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TRADING VOLUME: DEFINITIONS, DATA ANALYSIS, AND IMPLICATIONS OF PORTFOLIO THEORY

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ABSTRACT

We examine the implications of portfolio theory for the cross-sectional behavior of equity trading volume. Two-fund separation theorems suggest a natural definition for trading activity: share turnover. If two-fund separation holds, share turnover must be identical for all securities. If (K+1)-fund separation holds, we show that turnover satisfies an approximately linear K-factor structure. These implications are examined empirically using individual weekly turnover data for NYSE and AMEX securities from 1962 to 1996. We find strong evidence against two-fund separation, and a principal-components decomposition suggests that turnover is well approximated by a two-factor linear model.

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Contents

1 Introduction

If price and quantity are the fundamental building blocks of any theory of market interactions, the importance of trading volume in modeling asset markets is clear. Although most models of asset markets have focused on the behavior of returns—predictability, variability, and information content—their implications for trading volume have received far less attention.

In this paper we derive the implications of various asset-market models for volume and quantify their importance using recently available volume data for individual securities from the Center for Research in Security Prices (CRSP). Although the volume literature is voluminous,[1] we hope to add to this literature in two ways.

First, we develop the volume implications of popular asset-market models rather than construct more specialized, and often "stylized", models to explain volume behavior. Given the far-reaching impact of mutual-fund separation theorems, the CAPM, and the intertemporal CAPM (ICAPM), the volume implications of these paradigms may have important consequences. In contrast to much of the existing volume literature's focus on the time-series behavior of volume—price/volume and volatility/volume relations, for example—in this paper we focus instead on the *cross-sectional* variation in volume. How does trading activity vary from stock to stock, and why? The fact that popular asset-market models have strong implications for the cross section of expected returns suggests that they may also have implications for the cross section of volume. By turning our attention to a new set of testable implications for these well-worn models, we hope to gain new insights into some old unresolved issues.

Second, we empirically estimate the volume relations suggested by these asset-market models using both cross-section and time-series data for individual securities, examining both the behavior of aggregate and individual volume over the sample period from 1962 to 1996 and across thousands of securities. Until recently, individual volume data for a broad

[1]At last count, our volume citation list numbered 190 articles spanning several fields of study, including economics, finance, and accounting. Within the finance literature, volume is studied in several distinct subfields: market microstructure, price/volume relations, volume/volatility relations, models of asymmetric information, and so on. Therefore, even a cursory literature review cannot do full justice to the breadth and depth of the volume literature. See Table 1 and the discussion below for a list of the most relevant articles for our current purposes.

cross section of securities was not readily available. In much the same way that models such as the CAPM and ICAPM have guided empirical investigations of the time-series and cross-sectional properties of asset returns, we show that the volume implications of these models provide similar guidelines for investigating the behavior of volume.

We begin in Section 2 with the basic definitions and notational conventions of our volume investigation—not a trivial task given the variety of volume measures used in the extant literature, e.g., shares traded, dollars traded, number of transactions, etc. We argue that turnover—shares traded divided by shares outstanding—is a natural measure of trading activity when viewed in the context of standard portfolio theory. In particular, in Section 3 we show that a two-fund separation theorem implies that turnover is identical across all assets, and a $(K+1)$-fund separation theorem implies that turnover has an approximate linear K-factor structure.

Using weekly turnover data for individual securities on the New York and American Stock Exchanges from 1962 to 1996—recently made available by the Center for Research in Securities Prices—we document in Section 4 the time-series and cross-sectional properties of turnover indexes, individual turnover, and portfolio turnover. Turnover indexes exhibit a clear time trend from 1962 to 1996, beginning at less than 0.5% in 1962, reaching a high of 4% in October 1987, and dropping to just over 1% at the end of our sample in 1996.

The cross section of turnover also varies through time, fairly concentrated in the early 1960's, much wider in the late 1960's, narrow again in the mid 1970's, and wide again after that. There is some persistence in turnover deciles from week to week—the largest- and smallest-turnover stocks in one week are often the largest- and smallest-turnover stocks, respectively, the next week—however, there is considerable diffusion of stocks across the intermediate turnover-deciles from one week to the next.

To investigate the cross-sectional variation of turnover in more detail, in Section 5 we perform cross-sectional regressions of average turnover on several regressors related to expected return, market capitalization, and trading costs. With R^2's ranging from 29.6% to 44.7%, these regressions show that stock-specific characteristics do explain a significant portion of the cross-sectional variation in turnover. This suggests the possibility of a parsimonious linear-factor representation of the turnover cross-section.

To investigate this possibility and the implications of standard portfolio theory, i.e.,

(K+1)-fund separation, we perform a principal-components decomposition of the covariance matrix of the turnover of ten portfolios, where the portfolios are constructed by sorting on turnover betas. Across five-year subperiods, we find that a one-factor model for turnover is a reasonable approximation, at least in the case of turnover-beta-sorted portfolios, and that a two-factor model captures well over 90% of the time-series variation in turnover.

We conclude in Section 6 with some suggestions for future research directions.

2 Definitions and Notation

The literature on trading activity in financial markets is extensive and a number of measures of volume have been proposed and studied.[2] Some studies of aggregate trading activity use the total number of shares traded on the NYSE as a measure of volume (see Epps and Epps (1976), Gallant, Rossi, and Tauchen (1992), Hiemstra and Jones (1994), and Ying (1966)). Other studies use aggregate *turnover*—the total number of shares traded divided by the total number of shares outstanding—as a measure of volume (see Campbell, Grossman, Wang (1993), LeBaron (1992), Smidt (1990), and the *1996 NYSE Fact Book*). Individual share volume is often used in the analysis of price/volume and volatility/volume relations (see Andersen (1996), Epps and Epps (1976), and Lamoureux and Lastrapes (1990, 1994)). Studies focusing on the impact of information events on trading activity use individual turnover as a measure of volume (see Bamber (1986, 1987), Lakonishok and Smidt (1986), Morse (1980), Richardson, Sefcik, Thompson (1986), Stickel and Verrecchia (1994)). Alternatively, Tkac (1996) considers individual dollar volume normalized by aggregate market dollar-volume. And even the total number of trades (Conrad, Hameed, and Niden (1994)) and the number of trading days per year (James and Edmister (1983)) have been used as measures of trading activity. Table 1 provides a summary of the various measures used in a representative sample of the recent volume literature. These differences suggest that different applications call for different volume measures.

After developing some basic notation in Section 2.1, we review several volume measures in Section 2.2 and provide some economic motivation for turnover as a canonical measure of trading activity. Formal definitions of turnover—for individual securities, portfolios, and in

[2]See Karpoff (1987) for an excellent introduction to and survey of this burgeoning literature.

the presence of time aggregation—are given in Sections 2.3–2.4.

2.1 Notation

Our analysis begins with I investors indexed by $i = 1, \ldots, I$ and J stocks indexed by $j = 1, \ldots, J$. We assume that all the stocks are risky and non-redundant. For each stock j, let N_{jt} be its total number of shares outstanding, D_{jt} its dividend, and P_{jt} its ex-dividend price at date t. For notational convenience and without loss of generality, we assume throughout that the total number of shares outstanding for each stock is constant over time, i.e., $N_{jt} = N_j$, $j = 1, \ldots, J$.

For each investor i, let S^i_{jt} denote the number of shares of stock j he holds at date t. Let $\mathbf{P}_t \equiv [\ P_{1t}\ \cdots\ P_{Jt}\ ]^\top$ and $\mathbf{S}_t \equiv [\ S_{1t}\ \cdots\ S_{Jt}]^\top$ denote the vector of stock prices and shares held in a given portfolio, where $\mathbf{A}^\top$ denotes the transpose of a vector or matrix $\mathbf{A}$. Let the return on stock j at t be $R_{jt} \equiv (P_{jt} - P_{jt-1} + D_{jt})/P_{jt-1}$. Finally, denote by X_{jt} the total number of shares of security j traded at time t, i.e., share volume, hence

$$X_{jt} \;=\; \frac{1}{2}\sum_{i=1}^{I} |S^i_{jt} - S^i_{jt-1}| \tag{2.1}$$

where the coefficient $\frac{1}{2}$ corrects for the double counting when summing the shares traded over all investors.

2.2 Motivation

To motivate the definition of volume used in this paper, we begin with a simple numerical example drawn from portfolio theory.[3] Consider a stock market comprised of only two securities, A and B. For concreteness, assume that security A has 10 shares outstanding and is priced at \$100 per share, yielding a market value of \$1000, and security B has 30 shares outstanding and is priced at \$50 per share, yielding a market value of \$1500, hence $N_{at} = 10$, $N_{bt} = 30$, $P_{at} = 100$, $P_{bt} = 50$. Suppose there are only two investors in this market—call them investor 1 and 2—and let two-fund separation hold so that both investors hold a combination of risk-free bonds and a stock portfolio with A and B in the same relative

[3] A more formal motivation is provided later in Section 3, namely, mutual-fund separation theorems and the cross-sectional properties of volume.

proportion. Specifically, let investor 1 hold 1 share of A and 3 shares of B, and let investor 2 hold 9 shares of A and 27 shares of B. In this way, all shares are held and both investors hold the same *market* portfolio (40% A and 60% B).

Now suppose that investor 2 liquidates $750 of his portfolio—3 shares of A and 9 shares of B—and assume that investor 1 is willing to purchase exactly this amount from investor 2 at the prevailing market prices.[4] After completing the transaction, investor 1 owns 4 shares of A and 12 shares of B, and investor 2 owns 6 shares of A and 18 shares of B. What kind of trading activity does this transaction imply?

For individual stocks, we can construct the following measures of trading activity:

- Number of trades per period
- Share volume, X_{jt}
- Dollar volume, $P_{jt}X_{jt}$
- Relative dollar volume, $P_{jt}X_{jt}/\sum_j P_{jt}X_{jt}$
- Share turnover,

$$\tau_{jt} \equiv \frac{X_{jt}}{N_{jt}}$$

- Dollar turnover,

$$\nu_{jt} \equiv \frac{P_{jt}X_{jt}}{P_{jt}N_{jt}} = \tau_{jt}$$

where $j = a, b$.[5] To measure aggregate trading activity, we can define similar measures:

- Number of trades per period
- Total number of shares traded, $X_{at}+X_{bt}$
- Dollar volume, $P_{at}X_{at}+P_{bt}X_{bt}$
- Share-weighted turnover,

$$\tau_t^{SW} \equiv \frac{X_{at}+X_{bt}}{N_a+N_b} = \frac{N_a}{N_a+N_b}\tau_{at} + \frac{N_b}{N_a+N_b}\tau_{bt}$$

[4]This last assumption entails no loss of generality but is made purely for notational simplicity. If investor 1 is unwilling to purchase these shares at prevailing prices, prices will adjust so that both parties are willing to consummate the transaction, leaving two-fund separation intact.

[5]Although the definition of dollar turnover may seem redundant since it is equivalent to share turnover, it will become more relevant in the portfolio case below (see Section 2.3).

- Equal-weighted turnover,

$$\tau_t^{EW} \equiv \frac{1}{2}\left(\frac{X_{at}}{N_a} + \frac{X_{bt}}{N_a}\right) = \frac{1}{2}(\tau_{at} + \tau_{bt})$$

- Value-weighted turnover,

$$\tau_t^{VW} \equiv \frac{P_{at}N_a}{P_{at}N_a + P_{bt}N_b}\frac{X_{at}}{N_a} + \frac{P_{bt}N_b}{P_{at}N_a + P_{bt}N_b}\frac{X_{bt}}{N_b} = \omega_{at}\tau_{at} + \omega_{bt}\tau_{bt} .$$

Table 2 reports the values that these various measures of trading activity take on for the hypothetical transaction between investors 1 and 2. Though these values vary considerably—2 trades, 12 shares traded, $750 traded—one regularity does emerge: the turnover measures are all identical. This is no coincidence, but is an implication of two-fund separation. If all investors hold the same relative proportions of risky assets at all times, then it can be shown that trading activity—as measured by turnover—must be identical across all risky securities (see Section 3). Although the other measures of volume do capture important aspects of trading activity, if the focus is on the relation between volume and equilibrium models of asset markets (such as the CAPM and ICAPM), turnover yields the sharpest empirical implications and is the most natural measure. For this reason, we will focus on turnover throughout this paper.

2.3 Defining Individual and Portfolio Turnover

For each individual stock j, let turnover be defined by:

Definition 1 *The turnover τ_{jt} of stock j at time t is*

$$\tau_{jt} \equiv \frac{X_{jt}}{N_j} \tag{2.2}$$

where X_{jt} is the share volume of security j at time t and N_j is the total number of shares outstanding of stock j.

Although we define the turnover ratio using the total number of shares traded, it is obvious that using the total dollar volume normalized by the total market value gives the same result.

Given that investors, particularly institutional investors, often trade portfolios or *baskets* of stocks, a measure of portfolio trading activity would be useful. But even after settling on turnover as the preferred measure of an individual stock's trading activity, there is still some ambiguity in extending this definition to the portfolio case. In the absence of a theory for which portfolios are traded, why they are traded, and how they are traded, there is no natural definition of portfolio turnover.[6] For the specific purpose of investigating the implications of portfolio theory for trading activity (see Section 3), we propose the following definition:

Definition 2 *For any portfolio p defined by the vector of shares held* $\mathbf{S}_t^p = [\, S_{1t}^p \;\cdots\; S_{Jt}^p]^\top$ *with non-negative holdings in all stocks, i.e.,* $S_{jt}^p \geq 0$ *for all j, and strictly positive market value, i.e.,* $\mathbf{S}_t^{p\top}\mathbf{P}_t > 0$*, let* $\omega_{jt}^p \equiv S_{jt}^p P_{jt}/(\mathbf{S}_t^{p\top}\mathbf{P}_t)$ *be the fraction invested in stock j,* $j = 1, \ldots, J$*. Then its turnover is defined to be*

$$\tau_t^p \equiv \sum_{j=1}^{J} \omega_{jt}^p \tau_{jt} \,. \tag{2.3}$$

Under this definition, the turnover of value-weighted and equal-weighted indexes are well-defined

$$\tau_t^{VW} \equiv \sum_{j=1}^{J} \omega_{jt}^{VW} \tau_{jt} \quad , \quad \tau_t^{EW} \equiv \frac{1}{J}\sum_{j=1}^{J} \tau_{jt} \tag{2.4}$$

respectively, where $\omega_{jt}^{VW} \equiv N_j P_{jt}/\left(\sum_j N_j P_{jt}\right)$, for $j = 1, \ldots, J$.

Although (2.3) seems to be a reasonable definition of portfolio turnover, some care must be exercised in interpreting it. While τ_t^{VW} and τ_t^{EW} are relevant to the theoretical implications derived in Section 3, they should be viewed only as particular weighted averages of individual turnover, not necessarily as the turnover of any specific trading strategy.

In particular, Definition 2 cannot be applied too broadly. Suppose, for example, shortsales are allowed so that some portfolio weights can be negative. In that case, (2.3) can be quite misleading since the turnover of short positions will offset the turnover of long positions. We

[6]Although it is common practice for institutional investors to trade baskets of securities, there are few regularities in how such baskets are generated or how they are traded, i.e., in piece-meal fashion and over time or all at once through a *principal bid*. Such diversity in the trading of portfolios makes it difficult to define single measure of portfolio turnover.

can modify (2.3) to account for short positions by using the absolute values of the portfolio weights

$$\tau_t^p \equiv \sum_{j=1}^{J} \frac{|\omega_{jt}^p|}{\sum_k |\omega_{kt}^p|} \tau_{jt} \tag{2.5}$$

but this can yield some anomalous results as well. For example, consider a two-asset portfolio with weights $\omega_{at} = 3$ and $\omega_{bt} = -2$. If the turnover of both stocks are identical and equal to τ, the portfolio turnover according to (2.5) is also τ, yet there is clearly a great deal more trading activity than this implies. Without specifying *why* and *how* this portfolio is traded, a sensible definition of portfolio turnover cannot be proposed.

