

THE ALLEGED INSTABILITY OF  
NOMINAL INCOME TARGETING

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The Alleged Instability of Nominal Income Targeting

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### **ABSTRACT**

Recently it has been argued that a monetary policy of nominal income targeting would result in dynamically unstable processes for output and inflation. That result holds in a theoretical model that includes backward-looking IS and Phillips curve relations, but these are rather special and theoretically unattractive. The present paper demonstrates that replacement of the special Phillips curve with one of several more plausible specifications overturns the instability result, whether or not the IS equation is replaced with a forward-looking version. Thus the instability result is quite fragile and therefore provides almost no basis for a negative judgment regarding nominal income targeting.

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## I. Introduction

Recently it has been argued by Ball (1997) that a monetary policy of nominal income targeting would be destabilizing for output and inflation, even though it succeeded in stabilizing nominal income (in either a growth-rate or growing-level fashion). A similar result has also been presented by Svensson (1997b). While the latter's discussion is quite circumspect, Ball (1997, abstract) expresses the claim in strong language: "Finally, nominal income targets are not merely inefficient, but also disastrous: they imply that output and inflation have infinite variances." This seems a rather startling proposition, given that a number of economists have previously presented both theoretical<sup>1</sup> and empirical<sup>2</sup> analyses in which nominal income targeting yields stable behavior for output and inflation. Evidently the Ball-Svensson conclusions must stem from the use of a model specification that is somewhat unusual—but the model consists merely of a dynamic IS equation and an accelerationist Phillips curve relation, both of which give the appearance of being rather orthodox.

In this case, however, appearances are misleading. Although both of the model's behavioral relations are similar in some respects to specifications that have recently been given considerable theoretical support,<sup>3</sup> they differ in subtle but crucial ways.<sup>4</sup> The purpose of the present paper, accordingly, is to determine whether the instability result is robust to minor alterations in the specification of the Ball-Svensson model.<sup>5</sup> The findings indicate that replacement of the Ball-Svensson Phillips curve with the mentioned alternative results in a model in which both output and inflation are dynamically stable under nominal income targeting, whether or not the IS relations is respecified. Furthermore, consideration of additional Phillips curve specifications indicates that the instability result is highly fragile, i.e., it obtains only in quite special—and dubious—cases.

## II. Specifications

The behavioral relations used by Ball (1997) and Svensson (1997a, 1997b) may be written as follows:

$$(1) \quad y_{t+1} = \lambda y_t - \beta r_t + \varepsilon_{t+1} \quad \beta > 0 < \lambda < 1$$

$$(2) \quad \pi_{t+1} = \pi_t + \alpha y_t + \eta_{t+1} \quad \alpha > 0$$

Here  $y_t$  is the log of output relative to some capacity measure (the output “gap”),  $r_t$  is a one-period real interest rate,  $\pi_t = \Delta p_t$  with  $p_t$  being the log of the price level, and  $\varepsilon_t, \eta_t$  are white-noise shocks. Clearly, (1) is a backward-looking relation of the IS type, in which each period’s output demand depends on its previous value and the previous period’s real interest rate. Equation (2) is a backward-looking NAIRU type of Phillips curve in which inflation is determined by its own previous value and the previous period’s output gap. The model is closed by supposing that the central bank conducts policy by manipulating  $r_t$  to achieve its objectives, whatever they be, with emphasis given to Taylor-style (1993) rules of the form

$$(3) \quad r_t = \mu_0 + \mu_1 y_t + \mu_2 \pi_t.$$

Since the central bank is assumed to observe current values of  $y_t$  and  $\pi_t$ ,<sup>6</sup> it can in effect observe current inflation expectations  $E_t \pi_{t+1}$  and thereby use its control over the nominal interest rate  $R_t = r_t + E_t \pi_{t+1}$  to set  $r_t$  at its desired level. Expectations are rational, with  $E_t z_{t+j}$  denoting the conditional expectation of  $z_{t+j}$  formed in  $t$  on the basis of full information regarding periods  $t, t-1, \dots$

The foregoing represents the Ball-Svensson model. But Roberts (1995), in a useful exposition and synthesis, has shown that currently the closest thing to a standard model of the Phillips curve is a relation of the form

$$(4) \quad \pi_t = E_t \pi_{t+1} + a y_t + u_t \quad a > 0$$

which can be thought of as representing (or closely approximating) well-known models developed by Rotemberg (1982), Calvo (1983), and Taylor (1980).<sup>7</sup> It will be noted that (4) is rather like a rearranged version of (2) with  $E_t$  attached to  $\pi_{t+1}$ , but there are two important differences. First, (4) says that  $\pi_t$  depends upon expectations of future inflation, not on the realization of past inflation. Second,  $a > 0$  in (4) would imply  $\alpha < 0$  under this arrangement.<sup>8</sup> Of course the Rotemberg, Calvo, and Taylor models have been criticized on some points, but they certainly represent more careful and explicit theoretical analysis than does equation (2).

