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Working Paper **6494**

NBER WORKING PAPER SERIES

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Working Paper 6494
http://www.nber.org/papers/w6494

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 1998

We have benefitted from very helpful comments from Michael Dalton, Michael Grubb, Chad Jones, Alan Manne, William Nordhaus, Richard Richels, David Wheeler, and seminar participants at Stanford University, the National Bureau of Economic Research, and the World Bank. Financial support from the Department of Energy (Grant DE-FG03-95ER62104) and the National Science Foundation (Grant SBR 9310362) is gratefully acknowledged. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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NBER Working Paper No. 6494
April 1998
JEL Nos. H21, D99, Q28

**ABSTRACT**

This paper explores the significance of policy-induced technological change for the design of carbon-abatement policies. We derive analytical expressions characterizing optimal $CO_2$ abatement and carbon tax profiles under different specifications for the channels through which technological progress occurs. We consider both R&D-based and learning-by-doing-based knowledge accumulation, and examine each specification under both a cost-effectiveness and a benefit-cost policy criterion.

We show analytically that the presence of induced technological change (ITC) implies a lower time profile of optimal carbon taxes. The impact of ITC on the optimal abatement path varies. When knowledge is gained through R&D investments, the presence of ITC justifies shifting some abatement from the present to the future. However, when knowledge is generated through learning-by-doing, the impact on the timing of abatement is analytically ambiguous.

Illustrative numerical simulations indicate that the impact of ITC upon overall costs and optimal carbon taxes can be quite large in a cost-effectiveness setting but typically is much smaller under a benefit-cost policy criterion. The impact of ITC on the timing of abatement is very weak, and the effect (applicable in the benefit-cost case) on total abatement over time is generally small as well, especially when knowledge is accumulated via R&D.

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# 1 Introduction

Over the past decade considerable efforts have been directed toward evaluating alternative policies to reduce the atmospheric accumulation of greenhouse gases, particularly carbon dioxide ($CO_2$). Initial assessments tended to disregard interconnections between technological change and $CO_2$-abatement policies, treating the rate of technological progress as autonomous—that is, unrelated to policy changes or associated changes in relative prices. Recently, however, several researchers have emphasized that the rate of technological change and $CO_2$ policies are connected: to the extent that public policies affect the prices of carbon-based fuels, they affect incentives to invest in research and development (R&D) aimed at bringing alternative fuels on line earlier or at lower cost. Such policies may also prompt R&D oriented toward the discovery of new production methods that are less energy-intensive overall. Moreover, climate policies can affect the growth of knowledge through impacts on learning by doing (LBD): to the extent that climate policies affect producers' experience with alternative energy fuels or energy-conserving processes, they can influence the rate of advancement of knowledge.

Thus, through impacts on patterns of both R&D spending and learning by doing, climate policy can alter the path of knowledge acquisition. What does this connection imply for the design of $CO_2$-abatement policy? In particular, how do the optimal timing and extent of carbon emissions abatement, as well as the optimal time path of carbon taxes, change when we recognize the possibility of induced technological change (ITC)?

Policymakers and researchers are divided on these questions. Wigley, Richels, and Edmonds (1996) recently have argued that the prospect of technological change justifies relatively little current abatement of $CO_2$ emissions: better to wait until scientific advances make such abatement less costly. In contrast, Ha-Duong, Grubb, and Hourcade (1996) have maintained that the potential for ITC justifies relatively *more* abatement in the near term, in light of the ability of current abatement activities to contribute to learning by doing. Still others have claimed that the possibility of ITC makes it optimal to increase abatement in all periods and thus achieve more ambitious overall targets for atmospheric $CO_2$ concentrations.

As regards the optimal carbon tax profile, some authors have claimed that induced technological change justifies a higher carbon tax trajectory than would be optimal in the absence of ITC. The argument is that in the presence of ITC, carbon taxes not only confer the usual environmental benefit by forcing agents to internalize the previously external costs from $CO_2$ emissions, but also yield the benefit of faster innovation, particularly in the supply of alternative energy technologies.[1]

[1] Some have suggested that the innovation-related benefits from a carbon tax might be as large as the direct abatement costs associated with carbon taxes. If this were the case, then the overall cost (ignoring environmental benefits) of a carbon tax would be zero. Porter and van der Linde (1995) advance a general argument consistent with this view, maintaining that environmental regulation often stimulates substantial

Others, however, counter that with technological progress, a lower carbon tax profile is all that is needed to achieve desired levels of abatement.

This paper aims to clarify the issues underlying these debates. We derive analytical expressions characterizing the optimal paths of emissions abatement and carbon taxes under different specifications for the channels through which knowledge is accumulated, considering both *R&D-based* and *learning-by-doing-based* knowledge accumulation. We examine each of these specifications under two different optimization criteria: (1) the *cost-effectiveness* criterion of obtaining by a specified date and thereafter maintaining, at minimum cost, a given target for the atmospheric $CO_2$ concentration; and (2) the *benefit-cost* criterion under which we also choose the *optimal* target to achieve, thus obtaining the path of carbon abatement that maximizes the benefits from avoided climate damages net of abatement costs.[2] In order to gain a sense of plausible magnitudes, we also perform illustrative numerical simulations.

Our analysis is in the spirit of two studies by Nordhaus (1980, 1982)—the first to obtain analytical expressions for the optimal carbon tax trajectory—as well as more recent work by Farzin and Tahvonnen (1996), Farzin (1996), Peck and Wan (1996), Sinclair (1994), and Ulph and Ulph (1994). Our paper also complements work by Nordhaus (1994, 1996) and Peck and Teisberg (1992, 1994), in which numerical methods are used to obtain the optimal carbon abatement and carbon tax profiles under different exogenous technological specifications.[3] Another related paper is by Kolstad (1996), who solves numerically for optimal emissions trajectories in the presence of endogenous learning. Kolstad's paper differs from ours, however, in that it focuses on learning that reduces uncertainty about $CO_2$-related damages, rather than on learning that improves abatement technologies and thus reduces abatement costs. Finally, our paper is closely related to the previously mentioned studies by Wigley, Richels, and Edmonds (1996) and Ha-Duong, Grubb, and Hourcade (1996), as well as to recent working papers by Grubb (1996), Goulder and Schneider (1998), and Nordhaus (1997) that analyze the implications of induced technological change for optimal climate policy.

The present investigation differs from each of these other studies in three ways. First, it derives *analytical* results revealing the impact of ITC on optimal time profiles for carbon taxes and carbon abatement. Second, it considers, in a unified framework, two channels for knowledge accumulation (R&D activity and learning by doing) and two policy criteria (cost effectiveness and net-benefit maximization). In the model, policymakers (or the social planner) choose optimal paths of carbon

technological progress and leads to significant long-run cost savings that make the overall costs of regulation trivial or even negative.

[2]This is equivalent to minimizing the sum of abatement costs and $CO_2$-related damages to the environment.

[3]The present paper also complements that of Manne and Richels (1992), who employ a multi-region computable general equlibrium model to solve for Pareto-efficient paths of carbon abatement and taxes.

abatement and carbon taxes, taking into account the impact of these taxes on technological progress and future abatement costs. Finally, it employs both analytical and numerical methods in an integrated, complementary way.

The analytical model reveals (contrary to what some analysts have suggested) that the presence of ITC implies a lower time profile of optimal carbon taxes. The impact of ITC on the optimal abatement path varies. When knowledge is gained through R&D investments, the presence of ITC justifies shifting some abatement from the present to the future. However, when knowledge reflects learning by doing, the impact on the timing of abatement is analytically ambiguous.

When the government employs the benefit-cost policy criterion, the presence of ITC justifies greater overall (cumulative) abatement than would be warranted in its absence. However, this does not imply that abatement rises in *every* period: when knowledge accumulation results from R&D expenditure, the presence of ITC implies a reduction of near-term abatement efforts, despite the overall increase in the scale of abatement over time.

Our numerical simulations reinforce the qualitative predictions of the analytical model. The quantitative impact on overall costs and optimal carbon taxes can be quite large in a cost-effectiveness setting but typically is much smaller under a benefit-cost policy criterion. The weak effect on the tax rate in the benefit-cost case reflects the relatively trivial impact of ITC on optimal $CO_2$ concentrations, associated marginal damages, and (hence) the optimal tax rate. As for the optimal abatement path, the impact of ITC on the timing of abatement is very weak, and the effect on overall abatement (which applies in the benefit-cost case) is generally small as well, especially when knowledge is accumulated via R&D.

The rest of the paper is organized as follows. Section 2 lays out the analytical model and applies it to the case in which the policy criterion is cost-effectiveness. Section 3 applies the model to the situation in which policymakers employ the (broader) benefit-cost criterion. Section 4 presents and interprets results from numerical simulations and includes a sensitivity analysis. The final section offers conclusions and indicates directions for future research.

## 2 Optimal Policy under the Cost-Effectiveness Criterion

In this section we consider optimal abatement when the policy criterion is cost-effectiveness (CE): achieving at minimum cost a target atmospheric $CO_2$ concentration by a specified future date, and maintaining it thereafter. We assume that producers are competitive and minimize costs. Let $C(A_t, H_t)$ be the economy's (aggregate) abatement-cost function, where $A_t$ is abatement at time $t$ and $H_t$ is the stock of knowledge characterizing technology at time $t$. We assume that $C(\cdot)$ has the following properties: $C_A(\cdot) > 0$, $C_{AA}(\cdot) > 0$, $C_H(\cdot) < 0$, and $C_{AH}(\cdot) < 0$. The last two properties imply that increased knowledge reduces, respectively, total and marginal costs of abatement. Later on, we consider the implications of alternative assumptions. We also allow for the possibility that costs may depend on the relative amount of abatement ($\frac{A_t}{E_t^0}$) rather than the absolute level ($A_t$). In this case baseline emissions become an argument of the cost function. For expositional simplicity, however, we usually suppress $E_t^0$ from the cost function in the main text.

## 2.1 Technological Change via R&D

### 2.1.1 The Problem and Basic Characteristics of the Solution

Within our cost-effectiveness analysis, we consider two modes of knowledge accumulation. The first specification assumes that in order to accumulate knowledge, the economy must devote resources to research and development. We refer to this as the CE_R specification (where "R" indicates that the channel for knowledge accumulation is R&D). The planner's problem is to choose the time-paths of abatement and R&D investment that minimize the costs of achieving the concentration target.[4] Formally, the optimization problem is:

$$\min_{A_t, I_t} \int_0^{\infty} \Big(C(A_t, H_t) + p(I_t)I_t\Big)e^{-rt}dt \tag{1}$$

$$\text{s.t. } \dot{S}_t = -\delta S_t + E_t^0 - A_t \tag{2}$$

$$\dot{H}_t = k\Psi(I_t, H_t) \tag{3}$$

$$S_0, H_0 \text{ given}$$

$$\text{and } S_t \leq \bar{S} \ \ \forall\, t \geq T \tag{4}$$

where $A_t$ is abatement, $I_t$ is investment in knowledge (i.e., R&D expenditure), $S_t$ is the $CO_2$ concentration, $H_t$ is the knowledge stock, $p(\cdot)$ is the real price of investment resources, $r$ is the interest rate, $\delta$ is the natural rate of removal of atmospheric $CO_2$, $E_t^0$ is baseline emissions, and $k$ is a parameter that governs the ease with which knowledge can be accumulated through R&D investment. We use $k = 0$ to represent a world in which no induced technological change is possible (the "NITC" case); we shall contrast results from this case with those from the "ITC" case in which $k > 0$ and induced technological change is present.[5]

Expression (1) indicates that the objective is to minimize the discounted sum of abatement costs and expenditure on R&D into the infinite future. Expression (2) states that the change in the $CO_2$

[4]Our analysis focuses on the social planner's problem. We disregard the market failure associated with knowledge spillovers, that is, with the inability of firms to appropriate the full social returns on their investments in knowledge. Implicitly, we assume that this market failure is fully addressed by policymakers through R&D subsidies.

[5]$k = 0$ does not necessarily imply that there is no technological change. We allow for the possibility that firms enjoy improvements in knowledge in the absence of a $CO_2$-abatement policy. Such improvements may stem from R&D expenditure that is justified on other grounds, or from learning by doing. Thus the baseline against which we compare policy impacts includes *non-carbon-tax-induced* technological change. Likewise, $H_t - H_0$ represents the portion of the knowledge stock whose accumulation is induced by the carbon tax.

concentration is equal to the contribution from current emissions $(E_t^0 - A_t)$ net of natural removal $(\delta S_t)$.[6] Expression (3) indicates that the change in the stock of knowledge $(H_t)$ is a function of the current knowledge stock and current investment $(I_t)$ in R&D. Expression (4) shows that the target $CO_2$ concentration, $\bar{S}$, must be met by time $T$ and maintained after that point in time. We assume $p(\cdot)$ is non-decreasing in $I_t$; that is, the average cost of R&D investment increases with the level of R&D. This captures in reduced form the idea that there is an increasing opportunity cost (to other sectors of the economy) of employing scientists and engineers to devise new abatement technologies.[7] We also assume that the knowledge-accumulation function $\Psi(\cdot)$ has the following properties: $\Psi(\cdot) > 0$, $\Psi_I(\cdot) > 0$, and $\Psi_{II}(\cdot) < 0$.

The current-value Hamiltonian associated with the optimization problem for $t < T$ is:[8]

$$\mathcal{H}_t = -\Big(C(A_t, H_t) + p(I_t)I_t\Big) + \lambda_t(-\delta S_t + E_t^0 - A_t) + \mu_t k \Psi(I_t, H_t)$$

For $t \geq T$, however, we must form the following Lagrangian:

$$\mathcal{L}_t = \mathcal{H}_t + \eta_t(\bar{S} - S_t)$$

From the maximum principle, we obtain a set of first-order conditions, assuming an interior solution, as well as costate equations, state equations, and transversality conditions. Two key equations are:

$$C_A(\cdot) = -\lambda_t \tag{5}$$

$$\text{and } \dot{\lambda}_t = \begin{cases} (r+\delta)\lambda_t & \text{for } t < T \\ (r+\delta)\lambda_t + \eta_t & \text{for } t \geq T \end{cases} \tag{6}$$

The variable $\lambda_t$ is the shadow value of a small additional amount of $CO_2$ at time $t$. $\lambda_t$ is negative, since $CO_2$ is a "bad" from the policymaker's perspective. Thus $-\lambda_t$ represents the shadow *cost* of $CO_2$ or, equivalently, the benefit from an incremental amount of abatement (a small reduction in the $CO_2$ concentration). In a decentralized competitive economy in which all other market failures have been corrected, the optimal carbon tax is $-\lambda_t$, the shadow cost of $CO_2$. By equation (5), this is equal to the marginal abatement cost at the optimal level of abatement. Equation (5) states that abatement should be pursued up until the point at which marginal cost equals marginal benefit, while equation (6) states that the optimal carbon tax grows at the rate $(r + \delta)$ (at least for points

[6]For analytical convenience, we postulate a simple stock-flow relationship here. A more complicated equation of motion, such as the one introduced in the numerical simulations, would not alter the qualitative analytical results obtained here.

