. In addition, and due to the reduced-form nature of their model, Flood and Jeanne (2000) cannot address the issue of whether it is optimal or not to mount an active interest rate defense of a peg.[9]

On the empirical side, analyses of the effects of higher interest rates include Dekle, Hsiao, and Wang (1999), Goldfajn and Gupta (1999), Gould and Kamin (2000), Kraay (1999), and Zettelmeyer (2000). In particular, and based on a large sample of speculative attacks, Kraay (1999) concludes that there is no systematic association between interest rates and the outcome of speculative attacks. These empirical results are, in principle, consistent with our theoretical "non-linear" results which suggest that, in a pooled sample, detecting any systematic effect of higher interest rates in delaying the crisis may be problematic since some countries may have raised interest rates "too much". Dekle, Hsiao, and Wang (1999), on the other hand, find that high interest rates had the desired effect of appreciating the exchange rate in Korea, Malaysia, and Thailand during the recent crises. In the same vein, Zettelmeyer (2000) concludes that the experience of Australia, Canada, and New Zealand in the 1990s is, by and large, consistent with the idea that higher interest rates tend to appreciate the domestic currency. Finally, Gould and Kamin (2000) fail to find any significant effect of monetary policy on exchange rates in Mexico and five Asian countries but stress the fact that, in light of the endogeneity of interest rates with respect to exchange rates and expectations, it should prove extremely difficult to identify the impact of monetary policy on exchange rates even when it is indeed present.

The paper proceeds as follows. Section 2 presents the model. For conceptual clarity, Section 3 abstracts from the fiscal effect and focuses exclusively on the money demand effect. Section 4 derives the key results

[9]Related work includes Flood, Garber, and Kramer (1996) and Kumhof (1998) who focus on the central bank's ability to sterilize the monetary effects of a currency attack; Burnside, Eichenbaum, and Rebelo (1998) who study the dynamics of crises generated by future fiscal deficits; and Velasco (1993) who examines the fiscal effects of high real interest rates during stabilization.

of the paper by combining the fiscal and the money demand effects. Section 5 concludes.

2 The model

Consider a small open economy that is perfectly integrated with the rest of the world in both goods and capital markets. The economy is inhabited by an infinitely-lived representative consumer who receives a constant flow endowment y of a perishable good. Transactions are facilitated by resorting to two liquid assets: non-interest bearing money and a (non-traded) liquid bond. The world price of the good in terms of the foreign currency is given and assumed to be unity. Free mobility of goods across borders implies that the law of one price holds. The consumer can also buy and sell a pure (i.e., non-liquid) bond in perfectly competitive world capital markets.

2.1 Consumer

The representative consumer's lifetime utility (W) is given by

$$W \equiv \int_0^\infty u(c_t)e^{-\beta t}dt, \tag{1}$$

where c denotes consumption and $\beta(>0)$ is the rate of time preference. Real financial wealth at time t is given by

$$a_t = b_t + h_t + z_t,$$

where b are real holdings of an internationally-traded bond which bears an interest rate of r, h denotes holdings of non-interest bearing money (hereafter referred to as "cash"), and z are holdings of a liquid bond (i.e., an interest-bearing money).[10] The nominal interest rate borne by the "pure" (i.e., non-liquid bond) is i, while that borne by the liquid bond is i^g.

The consumer's flow budget constraint is thus given by

$$\dot{a}_t = ra_t + y + \tau_t - c_t - i_t h_t - (i_t - i_t^g)z_t - \phi(h_t, z_t), \tag{2}$$

[10] There is by now abundant evidence on the importance of liquid financial assets other than cash (such as short-term government debt, indexed bonds, and foreign currency deposits) in satisfying part of households' liquidity needs in developing countries. See, for example, Arrau, De Gregorio, Reinhart, and Wickham (1995), Easterly, Mauro, and Schmidt-Hebbel (1995) and Savastano (1996). Alternatively, as explained below, liquid bonds could be interpreted as interest-bearing demand deposits (say, money market accounts) held in banks.

where τ_t denotes lump-sum transfers from the government and $\phi(h_t, z_t)$ is a strictly decreasing and strictly convex transactions technology. To simplify the presentation, we will assume that $\phi(h_t, z_t)$ is separable in the two monies. Formally, we assume that:

$$\phi(h_t, z_t) = K - v(h_t) - w(z_t) \geq 0, \tag{3}$$

where $v(h)$ and $w(z)$ are strictly increasing and strictly concave functions and K is an arbitrary constant whose only role is to ensure that transactions costs are non-negative.[11] [12] Hence:

$$\begin{aligned}
\phi_h(h_t, z_t) &= -v'(h_t) < 0, \\
\phi_z(h_t, z_t) &= -w'(z_t) < 0, \\
\phi_{hh}(h_t, z_t) &= -v''(h_t) > 0, \\
\phi_{zz}(h_t, z_t) &= -w''(h_t) > 0.
\end{aligned}$$

Integrating (2) and imposing the standard transversality condition yields the consumer's lifetime budget constraint:

$$a_0 + \frac{y}{r} + \int_0^\infty \tau_t e^{-rt} dt = \int_0^\infty \left[c_t + i_t h_t + (i_t - i_t^g) z_t + \phi(h_t, z_t)\right] e^{-rt} dt. \tag{4}$$

The consumer chooses perfect foresight paths for $\{c_t, h_t, z_t\}$ to maximize lifetime utility (1) subject to (3) and (4), taking as given τ, i^g, i, r, y, and $a_{0.}$ Assuming interior solutions, the first-order conditions are given by [13]

$$\begin{aligned}
u'(c_t) &= \lambda, && (5) \\
v'(h_t) &= i_t, && (6) \\
w'(z_t) &= i_t - i_t^g, && (7)
\end{aligned}$$

[11] The transactions technology (3) is standard in the literature (see, for instance, McCallum and Goodfriend (1987) and Lucas (1993)), except for the fact that it does not depend on consumpion. As we will see, this implies that the time path of consumption is not subject to intertemporal distortions. Since we do not need such intertemporal distortions to make our points, we choose the simplest possible specification. Below we will show simulations that suggest that introducing this additional effect does not alter our main results.

[12] As a check of the theoretical robustness of our results, Appendices E and F analyze the cases of money in the utility function (MIUF) and cash in advance, respectively. The MIUF case turns out to deliver identical results. In the cash in advance case, the fixed velocity actually makes it easier for policymakers to delay the crisis, but leaves unchanged our welfare results.

[13] As usual, we assume that $\beta = r$ to eliminate inessential dynamics.

where λ is the (time-invariant) Lagrange multiplier associated with constraint (4).[14] Equations (6) and (7) implicitly define the demand for cash and liquid bonds, given by:

$$h_t = \tilde{h}(\underset{-}{i_t}), \quad (8)$$

$$z_t = \tilde{z}(i_t \underset{-}{-} i_t^g), \quad (9)$$

where a sign under a variable denotes its partial derivative. Equation (8) is the demand for non-interest bearing money, which depends negatively on the opportunity cost of holding real cash balances, given by i. Equation (9) captures the demand for liquid bonds, whose opportunity cost is given by the interest rate differential $i - i^g$.

In this model, we will think of "money" as the sum of cash and liquid bonds. This is, of course, the natural definition of money in this model because, by construction, both cash and liquid bonds provide liquidity services. Formally, $m \equiv h + z$. Hence, using (8) and (9), the demand for money can be written as

$$m_t = \tilde{h}(i_t) + \tilde{z}(i_t - i_t^g) \equiv \tilde{m}(\underset{-}{i_t}, \underset{+}{i_t^g}). \quad (10)$$

Notice, from (10), that money demand is a decreasing function of i but an *increasing* function of i^g. Hence, an increase in i^g will, other things being equal, increase the demand for liquid assets denominated in domestic currency. As will become clear below, the ability to affect the demand for liquid assets will enable policymakers to mount an active defense of an exchange rate peg.

2.2 Government constraints

The government comprises the monetary and the fiscal authority. For formal simplicity, it will be assumed that the monetary authority issues both cash and liquid bonds.[15] The monetary authority also pays interest on these

[14] It will be assumed throughout the paper that $i - i^g \geq 0$. This ensures an interior solution in which all three assets – pure bonds, liquid bonds, and cash – are held. Of course, for $i - i^g = 0$ to be a feasible equilibrium, the transactions technology must be such that the consumer can be satiated with liquid bonds.

[15] An alternative (and formally identical) set-up is one in which liquid bonds are interpreted as interest-bearing demand deposits and the monetary authority issues only non-interest bearing money but requires banks to hold 100 percent reserves. By paying

liquid bonds and holds interest-bearing foreign exchange reserves. The fiscal authority makes lump-sum transfers to the public. The government's budget constraint is thus given by

$$\dot{R}_t = rR_t + \dot{h}_t + \dot{z}_t + \varepsilon_t h_t + (\varepsilon_t - i_t^g)z_t - \tau_t, \tag{11}$$

where R is the government's (monetary authority's) stock of net foreign assets (i.e., international reserves). Integrating forward (11) and imposing the transversality condition $\lim_{t\to\infty} R_t e^{-rt} = 0$ yields the government's intertemporal budget constraint:

$$\int_0^\infty \tau_t e^{-rt} dt = R_0 + \int_0^\infty [\dot{h}_t + \dot{z}_t + \varepsilon_t h_t + (\varepsilon_t - i_t^g)z_t]e^{-rt}dt + \triangle(h_T + z_T)e^{-rT}, \tag{12}$$

where the last term on the right-hand side (RHS) allows for the possibility of a discrete change in real money balances at some time $t = T$.[16] Equation (12) makes clear that the government must finance the present discounted value of transfers (left-hand side (LHS)) with the initial stock of international reserves and the present discounted value of proceeds from money creation (RHS). The inflation tax is given by $\varepsilon_t h_t$ in the case of cash and $(\varepsilon_t - i_t^g)z_t$ in the case of liquid bonds. For further reference, note that constraint (12) can be simplified to read (imposing the additional transversality condition $\lim_{t\to\infty} m_t e^{-rt} = 0$):

$$\int_0^\infty \tau_t e^{-rt} dt = R_0 - (h_0 + z_0) + \int_0^\infty [i_t h_t + (i_t - i_t^g)z_t]e^{-rt}dt. \tag{13}$$

Denoting by μ_t the rate of growth of domestic credit, it follows that:

$$\frac{\dot{D}_t}{D_t} = \mu_t, \tag{14}$$

where D denotes nominal domestic credit. Let E denote the nominal exchange rate; that is, the price of foreign currency in terms of domestic currency. From the central bank's balance sheet, $\dot{R}_t = \dot{m}_t - \dot{d}_t$, where $d = D/E$.

interest on these reserves, the central bank would indirectly control the interest rate paid by the banks on demand deposits.

[16] Throughout the paper, we denote a discrete change in, say, variable x as $\Delta x_T \equiv x_T - x_{T^-}$. Of course, if real money balances were to jump at other times as well, this should also be accounted for. Equation (13) below, however, would not change. We also assume that $R_0 > 0$.

Further, note that $\dot{d}_t = (\mu_t - \varepsilon_t)d_t$. Using these two facts, equation (11) yields the path of government transfers:

$$\tau_t = rR_t + (\mu_t - \varepsilon_t)d_t + \varepsilon_t h_t + (\varepsilon_t - i_t^g)z_t. \tag{15}$$

So far, we have only looked at accounting identities. At this point, we would need to take a stand on whether (i) the monetary authority moves first – by setting an exogenous path of μ – and the fiscal authority passively accommodates such a path by letting transfers adjust, or (ii) the fiscal authority moves first – by setting an exogenous path of transfers – and the monetary authority accommodates this policy by letting the rate of domestic credit growth adjust. Clearly, the latter assumption is the most relevant for policy purposes. For conceptual purposes, however, we will begin our analysis in Section 3 by examining the first scenario. Having this simpler scenario as a convenient benchmark, Section 4 will look at the empirically more relevant case in which the fiscal authority sets an exogenous path of transfers.

2.3 Equilibrium conditions

Combining the consumer's and the government's flow constraints (equations (2) and (11), respectively) yields:

$$\dot{k}_t = rk_t + y - c_t - \phi(h_t, z_t), \tag{16}$$

where $k(\equiv b + R)$ denotes the economy's stock of net foreign assets. The RHS of equation (16) is the current account balance. Combining (4) and (13) yields the economy's resource constraint:

$$k_0 + \frac{y}{r} = \int_0^\infty [c_t + \phi(h_t, z_t)]e^{-rt}dt. \tag{17}$$

Equations (5) and (17) imply that, along a perfect foresight equilibrium path, consumption will be time-invariant (and denoted by $\bar{c}$) and equal to *net* permanent income (i.e., permanent income net of "permanent" transactions costs):

$$\bar{c} = rk_0 + y - r\int_0^\infty \phi(h_t, z_t)e^{-rt}dt. \tag{18}$$

Finally, given the assumption of perfect capital mobility, interest parity holds (recall that foreign inflation is assumed to be zero):

$$i_t = r + \varepsilon_t. \tag{19}$$

2.4 Exchange rate and interest rate policy

Exchange rate policy The exchange rate policy followed by the monetary authority is the same as in standard models à la Krugman. As of $t = 0$, the exchange rate is fixed at the level $\bar{E}$. Hence, by (19), $i_t = r$. In addition, it is assumed that there is a known lower bound for international reserves (say, $R_t = 0$). If that level is reached, the central bank ceases to intervene in the foreign exchange market and allows the exchange rate to float freely.

