DISCRETENESS AND THE WELFARE COST OF LABOR SUPPLY TAX DISTORTIONS

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Discreteness and the Welfare Cost of Labor
Supply Tax Distortions
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ABSTRACT

We discuss the role played by discrete labor supply (leisure consumption) choice in affecting measures of the welfare cost of labor supply tax distortions. We construct comparable continuous and discrete choice models, each calibrated to have similar aggregate (uncompensated) labor supply elasticities. In the former, there is a single representative consumer; in the latter there is a distribution of individuals across preference parameters. In the discrete model, taxes induce a large response from a subset of the population, while the majority of the population shows unchanged behavior. Welfare costs of similar taxes in continuous models can substantially exceed those in discrete models or vice versa, depending upon the formulation used. Experiments are also reported for a two labor type household model with one continuous variable (secondary labor) and one discrete variable (primary labor), and calculations are also made using an empirically based model specification calibrated to UK data. Model results clearly show that discrete choice matters in the assessment of the cost of labor supply tax distortions.

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1. INTRODUCTION

The influence that discrete choice in labour supply decisions can have both on the estimation of labour supply elasticities and on the ability of real business cycle models to calibrate to time series on output and employment has been investigated in labour- econometric (Killingsworth (1983)) and real business cycle literature (Hansen (1987)). However, public finance literature has not analyzed how discreteness can influence estimates of the welfare costs of tax distortions of labour supply.[2] In part, this is because the significance of discrete choice in labour supply for welfare cost estimates is an issue that public finance theorists would not be drawn to, since the issues are quantitative rather than qualitative. But the intuition is that it clearly matters, since in a heterogeneous world, tax induced adjustments will occur discretely for a small number of agents, rather than continuously for all.

Here we use numerical simulation techniques to explore the significance of discrete choice for calculations of the welfare cost of tax distortions of labour supply by constructing observationally equivalent discrete and continuous choice labour supply models.[3] The discrete analogue models we use embody varying forms of agent heterogeneity (over share parameters in preferences, substitution elasticities, endowments), while maintaining equivalence to identical single agent continuous models through similar model generated aggregate uncompensated labour supply elasticities. Issues arise as to how discrete choice is modelled in the with tax case, and we present two alternative formulations. In one, tax revenues are returned

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[2]There seems to be no discussion of this in existing surveys of taxation and labour supply, including Blundell (1992), and the well known earlier pieces on taxes and labour supply by Rosen (1980), Hausman (1984), and MaCurdy, Green, and Paarsch (1990).

[3]See the discussion of the related but different issue of non linear budget constraints and the welfare cost of taxes in Preston and Walker (1992).

to those who pay the tax; no interagent redistribution occurs. In the other, households have proportional claims on revenues generated by the tax, and redistribution occurs. The discrete analogue forms we use here both asymptotically approach the continuous case as the grid over which discrete choice occurs becomes everywhere dense.

Results from these exercises are striking. In simple numerical examples using under our first discrete formulation (taxes recycled to those who pay the tax), the discrete choice model produces sharply lower welfare costs of taxes if model calibration is made to the same uncompensated labour supply elasticities in both discrete and continuous models. Welfare costs of similar ad valorem labour supply tax distortions differ between models by factors of around 5, being sharply lower in the discrete formulation. On the other hand, under our second discrete choice formulation, which maintains fixed shares of households in tax revenues, the discrete choice model produces sharply higher welfare cost estimates. This is because households who switch from high to low labour supply inflict a fiscal externality on those remaining in their original state, since revenues fall. For an individual close to indifferent between high and low discrete labour supply states, in switching to the low labour supply state they experience little or no gain, while all other individuals collectively experience a loss represented by the tax revenue forgone. The factors of proportionality in results between discrete and continuous model in these examples change as the form of discreteness changes (step size across uniform parameter distributions, number of individuals, model parameters used for the population distributions), but not markedly.

