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IDENTIFICATION THROUGH HETEROSKEDASTICITY: MEASURING "CONTAGION" BETWEEN ARGENTINEAN AND MEXICAN SOVEREIGN BONDS

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ABSTRACT

In this paper, I develop a new identification method to solve the problem of simultaneous equations, based on heteroskedasticity of the structural shocks. I show that if the heteroskedasticity can be described as a two-regime process, then the system is just identified under relatively weak conditions. Identification is also discussed under more than two regimes, when the residuals exhibit ARCH behavior, and when there are aggregate shocks.

This methodology is applied to measure contagion across sovereign bonds between Argentina and Mexico. The estimates of the simultaneous parameters are relatively to different definitions of the regimes.

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1 Introduction

The problem of identification when the data is generated by simultaneous equations has been studied for several decades now.[1] It can be oversimplified to one in which the parameters of interest (unknowns) belong to a system of simultaneous equations (structural form) that cannot be estimated. Only a linear transformation of it, which is known as the reduced form, can. The problem of identification is one where the number of equations obtained from the reduced form is smaller than the number of unknowns.

The literature has solved the problem by offering additional constraints that have expanded the number of equations. Some of those assumptions are relatively not controversial, such as normalization and independence of the structural shocks. In general, these assumptions are not enough to fully solve the problem and the literature has developed other restrictions that depend on prior knowledge about the system of equations. Within this class of assumptions, the most commonly used are: exclusion, sign, long run restrictions, and covariance constraints. These restrictions had proven to be very useful. There are several circumstances, however, where they are difficult to justify.

In this paper, I discuss a new identification procedure to solve the problem of endogenous variables. This restriction is based on heteroskedasticity of the structural shocks, and the conditions in which it can be used are relatively general.

I apply the methodology to measure the "contagion", or propagation, between Argentina and Mexico's Brady Bonds. This is a case in which non of the standard assumptions can be defended, thus leaving the problem of identification unsolved. Using the fact that Brady Bonds exhibit important conditional heteroskedasticity, however, it is possible to estimate the contemporaneous relationship.

The paper is organized as follows: In section 2, the typical problem of identification is specified, and the new identification assumption is discussed in the context of two regimes. The conditions under which the assumption can be used are developed. An application of the methodology is implemented to measure the propagation of shocks between sovereign bonds from Argentina and Mexico. In section 3, the case when there are more than two regimes is analyzed. In section 4,

[1] See Fisher [1976] for the most comprehensive treatment of the subject. See Haavelmo [1947] and Koopmans, et.al. [1950] for the seminal contributions.

necessary conditions for identification are derived for multivariate processes with common shocks. In section 5, two alternative procedures to define the regimes are highlighted. Finally, in section 6, conclusions and extensions are presented.

2 New Identification Procedure

To illustrate the problem of identification assume we are interested in estimating the following system of equations:

$$\begin{aligned} p_t &= \beta q_t + \varepsilon_t, \qquad (1) \\ q_t &= \alpha p_t + \eta_t, \qquad (2) \end{aligned}$$

where (1) is the demand equation, (2) is the supply equation, p_t are the observed prices, q_t are the observed quantities, and ε_t and η_t are the structural shocks. The parameters of interest are α, β, and the variance covariance matrix of the structural shocks: σ_ε^2, σ_η^2. In this system of equations normalization has already been assumed[2], and the orthogonalization of structural shocks, $\sigma_{\varepsilon\eta} = 0$.[3] As was indicated above, these assumptions are relatively not controversial. However, they are not enough to identify the parameters. Equations (1) and (2) cannot be directly estimated because of simultaneous equation bias: if α and β are different from zero, then the right hand side variables are correlated with the errors in both equations. In this example, the only statistic that can be obtained is the variance covariance matrix of the reduced form. Solving for p_t and q_t in equations (1) and (2):

$$\begin{aligned} p_t &= \frac{1}{1-\alpha\beta}\left[\beta\eta_t + \varepsilon_t\right], \qquad (3) \\ q_t &= \frac{1}{1-\alpha\beta}\left[\eta_t + \alpha\varepsilon_t\right], \qquad (4) \end{aligned}$$

[2]The coefficients of p_t in the first equations, and of q_t in the second one are equal to one. This assumption is innocuous in the sense that it changes the units in which the disturbances are measured.

[3]In general, this constraint is imposed because the definition, or the economic interpretation, of the shocks in the structural equations implies their independence: for example, nominal versus real shocks, permanent versus transitory shocks, or supply versus demand shocks.

which implies a variance covariance matrix,

$$\Omega = \frac{1}{(1-\alpha\beta)^2}\left[\begin{array}{cc} \beta^2\sigma_\eta^2+\sigma_\varepsilon^2 & \beta\sigma_\eta^2+\alpha\sigma_\varepsilon^2 \\ . & \sigma_\eta^2+\alpha^2\sigma_\varepsilon^2 \end{array}\right].$$

The problem of identification is that the variance covariance matrix only provides three equations (the variance of p_t, the variance of q_t, and the covariance between p_t and q_t) and there are four unknowns: α, β, σ_η^2, σ_ε^2.

The literature has solve this by imposing additional parameter constraints: (i) exclusion restriction. This accounts to impose either $\alpha = 0$, or $\beta = 0$.[4] (ii) sign restrictions. The imposition of the sign on the slopes of the structural equations can achieve partial identification because the two inequalities imply a region of admissible parameters.[5] (iii) long run constraints. When the structural form includes lag dependent variables, it is possible to constraint the long run behavior of a particular shock. This assumptions is equivalent to impose that the sum of some of the lag coefficients is equal to zero. [6] Finally, covariance constraints, for example that $\sigma_\eta^2/\sigma_\varepsilon^2$ is equal to some constant.

These restrictions have proven to be very useful in several applications. However, there are important economic problems in which non of the previous assumptions can be justified.

The purpose of this section is to offer a new identification method that is based on heteroskedasticity, and the two standard assumptions of normalization and independence of the structural shocks. Here, I discuss the case in which the heteroskedasticity can be described as two regimes. In the following sections, I discuss some generalizations.

[4] Technically this is implemented as a Cholesky decomposition of the variance covariance matrix. This restriction has been used in several applications, specially in the measurement of the transmission of monetary policy in US. The restriction, however, is assuming the problem of simultaneous equations away; if any of the contemporaneous parameters is zero then there is no problem of endogenous bias, and the estimation could have been done directly in the structural equations. Circumstances in which this assumption can be used are, therefore, limited.

[5] Even though a unique estimate cannot be obtained, at least an admissible interval is derived. Blanchard and Diamond [1989] were the first ones to use this restriction.

[6] If it is known that one shocks does not have permanent effects, then under some conditions, it is possible to obtain identification. For example, the assume that nominal shocks are short lived, while real shocks are permanent. Imposing this constraint Blanchard and Quah [1989] and Shapiro and Watson [1988] were able to estimate the effects of aggregate shocks.

2.1 Identification under two regimes.

Assume that the data exhibits two regimes in the second moments; assume that the parameters (α and β) are stable through out the two regimes; and assume that it is known when the shift between the regimes occurs.[7] Additionally, without loss of generality, assume that the first moments are equal to zero.

Denote the regimes as $s \in \{1,2\}$, and define the variances of the structural shocks in regime s as $\sigma_{\varepsilon,s}$ and $\sigma_{\eta,s}$ then variance covariance matrix of the reduced form for each regime is,

$$\Omega_s \equiv \begin{bmatrix} \omega_{11,s} & \omega_{12,s} \\ & \omega_{22,s} \end{bmatrix} = \frac{1}{(1-\alpha\beta)^2} \begin{bmatrix} \beta^2\sigma^2_{\eta,s} + \sigma^2_{\varepsilon,s} & \beta\sigma^2_{\eta,s} + \alpha\sigma^2_{\varepsilon,s} \\ . & \sigma^2_{\eta,s} + \alpha^2\sigma^2_{\varepsilon,s} \end{bmatrix}, \quad s \in \{1,2\} \tag{5}$$

where Ω_s is the variance covariance matrix of the regime s. Note that now the unknowns are six: α, β, $\sigma^2_{\eta,1}$, $\sigma^2_{\varepsilon,1}$, $\sigma^2_{\eta,2}$, and $\sigma^2_{\varepsilon,2}$. But the two variance covariance matrixes provide six equations. Therefore, this is a non linear system with six equations and six unknowns. If they are not a combination then the system is just identified. Furthermore, note that no additional assumption is required to identify the structural form; there are no parameter restrictions, other than the stability of the coefficients.