Interest rate policy The key departure of our model relative to the standard Krugman model is that the monetary authority can engage in *active interest rate policy*. In other words, in addition to fixing the exchange rate, the monetary authority can also set the interest rate on liquid bonds, i^g. The monetary authority has thus two policy instruments: the nominal exchange rate and the interest rate on liquid bonds. By setting i^g, the monetary authority lets the *composition* of its liabilities (cash and liquid bonds) be market-determined.[17] [18]

To see more clearly the logic behind active interest rate policy, note that the central bank issues both cash (H) and liquid bonds (Z). Hence, money market equilibrium is given by (taking into account (10))

$$\frac{H_t + Z_t}{E_t} = \tilde{m}(i_t, i_t^g). \tag{20}$$

The second relevant equilibrium condition is the relative demand for cash and liquid bonds, which follows from (6) and (7):

$$\frac{v'(h_t)}{w'(z_t)} = \frac{i_t}{i_t - i_t^g}. \tag{21}$$

For a given level of the exchange rate and i^g set by the monetary authority, the private sector chooses real money demand which, through (20), deter-

[17] Alternatively, the monetary authority could set the level of both liabilities and let i^g be market-determined.

[18] Notice that, in a first-best world (i.e., a world not subject to the Krugman distortion of inconsistent policies), the optimal monetary policy would be to either (i) set $i = i^g = 0$ (i.e., the Friedman rule) and satiate the consumer with both cash and liquid bonds in the case that τ is endogenous or (ii) follow the Ramsey rule of optimal taxation in the case of an exogenous τ (which calls for setting the tax rate in inverse proportion to the price elasticity). In the latter case, if $w(z)$ and $v(h)$ were identical functions, then the optimal i^g is zero (so as to tax both monies at the same rate). If the demand for cash were always more inelastic, then the optimal i^g is positive (liquid bonds should be taxed less heavily than cash).

mines the nominal money supply $(H+Z)$. Given this level of the nominal money supply, equation (21) determines the levels of H and Z. Other things being equal, a rise in i^g increases real money demand by inducing a higher real demand for liquid bonds. In this model, therefore, an active interest rate defense of the peg will involve raising the policy-controlled interest rate in order to increase the demand for domestic currency denominated assets.

An alternative way of thinking about interest rate policy in this model – which is in the spirit of traditional portfolio models (see, for example, Flood, Garber and Kramer (1996)) – is to derive an interest parity condition between liquid bonds and pure bonds by combining (6) and (7) to obtain

$$i_t^g = i_t - \left[\frac{w'(z_t)}{v'(h_t)}\right] i_t. \tag{22}$$

This condition says that, in equilibrium, the return on the liquid bond issued by the central bank (i^g) must equal the return on the pure bond (i_t) minus a liquidity premium (last term on the RHS). For a given level of cash, the liquidity premium is a decreasing function of the level of liquid bonds.

Condition (22) is illuminating in several respects. First, it makes clear that the assumption that the bonds issued by the central bank provide liquidity services is a critical one. If they did not (i.e., if $w'(z_t)=0$), then i^g could not be different from i (assuming, of course, an interior solution). Second, given that these bonds provide liquidity services, interest rate policy operates by inducing changes in the liquidity premium. In other words, a higher i^g must be associated with a lower liquidity premium and viceversa.

Interest rate policy rule In practice, the monetary authority raises interest rates in response to shocks that may induce the public to switch from domestic to foreign assets. By making domestic assets more attractive, the monetary authority hopes to protect its stock of international reserves. In this spirit – and to capture an active interest rate defense in as simple a setting as possible – it will be assumed that the central bank announces at time $t=0$ that if the nominal interest rate i changes at any point in time t, then it will adjust i^g according to the following rule:

$$\triangle i_t^g = \gamma \triangle i_t, \qquad \gamma \in [0, \gamma^*], \tag{23}$$

where γ is a policy parameter (to be optimally chosen by policymakers) that captures the degree of interest rate activism.[19] In particular, $\gamma = 0$ is the

[19] We restrict γ to be non-negative to fix ideas. In principle, γ could be negative. When such a case is economically relevant, it will be discussed below. We will also assume that,

standard Krugman case analyzed in the literature in which the central bank follows a completely passive interest rate policy. At the other extreme, the monetary authority can set a value of γ, denoted by γ^*, such that money demand will *not* change even if i rises.[20] Intuitively, it should be clear that $\gamma^* > 1$. The reason is that in response to a rise in i, demand for cash will always fall. Hence, to leave real money demand unchanged, the opportunity cost of holding bonds needs to fall in order to induce consumers to hold more liquid bonds. This, in turn, requires that $\gamma > 1$, so that the opportunity cost of holding liquid bonds falls as i rises (i.e., $i_t - i_t^g$ falls). While highly stylized, policy rule (23) provides an easy and convenient way of parameterizing interest rate policy, which will enable us to study the effectiveness and optimality of raising interest rates to defend a fixed exchange rate peg.

3 Interest rate policy in the absence of an exogenous fiscal constraint

As a convenient benchmark, this section analyzes the effectiveness and optimality of active interest rate policy in the case in which the monetary authority sets a constant rate of growth of domestic credit, μ, and the fiscal authority passively accommodates such a policy by letting the level of transfers adjust endogenously.

3.1 Equilibrium paths

Consider first the equilibrium path associated with a fixed exchange rate. By interest parity (given by (19)), $i_t = r$. Since i is constant over time, policy rule (23) implies that i^g will also be constant over time. Hence, $i - i^g$ will also be constant over time. This, in turn, implies that real money demand, given by (10), will also be constant over time. From the central bank balance sheet, it follows that $\dot{R}_t = \dot{m}_t - \dot{d}_t$. The fixed exchange rate combined with the domestic credit policy given by (14) implies that $\dot{R}_t = \dot{m}_t - \mu d_t$. Since money demand is constant along paths with a constant i (i.e., $\dot{m}_t = 0$), the equilibrium evolution of international reserves is given by

$$\dot{R}_t = -\mu d_0 e^{\mu t}. \tag{24}$$

when optimally choosing γ, policymakers take as given the initial level of i^g, i_0^g. Unless otherwise noticed, the value of i_0^g is irrelevant for our analysis because it only affects the initial level of real money demand through its effect on the initial demand for liquid bonds.

[20] As will become clear below, while γ^* provides an upper bound for γ, in some cases it is not possible for the monetary authority to set such a value of γ.

Equation (24) shows that along paths with a fixed exchange rate and expanding domestic credit (i.e., $\mu > 0$), international reserves at the central bank will be falling at an increasing rate. Since the lower bound for international reserves will be reached in finite time, the fixed exchange rate regime is unsustainable. The central bank will thus be forced to abandon the peg at some point in time, T (to be determined endogenously). Private agents expect the central bank to allow the exchange rate to float from time T onwards, while leaving its domestic credit policy unchanged. Hence they expect that at time T the economy will jump to its long run steady-state with the domestic currency depreciating at the constant rate of monetary expansion, μ.[21] Formally, the expected path for the rate of devaluation/depreciation is given by

$$\varepsilon_t = \begin{cases} 0, & 0 \leq t < T, \\ \mu, & t \geq T. \end{cases}$$

The private sector also knows that from time T onwards the nominal interest rate will be given by $i_T = r + \mu$. Furthermore, given policy rule (23), it is also the case that $i_T^g = i_0^g + \gamma\mu$. Hence, $i_T - i_T^g = r - i_0^g + (1-\gamma)\mu$.

Given the time paths of i_t and i_t^g, we can now determine γ^*; that is, the value of γ for which real money demand at T will not jump. Formally, γ^* satisfies

$$\tilde{h}(r) - \tilde{h}(r+\mu) = \tilde{z}[r - i_0^g + (1-\gamma^*)\mu] - \tilde{z}(r - i_0^g). \tag{25}$$

For a given $\mu(> 0)$,the LHS of equation (25) is a positive constant, whereas the RHS is a strictly increasing function of γ (and negative for $\gamma = 0$). Hence, γ^* exists and is unique.[22]

[21] This is a perfect foresight monetary model with no intrinsic dynamics. It can be easily shown that the perfect foresight equilibrium path under flexible exchange rates is stationary with the rate of currency depreciation, ε, being equal to the rate of monetary expansion, μ.

[22] This statement assumes that the condition $i_T - i_T^g = r - i_0^g + (1-\gamma^*)\mu \geq 0$ is satisfied. In other words, it assumes that $\gamma^* \leq \hat{\gamma}$, where $\hat{\gamma}$ is the value of γ that satisfies $r - i_0^g + (1-\hat{\gamma})\mu = 0$ (i.e, $\hat{\gamma} = 1 + \frac{r - i_0^g}{\mu}$). In general, whether $\gamma^* \leq \hat{\gamma}$ holds depends – in addition to the specification of preferences – on the parameter configuration. If this condition does not hold, all our main results still go through, but there is a limit to how much the central bank can raise interest rates. For expositional purposes, this section will assume that this condition holds unless otherwise noted.

3.2 The timing of the crisis

In order to tie down the time of collapse, note that at T the exchange rate (E) cannot jump. If it did, there would be infinite arbitrage opportunities. Since the exchange rate cannot jump at T, money market equilibrium at T is given by

$$\tilde{h}(r+\mu)+\tilde{z}[r-i_0^g+(1-\gamma)\mu]=\frac{D_0e^{\mu T}}{\bar{E}}, \tag{26}$$

where the LHS of equation (26) denotes real money demand at time T, whereas the RHS indicates real money supply at time T. Equation (26) implicitly defines the time of collapse T as a function of γ. In what follows, we denote all pre-collapse variables by the subscript 0 and post-collapse variables by the subscript T. Letting T^- denote the instant before the run, the discrete change in real money demand at the moment of the crisis T is given by

$$\Delta m_T \equiv m_T - m_{T^-} = \tilde{m}(r+\mu, i_T^g) - \tilde{m}(r, i_0^g) \leq 0, \tag{27}$$

which corresponds to the loss in international reserves since $\Delta R_T = \Delta m_T$.[23] The weak inequality in (27) follows from the fact that, as discussed above, real money demand will remain unchanged for γ^*.

3.3 Effectiveness of active interest rate policy

The first question to ask regarding active interest rate policy is whether a tighter interest rate policy succeeds in postponing the time of the crisis. In other words, is there a monotonic and positive relationship between the time of the crisis (T) and the degree of policy activism (γ)?

The answer to this question follows directly from equation (26). This equation defines T as a strictly increasing function of γ; that is, $T=\tilde{T}(\gamma)$, with the slope being given by

$$\tilde{T}'(\gamma)=\frac{-\tilde{z}'[r-i_0^g+(1-\gamma)\mu]}{d_0e^{\mu T}}>0. \tag{28}$$

Furthermore, it should be clear that $\gamma=0$ corresponds to the Krugman case, where interest rate policy is totally passive. In that case, the crisis occurs at time T^k in Figure 2. In other words, $\tilde{T}(0)=T^k$. At the other extreme, if $\gamma=\gamma^*$, then there is no run when the crisis occurs. This follows

[23] Note that from time T onward, real domestic credit remains constant since D and E both rise at the common rate μ.

immediately from the fact that, by construction, if $\gamma = \gamma^*$ there is no change in real money demand when the crisis finally occurs. Denoting by T^* the no-run value of T, this implies that $\tilde{T}(\gamma^*) = T^*$. Put differently, the mere announcement of policy (23) with $\gamma = \gamma^*$ at time 0 enables the central bank to delay the crisis by as long as it is possible under the circumstances (from T^k to T^* in Figure 2). In other words, even though there is a crisis and the exchange rate peg is abandoned at time T^*, there is no run on international reserves.

We collect these results in the following proposition:

Proposition 1 *In the absence of an exogenous fiscal constraint, the timing of the crisis is a strictly increasing function of the degree of interest rate policy activism (i.e., $\tilde{T}'(\gamma) > 0$) . Furthermore, $\tilde{T}(0) = T^k$ and $\tilde{T}(\gamma^*) = T^*$.*

The intuition behind the central bank's ability to delay the crisis is straightforward. The more aggressive is interest rate policy (i.e., the higher is γ), the smaller will be the desired change in real money demand when the crisis occurs, because the change in the opportunity cost of holding liquid bonds will be smaller. Since the desired change in real money demand determines the size of the speculative attack – and hence the time of the crisis – a higher γ implies that the size of the speculative attack will be smaller and hence that it will occur later.[24]

3.4 Optimality of active interest rate policy

Having established that a more active interest rate policy always succeeds in postponing the crisis, the natural follow-up question is whether it is optimal to do so. Formally, we ask the following question: is there a monotonic and positive relationship between the consumer's welfare and the degree of policy activism (as captured by γ)?

To answer this question, recall from (18) that, along a perfect foresight equilibrium path, $c_t = \bar{c}$. Equation (1) thus reduces to:

$$W \equiv \frac{u(\bar{c})}{r}.$$

[24] As suggested above, γ could be greater than γ^* or negative. If $\gamma < 0$, then the crisis would be brought forward relative to the Krugman case. In terms of Figure 2, it would occur for $T < T^k$. The intuition is simply that the central bank's interest rate policy now *reduces* money demand at the time of the crisis. If $\gamma > \gamma^*$, the crisis would still occur at time T^*, but the central bank would need to allow for a once-and-for-all remonetization of the economy as real money demand would now increase once the crisis occurs.