[7]This issue is discussed in greater detail by Goulder and Schneider (1998).

[8]This Hamiltonian actually corresponds to the problem of maximizing negative costs. This formulation is useful because it yields shadow prices with signs that match intuition.

in time up until $T$).[9] The two equations together imply that in an optimal program, the discounted marginal costs of abatement must be equal at all points in time (up to $T$), where the appropriate discount rate is $(r + \delta)$.[10] In the appendix we demonstrate that this corresponds to an optimal abatement profile that slopes upwards over time (whether or not there is induced technological change) so long as baseline emissions are not declining "too rapidly."

### 2.1.2 Implications of ITC

We now examine the effect of ITC on abatement costs and on the optimal carbon tax and abatement profiles. We do this by considering the significance of a change in the parameter $k$. As mentioned above, the case of $k = 0$ corresponds to a scenario with no induced technological change (the NITC scenario), while positive values of $k$ imply the presence of induced technological change (the ITC scenario). Our analysis ill focus on incremental increases in $k$ from the point $k = 0$.[11]

If (as is assumed) $C_H(\cdot) < 0$, then additional knowledge is valuable (i.e., the multiplier $\mu_t$ is positive). When $k = 0$, the stock of knowledge cannot grow above the initial level $H_0$, but for strictly positive values of $k$, the social planner will find it optimal to accumulate at least some additional knowledge, assuming an interior solution.[12] This additional knowledge causes a decrease[13] in optimized costs to a degree dictated by $\mu_t$. Thus, the introduction of the ITC option has a negative (or at least nonpositive) effect on optimized costs.

Next we examine the impact of introducing ITC on the optimal time profiles of abatement and

[9] After $T$, matters are complicated by the $\eta_t$ term in equation (6).

[10] The appropriate discount rate is not simply $r$. Consider an arbitrary path of emissions leading to a given concentration $S_T$ at time $T$. Since $CO_2$ is removed naturally, altering this path by increasing emissions slightly at time $t$ and reducing emissions slightly at a later time $t'$ leads to greater overall removal and thus leads to a $CO_2$ concentration at time $T$ that is less than $S_T$. Equivalently (as seen in the sensitivity analysis in Section 3), $S_T$ can be achieved with less cumulative emissions abatement if the path of abatement is oriented more toward the future. Hence there is a value to postponing abatement beyond that implied by interest rate, $r$; this additional value is captured in the appearance of $\delta$ in the discount rate.

[11] The focus here on differential changes does not limit the generality of the analysis. Our analytical results here are independent of the initial value of $k$. Given the smooth nature of our problem, results that hold for small changes in $k$ around any initial value will carry over qualitatively for *large* changes around the point 0. This is confirmed in the numerical simulations.

[12] A corner solution arises if even the first increment of knowledge has marginal returns smaller than marginal costs. In this case, the social planner does not invest in additional knowledge, but even here we know that knowledge at least will not *decrease* from the initial level.

[13] Throughout, when we use the words "increase" and "decrease" we will mean *non-strict* increases and decreases, thus including the possibility that the variable stays constant.

carbon taxes. Differentiating equation (5) with respect to $k$ and rearranging, we obtain:

$$\frac{dA_t}{dk} = \frac{\frac{d(-\lambda_t)}{dk} - C_{AH}(\cdot)\frac{dH_t}{dk}}{C_{AA}(\cdot)} \tag{7}$$

For the moment, assume that the first term in the numerator is zero, i.e., that ITC has no impact on the shadow cost of $CO_2$. Under this assumption, we are left only with what we shall refer to as the *knowledge-growth effect*: to the extent that knowledge has increased as a result of ITC ($\frac{dH_t}{dk} > 0$) and has thus reduced marginal abatement costs ($-C_{AH}(\cdot) > 0$), abatement tends to rise.[14]

The knowledge-growth effect is represented in Figure 1 by the pivoting of abatement upward from the initial time path 1 to path 2. Path 2 coincides with the initial path at time 0 because knowledge is initially fixed at $H_0$: there can be no knowledge-growth effect at time 0. The distance between paths 1 and 2 grows over time, representing the fact that the knowledge-growth effect grows larger over time. This follows from $\frac{d}{dt}(\frac{dH_t}{dk}) \geq 0$, which in turn results from the fact that there is no depreciation of knowledge in our model: whatever additional knowledge was induced by ITC at time $t$ still remains at time $t' > t$, and there might have been a further increment to knowledge at this later time.

Note that path 2 involves more abatement in every period than does the first path. Given that the same $\bar{S}$ constraint holds and that the initial path satisfied this constraint, path 2 clearly cannot be optimal. Path 2 was obtained under the assumption that the introduction of ITC had no impact on the shadow cost of $CO_2$. In fact, however, as shown in the appendix under the maintained assumption that $C_{AH}(\cdot) < 0$, the shadow cost of $CO_2$ at all points in time decreases in magnitude in the presence of ITC: $\frac{d(-\lambda_t)}{dk} \leq 0 \;\; \forall\, t$. The basic explanation for this *shadow-cost effect* is as follows. If we are armed with advanced technologies, the prospect of being given an additional amount of $CO_2$ at time $t$ and still being expected to meet the $\bar{S}$ constraint by time $T$ is less worrisome than it would be if we had only primitive abatement technologies at our disposal. Note that since the optimal carbon tax is the shadow cost of $CO_2$, it follows that the presence of ITC lowers carbon taxes.

This result contradicts the notion that the induced-innovation benefit from carbon taxes justifies a higher carbon tax rate. Figure 2 demonstrates our result heuristically by offering a static representation of this dynamic problem. Cost-effective abatement (depicted in the upper panel) is achieved by a carbon tax set equal to the marginal abatement cost (MC) at the desired level of abatement. Technological progress causes the MC curve to pivot down, thus implying a lower optimal tax: it now takes a lower tax to yield the same amount of abatement. Note that this result depends on the assumption that *marginal* abatement costs are lowered by technological progress; i.e., $C_{AH} < 0$. It is possible to conceive of new technologies that involve higher marginal abatement costs but that are nonetheless attractive because of lower fixed (and overall) abatement costs;

[14]Note that the denominator of equation (7) is positive by assumption.

however, this seems to be an unusual case.

Now we return to our analysis of the impact of ITC on abatement. The shadow-cost effect, reflected in the first term of the numerator in equation (7), shows up in Figure 1 as the downward shift from path 2 to path 3. The shift is not parallel: as shown in the appendix, tax rates at later points in time fall by greater absolute amounts than do early taxes, in such a way as to preserve the carbon tax growth rate at $(r + \delta)$. The downward shift is of a magnitude such that path 3 lies neither completely above nor completely below path 1: if it did, it would imply either overshooting or undershooting the constraint $\bar{S}$, which is likely to be suboptimal.[15] Together, the knowledge-growth and shadow-cost effects imply a new optimal abatement path that is steeper than the initial one: abatement is postponed from the present into the future.[16] Intuitively, ITC reduces the cost of future abatement relative to current abatement and thus makes postponing (some) current abatement more attractive. Thus, in a cost-effectiveness setting and with R&D-based technological change, our analysis supports the claim of Wigley, Richels, and Edmonds (1996) that ITC justifies a more gradual approach to abatement.

At any given time $t$, we cannot be sure whether abatement rises or falls—this depends on whether the knowledge-growth effect or the shadow-cost effect dominates at that particular moment. But we can say something definitive about abatement at time 0. Because knowledge is initially fixed at $H_0$, only the shadow-cost effect comes into play at time 0:

$$\frac{dA_0}{dk} = \frac{\frac{d(-\lambda_0)}{dk}}{C_{AA}(\cdot)} \tag{8}$$

Thus, *initial* abatement weakly declines as a result of ITC.

These results depend on our assumption that $C_{AH}(\cdot) < 0$—that knowledge lowers marginal abatement costs. However, the possibility that $C_{AH}(\cdot) > 0$ cannot be ruled out. In this case (which we find somewhat implausible), ITC lowers total costs through greatly reduced sunk costs, even though it raises marginal costs. Under these circumstances, the shadow-cost effect is positive and the presence of ITC raises the optimal carbon tax. The net effect of an increase in $k$ on abatement at any arbitrary time $t$ is (again) ambiguous, but initial abatement unambiguously rises.

[15] If baseline emissions were to rise sharply after $T$, then given the convexity of the abatement cost function, it might be optimal to more than meet the $\bar{S}$ requirement at time $T$ in order to reduce the amount of abatement required afterwards. However, it is the case nevertheless that one curve cannot lie above or below another over the entire infinite horizon: perpetual over- or undershooting of the constraint cannot be optimal.

[16] In characterizing the path as "steeper" we do not mean that the slope of the new path is everywhere greater than that of the old path. In fact, in the numerical simulations we will see that this is often not the case. We simply mean that, loosely speaking, less abatement is undertaken early on, and more later on.

### 2.1.3 Summary

Our results to this point are summarized in the "Cost-Effectiveness, R&D" row of Table 1. The key results are, first, that the solution to the cost-minimization problem (for any value of $k$) involves carbon taxes that rise over time at the rate $(r+\delta)$ for $t<T$, but grow slower, and perhaps even decline, afterwards. The optimum is also characterized by an abatement profile that is upward sloping for $t<T$, as long as baseline emissions are not too steeply declining. Second, assuming that ITC reduces marginal (and total) abatement costs, opening the ITC option causes optimized costs to fall, makes the entire carbon tax path fall (and by an equal proportion at all $t$), and causes initial abatement to fall and later abatement to rise.

## 2.2 Technological Change via Learning by Doing

### 2.2.1 The Problem and Basic Characteristics of the Solution

Here we analyze a variant of the model presented above; now abatement itself yields improvements in technology. This is the "CE_L" model, where the "L" refers to learning by doing. As we shall see, some of our earlier results are altered in this new setting. The optimization problem is now:

$$\begin{aligned} \min_{A_t} \int_0^\infty & C(A_t, H_t) e^{-rt} dt \\ \text{s.t. } \dot{S}_t &= -\delta S_t + E_t^0 - A_t \\ \dot{H}_t &= k\Psi(A_t, H_t) \\ & S_0, H_0 \text{ given} \\ \text{and } & S_t \leq \bar{S} \quad \forall\, t \geq T \end{aligned}$$

This problem is virtually the same as the CE_R model of the previous section, except for a change in the $\Psi(\cdot)$ function: now knowledge growth is a function of the current level of abatement rather than R&D investment. Equivalently, current knowledge depends on cumulative abatement, which is regarded as a measure of experience. The first-order condition for abatement is now given by:

$$C_A(\cdot) - \mu_t k \Psi_A(\cdot) = -\lambda_t \tag{9}$$

Equation (9) states that the marginal benefit of abatement ($-\lambda_t$, the value of the implied reduction in the $CO_2$ concentration) should equal the gross marginal cost of abatement ($C_A(\cdot)$) adjusted for the cost-reduction associated with the learning by doing stemming from that abatement ($\mu_t k \Psi_A(\cdot)$).

In this model, just as in the CE_R model, the optimal carbon tax is equal to $-\lambda_t$.[17] Since the costate equation for $\lambda_t$ is unchanged from before, we can refer to earlier results and conclude that

[17] We have assumed no spillovers in the model; the cost-reduction from learning by doing is fully appropriated by agents.

the carbon tax grows at the rate $(r+\delta)$ for $t < T$, but grows more slowly, and perhaps declines, thereafter.

Although the CE_R and CE_L models are similar as regards the carbon tax path, they differ with respect to the characteristics of the optimal abatement path. In particular, it is no longer unambiguously true that the abatement path is positively sloped for $t < T$, even in the case in which baseline emissions are growing over time. This is demonstrated in the appendix; the basic reason is that the cost-reduction due to learning by doing does not necessarily grow with time.[18]

### 2.2.2 Implications of ITC

Now let us consider what happens to the optimal tax and abatement paths when we introduce ITC, i.e., increase $k$ from the point $k = 0$. Again we find, assuming $C_H(\cdot) < 0$, that the presence of ITC causes optimized costs to fall. Under the assumption that $C_{AH}(\cdot) < 0$, we again find that the presence of ITC causes the shadow cost of the $CO_2$ concentration, and thus, the optimal carbon tax, to decline (and increasingly so for higher $t$).

In order to analyze the impact of ITC on the abatement path, we differentiate equation (9) with respect to $k$. Evaluating this at $k = 0$ yields:

$$\frac{dA_t}{dk} = \frac{\frac{d(-\lambda_t)}{dk} + \mu_t \Psi_A(\cdot) - C_{AH}(\cdot)\frac{dH_t}{dk}}{C_{AA}(\cdot)} \tag{10}$$

As in the CE_R model, we observe the negative shadow-cost effect $(\frac{d(-\lambda_t)}{dk})$ and the positive knowledge-growth effect $(-C_{AH}(\cdot)\frac{dH_t}{dk})$. In our LBD specification, however, the presence of ITC has an additional, positive effect on abatement which we term the *learning-by-doing effect* $(\mu_t \Psi_A(\cdot))$. This effect reflects the fact that learning by doing offers an additional marginal benefit (the learning) from abatement. Other things equal, this further marginal benefit justifies additional abatement. Thus, under this specification the presence of ITC has three effects on abatement, one negative (the shadow-cost effect), and two positive (the knowledge-growth and learning-by-doing effects).[19] The net effect is ambiguous; even at time 0, when the knowledge-growth effect does not come into play, we are still left with the opposing shadow-cost and learning-by-doing effects:

$$\frac{dA_0}{dk} = \frac{\frac{d(-\lambda_0)}{dk} + \mu_t \Psi_A(\cdot)}{C_{AA}(\cdot)}$$

[18] In an NITC scenario, the abatement path will unambiguously slope upward for $t < T$, given that baseline emissions do not decline too rapidly. See appendix for details.