After solving for the variances in equations (5), α and β satisfy the following non-linear system of equations:

$$\beta = \frac{\omega_{12,s} - \alpha \cdot \omega_{11,s}}{\omega_{22,s} - \alpha \cdot \omega_{12,s}}, \quad s \in \{1,2\} \tag{6}$$

For the purpose of inference, the small sample distributions of α and β are derived by bootstrapping, taken as given the asymptotic distributions of the variance covariance matrixes.[8] The following proposition summarizes the conditions under which α and β are identified. In other words, when the non linear system of equations are indeed independent.

[7] Several of these assumptions are relaxed in the following sections.

[8] See pages 300 and 301 in Hamilton [1994] for the distribution of the variance covariance matrix.

Proposition 1 *Let p_t and q_t be realizations of a bivariate random variable governed by equations (1) and (2), where the parameters (α and β) determining the law of motion are stable through out all realizations, and where the disturbances exhibit heteroskedasticity that can be described as two regimes in the variance covariance matrix.*

Then, if the variance covariance matrixes satisfy the following two properties, the structural form is just identified.

$$\det\left[\Omega_2 - \Omega_1\right] \neq 0 \tag{7}$$

and

$$\det\left[\Omega_2 - \frac{w_{11,2}}{w_{11,1}}\Omega_1\right] \neq 0 \quad or \quad w_{12,2} - \frac{w_{11,2}}{w_{11,1}}w_{12,1} \neq 0 \tag{8}$$

Finally, α and β satisfy equations (6)

From now on the first condition (equation 7) is referred as the "determinant of the difference", while the second condition (equation 8) is the "determinant of the weighted difference" or the "covariance of the weighted difference".

Proof. Because equations (5) form a non linear system of six equations, the conditions in which they imply six linearly independent equations cannot be trivially stated. However, by inspection, there are only two cases in which the system fails to solve for both α and β.

The first one is when the heteroskedasticity does not implies a change in the relative importance of the structural shocks: in other words, when $\Omega_2 = a\Omega_1$, for some scalar a. In this case, the system lacks of one equation. Note that if $\Omega_2 = a\Omega_1$ then $\det\left[\Omega_2 - a\,\Omega_1\right] = 0$, which can be tested by computing whether or not

$$\det\left[\Omega_2 - \frac{w_{11,2}}{w_{11,1}}\,\Omega_1\right] \stackrel{?}{=} 0.$$

By construction this is equivalent to ask if the covariance of the normalized difference is equal to zero: $w_{12,2} - \frac{w_{11,2}}{w_{11,1}} w_{12,1} \overset{?}{=} 0$. The asymptotic properties of this statistic are better behaved than the one from the determinant, and in the empirical section this is what is implemented. Note that this condition is not satisfied if the residuals on the estimation of Ω_2 are exactly the same, and thus, perfectly correlated.

The second instance when there is no identification of both parameters is when only one of the variances changes. In this case, the system is only partially identified. For example, assume that $\sigma^2_{\eta,2} > \sigma^2_{\eta,1}$ and $\sigma^2_{\varepsilon,2} = \sigma^2_{\varepsilon,1}$. The difference between the two matrixes is

$$\frac{\sigma^2_{\eta,2} - \sigma^2_{\eta,1}}{(1-\alpha\beta)^2} \begin{bmatrix} \beta^2 & \beta \\ \beta & 1 \end{bmatrix},$$

which provides two procedures to estimate β, but none to compute α. The opposite occurs when only σ^2_ε changes. This problem can also be tested given that the determinant of the difference of the variance covariance matrixes is equal to zero,

$$\det[\Omega_2 - \Omega_1] = \frac{\sigma^2_{\eta,2} - \sigma^2_{\eta,1}}{(1-\alpha\beta)^2} \cdot \det \begin{bmatrix} \beta^2 & \beta \\ \beta & 1 \end{bmatrix} \overset{?}{=} 0.$$

This determinant is equal to zero if the residauls of the estimate of Ω_2 and Ω_1 are perfectly correlated.

In conclusion, to violate the conditions it is required that either the changes in the variance covariance are fully explained by only one shock, or that the changes are proportional. Both imply a perfect correlation in the residuals on the estimation of Ω_1 and Ω_2. On the contrary, the identification is valid if both variances shift, and if the relative importance of the shocks changes across the regimes. This is the case, for example, of almost all macro-applications where ARCH is used. ■

Corollary 2 *When the condition (7), the "determinant of difference", is not satisfied, but condition (8), the "covariance of the weighted difference" is, then the system is partially identified.*

Proof. As is argued in the proof of proposition 1, when $\sigma^2_{\eta,2} > \sigma^2_{\eta,1}$ and $\sigma^2_{\varepsilon,2} = \sigma^2_{\varepsilon,1}$ then there are two methods to estimate β. Defime $\Delta\Omega = \Omega_2 - \Omega_1$, then

$$\beta = \frac{\Delta\Omega_{11}}{\Delta\Omega_{12}} \text{ and } \beta = \frac{\Delta\Omega_{12}}{\Delta\Omega_{22}}$$

similarly, when $\sigma^2_{\eta,2} = \sigma^2_{\eta,1}$ and $\sigma^2_{\varepsilon,2} > \sigma^2_{\varepsilon,1}$, α can be estimated as

$$\alpha = \frac{\Delta\Omega_{12}}{\Delta\Omega_{11}} \text{ and } \alpha = \frac{\Delta\Omega_{22}}{\Delta\Omega_{12}}$$

Note that in both cases there is an extra equation to estimate the coefficient. This means that the model can be tested. In particular, the hypothesis that the parameters are stable across the regimes can be tested. ■

The intuition of why identification is achieved is that the heteroskedasticity changes the region in which the errors are distributed; when the parameters are stable, the heteroskedasticity rotates the errors along the structural equations. This is what allows one to solve for the system of equations. The simplest intuition can be developed by first analyzing the case in which only one shock is increasing its variance. As is mentioned before, this achieves partial identification; however, the intuition can be extended afterwards to the case studied here. Assume that it is known that at some point in time there is an increase in the variance of the supply shocks. During that period, the "cloud" of realizations is going to widen along the demand curve as is depicted in figure 1. Comparing how the ellipse of the realizations has changed across the two samples allows one to determine the slope of the demand curve.

This explanation has a very simple intrumental variable interpretation. Oversimplifying the intuition, a valid instrument is a variable that affects one equation and not the other one. The idea is that it "moves" one equation allowing to estimate the other one. The heteroskedasticity, in this example, is exactly doing that. Spliting the sample between those times in which the supply shocks are small and large separates those times in which the supply is "moving" relatively more. The change in the size of the shocks in the supply curve is equivalent to find an instrument for the demand curve.[9]

[9] In an earlier paper, I use the partial identification procedure (only one variance changes) to test for the stability of the parameter estimated. I implement the test as an Instrumental Variable. See Rigobon [1999].

When both variances shift, then there is an expansion along both schedules. If the relative importance of shocks remains the same, then the two ellipses are proportional and no additional information is obtained from the heteroskedasticity, other than the magnitude of the shift. On the contrary, if the relative importance of the shocks changes, then the ellipse widens more along one of the schedules than the other and the problem is solved as before. There is only one combination of α and β in which the increases in the variances are consistent with the expansion of the ellipse along the two dimension.

[Figure 1 here]

The following three remarks are in order:

Remark 1 *It is not necessary to know which shocks becomes more important across the regimes. The fact that there is change is enough.*

The proof of proposition 1 only requires the variances to shift, not the knowledge of the direction of the shifts.

Remark 2 *The assumption of knowing when the regime shift occurs can be relaxed. The two variance covariance matrixes can be estimated by implementing a regime shift regression as in Hamilton [1990].*

First, to obtain identification no assumption about the relationship between the two variance covariance matrixes was imposed. If the variance covariances are correlated across the regimes, but satisfy conditions (7) and (8), then they still provide enough equations.

Second, the most efficient procedure is to impose the windows exogenously. However, if the windows are wrongly assigned, then the estimates of the variance covariance matrixes are biased, inconsistent, inefficient, and their difference likely not to be statistically significant. Thus, one of the determinant conditions (and maybe both) is rejected and the identification cannot be used. An alternative is to define the windows endogenously. The estimates are consistent, but might be inefficient. If conditions (7) and (8) are satisfied, then the procedure for identification can be used.