Since $u(\bar{c})$ is a strictly increasing function of $\bar{c}$, welfare depends only on how $\bar{c}$ varies with γ. In fact, $\bar{c}$ will depend on γ directly as well as indirectly through $\tilde{T}(\gamma)$. Formally, from (18) (and taking into account (3)), it follows that

$$\bar{c}[\gamma, \tilde{T}(\gamma)] = rk_0 + y - K + (1 - e^{-r\tilde{T}(\gamma)})[v(h_0) + w(z_0)] + e^{-r\tilde{T}(\gamma)}[v(h_T) + w(z_T)], \tag{29}$$

where

$$h_0 = \tilde{h}(r), \tag{30}$$
$$z_0 = \tilde{z}(r - i_0^g), \tag{31}$$
$$h_T = \tilde{h}(r + \mu), \tag{32}$$
$$z_T = \tilde{z}[r - i_0^g + (1 - \gamma)\mu]. \tag{33}$$

From (29), it follows that

$$\frac{d\bar{c}}{d\gamma} = \frac{\partial \bar{c}}{\partial \gamma} + \frac{\partial \bar{c}}{\partial \tilde{T}} \tilde{T}'(\gamma). \tag{34}$$

As (34) makes clear, γ affects welfare through two channels. First, for a given T, a higher γ raises the post-crisis demand for liquid bonds, thus reducing transactions costs and increasing consumption and welfare (first term on the RHS of (34)). In effect, as follows directly from (29), $\frac{\partial \bar{c}}{\partial \gamma} \geq 0$ (with the equality holding only for $\gamma = \hat{\gamma}$). Second, for a given post-crisis demand for liquid bonds, a higher γ postpones the crisis (i.e., $\tilde{T}'(\gamma) > 0$, as established in Proposition 1). In turn, a larger T should increase welfare because consumers hold a higher level of real money balances for a longer period of time, which reduces transaction costs and increases consumption. This is obvious for the case in which $\gamma \leq 1$, because in that case demand for cash falls at T while demand for bonds either falls or remains constant. It is less obvious for values of $\gamma > 1$, in which case the demand for cash falls at T but the demand for liquid bonds increases. We show in Appendix A, however, that as long as the real demand for money falls (or remains constant) at T, consumption (and hence utility) always increases on this account (i.e., we formally show that $\frac{\partial \bar{c}}{\partial \tilde{T}} > 0$). Given that welfare is a strictly increasing function of γ, the optimal interest rate policy is to set $\gamma = \gamma^*$. In other words, welfare is maximized by delaying the balance of payment crisis as much as possible. Notice, incidentally, that the worst possible policy is not to increase interest rates at all (i.e., the Krugman case).[25]

In this light, we can summarize the welfare results in the following proposition:[26]

Proposition 2 *In the absence of an exogenous fiscal constraint, consumer's welfare is a strictly increasing function of the degree of interest rate activism at the time of the crisis. The optimal interest rate policy is thus to prevent a run from taking place when the peg is abandoned.*

4 Exogenous fiscal constraint

As a convenient benchmark, Section 3 assumed that the fiscal authority endogenously adjusts government spending (transfers) in response to changes in available revenues. This implies that the fiscal authority is willing to effect a sharp contraction in spending at the time of the crisis to accommodate the higher debt service. In practice, of course, such an adjustment is difficult to implement and inflation must typically rise to provide additional revenues to the treasury. To capture such a scenario, this section analyzes the case in which the monetary authority must finance an exogenously-given flow of transfers. In this context, the inflationary consequences of higher interest rates will play a critical role in determining the effectiveness and optimality of an active interest rate policy in defense of the peg.

4.1 Preliminaries

The government now faces an exogenous spending constraint given by

$$\tau_t = \bar{\tau}, \qquad t \in [0, \infty), \tag{35}$$

which says that real government transfers remain fixed at $\bar{\tau}$ at all points in time. Under this assumption, the monetary authority endogenously adjusts the rate of domestic credit expansion (μ) to balance the budget at all times.

[25] If $\hat{\gamma} < \gamma^*$, then it is optimal to set $\gamma = \hat{\gamma}$; that is, to raise interest rate as much as possible.

[26] Simulations of the model (available from the authors upon request) suggest that, as one would expect, both Propositions 1 and 2 hold for cases in which the transactions costs technology depends on consumption, in which case consumption is no longer flat along a perfect foresight equilibrium path (i.e., consumption falls at T).

Hence, unlike in the previous section, μ is now an endogenous variable which must satisfy the fiscal constraint (substituting (35) into (15)):

$$\bar{\tau} = rR_t + (\mu_t - \varepsilon_t)d_t + \varepsilon_t h_t + (\varepsilon_t - i_t^g)z_t. \tag{36}$$

The two crucial differences between this specification and the previous one (with an endogenous τ) are that (i) μ_T is now an endogenous variable since it is no longer specified by a domestic credit rule; and (ii) the money market equilibrium condition at time T must also take into account the fiscal constraint at time T, i.e., $\bar{\tau} = \mu_T d_T - i_T^g z_T$ (which follows from (36)). Our interest, as before, is in the impact of interest rate policy on the dynamics of a crisis.

As in the previous case, since the exchange rate floats from the date of the crisis, T, and the economy is completely stationary, the rate of depreciation of the domestic currency from time T is given by μ_T, the stationary rate of domestic credit creation from time T onwards. Thus, the nominal interest rate is given by $i_t = r$ for $0 \leq t < T$ and $i_t = r + \mu_T$ for $t \geq T$. Given policy rule (23), the implied path for the policy-controlled interest rate on liquid bonds is $i_t^g = i_0^g$ for $0 \leq t < T$ and $i_t^g = i_0^g + \gamma\mu_T$ for $t \geq T$.[27] Hence, the lifetime budget constraint for the government given by equation (13) reduces to

$$\bar{\tau} = rR_0 + e^{-rT}\left[\mu_T h_T + \mu_T(1-\gamma)z_T - r(z_0 - z_T + h_0 - h_T)\right]. \tag{37}$$

This equation says that in order to meet the government's intertemporal budget constraint, the constant level of government spending has to be equal to interest earnings on initial reserves plus the present discounted value of inflation tax revenues (which are zero until time T and positive afterwards) minus the present discounted value of interest earnings on the fall in the stock of reserves at time T due to the reduction in cash and liquid bond holdings.

At $t = 0$, equation (36) implies (noting that $\varepsilon_t = i_0^g = 0$) that

$$\bar{\tau} - rR_0 = \mu_0 d_0. \tag{38}$$

We will assume, of course, that $\bar{\tau} - rR_0 > 0$, which implies that $\mu_0 > 0$. In other words, for this economy to suffer a balance of payments crisis we need reserves to be falling from time 0 onwards which occurs only if there is

[27] For simplicity, we will assume in what follows that $i_0^g = 0$. Unless otherwise noticed, this assumption is irrelevant for all our results.

an initial deficit.[28] Thus, the government must be running a budget deficit from time 0 onwards, which will require an expansion of domestic credit.

Since reserves go to zero at the time of the speculative attack, the money market equilibrium condition at time T dictates that real domestic credit must equal real money demand at that time. Hence, $d_T = h_T + z_T$ where h_T and z_T are given by equations (32) and (33), respectively. Recall also that, at time T, $\bar{\tau} = \mu_T d_T - i_T^g z_T$. Substituting the money market equilibrium into this equation, using (32) and (33), and noting that $\mu_T - i_T^g = \mu_T(1-\gamma)$ (given that $i_0^g = 0$) yields

$$\bar{\tau} = \mu_T \tilde{h}(r + \mu_T) + \hat{\mu}_T \tilde{z}(r + \hat{\mu}_T), \tag{39}$$

where $\hat{\mu}_T \equiv \mu_T(1-\gamma)$. The RHS of equation (39) denotes net inflation tax revenues; that is, revenues from the inflation tax *net* of interest payments on liquid bonds.

Finally, as in standard BOP crisis models, we also wish to rule out the possibility that the economy is operating on the "wrong side" of the relevant Laffer curves. In this set-up with two monies, we need to impose two such restrictions. First, we will assume that the economy is operating on the "correct" side of the Laffer curve for net inflation tax revenues (given by the RHS of (39)). In this model, there is actually a different Laffer curve for any given γ (as illustrated in Figure 3). The higher the value of γ, the lower are net inflation tax revenues generated by any given inflation rate. Hence, we will assume that, for a given γ, the marginal increase in net inflation tax revenues in response to a marginal increase in the inflation rate is non-negative (for further reference, we denote this expression by F):

$$\frac{d[\mu_T \tilde{h}(r + \mu_T) + \hat{\mu}_T \tilde{z}(r + \hat{\mu}_T)]}{d\mu_T} \equiv F(\gamma) =$$
$$\begin{cases} h_T(1 - \frac{\mu_T}{r+\mu_T}\eta_T^h) + (1-\gamma) z_T(1 - \frac{\hat{\mu}_T}{r+\hat{\mu}_T}\eta_T^z) > 0, & \gamma \in [0, \gamma^L), \\ 0, & \gamma = \gamma^L, \end{cases} \tag{40}$$

where

[28] In terms of equation (37), this means that for a balance of payments crisis to be feasible, the second term on the RHS must be positive. As we analyze in detail elsewhere (Lahiri and Végh (1998)), the feasibility of a balance of payments crisis under an exogenous path of government transfers depends on the particular form of money demand and, hence, on preferences. Since this issue is unrelated to the existence of interest rate policy, we abstract from it in the current analysis and focus on cases in which this feasibility condition holds.

$$
\begin{aligned}
\eta_T^h &\equiv -\frac{\tilde{h}'(i_T)i_T}{\tilde{h}(i_T)} = -\frac{\tilde{h}'(r+\mu_T)(r+\mu_T)}{\tilde{h}(r+\mu_T)}, \qquad (41)\\
\eta_T^z &\equiv -\frac{\tilde{z}'(i_T-i_T^g)(i_T-i_T^g)}{\tilde{z}(i_T-i_T^g)} = -\frac{\tilde{z}'(r+\hat{\mu}_T)(r+\hat{\mu}_T)}{\tilde{z}(r+\hat{\mu}_T)}, \qquad (42)
\end{aligned}
$$

denote the (absolute value of the) opportunity cost elasticities of the demand for cash and liquid bonds, respectively, from time T onwards.

It should also be clear that for standard money demands, there will always exist a value of γ for which a Laffer curve exists even if η^h and η^z are less than one.[29] The reason is that, if $\gamma > 1$, a higher μ_T *reduces* net inflation tax revenues on liquid bonds and could thus more than offset higher inflation tax revenues on cash. In addition, based on standard arguments, a Laffer curve may exist for values of $\gamma < 1$. The value γ^L ($\gtreqless 1$) is the value of γ for which net inflation tax revenues as a function of μ reach a maximum value of $\bar{\tau}$ (see Figure 3).[30] In other words, γ^L is the maximum amount by which policymakers can raise interest rates and still be able to finance the exogenous level of expenditures, $\bar{\tau}$.[31]

Second, we will also assume that, at least in the relevant range, the opportunity-cost elasticity of the demand for liquid bonds is less than unity; that is, $\eta^z < 1$. This is a sufficient, though not necessary, condition to ensure that $z_T(1 - \frac{\hat{\mu}_T}{r+\hat{\mu}_T}\eta_T^z) > 0$; that is, net revenues from liquid bonds are an increasing function of the effective inflation tax rate on bonds, $\hat{\mu}_T$.[32]

[29] By "standard" money demands, we mean money demands whose interest rate elasticity is non-decreasing in the interest rate. Within this family, we have, on the one hand, constant elasticity money demands and, on the other hand, Cagan and linear money demands (in which case the elasticity is an increasing function of the corresponding opportunity cost). These money demands will be discussed in detail below. The main simulations will be conducted using the Cagan money demand.

[30] Whether $\gamma^L \gtreqless 1$ will depend on the underlying money demands. For instance, for constant elasticity money demands such that η^h and η^z are both less than 1, $\gamma^L > 1$. For varying elasticity money demands, γ^L could be equal or less than one. This will depend on the value of $\bar{\tau}$ (the higher is $\bar{\tau}$, the lower is γ^L).

[31] As explained below, however, there are other constraints on γ which may imply that it is not possible for policymakers to set $\gamma = \gamma^L$.

[32] Notice that we are not imposing the restriction that the interest rate elasticity of cash be less than one. In fact, as shown below, there will be equilibrium paths along which this will not be the case.

4.2 Inflationary effects of interest rate policy

One logical concern of policymakers regarding the use of an active interest rate policy to combat speculative pressures is the likely fiscal effect of such policies. In particular, when governments face constraints on enacting spending cuts, the fiscal costs of raising interest rates (in term of the resulting higher debt service) is likely to imply higher inflation. This logic is easy to formalize in the context of this model.

Notice that equation (39) implicitly defines μ_T as a function of γ, for a given value of $\bar{\tau}$:

$$\mu_T = \tilde{\mu}_T(\gamma; \bar{\tau}), \qquad \gamma \in [0, \gamma^L]. \tag{43}$$

Furthermore, as follows from (39), μ_T is a strictly increasing function of γ:

$$\begin{array}{l} \frac{d\tilde{\mu}_T}{d\gamma} = \frac{\mu_T z_T (1 - \frac{\hat{\mu}_T}{r + \hat{\mu}_T} \eta_T^z)}{F(\gamma)} > 0, \quad \gamma \in [0, \gamma^L), \\ \lim\limits_{\gamma \to \gamma^L} \frac{d\tilde{\mu}_T}{d\gamma} = \infty. \end{array} \tag{44}$$

Figure 4, Panel A illustrates μ_T as a function of γ. We have thus shown the following result:

Proposition 3 *Under an exogenous path of fiscal spending, a more aggressive interest rate policy leads to a higher post-crisis rate of inflation.*

The intuition behind this result is straightforward. A more aggressive interest rate policy (i.e., a higher γ) implies a larger debt service once the crisis has occurred. Since non-interest government spending is given, the only way to finance this additional government spending is to resort to higher inflation. This *fiscal effect* of higher interest rate policy – which was absent in the benchmark case studied in Section 3 – will clearly affect both the feasibility and optimality of an active interest rate defense of the peg.