Neither (2.3) or (2.5) are completely satisfactory measures of trading activities of a portfolio in general. Until we introduce a more specific context in which trading activity is to be measured, we shall have to satisfy ourselves with Definition 2 as a measure of trading activities of a portfolio.

2.4 Time Aggregation

Given our choice of turnover as a measure of volume for individual securities, the most natural method of handling time aggregation is to sum turnover across dates to obtain time-aggregated turnover. Although there are several other alternatives, e.g., summing share volume and then dividing by average shares outstanding, summing turnover offers several advantages. Unlike a measure based on summed shares divided by average shares outstanding, summed turnover is cumulative and linear, each component of the sum corresponds to the actual measure of trading activity for that day, and it is unaffected by "neutral" changes of units such as stock splits and stock dividends.[7] Therefore, we shall adopt this measure of time aggregation in our empirical analysis below.

[7]This last property requires one minor qualification: a "neutral" change of units is, by definition, one where trading activity is unaffected. However, stock splits can have non-neutral effects on trading activity such as enhancing liquidity (this is often one of the motivations for splits), and in such cases turnover will be affected (as it should be).

Definition 3 *If the turnover for stock j at time t is given by τ_{jt}, the turnover between $t-1$ to $t+q$, for any $q \geq 0$ is given by:*

$$\tau_{jt}(q) \equiv \tau_{jt} + \tau_{jt+1} + \cdots + \tau_{jt+q} . \tag{2.6}$$

3 Volume Implications of Portfolio Theory

The diversity in the portfolio holdings of individuals and institutions and in their motives for trade suggests that the time-series and cross-sectional patterns of trading activity can be quite complex. However, standard portfolio theory provides an enormous simplification: under certain conditions, *mutual-fund separation* holds, i.e., investors are indifferent between choosing among the entire universe of securities and a small number of mutual funds (see, for example, Cass and Stiglitz (1970), Markowitz (1952), Ross (1978), Tobin (1958), and Merton (1973)). In this case, all investors trade only in these *separating funds* and simpler cross-sectional patterns in trading activity emerge, and in this section we derive such cross-sectional implications.

While several models can deliver mutual-fund separation, e.g., the CAPM and ICAPM, we do not specify any such model in this study, but simply assert that mutual-fund separation holds. In particular, since the focus of this paper is primarily the cross-sectional properties of volume, we assume nothing about the behavior of asset prices, e.g., a factor structure for asset returns may or may not exist. As long as mutual-fund separation holds, the results in this section (in particular, Section 3.1 and 3.2) must apply.

The strong implications of mutual-fund separation for volume that we derive in this section suggest that the assumptions underlying the theory may be quite restrictive and therefore implausible (see, for example, Cass and Stiglitz (1970), Markowitz (1952), Ross (1978), and Tobin (1958)). For example, mutual-fund separation is often derived in static settings in which the motives for trade are not explicitly modeled. Also, most models of mutual-fund separation use a partial equilibrium framework with exogenously specified return distributions and strong restrictions on preferences. Furthermore, these models tend to focus on a rather narrow set of trading motives—changes in portfolio holdings due to changes in return distributions or preferences—ignoring other factors that may motivate individuals and institutions to adjust their portfolios, e.g., asymmetric information, idiosyncratic risk, transactions costs, taxes and other market imperfections. Finally, it has sometimes been argued that recent levels of trading activity in financial markets are simply too high to be attributable to the portfolio-rebalancing needs of rational economic agents.

A detailed discussion of these concerns is beyond the scope of this paper. Moreover, we are not advocating any particular structural model of mutual-fund separation here, but merely investigating the implications for trading volume when mutual-fund separation holds. Nevertheless, before deriving these implications in the following sections, it is important to consider how some of the limitations of mutual-fund separation may affect the interpretation of our analysis.

First, many of the limitations of mutual-fund separation theorems can be overcome to some degree. For example, extending mutual-fund separation results to dynamic settings is possible. As in the static case, restrictive assumptions on preferences and/or return processes are often required to obtain mutual-fund separation in a discrete-time setting. However, in a continuous-time setting—which has its own set of restrictive assumptions—Merton (1973) shows that mutual-fund separation holds for quite general preferences and return processes.

Also, it is possible to embed mutual-fund separation in a general equilibrium framework in which asset returns are determined endogenously. The CAPM is a well-known example of mutual-fund separation in a static equilibrium setting. To obtain mutual-fund separation in a dynamic equilibrium setting, stronger assumptions are required—Lo and Wang (1998) provide such an example.[8]

Of course, from a theoretical standpoint, no existing model is rich enough to capture the full spectrum of portfolio-rebalancing needs of all market participants, e.g., risk-sharing, hedging, liquidity, and speculation. Therefore, it is difficult to argue that current levels of trading activity are too high to be justified by rational portfolio rebalancing. Indeed, under the standard assumption of a diffusion information structure, volume is unbounded

[8]Tkac (1996) also attempts to develop a dynamic equilibrium model—a multi-asset extension of Dumas (1990)—in which two-fund separation holds. However, her specification of the model is incomplete. Moreover, if it is in the spirit of Dumas (1990) in which risky assets take the form of investments in linear production technologies (as in Cox, Ingersoll and Ross (1985)), the model has no volume implications for the risky assets since changes in investors' asset holdings involve changes in their own investment in production technologies, not in the trading of risky assets.

in absence of transaction costs. Moreover, from an empirical standpoint, little effort has been devoted to calibrating the level of trading volume within the context of a realistic asset-market model.

Despite the simplistic nature of mutual-fund separation, we study its volume implications for several reasons. One compelling reason is the fact that mutual-fund separation has become the workhorse of modern investment management. Although the assumptions of models such as the CAPM and ICAPM are known to be violated in practice, these models are viewed by many as a useful approximation for quantifying the trade-off between risk and expected return in financial markets. Thus, it seems natural to begin with such models in an investigation of trading activity in asset markets. Mutual-fund separation may seem inadequate—indeed, some might say irrelevant—for modeling trading activity, nevertheless it may yield an adequate approximation for quantifying the cross-sectional properties of trading volume. If it does not, then this suggests the possibility of important weaknesses in the theory, weaknesses that may have implications that extend beyond trading activity, e.g., preference restrictions, risk-sharing characteristics, asymmetric information, and liquidity. Of course, the virtue of such an approximation can only be judged by its empirical performance, which we examine in this paper.

Another reason for focusing on mutual-fund separation is that it can be an important benchmark in developing a more complete model of trading volume. The trading motives that mutual-fund separation captures (such as portfolio rebalancing) may be simple and incomplete, but they are important, at least in the context of models such as the CAPM and ICAPM. Using mutual-fund separation as a benchmark allows us to gauge how important other trading motives may be in understanding the different aspects of trading volume. For example, in studying the market reaction to corporate announcements and dividends, the factor model implied by mutual-fund separation can be used as a "market model" in defining the abnormal trading activity that is associated with these events (Tkac (1996) discusses this in the special case of two-fund separation).

Factors such as asymmetric information, idiosyncratic risks, transaction costs, and other forms of market imperfections are also likely to be relevant for determining the level and variability of trading activity. Each of these issues has been the focus of recent research, but only in the context of specialized models. To examine their importance in explaining volume, we need a more general and unified framework that can capture these factors. Unfortunately, such a model has not yet been developed.

For all these reasons, we propose to examine the implications of mutual-fund separation for trading activity. The theoretical implications serve as valuable guides for our data construction and empirical analysis, but it is useful to keep their limitations in mind. We view this as the first step in developing a more complete understanding of trading and pricing in asset markets and we hope to explore these other issues in future research (see Section 6).

In Section 3.1 we consider the case of two-fund separation in which one fund is the riskless asset and the second fund is a portfolio of risky assets. In Section 3.2 we investigate the general case of $(K+1)$-fund separation, one riskless fund and K risky funds. Mutual-fund separation with a riskless asset is often called *monetary separation* to distinguish it from the case without a riskless asset. We assume the existence of a riskless asset mainly to simplify the exposition, but for our purposes this assumption entails no loss of generality.[9] Thus, in what follows, we consider only cases of monetary separation without further qualification.

3.1 Two-Fund Separation

Without loss of generality, we normalize the total number of shares outstanding for each stock to one in this section, i.e., $N_j = 1$, $j = 1, \ldots, J$, and we begin by assuming two-fund separation, i.e., all investors invest in the same two mutual funds: the riskless asset and a stock fund. Market clearing requires that the stock fund is the "market" portfolio. Given our normalization, the market portfolio $\mathbf{S}^M$—measured in shares outstanding—is simply a vector of one's: $\mathbf{S}^M = [\,1 \;\cdots\; 1\,]^\top$. Two-fund separation implies that the stock holdings of any investor i at time t is given by:

$$\mathbf{S}_t^i \;=\; h_t^i \mathbf{S}^M \;=\; h_t^i \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}, \qquad i = 1, \ldots, I \tag{3.1}$$

where h_t^i is the share of the market portfolio held by investor i (and $\sum_i h_t^i = 1$ for all t). His holding in stock j is then $S_{jt}^i = h_t^i$, $j = 1, \ldots, J$. Over time, investor i may wish to adjust

[9] For example, if two-fund separation holds but both funds contain risky assets (as in Black's (1972) zero-beta CAPM), this is covered by our analysis of $(K+1)$-fund separation in Section 3.2 for $K = 2$ (since two of the three funds are assumed to contain risky assets).

his portfolio. If he does, he does so by trading only in the two funds (by the assumption of two-fund separation), hence he purchases or sells stocks in very specific proportions, as fractions of the market portfolio. His trading in stock j, normalized by shares outstanding, is: $S^i_{jt} - S^i_{jt-1} = h^i_t - h^i_{t-1}$, $i = 1, \ldots, I$. But this, in turn, implies $S^i_{jt} - S^i_{jt-1} = S^i_{j't} - S^i_{j't-1}$, $j, j' = 1, \ldots, J$. Thus, if two-fund separation holds, investor i's trading activity in each stock, normalized by shares outstanding, is identical across all stocks. This has an important implication for the turnover of stock j:

$$\tau_{jt} \;=\; \frac{1}{2}\sum_{i=1}^{I}|S^i_{jt} - S^i_{jt-1}| \;=\; \frac{1}{2}\sum_{i=1}^{I}|h^i_t - h^i_{t-1}| \, , \qquad j = 1, \ldots, J \tag{3.2}$$

which is given by the following proposition:

Proposition 1 *When two-fund separation holds, the turnover of all individual stocks are identical.*

Proposition 1 has strong implications for the turnover of the market portfolio. From the definition of Section 2.3, the turnover of the market portfolio is:

$$\tau^{VW}_t \;\equiv\; \sum_{j=1}^{J} w^{VW}_{jt}\tau_{jt} \;=\; \tau_{jt} \, , \qquad j = 1, \ldots, J.$$

The turnover of individual stocks is identical to the turnover of the market portfolio. This is not surprising given that individual stocks have identical values for turnover. Indeed, *all* portfolios of risky assets have the same turnover as individual stocks. For reasons that becomes apparent in Section 5, we can express the turnover of individual stocks as an exact linear one-factor model:

$$\tau_{jt} \;=\; b_j\tilde{F}_t \, , \qquad j = 1, \ldots, J \tag{3.3}$$

where $\tilde{F}_t = \tau^{VW}_t$ and $b_j = 1$.

Proposition 1 also implies that under two-fund separation the share volume of individual stocks is proportional to the total number of shares outstanding and dollar volume is proportional to market capitalization. Another implication is that each security's relative dollar-volume is identical to its relative market-capitalization for all t: $P_{jt}X_{jt}/\left(\sum_j P_{jt}X_{jt}\right) =$

$P_{jt}N_j / \left(\sum_j P_{jt}N_j\right)$. This relation is tested in Tkac (1996). Tkac (1996) derives this result in the context of a continuous-time dynamic equilibrium model with a special form of heterogeneity in preferences, but it holds more generally for any model that implies two-fund separation.[10]

3.2 $(K+1)$-Fund Separation

We now consider the more general case where $(K+1)$-fund separation holds. Let $\mathrm{S}_t^k = (S_{1t}^k, \ldots, S_{Jt}^k)^\top$, $k = 1, \ldots, K$, denote the K separating stock funds, where the separating funds are expressed in terms of the number of shares of their component stocks. The stock holdings of any investor i are given by

$$\begin{pmatrix} S_{1t}^i \\ \vdots \\ S_{Jt}^i \end{pmatrix} = \sum_{k=1}^{K} h_{kt}^i \mathrm{S}_t^k \,, \qquad i = 1, \ldots, I \,. \tag{3.4}$$

In particular, his holding in stock j is $S_{jt}^i = \sum_{k=1}^K h_{kt}^i S_{jt}^k$. Therefore, the turnover of stock j at time t is

$$\tau_{jt} = \frac{1}{2}\sum_{i=1}^{I} \left| S_{jt}^i - S_{jt-1}^i \right| = \frac{1}{2}\sum_{i=1}^{I} \left| \sum_{k=1}^{K} \left(h_{kt}^i S_{jt}^k - h_{kt-1}^i S_{jt}^k \right) \right| , \quad j = 1, \ldots, J \,. \tag{3.5}$$

To simplify notation, we define $\Delta h_{kt}^i \equiv h_{kt}^i - h_{kt-1}^i$ as the change in investor i's holding of fund k from $t-1$ to t.

We now impose the following assumption on the separating stock funds:

Assumption 1 *The separating stock funds, S_t^k, $k = 1, \ldots, K$, are constant over time.*

Given that, in equilibrium, $\sum_{i=1}^I \mathrm{S}_{i,t} = \mathrm{S}^M$ for all t, we have

$$\sum_{k=1}^{K} \left(\sum_{i=1}^{I} h_{kt}^i \right) \mathrm{S}^k = \mathrm{S}^M \,.$$

Therefore, without loss of generality, we can assume that the market portfolio S^M is one of the separating stock funds, which we label as the first fund. Following Merton (1973), we

[10] To see this, substitute $\tau_t N_j$ for X_{jt} in the numerator and denominator of the left side of the equation and observe that τ_t is constant over j hence it can be factored out of the summation and canceled.

call the remaining stock funds *hedging* portfolios.[11]

In addition, we assume that the amount of trading in the hedging portfolios is small for all investors:

Assumption 2 *For $k = 1, \ldots, K$, and $i = 1, \ldots, I$, $\Delta h^1_{1t} = \tilde{h}^1_{1t}$, $\Delta h^i_{kt} = \lambda \tilde{h}^i_{kt}$ $(k \neq 1)$, where $|\tilde{h}^i_{kt}| \leq H < \infty$, $0 < \lambda \ll 1$ and h^i_{1t}, h^i_{2t}, ..., h^i_{Jt} have a continuous joint probability density.*

We then have the following result (see the Appendix for the proof):

Lemma 1 *Under Assumptions 1–2, the turnover of stock j at time t can be approximated by*

$$\tau_{jt} \approx \frac{1}{2}\sum_{i=1}^{I}|\Delta h^i_{1t}| + \frac{1}{2}\sum_{k=2}^{K}\left[\sum_{i=1}^{I}\text{sgn}\left(\Delta h^i_{1t}+\Delta h^i_{kt}\right)\Delta h^i_{kt}\right]S^k_j\ , \quad j = 1, \ldots, J \qquad (3.6)$$

and the n-th absolute moment of the approximation error is $\mathrm{o}(\lambda^n)$.