[19] Evaluating at an arbitrary nonzero initial value of $k$ adds extra terms which are difficult to sign. Unlike in the R&D-specification, here we cannot be fully confident that our differential analysis around the point $k = 0$ carries over to the case of large increases in $k$ from 0. However, the numerical simulations below indicate that the qualitative results obtained here carry through even for large changes in $k$.

Thus, in contrast to the CE_R model, the presence of ITC no longer implies unambiguously that initial abatement will fall. If the learning-by-doing effect is strong enough, initial abatement rises. (This in fact happens in most of the numerical simulations presented in Section 4.) These results offer partial support for Ha-Duong, Grubb, and Hourcade's (1996) claim that because of learning by doing, ITC justifies higher initial abatement. Higher initial abatement may be justified, but this is not always the case.

#### 2.2.3 Summary

Thus, the following results (summarized in Table 1) were obtained in the CE_L case. First, the optimal carbon tax grows at the rate $(r + \delta)$ for $t < T$, but will grow more slowly, and perhaps even decline, after that. The slope of the optimal abatement path is of ambiguous sign throughout (unless we are in an NITC scenario, in which case abatement unambiguously rises over time, at least for $t < T$, if baseline emissions do not decline too rapidly). Second, although introducing the ITC option lowers optimized costs and makes the entire carbon tax path fall by an equal proportion at all $t < T$, the impact on initial abatement is analytically ambiguous.

## 3 Optimal Policy under the Benefit-Cost Criterion

We now analyze optimal tax and abatement profiles in a benefit-cost (BC) framework. No longer is there an exogenously given concentration target; rather, the objective is to minimize the sum of abatement costs, investment costs (in the R&D model), *and damages from* $CO_2$ over an infinite horizon.

### 3.1 Technological Change via R&D

#### 3.1.1 The Problem and Basic Characteristics of the Solution

In the R&D-based specification (hereafter referred to as the BC_R model), we have the following problem:

$$\begin{aligned} \min_{A_t, I_t} \int_0^\infty (C(A_t, H_t) + p(I_t)I_t + D(S_t))e^{-rt}dt \\ \text{s.t. } \dot{S}_t = -\delta S_t + E_t^0 - A_t \\ \dot{H}_t = k\Psi(I_t, H_t) \\ \text{and } S_0, H_0 \text{ given} \end{aligned} \tag{11}$$

where $D(S_t)$ is the damage function, assumed to have the following properties: $D'(\cdot) > 0$ and $D''(\cdot) > 0$.[20]

The current-value Hamiltonian associated with the optimization problem is:

$$\mathcal{H}_t = -\Big(C(A_t, H_t) + p(I_t)I_t + D(S_t)\Big) + \lambda_t(-\delta \dot{S}_t + E_t^0 - A_t) + \mu_t k \Psi(I_t, H_t)$$

From the maximum principle, assuming an interior solution, we obtain a set of necessary conditions, of which the most important to us are:

$$C_A(\cdot) = -\lambda_t \tag{12}$$

$$\text{and } \dot{\lambda_t} - r\lambda_t = \delta\lambda_t + D'(\cdot) \tag{13}$$

As before, $\lambda_t$ is the negative shadow value of a small additional amount of $CO_2$. Hence $-\lambda_t$ again represents the marginal benefit of abatement. Equation (12) states that abatement should be pursued up to the point at which marginal cost equals marginal benefit. Equation (13) can be integrated, using the relevant transversality condition as a boundary condition, to obtain:

$$-\lambda_t = \int_t^\infty D'(S_s)e^{-(r+\delta)(s-t)}ds \tag{14}$$

Equation (14) states that the shadow cost of an increment to the $CO_2$ concentration equals the discounted sum of marginal damages that this increment would inflict over all future time. Alternatively, the marginal benefit from incremental $CO_2$ abatement equals the discounted sum of the avoided damages attributable to such abatement.

As in the CE_R model, the optimal carbon tax is equal to $-\lambda_t$, and thus, by equation (12), to the marginal abatement cost at the optimum. Using equation (14), we demonstrate in the appendix that in the BC_R model, the optimal carbon tax may either rise or fall over time. This contrasts with the results from the cost-effectiveness models, in which the optimal carbon tax rose at the rate $(r + \delta)$ (at least for $t < T$). The reason for the ambiguity is that although there is a tendency for the BC_R shadow cost to grow at the rate $(r + \delta)$, there is also a tendency for it to decline over time because an extra amount of $CO_2$ later on would inflict marginal damages over a shorter time horizon. The appendix shows that given the convex damage function which we have assumed, a sufficient condition ensuring that the tax path slopes upward is that the optimized path of $CO_2$ also slopes upward.

Given rising taxes and a baseline emissions path that rises (or at least does not fall too rapidly), we can also demonstrate that optimal abatement rises; otherwise, the slope of the abatement path is ambiguous. (See appendix for details.)

[20]This is not completely uncontroversial. Although most would accept that damages are a convex function of climate change, it is also widely felt—see, e.g., Dickinson and Cicerone (1986)—that climate change is a *concave* function of changes in the atmospheric $CO_2$ concentration. Thus our $D(\cdot)$ function—relating damages to concentrations—could be concave.

### 3.1.2 Implications of ITC

As before, the presence of ITC leads to lower optimized total costs (where these now include $CO_2$-related damages as well as abatement and investment costs). Just as before (and as proven in the appendix), if we assume that knowledge reduces the marginal costs of abatement, the shadow cost of $CO_2$ declines in the presence of ITC: $\frac{d(-\lambda_t)}{dk} \leq 0$. The intuition is similar to what it was in both the CE_R and CE_L models. Technological progress makes marginal abatement cheaper. Thus, when R&D investments yield advanced technologies ($k > 0$), the prospect of being given an additional amount of $CO_2$ is less worrisome than it would be if we knew we would always have only primitive abatement technologies available ($k = 0$). Since the optimal carbon tax is the shadow cost of $CO_2$, the presence of ITC lowers carbon taxes (the shadow-cost effect).[21] We can also provide intuition for the shadow-cost effect in terms of equation (14). When ITC gives us the prospect of having more advanced technologies at our disposal, it makes sense that we would aim for more ambitious $CO_2$ concentration targets. Given a convex damage function, this would imply that marginal damages would be lower in the ITC world, and thus, by equation (14), optimal carbon taxes would be lower as well.

This result is perhaps surprising. Earlier, in a cost-effectiveness setting, we dismissed the claim that the presence of ITC should increase optimal taxes by appealing to a simple static graph; this graph showed that with ITC, it took a lower tax to achieve the same required level of abatement. But one might still have expected that in the broader, benefit-cost setting, if technology progressed sufficiently, it would make sense to *increase* the amount of abatement, and thus the optimal tax would increase.

The lower panel of Figure 2 heuristically indicates that this notion is incorrect, at least under the assumption that the damage function is convex in the $CO_2$ concentration. The optimal amount of abatement and the optimal carbon tax are given by the intersection of the upward sloping MC curve and the downward sloping marginal abatement benefit (MB) curve.[22] If the MC curve were to pivot downward as a result of technological progress, the optimal amount of abatement would increase, but *the optimal carbon tax would fall* because we move to a lower point on the marginal benefit (marginal damage) curve.

If the damage function were linear, implying a flat marginal damage schedule, then the MC pivot would increase the optimal amount of abatement while leaving the optimal carbon tax unchanged. On the other hand, if damages were concave in the $CO_2$ concentration, then the MB curve would be upward sloping, and it is possible to envision a scenario in which a technology-driven fall in the

[21]Unlike in the cost-effectiveness models, however, it is not necessarily true that taxes later on fall by greater amounts than do early taxes. See appendix.

[22]This MB curve conveys the same information as the schedule of marginal damages from additions to the stock of $CO_2$.

MC schedule could actually increase the optimal carbon tax. This situation seems unlikely.[23]

Next we examine the implications of increasing $k$. Using the same approach as in the CE_R model, we obtain:

$$\frac{dA_t}{dk} = \frac{\frac{d(-\lambda_t)}{dk} - C_{AH}(\cdot)\frac{dH_t}{dk}}{C_{AA}(\cdot)} \tag{15}$$

Once again, the impact of ITC on abatement at time $t$ is ambiguous because the shadow-cost effect and the knowledge-growth effect oppose one another. At time 0, however, the stock of knowledge is fixed at $H_0$, and thus only the shadow-cost effect comes into play:

$$\frac{dA_0}{dk} = \frac{\frac{d(-\lambda_0)}{dk}}{C_{AA}(\cdot)} \tag{16}$$

Thus initial abatement declines as a result of ITC (although this result is reversed if $C_{AH}(\cdot) > 0$).

In the cost-effectiveness analyses, where we had a fixed terminal constraint, $\bar{S}$, we knew that over the entire time horizon, cumulative abatement would be approximately the same under both ITC and NITC scenarios.[24] This implied that the shadow-cost and knowledge-growth effects would approximately balance one another out over the entire horizon; in terms of Figure 1, the area under path 1 would roughly approximate the area under path 3.

In the benefit-cost framework, however, this is not the case. As demonstrated in the appendix, the overall scale of abatement over the entire infinite horizon increases; that is to say, the knowledge-growth effect dominates the shadow-cost effect on average. Since $CO_2$ inflicts environmental damages, it seems reasonable that in the presence of ITC, which makes emissions abatement cheaper, the optimal balance of benefits and costs of emissions abatement would be struck at a higher level of abatement (on average) than would be optimal in the NITC scenario. This result is not very surprising. A more unexpected result is that *initial* abatement still falls, no matter how "large" or powerful the ITC option. Equation (15) indicates that this occurs because there is no separate analytical term representing an upward shift of abatement at all points in time. Rather, the increased scale of abatement is reflected completely in the steepening of the abatement path resulting from the interaction between the knowledge-growth and shadow-cost effects.

[23]See Repetto (1987) for a discussion of an example with non-convex damages. Also note that, as before, if technological progress were to *raise* the MC schedule, then even with convex damages, the optimal carbon tax would rise (and the optimal scale of abatement would fall). This is confirmed in the appendix.

[24]We say "approximately" because natural removal implies that two abatement paths leading to $\bar{S}$ need not involve *exactly* the same cumulative abatement. In fact, as will be seen in the sensitivity analysis in Section 4, paths which concentrate relatively more abatement in the future need less cumulative abatement to reach the same $\bar{S}$ constraint because they take better advantage of natural removal than do more heavily "frontloaded" abatement paths.

### 3.1.3 Summary

We have obtained the following main results for this BC_R case (see also Table 1). First, the optimal carbon tax may either rise or fall over time, but if concentrations of $CO_2$ are increasing through time, then (given a convex damage function) the optimal carbon tax rises as well. Optimal abatement may either rise or fall over time, but, as long as baseline emissions are not falling too rapidly over time, it will rise if the carbon tax is rising. Second, introducing the ITC option lowers optimized net costs and causes the entire carbon tax path to fall. Initial abatement also falls, but cumulative abatement over the entire horizon rises; hence ITC implies a "steeper" abatement path.

## 3.2 Technological Change via Learning by Doing

Finally, we examine an LBD specification in a benefit-cost framework (the BC_L model).

### 3.2.1 The Problem and Basic Characteristics of the Solution

The optimization problem is now:

$$\min_{A_t} \int_0^\infty (C(A_t, H_t) + D(S_t))e^{-rt}dt$$
$$\text{s.t. } \dot{S}_t = -\delta S_t + E_t^0 - A_t$$
$$\dot{H}_t = k\Psi(A_t, H_t)$$
$$\text{and } S_0, H_0 \text{ given}$$

Thus, $CO_2$-related damages are part of the minimand, and abatement effort contributes to the change in the knowledge stock. The optimality conditions are the same as in the BC_R model, with one major change: the first-order condition for abatement is now

$$C_A(\cdot) - \mu_t k \Psi_A(\cdot) = -\lambda_t \tag{17}$$

which is just as it was in the CE_L model (equation (9)).

As in the BC_R model, the slope of the carbon tax path is ambiguous (though it will be positive if the optimized $CO_2$ concentration rises over time, given convex damages). Thus the slope of the abatement path is ambiguous as well.

### 3.2.2 Implications of ITC

As always, the presence of ITC lowers overall optimized costs as well as the profile of optimal carbon taxes (assuming $C_H(\cdot) < 0$ and $C_{AH}(\cdot) < 0$). The impact of ITC on abatement is given by:[25]

$$\frac{dA_t}{dk} = \frac{\frac{d(-\lambda_t)}{dk} + \mu_t \Psi_A(\cdot) - C_{AH}(\cdot)\frac{dH_t}{dk}}{C_{AA}(\cdot)}$$

As in the CE_L model, ITC has three effects on abatement: the negative shadow-cost effect $(\frac{d(-\lambda_t)}{dk})$, the positive learning-by-doing effect $(\mu_t \Psi_A(\cdot))$, and the positive knowledge-growth effect $(-C_{AH}(\cdot)\frac{dH_t}{dk})$. The net effect on abatement at an arbitrary point in time $t$ (including $t = 0$) is clearly ambiguous. At $t = 0$, in particular, the knowledge-growth effect drops out, leaving the shadow-cost and learning-by-doing effects:

$$\frac{dA_0}{dk} = \frac{\frac{d(-\lambda_0)}{dk} + \mu_t \Psi_A(\cdot)}{C_{AA}(\cdot)}$$

and we cannot even claim that *initial* abatement declines unambiguously.

Although the components of the analysis here are the same as in the corresponding cost-effectiveness case, their overall impact is different. In the CE_L model, since the overall scale of abatement was approximately the same in both ITC and NITC scenarios, all three effects roughly balanced out over the entire time horizon. In contrast, in this benefit-cost case, the overall scale of abatement increases.[26] Thus, on average the learning-by-doing and knowledge-growth effects dominate the shadow-cost effect.

### 3.2.3 Summary

The key results are as follows (see also Table 1). The slope of the optimal carbon tax path is ambiguous. However, if the optimized $CO_2$ concentration rises (given a convex damage function), then so does the tax. These results are similar to those in the CE_L model. Moreover, the slope of the optimal abatement path is of ambiguous sign throughout (unless we are in an NITC world with rising taxes and baseline emissions that are not declining too rapidly). Although introducing the ITC option makes overall costs and the entire carbon tax path fall, it could lead to an increase in initial abatement. Furthermore, cumulative abatement over the entire time horizon increases.