While the positive money demand effect isolated in Section 3 is still operative, it can actually be overwhelmed by the fiscal effect. To see this, totally differentiate $\hat{\mu}_T \equiv \mu_T(1-\gamma)$ with respect to γ (taking into account (44)) to obtain:

$$\frac{\partial \hat{\mu}_T}{\partial \gamma} = \frac{-\mu_T h_T (1 - \frac{\mu_T}{r + \mu_T} \eta_T^h)}{F}, \quad \gamma \in [0, \gamma^L). \tag{45}$$

Hence, as long as $\eta_T^h < \frac{r+\mu_T}{\mu_T}$, $\frac{\partial \hat{\mu}_T}{\partial \gamma} < 0$; that is, a higher γ reduces $\hat{\mu}_T$. In that range, therefore, higher interest rates increase the demand for liquid bonds (recall that the opportunity cost of holding bonds is $i_T - i_T^g =$

$r + \hat{\mu}_T$). As in the previous section, therefore, this direct money demand effect tends to increase post-collapse money demand and thus postpone the crisis. Hence, whether higher interest rates can delay an impending crisis or not will depend on the relative strength of the money demand and fiscal effects. Interestingly, however, when $\eta_T^h > \frac{r+\mu_T}{\mu_T}$, the marginal increase in the inflation rate is so large that a higher γ will actually increase $\hat{\mu}_T$. In this range, even the positive money demand effect disappears and hence interest rate policy will unambiguously tend to bring forward, rather than delay, a crisis.

Figure 4, Panel B, depicts $\hat{\mu}_T$ as a function of γ for the case in which $\gamma^L < 1$, $\eta_T^h < \frac{r+\mu_T}{\mu_T}$ for $\gamma = 0$, and η_T^h is an increasing function of μ_T (as would be the case for a Cagan money demand, as discussed in detail below).[33] If $\gamma^L < 1$, then $F(\gamma^L) = 0$ implies, from (40), that $1 - \frac{\mu_T}{r+\mu_T}\eta_T^h < 0$ for $\gamma = \gamma^L$. Hence, it follows from (45) that, for some $\gamma < \gamma^L$, $\hat{\mu}_T$ will reach a minimum beyond which the marginal increase in μ_T is so large that it more than offsets the direct effect of a higher γ on $\hat{\mu}_T$. This is already a good illustration of how powerful the fiscal effect can be in this model.

4.3 Effectiveness of active interest policy

The first question in our agenda is whether, once fiscal costs are taken into account, higher interest rates can delay a BOP crisis. To answer this question, we first establish that, as in the previous case, the time path of international reserves prior to the crisis is independent of the announced interest rate policy (i.e., independent of γ). To see this, notice that from the central bank's balance sheet and the fact that both h and z are constant prior to the crisis, it follows that $\dot{R}_t = -\dot{d}_t$ for $t \in [0, T)$. Furthermore, from (36), we have that, prior to the crisis, $\bar{\tau} = rR_t + \mu_t d_t$. Time-differentiating the latter expression and combining it with the former yields $\dot{\mu}_t = (r - \mu_t)\mu_t$. This differential equation gives the time path of the rate of domestic credit expansion which is consistent with the fiscal constraint (36), given that real money demand is constant. Since this differential equation is independent of the interest rate policy parameter γ, it follows that the time path of international reserves R prior to the crisis does not depend on γ.[34]

[33] As will become clear below, the way to interpret the condition that $\gamma^L < 1$ is as corresponding to a case in which fiscal spending (i.e, $\bar{\tau}$) is high.

[34] Notice that the initial condition, μ_0, is also independent of γ. This follows from the fact that (i) $\bar{\tau} - rR_0 = \mu_0 d_0$ and (ii) d_0 is determined by real money demand and the initial stock of international reserves (i.e., $d_0 = m_0 - R_0$) and is thus also independent of γ.

Since, by construction, international reserves fall to zero when the crisis occurs, the size of the speculative attack at time T is given by R_{T^-}. (Since $\Delta R_T \equiv R_T - R_{T^-} = -R_{T^-}$, the size of the run is $-\Delta R_T$. In what follows we denote the size of the run by S; i.e., $S_T \equiv -\Delta R_T$.) Hence, for a *given* time path for reserves prior to the crisis and an exogenously-given level of initial reserves R_0, the crisis will happen earlier (later), if and only if the size of the run at time T is bigger (smaller). We have already established that starting from any given initial level of reserves, the exogenously-specified fiscal spending $\bar{\tau}$ induces a rate of loss of reserves which is independent of γ. Thus, if we can determine the effect of the interest rate policy parameter γ on the *size* of the speculative attack, then we will have also pinned down the effect of interest rate policy on the *timing* of the crisis.

Formally, the timing of the crisis can be determined from the government's intertemporal budget constraint, equation (37). Solving (37) for T gives (taking into account (39) and recalling that $\Delta m_T = \Delta R_T$)

$$T = \frac{\log(\bar{\tau} - rS_T)}{r} - \frac{\log(\bar{\tau} - rR_0)}{r}.$$

Differentiating this with respect to γ then yields

$$\frac{\partial T}{\partial \gamma} = \frac{-1}{\bar{\tau} + r\Delta R_T}\frac{\partial S_T}{\partial \gamma}, \tag{46}$$

which proves our previous claim that the time of the run and the size of the run are inversely related.[35]

The size of the run at time T is simply the fall in total money demand at that time. Hence,

$$S_T = (h_0 + z_0) - (h_T + z_T) \equiv m_0 - m_T. \tag{47}$$

Since aggregate money demand before the collapse, m_0, depends only on r and is thus independent of γ, interest rate policy affects the size of the speculative attack only through its effect on post-collapse money demand m_T.[36] In particular, equation (47) says that the run will be smaller the higher is m_T. Intuitively, the higher the post-collapse money demand the smaller is the size of the run at time T. Thus, if interest rate policy were

[35] Notice that $\bar{\tau} + r\Delta R_T > 0$, as follows from (36).

[36] This would, of course, not be true if consumption were affected by changes in γ (which would be the case if consumption entered the transactions technology). In such a set up, consumption before T would be affected by γ, which would in turn affect real money demand before T.

to increase money demand at time T, then it would also reduce the size of the speculative attack and, hence, delay the crisis.

After substituting equations (32) and (33) into $m_T = h_T + z_T$, it follows that (using (44))

$$\frac{dm_T}{d\gamma} = \frac{\mu_T h_T z_T}{F(r+\mu_T)(r+\hat{\mu}_T)} \left[(\eta_T^z - \eta_T^h)(r + \mu_T) + \gamma \eta_T^h (1 - \eta_T^z) \mu_T \right], \quad \gamma \in [0, \gamma^L), \tag{48}$$

where F is given by (40).

It is straightforward to verify from equation (48) that

$$\frac{dm_T}{d\gamma} \gtreqless 0 \qquad \text{as} \qquad \gamma \gtreqless \frac{(\eta_T^h - \eta_T^z)(r + \mu_T)}{\eta_T^h (1 - \eta_T^z) \mu_T}. \tag{49}$$

Recall that, by assumption, $\eta_T^z < 1$. Since γ is non-negative, equation (49) says that $\eta_T^z > \eta_T^h$ is a sufficient condition for a more aggressive interest rate policy (a higher γ) to reduce the size of the attack and hence postpone the time of the attack. In other words, if the demand for cash is less interest elastic than the corresponding demand for liquid bonds, a more aggressive interest rate policy is successful in delaying a crisis.

Intuitively, a rise in γ reduces the opportunity cost of holding bonds by reducing $\hat{\mu}_T$ (recall (45)). This effect raises the post-collapse demand for liquid bonds through the direct money demand effect. However, as we showed earlier, a higher γ also increases μ_T through the fiscal effect and thereby raises the pure interest rate i which reduces the demand for cash. If bonds are more interest elastic than cash, then the rise in bond holdings that is induced by the higher γ outweighs the decline in cash demand that occurs due to the associated rise in i. In this event, the overall post-collapse demand for money rises. Hence, the size of the run is smaller and the run occurs later.

Interestingly, equation (49) also shows how interest rate activism can actually hasten a crisis. Suppose that $\eta_T^z < \eta_T^h$ and start from an initial situation in which $\gamma < \frac{(\eta_T^h - \eta_T^z)(r+\mu_T)}{\eta_T^h(1-\eta_T^z)\mu_T}$. Then a marginal increase in γ causes the size of the run to become bigger and, hence, brings the crisis forward in time. The logic behind this result follows the one just described. When cash is more interest elastic than bonds, a higher γ induces a fall in the demand for cash that is larger than the increase in the demand for bonds as long as the direct effect of γ is not too large relative to this differential effect. In this event, greater interest rate activism increases the size of the attack and hastens the crisis. We collect these results in the following proposition:

Proposition 4 *A sufficient (but not necessary) condition for a more aggressive interest rate policy to delay the time of the BOP crisis is that the elasticity of demand for cash be smaller than the elasticity of demand for liquid bonds (i.e., $\eta_T^h < \eta_T^z$). On the other hand, bonds being less elastic than cash (i.e., $\eta_T^z < \eta_T^h$) is a necessary (but not sufficient) condition for a more aggressive interest rate policy to bring forward the time of the attack.*

Recall that, in the absence of an exogenous fiscal constraint, a more aggressive interest rate policy *always* succeeds in delaying the crisis (Proposition 1). The introduction of an exogenous fiscal constraint is therefore critical in raising the possibility that a more aggressive interest rate policy may actually bring forward the time of the crisis. This supports the widely-held (but seldom formalized) notion that taking into effect the fiscal costs of higher interest rates is a critical element in assessing the effectiveness of an active interest rate defense of a peg.

We now go further and provide necessary and sufficient conditions for two empirically relevant cases. First, by setting $\gamma = 0$ in (48), the following proposition is self-evident:

Proposition 5 *For small values of γ, cash being less elastic than bonds (i.e., $\eta_T^h < \eta_T^z$) is a necessary and sufficient condition for a more aggressive interest rate policy to delay a BOP crisis.*

We consider the case of $\eta_T^h < \eta_T^z$ for $\gamma = 0$ to be the most relevant from an empirical point of view. The reason is that for $\gamma = 0$ the inflation rate is relatively low (recall Figure 4, Panel A). Intuitively, for low inflation rates, we expect the elasticity of cash to be smaller than that of bonds. The idea is that cash is kept mainly for transactions and is relatively interest rate inelastic. This intuition is confirmed by evidence available for the United States by Moore, Porter, and Small (1990).[37]

Second, Proposition 4 raises the intriguing possibility of a non-monotonic relationship between the time of the crisis and the degree of interest rate activism γ. In particular, if $\eta_T^z > \eta_T^h$ for small values of γ but $\eta_T^z < \eta_T^h$ for higher values of γ (which would obviously require money demands with non-constant elasticities), then the timing of the BOP crisis could bear a non-monotonic relationship with γ. For low values of γ, a more aggressive

[37] They conclude that the (absolute value of the) semi-elasticity of money market funds with respect to their opportunity cost is 0.277 and that of money market deposits 0.316 (and significant in both cases). These interest-bearing money semi-elasticities are much higher than the one for checkable and saving deposits (a good proxy for cash), which is only 0.096 (and also significant).

interest rate policy would be successful in delaying the crisis but once γ surpasses a certain threshold, greater activism could in fact hasten the crisis.

Equation (49) points to the specific conditions that one needs for such a case to occur. For T to be non-monotonic (initially rising and then declining) in γ, η_T^z should be falling in γ while η_T^h should be increasing in γ. Since i rises and $i - i^g$ falls as γ rises, this implies that cash demand should become more elastic as the nominal interest rate rises while bond demand should become less elastic as $i - i^g$ falls. In other words, the interest rate elasticities of both cash and bond demand must be increasing functions of the relevant opportunity cost. As an example below will make clear, Cagan-type money demands exhibit precisely such a property. Since Cagan-type money demands seem to provide the best econometric fit for developing countries (see, for instance, Easterly, Mauro, and Schmidtt-Hebbel (1995)), we see the case of non-constant elasticities as the most relevant from a practical point of view.

In fact, we can formally show that for Cagan-type money demands and high levels of fiscal spending (i..e, high levels of $\bar{\tau}$), T will indeed be a non-monotonic function of γ.

Proposition 6 *Suppose that (i) $\eta_T^h < \eta_T^z < 1$ for $\gamma = 0$; (ii) η_T^z and η_T^h are strictly increasing functions of their respective opportunity costs, and (iii) $\gamma^L \leq 1$ (which corresponds to a high level of $\bar{\tau}$). Then, T is a non-monotonic function of γ.*

Proof: See Appendix B.

Figure 4, Panel C illustrates the behavior of T for this case. The intuition should be clear from the above discussion. For small values of γ, the demand for cash is less elastic than that of bonds and therefore a higher interest rate raises overall money demand, thus delaying the crisis. The inflationary effects of higher interest rates, however, imply that cash demand is becoming more elastic over time while liquid bond demand is becoming less elastic (as its opportunity cost falls). At some point, the fact that bond demand becomes less elastic than cash demand implies that higher interest rates reduce money demand, thus bringing forward the crisis.

It is important to clarify the role of condition (iii) in Proposition 6. First, the way to think about $\gamma^L \leq 1$ is as corresponding to a high level of government expenditures (i.e., a high value of $\bar{\tau}$). It should be clear that one can always choose $\bar{\tau}$ such that $\gamma^L \leq 1$ [38] Second, the role of this

[38] For instance, to have $\gamma^L = 1$ one just needs to choose $\bar{\tau}$ as the maximum value of the Laffer curve corresponding to revenues from cash, as equation (40) makes clear.

condition is simply to ensure that the binding upper bound on γ is provided by γ^L. To understand this, notice that there are two other upper bounds on γ. One is the value of γ (denoted by γ^* in the previous section) for which there would be no run. If it exists, however, $\gamma^* > 1$ because the demand for liquid bonds at T must increase to compensate for the fall in the demand for cash. The remaining upper bound on γ is provided by the non-negativity constraint on $i_T - i_T^g$, which implies that $r + (1-\gamma)\mu_T \geq 0$. Again, this restriction will only be binding for values of $\gamma > 1$. Hence, choosing $\bar{\tau}$ so that $\gamma^L \leq 1$ ensures that the upper bound on γ faced by policymakers is γ^L. If $\gamma^L > 1$, then which restriction on γ becomes the binding one will depend on the specific parameter configuration.[39]

We now illustrate the above propositions with some examples using different money demands.