Now define the following "factors":

$$\begin{aligned} \tilde{F}_{1t} &\equiv \frac{1}{2}\sum_{i=1}^{I}\left|\Delta h^i_{1t}\right| \\ \tilde{F}_{kt} &\equiv \frac{1}{2}\sum_{i=1}^{I}\text{sgn}\left(\Delta h^i_{1t}+\Delta h^i_{kt}\right)\Delta h^i_{kt}\ , \qquad k = 2, \ldots, K\ . \end{aligned}$$

Then the turnover of each stock j can be represented by an approximate K-factor model

$$\tau_{jt} = \tilde{F}_{1t} + \sum_{k=2}^{K} S^k_j \tilde{F}_{kt} + \mathrm{o}(\lambda)\ , \qquad j = 1, \ldots, J\ . \qquad (3.7)$$

In summary, we have:

Proposition 2 *Suppose that the riskless security, the market portfolio, and $K-1$ constant hedging portfolios are separating funds, and the amount of trading in the hedging portfolios is small. Then the turnover of each stock has an approximate K-factor structure.*

[11] In addition, we can assume that all the separating stock funds are mutually orthogonal, i.e., $\mathbf{S}^{k\top}\mathbf{S}^{k'} = 0$, $k = 1, \ldots, K$, $k' = 1, \ldots, K$, $k \neq k'$. In particular, $\mathbf{S}^{M\top}\mathbf{S}^k = \sum_{j=1}^{J} S^k_j = 0$, $k = 2, \ldots, K$, hence the total number of shares in each of the hedging portfolios sum to zero under our normalization. For this particular choice of the separating funds, h^i_{kt} has the simple interpretation that it is the projection coefficient of $\mathbf{S}^i_t$ on $\mathbf{S}^k$. Moreover, $\sum_{i=1}^{I} h^i_{1t} = 1$ and $\sum_{i=1}^{I} h^i_{kt} = 0$, $k = 2, \ldots, K$.

4 Exploratory Data Analysis

Having defined our measure of trading activity as turnover, we use the CRSP Daily Master File to construct *weekly* turnover series for individual NYSE and AMEX securities from July 1962 to December 1996 (1,800 weeks) using the time-aggregation method discussed in Section 2.4.[12] We choose a weekly horizon as the best compromise between maximizing sample size while minimizing the day-to-day volume and return fluctuations that have less direct economic relevance. And since our focus is the implications of portfolio theory for volume behavior, we confine our attention to ordinary common shares on the NYSE and AMEX (CRSP sharecodes 10 and 11 only), omitting ADRs, SBIs, REITs, closed-end funds, and other such exotica whose turnover may be difficult to interpret in the usual sense.[13] We also omit NASDAQ stocks altogether since the differences between NASDAQ and the NYSE/AMEX (market structure, market capitalization, etc.) have important implications for the measurement and behavior of volume (see, for example, Atkins and Dyl (1997)), and this should be investigated separately.

Throughout our empirical analysis, we report turnover and returns in units of percent per week—they are *not* annualized.

Finally, in addition to the exchange and sharecode selection criteria imposed, we also discard 37 securities from our sample because of a particular type of data error in the CRSP

[12]To facilitate research on turnover and to allow others to easily replicate our analysis, we have produced daily and weekly "MiniCRSP" dataset extracts comprised of returns, turnover, and other data items for each individual stock in the CRSP Daily Master file, stored in a format that minimizes storage space and access times. We have also prepared a set of access routines to read our extracted datasets via either sequential and random access methods on almost any hardware platform, as well as a user's guide to MiniCRSP (see Lim et al. (1998)). More detailed information about MiniCRSP can be found at the website `http://lfe.mit.edu/volume/`.

[13]The bulk of NYSE and AMEX securities are ordinary common shares, hence limiting our sample to securities with sharecodes 10 and 11 is not especially restrictive. For example, on January 2, 1980, the entire NYSE/AMEX universe contained 2,307 securities with sharecode 10, 30 securities with sharecode 11, and 55 securities with sharecodes other than 10 and 11. Ordinary common shares also account for the bulk of the market capitalization of the NYSE and AMEX (excluding ADRs of course).

volume entries.[14]

4.1 Secular Trends

Although it is difficult to develop simple intuition for the behavior of the entire time-series/cross-section volume dataset—a dataset containing between 1,700 and 2,200 individual securities per week over a sample period of 1,800 weeks—some gross characteristics of volume can be observed from value-weighted and equal-weighted turnover indexes.[15] These characteristics are presented in Figures 1–3, and in Tables 3 and 4.

Figure 1a shows that value-weighted turnover has increased dramatically since the mid-1960's, growing from less than 0.20% to over 1% per week. The volatility of value-weighted turnover also increases over this period. However, equal-weighted turnover behaves somewhat differently: Figure 1b shows that it reaches a peak of nearly 2% in 1968, then declines until the 1980's when it returns to a similar level (and goes well beyond it during October 1987). These differences between the value- and equal-weighted indexes suggest that smaller-capitalization companies can have high turnover.

Since turnover is, by definition, an asymmetric measure of trading activity—it cannot be negative—its empirical distribution is naturally skewed. Taking natural logarithms may provide more (visual) information about its behavior and this is done in Figures 1c and 1d. Although a trend is still present, there is more evidence for cyclical behavior in both indexes.

Table 3 reports various summary statistics for the two indexes over the 1962–1996 sample period as well as over five-year subperiods. Over the entire sample the average weekly turnover for the value-weighted and equal-weighted indexes is 0.78% and 0.91%, respectively.

[14]Briefly, the NYSE and AMEX typically report volume in round lots of 100 shares—"45" represents 4500 shares—but on occasion volume is reported in shares and this is indicated by a "Z" flag attached to the particular observation. This Z status is relatively infrequent, usually valid for at least a quarter, and may change over the life of the security. In some instances, we have discovered daily share volume increasing by a factor of 100, only to decrease by a factor of 100 at a later date. While such dramatic shifts in volume is not altogether impossible, a more plausible explanation—one that we have verified by hand in a few cases—is that the Z flag was inadvertently omitted when in fact the Z status was in force. See Lim et al. (1998) for further details.

[15]These indexes are constructed from weekly individual security turnover, where the value-weighted index is re-weighted each week. Value-weighted and equal-weighted return indexes are also constructed in a similar fashion. Note that these return indexes do not correspond exactly to the time-aggregated CRSP value-weighted and equal-weighted return indexes because we have restricted our universe of securities to ordinary common shares. However, some simple statistical comparisons show that our return indexes and the CRSP return indexes have very similar time series properties.

The standard deviation of weekly turnover for these two indexes is 0.48% and 0.37%, respectively, yielding a coefficient of variation of 0.62 for the value-weighted turnover index and 0.41 for the equal-weighted turnover index. In contrast, the coefficients of variation for the value-weighted and equal-weighted *returns* indexes are 8.52 and 6.91, respectively. Turnover is not nearly so variable as returns, relative to their means.

Table 3 also illustrates the nature of the secular trend in turnover through the five-year subperiod statistics. Average weekly value-weighted and equal-weighted turnover is 0.25% and 0.57%, respectively, in the first subperiod (1962–1966); they grow to 1.25% and 1.31%, respectively, by the last subperiod (1992–1996). At the beginning of the sample, equal-weighted turnover is three to four times more volatile than value-weighted turnover (0.21% versus 0.07% in 1962–1966, 0.32% versus 0.08% in 1967–1971), but by the end of the sample their volatilities are comparable (0.22% versus 0.23% in 1992–1996).

The subperiod containing the October 1987 crash exhibits a few anomalous properties: excess skewness and kurtosis for both returns and turnover, average value-weighted turnover slightly higher than average equal-weighted turnover, and slightly higher volatility for value-weighted turnover. These anomalies are consistent with the extreme outliers associated with the 1987 crash (see Figures 1a,b).

4.2 Nonstationarity and Detrending

Table 3 also reports the percentiles of the empirical distributions of turnover and returns which document the skewness in turnover that Figure 1 hints at, as well as the first 10 autocorrelations of turnover and returns and the corresponding Box-Pierce Q-statistics. Unlike returns, turnover is highly persistent, with autocorrelations that start at 91.25% and 86.73% for the value-weighted and equal-weighted turnover indexes, respectively, decaying very slowly to 84.63% and 68.59%, respectively, at lag 10. This slow decay suggests some kind of nonstationarity in turnover—perhaps a stochastic trend or *unit root* (see Hamilton (1994), for example)—and this is confirmed at the usual significance levels by applying the Kwiatkowski et al. (1992) Lagrange Multiplier test of stationarity versus a unit root to the two turnover indexes.[16]

[16] In particular, two LM tests were applied: a test of the level-stationary null, and a test of the trend-stationary null, both against the alternative of difference-stationarity. The test statistics are 17.41 (level) and 1.47 (trend) for the value-weighted index and 9.88 (level) and 1.06 (trend) for the equal-weighted index.

For these reasons, many empirical studies of volume use some form of detrending to induce stationarity. This usually involves either taking first differences or estimating the trend and subtracting it from the raw data. To gauge the impact of various methods of detrending on the time-series properties of turnover, we report summary statistics of detrended turnover in Table 4 where we detrend according to the following six methods:

$$\tau_{1t}^d = \tau_t - \left(\hat{\alpha}_1 + \hat{\beta}_1 t \right) \tag{4.8}$$

$$\tau_{2t}^d = \log \tau_t - \left(\hat{\alpha}_2 + \hat{\beta}_2 t \right) \tag{4.9}$$

$$\tau_{3t}^d = \tau_t - \tau_{t-1} \tag{4.10}$$

$$\tau_{4t}^d = \frac{\tau_t}{(\tau_{t-1} + \tau_{t-2} + \tau_{t-3} + \tau_{t-4})/4} \tag{4.11}$$

$$\begin{aligned} \tau_{5t}^d = \tau_t - \Big(& \hat{\alpha}_4 + \hat{\beta}_{3,1} t + \hat{\beta}_{3,2} t^2 + \\ & \hat{\beta}_{3,3}\mathrm{DEC1}_t + \hat{\beta}_{3,4}\mathrm{DEC2}_t + \hat{\beta}_{3,5}\mathrm{DEC3}_t + \hat{\beta}_{3,6}\mathrm{DEC4}_t + \\ & \hat{\beta}_{3,7}\mathrm{JAN1}_t + \hat{\beta}_{3,8}\mathrm{JAN2}_t + \hat{\beta}_{3,9}\mathrm{JAN3}_t + \hat{\beta}_{3,10}\mathrm{JAN4}_t + \\ & \hat{\beta}_{3,11}\mathrm{MAR}_t + \hat{\beta}_{3,12}\mathrm{APR}_t + \cdots + \hat{\beta}_{3,19}\mathrm{NOV}_t \Big) \end{aligned} \tag{4.12}$$

$$\tau_{6t}^d = \tau_t - \hat{\mathrm{K}}(\tau_t) \tag{4.13}$$

where (4.8) denotes linear detrending, (4.9) denotes log-linear detrending, (4.10) denotes first-differencing, (4.11) denotes a four-lag moving-average normalization, (4.12) denotes linear-quadratic detrending and deseasonalization (in the spirit of Gallant, Rossi, and Tauchen (1994)),[17] and (4.13) denotes nonparametric detrending via kernel regression (where the bandwidth is chosen optimally via cross validation).

The summary statistics in Table 4 show that the detrending method can have a substantial impact on the time-series properties of detrended turnover. For example, the skewness of

The 1% critical values for these two tests are 0.739 and 0.216, respectively. See Hamilton (1994) and Kwiatkowski et al. (1992) for further details concerning unit root tests, and Andersen (1996) and Gallant, Rossi, and Tauchen (1992) for highly structured (but semiparametric) procedures for detrending individual and aggregate daily volume.

[17]In particular, in (4.12) the regressors $\mathrm{DEC1}_t, \ldots, \mathrm{DEC4}_t$ and $\mathrm{JAN1}_t, \ldots, \mathrm{JAN4}_t$ denote weekly indicator variables for the weeks in December and January, respectively, and $\mathrm{MAR}_t, \ldots, \mathrm{NOV}_t$ denote monthly indicator variables for the months of March through November (we have omitted February to avoid perfect collinearity). This does not correspond exactly to the Gallant, Rossi, and Tauchen (1994) procedure—they detrend and deseasonalize the volatility of volume as well.

detrended value-weighted turnover varies from 0.09 (log-linear) to 1.77 (kernel), and the kurtosis varies from −0.20 (log-linear) to 29.38 (kernel). Linear, log-linear, and Gallant, Rossi, and Tauchen (GRT) detrending seem to do little to eliminate the persistence in turnover, yielding detrended series with large positive autocorrelation coefficients that decay slowly from lags 1 to 10. However, first-differenced value-weighted turnover has an autocorrelation coefficient of −34.94% at lag 1, which becomes positive at lag 4, and then alternates sign from lags 6 through 10. In contrast, kernel-detrended value-weighted turnover has an autocorrelation of 23.11% at lag 1, which becomes negative at lag 3 and remains negative through lag 10. Similar disparities are also observed for the various detrended equal-weighted turnover series.

Despite the fact that the R^2's of the six detrending methods are comparable for the value-weighted turnover index—ranging from 70.6% to 88.6%—the basic time-series properties vary considerably from one detrending method to the next.[18] To visualize the impact that various detrending methods can have on turnover, compare the various plots of detrended value-weighted turnover in Figure 2a, and detrended equal-weighted turnover in Figure 2b.[19] Even linear and log-linear detrending yield differences that are visually easy to detect: linear detrended turnover is smoother at the start of the sample and more variable towards the end, whereas loglinearly detrended turnover is equally variable but with lower-frequency fluctuations. The moving-average series looks like white noise, the log-linear series seems to possess a periodic component, and the remaining series seem heteroskedastic.

For these reasons, we shall continue to use raw turnover rather than its first difference or any other detrended turnover series in much of our empirical analysis (the sole exception is the eigenvalue decomposition of the first differences of turnover in Table 8). To address the problem of the apparent time trend and other nonstationarities in raw turnover, the empirical analysis of Section 5 is conducted within five-year subperiods only (the exploratory data

[18]The R^2 for each detrending method is defined by

$$R_j^2 \equiv 1 - \frac{\sum_t (\tau_{jt}^d - \overline{\tau}_j^d)^2}{\sum_t (\tau_t - \overline{\tau})^2} .$$

Note that the R^2's for the detrended equal-weighted turnover series are comparable to those of the value-weighted series except for linear, log-linear, and GRT detrending—evidently, the high turnover of small stocks in the earlier years creates a "cycle" that is not as readily explained by linear, log-linear, and quadratic trends (see Figure 1).

[19]To improve legibility, only every 10th observation is plotted in each of the panels of Figures 2a and 2b.

analysis of this section contains entire-sample results primarily for completeness).[20] This is no doubt a controversial choice and, therefore, requires some justification.

From a purely statistical point of view, a nonstationary time series is nonstationary over *any* finite interval—shortening the sample period cannot induce stationarity. Moreover, a shorter sample period increases the impact of sampling errors and reduces the power of statistical tests against most alternatives.