Remark 3 *The assumption that the structural shocks have to be independent can be relaxed, and, at least, the relative variances of the structural shocks in each of the regimes can still be estimated. Assume the model is described as follows:*

$$
\begin{aligned}
p_t &= \beta q_t + \varepsilon_t + \gamma \eta_t, \\
q_t &= \alpha p_t + \eta_t + \delta \varepsilon_t,
\end{aligned}
$$

whose reduced form is

$$
\underbrace{\begin{bmatrix} 1 & -\beta \\ -\alpha & 1 \end{bmatrix}}_{A} \begin{bmatrix} p_t \\ q_t \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \gamma \\ \delta & 1 \end{bmatrix}}_{B} \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix}
$$

$$
B^{-1} A \begin{bmatrix} p_t \\ q_t \end{bmatrix} = \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix}.
$$

In this case, the procedure identifies a normalization of $C \equiv B^{-1}A$. *The relative importance of the shocks in each of the regimes still can be obtained* $\left(\sigma_\varepsilon / \sigma_\eta\right)_s$. *The structural parameters, however, cannot be recovered. Further assumptions on how the structural shocks are interrelated are required in order to identify the whole system.*

2.2 Brady Bonds: An example

In this section, an application of the identification methodology is implemented to estimate the propagation of shocks across sovereign risk bonds between Argentina and Mexico. This issue has been discussed in the literature as "contagion" and the purpose here is to measure the strength of the channels.[10]

It should be clear that if Mexican shocks affect Argentina (through trade, for example), then Argentinean shocks should affect Mexico. This means that the price of both sovereign bonds are determined simultaneously. In this application, the two standard assumptions are easy to justify: normalization is just changing the units in which shocks are measured, and the idiosyncratic shocks to each country (elections, social demonstrations, natural disasters, etc.) can be considered

[10] See Pristker [1999] for a survey on the theoretical literature and Forbes and Rigobon [1999] for a survey on the empirical literature.

independent across them. The other identification assumptions, however, are difficult to defend: (i) it is not reasonable to assume exclusion restrictions in one direction and not in the other one; (ii) nor it makes sense to assume that one transmission is positive while the other one is negative; (iii) there are no good reasons to assume that the shocks to one country are more persistent than the shocks to the other one; (iv) and finally, it is difficult to substantiate an assumption about the relative importance of idiosyncratic shocks across the countries. This leaves the problem of identification unsolved with the standard techniques.

The data consists of the daily Brady Bond returns for Mexico and Argentina between January 1994 to December 1998 obtained from the EMBI+ index constructed by JPMorgan. In figure 2, the two indexes are shown. As can be seen, the indexes are highly correlated; more than 99 percent in levels and more than 80 percent in daily returns.

[Figure 2 here]

In figure 3, the rolling variances and covariance of daily returns is shown, where a 20 days rolling window is used. Note that the moments change significantly through time. Moreover, it is easy to identify the periods of crisis by looking at the large changes in the second moments.[11]

[Figure 3 here]

The very large conditional heteroskedasticity present in the data grants the use of the procedure here described to solve the problem of identification.

Brady Bond returns might be affected by common shocks, such as the international interest rate, therefore, to control for them and lags the following specification is estimated first:

$$x_t = z_t + \phi(L)\, x_t + \Phi(L)\, z_t + \nu_t \tag{9}$$

where x_t is the vector of Argentinean and Mexican daily returns, z_t is the return on the US 10 years bond, $\phi(L)$ and $\Phi(L)$ are lag operators (usually 5 or 10 lags), and ν_t represent the residuals. The 10 years bond is used given that this is close to the average maturity of outstanding Brady bonds.

[11] The Mexican crisis implied an increase in the conditional variance of both countries by more than 15 times. The same for the Russian crisis. The Asian crises, though, did not have a large impact on the conditional variances. As is shown below, this is reflected in the quality of the estimates.

Several sensitivity analysis were performed, and choosing a different US bond did not change the results.

After regression (9) is estimated, the second step is to take the residuals and compute their contemporaneous relationship using the procedure here developed. Note that under the assumption of endogeneity, the residuals from regression (9) can be written as follows:

$$\begin{aligned}\nu_{1,t} &= \frac{1}{1-\alpha\beta}\left[\beta\eta_t + \varepsilon_t\right],\\ \nu_{2,t} &= \frac{1}{1-\alpha\beta}\left[\eta_t + \alpha\varepsilon_t\right].\end{aligned}$$

I define the crisis and tranquility window exogenously. Five cases are studied: the Mexican crisis alone, the Asian crises, the Mexican and Asian crises together, the Russian crisis alone, and All three recent crises together; Mexican, Asian, and Russian.

The crises periods are as follows: The Mexican crisis started in December 19, 1994 when the abandonment of the fixed exchange rate occurred. The end of the crisis is set at March 31, 1995 when the markets calmed down after the US bailout. The South East Asian crises started with the speculative attack against Thailand (June 01, 1997) and ended after the Korean crisis calmed down at the end of January 1998 (January 31, 1998). Finally, the Russian crisis started with the collapse in the bond market at the beginning of August (August 01, 1998) and lasted until the end of the month (August 31, 1998).

The sample for each exercise is as follows: for the Mexican crisis, the sample runs from January 01, 1994 until March 31, 1995; for the Asian crises the sample starts in April 01, 1995 and ends in January 31, 1998; while for the Russian crisis the sample is from February 01, 1998 until December 31, 1998.

Table 1, presents the results of estimating the variance covariance matrix for both regimes. The columns it is highlighted the crisis under consideration. The first set of three rows indicates the variance covariance matrix in the low variance regime; the next three rows are the variance covariance matrix in the high volatility regime; and the bottom three rows show the increase across the two regimes. Note that the increase in the moments is appreciable across the regimes.

The next step is to determine if the estimates satisfy conditions (7) and (8) in proposition

	Mexico	Asia	Mexico Asia	Russia	Mexico Asia Russia
Low Variance Regime					
Variance Argentina $(\cdot 10^{-4})$	0.6324	0.1248	0.5432	1.7854	0.7923
Covariance $(\cdot 10^{-4})$	0.3676	0.0962	0.3197	1.0827	0.4713
Variance Mexico $(\cdot 10^{-4})$	0.4113	0.1297	0.3616	0.8420	0.4584
Hig Variance Regime					
Variance Argentina $(\cdot 10^{-4})$	8.9847	0.5979	3.1016	7.4546	3.5428
Covariance $(\cdot 10^{-4})$	7.2500	0.4201	2.4627	4.9882	2.7200
Variance Mexico $(\cdot 10^{-4})$	8.9327	0.4165	2.9720	3.6873	3.0612
Change					
Variance Argentina	14.21	4.79	5.71	4.18	4.47
Covariance	19.72	4.37	7.70	4.61	5.77
Variance Mexico	21.72	3.21	8.22	4.38	6.68

Table 1: Change in the variance covariance matrix during several crises. Exogenous crisis window.

1. Taking the change in the second moments as given, the distribution of the two conditions is obtained using a bootstrap; 1000 series of residuals are computed using the asymptotic distribution of the variance covariance matrixes estimated is table 1.[12] For each run, the "determinant of the difference" and the "covariance of the weighted difference" is calculated. Table 2 summarizes some of the statistics of the bootstrapped distribution.

	Mexico	Asia	Mexico Asia	Russia	Mexico Asia Russia
Determinant of the Difference					
Average $(\cdot 10^{-8})$	23.3756	0.0299	2.0662	0.6977	2.0958
Standard Deviation $(\cdot 10^{-8})$	6.0336	0.0082	0.3403	0.7863	0.3420
Quasi t_stat	3.87	3.64	6.07	0.89	6.13
Mass below zero	0.0%	0.0%	0.0%	15.9%	0.0%
Covariance of the Weighted Difference					
Average $(\cdot 10^{-4})$	0.2057	0.0047	0.0637	0.0522	0.0610
Standard Deviation $(\cdot 10^{-4})$	0.0756	0.0032	0.0144	0.0344	0.0137
Quasi t_stat	2.72	1.44	4.42	1.52	4.45
Mass below zero	0.00%	0.00%	0.00%	0.00%	0.00%

Table 2: Test of the two conditions. Exogenous crisis window.

As can be seen, if the quasi t_stats are used (remember that the small sample distributions are not normal) the "determinant of the difference" condition is not statistically different from zero during the Asian and the Russian crises. Using the mass below zero as the criteria for rejection, then only the Russian crisis fails to satisfy this condition. On the other hand, the "covariance of the weighted difference" cannot be rejected to be equal to zero during Asia and Russia using the

[12]The distributions of the bootstrapps are not shown for brevety. The results are provided upon request and can be found in my web page.

quasi t_stats, but both have no mass below zero. In other words, only during the Russian crisis it is possible to be sure that the identification cannot be used. During the Asian crises, the results have to be taken cautiously though.[13]

In table 3, the estimates of $\hat{\alpha}$ and $\hat{\beta}$ solving equations (6) are shown. Again, the distribution is obtained by bootstrapping.