Example 1: The constant elasticity case To fix ideas, we begin with the case in which the demands for cash and liquid bonds exhibit constant interest rate elasticity. Let

$$v(h_t) + w(z_t) = \frac{(h_t)^{1-\theta}}{1-\theta} + \frac{(z_t)^{1-\alpha}}{1-\alpha}, \qquad \alpha > 1,\ \theta > 1. \tag{50}$$

Using (6) and (7), the demands for cash and liquid bonds are given by

$$h_t = \left(\frac{1}{i_t}\right)^{\frac{1}{\theta}}, \tag{51}$$

$$z_t = \left(\frac{1}{i_t - i_t^g}\right)^{\frac{1}{\alpha}}. \tag{52}$$

It follows that $\eta^h = 1/\theta$ and $\eta^z = 1/\alpha$. Substituting these elasticities into equation (48), we conclude that

$$\frac{\partial m_T}{\partial \gamma} \gtreqless 0 \qquad \text{as} \qquad \gamma \gtreqless \left(\frac{\alpha-\theta}{\alpha-1}\right)\left(\frac{r+\mu_T}{\mu_T}\right). \tag{53}$$

For the relevant range (i.e., $\gamma \geq 0$), it is easy to see that when bonds are more interest elastic than cash (i.e., $\alpha < \theta$), a more aggressive interest rate policy always increases money demand post-collapse and, hence, postpones the crisis. In this case, therefore, a more aggressive interest rate policy always succeeds in postponing the crisis in spite of the rising inflationary effects.

[39] Notice, of course, that $\gamma^L \leq 1$ is a sufficient, but not necessary, condition for the non-monotonicity of T. As long as γ^L is the binding constraint, T will be non-monotonic even if $\gamma^L > 1$.

Example 2: The Cagan case The second example focuses on the well-known Cagan-type money demands. To this end, let

$$v(h_t) + w(z_t) = h(F - G\log h) + z(A - B\log z). \tag{54}$$

Under this specification, the demands for cash and bonds are given by

$$h = Ke^{-\frac{i}{G}}, \quad K \equiv e^{\frac{F-G}{G}}, \tag{55}$$

$$z = Me^{-\frac{i-i^g}{B}}, \quad M \equiv e^{\frac{A-B}{B}}, \tag{56}$$

which are the standard Cagan demand functions. The corresponding interest rate elasticities for this case are

$$\eta^h = \frac{i}{G}, \tag{57}$$

$$\eta^z = \frac{i-i^g}{B}. \tag{58}$$

The crucial feature of the Cagan case is that interest elasticities are no longer constant, but rather *increasing* functions of their respective opportunity cost. Moreover, we know from (44) and (45) that as γ rises, μ_T (and hence i_T) increases but $\hat{\mu}_T$ (and hence $i_T - i_T^g$) falls (for values of $\eta_T^h < \frac{r+\mu_T}{\mu_T}$). Hence, as γ rises, η^h increases while η^z falls, which is precisely the feature that may give rise to a non-monotonic relationship between γ and T (with T first rising and then falling)

To see this, substitute (57) and (58) into (49) to obtain

$$\frac{\partial m_T}{\partial \gamma} \gtreqless 0 \qquad \text{as} \qquad \gamma \lesseqgtr \frac{G-B}{\mu_T}. \tag{59}$$

Consider the case in which $G > B$; that is, for the same opportunity cost, the demand for cash is less elastic than that for liquid bonds. Then, for $\gamma^L \leq 1$, Proposition 6 applies. Figure 5 shows a numerical simulation for this case, with $G = 5$ and $B = 3$.[40] As shown in Panel A, the time of the crisis is indeed a non-monotonic function of the degree of interest rate activism γ. Panel B shows how the post-crisis inflation rate increases with γ (as formally shown in Proposition 3).[41] Figure 5 thus makes clear that,

[40] The remaining parameters are $\bar{\tau} = 0.35$ (as a proportion of GDP), $A = F = 10$, $r = 0.03$ and $R_0 = 0.5$ (as a proportion of GDP). The time unit is years. It should be noted that we are only interested in the qualitative nature of the results; there is thus no attempt at replicating any particular economy.

[41] For further reference, Panel C emphasizes the fact that the path of consumption is flat along a perfect foresight path, and hence the fall in consumption at time T (in percentage terms) is zero for all γ. Panel D will be discussed later.

for high levels of fiscal spending, there is a certain degree of interest rate activism (i.e., some value of γ) that maximizes the delay. To illustrate the role of fiscal spending, Figure 6 illustrates the same case but for a lower value of fiscal spending ($\bar{\tau}$ is now 0.1 down from 0.35 in Figure 5). In this case, T is always an increasing function of γ.[42] In other words, the higher post-crisis inflation rate (the fiscal effect) is always dominated by the money demand effect.

Example 3: The quadratic case As a final example, we consider the quadratic specification given by

$$v(h_t) + w(z_t) = -\frac{G}{2}(\bar{h} - h)^2 - \frac{B}{2}(\bar{z} - z)^2. \tag{60}$$

It is easy to check that the corresponding interest rate elasticities of cash and bonds are now given by

$$\eta^h = \frac{1}{\frac{G\bar{h}}{i} - 1}, \tag{61}$$

$$\eta^z = \frac{1}{\frac{B\bar{z}}{i - i^g} - 1}. \tag{62}$$

As can be seen from (61) and (62), this case is very similar to the Cagan case just analyzed since the elasticities are rising in their respective opportunity cost. Substituting (61) and (62) into (48) yields

$$\frac{\partial m_T}{\partial \gamma} \gtreqless 0 \qquad \text{as} \qquad \gamma \lesseqgtr \frac{G\bar{h} - B\bar{z}}{2\mu_T}. \tag{63}$$

Hence, for $\gamma^L \leq 1$, Proposition (6) applies, as it did for the Cagan case.

4.4 Optimality of active interest rate policy

The preceding analysis has focused on the conditions under which a more aggressive interest rate policy could delay an impending balance of payments crisis. Of course, the fact that policymakers may have the ability to delay a crisis by raising interest rates does not necessarily mean that it is optimal to do so. To address this issue, we now analyze the optimality of raising interest rates to delay a crisis.

[42] In this case, the terminal condition is provided by the non-negativity constraint on $i_T - i_T^g$.

The starting point of the welfare analysis is equation (34), which remains valid for this case. Equation (34) decomposes the change in the time-invarying level of consumption (and hence welfare) that results from an increase in γ into two terms. The first term indicates how, for a given T, an increase in γ directly affects consumption. The second term captures the indirect effect of γ on consumption through changes in T.

Let us first look at the second term on the RHS of (34). As shown in Appendix A, $\frac{\partial \bar{c}}{\partial \tilde{T}} > 0$.[43] In other words, other things being equal, a higher T increases consumption by allowing consumers to reap the benefits of a higher pre-crisis liquidity services for a longer period of time. This implies that the sign of the second term on the RHS of (34) depends solely on the sign of $\tilde{T}'(\gamma)$. On this account, therefore, delaying a crisis is welfare improving.

The sign of the first term on the right-hand side of (34), however, is now ambiguous (unlike the previous case in which μ was exogenously given). The reason is that, for a given T, a higher γ may actually reduce both the demand for liquid bonds (which, from (45), happens for $\eta_T^h > \frac{r+\mu_T}{r}$) and, by raising inflation, the demand for cash. This would raise the present discounted value of transactions costs and hence reduce $\bar{c}$. On this account, therefore, raising interest rates is not necessarily welfare improving.

To get a better intuitive sense of the trade-offs involved, it is useful to rewrite (34) as:[44]

$$\frac{d\bar{c}}{d\gamma} = \frac{e^{-r\tilde{T}}}{r}\left[\frac{dm_T}{d\gamma}\left(r+\mu_T+\frac{\Phi}{\frac{\bar{\tau}}{r}+\Delta R_T}\right)-\gamma\mu_T\tilde{z}'\frac{d\hat{\mu}_T}{d\gamma}\right], \tag{64}$$

where $\Phi \equiv v(h_0)+w(z_0)-v(h_T)-w(z_T) > 0$ (as shown in appendix A) and $\frac{\bar{\tau}}{r}+\Delta R_T > 0$ (as remarked earlier). Equation (64) makes clear that a higher real money demand in response to a higher γ (i.e., a positive $\frac{dm_T}{d\gamma}$) is always welfare improving. Intuitively, a higher real money demand raises welfare on two counts: (i) the crisis is delayed and (ii) the higher level of real money balances at T ensures a lower present discounted value of transactions costs. The second term in the square brackets on the RHS of (64) captures a negative effect on welfare that results from substituting cash for bonds. As first-order conditions (6) and (7) make clear, cash is, at the margin, more productive than liquid bonds. Hence, by itself, the substitution away from cash and toward liquid bonds is welfare decreasing because it leads to higher transactions costs. This explains why, a higher γ, which reduces the

[43] Note that this proof remains valid since it applies to any arbitrary γ and (small) μ_T.
[44] See Appendix C for the derivation of this equation.

opportunity cost of bonds (i.e., $\frac{d\hat{\mu}_T}{d\gamma} < 0$), also reduces welfare (as evidenced by the fact that the second term in the square brackets is negative).

As should be clear from (64), the effect of a more aggressive interest rate policy on consumption (and, hence, welfare) is, in general, ambiguous. However, even at this level of generality, we can (i) characterize the behavior of welfare for small values of γ (i.e., around $\gamma = 0$), and (ii) provide sufficient conditions for welfare to be a non-monotonic function of γ.

To study the first issue, evaluate (64) around $\gamma = 0$ to obtain:

$$\left.\frac{d\bar{c}}{d\gamma}\right|_{\gamma=0} = \frac{e^{-r\tilde{T}}}{r}\frac{dm_T}{d\gamma}\left(r + \mu + \frac{\Phi}{\frac{\bar{\tau}}{r} + \Delta R_T}\right). \tag{65}$$

Equation (65) thus says that, around $\gamma = 0$, $\bar{c}$ (and hence welfare) will change in the same direction as T does. We know from Proposition 5 that, around $\gamma = 0$, $\eta_T^h < \eta_T^z$ is a necessary and sufficient condition for a higher γ to raise m_T and thus T. We have thus shown the following:

Proposition 7 *Around $\gamma = 0$, a necessary and sufficient condition for welfare to be rising in γ is that the demand for cash be less interest rate elastic than the demand for liquid bonds (i.e., $\eta_T^h < \eta_T^z$).*

Intuitively, if $\gamma = 0$, cash and liquid bonds are, at the margin, equally productive (recall that $i_0^g = 0$).[45] Hence, based on the above discussion, the only relevant channel for welfare is the change in post-crisis real money balances. The condition $\eta_T^h < \eta_T^z$ ensures that real money balances increase, which raises welfare. From a policy point of view, then, Proposition 7 says that, under the most plausible parameter configuration, it is always optimal to raise interest rates by some amount to defend the peg. In other words, sitting still (as implied by the Krugman model) is not optimal.

We now show that the assumptions specified in Proposition (6) provide sufficient conditions for welfare to be a non-monotonic function of γ.

Proposition 8 *Suppose that (i) $\eta_T^z > \eta_T^h$ for $\gamma = 0$; (ii) η_T^z and η_T^h are strictly increasing functions of their respective opportunity costs, and (iii)*

[45] If $i_0^g > 0$, this proposition does not go through because it would still be the case that, for $\gamma = 0$, cash is more productive, at the margin, than liquid bonds. In other words, this substitution effect between the two monies could, in principle, more than offset the money demand effect. In this case, we have established existence of the result in Proposition 7 by resorting to simulations. For the parameterization underlying the simulation in Figure 5, we find that this result still goes through for positive values of i_0^g, with the only effect being that of reducing the value of γ for which welfare reaches a maximum (as one would have expected).

$\gamma^L \leq 1$ (which corresponds to a high level of $\bar{\tau}$). Then, W is a non-monotonic function of γ and reaches a maximum before T does.

Proof: See Appendix D.

Figure 4, Panel D illustrates the path of welfare for the case considered in Proposition 8 and Figure 5, Panel D shows the simulated path.[46] What is the intuition behind these results? As suggested by equation (64), once real money balances begin to fall, welfare is negatively affected because (i) the crisis is being brought forward and (ii) the post-crisis level of real money balances is lower. This negative effect on welfare is compounded by the switch from cash to liquid bonds (as long, of course, as $\hat{\mu}_T$ is falling). The next question is then why welfare begins to fall before T does. Again, equation (64) suggests an answer. As discussed above, welfare (unlike T) is also affected by the substitution between the two monies. Even as post-crisis real money balances go up, the substitution away from the more productive liquid asset (cash) is, all else equal, reducing welfare. Hence, welfare must begin to fall before total real money demand (and hence T) does.[47]

Let us emphasize the policy implications of Proposition 8. First, in the presence of a high level of fiscal expenditures, raising interest rates to defend a peg is optimal but only up to a point. The fiscal costs of higher interest rates make it sub-optimal to raise interest rates too much. Second, as a comparison of panels A and D in Figure 5 illustrates, there is a whole range of interest rate increases for which it is *feasible* to delay a BOP crisis but *not optimal* to do so. In other words, the mere fact that higher interest rates are being effective in postponing the crisis does not imply that this is an optimal policy.

Finally, the simulation reported in Figure 6 illustrates the fact that the non-monotonicity of T is a sufficient but not necessary condition for welfare to be non-monotonic. While T is rising throughout the whole relevant range of γ (Panel A), welfare is non-monotonic (Panel D). This occurs when the financing needs of the fiscal authority are substantially lower than in Figure 5.[48] This reinforces the idea that, even when a more aggressive interest

[46] Welfare is reported as the percentage change in welfare relative to the $\gamma = 0$ case.