However, from an empirical point of view, confining our attention to five-year subperiods is perhaps the best compromise between letting the data "speak for themselves" and imposing sufficient structure to perform meaningful statistical inference. We have very little confidence in our current understanding of the trend component of turnover, yet a well-articulated model of the trend is a pre-requisite to detrending the data. Rather than filter our data through a specific trend process that others might not find as convincing, we choose instead to analyze the data with methods that require minimal structure, yielding results that may be of broader interest than those of a more structured analysis.[21]

Of course, *some* structure is necessary for conducting any kind of statistical inference. For example, we must assume that the mechanisms governing turnover is relatively stable over five-year subperiods, otherwise even the subperiod inferences may be misleading. Nevertheless, for our current purposes—exploratory data analysis and tests of the implications of portfolio theory—the benefits of focusing on subperiods are likely to outweigh the costs of larger sampling errors.

4.3 The Cross Section of Turnover

To develop a sense for cross-sectional differences in turnover over the sample period, we turn our attention from turnover indexes to the turnover of individual securities. Figures 3a–d provide a graphical representation of the cross section of turnover: Figure 3a plots

[20]However, we acknowledge the importance of stationarity in conducting formal statistical inferences—it is difficult to interpret a t-statistic in the presence of a strong trend. Therefore, the summary statistics provided in this section are intended to provide readers with an intuitive feel for the behavior of volume in our sample, not to be the basis of formal hypothesis tests.

[21]See Andersen (1996) and Gallant, Rossi, and Tauchen (1992) for an opposing view—they propose highly structured detrending and deseasonalizing procedures for adjusting raw volume. Andersen (1996) uses two methods: nonparametric kernel regression and an equally weighted moving average. Gallant, Rossi, and Tauchen (1992) extract quadratic trends and seasonal indicators from both the mean and variance of log volume.

the deciles for the turnover cross-section—nine points, representing the 10-th percentile, the 20-th percentile, and so on—for each of the 1,800 weeks in the sample period; Figure 3b simplifies this by plotting the deciles of the cross section of *average* turnover, averaged within each year; and Figures 3c and 3d plot the same data but on a logarithmic scale.

Figures 3a–b show that while the median turnover (the horizontal bars with vertical sides in Figure 3b) is relatively stable over time—fluctuating between 0.2% and just over 1% over the 1962–1996 sample period—there is considerable variation in the cross-sectional dispersion over time. The range of turnover is relatively narrow in the early 1960's, with 90% of the values falling between 0% and 1.5%, but there is a dramatic increase in the late 1960's, with the 90-th percentile approaching 3% at times. The cross-sectional variation of turnover declines sharply in the mid-1970's and then begins a steady increase until a peak in 1987, followed by a decline and then a gradual increase until 1996.

The logarithmic plots in Figures 3c–d seem to suggest that the cross-sectional distribution of log-turnover is similar over time up to a location parameter. This implies a potentially useful statistical or "reduced-form" description of the cross-sectional distribution of turnover: an identically distributed random variable multiplied by a time-varying scale factor.

5 Cross-Sectional Analysis of Turnover

The implications of standard portfolio theory for turnover provide a natural direction for empirical analysis: look for linear factor structure in the turnover cross-section. If two-fund separation holds, turnover should be identical across all stocks, i.e., a one-factor linear model where all stocks have identical factor loadings. If $(K+1)$-fund separation holds, turnover should satisfy a K-factor linear model. We examine these hypotheses in Sections 5.1 and 5.4. Throughout this section, we shall focus our attention on the sample of CRSP weekly turnover and returns data from July 1962 to December 1996 described in Section 4.

5.1 Specification of Cross-Sectional Regressions

It is clear from Section 4.3 and Figure 3 that turnover varies considerably in the cross section, hence two-fund separation may be rejected out of hand. However, the turnover implications of two-fund separation might be *approximately* correct in the sense that the cross-sectional variation in turnover may be "idiosyncratic" white noise, e.g., cross-sectionally uncorrelated and without common factors. We shall test this, and the more general $(K+1)$-fund separation hypothesis, in Section 5.4, but before doing so, we first consider a less formal, more exploratory analysis of the cross-sectional variation in turnover. In particular, we wish to examine the explanatory power of several economically motivated variables such as expected return, volatility, and trading costs in explaining the cross section of turnover.

To do this, we estimate cross-sectional regressions over five-year subperiods where the dependent variable is the median turnover $\tilde{\tau}_j$ of stock j and the explanatory variables are the following stock-specific characteristics:[22]

$\hat{\alpha}_{r,j}$: Intercept coefficient from the time-series regression of stock j's return on the value-weighted market return.

$\hat{\beta}_{r,j}$: Slope coefficient from the time-series regression of stock j's return on the value-weighted market return.

$\hat{\sigma}_{\epsilon,r,j}$: Residual standard deviation of the time-series regression of stock j's return on the value-weighted market return.

v_j: Average of natural logarithm of stock j's market capitalization.

p_j: Average of natural logarithm of stock j's price.

d_j: Average of dividend yield of stock j, where dividend yield in week t is defined by

$$d_{jt} = \max\left[\,0\,,\ \log\Big((1+R_{jt})V_{jt-1}/V_{jt}\Big)\,\right]$$

and V_{jt} is j's market capitalization in week t.

SP500_j Indicator variable for membership in the S&P 500 Index.

$\hat{\gamma}_{r,j}(1)$ First-order autocovariance of returns.

[22]We use median turnover instead of mean turnover to minimize the influence of outliers (which can be substantial in this dataset). Also, within each five-year period we exclude all stocks that are missing turnover data for more than two-thirds of the subsample.

The inclusion of these regressors in our cross-sectional analysis is loosely motivated by various intuitive "theories" that have appeared in the volume literature.

The motivation for the first three regressors comes partly from linear asset-pricing models such as the CAPM and APT; they capture excess expected return ($\hat{\alpha}_{r,j}$), systematic risk ($\hat{\beta}_{r,j}$), and residual risk ($\hat{\sigma}_{\epsilon,r,j}$), respectively. To the extent that expected excess return ($\hat{\alpha}_{r,j}$) may contain a premium associated with liquidity (see, for example, Amihud and Mendelson (1986a,b) and Hu (1997)) and heterogeneous information (see, for example, He and Wang (1995) and Wang (1994)), it should also give rise to cross-sectional differences in turnover. Although a higher premium from lower liquidity should be inversely related to turnover, a higher premium from heterogeneous information can lead to either higher or lower turnover, depending on the nature of information heterogeneity. The two risk measures of an asset, $\hat{\beta}_{r,j}$ and $\hat{\sigma}_{\epsilon,r,j}$, also measure the volatility in its returns that is associated with systematic risk and residual risk, respectively. Given that realized returns often generate portfolio-rebalancing needs, the volatility of returns should be positively related to turnover.

The motivation for log-market-capitalization (v_j) and log-price (p_t) is two-fold. On the theoretical side, the role of market capitalization in explaining volume is related to Merton's (1987) model of capital market equilibrium in which investors hold only the assets they are familiar with. This implies that larger-capitalization companies tend to have more diverse ownership, which can lead to more active trading. The motivation for log-price is related to trading costs. Given that part of trading costs comes from the bid-ask spread, which takes on discrete values in dollar terms, the actual costs in percentage terms are inversely related to price levels. This suggests that volume should be positively related to prices.

On the empirical side, there is an extensive literature documenting the significance of log-market-capitalization and log-price in explaining the cross-sectional variation of expected returns, e.g., Banz (1981), Black (1976), Brown, Van Harlow, and Tinic (1993), Marsh and Merton (1987), and Reinganum (1992). If size and price are genuine factors driving expected returns, they should drive turnover as well (see Lo and Wang (1998) for a more formal derivation and empirical analysis of this intuition).

Dividend yield (d_j) is motivated by its (empirical) ties to expected returns, but also by *dividend-capture* trades—the practice of purchasing stock just before its ex-dividend date and then selling it shortly thereafter.[23] Often induced by differential taxation of dividends versus capital gains, dividend-capture trading has been linked to short-term increases in trading activity, e.g., Karpoff and Walking (1988, 1990), Lakonishok and Smidt (1986), Lakonishok and Vermaelen (1986), Lynch-Koski (1996), Michaely (1991), Michaely and Murgia (1995), Michaely and Vila (1995, 1996), and Stickel (1991). Stocks with higher dividend yields should induce more dividend-capture trading activity, and this may be reflected in higher median turnover.

The effects of membership in the S&P 500 have been documented in many studies, e.g., Dhillon and Johnson (1991), Goetzmann and Garry (1986), Harris and Gurel (1986), Jacques (1988), Jain (1987), Lamoureux and Wansley (1987), Pruitt and Wei (1989), Shleifer (1986), Tkac (1996), and Woolridge and Ghosh (1986). In particular, Harris and Gurel (1986) document increases in volume just after inclusion in the S&P 500, and Tkac (1996) uses an S&P 500 indicator variable to explain the cross-sectional dispersion of relative turnover (relative dollar-volume divided by relative market-capitalization). The obvious motivation for this variable is the growth of indexation by institutional investors, and by the related practice of *index arbitrage*, in which disparities between the index futures price and the spot prices of the component securities are exploited by taking the appropriate positions in the futures and spot markets. For these reasons, stocks in the S&P 500 index should have higher turnover than others. Indexation began its rise in popularity with the advent of the mutual-fund industry in the early 1980's, and index arbitrage first became feasible in 1982 with the introduction of the Chicago Mercantile Exchange's S&P 500 futures contracts. Therefore, the effects of S&P 500 membership on turnover should be more dramatic in the later subperiods. Another motivation for S&P 500 membership is its effect on the publicity of member companies, which leads to more diverse ownership and more trading activity in the context of Merton (1987).

The last variable, the first-order return autocovariance ($\hat{\gamma}_{r,j}(1)$), serves as a proxy for trading costs, as in Roll's (1984) model of the "effective" bid/ask spread. In that model, Roll shows that in the absence of information-based trades, prices bouncing between bid and ask prices implies the following approximate relation between the spread and the first-order

[23] Our definition of d_j is meant to capture net corporate distributions or outflows (recall that returns R_{jt} are inclusive of all dividends and other distributions). The purpose of the non-negativity restriction is to ensure that inflows, e.g., new equity issues, are not treated as negative dividends.

return autocovariance:

$$\frac{s_{r,j}^2}{4} \approx -\operatorname{Cov}[R_{jt}, R_{jt-1}] \equiv -\gamma_{r,j}(1) \tag{5.1}$$

where $s_{r,j} \equiv s_j/\sqrt{P_{aj}P_{bj}}$ is the percentage effective bid/ask spread of stock j as a percentage of the geometric average of the bid and ask prices P_{bj} and P_{aj}, respectively, and s_j is the dollar bid/ask spread.

Rather than solve for $s_{r,j}$, we choose instead to include $\hat{\gamma}_{r,j}(1)$ as a regressor to sidestep the problem of a positive sample first-order autocovariance, which yields a complex number for the effective bid/ask spread. Of course, using $\hat{\gamma}_{r,j}(1)$ does not eliminate this problem, which is a symptom of a specification error, but rather is a convenient heuristic that allows us to estimate the regression equation (complex observations for even one regressor can yield complex parameter estimates for all the other regressors as well!). This heuristic is not unlike Roll's method for dealing with positive autocovariances, however, it is more direct.[24]

Under the trading-cost interpretation for $\hat{\gamma}_{r,j}(1)$, we should expect a positive coefficient in our cross-sectional turnover regression—a large negative value for $\hat{\gamma}_{r,j}(1)$ implies a large bid/ask spread, which should be associated with lower turnover. Alternatively, Roll (1984) interprets a positive value for $\hat{\gamma}_{r,j}(1)$ as a negative bid/ask spread, hence turnover should be higher for such stocks.

These eight regressors yield the following regression equation to be estimated:

$$\begin{aligned}\tilde{\tau}_j \;=\; \gamma_0 + \gamma_1\hat{\alpha}_{r,j} + \gamma_2\hat{\beta}_{r,j} + \gamma_3\hat{\sigma}_{\epsilon,r,j} + \gamma_4 v_j + \gamma_5 p_j + \gamma_6 d_j \;+ \\ \gamma_7\mathrm{SP500}_j + \gamma_8\hat{\gamma}_{r,j}(1) + \epsilon_j\,.\end{aligned} \tag{5.2}$$

[24]In a parenthetical statement in footnote a of Table I, Roll (1984) writes "The sign of the covariance was preserved after taking the square root".

5.2 Summary Statistics For Regressors

Table 5 reports summary statistics for these regressors, as well as for three other variables relevant to Section 5.4:

$\hat{\alpha}_{\tau,j}$: Intercept coefficient from the time-series regression of stock j's turnover on the value-weighted market turnover.

$\hat{\beta}_{\tau,j}$: Slope coefficient from the time-series regression of stock j's turnover on the value-weighted market turnover.

$\hat{\sigma}_{\epsilon,\tau,j}$: Residual standard deviation of the time-series regression of stock j's turnover on the value-weighted market turnover.

These three variables are loosely motivated by a one-factor linear model of turnover, though as we saw in Section 3.1 the single-factor model for turnover implied by two-fund separation is a degenerate one. However, we shall make use of turnover betas in our empirical analysis of $(K+1)$-fund separation in Section 5.4, hence we summarize their empirical properties here.

Table 5 contains means, medians, and standard deviations for these variables over each of the seven subperiods. The entries show that return betas are approximately 1.0 on average, with a cross-sectional standard deviation of about 0.5. Observe that return betas have approximately the same mean and median in all subperiods, indicating an absence of dramatic skewness and outliers in their empirical distributions.

In contrast, turnover betas have a considerably higher means, starting at 2.2 in the first subperiod (1962–1966) to an all-time high of 3.1 in the second subperiod (1967–1971), and declining steadily thereafter to 0.7 (1987–1991) and 0.8 (1992–1996). Also, the means and medians of turnover betas differ dramatically, particularly in the earlier subperiods, e.g., 2.2 mean versus 0.7 median (1962–1966) and 3.1 mean versus 1.9 median (1967–71), implying a skewed empirical distribution with some outliers in the right tail. Turnover betas are also more variable than return betas, with cross-sectional standard deviations that range from twice to ten times those of return betas.

The summary statistics for the first-order return autocovariances show that they are negative on average, which is consistent with the trading-cost interpretation, though there is considerable skewness in their distribution as well given the differences between means and medians. The means and medians vary from subperiod to subperiod in a manner also consistent with the trading-cost interpretation—the higher the median of median turnover $\tilde{\tau}_j$, the closer to 0 is the median autocovariance.[25] In particular, between the first and second subperiods, median autocovariance decreases (in absolute value) from -0.851 to -0.623, signaling lower trading costs, while median turnover increases from 0.272 to 0.446. Between the second and third subperiods, median autocovariance increases (in absolute value) from -0.623 to -1.007 while median turnover decreases from 0.446 to 0.291, presumably due to the Oil Shock of 1973–1974 and the subsequent recession. The 1977–1981 subperiod is the first subperiod after the advent of negotiated commissions (May 1, 1975), and median turnover increases to 0.449 while median autocovariance decreases (in absolute value) to -0.622. During the 1982–1986 subperiod when S&P 500 index futures begin trading, median autocovariance declines (in absolute value) to -0.573 while median turnover increases dramatically to 0.704. And during the 1987–1991 subperiod which includes the October 1987 Crash, median turnover is essentially unchanged (0.708 versus 0.704 from the previous subperiod), median autocovariance decreases (in absolute value) from -0.573 in the previous subperiod to -0.386, but mean autocovariance increases (in absolute value) dramatically from -1.627 in the previous subperiod to -5.096, indicating the presence of outliers with very large trading costs.