With respect to the IS relation, recent papers by Kerr and King (1996), McCallum and Nelson (1997), and Woodford (1996), among others, have shown that an optimizing analysis in a stripped-down framework of the Sidrauski-Brock type gives rise to a relation of the form

$$(5) \quad y_t = E_t y_{t+1} - b r_t + v_t \quad b > 0.$$

The relationship of the latter to equation (1) is analogous to that of (4) to (2). And while the derivation of (5) requires some simplifying assumptions, its theoretical basis would seem to be much stronger than that of (1).

### III. Analysis

Let us consider, then, the outcome of nominal income targeting in a model in which (4) and (5) replace the comparatively ad hoc relations (1) and (2). Thus we suppose that the central bank sets  $r_t$  each period so as to make  $\Delta x_t = 0$ , where  $x_t = y_t + p_t$ , the object being to keep nominal income growth equal to a target value  $\Delta x^*$  which is, for simplicity, set to zero. Then substituting (4) and (5) into  $\pi_t + y_t - y_{t-1} = 0$  yields

$$(6) \quad E_t \pi_{t+1} + a y_t + u_t + E_t y_{t+1} - b r_t + v_t - y_{t-1} = 0$$

and solving this gives a rule for  $r_t$  that will stabilize  $\Delta x_t$  at zero. Putting that expression back into (5) yields

$$(7) \quad (1+a) y_t = y_{t-1} - E_t \pi_{t+1} - u_t.$$

Thus our task is to determine how  $y_t$  and  $\pi_t$  behave in as system consisting of (7) and (4).

In this system the relevant state variables are  $y_{t-1}$  and  $u_t$  so it is apparent that the bubble-free solution obtained via the minimal-state-variable (MSV) procedure (McCallum, 1983) will be of the form

$$(8) \quad y_t = \phi_{11} y_{t-1} + \phi_{12} u_t$$

$$(9) \quad \pi_t = \phi_{21} y_{t-1} + \phi_{22} u_t$$

in which case  $E_t \pi_{t+1} = \phi_{21}(\phi_{11} y_{t-1} + \phi_{12} u_t)$ . Substituting these into (7) and (4) yields the two relations

$$(10) \quad (1+a) (\phi_{11} y_{t-1} + \phi_{12} u_t) = y_{t-1} - \phi_{21} (\phi_{11} y_{t-1} + \phi_{12} u_t) - u_t$$

$$(11) \quad \phi_{21} y_{t-1} + \phi_{22} u_t = (\phi_{21} + a)(\phi_{11} y_{t-1} + \phi_{12} u_t) + u_t$$

Together these imply the undetermined-coefficient requirements

$$(12) \quad (1+a) \phi_{11} = 1 - \phi_{21} \phi_{11}$$

$$(1+a) \phi_{12} = -\phi_{21} \phi_{12} - 1$$

$$\phi_{21} = (\phi_{21} + a) \phi_{11}$$

$$\phi_{22} = (\phi_{21} + a) \phi_{12} + 1$$

The first and third of equations (12) give  $\phi_{11}^2 - (2+a)\phi_{11} + 1 = 0$  so the MSV value of  $\phi_{11}$  is<sup>9</sup>

$$(13) \quad \phi_{11} = (1/2) (2 + a - \sqrt{a^2 + 4a}).$$

For any  $a > 0$ , the foregoing implies  $0 < \phi_{11} < 1$  so we see from (8) that the behavior of  $y_t$  is dynamically stable. Then putting  $\phi_{11}$  back into the third of equations (12) shows that  $\phi_{21} = a\phi_{11}/(1-\phi_{11})$  is positive

and finite. Finally  $\phi_{12} = -1/(1 + a + \phi_{21}) < 0$  and  $\phi_{22} = -\phi_{12}$ . From the second of equations (8), stability of  $y_t$  implies stability of  $\pi_t$ .