[47] In the special case of constant interest elasticity of demand for both cash and bonds, one can show that a necessary and sufficient condition for welfare to be monotonically rising (falling) in the degree of interest rate activism γ is that the demand for non-interest bearing cash be less (more) interest elastic than liquid bonds. Welfare becomes independent of interest rate policy in the special case where the interest elasticities of cash and bonds are equal.

[48] The only change in the parameter configuration relative to Figure 5 is that now $\bar{\tau} = 0.1$ (down from 0.35).

rate policy may succeed in delaying the crisis, it may not be optimal to do so.

4.5 Time-varying consumption

Consider now the case in which the time path of consumption is directly influenced by the path of interest rates. This is crucial in order to check whether the basic insights gained so far are robust to relaxing the assumption that consumption does not change at T. To this effect, we now let the transactions technology depend on consumption and hence be given by $\phi(h, z, c)$. We assume that $\phi(.)$ is strictly decreasing in h and z and strictly increasing in c. With this formulation, the first-order conditions (5), (6), and (7) become:

$$u'(c_t) = \lambda[1 + \phi_c(h, z, c)], \tag{66}$$

$$-\phi_h(h_t, z_t, c_t) = i_t, \tag{67}$$

$$-\phi_z(h_t, z_t, c_t) = i_t - i_t^g. \tag{68}$$

Under this specification, it is easy to check analytically that, for a given γ, consumption falls at T. However, in order to study the effects of changes in γ on the timing of the crisis and welfare (which is our main focus), the model becomes rather unwieldy because the fall in consumption at T is itself a function of γ and hence affects how T responds to changes in γ. In a similar vein, welfare now also depends on how the fall in consumption varies with γ. In view of this, we proceed instead by numerically simulating the model.

To this effect, let the transactions technology take the Cagan-type specification:

$$\phi(h, z, c) = c^q[K - \alpha h(F - G \log h) - (1 - \alpha)z(A - B \log z)].$$

The case analyzed above corresponds to the $q = 0$ case.

Figure 7 reports a typical simulation.[49] As can be seen, both T and welfare are non-monotonic functions of γ. In fact, the paths look similar to those obtained in the separable case (Figure 5). This suggests that non-

[49] The parameter configuration is the same as that used for the separable case reported in Figure 5, except for q which is now 1. Panel C, reports the fall in consumption at time T in percentage terms.

separability does not alter the essential logic behind our results.[50] In this case, of course, the path of consumption is no longer flat. As shown by Panel C in Figure 7, the fall in consumption at time T mirrors the behavior of welfare. The fall in consumption is reduced early on as γ increases, and then rises sharply.

5 Conclusions

In the classical approach to balance of payments crises inspired by Krugman (1979), crises are caused by inconsistent policies. In particular, countries which attempt to expand money supply while simultaneously fixing their exchange rate end up losing foreign exchange reserves thus opening the door to speculative attacks. A notable – but rarely discussed – assumption of Krugman-type models is that the central bank sits passively as its international reserves are depleted, and thus takes no measures to fight a speculative attack. Of course, this is rarely the case in practice since central banks actively try to defend the peg by raising short-term interest rates, thus making more attractive domestic currency denominated assets.

This paper has analyzed the feasibility and optimality of delaying an impending crisis by raising interest rates. We have shown that, under certain conditions, raising interest rates succeeds in delaying a crisis and is in fact the optimal (i.e., welfare-maximizing) policy. Higher interest rates, however, increase the service burden of the public debt and imply higher inflation. This fiscal cost of using higher interest rates to defend a peg implies that, for a high level of government spending, there is a certain rise in interest rates which maximizes how much the crisis can be delayed. In the same vein, there is a certain increase in interest rates that maximizes welfare. In fact, there is a range of interest rate increases for which the crisis could be further delayed but it is not optimal to do so. We also show in the appendix that our welfare results do not depend on the way in which money is introduced into the model (transaction costs, MIUF, or cash-in advance). Hence, the main message of the paper that it may be feasible to delay a BOP crisis but not optimal to do so is robust across different theoretical specifications.

Our results help in providing a conceptual framework for one of the

[50] As a robustness check, we performed simulation for values of q between 0 and 10 and obtained identical qualitative results. When we reduced $\bar{\tau}$ to 0.10 (as in the simulation reported in Figure 6), we also obtained the same results as in Figure 6 for the same range of q (between 0 and 10).

more hotly debated policy issues in recent years. As is well-known, IMF-supported programs in Asia and Latin America typically require a policy of high interest rates to defend the currency. This policy recommendation has been attacked by outside analysts. While highly stylized, our model should at least provide some intellectual food for thought on these issues and hopefully serve as a framework that may guide econometric efforts in this area. Our results suggest that *some* active defense of a peg is optimal but that very high interest rate increases are likely to do more harm than good by generating higher inflation in the future, a result in the spirit of Sargent and Wallace's (1981) famous unpleasant monetarist arithmetic.

From a theoretical point of view, there are several issues related to the model worth mentioning. First, to keep the analysis tractable, we have focused on interest rate policies that involve raising interest rates when a crisis actually occurs, as opposed to a policy that would raise interest rates *before* the crisis occurs. Notice, however, that the set-up that we have chosen is not restrictive for the issues at hand since, as is clear from the analysis, the mere *announcement* of such a policy has the effect of delaying the crisis (which is the policymakers' intent). Alternatively, one could assume that policymakers actually raise interest rates sometime before the crisis is expected to happen (as in Lahiri and Végh (1999)). As a result, demand for liquid bonds (and, hence, money demand) would increase at that point (which would tend to delay the crisis) but, due to the higher fiscal deficit, the rate at which international reserves fall would increase and post-crisis money demand would fall (which would tend to bring forward the crisis). It seems intuitive that, depending on the relative strength of both effects, similar results would obtain. A more complicated exercise would be to posit a "leaning-against-the-wind" interest rate rule whereby interest rates would be raised as a function of the loss of international reserves. While this set-up would considerably complicate the analysis, the outcome would be the same insofar as such a policy would also delay the crisis by mitigating the loss of reserves. In sum, we believe that the essential insights derived from our current set-up are robust to alternative interest rate rules.

Secondly, in our analysis, the crisis will always occur. The only question is whether higher interest rates can delay it and whether such a policy is optimal. In practice, of course, the ability to delay a potential crisis buys precious time that may allow the fiscal authority to put its house in order (see Guidotti and Végh (1999)). One could incorporate such a feature in our model by, say, assuming that there is certain probability at each point in time that the fiscal authority will indeed be able to solve the fiscal problems This would clearly increase the benefits of delaying and may, in fact, help avoiding the crisis altogether. In this sense, therefore, our current framework underestimates the benefits of delaying by not taking into account the possibility that the fiscal authority will be able to solve its problems.

Thirdly, an important caveat to our analysis is that we have focused exclusively on the *fiscal* costs of higher interest rates and abstracted from potential *output* costs. In related work (Lahiri and Végh (1999)), we abstract from fiscal costs and incorporate output costs into the picture by introducing a banking system and, hence, supply and demand for bank credit. In this set-up, higher interest rates exert a contractionary effect on output by reducing bank credit (i.e., working capital) to firms. This negative output cost will tend to offset the positive money demand effect and thus generate trade-offs in the monetary authority's ability to delay a crisis and in the optimality of using higher interest rates. In line with the results in this paper, our results indicate that some active interest rate defense will also be optimal when output costs are taken into account but that, beyond certain point, raising interest rates further is welfare reducing.

Finally, we should also note that our results should carry through to a world of flexible exchange rates (this is the next issue in our research agenda on this topic). In that context, higher interest rates would affect the level of the exchange rate rather than the level of reserves. Hence, higher interest rates should lead to an initial exchange rate appreciation but, if raised beyond certain point, the exchange rate would begin to depreciate due to the higher inflation rate induced by fiscal costs.

Appendices

A Proof of proposition 2

As argued in the text, the fact that welfare is an increasing function of γ for $\gamma > 1$ is not formally obvious. In light of (34) and the fact that $\frac{\partial \bar{c}}{\partial \gamma} \geq 0$ and $\tilde{T}'(\gamma) > 0$, we need to show that

$$\frac{\partial \bar{c}}{\partial \tilde{T}} = re^{-r\tilde{T}} \left\{ v[\tilde{h}(r)] + w[\tilde{z}(r - i_0^g)] - v[\tilde{h}(r + \mu)] - w[\tilde{z}(r - i_0^g + (1-\gamma)\mu)] \right\} > 0. \tag{69}$$

This appendix proves this claim for the case in which $v(h)$ and $w(z)$ are quadratic functions (i.e., $v'(h) > 0$, $v''(h) < 0, v'''(h) = 0, w'(h) > 0$, $w''(h) < 0$, and $w'''(h) = 0$).[51]

Given (69), it must be true that

$$f(\mu) \equiv v[\tilde{h}(r)] + w[\tilde{z}(r - i_0^g)] - v[\tilde{h}(r + \mu)] - w[\tilde{z}(r - i_0^g + (1-\gamma)\mu)] > 0.$$

Using a Taylor expansion around $\mu = 0$ (taking into account that $\tilde{h}'(i) = \frac{1}{v''[\tilde{h}(i)]}$, $\tilde{z}'(i - i^g) = \frac{1}{w''[\tilde{z}(i-i^g)]}$, and $f(0) = 0$):

$$f(\mu_T) \approx f'(0)\mu + f''(0)\frac{\mu^2}{2}, \tag{70}$$

where

$$\begin{aligned} f'(0) &= -v'[\tilde{h}(r)]\tilde{h}'(r) - w'[\tilde{z}(r - i_0^g)]\tilde{z}'(r - i_0^g)(1-\gamma), \\ f''(0) &= -\frac{1}{v''[\tilde{h}(r)]} - \frac{(1-\gamma)^2}{w''[\tilde{z}(r - i_0^g)]}. \end{aligned}$$

Since $f''(0) > 0$, for $f(\mu) > 0$ we need to show that $f'(0) \geq 0$. This is, of course, obvious for $\gamma \leq 1$. In general, notice that for $f'(0) \geq 0$, it must be true that

$$-v'\tilde{h}'(r) \geq w'(1-\gamma)\tilde{z}'(r - i_0^g). \tag{71}$$

[51] A general proof would involve third derivatives of $v(h)$ and $w(z)$. Since theory does not provide us with plausible restrictions on third derivatives, one would need to proceed on a case-by-case basis. Alternatively, this same proof would hold for general separable transactions technologies for the corresponding quadratic approximations.

From first order conditions (6) and (7), we know that $v' \geq w'$. Furthermore, the condition that money demand not increase at T implies that (for small μ)

$$\begin{aligned} \frac{dm}{d\mu} &= h' + (1-\gamma)z' \leq 0 \\ &\Rightarrow -h' \geq (1-\gamma)z'. \end{aligned}$$

Given the last inequality and that $v' \geq w'$, (71) holds. This proves the claim.

B Proof of proposition 6

First, notice that $\frac{dm_T}{d\gamma} > 0$ for $\gamma = 0$, as follows directly from Proposition 4. Second, notice that we can rewrite (48) as

$$\frac{dm_T}{d\gamma} = \frac{d\tilde{\mu}_T}{d\gamma}[\tilde{h}' + (1-\gamma)\tilde{z}'] - \mu_T \tilde{z}', \tag{72}$$

which implies, given (48) that, for $\gamma^L \leq 1$,

$$\lim_{\gamma \longrightarrow \gamma^L} \frac{dm_T}{d\gamma} = -\infty. \tag{73}$$

Hence, there exists at least one value of $\gamma \in (0, \gamma^L)$ such that $\frac{dm_T}{d\gamma} = 0$. We will assume (without proving it) that this value is unique (as all our simulations suggest) and denote it by γ^T.

C Derivation of equation (64)

From (29), it follows that

$$\frac{\partial \bar{c}}{\partial \gamma} = \frac{e^{-rT}}{r} \left(v'h' \frac{d\mu_T}{d\gamma} + w'z' \frac{d\hat{\mu}_T}{d\gamma} \right). \tag{74}$$

Taking into account first-order conditions (6) and (7) and that, from the definition of real money balances, $\frac{dm_T}{d\gamma} = h' \frac{d\mu_T}{d\gamma} + z' \frac{d\hat{\mu}_T}{d\gamma}$, it follows that

$$\frac{\partial \bar{c}}{\partial \gamma} = \frac{e^{-rT}}{r} \left[(r + \mu_T) \frac{dm_T}{d\gamma} - \gamma \mu_T z' \frac{d\hat{\mu}_T}{d\gamma} \right]. \tag{75}$$

In addition, notice that from (29), it also follows that

$$\frac{\partial \bar{c}}{\partial T} = e^{-rT}\left[v(h_0) + w(z_0) - v(h_T) - w(z_T)\right] \equiv e^{-rT}\Phi. \tag{76}$$

Also notice that, by definition, (46) can be rewritten as:

$$T' = \frac{1}{\bar{\tau} + r\Delta R_T}\frac{dm_T}{d\gamma}. \tag{77}$$

Substituting (75), (76), and (77) into (34) yields (64).