	Mexico	Asia	Mexico Asia	Russia	Mexico Asia Russia
$\hat{\alpha}$					
Mean	0.3247	0.1911	0.3139	0.7109	0.3088
Standard Deviation	0.1470	0.0747	0.0791	0.5512	0.0871
Quasi t_stat	2.21	2.56	3.97	1.29	3.55
Mass below zero	2.60%	0.50%	0.20%	14.60%	0.20%
$\hat{\beta}$					
Mean	0.6474	0.6586	0.6495	0.4637	0.6493
Standard Deviation	0.1133	0.0468	0.0609	0.2439	0.0669
Quasi t_stat	5.72	14.08	10.67	1.90	9.70
Mass below zero	0.00%	0.00%	0.00%	5.60%	0.00%

Table 3: Estimation of the structural parameters. Exogenous Windows.

First note that confirming the results from table 2, the parameters estimated during the Russian crisis are not significantly different from zero. Moreover, as can be seen, it is not the case that the coefficient estimated is closer to zero, but that the standard deviation is large (in comparison to the estimates in the other subsamples).

Second, the estimates are remarkably stable across sub-samples. The estimates are obviously correlated, however, it is not clear how this should be taken into account. In order to provide some idea of the robustness of the parameters two cases were tested; one assuming that the estimates are independent, and the second one using the correlation that maximizes the possibility of rejection. When the estimates are assumed to be independent, then it is not possible to reject the hypothesis that the estimates are the same across all samples. When the minimum standard deviation is computed, however, the estimates of $\hat{\alpha}$ between Asia vs. Mexico-Asia, and Asia vs. All samples, are statistically different. For $\hat{\beta}$, again, it is not possible to reject the hypothesis that the estimates are the same.

In table 4, the results of these tests are shown. The top panel describes the mean and standard

[13]The implication of failling the conditions is that the confidence intervals of the parameters should be infinitely wide. This is because there exists a continuum of solutions to the system of equations. I owe this interpretation to Mark Watson.

deviation of the test for $\hat{\alpha}$: the upper triangle show the absolute value of the difference in the parameters, while the lower triangle are the minimum standard deviations. The bottom panel are the results for the $\hat{\beta}$ estimates.

	Mexico	Asia	Mexico-Asia	Russia	All
$\hat{\alpha}$					
Mexico		0.13360	0.01078	0.38627	0.01583
Asia	0.07234		0.12282	0.51987	0.11777
Mexico-Asia	0.06796	0.00438		0.39705	0.00505
Russia	0.40417	0.47651	0.47213		0.40210
All	0.05991	0.01243	0.00805	0.46408	
$\hat{\beta}$					
Mexico		0.01121	0.00214	0.18372	0.00194
Asia	0.06651		0.00907	0.19493	0.00927
Mexico-Asia	0.05239	0.01412		0.18586	0.00020
Russia	0.13063	0.19714	0.18302		0.18566
All	0.04633	0.02017	0.00606	0.17697	

Table 4: Test of stability of parameters. Exogenous Windows.

3 Identification under more than two regimes.

Generalizations of the identification problem to more than two regimes should be straight forward. First, I study the case in which there are S regime shifts. Second, I discuss the case in which there is a continuum of states, or ARCH.

3.1 S regimes.

When the data exhibits multiple regimes, there is a variance covariance matrix for each of them, giving rise to the following set of equations:

$$\Omega_s \equiv \begin{bmatrix} \omega_{11,s} & \omega_{12,s} \\ & \omega_{22,s} \end{bmatrix} = \frac{1}{(1-\alpha\beta)^2} \begin{bmatrix} \beta^2\sigma^2_{\eta,s} + \sigma^2_{\varepsilon,s} & \beta\sigma^2_{\eta,s} + \alpha\sigma^2_{\varepsilon,s} \\ . & \sigma^2_{\eta,s} + \alpha^2\sigma^2_{\varepsilon,s} \end{bmatrix}, \quad s \in \{1,..,S\} \tag{10}$$

where this system has $3S$ equations (one variance covariance matrix per regime) and $2S+2$ unknowns: S times two variances for each regime, plus two parameters (α and β).

For S larger than two the system has more equations than unknowns. This can have three interpretations: Firstly, those equations can be thought as overidentifying restrictions, and the model can be tested.[14] Secondly, it can be a factor regression model. The left hand side variables of equations (10) are the estimates (or observables), the variances ($\sigma^2_{\eta,s}$ and $\sigma^2_{\varepsilon,s}$)are the unobservable factors, and the coefficients are the weights or factor loadings.[15] Thirdly, it has a minimum distance interpretation, either as a NLS or as a GMM. Solving in equations (10) for α and β, the same relationship as equation (6) is obtained in each state.

$$\beta = \frac{\omega_{12,s} - \alpha \cdot \omega_{11,s}}{\omega_{22,s} - \alpha \cdot \omega_{12,s}}, \ s \in \{1, ..., S\} \tag{11}$$

The parameters can be estimated as of to reduce the distance between the left hand side and the right hand side of equation (11) as a NLS,

$$\sum_s \left[\beta - \frac{\omega_{12,s} - \alpha \cdot \omega_{11,s}}{\omega_{22,s} - \alpha \cdot \omega_{12,s}}\right]^2 = 0, \tag{12}$$

or as GMM,

$$E_s \left[\beta - \frac{\omega_{12,s} - \alpha \cdot \omega_{11,s}}{\omega_{22,s} - \alpha \cdot \omega_{12,s}}\right] = 0. \tag{13}$$

Both the NLS and the GMM require important assumptions on the innovations of the system of equations (10) in order to assure that equations (12) or (13) make econometric sense; for example, these assumptions have to guarantee that the deviations $\left(\beta - \frac{\omega_{12,s}-\alpha\cdot\omega_{11,s}}{\omega_{22,s}-\alpha\cdot\omega_{12,s}}\right)$ are orthogonal.

3.2 ARCH Residuals.

Finally, it is possible to estimate a bivariate ARCH or GARCH model and construct a system similar to (10), where there is a set of equations for each period.

Again, this has the same interpretations as before, even though the overidentification interpre-

[14]For example, the underlying assumption that α and β are stable through time can be tested if at least three regimes are present in the data.

[15]Factor analysis usually assumes that the $\omega_{ij,s}$'s are independent. It is unlikely, however, that this is the case in this setup. Therefore proper corrections have to be taken into consideration in the estimation procedure.

tation assumes a very large set of constraints that are not independent. Note that by construction the observations are serially correlated; however, one advantage of the ARCH structure is that it indicates the form of the dependence.

3.3 Brady Bonds: $S = 4$ regimes.

Going back to the Brady Bonds example, assume there are four regimes: three crises and one tranquil period. The sample period runs from January 94 to December 98, the crises windows are defined as before, and the tranquil time is the rest of the sample. It is assumed that each crisis coincides with a different regime. After running the first step, four variance covariance matrices are estimated (see table 5).

Regimes	Tranquil	Mexico	Asia	Russia
Variance Argentina $(\cdot 10^{-4})$	0.7922	8.9847	1.1502	7.4546
Covariance $(\cdot 10^{-4})$	0.4713	7.2499	0.7844	4.9882
Variance Mexico $(\cdot 10^{-4})$	0.4584	8.9327	0.6756	3.6873

Table 5: Variance Covariance Matrix for all the regimes.

Similarly as before, the Mexican and Russian crises imply important increases in the second moments, while the Asian crises mildly affected the variance covariance matrix.The estimates of the variances are similar to those obtained in the previous section.

The four variance covariance matrices provide more equations than unknowns. The parameters, therefore, are estimated to reduced the distance in equations (11). This is the non linear least square approach: minimize equation (12). The estimates and distribution of $\hat{\alpha}$ and $\hat{\beta}$ are obtained by bootstrapping. In table 6 the results are summarized.

	$\hat{\alpha}$	$\hat{\beta}$
Mean	0.317345	0.639383
Standard Deviation	0.1919	0.0858
Quasi t-stat	1.65	7.52
Mass below zero	5.60%	0.20%

Table 6: Summary statistics of the bootstrapped distributions.

Note that the estimates are close to those obtained in the previous section. Moreover, it is not possible to reject the hypothesis that the parameters are the same between these ones and all previous five exercises even using the minimum standard deviation.