#### IV. Extensions

Let us now explore the Ball-Svensson result a bit, to understand its source, before going on to other model specifications. As it happens, the result is purely a consequence of the particular Phillips curve specification (2), i.e., it is independent of aggregate demand behavior. What is relevant in that regard is merely the assumption that the central bank can--with whatever instrument it chooses!--keep nominal income equal to its target value plus a random control error. For present purposes, we suppose that the target is  $\Delta x^* = 0$  and we neglect the random control error, which is irrelevant for the issue of dynamic stability. Then let us write  $\Delta x_t = 0$  as

$$(14) \quad y_t - y_{t-1} + \pi_t = 0$$

and consider this equation together with the Ball-Svensson Phillips relation, which we write now as

$$(2') \quad \pi_t = \pi_{t-1} + \alpha y_{t-1} + \eta_t.$$

Combining these we easily obtain

$$(15) \quad y_t - (2-\alpha)y_{t-1} + y_{t-2} = \eta_t.$$

But it is well-known that for stability of a second-order linear difference equation, one necessary condition is that the coefficient on  $y_{t-2}$  must be less than 1.0 in absolute value—see, e.g., Sargent (1987, p. 189). So with relation (2), stability of  $y_t$  cannot prevail if nominal income targeting is successful. That is Ball's result.

But relation (2') is, to reiterate, quite special and rather problematic. First note that if we attach a coefficient  $\gamma$  to the lagged inflation term, then the counterpart of (15) would have  $\gamma$  attached to  $y_{t-2}$ . So if  $\gamma$  were 0.99, rather than 1.00, stability would prevail. This observation is relevant



because typical Phillips-curve specifications include  $E_{t-1}\pi_t$  rather than  $\pi_{t-1}$ , which we will do momentarily. Another common difference relative to (2') is that the role of output involves  $y_t$ , or  $y_t$  and  $y_{t-1}$ , instead of  $y_{t-1}$  alone. Suppose, then that instead of (2') we adopt the relation

$$(16) \quad \pi_t = \pi_{t-1} + \alpha_1 y_t + \alpha_2 y_{t-1} + \eta_t$$

where  $\alpha_1 > 0$  and  $|\alpha_2| < \alpha_1$ . Then combining (16) with (14) gives

$$(17) \quad y_t - [(2-\alpha_2)/(1+\alpha_1)]y_{t-1} + [1/(1+\alpha_1)]y_{t-2} = \eta_t,$$

in which all the conditions sufficient for stability are satisfied.<sup>10</sup>

Continuing with our exploration, let us next, as promised above, replace  $\pi_{t-1}$  in (16) with  $E_{t-1}\pi_t$ . This results in a specification like that of Lucas (1973), one which is shown by Svensson (1996) also to represent a generalization of the P-bar model promoted by McCallum (1994).<sup>11</sup> Then instead of (16) we have

$$(18) \quad \pi_t = E_{t-1} \pi_t + \alpha_1 y_t + \alpha_2 y_{t-1} + \eta_t.$$

But since  $\pi_t - E_{t-1} \pi_t$  is a linear combination of  $\eta_t$  and any control error in  $\Delta x_t$ , (18) immediately implies

$$(19) \quad y_t = (-\alpha_2/\alpha_1) y_{t-1} + \text{white noise},$$

which with  $|\alpha_2| < \alpha_1$  is unambiguously stable.

Next, in order to include one Phillips curve specification in which  $p_t$  is entirely predetermined (i.e.,  $p_t = E_{t-1} p_t$ ), let us consider the basic P-bar model without Svensson's simplified specification of aggregate demand or his inclusion of within-period effects on  $p_t$ .<sup>12</sup> Thus we assume that

$$(20) \quad p_t - p_{t-1} = (1 - \psi) (\bar{p}_{t-1} - p_{t-1}) + E_{t-1} (\bar{p}_t - \bar{p}_{t-1})$$

with  $0 < \psi < 1$ , where  $\bar{p}_t$  is the value of  $p_t$  that would prevail in  $t$  if there were no nominal stickiness.