We have also estimated correlations among the variables in Table 5 but to conserve space, we shall summarize the main features of those correlations and refer readers to Lim et al. (1998) for further details.

Median turnover is highly correlated with both turnover beta and return beta, with correlations that exceed 50% in most subperiods, hinting at the prospect of two or more factors driving the cross-sectional variation in turnover. We shall address this issue more formally in Section 5.4.

Median turnover is not particularly highly correlated with S&P 500 membership during the first four subperiods, with correlations ranging from -10.6% (1967–1971) to 8.6% (1972–1976). However, with the advent of S&P 500 futures and the growing popularity of indexation in the early 1980's, median turnover becomes more highly correlated with S&P 500

[25]Recall that $\tilde{\tau}_j$ is the median turnover of stock j during the five-year subperiod; the median of $\tilde{\tau}_j$ is the median across all stocks j in the five-year subsample.

membership, jumping to 22.7% in 1982–1986, 25.4% in 1987–1991, and 15.9% in 1992–1996.

Turnover betas and return betas are highly positively correlated, with correlations ranging from 25.5% (1987–1991) to 55.4% (1967–1971). Not surprisingly, log-price p_j is highly positively correlated with log-market-capitalization v_j, with correlations exceeding 75% in every subperiod. Dividend yield is positively correlated with both log price and log market capitalization, though the correlation is not particularly large. This may seem counterintuitive at first but recall that these are cross-sectional correlations, not time-series correlations, and the level of dividends per share varies cross-sectionally as well as average log-price.

5.3 Regression Results

Table 6 contains the estimates of the cross-sectional regression model (5.2). We estimated three regression models for each subperiod: one with all eight variables and a constant term included, one excluding log market-capitalization, and one excluding log price. Since the log price and log market-capitalization regressors are so highly correlated (see Lim et al. (1998)), regressions with only one or the other included were estimated to gauge the effects of multicollinearity. The exclusion of either variable does not affect the qualitative features of the regression—no significant coefficients changed sign other than the constant term—though the quantitative features were affected to a small degree. For example, in the first subperiod v_j has a negative coefficient (-0.064) and p_j has a positive coefficient (0.150), both significant at the 5% level. When v_j is omitted the coefficient of p_j is still positive but smaller (0.070), and when p_j is omitted the coefficient of v_j is still negative and also smaller in absolute magnitude (-0.028), and in both these cases the coefficients retain their significance.

The fact that size has a negative impact on turnover while price has a positive impact is an artifact of the earlier subperiods. This can be seen heuristically in the time-series plots of Figure 1—compare the value-weighted and equal-weighted turnover indexes during the first two or three subperiods. Smaller-capitalization stocks seem to have higher turnover than larger-capitalization stocks.

This begins to change in the 1977–1981 subperiod: the size coefficient is negative but not significant, and when price is excluded, the size coefficient changes sign and becomes significant. In the subperiods after 1977–1981, both size and price enter positively. One explanation of this change is the growth of the mutual fund industry and other large institutional investors in the early 1980's. As portfolio managers manage larger asset bases, it becomes more difficult to invest in smaller-capitalization companies because of liquidity and corporate-control issues. Therefore, the natural economies of scale in investment management coupled with the increasing concentration of investment capital make small stocks less actively traded than large stocks. Of course, this effect should have implications for the equilibrium return of small stocks versus large stocks, and we investigate such implications in Lo and Wang (1998).

The first-order return autocovariance has a positive coefficient in all subperiods except the second regression of the last subperiod (in which the coefficient is negative but insignificant), and these coefficients are significant at the 5% level in all subperiods except 1972–1976 and 1992–1996. This is consistent with the trading-cost interpretation of $\hat{\gamma}_{r,j}(1)$: a large negative return autocovariance implies a large effective bid/ask spread which, in turn, should imply lower turnover.

Membership in the S&P 500 also has a positive impact on turnover in all subperiods as expected, and the magnitude of the coefficient increases dramatically in the 1982–1986 subperiod—from 0.013 in the previous period to 0.091—also as expected given the growing importance of indexation and index arbitrage during this period, and the introduction of S&P 500 futures contracts in April 1982. Surprisingly, in the 1992–1996 subperiod, the S&P 500 coefficient declines to 0.029, perhaps because of the interactions between this indicator variable and size and price (all three variables are highly positively correlated with each other; see Lim et al. (1998) for further details). When size is omitted, S&P 500 membership becomes more important, yet when price is omitted, size becomes more important and S&P 500 membership becomes irrelevant. These findings are roughly consistent with those in Tkac (1996).[26]

Both systematic and idiosyncratic risk—$\hat{\beta}_{r,j}$ and $\hat{\sigma}_{\epsilon,r,j}$—have positive and significant impact on turnover in all subperiods. However, the impact of excess expected returns $\hat{\alpha}_{r,j}$ on turnover is erratic: negative and significant in the 1977–1981 and 1992–1996 subperiods, and

[26]In particular, she finds that S&P 500 membership becomes much less significant after controlling for the effects of size and institutional ownership. Of course, her analysis is not directly comparable to ours because she uses a different dependent variable (monthly relative dollar-volume divided by relative market-capitalization) in her cross-sectional regressions, and considers only a small sample of the very largest NYSE/AMEX stocks (809) over the four year period 1988–1991.

positive and significant in the others.

The dividend-yield regressor is insignificant in all subperiods but two: 1982–1986 and 1992–1996. In these two subperiods, the coefficient is negative, which contradicts the notion that dividend-capture trading affects turnover.

In summary, the cross-sectional variation of turnover does seem related to several stock-specific characteristics such as risk, size, price, trading costs, and S&P 500 membership. The explanatory power of these cross-sectional regressions—as measured by R^2—range from 29.6% (1992–1996) to 44.7% (1967–1971), rivaling the R^2's of typical cross-sectional return regressions. With sample sizes ranging from 2,073 (1962–1966) to 2,644 (1982–1986) stocks, these R^2's provide some measure of confidence that cross-sectional variations in median turnover are not purely random but do bear some relation to economic factors.

5.4 Tests of $(K+1)$-Fund Separation

Since two-fund and $(K+1)$-fund separation imply an approximately linear factor structure for turnover, we can investigate these two possibilities by using principal components analysis to decompose the covariance matrix of turnover (see Muirhead (1982) for an exposition of principal components analysis). If turnover is driven by a linear K-factor model, the first K principal components should explain most of the time-series variation in turnover. More formally, if

$$\tau_{jt} \;=\; \alpha_j + \delta_1 F_{1t} + \cdots + \delta_K F_{Kt} + \epsilon_{jt} \tag{5.3}$$

where $\mathrm{E}[\epsilon_{jt}\epsilon_{j't}] = 0$ for any $j \neq j'$, then the covariance matrix $\mathbf{\Sigma}$ of the vector $\boldsymbol{\tau}_t \equiv [\ \tau_{1t} \ \cdots \ \tau_{Jt}\ ]'$ can be expressed as

$$\mathrm{Var}[\boldsymbol{\tau}_t] \;\equiv\; \mathbf{\Sigma} \;=\; \mathbf{Q}\mathbf{\Theta}\mathbf{Q}^\top \tag{5.4}$$

$$\mathbf{\Theta} \;=\; \begin{pmatrix} \theta_1 & 0 & \cdots & 0 \\ 0 & \theta_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & \theta_N \end{pmatrix} \tag{5.5}$$

where Θ contains the eigenvalues of Σ along its diagonal and $\mathbf{Q}$ is the matrix of corresponding eigenvectors. Since Σ is a covariance matrix, it is positive semidefinite hence all the eigenvalues are nonnegative. When normalized to sum to one, each eigenvalue can be interpreted as the fraction of the total variance of turnover attributable to the corresponding principal component. If (5.3) holds, it can be shown that as the size N of the cross section increases without bound, exactly K normalized eigenvalues of Σ approach positive finite limits, and the remaining $N-K$ eigenvalues approach 0 (see, for example, Chamberlain (1983) and Chamberlain and Rothschild (1983)). Therefore, the plausibility of (5.3), and the value of K, can be gauged by examining the magnitudes of the eigenvalues of Σ.

The only obstacle is the fact that the covariance matrix Σ must be estimated, hence we encounter the well-known problem that the standard estimator

$$\widehat{\Sigma} \equiv \frac{1}{T}\sum_{t=1}^{T}(\tau_t - \overline{\tau})(\tau_t - \overline{\tau})^{\top}$$

is singular if the number of securities J in the cross section is larger than the number of time series observations T.[27] Since J is typically much larger than T—for a five-year subperiod T is generally 261 weeks, and J is typically well over 2,000—we must limit our attention to a smaller subset of stocks. We do this by following the common practice of forming a small number of portfolios (see Campbell, Lo, and MacKinlay (1997, Chapter 5)), sorted by turnover beta to maximize the dispersion of turnover beta among the portfolios.[28] In particular, within each five-year subperiod we form ten turnover-beta-sorted portfolios using betas estimated from the previous five-year subperiod, estimate the covariance matrix $\widehat{\Sigma}$ using 261 time-series observations, and perform a principal-components decomposition on

[27]Singularity by itself does not pose any problems for the computation of eigenvalues—this follows from the singular-value decomposition theorem—but it does have implications for the statistical properties of estimated eigenvalues. In some preliminary Monte Carlo experiments, we have found that the eigenvalues of a singular estimator of a positive-definite covariance matrix can be severely biased. We thank Bob Korajczyk and Bruce Lehmann for bringing some of these issues to our attention and plan to investigate them more thoroughly in ongoing research.

[28]Our desire to maximize the dispersion of turnover beta is motivated by the same logic used in Black, Jensen, and Scholes (1972): a more dispersed sample provides a more powerful test of a cross-sectional relationship driven by the sorting characteristic. This motivation should not be taken literally in our context because the theoretical implications of Section 3 need not imply a prominent role for turnover beta (indeed, in the case of two-fund separation, there is no cross-sectional variation in turnover betas!). However, given the factor structure implied by $(K+1)$-fund separation (see Section 3.2), sorting by turnover betas seems appropriate.

$\widehat{\Sigma}$. For purposes of comparison and interpretation, we perform a parallel analysis for returns, using ten return-beta-sorted portfolios. The results are reported in Table 7.

Table 7 contains the principal components decomposition for portfolios sorted on out-of-sample betas, where the betas are estimated in two ways: relative to value-weighted indexes (τ^{VW} and R^{VW}) and equal-weighted indexes (τ^{EW} and R^{EW}).[29] The first principal component typically explains between 70% to 85% of the variation in turnover, and the first two principal components explain almost all of the variation. For example, the upper-left subpanel of Table 7 shows that in the second five-year subperiod (1967–1971), 85.1% of the variation in the turnover of turnover-beta-sorted portfolios (using turnover betas relative to the value-weighted turnover index) is captured by the first principal component, and 93.6% is captured by the first two principal components. Although using betas computed with value-weighted instead of equal-weighted indexes generally yields smaller eigenvalues for the first principal component (and therefore larger values for the remaining principal components) for both turnover and returns, the differences are typically not large.

The importance of the second principal component grows steadily through time for the value-weighted case, reaching a peak of 15.6% in the last subperiod, and the first two principal components account for 87.3% of the variation in turnover in the last subperiod. This is roughly comparable with the return portfolios sorted on value-weighted return-betas—the first principal component is by far the most important, and the importance of the second principal component is most pronounced in the last subperiod. However, the lower left subpanel of Table 7 shows that for turnover portfolios sorted by betas computed against equal-weighted indexes, the second principal component explains approximately the same variation in turnover, varying between 6.0% and 10.4% across the six subperiods.

Of course, one possible explanation for the dominance of the first principal component is the existence of a time trend in turnover. Despite the fact that we have limited our analysis to five-year subperiods, within each subperiod there is a certain drift in turnover; might this account for the first principal component? To investigate this conjecture, we perform eigenvalue decompositions for the covariance matrices of the *first differences* of turnover for the 10 turnover portfolios.

[29]In particular, the portfolios in a given period are formed by ranking on betas estimated in the immediately preceding subperiod, e.g., the 1992–1996 portfolios were created by sorting on betas estimated in the 1987–1991 subperiod, hence the first subperiod in Table 7 begins in 1967, not 1962.

These results are reported in Table 8 and are consistent with those in Table 7: the first principal component is still the most important, explaining between 60% to 88% of the variation in the first differences of turnover. The second principal component is typically responsible for another 5% to 20%. And in one case—in-sample sorting on betas relative to the equal-weighted index during 1987–1991—the third principal component accounts for an additional 10%. These figures suggest that the trend in turnover is unlikely to be the source of the dominant first principal component.

In summary, the results of Tables 7 and 8 indicate that a one-factor model for turnover is a reasonable approximation, at least in the case of turnover-beta-sorted portfolios, and that a two-factor model captures well over 90% of the time-series variation in turnover. This lends some support to the practice of estimating "abnormal" volume by using an event-study style "market model", e.g., Bamber (1986), Jain and Joh (1988), Lakonishok and Smidt (1986), Morse (1980), Richardson, Sefcik, Thompson (1986), Stickel and Verrecchia (1994), and Tkac (1996).

As compelling as these empirical results are, several qualifications should be kept in mind. First, we have provided little statistical inference for our principal components decomposition. In particular, the asymptotic standard errors reported in Tables 7 and 8 were computed under the assumption of IID Gaussian data, hardly appropriate for weekly US stock returns and even less convincing for turnover (see Muirhead (1982, Chapter 9) for further details). Perhaps nonparametric methods such as the moving-block bootstrap can provide better indications of the statistical significance of our estimated eigenvalues. Monte Carlo simulations should also be conducted to check the finite-sample properties of our estimators.

More importantly, the economic interpretation of the first two principal components or, alternatively, identifying the specific factors is a challenging issue that principal components cannot resolve. More structure must be imposed on the data—in particular, a well-articulated dynamic economic model of trading activity—to obtain a better understanding for the sources of turnover variation, and we present such structure in Lo and Wang (1998).

6 Conclusion

Trading activity is fundamental to a deeper understanding of economic interactions, and in this paper we have provided definitions, data analysis, and theoretical implications of portfolio theory for trading activity in financial markets.

There are many issues that remain to be examined. Perhaps the most pressing is the time-series variation in volume and the relations between volume, prices, and other economic quantities. We turn to these issues in Lo and Wang (1998) by developing a formal dynamic equilibrium asset-market model in which volume, prices, and other state variables evolve through time together in an economically consistent way. By explicitly modeling the motives for trade as a function of preferences, endowments, and economic conditions, we obtain more likely explanations for the dynamic properties of volume and returns. Using the volume dataset developed in this paper, we examine the empirical relevance of these explanations.

Given the complexity of the trading process in financial markets, some insights may also be garnered from a more "reduced-form" analysis of trading patterns. In particular, in Lo, Mamaysky and Wang (2000) we investigate the relevance of heuristic models of trading activity, i.e., informal explanations of volume that may not be based on fully articulated economic models of optimizing agents. Such heuristics include technical analysis, market psychology, and trading folklore. While much of this literature is foreign in spirit and syntax to the culture of economics, much of its focus is the same as ours.

Through these three somewhat different lines of investigation, we hope to develop a more complete understanding of trading behavior in asset markets.