4 Aggregate shocks

In the previous section, the stochastic process is bivariate and there are no common shocks. In this section, I relax these assumptions and discuss the necessary conditions to achieve identification. I continue to assume normalization of all structural shocks and aggregate shocks, as well as the independence of them. Modify the structural form as follows:

$$p_t = \beta q_t + \gamma z_t + \varepsilon_t, \tag{14}$$

$$q_t = \alpha p_t + z_t + \eta_t, \tag{15}$$

where z_t is an unobservable common shock whose variance is σ_z^2. The reduced form is then

$$p_t = \frac{1}{1-\alpha\beta}\left[\beta\eta_t + \varepsilon_t + (\beta+\gamma) z_t\right], \tag{16}$$

$$q_t = \frac{1}{1-\alpha\beta}\left[\eta_t + \alpha\varepsilon_t + (1+\alpha\gamma) z_t\right], \tag{17}$$

and where the variance covariance matrix is

$$\Omega = \frac{1}{(1-\alpha\beta)^2}\begin{bmatrix} \beta^2\sigma_\eta^2 + \sigma_\varepsilon^2 + (\beta+\gamma)^2\sigma_z^2 & \beta\sigma_\eta^2 + \alpha\sigma_\varepsilon^2 + (1+\alpha\gamma)(\beta+\gamma)\sigma_z^2 \\ . & \sigma_\eta^2 + \alpha^2\sigma_\varepsilon^2 + (1+\alpha\gamma)^2\sigma_z^2 \end{bmatrix}.$$

In this case, the problem of identification cannot be solved using the heteroskedasticity. Each additional variance covariance matrix contributes with three equations, but also with three new unknowns; $\sigma_{\eta,s}^2$, $\sigma_{\varepsilon,s}^2$, and $\sigma_{z,s}^2$. Thus the system remains equally underidentified.

How to solve the problem when there are common shocks then? Allowing for more endogenous variables. Assume the structural form is written as follows:

$$A_{NxN}\begin{bmatrix} x_{1,t} \\ \vdots \\ x_{N,t} \end{bmatrix} = \Gamma_{NxK}\begin{bmatrix} z_{1,t} \\ \vdots \\ z_{K,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \vdots \\ \varepsilon_{N,t} \end{bmatrix} \tag{18}$$

where the assumption of orthogonalization implies,

$$\begin{aligned} E\left[z_{i,t}, z_{j,t}\right] &= 0 \quad \forall i \neq j, \quad 1 \leq i, j \leq K \\ E\left[\varepsilon_{i,t}, \varepsilon_{j,t}\right] &= 0 \quad \forall i \neq j, \quad 1 \leq i, j \leq N \\ E\left[z_{i,t}, \varepsilon_{j,t}\right] &= 0 \quad \forall i \neq j, \quad 1 \leq i \leq K, \quad 1 \leq j \leq N, \end{aligned}$$

and where $x_{n,t}$, $n \in \{1, ..., N\}$ are the N endogenous variables; $z_{k,t}$, $k \in \{1, ..., K\}$ are the K unobservable common (or aggregate) shocks, assumed to be independent but not identically distributed, and with variance $\sigma_{z,k,s}$ in state s; $\varepsilon_{n,t}$ are the structural shocks, assumed to be independent, but not identically distributed, and with variance $\sigma_{\varepsilon,n,s}$ in state s. As before, it is assumed that there are $s \in \{1, ..., S\}$ possible regimes. A_{NxN} is the matrix that describes the contemporaneous parameters, where the assumption of normalization has been already imposed:

$$A_{NxN} = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ a_{21} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & 1 \end{bmatrix}, \tag{19}$$

and Γ_{NxK} are the parameters from the common shocks. Normalization of the common shocks is also assumed; in this case, it implies a unitary impact on the first equation,

$$\Gamma_{NxK} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nk} \end{bmatrix}. \tag{20}$$

Proposition 3 *A multivariate system of N equations, with K unobservable aggregate shocks, described by equations (18), (19) and (20) can be identified if the number of states (S) is larger than* $2\frac{(N+K)(N-1)}{N^2-N-2K}$ *and if there is a minimum number of endogenous variables that satisfy* $N^2-N-2K>0$.

Proof. Note that the proposition states a necessary condition, but not a sufficient one. In other words, a necessary condition for identification requires that the number of unknowns is smaller that the number of equations, the sufficient condition indicates when the equations are also independent.

From equation (18), the number of equations is given by the variance covariance matrix in each regime. This provides $\frac{N(N+1)}{2}$ equations in each state. The number of equations is:

$$S \cdot \frac{N(N+1)}{2}$$

The total number of unknowns is as follows: The matrix A_{NxN} has $N(N-1)$ parameters; The matrix Γ_{NxK} has $K(N-1)$ parameters; the variances of the aggregate shocks in each state, K variances times S regimes; and the variances of the structural shocks in each regime, N variances times S regimes. Therefore, the total number of unknowns is:

$$N(N-1)+K(N-1)+S\cdot K+S\cdot N$$

Identification, then, requires

$$
\begin{aligned}
S \cdot \frac{N(N+1)}{2} &\geq N(N-1) + K(N-1) + S \cdot K + S \cdot N \\
S &\geq 2\frac{(N+K)(N-1)}{N^2 - N - 2K} \qquad (21)
\end{aligned}
$$

Inequality (21) indicates the minimum number of states required to obtain identification.

In order for (21) to make sense, there is a minimum number of endogenous variables that is required.

$$N^2 - N - 2K > 0 \qquad (22)$$

This minimum number of endogenous variables is necessary to solve the problem that has motivated this section. In particular, for $N = 2$ and $K = 1$ The inequality is not satisfied. In fact, in this case, the proposition implies that the number of states has to be larger than ∞. ■

There are several remarks that can be extracted from proposition 3.

Remark 4 *First, in the absence of aggregate shocks only two states are required to achieve identification. When $K = 0$, the right hand side of equation 21 is equal to 2, independently of the number of endogenous variables N.*

Remark 5 *Second, in the limit, when the number of endogenous variables is infinite, the system can be just identified with only two states, independently of any finite number of aggregate shocks. This is obtained by taking the limit in equation 21.*

Remark 6 *Finally, if $K > 0$ and N is finite, the number of states required to achieve identification is always larger than two. With two states the constraint is always violated.*

$$(N+K)(N-1) > N^2 - N - 2K \Rightarrow KN > -K$$

5 Additional methods to define the windows.

In this section, I study two alternative procedures to define the regime windows: First, I determine the regimes endogenously using a Markov switching model in the spirit of Hamilton [1990]. Second, I estimate the crisis window as those times in which the variance of the residuals is larger than 1.5 times the unconditional variance in the sample.

The idea is to determine how robust the parameters are to these changes in the definition of the windows. I give most of the attention to the first one, and mainly present the results for the last one.

5.1 Switching regime.

In this section, the two regimes are determined endogenously by estimating a switching regime model that allows for changes in variances.[16],[17] The sample is split in the same five subsamples studied before.

In table 7, the estimates of the variance covariance matrices are shown. The top row indicates the subsample considered. The next three rows are the variance covariance estimates in the low variance regime. The second set of three rows are the estimates for the high variance state. The last set of rows compute the increase across regimes.[18]

Note that, in general, the crisis regime is almost always 10 times more volatile than the tranquil state. Using the asymptotic distribution is straight forward to show that the variance covariance across the states are statistically different.

The first question that has to be answered is if the variance covariance matrixes satisfy the two conditions in proposition 1. As before, taking the change in the second moments as given, the distribution of the two conditions is obtained using a bootstrap. Table 8 summarizes some statistics of the bootstrapped distributions.

The first set of results summarizes the properties of the distribution of the "determinant of the difference" condition. The second set of rows presents the results for the "covariance of the weighted

[16]This model is estimated using the EM algorithm explained is Hamilton [1990] and in Hamilton [1994], chapter 22.

[17]This step was computed either by forcing the means across regimes to be the same, or by allowing them to change. Additionally, the lags were increased from 5 days to 10 days. The results are qualitatively the same.

[18]The estimates of the means are not shown given that they are not interesting for the purpose of this paper.

Switching Regime	Mexico	Asia	Mexico Asia	Russia	Mexico Asia Russia
Regime of low variance					
Variance Argentina $(\cdot 10^{-4})$	0.594	0.179	0.265	0.316	0.273
Covariance $(\cdot 10^{-4})$	0.341	0.116	0.165	0.247	0.179
Variance Mexico $(\cdot 10^{-4})$	0.343	0.131	0.183	0.255	0.196
Regime of high variance					
Variance Argentina $(\cdot 10^{-4})$	7.871	1.903	4.766	12.420	6.382
Covariance $(\cdot 10^{-4})$	6.038	1.274	3.504	7.510	4.352
Variance Mexico $(\cdot 10^{-4})$	7.449	1.432	4.211	5.245	4.466
Change					
Variance Argentina	12.26	9.62	16.96	38.29	22.39
Covariance	16.71	10.03	20.26	29.46	23.30
Variance Mexico	20.73	9.94	21.99	19.60	21.76

Table 7: Variance Covariance Matrix for several crises and combination of crises. Regime switches are estimated endogenously.