We also consider cases where there is discrete choice for one household member (the primary worker) and continuity of choice for the other (the secondary worker); and consider household rather than individual optimization. Differences in results between mixed discrete-continuous choice models are smaller, but other insights also emerge. These include the influence of discreteness of choice for one household member over the labour supply behaviour of the other, with implications for the literature on labour supply elasticities for primary and secondary workers. We conclude by investigating differences in model results empirically using UK data and literature based parameter estimates instead of numerical examples. Results indicate slightly smaller but still significant factors of proportionality to those which occur in the numerical examples. We conclude that discreteness in modelling labour supply behaviour matters not only for labour supply estimation, but also for the calculation of the welfare costs of labour supply taxation. The formulation chosen also critically affects whether estimates are raised or lowered.

2. DISCRETE AND CONTINUOUS MODELS OF THE WELFARE COST OF LABOUR SUPPLY TAX DISTORTIONS

As we note in our introduction above, the significance of discreteness (and non-linear budget constraints) for the estimation of labour supply elasticities is widely acknowledged in the literature (see Killingsworth (1983) and Hausman (1984)), but issues raised by discreteness in labour supply are little discussed when calculating the welfare costs of taxation. The closest to our discussion here that we are able to find is by Blinder and Rosen (1985) who analyze the consequences of discontinuities in welfare and related benefit programs. They do not, however, use the analytic formulations of the welfare costs of taxes that we suggest here, nor offer the observation that discreteness can substantially affect welfare cost estimates.

We begin our discussion with the representative single agent continuous case, in which labour supply behaviour in the presence of a labour income tax is characterized by the solution to an optimization problem

$$\max U(C, L) \tag{1}$$

$$s.t.\ PC = w(1-t)(\overline{L} - L) + R \tag{2}$$

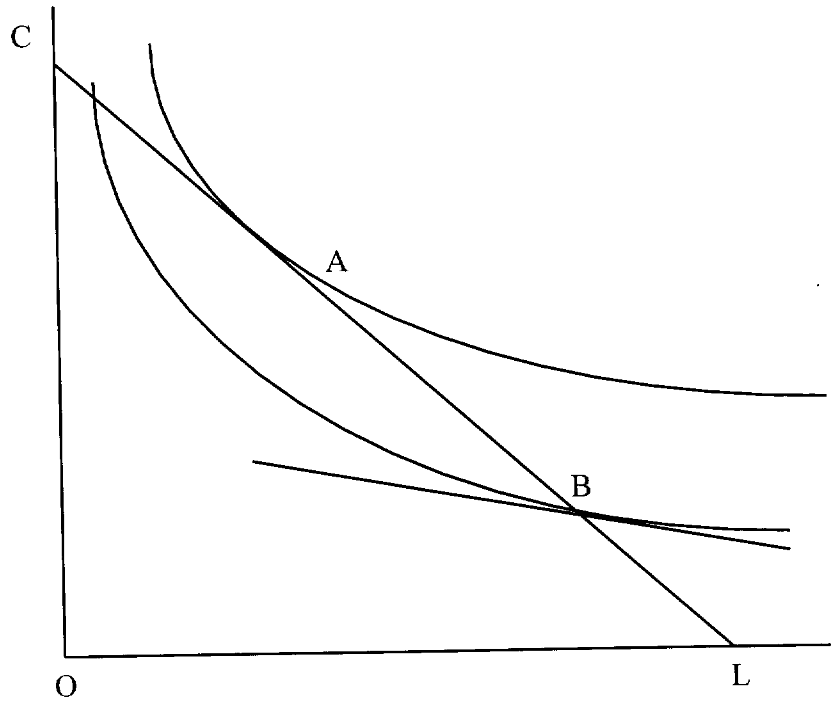
$$t.\, w.\, (\overline{L} - L) = R \tag{3}$$

where C is consumption of goods, L is leisure, $\overline{L}$ is the labour endowment, w is the wage rate, P is the price of the consumption good, t is the labour income tax rate, and R is tax revenue.