A Appendix – Proof of Lemma 1

For brevity, we only consider the case of $K = 2$. Let $x = h_{1t}$ and $y = h_{2t}$ (the superscript i is omitted for notational simplicity). From Assumption 2, x and y has joint probability density $f(x, y)$ such that (i) it is continuous and (ii) $P(|y| < H) = 1$. Let $\lambda > 0$ be a parameter. Rewrite the random variable $|x + \lambda y|$ as

$$|x + \lambda y| \;=\; |x| + \lambda\,\mathrm{sgn}(x)y \;+\; 1_{D(x,\lambda y)}\delta(x, \lambda y) \tag{A.1}$$

where the last term is defined by the functions $D(z_1, z_2) = \{z_1 z_2 < 0,\ |z_1| < |z_2|\}$ and δ is defined by $\delta(z_1, z_2) = -2|z_1| + 2|z_2|$. To assess the statistical error of the approximation

$$|x + \lambda y| \;\approx\; |x| \;+\; \lambda \mathrm{sgn}(x) y, \tag{A.2}$$

we compute the expected approximation error using any loss function $L(|.|)$ which is non-negative, nondecreasing on R^+, and of order $\alpha > 0$, i.e., $\lim_{\epsilon \to 0} L(|\epsilon|)/\epsilon^\alpha = A$ exists and is finite. Loss functions satisfying these conditions include the mean-absolute-error ($L(z) = |z|$, $\alpha = 1$) and mean-squared-error ($L(z) = z^2$, $\alpha = 2$) loss functions. We then have

$$\mathrm{E}\left[L(|1_{D(x,\lambda y)}\delta(x, \lambda y)|)\right] \;=\; P(D)\mathrm{E}[L(|\delta(x, \lambda y)|)|D] \tag{A.3}$$

where $P(D) = \int_0^H \int_0^{\lambda y} [f(-x, y) + f(x, -y)]\, dx\, dy$. Because $|\delta(x, \lambda y)| \leq 2\lambda |y| < 2\lambda H$ on D and $P(D) = \mathbf{O}(\lambda)$ with $\lim_{\lambda \to 0} P(D)/\lambda = \int_{-H}^{H} y f(0, y)\, dy < \infty$, we have

$$\mathrm{E}\left[L(|1_{D(x,\lambda y)}\delta(x, \lambda y)|)\right] \;<\; A\lambda^{\alpha+1}(2H)^\alpha \int_{-H}^{H} y f(0, y)\, dy \;+\; \mathbf{o}(\lambda^{\alpha+1}). \tag{A.4}$$

Thus, even though the pointwise properties of the approximation are not satisfactory (since $\sup_{x,|y|<H} |\delta(x, \lambda y)| = \lambda H$), the approximation error is small in a statistical sense for small λ, i.e., the expected loss decreases faster than $\lambda^{\alpha+1}$.

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Table 1

Selected volume studies grouped according to the volume measure used.

Volume Measure	Study
Aggregate Share Volume	Gallant, Rossi, and Tauchen (1992), Hiemstra and Jones (1994), Ying (1966)
Individual Share Volume	Andersen (1996), Epps and Epps (1976), James and Edmister (1983), Lamoureux and Lastrapes (1990, 1994)
Aggregate Dollar Volume	—
Individual Dollar Volume	James and Edmister (1983), Lakonishok and Vermaelen (1986)
Relative Individual Dollar Volume	Tkac (1996)
Individual Turnover	Bamber (1986, 1987), Hu (1997), Lakonishok and Smidt (1986), Morse (1980), Richardson, Sefcik, Thompson (1986), Stickel and Verrechia (1994)
Aggregate Turnover	Campbell, Grossman, Wang (1993), LeBaron (1992), Smidt (1990), NYSE Fact Book
Total Number of Trades	Conrad, Hameed, and Niden (1994)
Trading Days Per Year	James and Edmister (1983)
Contracts Traded	Tauchen and Pitts (1983)

Table 2

Volume measures for a two-asset two-investor numerical example assuming that two-fund separation holds.

Volume Measure	A	B	Aggregate
Number of Trades	1	1	2
Shares Traded	3	9	12
Dollars Traded	$300	$450	$750
Share Turnover	0.3	0.3	0.3
Dollar Turnover	0.3	0.3	0.3
Relative Dollar Turnover	0.4	0.6	1.0
Share-Weighted Turnover	—	—	0.3
Equal-Weighted Turnover	—	—	0.3
Value-Weighted Turnover	—	—	0.3

Table 3

Summary statistics for weekly value-weighted and equal-weighted turnover and return indexes of NYSE and AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume) for July 1962 to December 1996 (1,800 weeks) and subperiods. Turnover and returns are measured in percent per week and p-values for Box-Pierce statistics are reported in parentheses.

Statistic	τ^{VW}	τ^{EW}	R^{VW}	R^{EW}
Mean	0.78	0.91	0.23	0.32
Std. Dev.	0.48	0.37	1.96	2.21
Skewness	0.66	0.38	−0.41	−0.46
Kurtosis	0.21	−0.09	3.66	6.64
Percentiles:				
Min	0.13	0.24	−15.64	−18.64
5%	0.22	0.37	−3.03	−3.44
10%	0.26	0.44	−2.14	−2.26
25%	0.37	0.59	−0.94	−0.80
50%	0.64	0.91	0.33	0.49
75%	1.19	1.20	1.44	1.53
90%	1.44	1.41	2.37	2.61
95%	1.57	1.55	3.31	3.42
Max	4.06	3.16	8.81	13.68
Autocorrelations:				
ρ_1	91.25	86.73	5.39	25.63
ρ_2	88.59	81.89	−0.21	10.92
ρ_3	87.62	79.30	3.27	9.34
ρ_4	87.44	78.07	−2.03	4.94
ρ_5	87.03	76.47	−2.18	1.11
ρ_6	86.17	74.14	1.70	4.07
ρ_7	87.22	74.16	5.13	1.69
ρ_8	86.57	72.95	−7.15	−5.78
ρ_9	85.92	71.06	2.22	2.54
ρ_{10}	84.63	68.59	−2.34	−2.44
Box-Pierce Q_{10}	13,723.0	10,525.0	23.0	175.1
	(0.000)	(0.000)	(0.010)	(0.000)

Statistic	τ^{VW}	τ^{EW}	R^{VW}	R^{EW}
1962–1966 (234 weeks)				
Mean	0.25	0.57	0.23	0.30
Std. Dev.	0.07	0.21	1.29	1.54
Skewness	1.02	1.47	−0.35	−0.76
Kurtosis	0.80	2.04	1.02	2.50
1967–1971 (261 weeks)				
Mean	0.40	0.93	0.18	0.32
Std. Dev.	0.08	0.32	1.89	2.62
Skewness	0.17	0.57	0.42	0.40
Kurtosis	−0.42	−0.26	1.52	2.19
1972–1976 (261 weeks)				
Mean	0.37	0.52	0.10	0.19
Std. Dev.	0.10	0.20	2.39	2.78
Skewness	0.93	1.44	−0.13	0.41
Kurtosis	1.57	2.59	0.35	1.12
1977–1981 (261 weeks)				
Mean	0.62	0.77	0.21	0.44
Std. Dev.	0.18	0.22	1.97	2.08
Skewness	0.29	0.62	−0.33	−1.01
Kurtosis	−0.58	−0.05	0.31	1.72
1982–1986 (261 weeks)				
Mean	1.20	1.11	0.37	0.39
Std. Dev.	0.30	0.29	2.01	1.93
Skewness	0.28	0.45	0.42	0.32
Kurtosis	0.14	−0.28	1.33	1.19
1987–1991 (261 weeks)				
Mean	1.29	1.15	0.29	0.24
Std. Dev.	0.35	0.27	2.43	2.62
Skewness	2.20	2.15	−1.51	−2.06
Kurtosis	14.88	12.81	7.85	16.44
1992–1996 (261 weeks)				
Mean	1.25	1.31	0.27	0.37
Std. Dev.	0.23	0.22	1.37	1.41
Skewness	−0.06	−0.05	−0.38	−0.48
Kurtosis	−0.21	−0.24	1.00	1.30

Table 4

Impact of detrending on the statistical properties of weekly value-weighted and equal-weighted turnover indexes of NYSE and AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume) for July 1962 to December 1996 (1,800 weeks). Six detrending methods are used: linear, log-linear, first differencing, normalization by the trailing four-week moving average, linear-quadratic and seasonal detrending proposed by Gallant, Rossi, and Tauchen (1992) (GRT), and kernel regression.

Statistic	Raw	Linear	Log Linear	First Diff.	MA(4) Ratio	GRT	Kernel	Raw	Linear	Log Linear	First Diff.	MA(4) Ratio	GRT	Kernel
	Value-Weighted Turnover Index							Equal-Weighted Turnover Index						
R^2 (%)	—	70.6	78.6	82.6	81.9	72.3	88.6	—	36.9	37.2	73.6	71.9	42.8	78.3
Mean	0.78	0.00	0.00	0.00	1.01	0.00	0.00	0.91	0.00	0.00	0.00	1.01	0.00	0.00
Std. Dev.	0.48	0.26	0.31	0.20	0.20	0.25	0.16	0.37	0.30	0.35	0.19	0.20	0.28	0.17
Skewness	0.66	1.57	0.09	0.79	0.73	1.69	1.77	0.38	0.90	0.00	0.59	0.67	1.06	0.92
Kurtosis	0.21	10.84	−0.20	17.75	3.02	11.38	29.38	−0.09	1.80	0.44	7.21	2.51	2.32	6.67
Percentiles:														
Min	0.13	−0.69	−0.94	−1.55	0.45	−0.61	−0.78	0.24	−0.62	−1.09	−0.78	0.44	−0.59	−0.59
5%	0.22	−0.34	−0.51	−0.30	0.69	−0.32	−0.26	0.37	−0.44	−0.63	−0.32	0.70	−0.38	−0.27
10%	0.26	−0.29	−0.38	−0.19	0.76	−0.28	−0.15	0.44	−0.36	−0.43	−0.21	0.76	−0.32	−0.20
25%	0.37	−0.18	−0.21	−0.08	0.89	−0.17	−0.06	0.59	−0.19	−0.20	−0.09	0.88	−0.20	−0.10
50%	0.65	−0.01	−0.02	−0.00	1.00	−0.02	0.00	0.91	−0.04	−0.00	−0.00	1.01	−0.05	−0.01
75%	1.19	0.13	0.23	0.07	1.12	0.12	0.06	1.20	0.16	0.20	0.09	1.12	0.16	0.09
90%	1.44	0.30	0.41	0.20	1.25	0.29	0.16	1.41	0.42	0.46	0.21	1.25	0.38	0.21
95%	1.57	0.45	0.50	0.31	1.35	0.46	0.23	1.55	0.55	0.63	0.32	1.35	0.54	0.28
Max	4.06	2.95	1.38	2.45	2.48	2.91	2.36	3.16	2.06	1.11	1.93	2.44	2.08	1.73
Autocorrelations:														
ρ_1	91.25	70.15	74.23	−34.94	22.97	70.24	23.11	86.73	79.03	83.07	−31.94	29.41	77.80	39.23
ρ_2	88.59	61.21	66.17	−9.70	−6.48	64.70	0.54	81.89	71.46	77.27	−8.69	0.54	71.60	17.95
ρ_3	87.62	58.32	63.78	−4.59	−19.90	60.78	−6.21	79.30	67.58	74.25	−5.07	−13.79	66.89	8.05
ρ_4	87.44	58.10	63.86	1.35	−20.41	60.96	−5.78	78.07	65.84	72.60	1.45	−16.97	65.14	4.80
ρ_5	87.03	56.79	62.38	2.58	−6.12	60.31	−7.79	76.47	63.41	70.64	2.68	−4.87	62.90	−0.11
ρ_6	86.17	54.25	59.37	−10.96	−4.35	58.78	−12.93	74.14	59.95	67.29	−8.79	−4.23	60.03	−7.54
ρ_7	87.22	58.20	60.97	9.80	4.54	61.46	−1.09	74.16	60.17	66.27	4.60	0.17	59.28	−3.95
ρ_8	86.57	56.30	59.83	−0.10	1.78	59.39	−4.29	72.95	58.45	64.76	2.52	−0.37	57.62	−5.71
ρ_9	85.92	54.54	57.87	3.73	−2.43	59.97	−7.10	71.06	55.67	62.54	2.25	−2.27	56.48	−10.30
ρ_{10}	84.63	50.45	53.57	−11.95	−13.46	55.85	−15.86	68.59	51.93	58.81	−10.05	−10.48	53.06	−17.59

Table 5

Summary statistics of variables for cross-sectional analysis of weekly turnover of NYSE and AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume) for subperiods of the sample period from July 1962 to December 1996. The variables are: $\overline{\tau}_j$ (average turnover); $\tilde{\tau}_j$ (median turnover); $\hat{\alpha}_{\tau,j}$, $\hat{\beta}_{\tau,j}$, and $\hat{\sigma}_{\epsilon,\tau,j}$ (the intercept, slope, and residual, respectively, from the time-series regression of an individual security's turnover on market turnover); $\hat{\alpha}_{r,j}$, $\hat{\beta}_{r,j}$, and $\hat{\sigma}_{\epsilon,r,j}$ (the intercept, slope, and residual, respectively, from the time-series regression of an individual security's return on the market return); v_j (natural logarithm of market capitalization), p_j (natural logarithm of price); d_j (dividend yield); SP500$_j$ (S&P 500 indicator variable); and $\hat{\gamma}_{r,j}(1)$ (first-order return autocovariance).