Switching Regime	Mexico	Asia	Mexico Asia	Russia	Mexico Asia Russia
Determinant of the Difference					
Point Estimate $(\cdot 10^{-8})$	19.029	0.887	6.942	7.427	8.619
Standard Deviation $(\cdot 10^{-8})$	4.295	0.252	1.062	2.295	1.147
T-stat	4.43	3.52	6.54	3.24	7.52
Covariance of the Weighted Difference					
Point Estimate $(\cdot 10^{-4})$	1.530	0.118	0.550	2.153	0.228
Standard Deviation $(\cdot 10^{-4})$	0.578	0.088	0.219	0.734	0.168
T-stat	2.65	1.34	2.51	2.93	1.36

Table 8: Test of the determinant and the covariance of the weighted difference for several crises and combination of crises. Regime switches are estimated endogenously.

difference" condition. For each case, I report *quasi*-t-stats. Using the standard deviations it could be concluded that the first condition is satisfied in all five exercises, while the second condition is only statistically significant from zero in three cases of the five; it is not satisfied in the Asian and All the crises.

The distributions of $\hat{\alpha}$ and $\hat{\beta}$ are also determined by the bootstrapping. The estimates are obtained by solving the non linear system of equations described in equations (6). In table 9 these results are summarized.

Switching Regime	Mexico	Asia	Mexico Asia	Russia	Mexico Asia Russia
$\hat{\alpha}$					
Point Estimate	0.321	0.249	0.236	0.548	0.329
Standard Deviation	0.110	0.451	0.187	0.052	0.511
T-stat	2.92	0.55	1.27	10.53	0.64
$\hat{\beta}$					
Point Estimate	0.619	0.459	0.667	0.607	0.240
Standard Deviation	0.149	0.538	0.231	0.104	0.704
T-stat	4.17	0.85	2.89	5.83	0.34

Table 9: Estimation of the structural parameters for several crises and combination of crises. Regime switches are estimated endogenously.

As can be seen, during the Mexican crisis, both $\hat{\alpha}$ and $\hat{\beta}$ are statistically different from zero, while during the Asian crises the estimates are not. This confirms the results from table 8, the coefficients are "well" estimated during the Mexican, Mexico-Asia, and the Russian crises. The rejection from zero is not necessarily because the means are close to zero, but because the standard deviation of the estimates are almost 4 times larger than those obtained during the Mexican crisis.[19] For the subsample of Mexico-Asia the estimate of $\hat{\beta}$ is statistically different from zero, but not $\hat{\alpha}$. During Russia both estimates are statistically different from zero. Finally, when All the sample is used, the estimate of $\hat{\alpha}$ and $\hat{\beta}$ are not statistically different from zero.

5.1.1 Comparison of estimates across samples.

In this section, several comparision of the estimates are performed. First, a test to determine if the estimates of $\hat{\alpha}$ and $\hat{\beta}$ are statistically sensible to the choice of sample is performed. Second, comparisons between these estimates and those from section (2.2) are computed.

[19] Again, this result should be expected given the fact that in table 8 one of the conditions is not satisfied, which means that the system of equations lack of one equation, and therefore, there are a continuum of solutions. In practice, this implies that the distributions are flat and wide.

As it should become clear from the set up, it has to be the case that the estimates are correlated across the samples. As before, there is no easy solution for this problem given that $\hat{\alpha}$ and $\hat{\beta}$ are the result to a non linear system of equations, where their correlations comes from the correlation between the variance covariance matrixes estimated in the switching regime. In table 10, I present the results of testing if the estimates are statistically different assuming the worse possible correlation; the one that minimizes the standard deviation of the difference in the estimates, or in other words, the one that maximizes the chances of rejection.

	Mexico	Asia	Mexico-Asia	Russia	All
$\hat{\alpha}$					
Mexican		0.071448	0.084	0.227166	0.009
Asian	0.341		0.013	0.298614	0.080
Mexican-Asian	0.077	0.264347		0.311339	0.093
Russian	0.058	0.399	0.135		0.218
All	0.401	0.060	0.324	0.459	
$\hat{\beta}$					
Mexican		0.15966	0.048	0.012171	0.379
Asian	0.389		0.208	0.14749	0.219
Mexican-Asian	0.082	0.306691		0.060153	0.427
Russian	0.045	0.434	0.127		0.367
All	0.556	0.166	0.473	0.600	

Table 10: Tests to determine if the estimates are different across different sample choices. Regime switches are estimated endogenously.

In table 10, the tests for $\hat{\alpha}$ and $\hat{\beta}$ are performed. The table has two panels. In the first one, the difference between the $\hat{\alpha}$ estimators (the upper triangle) as well as the smallest possible standard deviation of the difference (lower triangle) are shown. The second panel reports the results for $\hat{\beta}$. In the case of $\hat{\alpha}$, there are two out of ten instances in which the estimates might be statistically different: between Mexico and Russia, and between Mexican-Asian combined crises and Russia. In the case of $\hat{\beta}$, the estimates are not statistically different from each other in all ten comparisons. If the estimates are assumed to be independent, then it is not possible to reject the hypothesis that the estimates are the same across all samples.

Finally, in table 11, comparisons between the estimates using the exogenous windows and the switching regime are preformed. Under the assumption that the exogenous specification is more efficient than the switching regime, the variance of the difference is equal to the difference in the variances.[20] However, the table is constructed with the smallest standard deviation. As can be

[20] A Hausman specification test like could be implemented. See Hausman [1978]. If this regression is well especified both methodologies are consistent, but the one that uses the prior knowledge achieves the lower variance. On the other hand, if the window is mispecified, the exogenous case is inconsistent but the switching regime is consistent.

seen, the hypothesis that the estimates are the same cannot be rejected.

	$\hat{\alpha}$		$\hat{\beta}$	
	Difference	Smallest Standard Deviation	Difference	Smallest Standard Deviation
Mexico	0.004099	0.037186	0.028286	0.035334
Asia	0.058056	0.376488	0.199158	0.490981
Mexico-Asia	0.077489	0.107759	0.017557	0.170172
Russia	0.163203	0.499199	0.143263	0.139875
All	0.020506	0.423731	0.409088	0.637265

Table 11: Tests to determine if the estimates are different between methodologies defining the windows. Exogenous windows vs. Regime switching.

5.2 Regimes as large changes in second moments.

Finally, in this section, the crisis windows are defined as large changes in the conditional second moments. The unconditional variance covariance matrix is computed from the residuals obtained from the first step. The conditional variances and covariances are computed along rolling windows of 20 days. The crisis periods are defined as those instances in which the conditional variances, or covariance, (any one of the three) are larger than the mean values (the unconditional) plus 1.5 times the standard deviation of the particular variable. In figure 4, the rolling window variance covariance together with the crises windows are shown. Notice that the Asian crises are not included given that they represented a relatively small shift in the variances, as it has being discussed before. This is in constrast with the two previous methods defining the regimes.

The variance covariance matrix in each regime is computed, and in table 12, the results are summarized. The two conditions, equations (7) and (8), are satisfied.

All crises	Variance Argentina	Covariance	Variance Mexico
Low variance regime ($\cdot 10^{-4}$)	1.16	0.68	0.65
High variance regime ($\cdot 10^{-4}$)	11.24	13.18	17.44
Increase in the variance	9.70	19.34	26.76
Test that estimates are different	3.23	3.31	3.47

Table 12: Change in the variance covariance matrix during the three crises using the change in the second moments to determine the crises windows: Mexican, South East Asian, and Russian crises.

In table 13, the values of $\hat{\alpha}$ and $\hat{\beta}$, as well as their standard deviations are shown.

The tests to determine if the estimates are statistically different from the previous two methodologies are shown in table 14. For the $\hat{\alpha}$ estimates, there are two cases in which the test is rejected;

	Endogenous Windows
$\hat{\alpha}$	
Point Estimate	0.315000
Standard Deviation	0.146255
T-stat	2.153771
$\hat{\beta}$	
Point Estimate	0.724991
Standard Deviation	0.431501
T-stat	1.680160

Table 13: Estimation of the structural parameters for the three crises: Mexican, South East Asian, and Russian crises. Crises windows are defined as 1.5 standard deviation increases in the conditional second moments.

with the Mexican crisis using exogenous windows, and with the Russian crisis estimated using the switching regime. For the $\hat{\beta}$ estimates, the estimates are statistically different only in the case between the Asian crises estimated with the switching regime. Remember that these rejections are obtained using the correlation that minimizes the variance, thus they should be taken cautiously.