## 4 Numerical Simulations

Here we perform numerical simulations to gauge the quantitative significance of our results. We postulate functional forms and parameter values and solve for optimal paths. We then conduct sensitivity analysis to assess the robustness of our results. The numerical simulations reinforce our analytical findings and also point up several striking empirical regularities, as discussed below. We

[25]As in the CE_L analysis, we restrict our attention to the neighborhood around $k = 0$.

[26]See appendix for details.

begin this section by describing the choice of functional forms and the methods used to calibrate the various parameters of the model. We then present and discuss the numerical results.

### 4.1 Functional Forms and Parameter Values

The numerical model is solved at ten-year intervals, with the year 2000 as the initial year. Although the planner's time horizon is infinite, we actually simulate over 41 periods (400 years) and impose steady-state conditions in the last simulated period. This enables us to project forward the values of this last period and thereby determine benefits and costs into the infinite future.[27]

The $CO_2$ concentration in 2000 is taken to be 360 parts per million by volume (ppmv), following the projections of the Intergovernmental Panel on Climate Change (IPCC (1995)). Baseline emissions for the period 2000 to 2100 roughly follow the IPCC's IS92(a) central scenario. After that time, we adopt a hump-shaped profile that peaks at 26 gigatons of carbon (GtC) in 2125 and flattens out to 18 GtC by 2200.[28]

In the analytical section, we assumed for expositional clarity that $CO_2$ in the atmosphere is naturally "removed" at a constant exponential rate. In the numerical simulations, we adopt Nordhaus' (1994) slightly more complex and realistic "two-box model," which applies short-term and long-term removal rates to the flow and "stock" of emissions, respectively:[29]

$$\dot{S}_t = \beta(E_t^0 - A_t) - \delta(S_t - PIL)$$
$$\text{where } \beta = 0.64$$
$$\text{and } \delta = 0.008$$

Thus, only 64 percent of current emissions actually contribute to the augmentation of atmospheric $CO_2$, and the portion of the current $CO_2$ concentration in excess of the pre-industrial level ($PIL =$ 278 ppmv) is removed naturally at a rate of 0.8 percent per annum.

For our benefit-cost simulations, we need to specify a $CO_2$ damage function. We assume this function to be quadratic and, following Nordhaus (1994), who reviewed damage estimates from a

[27]Specifically, we require abatement to level off so as to keep the $CO_2$ concentration steady. Our cost formulation implies that the return to investment in knowledge approaches zero as $H_t$ becomes very large. Hence in the R&D simulations we impose the steady-state constraint that investment goes to zero, and that thus the knowledge stock remains constant in the long run. In the LBD simulations, the continued positive abatement necessary to maintain the $CO_2$ concentration implies that the knowledge stock grows perpetually. We account for this by analytically deriving the value of the optimal program from the last simulation period to the infinite future and then incorporating this value in the finite-interval problem solved by the computer.

[28]This profile is patterned after a scenario used by Manne and Richels (1992).

[29]Some scholars endorse more sophisticated formulations, such as the five-box model of Maier-Reimer and Hasselmann (1987).

number of studies, calibrate the remaining scale parameter so that a doubling of the atmospheric $CO_2$ concentration implies a loss of 1.33 percent of world output each year. Thus we have:

$$D(S_t) = M_D {S_t}^{\alpha_D}$$
$$\text{where } M_D = 0.0012$$
$$\text{and } \alpha_D = 2$$

The functional form assumed for the abatement-cost function is:

$$C(A_t, H_t) = M_C \frac{A_t^{\alpha_{C1}}}{(E_t^0 - A_t)^{\alpha_{C2}}} \frac{1}{H_t}$$

This form has the properties assumed in the analytical model, including the feature that knowledge lowers marginal abatement costs ($C_{AH}(\cdot) < 0$). It also has the property that marginal costs tend to infinity as abatement approaches 100 percent of baseline emissions. We choose the parameters $M_C$, $\alpha_{C1}$, and $\alpha_{C2}$ to meet the requirements that: (1) a 25 percent emissions reduction in 2020 should cost between 0.5 and four percent of global GDP;[30] and (2) the present value (at a five percent discount rate) of global abatement costs for reaching $S_t = 550$ ppmv by 2200 (in an NITC world) should be roughly $600 billion (Manne and Richels (1997)). The parameter values that best meet these requirements are $M_C = 83$, $\alpha_{C1} = 3$, and $\alpha_{C2} = 2$.

The knowledge accumulation function exhibits the properties discussed in the analytical section and is given, in the R&D simulations, by

$$\Psi(I_t, H_t) = M_\Psi {I_t}^\gamma H_t^\phi$$
$$\text{where } M_\Psi = 0.0022$$
$$\gamma = 0.5$$
$$\text{and } \phi = 0.5$$

$H_0$, the initial knowledge stock, is normalized to unity. In the learning-by-doing simulations, the knowledge accumulation function is the same, with $A_t$ replacing $I_t$. The function we use is fairly standard in the endogenous growth literature.[31] $\gamma$ is chosen to be 0.5 to indicate diminishing returns to R&D investment,[32] while $\phi$, which dictates the intertemporal knowledge spillover, is set to 0.5, a central value of the range typically seen in the literature. As it is positive, it indicates

[30] calculations based on results of a literature review in EPRI (1994), and extrapolated to the global economy

[31] See, for example, Romer (1990), Jones (1996), or Jones and Williams (1996). We are grateful to William Nordhaus and Chad Jones for recommending this function and alerting us to its usefulness.

[32] Jones and Williams (1996) dub this the "stepping on toes effect," for "an increase in R&D effort induces duplication that reduces the average productivity of R&D."

that knowledge accumulation today makes future accumulation easier. This is the "standing on shoulders" case which has been used, for example, by Nordhaus (1997). It contrasts with the case where $\phi < 0$, which implies a limited pool of ideas which are slowly "fished out"—current knowledge accumulation makes future accumulation more difficult. $M_\Psi$ is calibrated so that the cost-savings from ITC are approximately 30 percent in the CE_R model. This is consistent with Manne and Richels (1992), who compare the costs of carbon abatement under different assumptions about technological progress.[33]

We assume that the price of investment funds is

$$p(I_t) = I_t$$

Thus the average cost of R&D investment increases with scale; as mentioned earlier, this captures the idea that drawing scientists away from R&D in other sectors involves increasing costs. Following Manne and Richels (1997), we take the discount rate[34] to be five percent. Finally, we model the NITC cases by setting $k = 0$ and the ITC cases with $k = 1$.

## 4.2 Central Cases

### 4.2.1 CE_R Simulation

In the cost-effectiveness cases (CE_R and CE_L), the concentration target ($\bar{S}$) is 550 ppmv, which must be reached by 2200. This scenario has received considerable attention in policy discussions.

We first consider results for the CE_R case, both with and without ITC. The upper-left panels of Figures 3, 4, and 5 depict, respectively, the optimal abatement, $CO_2$ concentration, and carbon tax paths in this case.

*Abatement.* As predicted by the analytical model, the optimal abatement paths slope upwards, at least until 2200, the year in which the constraint is first imposed.[35] Figure 3 shows that the

[33]In the work by Manne and Richels (1992, p. 64), GDP costs of abatement policy are approximately 90 percent lower in an optimistic technology scenario than in the central-case technology scenario. This difference in GDP costs does not account for the costs of developing the improved technologies that distinguish the optimistic scenario from the central-case scenario. We assume that R&D investments have a social rate of return of 50 percent (as in Nordhaus (1997)) and then calculate the net cost savings from technological progress to be roughly 30 percent. (The R&D costs that generate .90 of abatement-cost savings amount to (1/1.5).90. Thus, the net cost savings from technological progress is given by .90 – (1/1.5).90 = .30.) We assume that this figure is relevant to the *induced* technological change which we study in our paper, and we then choose $M_\Psi$ to generate this level of savings.

[34]representing, in this context, the marginal product of capital, rather than the pure rate of time preference

[35]In both the ITC and NITC cases, the level of abatement drops discontinuously in the year 2200 and stays constant thereafter, maintaining the $CO_2$ concentration at the level $\bar{S}$. The constraint on the year-2200 concentration forces this discontinuity.

presence of ITC leads to a slightly "steeper" abatement profile, with less abatement during the first 125 years and more abatement after that. However, the effect of ITC on abatement is almost imperceptible. The percentage impacts are very small, especially in later periods, when the level of abatement is higher. The minuteness of this "abatement-timing" effect is noteworthy, particularly in light of the fact that ITC lowers discounted average costs of abatement by nearly 30 percent. The sensitivity analysis below will show that the weakness of ITC's abatement-timing effect is robust to different parameter specifications.

*Concentrations.* The first panel of Figure 4 shows the optimized time profile of the $CO_2$ concentration in the presence and absence of ITC. The impact of ITC on the accumulation of $CO_2$ in the atmosphere reflects the abatement-timing effect from Figure 3: in the presence of ITC, the $CO_2$ concentration is allowed to build up to a (slightly) higher level before eventually being brought down more rapidly in order to meet the $\bar{S}$ constraint by the year 2200.

*Carbon Tax.* The upper-left panel of Figure 5 shows that the optimal carbon tax starts at a few dollars per ton and grows exponentially. Although not evident from the figure alone, the tax grows at the rate $(r + \delta)$, just as predicted by the analytical model. While ITC's impact on abatement was extremely small, its effect on the optimal tax is pronounced. The presence of ITC lowers the optimal carbon tax path at all points in time up to 2200 by about 35 percent, roughly in line with the 30-percent cost savings mentioned earlier.

### 4.2.2 CE_L Simulation

The upper-right panels of Figures 3-5 depict the abatement, concentration, and tax paths for the CE_L case. The results here are broadly similar to those in the CE_R case just discussed. Again the optimal abatement paths slope upwards,[36] the optimal carbon tax rises at the rate $(r+\delta)$, and the presence of ITC causes a slight steepening of the abatement path and a sizable downward shift in the tax path. Here ITC implies a reduction in total costs of about 40 percent, and a comparable (42 percent) lowering of the optimal carbon tax path.

Some differences between the CE_R and CE_L cases deserve mention. First, under learning by doing, the presence of ITC has an even smaller effect on the optimal abatement path than it does under R&D. This makes sense because the basic tendency toward postponing some abatement from the present to the future is offset in the CE_L case by the learning-by-doing effect, which prompts more abatement now in order to accumulate experience-based knowledge. A second difference is that ITC has a larger impact on taxes and costs in the CE_L case than it does in the CE_R case. This reflects the fact that under LBD-based ITC, technological progress comes about as a "free" by-product of abatement, rather than as a result of costly expenditures on R&D.

[36]Recall that the analytical model was unable to guarantee this result for the ITC scenario.

### 4.2.3 BC_R Simulation

Consider now the benefit-cost cases. The lower-left panels of Figures 3-5 depict the optimal abatement, concentration, and tax paths in the BC_R model.

*Abatement and Concentrations.* The analytical model indicated that as long as taxes were rising and baseline emissions not declining "too rapidly," the abatement path would rise. In our simulations, abatement rises over the interval 2000-2140 and falls after that, matching the pattern of baseline emissions. As shown in Figure 3, in the presence or absence of ITC, there is much less abatement here than in the CE cases (note the different scales used on the vertical axes). Correspondingly, Figure 4 shows that the $CO_2$ concentration in 2200 from the optimal abatement path is above 800 ppmv, considerably higher than the 550 ppmv imposed in the CE simulations. These differences imply (given the damage and cost functions and parameters employed here) that the 550-ppmv target in the cost-effective analysis—a target given much attention in policy discussions—is too stringent from an efficiency point of view. The presence of ITC implies a slight increase in the overall scale of abatement and steepening of the abatement path. Nonetheless, initial abatement *falls* (though only slightly). These outcomes all square with the predictions of the analytical model.

*Carbon Tax.* The lower left-panel of Figure 5 shows that the optimal carbon tax profile is roughly linear in this simulation. This contrasts with the exponential shape in the CE simulations and conforms with the analysis of Section 3. Recall that the shadow cost of the $CO_2$ concentration (i.e., the carbon tax) is given by the sum of marginal damages that a small additional amount of $CO_2$ would cause into the infinite future, discounted at the rate $(r + \delta)$. Although the shadow cost tends to rise at the rate $(r + \delta)$, this is offset by the fact that as time goes on, less time remains over which the incremental amount of $CO_2$ can inflict marginal damages. The combination of these two effects produces a linear carbon tax profile.

Again in striking contrast to the CE simulations, the impact of ITC on the optimal carbon tax path is virtually imperceptible in the BC_R central case. There are two basic reasons for the difference. First, as suggested by Figure 2, the adjustment due to ITC is both in the quantity (that is, abatement) and price (tax) dimensions; in the CE cases, in contrast, adjustment can only occur in the price dimension because of the constraint on the terminal $CO_2$ concentration. In our central case, the marginal damage curve is very flat over the relevant range. As a result, nearly all of the adjustment to ITC in the BC cases comes via changes in the level of abatement.[37]

The second reason is more subtle and relates to the fact that $\bar{S}$ constraint imposed in the cost-effectiveness scenarios is too stringent. Consequently, optimal levels of abatement are generally much higher in the cost-effectiveness cases, which implies that the potential gains from improved technology are also higher. Thus in the cost-effectiveness cases, it pays to accumulate more knowl-

[37]Here, because the MC curve is relatively steep, the effect on abatement is not very large.

edge than it does in the benefit-cost settings. This implies that the downward pivot of the MC curve and associated impact on the carbon tax are larger in these cases.

Finally, the presence of ITC has an extremely small impact on average costs of abatement in the benefit-cost cases, which, as before, contrasts with the result in the cost-effectiveness cases. Given that ITC in the benefit-cost cases has a small impact on the carbon tax—the marginal abatement cost at the optimum—it should not be surprising that it has a correspondingly small impact on average costs. Similarly, the ITC-induced percentage increase in the net benefits of climate policy is very small.

In a recent working paper, Nordhaus (1997) independently obtains the result that, in a benefit-cost context, the presence of ITC has an imperceptible impact on the optimal carbon tax and on the net benefits from carbon abatement policy. Our BC results conform to Nordhaus's, although, as discussed below, we obtain different results under alternative parameterizations.[38]

#### 4.2.4 BC_L Simulation

Finally, we consider the BC_L case. The results for this case are displayed in the lower-right panels of Figures 3-5. The effect of ITC on abatement (Figure 3) is similar to the effect in the BC_R case, although the impact is somewhat more pronounced.[39] Initial abatement rises, indicating that the learning-by-doing effect outweighs the shadow-cost effect. Once again, the presence of ITC has a virtually imperceptible impact on the optimal carbon tax path, average costs per unit of abatement, and net benefits. The explanation is the same as that which applied in the BC_R case.