In a log-linear model we have  $p_t - \bar{p}_t = -\theta y_t$  under the Ball-Svensson convention that  $y_t$  is the

relevant gap measure of output. (Of course  $\theta > 0$ .) In this case we can rearrange (20) as  $p_t - E_{t-1} \bar{p}_t =$

$\psi(p_{t-1} - \bar{p}_{t-1})$  and infer that

$$(21) \quad E_{t-1} y_t = \psi y_{t-1}.$$

But the latter implies that  $y_t$  is stable and with policy designed to yield  $\pi_t + y_t - y_{t-1} = 0$ ,  $\pi_t = (1 - \psi)y_{t-1}$  is also stable.<sup>13</sup>

Finally, mention should be made of the Fuhrer-Moore (1995) Phillips curve specification, which has not been given much in the way of theoretical support but which performs nicely from an empirical perspective. Analysis of its implications takes a bit more space than is appropriate here, but an adaptation of the result in Section VI of McCallum and Nelson (1997) indicates that stability of  $y_t$  and  $\pi_t$  prevail in a model that includes (5) and the Fuhrer-Moore relation when policy is designed to keep  $\Delta x_t$  constant.

## V. Level Targets

As in Ball (1997), we can also consider nominal income targeting in which the level rather than the growth rate is controlled. In place of  $\Delta x_t = \Delta x^*$ , we would have the objective  $x_t = x_t^*$ , with  $x_t^* = x_{t-1}^* + \Delta x^*$ . Letting  $\Delta x^* = 0$  as before, we have as the counterpart of (14) the relation

$$(22) \quad y_t + p_t = x^*.$$

Again we rewrite the Ball-Svensson Phillips curve, this time as

$$(2'') \quad p_t = p_{t-1} + p_{t-1} - p_{t-2} + \alpha y_{t-1} + \eta_t.$$

Combining (2'') and (20) yields

$$(23) \quad p_t - (2-\alpha)p_{t-1} + p_{t-2} = \alpha x^* + \eta_t.$$

Just as in the case of (15), this implies instability of  $p_t$  – and therefore, from (22), of  $y_t$ .

But suppose we replace (2'') with (18). Then we have  $p_t - E_{t-1} p_t$  in place of  $\pi_t - E_{t-1} \pi_t$  in (18), and results analogous to (19) obtain again. Using instead (16) with (22) we have

$$(24) \quad p_t - [(2-\alpha_2)/(1+\alpha_1)] p_{t-1} + [1/(1+\alpha_1)] p_{t-2} = (\alpha_1 + \alpha_2)x^* + \eta_t$$

and stability holds as with (17).

No new proof is needed for the P-bar case so, finally, let us consider the forward-looking specification (4), which we rewrite as

$$(25) \quad p_t - p_{t-1} = E_t p_{t+1} - p_t + ay_t + \eta_t$$

Using (22) this gives

$$(26) \quad (2 + a)p_t = p_{t-1} + E_t p_{t+1} + \alpha x^* + \eta_t$$

The MSV solution will be

$$(27) \quad p_t = \phi_0 + \phi_1 p_{t-1} + \phi_2 \eta_t$$

so  $E_t p_{t+1} = \phi_0 + \phi_1 (\phi_0 + \phi_1 p_{t-1} + \phi_2 \eta_t)$  and substitution into (25) gives rise to undetermined-coefficient relations (analogous to (12)) including

$$(28) \quad \phi_1^2 - (2 - a) \phi_1 + 1 = 0$$

The MSV solution for  $\phi_1$  is exactly the same as for  $\phi_{11}$  in (13), so  $0 < \phi_1 < 1$  and again stability prevails.<sup>14</sup>

## VI. Conclusion

What the foregoing demonstrates is that the Ball-Svensson instability result stems principally from the Phillips curve specification used in their analyses. That specification is neither standard nor theoretically attractive, and its replacement with a number of more attractive alternatives leads to a

reversal of the finding that nominal income targeting results in non-stationary processes for output and inflation.

This demonstration does not establish, of course, that nominal income targeting is preferable to inflation targeting or to other rules for monetary policy. To reach such a conclusion would require an extensive combination of theoretical and empirical analyses, conducted in a manner that gives due emphasis to the principle of robustness to model specification, plus attention to concerns involving policy transparency and communication with the public.<sup>15</sup> The point of the present paper is not to attempt any such ambitious undertaking, but merely to make it clear that there is very little basis for a negative judgment regarding nominal income targeting provided by the fragile results announced by Ball (1997).