Statistics	$\overline{\tau}_j$	$\tilde{\tau}_j$	$\hat{\alpha}_{\tau,j}$	$\hat{\beta}_{\tau,j}$	$\hat{\sigma}_{\epsilon,\tau,j}$	$\hat{\alpha}_{r,j}$	$\hat{\beta}_{r,j}$	$\hat{\sigma}_{\epsilon,r,j}$	v_j	p_j	d_j	SP500$_j$	$\hat{\gamma}_{r,j}(1)$
						1962 to 1966 (234 weeks)							
Mean	0.576	0.374	0.009	2.230	0.646	0.080	1.046	4.562	17.404	1.249	0.059	0.175	−2.706
Median	0.397	0.272	0.092	0.725	0.391	0.064	1.002	3.893	17.263	1.445	0.058	0.000	−0.851
Std. Dev.	0.641	0.372	1.065	5.062	0.889	0.339	0.529	2.406	1.737	0.965	0.081	0.380	8.463
						1967 to 1971 (261 weeks)							
Mean	0.900	0.610	−0.361	3.134	0.910	0.086	1.272	5.367	17.930	1.442	0.049	0.178	−1.538
Median	0.641	0.446	−0.128	1.948	0.612	0.081	1.225	5.104	17.791	1.522	0.042	0.000	−0.623
Std. Dev.	0.827	0.547	0.954	3.559	0.940	0.383	0.537	1.991	1.566	0.685	0.046	0.382	4.472
						1972 to 1976 (261 weeks)							
Mean	0.521	0.359	−0.025	1.472	0.535	0.085	0.986	6.252	17.574	0.823	0.072	0.162	−3.084
Median	0.420	0.291	0.005	1.040	0.403	0.086	0.955	5.825	17.346	0.883	0.063	0.000	−1.007
Std. Dev.	0.408	0.292	0.432	1.595	0.473	0.319	0.429	2.619	1.784	0.890	0.067	0.369	8.262
						1977 to 1981 (261 weeks)							
Mean	0.780	0.553	0.043	1.199	0.749	0.254	0.950	5.081	18.155	1.074	0.099	0.176	−1.748
Median	0.629	0.449	0.052	0.818	0.566	0.215	0.936	4.737	18.094	1.212	0.086	0.000	−0.622
Std. Dev.	0.561	0.405	0.638	1.348	0.643	0.356	0.428	2.097	1.769	0.805	0.097	0.381	5.100
						1982 to 1986 (261 weeks)							
Mean	1.160	0.833	0.005	0.957	1.135	0.113	0.873	5.419	18.63	1.143	0.090	0.181	−1.627
Median	0.998	0.704	0.031	0.713	0.902	0.146	0.863	4.813	18.51	1.293	0.063	0.000	−0.573
Std. Dev.	0.788	0.605	0.880	1.018	0.871	0.455	0.437	2.581	1.76	0.873	0.126	0.385	8.405
						1987 to 1991 (261 weeks)							
Mean	1.255	0.888	0.333	0.715	1.256	−0.007	0.977	6.450	18.847	0.908	0.095	0.191	−5.096
Median	0.995	0.708	0.171	0.505	0.899	0.014	0.998	5.174	18.778	1.108	0.062	0.000	−0.386
Std. Dev.	1.039	0.773	1.393	1.229	1.272	0.543	0.414	5.417	2.013	1.097	0.134	0.393	44.246
						1992 to 1996 (261 weeks)							
Mean	1.419	1.032	0.379	0.833	1.378	0.147	0.851	5.722	19.407	1.081	0.063	0.182	−3.600
Median	1.114	0.834	0.239	0.511	0.997	0.113	0.831	4.674	19.450	1.297	0.042	0.000	−1.136
Std. Dev.	1.208	0.910	1.637	1.572	1.480	0.482	0.520	3.901	2.007	1.032	0.095	0.386	21.550

Table 6

Cross-sectional regressions of median weekly turnover of NYSE and AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume) for subperiods of the sample period from July 1962 to December 1996. The explanatory variables are: $\hat{\alpha}_{r,j}$, $\hat{\beta}_{r,j}$, and $\hat{\sigma}_{\epsilon,r,j}$ (the intercept, slope, and residual, respectively, from the time-series regression of an individual security's return on the market return); v_j (natural logarithm of market capitalization), p_j (natural logarithm of price); d_j (dividend yield); SP500$_j$ (S&P 500 indicator variable); and $\hat{\gamma}_{r,j}(1)$ (first-order return autocovariance).

c	$\hat{\alpha}_{r,j}$	$\hat{\beta}_{r,j}$	$\hat{\sigma}_{\epsilon,r,j}$	v_j	p_j	d_j	SP500$_j$	$\hat{\gamma}_{r,j}(1)$	R^2 (%)
				1962 to 1966 (234 weeks, 2,073 stocks)					
0.742	0.059	0.354	0.043	−0.064	0.150	0.071	0.048	0.004	41.8
(0.108)	(0.019)	(0.014)	(0.006)	(0.006)	(0.014)	(0.081)	(0.018)	(0.001)	
−0.306	0.068	0.344	0.053	—	0.070	0.130	−0.006	0.006	38.8
(0.034)	(0.020)	(0.015)	(0.006)		(0.012)	(0.083)	(0.018)	(0.001)	
0.378	0.111	0.401	0.013	−0.028	—	0.119	0.048	0.005	38.7
(0.105)	(0.019)	(0.014)	(0.005)	(0.005)		(0.083)	(0.019)	(0.001)	
				1967 to 1971 (261 weeks, 2,292 stocks)					
0.289	0.134	0.448	0.095	−0.062	0.249	0.027	0.028	0.006	44.7
(0.181)	(0.024)	(0.023)	(0.009)	(0.010)	(0.023)	(0.235)	(0.025)	(0.002)	
−0.797	0.152	0.434	0.112	—	0.173	0.117	−0.026	0.007	43.7
(0.066)	(0.024)	(0.023)	(0.009)		(0.020)	(0.237)	(0.024)	(0.002)	
−0.172	0.209	0.507	0.057	−0.009	—	−0.108	0.023	0.011	41.9
(0.180)	(0.023)	(0.023)	(0.009)	(0.009)		(0.241)	(0.026)	(0.002)	
				1972 to 1976 (261 weeks, 2,084 stocks)					
0.437	0.102	0.345	0.027	−0.041	0.171	−0.031	0.031	0.001	38.0
(0.092)	(0.015)	(0.013)	(0.003)	(0.005)	(0.012)	(0.079)	(0.015)	(0.001)	
−0.249	0.111	0.320	0.032	—	0.114	−0.058	−0.007	0.002	36.5
(0.027)	(0.015)	(0.013)	(0.003)		(0.009)	(0.080)	(0.014)	(0.001)	
−0.188	0.141	0.367	0.008	0.008	—	−0.072	0.020	0.003	32.7
(0.085)	(0.015)	(0.014)	(0.003)	(0.004)		(0.082)	(0.015)	(0.001)	
				1977 to 1981 (261 weeks, 2,352 stocks)					
−0.315	−0.059	0.508	0.057	−0.001	0.139	0.015	0.013	0.005	44.2
(0.127)	(0.020)	(0.018)	(0.006)	(0.007)	(0.017)	(0.069)	(0.019)	(0.002)	
−0.344	−0.058	0.508	0.057	—	0.137	0.015	0.011	0.005	44.2
(0.035)	(0.019)	(0.017)	(0.005)		(0.013)	(0.069)	(0.018)	(0.002)	
−0.810	−0.008	0.534	0.040	0.037	—	−0.001	−0.001	0.009	42.6
(0.114)	(0.019)	(0.018)	(0.005)	(0.006)		(0.070)	(0.020)	(0.002)	
				1982 to 1986 (261 weeks, 2,644 stocks)					
−1.385	0.051	0.543	0.062	0.071	0.085	−0.223	0.091	0.006	31.6
(0.180)	(0.025)	(0.027)	(0.007)	(0.010)	(0.023)	(0.081)	(0.031)	(0.001)	
−0.193	0.018	0.583	0.057	—	0.170	−0.182	0.187	0.005	30.4
(0.051)	(0.024)	(0.027)	(0.007)		(0.020)	(0.081)	(0.028)	(0.001)	
−1.602	0.080	0.562	0.048	0.091	—	−0.217	0.085	0.006	31.3
(0.170)	(0.023)	(0.027)	(0.005)	(0.009)		(0.081)	(0.031)	(0.001)	
				1987 to 1991 (261 weeks, 2,471 stocks)					
−1.662	0.155	0.791	0.038	0.078	0.066	−0.138	0.131	0.003	31.9
(0.223)	(0.027)	(0.034)	(0.005)	(0.013)	(0.024)	(0.097)	(0.041)	(0.001)	
−0.313	0.153	0.831	0.035	—	0.158	−0.128	0.252	0.003	30.9
(0.052)	(0.027)	(0.033)	(0.005)		(0.019)	(0.098)	(0.036)	(0.001)	
−1.968	0.171	0.795	0.031	0.100	—	−0.122	0.119	0.003	31.7
(0.195)	(0.026)	(0.034)	(0.005)	(0.010)		(0.097)	(0.041)	(0.001)	
				1992 to 1996 (261 weeks, 2,520 stocks)					
−1.004	−0.087	0.689	0.077	0.040	0.262	−0.644	0.029	0.000	29.6
(0.278)	(0.034)	(0.033)	(0.007)	(0.016)	(0.033)	(0.164)	(0.049)	(0.001)	
−0.310	−0.095	0.708	0.076	—	0.314	−0.641	0.087	−0.001	29.4
(0.061)	(0.034)	(0.032)	(0.007)		(0.026)	(0.164)	(0.043)	(0.001)	
−2.025	−0.025	0.711	0.046	0.115	—	−0.590	−0.005	0.000	27.8
(0.249)	(0.034)	(0.033)	(0.006)	(0.012)		(0.166)	(0.049)	(0.001)	

Table 7

Eigenvalues $\hat{\theta}_i$, $i = 1, \ldots, 10$ of the covariance matrix of ten out-of-sample-beta-sorted portfolios of weekly turnover and returns of NYSE and AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume)—in percentages (where the eigenvalues are normalized to sum to 100%)—for subperiods of the sample period from July 1962 to December 1996. Turnover portfolios are sorted by out-of-sample turnover betas and return portfolios are sorted by out-of-sample return betas, where the symbols "τ^{VW}" and "R^{VW}" indicate that the betas are computed relative to value-weighted indexes, and "τ^{EW}" and "R^{EW}" indicate that they are are computed relative to equal-weighted indexes. Standard errors for the normalized eigenvalues are given in parentheses and are calculated under the assumption of IID normality.

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	Period	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}
Turnover-Beta-Sorted Turnover Portfolios (τ^{VW})											*Return-Beta-Sorted Return Portfolios* (R^{VW})									
85.1	8.5	3.6	1.4	0.8	0.3	0.2	0.1	0.0	0.0	1967 to 1971	85.7	5.9	2.0	1.4	1.4	1.1	0.8	0.7	0.5	0.4
(7.5)	(0.7)	(0.3)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)		(7.5)	(0.5)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)
82.8	7.3	4.9	2.0	1.4	0.8	0.5	0.2	0.1	0.1	1972 to 1976	90.0	3.8	1.8	1.0	0.9	0.7	0.6	0.6	0.4	0.3
(7.3)	(0.6)	(0.4)	(0.2)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)		(7.9)	(0.3)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)
83.6	8.6	2.3	2.0	1.2	0.8	0.6	0.4	0.4	0.1	1977 to 1981	85.4	4.8	4.3	1.4	1.3	0.9	0.6	0.5	0.4	0.3
(7.3)	(0.8)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)		(7.5)	(0.4)	(0.4)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)
78.9	7.9	3.6	2.9	2.4	1.4	1.3	0.8	0.5	0.4	1982 to 1986	86.6	6.1	2.4	1.6	1.0	0.6	0.5	0.5	0.4	0.3
(6.9)	(0.7)	(0.3)	(0.3)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)		(7.6)	(0.5)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)
80.1	6.2	5.2	2.4	1.6	1.3	1.0	1.0	0.8	0.5	1987 to 1991	91.6	2.9	1.7	1.1	0.7	0.6	0.6	0.4	0.3	0.2
(7.0)	(0.5)	(0.5)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)		(8.0)	(0.3)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)
71.7	15.6	4.5	2.9	1.8	1.2	0.9	0.8	0.5	0.3	1992 to 1996	72.4	11.6	4.4	3.5	2.2	1.8	1.5	1.1	0.8	0.6
(6.3)	(1.4)	(0.4)	(0.3)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)		(6.3)	(1.0)	(0.4)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)
Turnover-Beta-Sorted Turnover Portfolios (τ^{EW})											*Return-Beta-Sorted Return Portfolios* (R^{EW})									
86.8	7.5	3.0	1.3	0.6	0.5	0.2	0.1	0.1	0.0	1967 to 1971	87.8	4.3	2.2	1.5	1.0	0.9	0.8	0.5	0.5	0.5
(7.6)	(0.7)	(0.3)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)		(7.7)	(0.4)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)
82.8	6.0	5.4	2.9	1.2	1.0	0.4	0.2	0.1	0.0	1972 to 1976	91.6	4.1	0.9	0.8	0.6	0.5	0.4	0.4	0.3	0.3
(7.3)	(0.5)	(0.5)	(0.3)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)		(8.0)	(0.4)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
79.1	8.5	5.4	2.8	1.4	1.0	0.7	0.6	0.3	0.1	1977 to 1981	91.5	3.9	1.4	0.8	0.6	0.5	0.4	0.3	0.3	0.3
(6.9)	(0.7)	(0.5)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)		(8.0)	(0.3)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
78.0	10.4	3.1	2.3	2.0	1.3	1.3	0.8	0.6	0.4	1982 to 1986	88.9	4.4	2.3	1.3	0.7	0.7	0.6	0.5	0.4	0.4
(6.8)	(0.9)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)		(7.8)	(0.4)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)
82.5	4.8	3.2	2.4	2.0	1.4	1.3	0.9	0.9	0.6	1987 to 1991	92.7	3.0	1.2	0.7	0.7	0.4	0.4	0.4	0.3	0.2
(7.2)	(0.4)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)		(8.1)	(0.3)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
79.0	8.5	4.9	2.6	1.5	1.1	0.9	0.6	0.5	0.4	1992 to 1996	76.8	10.4	3.9	2.7	1.9	1.1	1.0	0.9	0.7	0.6
(6.9)	(0.7)	(0.4)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)		(6.7)	(0.9)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)

Table 8

Eigenvalues $\hat{\theta}_i$, $i = 1, \ldots, 10$ of the covariance matrix of the first-differences of the weekly turnover of ten out-of-sample-beta-sorted portfolios of NYSE and AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume)—in percentages (where the eigenvalues are normalized to sum to 100%)—for subperiods of the sample period from July 1962 to December 1996. Turnover betas are calculated in two ways: with respect to a value-weighted turnover index (τ^{VW}) and an equal-weighted turnover index (τ^{EW}). Standard errors for the normalized eigenvalues are given in parentheses and are calculated under the assumption of IID normality.

Period	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$	$\hat{\theta}_5$	$\hat{\theta}_6$	$\hat{\theta}_7$	$\hat{\theta}_8$	$\hat{\theta}_9$	$\hat{\theta}_{10}$
Out-of-Sample Turnover-Beta-Sorted Turnover-Differences Portfolios (τ^{VW})										
1967 to 1971	82.6	7.1	5.1	2.0	1.6	0.8	0.5	0.1	0.1	0.1
	(7.2)	(0.6)	(0.5)	(0.2)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)
1972 to 1976	81.2	6.8	4.7	2.8	2.0	1.0	0.9	0.4	0.2	0.1
	(7.1)	(0.6)	(0.4)	(0.2)	(0.2)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)
1977 to 1981	85.2	4.5	2.9	2.6	1.6	1.2	0.8	0.5	0.5	0.2
	(7.5)	(0.4)	(0.3)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)
1982 to 1986	81.3	5.1	3.5	2.7	2.2	1.7	1.3	0.9	0.7	0.6
	(7.1)	(0.4)	(0.3)	(0.2)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)
1987 to 1991	73.1	10.9	4.1	3.0	2.2	1.7	1.6	1.4	1.1	0.9
	(6.4)	(1.0)	(0.4)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)
1992 to 1996	78.4	8.6	4.0	2.8	2.1	1.2	1.0	0.9	0.6	0.4
	(6.9)	(0.8)	(0.4)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)
Out-of-Sample Turnover-Beta-Sorted Turnover-Differences Portfolios (τ^{EW})										
1967 to 1971	82.2	8.0	4.5	2.3	1.4	0.7	0.4	0.3	0.1	0.0
	(7.2)	(0.7)	(0.4)	(0.2)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)
1972 to 1976	79.3	7.5	4.8	4.0	1.9	1.3	0.6	0.4	0.2	0.1
	(7.0)	(0.7)	(0.4)	(0.4)	(0.2)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)
1977 to 1981	80.3	5.3	4.8	3.8	2.0	1.4	1.2	0.7	0.5	0.2
	(7.0)	(0.5)	(0.4)	(0.3)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)
1982 to 1986	82.6	5.0	3.0	2.6	2.0	1.7	1.1	0.9	0.7	0.4
	(7.3)	(0.4)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)
1987 to 1991	77.2	5.5	4.3	2.7	2.5	2.3	1.8	1.6	1.2	1.0
	(6.8)	(0.5)	(0.4)	(0.2)	(0.2)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)
1992 to 1996	80.4	6.4	4.6	2.6	1.7	1.4	1.1	0.7	0.5	0.4
	(7.1)	(0.6)	(0.4)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)