### 4.3 Sensitivity Analysis

Here we examine the sensitivity of the numerical results to changes in key parameters. For each of the four models, we examine five sets of variants of the central case. Table 2 presents summary

[38]Nordhaus obtains this result in a dynamic optimization model in which technological change is driven by R&D expenditure. Some differences between the Nordhaus study and the present study are worth noting. His analysis explicitly models a production function, while ours represents production (or ease of substitution) in reduced form through the abatement-cost function. This enables us to obtain analytical results where Nordhaus relies solely on numerical simulations. Another difference is that the Nordhaus study considers only the BC_R case. In contrast, the present study considers both the benefit-cost and cost-effectiveness cases, and considers learning-by-doing- as well as R&D-based technological change. The present paper's attention to alternative policy specifications and knowledge-generation channels, along with the broad sensitivity analysis below, enable it to map out more broadly the conditions under which ITC has (or does not have) a significant impact on economic outcomes.

[39]As when we compared the CE_R and CE_L models, this difference is due to the fact that ITC is "free" under an LBD specification but costly under R&D. However, in some of the variants tried in the sensitivity analysis, this result is reversed: ITC sometimes has a larger impact under R&D.

statistics describing, in each model variant, the percentage impact that ITC has on the abatement profile, the terminal $CO_2$ concentration, the total amount of abatement over the period 2000–2200, the tax profile, and overall costs per unit of abatement. We report the impacts on abatement and taxes in the years 2000, 2050, and 2200 (or 2190).[40] For the benefit-cost cases, we also report the percentage impact of ITC on the net benefits of optimal climate policy relative to a zero-abatement baseline.

A higher discount rate (Case 2a) reduces the importance of future benefits or costs relative to current ones. Since the costs of ITC are borne today, whereas the benefits are spread more uniformly through time, a higher discount rate tends to reduce the net benefits from ITC. This means that there will be less knowledge accumulation, which implies that the abatement-timing effect (the pivoting of the abatement path) is smaller. The reduced attractiveness of ITC implies, in the benefit-cost cases, that there will be a smaller impact on the overall scale of abatement as well. The opposite results hold under a lower discount rate (Case 2b).

The next variant involves changes either in the constraint on year-2200 concentrations (in the cost-effectiveness cases) or in the parameters of the damage function (in the benefit-cost cases). In a cost-effectiveness setting, a tighter concentration constraint (Case 3a) enforces greater overall abatement and therefore entails higher marginal costs of abatement. This confers higher value to ITC. The reverse applies when the constraint is more lax (Case 3b).[41] In the benefit-cost simulations, Case 3a imposes higher curvature on the damage function. Thus the marginal damage function is steeper in the relevant range. As a result, ITC has a larger impact on the optimal carbon tax and there is less impact on quantity (abatement). Case 3b imposes a linear damage function, so that the marginal damage schedule is perfectly flat. In this case, there is no impact on the optimal carbon tax profile. All of the adjustment occurs in quantity (abatement). Even in this case, however, the effect on abatement levels is quite small because the marginal cost curve is quite steep.

In cases 4a, 4b, and 4c we alter the curvature of the cost function such that the *marginal* cost curve is, respectively, more convex than in the central case, strictly linear (less convex), and concave (much less convex). For the CE models, $M_C$ in cases 4a, 4b, and 4c is calibrated such that the optimal tax path in the NITC world coincides with what it was in the central case.[42] For the BC models, $M_C$ is calibrated such that the total amount of NITC abatement over the period 2000–2200 stays constant. Changes in the curvature of the cost function are most important to the results of

[40]In the cost-effectiveness simulations, we report these results for the year 2190, the period that precedes the year-2200 downward spikes in abatement and taxes.

[41]In Case 3b, the value 837.67 is chosen in order to match the optimized value of the year-2200 $CO_2$ concentration in the NITC scenario of the BC_R simulation.

[42]Thus we are assuring, for comparability across the cases, that the MC curve always intersects the vertical constraint (in the upper panel of Figure 2) at the same point.

the BC simulations. In case 4a, the marginal cost function is convex and steep in the relevant range. As a result, the downward-pivot of this function caused by ITC does not greatly alter the optimal levels of abatement. In contrast, when the marginal cost function is linear or concave (cases 4b and 4c) and much flatter in the relevant range, ITC has pronounced effects on optimal abatement. Indeed, in the concave case, ITC implies a 23 percent increase in cumulative abatement in the BC_R model and a 106 percent increase in the BC_L model! These larger impacts on abatement are associated with significant effects on average costs (costs per unit of abatement) and on the net benefits from optimal abatement. Thus, even if ITC's impact on the tax profile is small (a result attributable to the flatness of the marginal damage schedule), it may have a significant impact on abatement levels, abatement costs, and net benefits if the marginal cost function is concave, and flat in the relevant range.

In variants 5a and 5b we change the ease of accumulating knowledge, when ITC is present, by altering the multiplicative parameter $M_\Psi$ in the $\Psi(\cdot)$ function. When the ITC option is made more powerful (Case 5a), the effects on ITC are magnified. The reverse occurs when the ITC option is made weaker (Case 5b).

Finally, in variants 6a and 6b we consider alternative values for $\phi$, which governs the intertemporal knowledge spillover. The central value is 0.5, indicating some degree of "standing on shoulders." Case 6a involves a value of 0.75 (a stronger positive intertemporal spillover); as expected, the effects of ITC are magnified, though only by a small amount. In case 6b, we set $\phi$ to $-0.5$ (which represents "fishing out"); here, the opposite holds, and the effects of ITC are (slightly) diminished.

# 5 Conclusions

This paper has employed analytical and numerical models to examine the implications of induced technological change for the optimal design of $CO_2$-abatement policy. We obtain optimal time profiles for carbon taxes and $CO_2$ abatement under two channels for knowledge accumulation—R&D-based and LBD-based technological progress—and under both a cost-effectiveness and a benefit-cost policy criterion.

The analytical model reveals, in contrast with some recent claims, that the presence of ITC lowers the time profile of optimal carbon taxes. The impact of ITC on the optimal abatement path varies: when knowledge is gained through R&D investments, some abatement is shifted from the present to the future, but if the channel for knowledge-growth is learning by doing, the impact on the timing of abatement is analytically ambiguous.

When the government employs the benefit-cost policy criterion, the presence of ITC justifies greater overall (cumulative) abatement than would be warranted in its absence. However, ITC does not always promote greater abatement in all periods. When knowledge accumulation results from R&D expenditure, the presence of ITC implies a reduction in near-term abatement, despite the increase in overall abatement.

The numerical simulations reinforce the qualitative predictions of the analytical model. The quantitative impacts depend critically on whether the government is adopting the cost-effectiveness criterion or the benefit-cost criterion. ITC's effect on overall costs and optimal carbon taxes can be quite large in a cost-effectiveness setting: thus, policy-evaluation models that neglect ITC can seriously overstate both the costs of reaching stipulated concentration targets and the carbon taxes needed to elicit the desired abatement. On the other hand, the impact on costs and taxes is typically much smaller under a benefit-cost policy criterion. The weak effect on the tax rate in the benefit-cost case reflects the relatively trivial impact of ITC on $CO_2$ concentrations, associated marginal damages, and (hence) the optimal tax rate. As for the optimal abatement path, the impact of ITC on the timing of abatement is very weak, and the effect (present in the benefit-cost case) on total abatement over time is generally small as well, especially when knowledge is accumulated via R&D.

Our work abstracts from some important issues. One is uncertainty. We have assumed both that knowledge-accumulation is a deterministic process and that the cost and damage functions are perfectly known. In doing so, we have avoided difficult issues of abatement timing relating to irreversibilities and the associated need to trade off the "sunk costs and sunk benefits" of abatement policy.[43]

Our model also abstracts from possible indivisibilities associated with physical capital stocks. Accounting for capital stock turnover could alter the effect of ITC on the optimal abatement path. In particular, the effects through time could be less smooth than those shown here.

Finally, in this model the sole policy instrument available to the decisionmaker (social planner) is a tax on $CO_2$ emissions. It would be useful to extend the model to include two instruments; viz., a carbon tax and a subsidy to R&D. This would allow explorations of public policies that simultaneously consider two market failures—one attributable to the external costs from emissions of $CO_2$, and one attributable to knowledge spillovers, which force a wedge between the social and private returns to R&D. In this broader model, one could investigate optimal combinations of carbon taxes and subsidies to R&D. It would also permit investigations of second-best policies: for example, optimal R&D subsidies in a situation in which the government is not able to levy a carbon tax. This approximates the situation implied by recent policy proposals of the Clinton Administration.

[43] See Pindyck (1993) and Ulph and Ulph (1997).

# A Appendix

## A.1 The Cost-Effectiveness Criterion

### A.1.1 Technological Change via R&D

We first demonstrate the basic characteristics of the slope of the optimal abatement path. We then go on to establish the implications of ITC. To determine how abatement changes over time, we differentiate the first-order condition governing abatement with respect to $t$. Note that the abatement-cost function is not necessarily time-stationary because costs may depend on baseline emissions, which usually vary through time. Differentiating equation (5) with respect to $t$ yields:

$$C_{AA}(\cdot)\dot{A}_t + C_{AH}(\cdot)\dot{H}_t + \frac{\partial C_A(\cdot)}{\partial t} = -\dot{\lambda}_t$$
$$\Longleftrightarrow \quad \dot{A}_t = \frac{-\dot{\lambda}_t - C_{AH}(\cdot)\dot{H}_t - C_{AE}(\cdot)\dot{E}_t^0}{C_{AA}(\cdot)}$$

We have established that for $t < T$, $-\dot{\lambda}_t > 0$ (see equation (6)), and we know that $\dot{H}_t \geq 0$. Previously we had assumed that $C_{AH}(\cdot) < 0$ and $C_{AA}(\cdot) > 0$. If costs do not depend on the level of emissions, then $C_{AE}(\cdot) = 0$ and equation (18) implies that abatement increases over time ($\dot{A}_t \geq 0$).

It is plausible that $C_{AE}(\cdot) < 0$, namely that the marginal cost of a fixed amount of abatement is greater, the lower the level of baseline emissions. This is consistent with the idea that abatement costs depend on relative, rather than absolute, levels of abatement. In this circumstance, $\dot{A}_t \geq 0$ so long as baseline emissions are not declining "too rapidly."

Next we move to the ITC/NITC comparison. Under the assumption that $C_{AH}(\cdot) < 0$, we prove the claim that $\frac{d(-\lambda_0)}{dk} \leq 0$. Suppose the opposite; i.e., suppose that

$$\frac{d(-\lambda_0)}{dk} > 0 \tag{18}$$

Equation (6) in the main text can be integrated, using the relevant transversality condition as a boundary condition, to obtain the following expression:

$$-\lambda_t = \int_{max[t,T]}^{\infty} \eta_s e^{-(r+\delta)(s-t)} ds \tag{19}$$

$\eta_t$, the multiplier on the $\bar{S}$ constraint, is zero if the constraint does not bind, and is typically positive, representing the shadow value of relaxing the constraint, if the constraint does bind. The multiplier can be interpreted as a measure of how binding the $\bar{S}$ constraint is. Thus, equation (19) states that the shadow cost of having a small additional amount of $CO_2$ at time $t$ is dictated by how binding the $\bar{S}$ constraints are into the infinite future.[44] Combining equation (19) with our supposition (18) yields (assuming the proper regularity conditions hold),

$$\int_T^{\infty} \frac{d\eta_s}{dk} e^{-(r+\delta)s} ds > 0 \tag{20}$$

[44] As noted in the main text, this is in contrast to the benefit-cost cases, in which the shadow cost is given by the discounted sum of marginal damages which a small additional amount of $CO_2$ would cause into the infinite future.

which states that overall, the $\bar{S}$ constraints from $T$ onwards become more binding, or costly. The supposition that $-\lambda_0$ rises implies, from equation (8), that $A_0$ also rises. Noting from equation (6) that $\lambda_t = \lambda_0 e^{(r+\delta)t}$ for $t < T$, we see that our supposition in fact implies that $-\lambda_t$ rises for all $t < T$. This in turn implies, from equation (7) (since $-C_{AH}(\cdot)\frac{dH_t}{dk}$ is clearly nonnegative), that $A_t$ strictly rises for all $t < T$. In fact, as we shall now show, $A_t$ strictly rises for *all* $t$, even beyond $T$. If abatement has strictly risen at every point in time up until $T$, then we know that $S_T$ is now strictly less than it used to be in the NITC scenario, and thus certainly strictly less than $\bar{S}$. This itself is acceptable: it is easy to imagine situations in which, given a convex abatement-cost function and an emissions baseline that rises sharply after time $T$, an optimal program involves undershooting the constraint at the first point in time when it is imposed. However, the fact that $S_T$ is now strictly less than $\bar{S}$ implies, by complementary slackness, that $\eta_T = 0$, and thus, since intuition tells us $\eta_T$ is nonnegative, that $\eta_T$ is less than or equal to its value before the increase in $k$. In other words, $\frac{d\eta_T}{dk} \leq 0$. But we know from equation (20), that the constraints from $T$ onwards are, on the whole, more binding, and thus we can now conclude, for sufficiently small $\epsilon'$ and all $\epsilon \in (0, \epsilon')$ as well, that

$$\int_{T+\epsilon}^{\infty} \frac{d\eta_s}{dk} e^{-(r+\delta)s} ds > 0$$

$$\iff \frac{d(-\lambda_{T+\epsilon})}{dk} > 0$$

$$\iff \frac{dA_{T+\epsilon}}{dk} > 0$$

Now we know that abatement has strictly risen for all $t < T + \epsilon$. The above argument can be repeated, in the style of a proof by induction, to show that $\frac{dS_{T+\epsilon}}{dk} < 0$, implying that $\frac{d\eta_{T+\epsilon}}{dk} \leq 0$, and that, in turn, $\frac{dA_t}{dk} > 0 \;\; \forall \; t$. Our supposition that $\frac{d(-\lambda_0)}{dk} > 0$ has led us to the conclusion that abatement rises at all points in time. Given that the initial program satisfied the constraints, a new program in which abatement is higher at every point clearly cannot be optimal. Thus we have a contradiction. We may conclude that $\frac{d(-\lambda_0)}{dk} \leq 0$, and that thus, $\frac{dA_0}{dk} \leq 0$. Since the multiplier simply grows at the constant rate $(r + \delta)$ until time $T$, we have also shown that $\frac{d(-\lambda_t)}{dk} \leq 0 \;\; \forall \, t < T$, and in fact, that the absolute fall in the multiplier increases with $t$ over this time range, but in such a way as to preserve the growth rate as $(r + \delta)$.