D Proof of proposition 8

Notice first that $\frac{d\bar{c}}{d\gamma} > 0$ for $\gamma = 0$, as shown in Proposition 7. From (64), and using (40), (45) and (48), it follows that:

$$\begin{aligned}
\lim \left.\frac{d\bar{c}}{d\gamma}\right|_{\gamma=\gamma^L<1} &= \frac{e^{-r\tilde{T}}}{r}[\left(r + \mu + \frac{\Phi}{\frac{\bar{\tau}}{r} + \Delta R_T}\right)\lim \left.\frac{dm_T}{d\gamma}\right|_{\gamma=\gamma^L<1} \\
-\gamma\mu_T\tilde{z}'\lim \left.\frac{d\hat{\mu}_T}{d\gamma}\right|_{\gamma=\gamma^L<1}] &= -\infty, \qquad (78) \\
\lim \left.\frac{d\bar{c}}{d\gamma}\right|_{\gamma=\gamma^L=1} &= \frac{e^{-r\tilde{T}}}{r}[\left(r + \mu + \frac{\Phi}{\frac{\bar{\tau}}{r} + \Delta R_T}\right)\lim \left.\frac{dm_T}{d\gamma}\right|_{\gamma=\gamma^L=1} \\
+\gamma\mu_T^2\tilde{z}'] &= -\infty. \qquad (79)
\end{aligned}$$

Hence, there exists at least one value of $\gamma \in (0, \gamma^L)$ such that $\frac{d\bar{c}}{d\gamma} = 0$. Again, we will assume (without proving it) that this value is unique (as all our simulations suggest) and denote it by γ^W.

We now show that $\gamma^W < \gamma^T$. To this effect, first notice that since $m_T = \tilde{h}(r + \mu_T) + \tilde{z}(r + \tilde{\mu}_T)$, it follows that

$$\frac{dm_T}{d\gamma} = \tilde{h}'\frac{d\mu_T}{d\gamma} + \tilde{z}'\frac{d\hat{\mu}_T}{d\gamma}. \tag{80}$$

Since $\left.\frac{dm_T}{d\gamma}\right|_{\gamma=\gamma^T} = 0$ and $\frac{d\mu_T}{d\gamma} > 0$ for all $\gamma \in [0, \gamma^L)$, it follows that $\left.\frac{d\hat{\mu}_T}{d\gamma}\right|_{\gamma=\gamma^T} < 0$. Using this piece of information, it follows from (64) that:

$$\left.\frac{dW}{d\gamma}\right|_{\gamma=\gamma^T} = -\frac{e^{-r\tilde{T}}}{r}\gamma\mu_T\tilde{z}'\left.\frac{d\hat{\mu}_T}{d\gamma}\right|_{\gamma=\gamma^T} < 0. \tag{81}$$

Hence $\gamma^W < \gamma^T$.

E The MIUF case

This appendix shows that all the key results in the text concerning the feasibility and optimality of delaying a crisis by raising interest rates obtain in the money-in-the-utility-function (MIUF) case. In fact, we show that the two cases (MIUF and transaction costs) are isomorphic in the sense that the conditions needed for the results to go through in either case are *identical*.

Let the utility function be given by:[52]

$$W \equiv \int_0^\infty [u(c_t) + v(h_t) + w(z_t)]\, e^{-\beta t} dt. \tag{82}$$

The intertemporal budget constraint is given by:

$$a_0 + \frac{y}{r} + \int_0^\infty \tau_t e^{-rt} dt = \int_0^\infty [c_t + i_t h_t + (i_t - i_t^g) z_t]\, e^{-rt} dt. \tag{83}$$

The first-order conditions for this model are thus (maximizing (82) subject to (83)):

$$\begin{aligned} u'(c_t) &= \lambda, & (84)\\ v'(h_t) &= \lambda i_t, & (85)\\ w'(z_t) &= \lambda (i_t - i_t^g). & (86) \end{aligned}$$

It is easy to check that the resource constraint is now given by:

$$k_0 + \frac{y}{r} = \int_0^\infty c_t e^{-rt} dt. \tag{87}$$

Given (84) and (87), it follows that $c_t = rk_0 + y$. In fact, consumption will take this value *regardless* of the path of interest rates (and hence of the value of γ). Hence, we can always define units such that $u'(rk_0 + y) = 1$ which implies, from (84) that $\lambda = 1$. Equations (85) and (86) then define the same money demands as in the transactions costs case, given by (8), (9), and (10). Since the money demands are identical to the transactions costs case, all the results related to the ability of higher interest rates to delay a BOP crisis (given by Propositions 1, 5, and 6) go though and the proofs would, in fact, be identical.

We now turn to the issue of optimality. Given (82) and the fact that $c_t (= rk_0 + y)$ does not depend on γ, welfare can be expressed as (ignoring constants):

[52] To facilitate comparison, we use the same notation as in the text.

$$W[\gamma, \tilde{T}(\gamma)] = \frac{[1 - e^{-r\tilde{T}(\gamma)}]}{r}[v(h_0) + w(z_0)] + \frac{e^{-r\tilde{T}(\gamma)}}{r}[v(h_T) + w(z_T)]$$

Ignoring constants, this expression is the same as (29). Hence, all our previous results, given by Propositions 2, 7, and 8, hold. The interpretation, however, is slightly different. In the transactions cost model in the text, changes in cash and liquid bonds affect utility indirectly through consumption. In this MIUF model, changes in cash and liquid bonds affect utility directly. This is the reason why the model in the text offers a more attractive interpretation, even though it is formally identical for our purposes to the MIUF case.

Finally, notice that, by assuming that the two monies enter separably in the utility function (82), we have dealt with the case in which nominal interest rates do not affect the path of consumption. This simplification makes sense because, as stressed before, our main results do not require that nominal interest rates affect the path of consumption. If the two monies entered non-separably into the utility functions, simulations of the model using Cagan money demands (available from the authors upon request) suggest that all the same results go through.

F The cash-in-advance case

In addition to the transactions costs and MIUF approaches, the third most common way of introducing money into optimizing models is through a cash in advance constraint. This appendix briefly discusses how our results would be affected by introducing money through a cash-in-advance constraint. In this set-up, consumers are subject to a "liquidity-in-advance constraint" of the form:

$$c_t = L(h_t, z_t), \tag{88}$$

where $L(h_t, z_t)$ is a strictly increasing and concave function.

Consumers thus maximize (1) subject to (83) and (88). The first-order conditions are given by

$$\begin{aligned} u'(c_t) &= \lambda\left[1 + \frac{i_t}{L_h(h_t, z_t)}\right], \\ \frac{L_h(h_t, z_t)}{L_z(h_t, z_t)} &= \frac{i_t - i_t^g}{i_t}. \end{aligned}$$

It is easy to check that in this model consumption falls at T when the crisis takes place. However, due to the fact that the path of consumption depends on interest rates, the model becomes rather unwieldy when it comes to study how T and welfare depend on γ. We therefore simulated the model for $L(h_t, z_t)$ taking the following specification (which leads to Cagan-type money demands):

$$L(h_t, z_t) = \alpha h(F - G \log h) + (1 - \alpha)z(A - B \log z).$$

Figure 8 shows a typical simulation for values of G higher than B.[53] The timing of the crisis is a strictly increasing function of γ (Panel A), while welfare follows an inverted-U shape (Panel D). Hence, it is always optimal to engage in some active interest rate defense of the peg. The benefits, however, soon vanish and increasing interest rates further is not optimal even though it does manage to postpone the crisis until T reaches a plateau. Except for the non-monotonicity of T, then, all the previous results seem to hold for the liquidity in advance case.

How robust are these results? We checked robustness for the two parameters that are the most important in our theoretical section: G (relative to B) and $\bar{\tau}$ (the level of fiscal spending).

Taking the simulation in Figure 8 as our benchmark, we varied G to check how the qualitative results change. It is only when G gets very close to B that any qualitative change occurs. For values of G close to B ($G \leq 3.05$), welfare decreases throughout. The pattern of an increasing T and falling welfare remains valid when G falls below B. Hence, as in the text, the relative elasticities of the demand for cash and liquid bonds appear to play an important role for the welfare results in the sense that for welfare to increase for small values of γ, the demand for cash needs to be less elastic than the demand for bonds.

Again with Figure 8 as our benchmark, we varied $\bar{\tau}$. In the benchmark, $\bar{\tau}(= 0.1)$ is "high" in the sense that, for $\gamma = 0$, the post-crisis inflation rate is 84 percent (Panel B). We reduced it to as low as 0.005 and, even though the post-crisis inflation rate for $\gamma = 0$ is now only 4 percent, the qualitative features remain the same. In sum, the results shown in Figure 8 seem to be robust to the level of fiscal spending.

[53] In this benchark simulation, $G = 5$, $B = 3$, $\bar{\tau} = .1$, $F = 0.000001$, $A = 10$, $r = 0.03$, $\sigma = 0.5$, $\alpha = 0.5$, and $R_0 = 0.07$.

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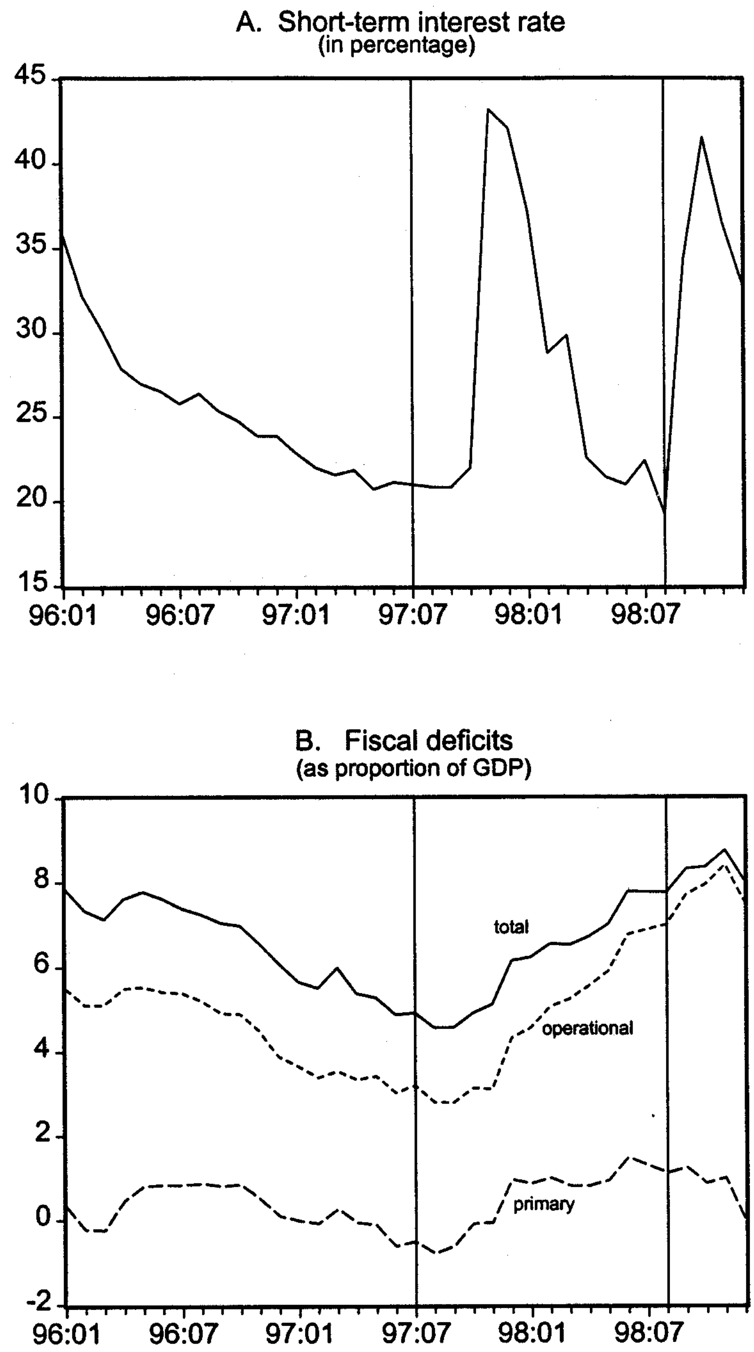
Figure 1. Brazil: Interest rate and fiscal deficits



Note: Vertical lines denote the onset of the Asian (July 1997) and Russian (August 1998) crises.
Source: Central Bank of Brazil