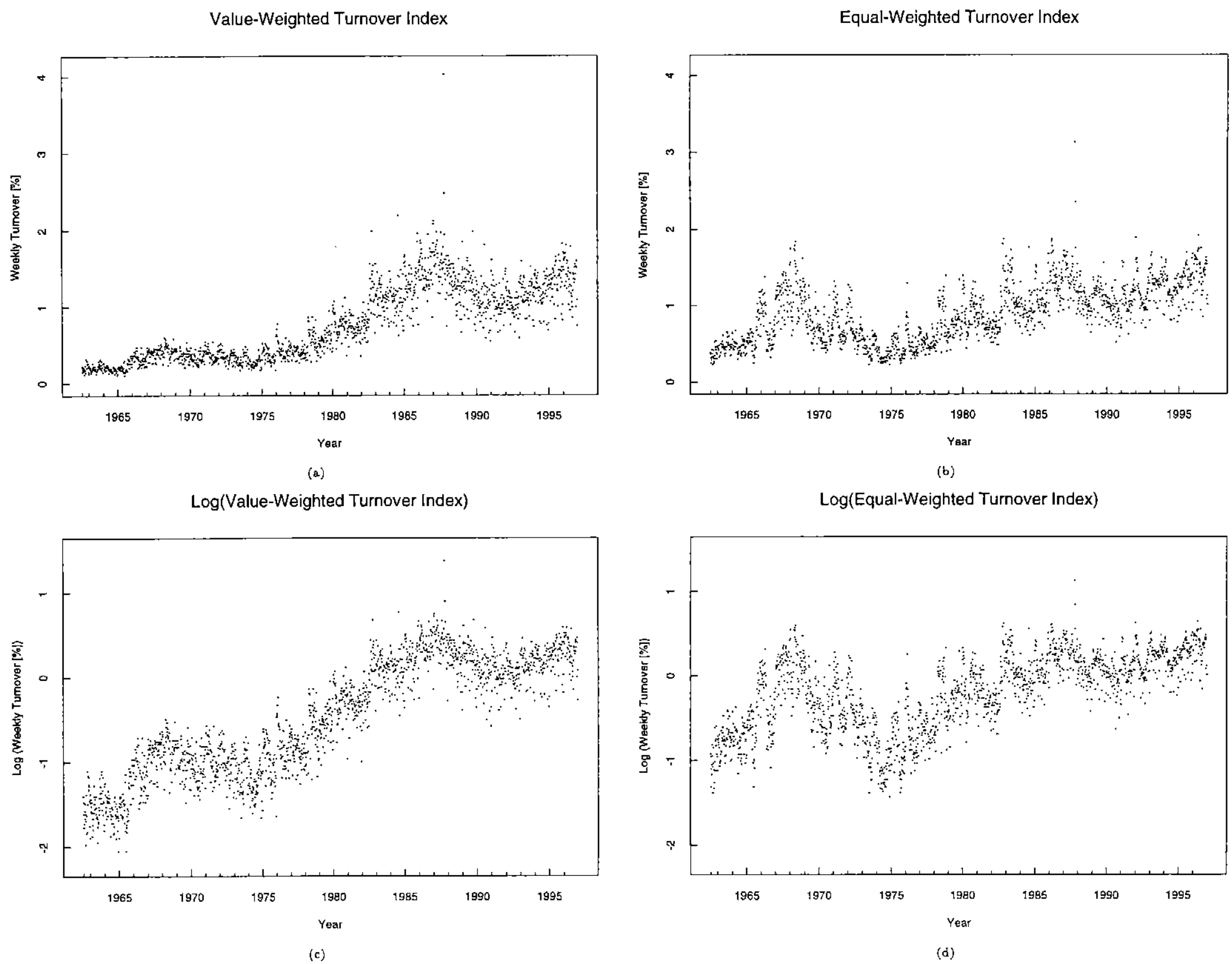

Figure 1. Weekly Value-Weighted and Equal-Weighted Turnover Indexes, 1962 to 1996.

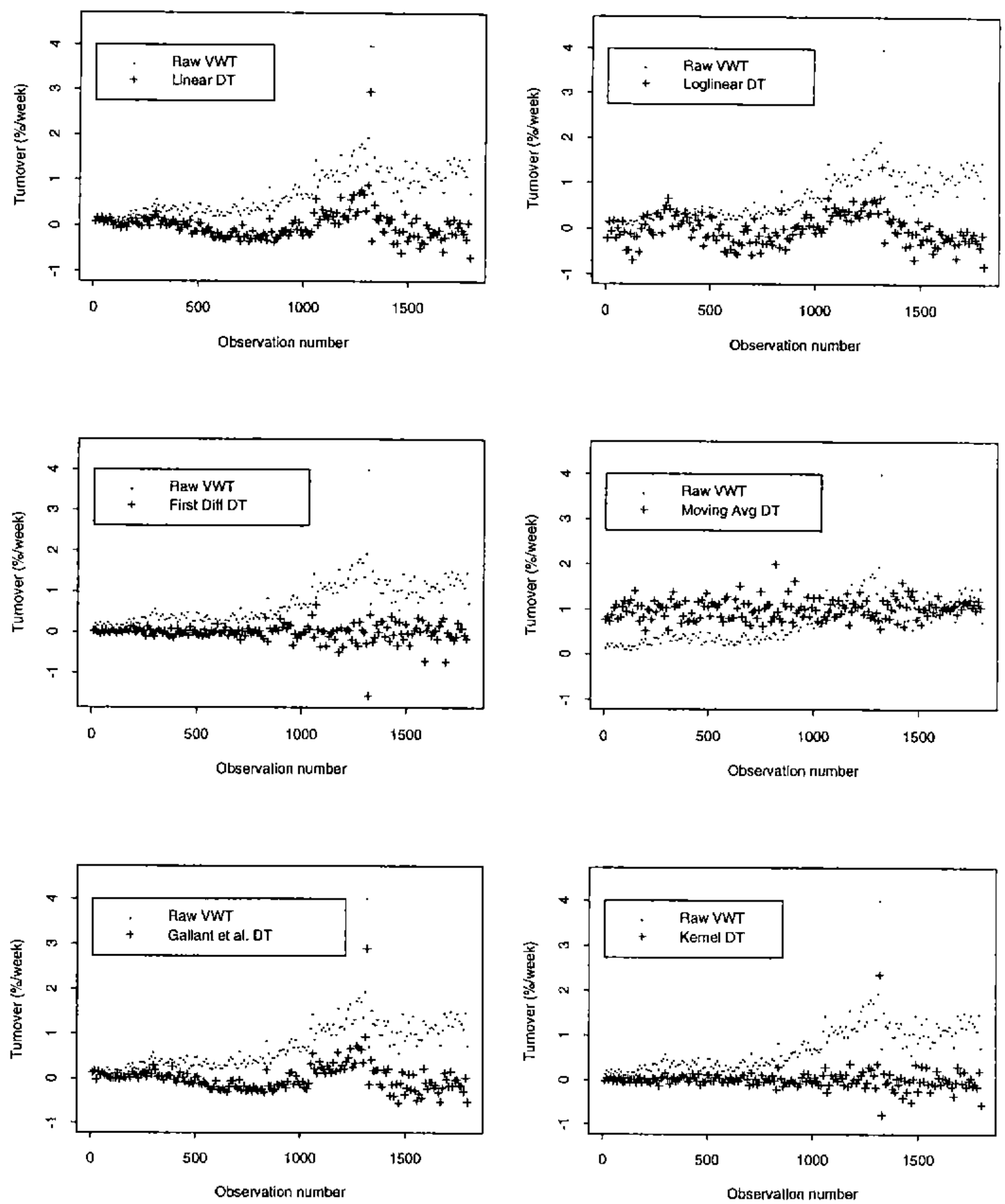

Figure 2a. Raw and Detrended Weekly Value-Weighted Turnover Indexes, 1962 to 1996.

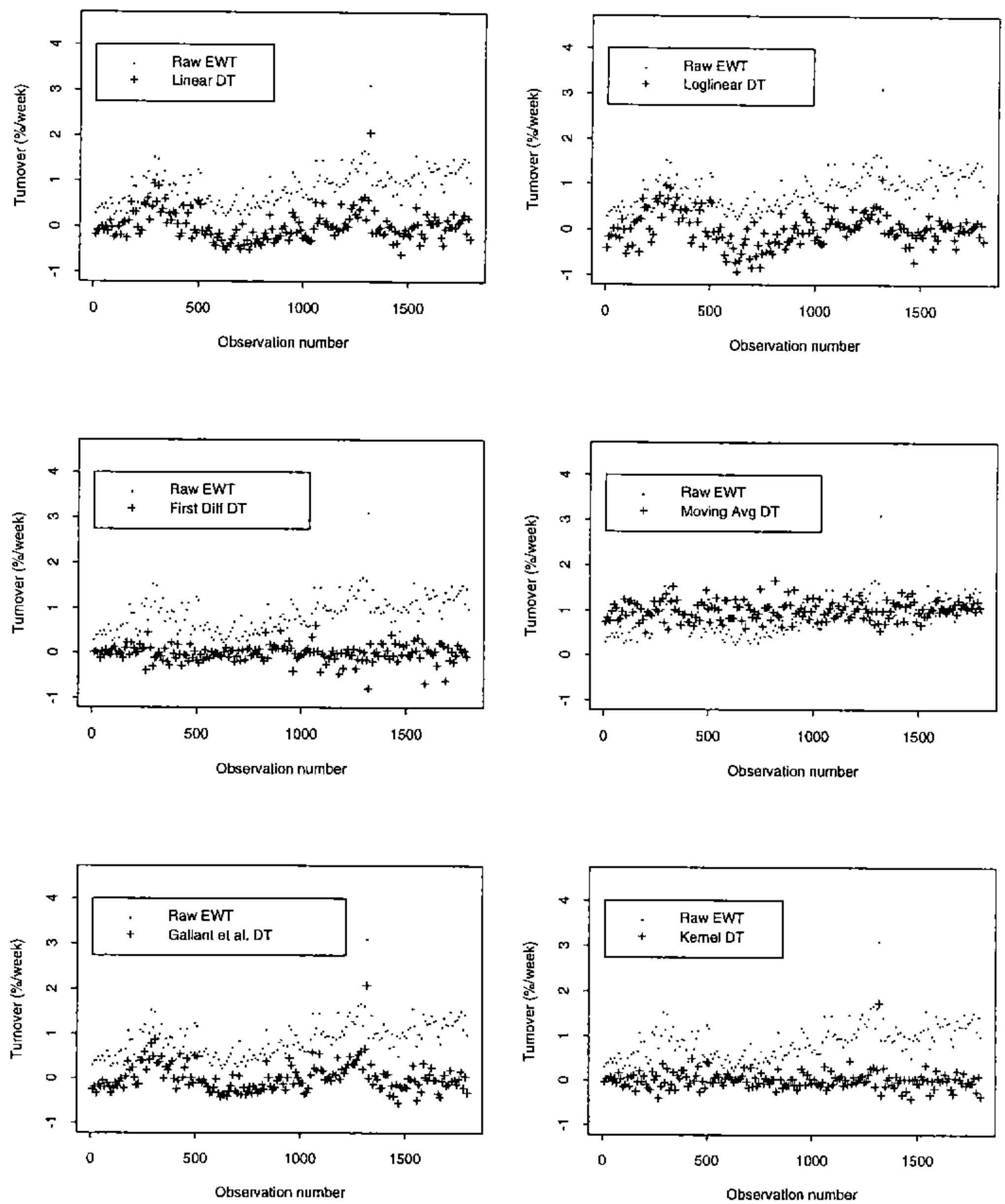

Figure 2b. Raw and Detrended Weekly Ealue-Weighted Turnover Indexes, 1962 to 1996.

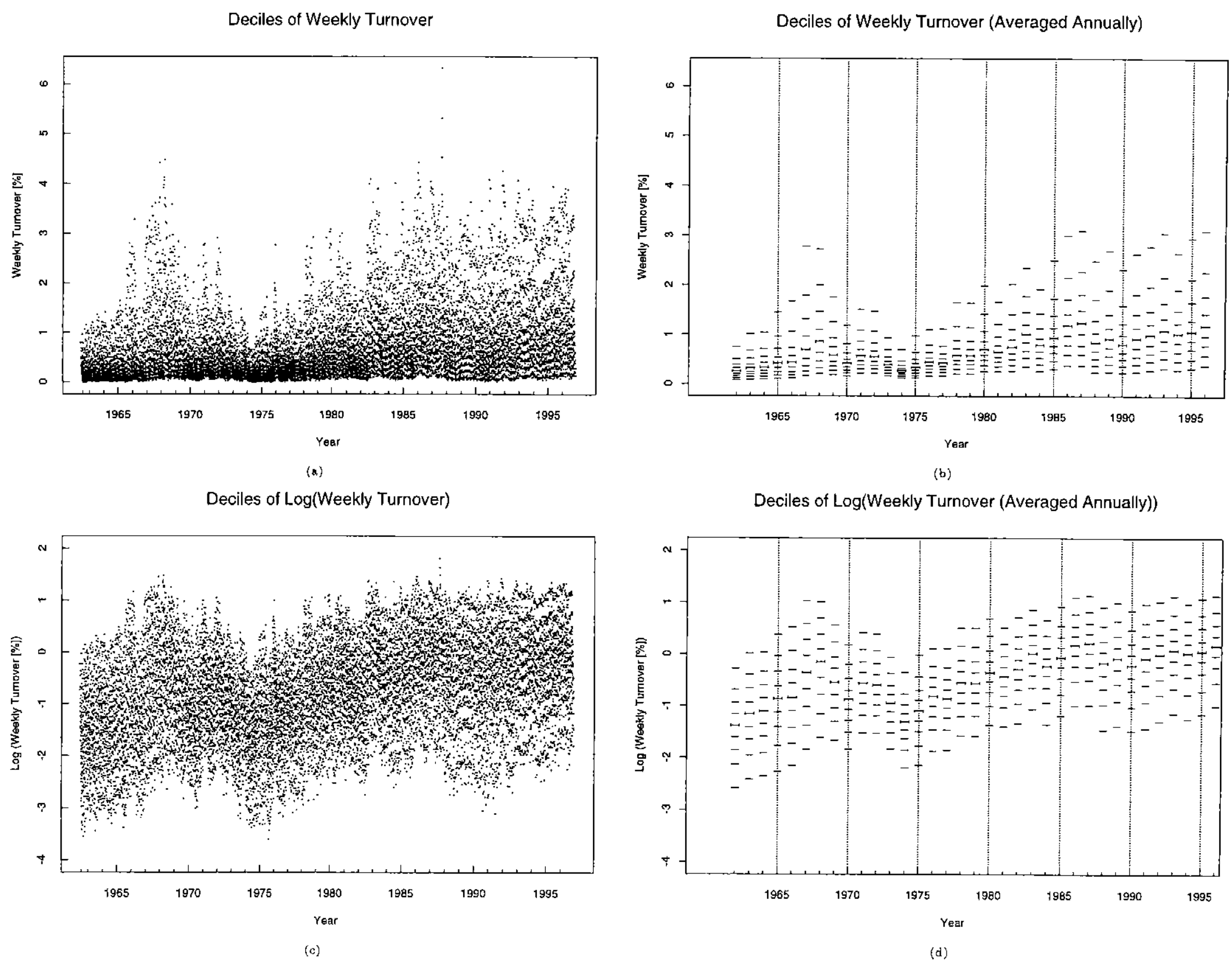

Figure 3. Cross Section of Weekly Turnover, 1962–1996.

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