Note that if we had assumed $C_{AH}(\cdot) > 0$, then the above proof could essentially be reversed to show that initial abatement and the entire tax path weakly rise. We would find that, in contrast to the normal case, ITC would cause a "flattening," rather than a "steepening" of the optimal abatement profile.

### A.1.2 Technological Change via Learning by Doing

We start by establishing the slope of the optimal abatement path. It is necessary, however, first to examine the movement of $\mu_t$, the shadow value of knowledge. The costate equation for $\mu_t$, which is the same in both the CE_R and CE_L models, states that:

$$\dot{\mu}_t = \mu_t(r - k\Psi_H(\cdot)) + C_H(\cdot)$$

The shadow value grows at $r$ because it is a *current-value* multiplier. Depending on the sign of $\Psi_H(\cdot)$—that is, depending on whether knowledge accumulation is characterized by "standing on shoulders" or "fishing out"—there is a second tendency for the shadow value either to fall or to rise over time. For example, when $\Psi_H(\cdot) < 0$—the "fishing out" case where further knowledge accumulation becomes more difficult the larger the current stock of knowledge—it is preferable to suffer this disadvantage over as short a time interval as possible. Thus in this case, the shadow value tends to rise over time. The opposite holds in the "standing on shoulders" case where $\Psi_H(\cdot) > 0$. Finally, there is a tendency ($C_H(\cdot)$) for the shadow value of knowledge to fall over time because we have a shorter time range over which the knowledge will serve to reduce abatement costs. These three effects combine to make the slope of the $\mu_t$ path ambiguous in sign.

We now focus on the slope of the optimal abatement path in the CE_L model. Differentiating equation (9) and rearranging, we obtain:

$$\dot{A}_t = \frac{-\dot{\lambda}_t + \dot{\mu}_t k \Psi_A(\cdot) + \Big(\Psi_{AH}(\cdot)\mu_t k - C_{AH}(\cdot)\Big)\dot{H}_t - C_{AE}(\cdot)\dot{E}_t^0}{C_{AA}(\cdot) - \mu_t k \Psi_{AA}(\cdot)}$$

The denominator is positive, but the numerator is of ambiguous sign because of the second and third terms. If we consider an NITC scenario in which $k = 0$ and $\dot{H}_t = 0$, then we obtain

$$\dot{A}_t = \frac{-\dot{\lambda}_t - C_{AE}(\cdot)\dot{E}_t^0}{C_{AA}(\cdot)}$$

which, at least for $t < T$, is clearly positive, as discussed above, as long as $\dot{E}_t^0$ is not too negative. In the general LBD case with ITC, however, the optimal abatement path may very well slope downwards (even if the emissions baseline is growing over time), in contrast to the R&D case.[45]

Now we examine the implications of ITC. The proof that $\frac{d(-\lambda_0)}{dk} \leq 0$ proceeds along the same lines as in the CE_R appendix. We suppose that $-\lambda_0$ strictly rises, and this implies that abatement rises for all $t$, which cannot be optimal. (The extra learning-by-doing effect in equation (10) is positive and thus only strengthens the link between $-\lambda_t$'s rising and $A_t$'s rising.[46]) We conclude, as in the CE_R model, that the entire path of carbon taxes falls, and increasingly so for higher $t$ (up to $T$). As noted in the text, however, this finding is not enough to assure us that initial abatement also falls.

## A.2 The Benefit-Cost Criterion

### A.2.1 Technological Change via R&D

First let us analyze the slope of the carbon tax path. We rearrange equation (13) to see that

$$\dot{\lambda}_t = (r + \delta)\lambda + D'(\cdot) \tag{21}$$

The first term on the right-hand side contributes to growth in (the absolute value of) $\lambda_t$, while the second contributes to its decline over time (an additional amount of $CO_2$ later on inflicts marginal damages over a shorter horizon). It is thus possible for the optimal carbon tax to decline over time.

Let us now consider the conditions under which the carbon tax will necessarily rise. Substituting equation (14) into equation (21) yields

$$-\dot{\lambda}_t = (r + \delta)e^{(r+\delta)t}\int_t^{\infty} D'(S_s)e^{-(r+\delta)s}ds - D'(S_t) \tag{22}$$

[45]Our numerical solutions confirm that typically, however, the abatement path *does* slope upward, even in the ITC learning-by-doing case.

[46]The learning-by-doing effect, however, prevents us from reversing the proof for the $C_{AH} > 0$ case. In that case, under a learning-by-doing specification, we cannot conclude anything about the impact of ITC on taxes or abatement.

If we had a linear damage function, such that $D'(S_s)$ were constant and equal to $D'(S_t)$ for all $s > t$, then the first term in equation (22) would reduce to $D'(S_t)$, and we would conclude that $-\dot{\lambda}_t = 0$; i.e., the optimal tax path would be flat. If, however, $D'(S_s) > D'(S_t) \ \forall \ s > t$, the first term in equation (22) would be larger than $D'(S_t)$, and the tax path would be upward sloping. Given the convex damage function which we (and others, typically) assume, having an optimized $S_t$ path that slopes upward ensures $D'(S_s) > D'(S_t) \ \forall \ s > t$, and is thus a sufficient condition for having an upward sloping tax path. Given that many other authors' simulations involve a steadily increasing optimized $CO_2$ concentration, it is easy to see why the literature frequently obtains optimal carbon taxes that forever rise.

What can we say about the slope of the abatement path in the BC_R model? Differentiating equation (12) with respect to $t$ and rearranging, we obtain, as in the CE_R model,

$$\dot{A}_t = \frac{-\dot{\lambda}_t - C_{AH}(\cdot)\dot{H}_t - C_{AE}(\cdot)\dot{E}_t^0}{C_{AA}(\cdot)}$$

The denominator and the second term in the numerator[47] are clearly positive, but our ambiguity about the slope of the optimal carbon tax path prevents us from concluding that optimized abatement must always rise over time. Once again, if the optimized $S_t$ path is rising and the damage function is convex, then taxes rise, and thus so does abatement, as long as the emissions baseline is not declining too rapidly. As we see in our numerical simulations of the BC_R model, even though taxes are always rising, the optimal abatement profile actually slopes down during the time when baseline emissions are steeply decreasing.

Now we turn to the analysis of the implications of ITC; i.e., the effects of increasing $k$. We shall prove that $\frac{d(-\lambda_0)}{dk} \leq 0$, that $\frac{dA_0}{dk} \leq 0$, that the overall scale of abatement increases when we raise $k$, that the abatement path thus becomes steeper, and that $\frac{d(-\lambda_t)}{dk} \leq 0 \ \ \forall \ t$. If we were to assume that knowledge ***raises*** marginal abatement costs, ($C_{AH}(\cdot) \geq 0$), then the entire proof could be reversed to demonstrate that taxes rise, initial abatement rises, the overall scale of abatement falls, and the abatement path thus becomes flatter.

Suppose that $-\lambda_0$ rises, and that thus, by equation (16), $A_0$ rises as well. We have, using equations (14) and (11),

$$\begin{aligned} \frac{d(-\lambda_0)}{dk} &> 0 && (23) \\ \iff \int_0^\infty \frac{dD'(S_s)}{dk} e^{-(r+\delta)s} ds &> 0 \\ \iff \int_0^\infty D''(S_s) \frac{dS_s}{dk} e^{-(r+\delta)s} ds &> 0 \\ \iff \int_0^\infty D''(S_s) e^{-(r+\delta)s} \int_0^s \frac{dA_m}{dk} e^{-\delta(s-m)} dm \ ds &< 0 && (24) \end{aligned}$$

Given our convex damage function, this last equation (24) means that the overall scale of abatement becomes less ambitious when $k$ rises.

For $t > 0$, we can use similar steps to obtain

$$\frac{d(-\lambda_t)}{dk} = -\int_t^\infty D''(S_s) e^{-(r+\delta)(s-t)} \int_0^s \frac{dA_m}{dk} e^{-\delta(s-m)} dm \ ds$$

[47] assuming $C_{AH}(\cdot) < 0$

$$\iff \frac{d}{dt}(\frac{d(-\lambda_t)}{dk}) = -(r+\delta)\int_t^\infty D''(S_s)e^{-(r+\delta)(s-t)}\int_0^s \frac{dA_m}{dk}e^{-\delta(s-m)}dm\ ds + D''(S_t)\int_0^t \frac{dA_m}{dk}e^{-\delta(t-m)}dm \quad (25)$$

Note that $\frac{d}{dt}(\frac{d(-\lambda_t)}{dk})$ is clearly positive if each of the two terms on the righthand side of equation (25) is positive. Given equation (24), the first term is definitely nonnegative if

$$\int_0^t D''(S_s)e^{-(r+\delta)s}\int_0^s \frac{dA_m}{dk}e^{-\delta(s-m)}dm\ ds \geq 0$$

The second term is clearly positive, assuming a convex damage function, if

$$\int_0^t \frac{dA_m}{dk}e^{-\delta(t-m)}dm \geq 0$$

We thus are led to the following lemma. Given our assumption that the initial tax rises (equation (23)), and assuming a convex damage function, then

$$\int_0^t D''(S_s)e^{-(r+\delta)s}\int_0^s \frac{dA_m}{dk}e^{-\delta(s-m)}dm\ ds \geq 0 \quad (26)$$

and

$$\int_0^t \frac{dA_m}{dk}e^{-\delta(t-m)}dm \geq 0 \quad (27)$$

together are sufficient for us to conclude that

$$\frac{d}{dt}(\frac{d(-\lambda_t)}{dk}) \geq 0$$

Equations (23) and (16) together tell us that $\frac{dA_0}{dk} > 0$. This means (using inductive reasoning much like that used in the CE_R appendix),[48] that sufficiency conditions (26) and (27) hold for $t = \epsilon$ sufficiently close to 0, and also for $t = \epsilon' \ \ \forall\, \epsilon' \in (0, \epsilon)$. Thus we conclude that

$$\frac{d}{dt}(\frac{d(-\lambda_\epsilon)}{dk}) \geq 0$$
$$\iff \frac{d(-\lambda_\epsilon)}{dk} \geq \frac{d(-\lambda_0)}{dk} > 0$$
$$\iff \frac{dA_\epsilon}{dk} > 0$$

where this last implication is only strengthened by the $-C_{AH}(\cdot)\frac{dH_\epsilon}{dk}$ effect in equation (16). The whole chain of reasoning can be repeated inductively to imply that abatement strictly rises at every point in time. This, however, clearly contradicts equation (24), which says that the overall scale of abatement is less ambitious. Thus, our supposition must be wrong. We may thus conclude that $-\lambda_0$ (weakly) falls, $A_0$ falls, a more ambitious overall scale of abatement is adopted, and the abatement path becomes "steeper," all as a result of the increase in $k$. That is to say:

$$\frac{d(-\lambda_0)}{dk} < 0$$
$$\frac{dA_0}{dk} < 0$$
$$\text{and} \int_0^\infty D''(S_s)e^{-(r+\delta)s}\int_0^s \frac{dA_m}{dk}e^{-\delta(s-m)}dm\ ds \geq 0 \quad (28)$$

[48]where we used a small $\epsilon$ and appealed to continuity to justify an inductive proof in a continuous-time problem

Using arguments similar to those used above, it is also possible to demonstrate that the entire path of carbon taxes must weakly fall: $\frac{d(-\lambda_t)}{dk} \leq 0 \ \ \forall \ t$. This does not mean, however, that *abatement* always weakly falls; the growth in $H_t$ as a result of $k$ counters the effect of the weakly falling carbon taxes, and we know, in fact, that overall we end up with a weakly more ambitious abatement path.

### A.2.2 Technological Change via Learning by Doing

Using methods virtually identical to those in the previous sections, we prove that: (1) the carbon tax falls at all points in time, including time 0; (2) the impact on $A_0$ is ambiguous; and (3) the overall scale of abatement increases. Please refer to earlier sections of the appendix corresponding to the CE_L and BC_R models; the proofs here are not substantively different.