With revenues recycled in lump sum form, the effect of a labour income tax is to change the slope of the budget constraint, and hence change labour supply behaviour. This is shown in figure 1, where A is the tangency point in the no tax case, and B represents a tangency to a net of tax budget constraint with revenues recycled so as that the consumer remains on the original no-tax constraint. The social cost of the labour income tax distortion is reflected in reduced welfare at point B relative to that achieved at point A.

Figure 1

The Effects of an Income Tax on Labour Supply: Continuous Case



However, to generate an outcome at B in the presence of taxes, using the equation representation of the model (equations (1), (2) and (3)) displayed in Figure 1, some additional structure is needed. This is because if (1) is maximised subject to (2) and (3), substituting (3) into (2) simply returns the solution (even in the presence of taxes) to the no tax solution, A. Something else is needed to generate the with tax solution at point B.

One way to proceed is to add a tax distorted first order condition

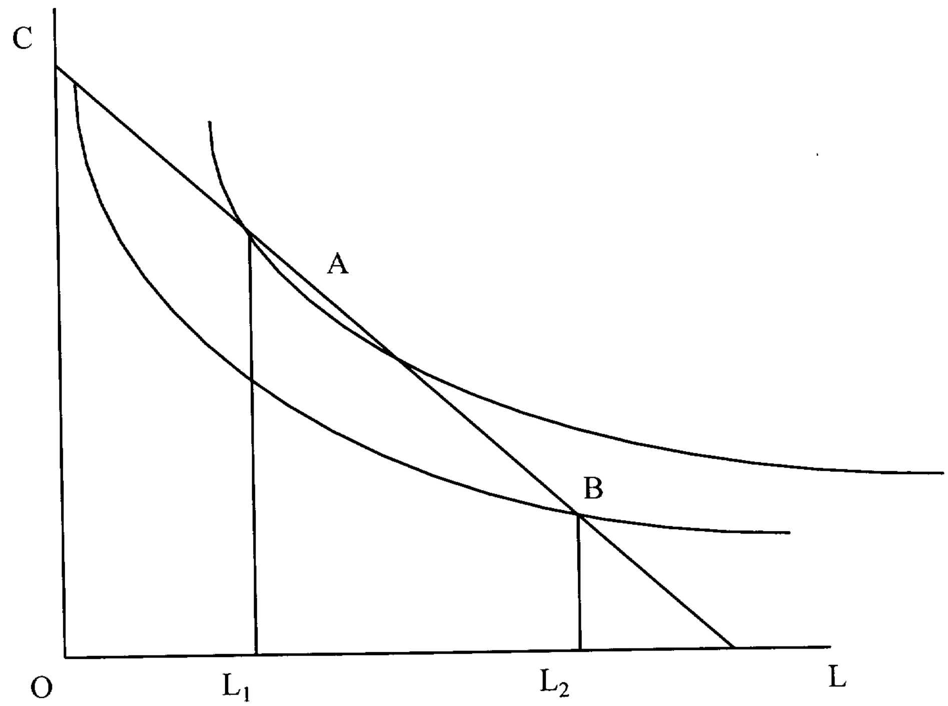
$$\frac{U_c}{U_L} = \frac{P}{w(1-t)} \tag{4}$$

as an additional constraint to the optimization problem (1), (2), and (3). This forces the solution in the presence of taxes to move from point A to point B. Another way of proceeding is to treat R in (2) as parametric, and require it to be consistent with its value in (3), rather than directly substituting between the equations. This again forces the slope of the with tax budget constraint to be $\frac{P}{w(1-t)}$, and optimizing behaviour in the presence of taxes to be at point B.

With tax versions of the discrete choice labour supply model are, however, more difficult to formulate than in the continuous case, since at neither point A nor B in figure 2 is the slope of the indifference surface equal to that of the net of tax budget constraint, and no first order condition similar to (4) holds. Because of this, we use two alternative discrete versions of the with tax model, each of which reverts to the analogous continuous case as the discreteness becomes small. They differ in their treatment of tax revenues. In one, taxes are returned in lump sum form to those who pay the tax (the traditional public finance treatment). But because this formulation includes choices involving infeasible off budget points, we also use an alternative formulation in which, given tax rates, tax revenues are distributed between households in fixed proportion lump sum form. This avoids any need to consider infeasible points in the choice set, but can result in significant redistribution between households in the event of a tax change, depending upon whether or not individual households respond by switching between discrete labour supply values.