	$\hat{\alpha}$		$\hat{\beta}$	
	Difference	Smallest Standard Deviation	Difference	Smallest Standard Deviation
Exogenous Windows				
Mexico	0.009665	0.000767	0.077117	0.318229
Asia	0.123938	0.071575	0.065906	0.384737
Mexico-Asia	0.001118	0.067193	0.074978	0.370618
Russia	0.395935	0.404938	0.260837	0.187596
All	0.006166	0.059143	0.075178	0.364562
Switching Regime				
Mexico	0.005566	0.03642	0.105403	0.282894
Asia	0.065882	0.304913	0.265064	0.106244
Mexico-Asia	0.078607	0.040566	0.057421	0.200447
Russia	0.232732	0.094261	0.117574	0.327471
All	0.01434	0.364588	0.484266	0.272703

Table 14: Tests to determine if the estimates are different among methodologies defining the windows. Endogenous vs. Exogenous windows and Regime switching.

In conclusion, it is fair to say that the estimates are remarkably stable across samples and methodologies to define the windows. This should be surprising taken into consideration that the estimates in this section are not even considering the Asian crises at all as part of the crisis regime.

6 Conclusions and extensions

This paper develops a new identification assumption that solves the problem of simultaneous equations. The restriction is based on heteroskedasticity of the structural shocks, and the circumstances in which it can be used are relatively weak. As is discussed in proposition 1, identification requires four assumptions: firstly, normalization; secondly, independence of the structural shocks, which both are almost always used in this type of problems; thirdly, the stability of the structural parameters (α and β in my model); and finally, the existence of at least two regimes of different variances. These assumptions should be relatively easy to justify in several macro and finance applications. Additionally, to these assumptions, in order to achieve identification, the variance covariance matrixes have to satisfy two testable conditions; the "determinant of the difference" and the "covariance of the weighted difference".

Furthermore, when more than two regimes are present in the data (either countable or not), the system of equations generally has more equations than unknowns. This can have three interpretations: the extra equations can be used as overidentifying restrictions, or the system of equations can be interpreted either as a factor analysis regression, where the parameters are the factor loadings and the structural variances the factors, or as a GMM or NLS regression. Finally, the paper discusses the necessary conditions for identification under multivariate simultaneous equations, and unobservable common shocks.

An application of the identification is implemented to measure the "contagion" between sovereign bonds of Argentina and Mexico. The case when there are only two regimes, and four regimes are studied. The estimates are remarkably stable to different definitions of the regimes, and to the introduction of alternative controls.

The identification assumption described has several applications in financial economics where the data exhibits conditional heteroskedasticity and important problems of endogeneity exists. For example, in the stock market, prices and volumes are jointly determined. Using this identification assumption it is possible to estimate the price impact of order flows. Moreover, the methodology has applications in macroeconomics. Changes in the variance of interest rates could be used to determine the importance of the different channels of monetary policy propagation, as to shed light on the monetary policy reaction function.

In general, the methodology here developed can help to solve the problem of endogenous variables in those circumstances in which the researcher believes the model is stable, and it exhibits heteroskedasticity.

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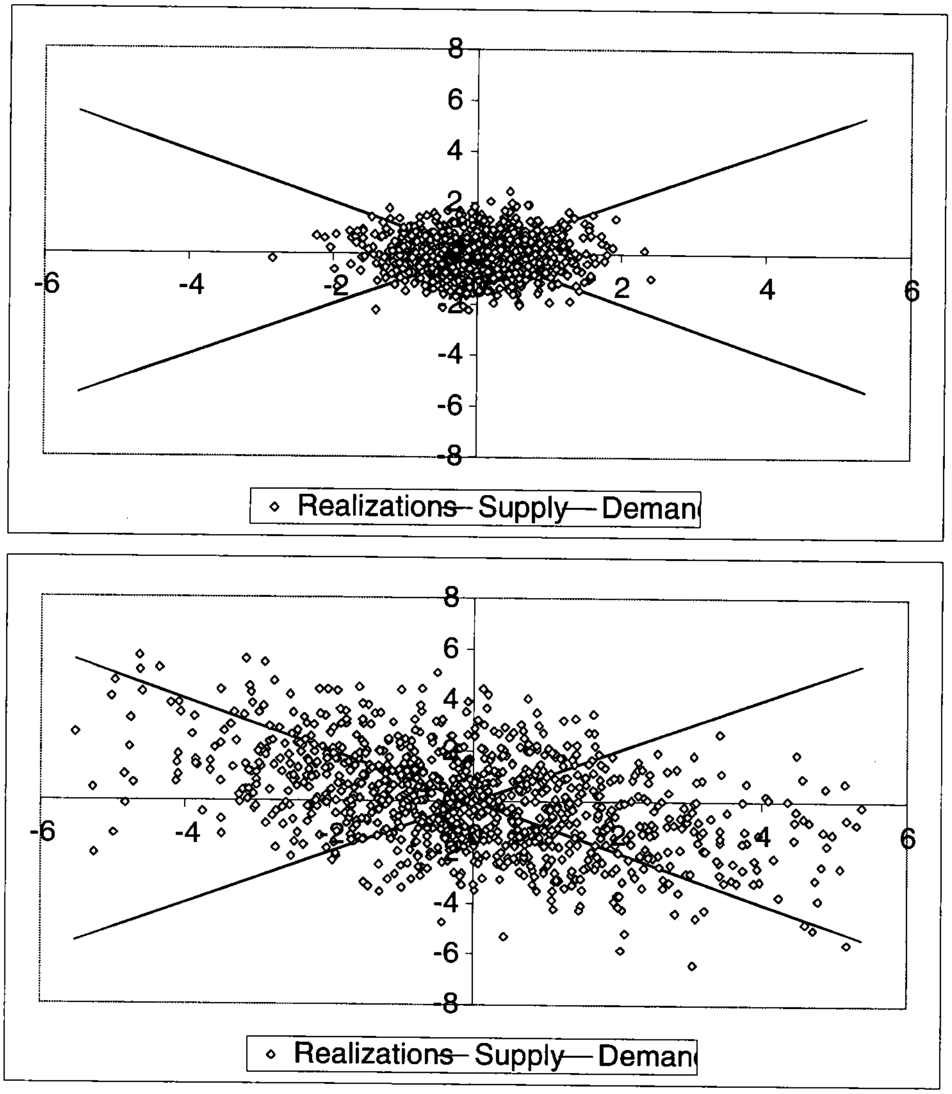

Figure 1: Identification Problem.