Figure 2. Timing of balance of payments crisis

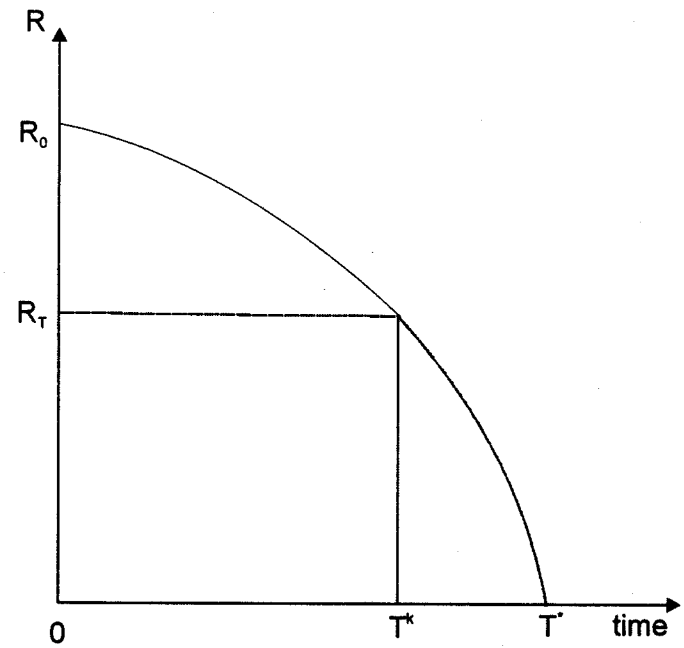
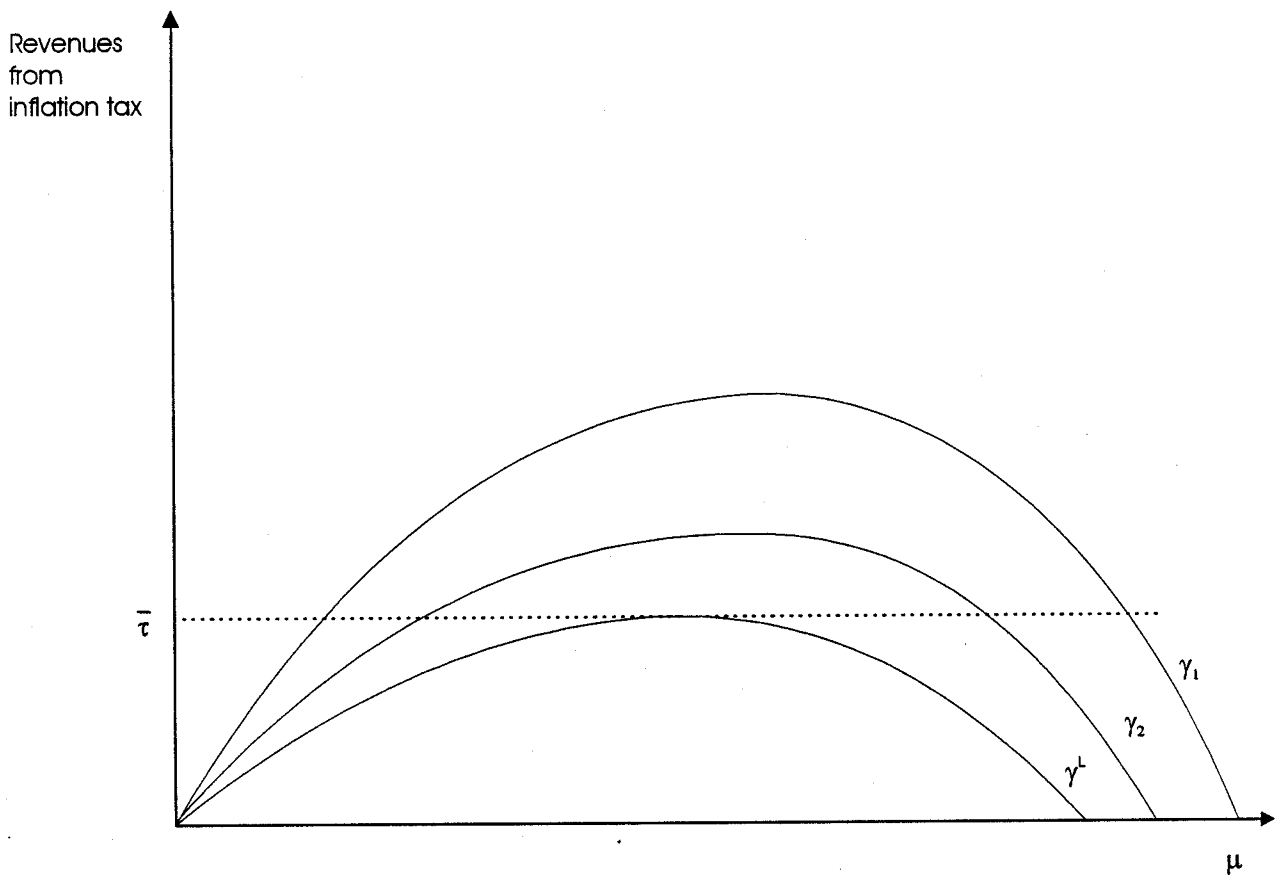


Figure 3. Laffer curves



Note: $\gamma^L > \gamma_2 > \gamma_1$

Figure 4. High fiscal spending case

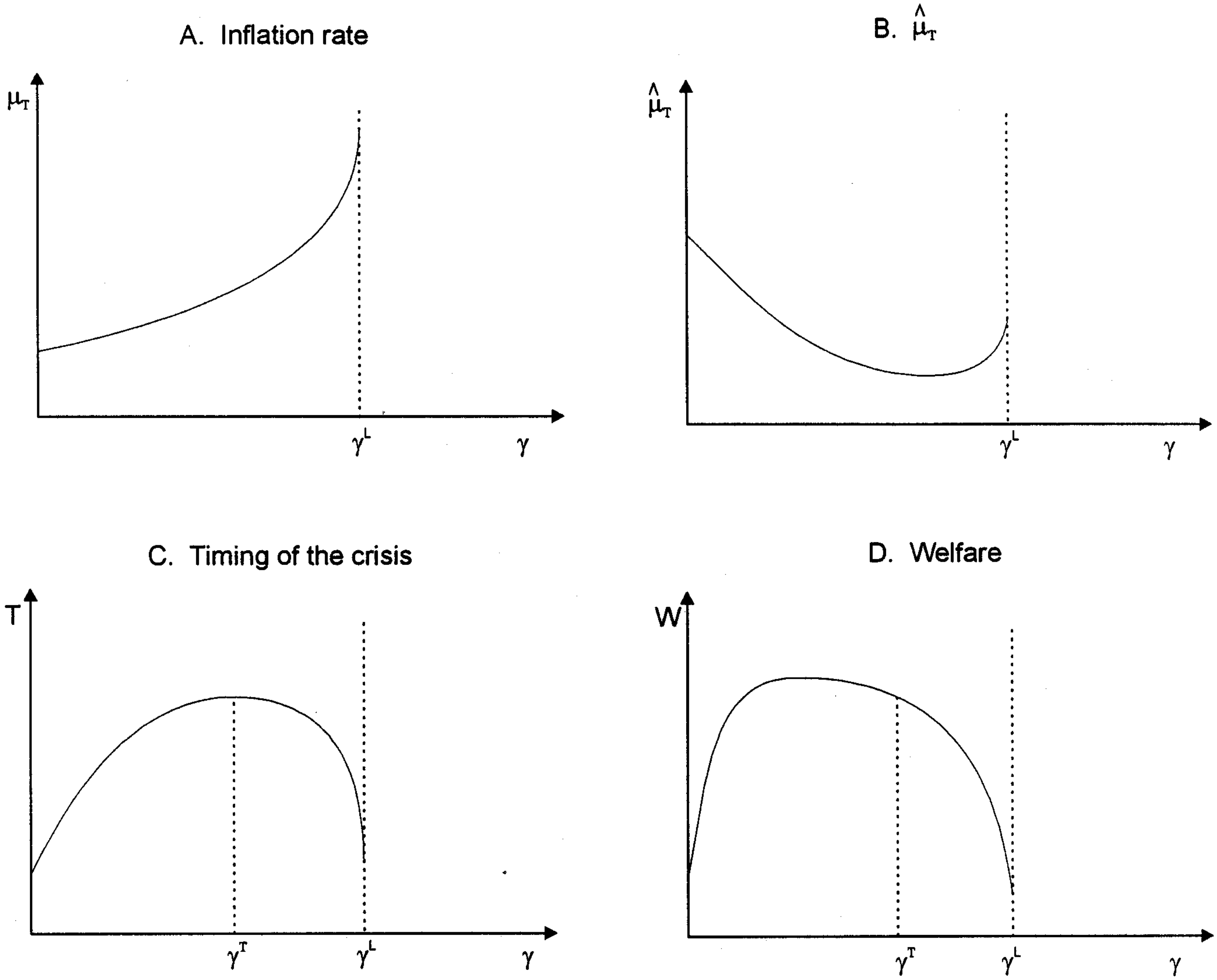


Figure 5. Separable case with high fiscal spending

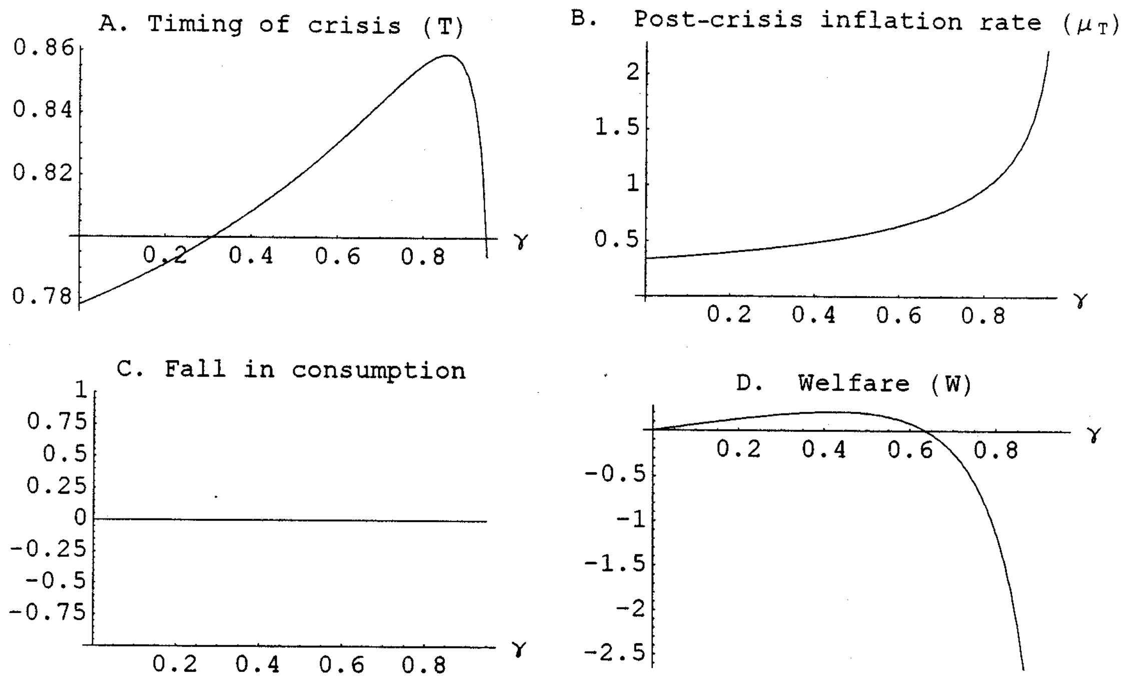


Figure 6. Separable case with low fiscal spending

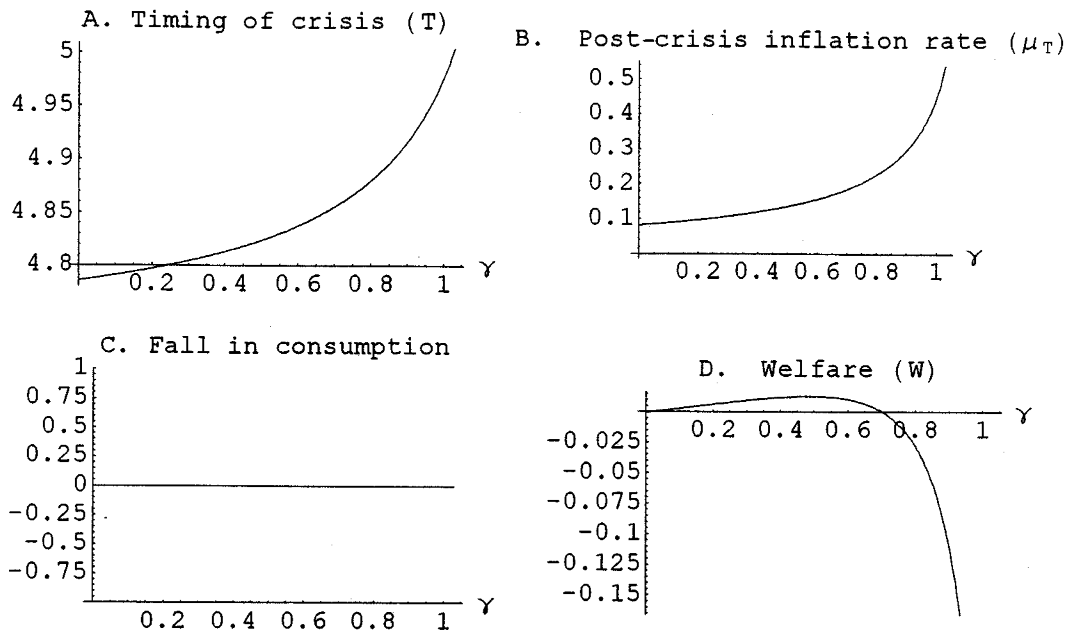


Figure 7. Non-separable case with high fiscal spending

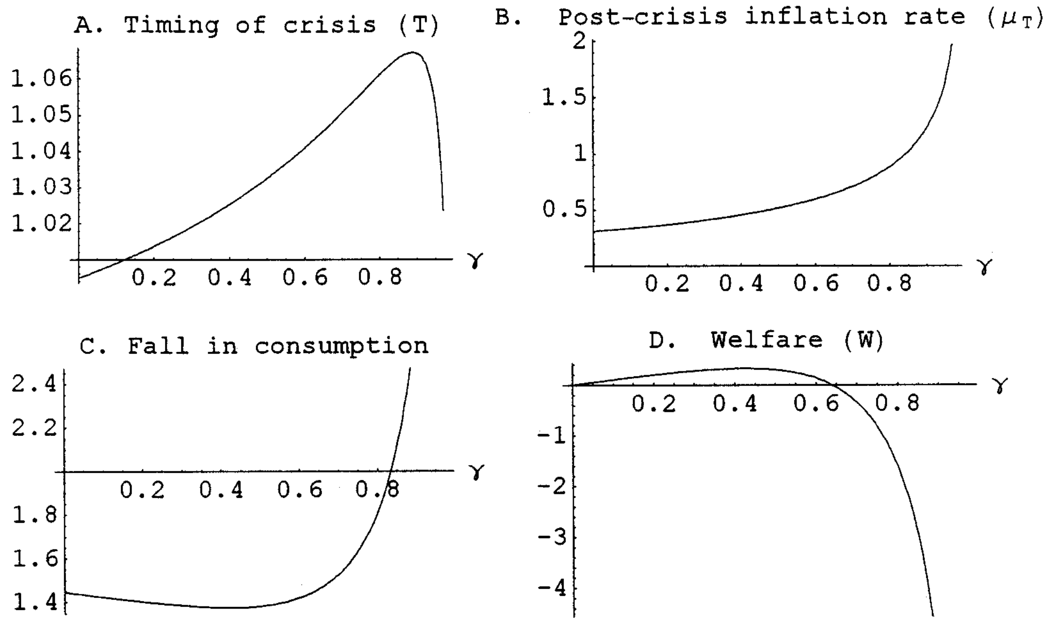


Figure 8. Liquidy - in - advance case