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**FIGURE 1: KNOWLEDGE-GROWTH AND SHADOW-COST EFFECTS**
**(drawn for CE_R model)**

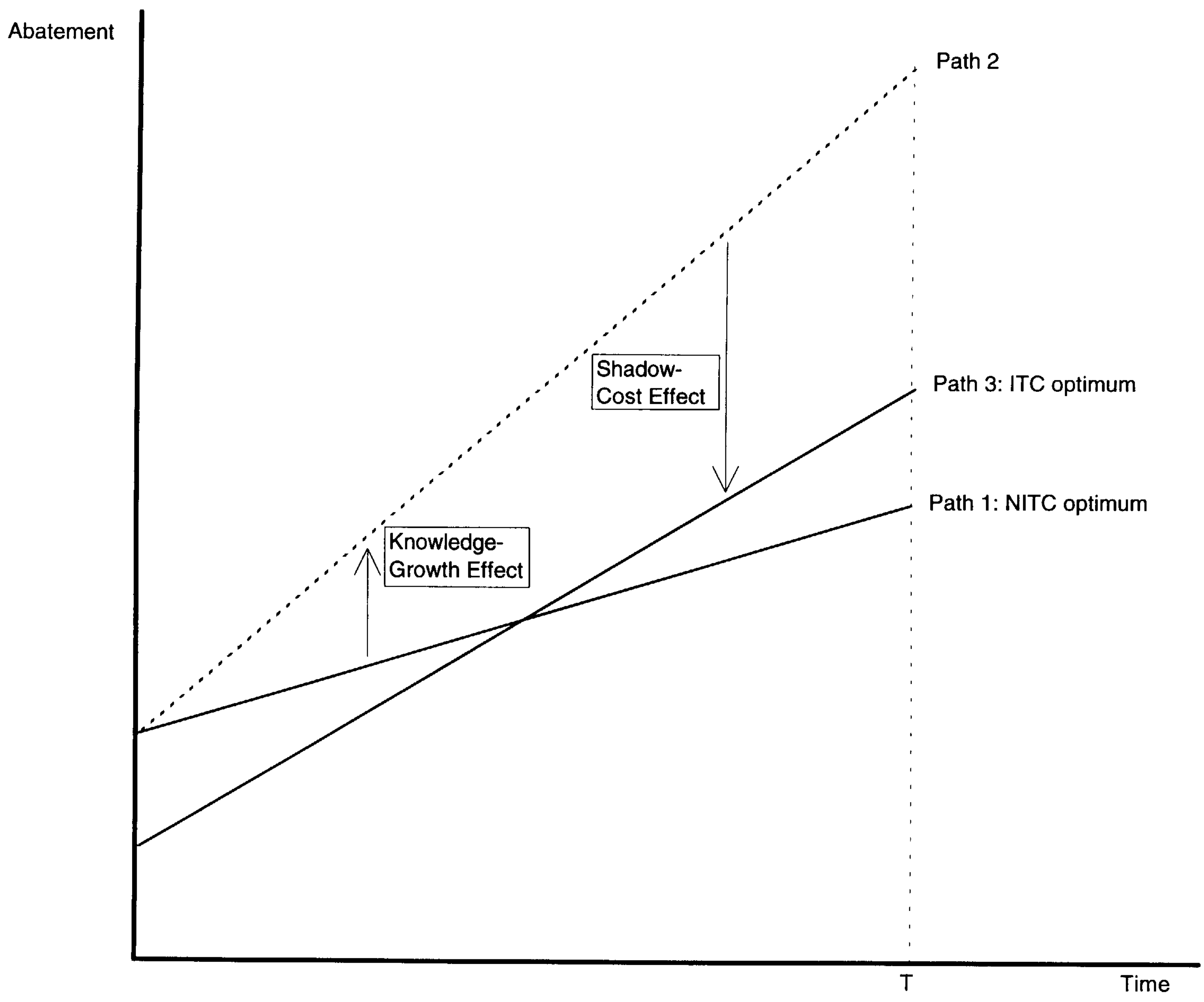
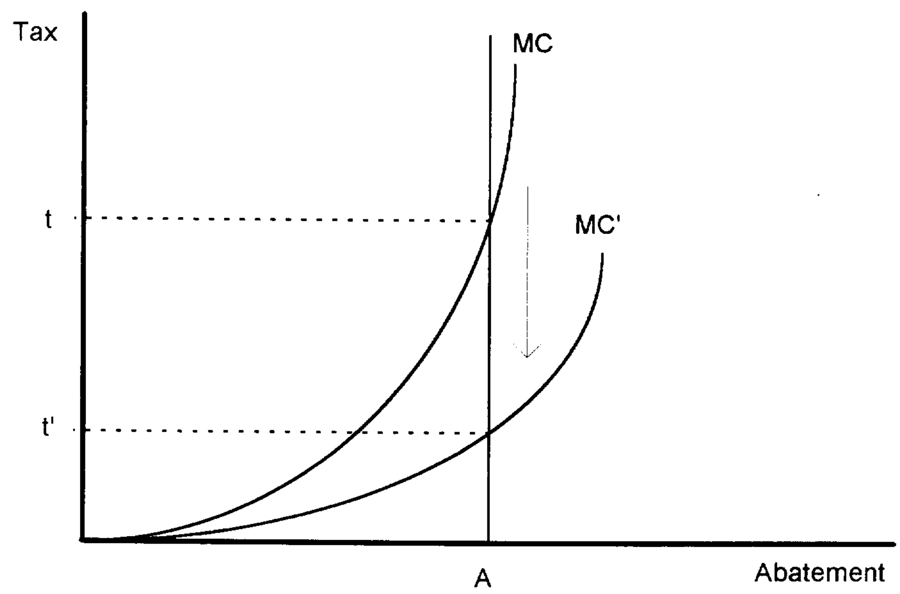


**FIGURE 2: OPTIMAL CLIMATE POLICY IN A STATIC SETTING**

**Cost-Effectiveness Case**




**Benefit-Cost Case**

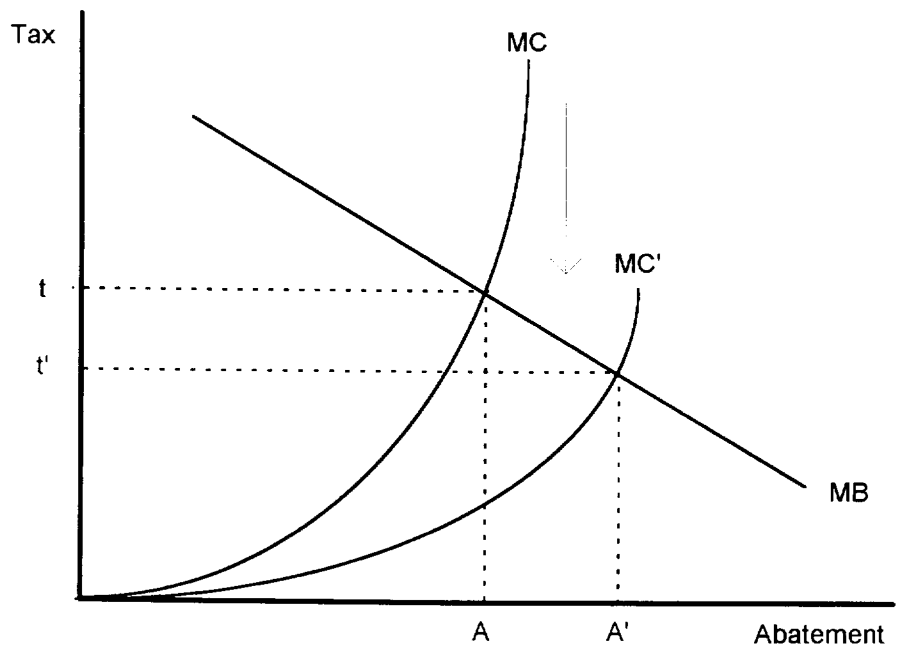


**TABLE 1: SUMMARY OF ANALYTICAL RESULTS**

| **Policy Criterion** | **Channel for Technological Change** | **Basic Characteristics of Optimal Solution** | | **Impacts of Induced Technological Change on Optimal Solution** | |
|---|---|---|---|---|---|
| | | *Tax Path* | *Abatement Path* *(assuming baseline emissions do not decline too rapidly)* | *Tax Path* | *Abatement Path* |
| *Cost-Effectiveness* | R&D | grows at rate $(r+\delta)$ at least until T | slopes upwards at least until T | falls by an equal proportion at all t | $A_0$ falls and later A rises; "steepening" of path |
| | LBD | " | ambiguous slope; rises in NITC case | " | ambiguous effect on $A_0$ |
| *Benefit-Cost* | R&D | ambiguous slope; rises if S* rises and damages are convex | ambiguous slope; rises if tax rises | falls | $A_0$ falls; overall scale of abatement rises; "steepening" of path |
| | LBD | " | ambiguous slope; rises in NITC case if tax is rising | falls | ambiguous effect on $A_0$; overall scale increases |

## FIGURE 3: OPTIMAL ABATEMENT PATHS

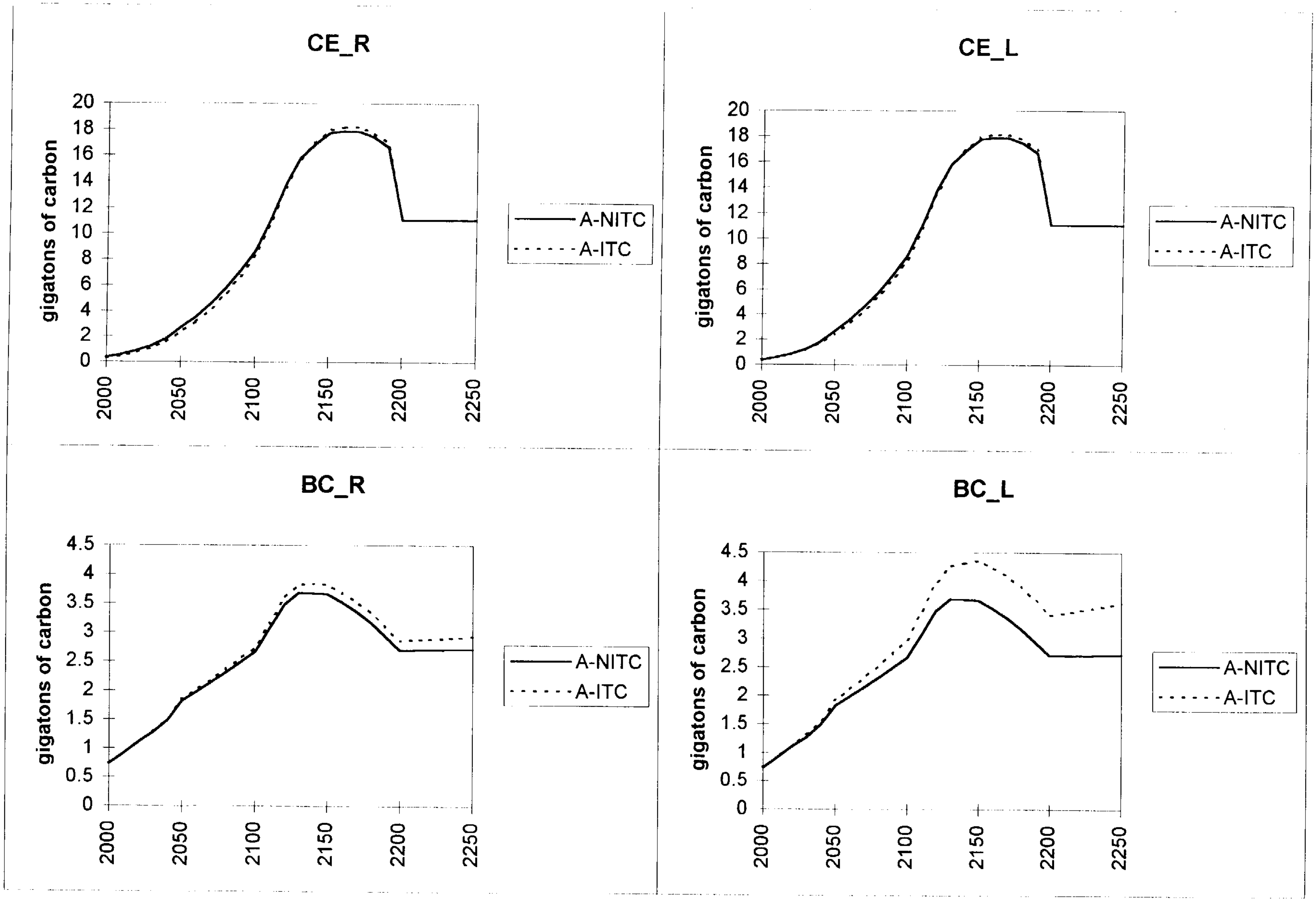
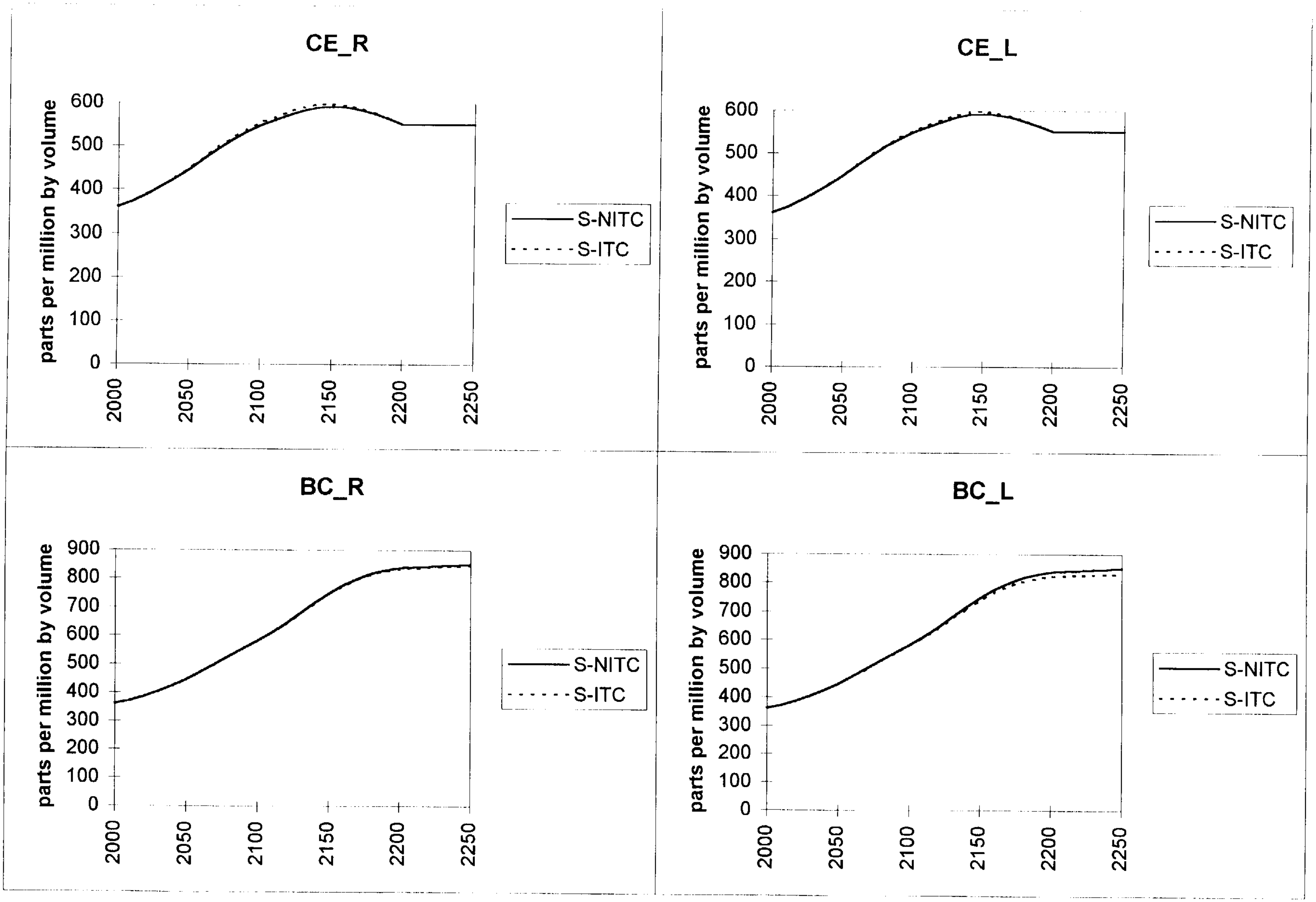
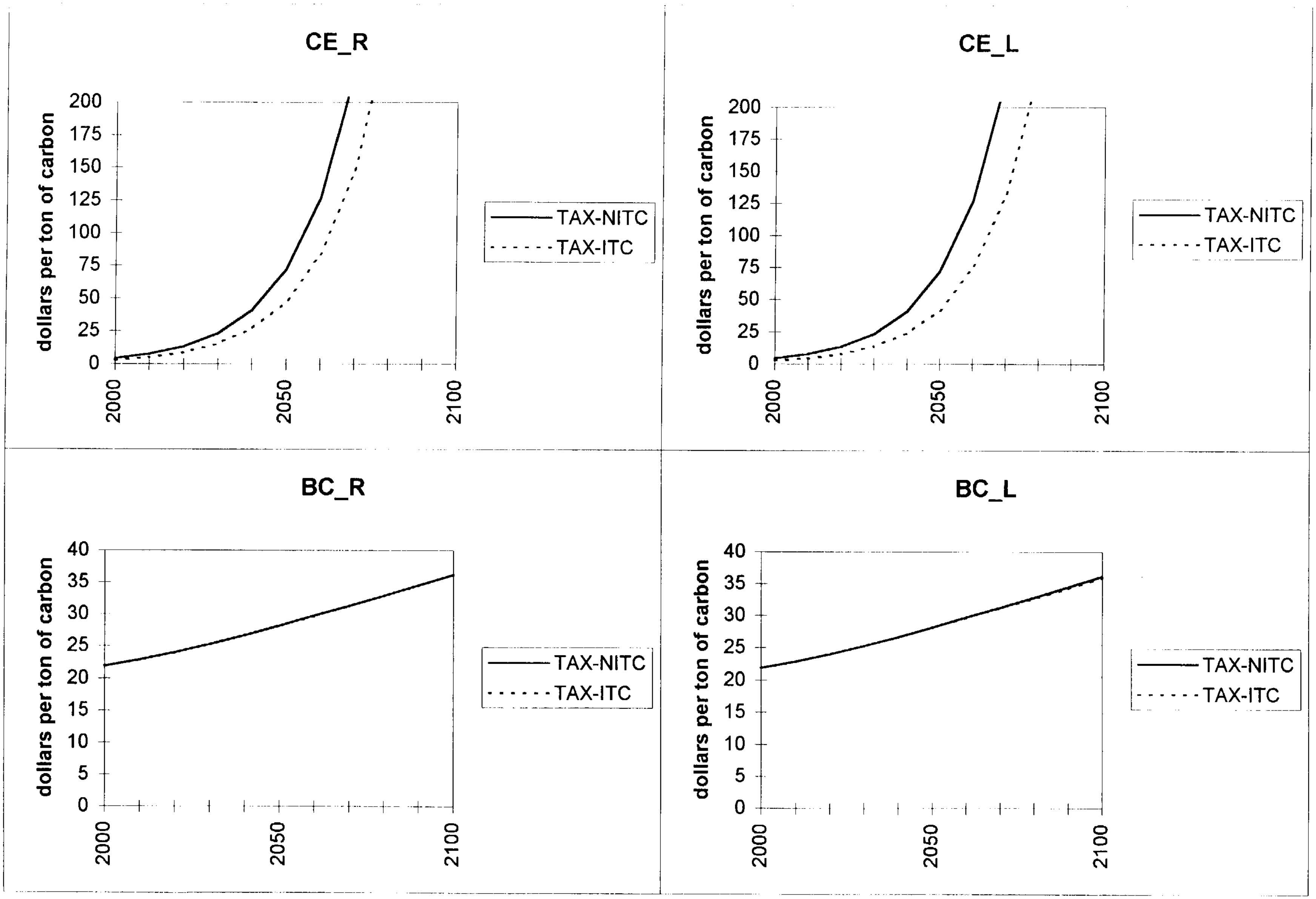