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## Endnotes

- <sup>1</sup> See, for example, Bean (1983), Frankel and Chinn (1995), and Ratti (1997).
- <sup>2</sup> See, for example, Taylor (1985), McCallum (1990), Bryant, Hooper, and Mann (1993), Feldstein and Stock (1994), and Hall and Mankiw (1994).
- <sup>3</sup> This is recognized by Svensson (1997b, pp. 21-23) but not Ball (1997). References are provided below.
- <sup>4</sup> A major difference is that the Ball-Svensson specification is “backward looking” whereas the theoretically cleaner alternative relations are “forward looking.” Svensson (1997b, pp. 22-23) also discusses an intermediate specification, mentioned below in footnote 11.
- <sup>5</sup> Svensson (1997b, p. 21) expresses some concern regarding the possibility of nonrobustness.
- <sup>6</sup> This assumption is in my opinion quite unrealistic; actual central banks must set nominal rates  $R_t$  in response to data from  $t-1$  and earlier. That fact increases the likelihood of instrument instability, which in the case at hand would mean instability of nominal income. But this is an entirely different kind of concern, one that is not considered in the Ball or Svensson analyses, so we shall abstract from it here. For a brief discussion, see McCallum (1997, p. 39).
- <sup>7</sup> In (4),  $u_t$  is a stochastic behavioral shock that we take to be white noise for simplicity and for comparability with the Ball and Svensson analyses. The same statement applies to the  $v_t$  term in (5) below. In realistic models these shocks are probably serially correlated, and their properties may be important for comparisons of nominal income targeting to other schemes. But the assumption of white noise is acceptable for the issue of dynamic stability, which is the only concern in the present paper.



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<sup>8</sup> Roberts (1995) presents econometric evidence for the United States that is supportive of the hypothesis  $a > 0$ .

<sup>9</sup> Here we know that it is  $-\sqrt{a^2 + 4a}$  that is relevant by the MSV criterion developed in McCallum (1983, pp. 146-7). The more common dynamic stability criterion of (e.g.) Blanchard and Kahn (1980) would give the same result but seems somewhat less appropriate given the issue at hand.

<sup>10</sup> Using  $c_1$  and  $c_2$  to denote the coefficients on  $y_{t-1}$  and  $y_{t-2}$  in (17), the conditions are  $1 + c_1 + c_2 > 0$ ,  $1 - c_1 + c_2 > 0$ , and  $1 - c_2 > 0$ . In the case at hand, we multiply by  $1 + \alpha_1$  and then the first of these is  $(1 + \alpha_1) - (2 + \alpha_2) + 1 = \alpha_1 + \alpha_2 > 0$ , the second is  $(1 + \alpha_1) + (2 - \alpha_2) + 1 = 4 + \alpha_1 - \alpha_2 > 0$ , and the third is  $1 + \alpha_1 - 1 = \alpha_1 > 0$ .

<sup>11</sup> Svensson's (1996) equation (A.1) represents a generalization in the sense that current-period conditions are permitted to modify the price—totally predetermined—given by the P-bar model. This result requires, however, a particularly simple specification of aggregate demand.

<sup>12</sup> The P-bar model, incidentally, has the theoretically attractive property of satisfying the strict version of the natural rate hypothesis (McCallum, 1994, pp. 259-261). Of the seven Phillips curve specifications considered here or in Section III, only it and the Lucas (1973) model—i.e., (16) but with  $E_{t-1}\pi_t$  replacing  $\pi_{t-1}$ —have that property.

<sup>13</sup> In realistic applications, the P-bar model implies that  $y_t$  differs from  $E_{t-1} y_t$ , but the Ball-Svensson assumption that current  $y_t$  is observed by the central bank when setting  $r_t$  implies that the  $v_t$  shock in (5) is precisely offset.

<sup>14</sup> Another specification mentioned by Svensson (1997b, p. 22) is (4) but with  $E_t\pi_{t+1}$  replaced by a weighted average of  $E_t\pi_{t+1}$  and  $\pi_{t-1}$ . Svensson then modifies that by replacing the right-hand-side terms by their expectations from two periods before. I can see little justification for this last step, but

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the previous specification is one that might merit consideration. Its analysis is more difficult than for the specifications considered in Section IV.

<sup>15</sup> Some of the difficulties of such a study are discussed in McCallum (1997).