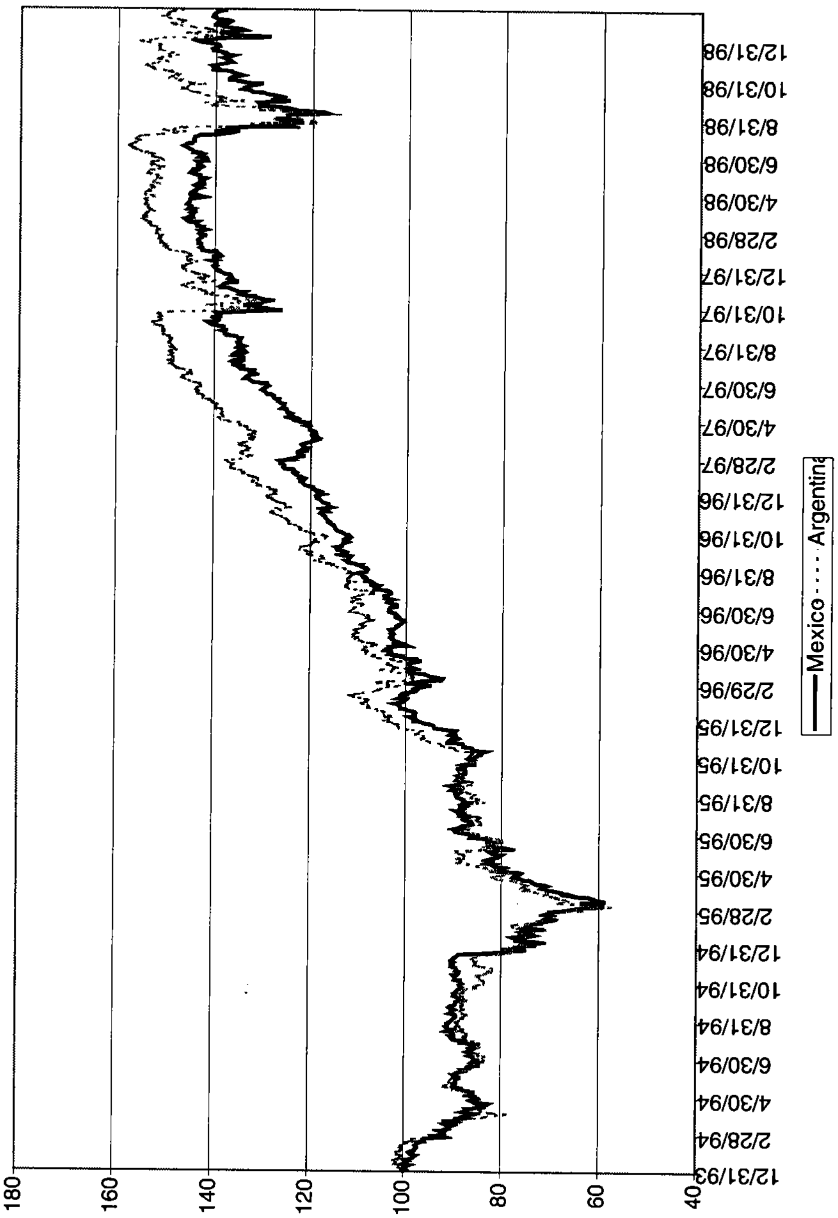

Figure 2: Argentina and Mexico Brady Index. Source JPMorgan.

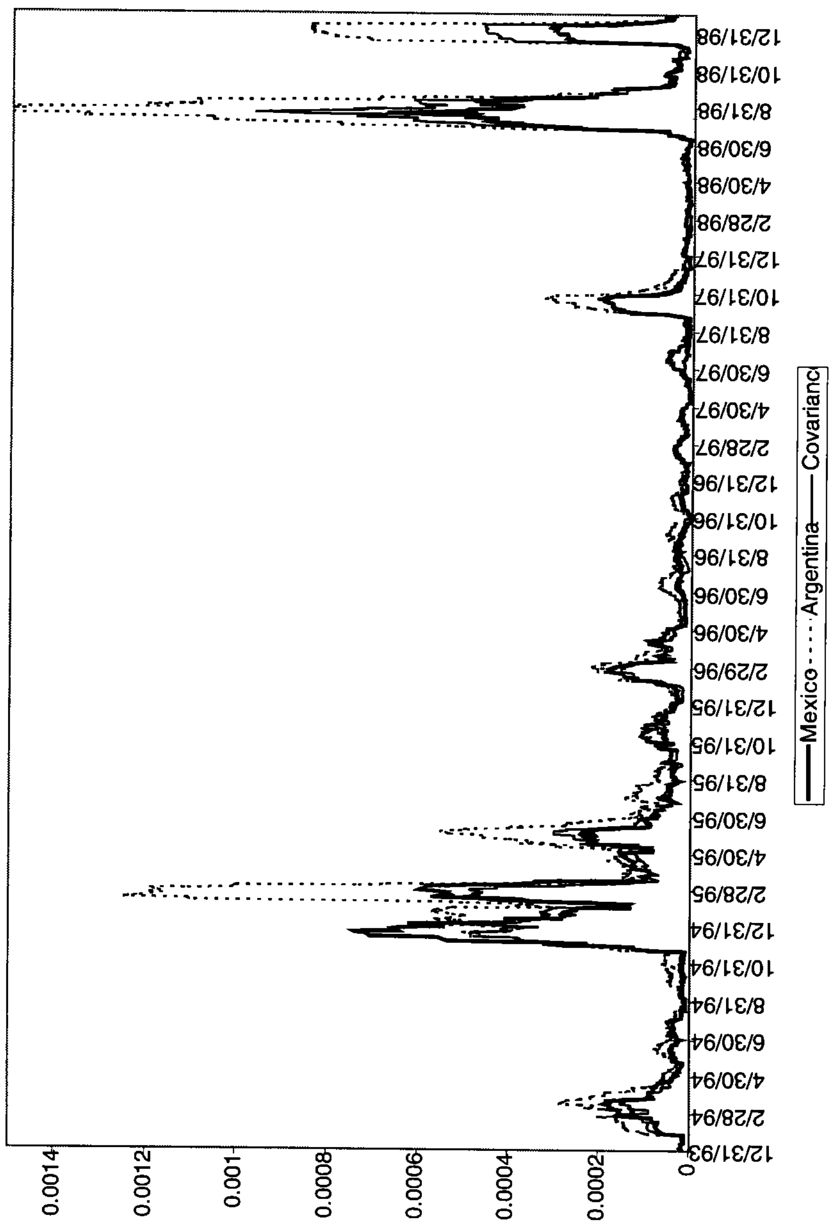

Figure 3: Rolling variances and covariance of daily returns on Argentinean and Mexican Brady Bonds. Window = 20 days.

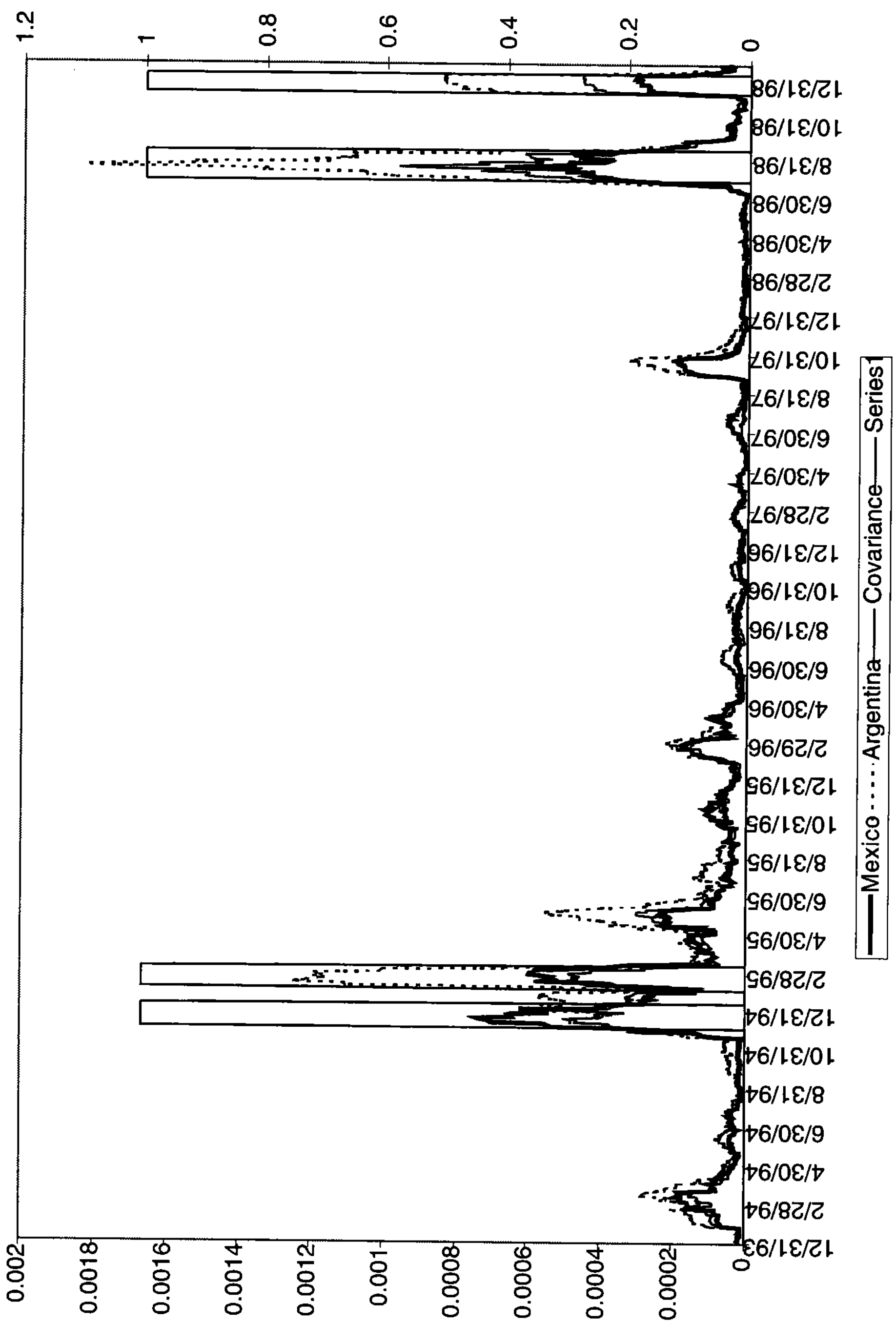

Figure 4: Crisis window defined as large shifts in second moments.