**FIGURE 4: OPTIMAL CO2 CONCENTRATION PATHS**



## FIGURE 5: OPTIMAL CARBON TAX PATHS



# TABLE 2: SENSITIVITY ANALYSIS

*(ITC-induced % changes in relevant variables under different numerical specifications)*

### COST-EFFECTIVENESS, R&D (CE_R Model)

| | Abatement | | | $CO_2$ | Total | Tax | | | Avg. |
|---|---|---|---|---|---|---|---|---|---|
| | 2000 | 2050 | 2190 | 2200 | Abate. | 2000 | 2050 | 2190 | Cost |
| 1. *Central Case* | -17.93 | -12.46 | 1.91 | 0.00 | -1.04 | -34.39 | -34.39 | -34.39 | -29.52 |
| 2a. $r = 0.075$ | -15.09 | -12.40 | 1.36 | 0.00 | -0.71 | -28.26 | -28.26 | -28.26 | -24.67 |
| 2b. $r = 0.025$ | -19.73 | -11.01 | 2.73 | 0.00 | -1.49 | -43.09 | -43.09 | -43.09 | -36.38 |
| 3a. $\bar{S} = 350$ | -16.99 | -4.37 | 0.65 | 0.00 | -0.81 | -55.06 | -55.06 | -55.06 | -50.74 |
| 3b. $\bar{S} = 837.67$ | -4.98 | -4.42 | 1.34 | 0.00 | -0.34 | -9.74 | -9.74 | -9.74 | -7.20 |
| 4a. $C = 6.19*A^4/((E-A)^2*H)$ | -11.65 | -8.50 | 1.97 | 0.00 | -1.03 | -34.42 | -34.42 | -34.42 | -29.16 |
| 4b. $C = 53*A^2/H$ | -38.17 | -32.13 | 0.00 | 0.00 | -0.35 | -38.17 | -38.17 | -38.17 | -36.23 |
| 4c. $C = 225*A^{1.5}/H$ | 0.00 | -51.60 | 0.00 | 0.00 | -0.08 | -36.66 | -36.66 | -36.66 | -36.62 |
| 5a. $M_\psi = 0.0044$ | -30.46 | -21.30 | 2.99 | 0.00 | -1.74 | -53.74 | -53.74 | -53.74 | -48.31 |
| 5b. $M_\psi = 0.0011$ | -9.13 | -6.32 | 1.04 | 0.00 | -0.54 | -18.53 | -18.53 | -18.53 | -15.28 |
| 6a. $\phi = 0.75$ | -19.24 | -13.56 | 2.19 | 0.00 | -1.16 | -36.59 | -36.59 | -36.59 | -31.23 |
| 6b. $\phi = -0.5$ | -14.42 | -9.56 | 1.28 | 0.00 | -0.76 | -28.29 | -28.29 | -28.29 | -24.67 |

### COST-EFFECTIVENESS, LBD (CE_L Model)

| | Abatement | | | $CO_2$ | Total | Tax | | | Avg. |
|---|---|---|---|---|---|---|---|---|---|
| | 2000 | 2050 | 2190 | 2200 | Abate. | 2000 | 2050 | 2190 | Cost |
| 1. *Central Case* | 9.73 | -7.58 | 1.68 | 0.00 | -0.70 | -41.62 | -41.62 | -41.62 | -39.68 |
| 2a. $r = 0.075$ | 32.09 | -4.55 | 1.09 | 0.00 | -0.46 | -37.17 | -37.17 | -37.17 | -35.69 |
| 2b. $r = 0.025$ | -3.64 | -7.93 | 2.62 | 0.00 | -1.06 | -46.32 | -46.32 | -46.32 | -43.75 |
| 3a. $\bar{S} = 350$ | -8.49 | -2.54 | 0.38 | 0.00 | -0.45 | -41.65 | -41.65 | -41.65 | -41.87 |
| 3b. $\bar{S} = 837.67$ | 56.38 | 9.97 | 2.35 | 0.00 | -0.19 | -26.59 | -26.59 | -26.59 | -23.57 |
| 4a. $C = 6.19*A^4/((E-A)^2*H)$ | -3.01 | -6.77 | 1.73 | 0.00 | -0.80 | -44.23 | -44.23 | -44.23 | -42.52 |
| 4b. $C = 53*A^2/H$ | 417 | 93.08 | 0.00 | 0.00 | 0.30 | -28.88 | -28.88 | -28.88 | -28.01 |
| 4c. $C = 225*A^{1.5}/H$ | $\infty$ | 1964 | 0.00 | 0.00 | 0.22 | -22.72 | -22.72 | -22.72 | -21.18 |
| 5a. $M_\psi = 0.0044$ | 6.50 | -13.24 | 2.63 | 0.00 | -1.20 | -60.40 | -60.40 | -60.40 | -58.36 |
| 5b. $M_\psi = 0.0011$ | 8.68 | -4.07 | 0.99 | 0.00 | -0.38 | -25.58 | -25.58 | -25.58 | -24.13 |
| 6a. $\phi = 0.75$ | 10.79 | -7.99 | 1.91 | 0.00 | -0.77 | -43.76 | -43.76 | -43.76 | -41.64 |
| 6b. $\phi = -0.5$ | 6.85 | -6.31 | 1.14 | 0.00 | -0.53 | -35.29 | -35.29 | -35.29 | -33.83 |

### BENEFIT-COST, R&D (BC_R Model)

| | Abatement | | | $CO_2$ | Total | Tax | | | Avg. | Net |
|---|---|---|---|---|---|---|---|---|---|---|
| | 2000 | 2050 | 2200 | 2200 | Abate. | 2000 | 2050 | 2200 | Cost | Ben. |
| 1. *Central Case* | -0.00 | 1.29 | 6.15 | -0.42 | 3.40 | -0.01 | -0.06 | -0.49 | -2.29 | 0.56 |
| 2a. $r = 0.075$ | -0.00 | 0.86 | 4.31 | -0.24 | 2.35 | -0.00 | -0.03 | -0.26 | -1.94 | 0.19 |
| 2b. $r = 0.025$ | -0.03 | 2.34 | 10.25 | -0.96 | 5.75 | -0.07 | -0.23 | -1.07 | -1.42 | 2.59 |
| 3a. $D = 1.17E\text{-}6*S^3$ | -0.01 | 1.15 | 5.93 | -0.42 | 3.32 | -0.02 | -0.11 | -0.98 | -2.10 | 0.65 |
| 3b. $D = 1.6*S$ | -0.00 | 1.44 | 6.40 | -0.42 | 3.49 | 0.00 | 0.00 | 0.00 | -2.44 | 0.51 |
| 4a. $C = 24.2*A^4/((E-A)^2*H)$ | -0.00 | 0.85 | 4.06 | -0.27 | 2.16 | -0.01 | -0.04 | -0.31 | -1.41 | 0.32 |
| 4b. $C = 1.315*A^2/H$ | -0.03 | 3.71 | 18.47 | -1.17 | 9.36 | -0.03 | -0.19 | -1.48 | -6.85 | 1.19 |
| 4c. $C = 2.845*A^{1.5}/H$ | -0.08 | 7.55 | 41.07 | -2.98 | 22.73 | -0.04 | -0.31 | -3.85 | -15.17 | 2.98 |
| 5a. $M_\psi = 0.0044$ | -0.01 | 3.20 | 14.94 | -1.04 | 8.35 | -0.02 | -0.15 | -1.18 | -5.34 | 1.41 |
| 5b. $M_\psi = 0.0011$ | -0.00 | 0.51 | 2.48 | -0.17 | 1.36 | -0.00 | -0.02 | -0.20 | -0.94 | 0.22 |
| 6a. $\phi = 0.75$ | -0.00 | 1.29 | 6.32 | -0.43 | 3.46 | -0.01 | -0.06 | -0.50 | -2.34 | 0.57 |
| 6b. $\phi = -0.5$ | -0.00 | 1.26 | 5.60 | -0.39 | 3.19 | -0.01 | -0.06 | -0.45 | -2.12 | 0.55 |

### BENEFIT-COST, LBD (BC_L Model)

| | Abatement | | | $CO_2$ | Total | Tax | | | Avg. | Net |
|---|---|---|---|---|---|---|---|---|---|---|
| | 2000 | 2050 | 2200 | 2200 | Abate. | 2000 | 2050 | 2200 | Cost | Ben. |
| 1. *Central Case* | 0.37 | 5.13 | 26.31 | -1.77 | 14.15 | -0.03 | -0.24 | -2.04 | -9.29 | 2.79 |
| 2a. $r = 0.075$ | 0.18 | 4.63 | 24.92 | -1.35 | 13.36 | -0.01 | -0.14 | -1.52 | -10.25 | 1.30 |
| 2b. $r = 0.025$ | 1.22 | 6.14 | 27.75 | -2.56 | 15.22 | -0.19 | -0.63 | -3.01 | -5.77 | 8.08 |
| 3a. $D = 1.17E\text{-}6*S^3$ | 0.45 | 4.80 | 24.47 | -1.73 | 13.61 | -0.07 | -0.45 | -3.99 | -8.46 | 3.29 |
| 3b. $D = 1.6*S$ | 0.32 | 5.49 | 28.27 | -1.79 | 14.63 | 0.00 | 0.00 | 0.00 | -10.00 | 2.41 |
| 4a. $C = 24.2*A^4/((E-A)^2*H)$ | 0.19 | 3.94 | 18.64 | -1.22 | 9.93 | -0.03 | -0.20 | -1.41 | -6.80 | 1.86 |
| 4b. $C = 1.315*A^2/H$ | 1.33 | 16.13 | 85.56 | -5.20 | 41.41 | -0.15 | -0.85 | -6.66 | -23.05 | 6.15 |
| 4c. $C = 2.845*A^{1.5}/H$ | 3.60 | 30.30 | 213.38 | -14.11 | 105.60 | -0.21 | -1.31 | -18.94 | -42.05 | 13.43 |
| 5a. $M_\psi = 0.0044$ | 0.73 | 10.20 | 51.23 | -3.47 | 27.84 | -0.07 | -0.48 | -3.99 | -16.33 | 5.56 |
| 5b. $M_\psi = 0.0011$ | 0.19 | 2.57 | 13.30 | -0.89 | 7.12 | -0.02 | -0.12 | -1.03 | -4.98 | 1.40 |
| 6a. $\phi = 0.75$ | 0.38 | 5.20 | 28.46 | -1.87 | 14.89 | -0.03 | -0.25 | -2.17 | -9.81 | 2.84 |
| 6b. $\phi = -0.5$ | 0.35 | 4.87 | 20.57 | -1.47 | 11.95 | -0.03 | -0.23 | -1.66 | -7.75 | 2.63 |