Figure 2

Optimizing Labour Supply Behaviour, No Taxes: Discrete Case



The no tax discrete case is shown in Figure 2. For simplicity, two values of leisure consumption (labour supply), L_1 and L_2 are taken to be admissible. Restricting labour supply choices to discrete values in this way may be thought of as reflecting the technological requirement that all workers must supply labour time together because they are members of a team; and/or the feature that workers (members of families) jointly consume leisure at the same time (such as weekends or holidays).[4] Optimal behaviour with two point discrete labour supply (or leisure consumption) then involves a simple comparison of welfare levels across the two

[4]See also the discussion of fixed costs and labour supply in Cogan (1981), and hours restrictions and labour supply in Dickens and Lundberg (1993), Tummers and Woittiez (1991), and Khan and Lang (1991).

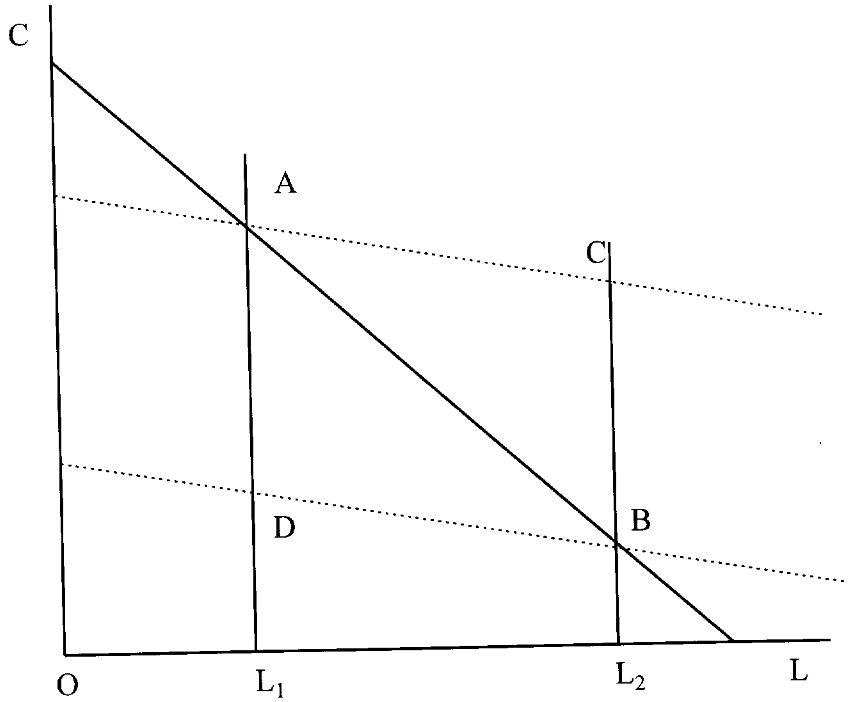
points on the budget constraint corresponding to the values L_1 and L_2. Optimising behaviour yields an outcome at point A in Figure 2, with labour supply L_1.

The first of our with tax formulations is illustrated in Figure 3. Two discrete values of labour supply, L_1 and L_2, are again assumed. In the presence of a labour income tax at rate *t*, we construct with tax budget constraints through points A and B reflecting the common public finance treatment that taxes are returned in lump-sum form to those who pay the tax. The implication of these two additional budget constraints in this case is that in the presence of taxes consumers attempt to move along a with tax budget constraint with slopes given by net of tax rather than gross of tax prices, comparing utility from their current goods-leisure consumption bundle to that obtained where the discrete variable takes its other value on the constructed net of tax budget constraint.

These comparison points are, however, not sustainable if consumers try to move to them because only points on the original (full) budget constraint can be supported in equilibrium. Thus if consumers try to move along the constructed with tax budget line through their current no tax consumption point, they will in practice move to the other discrete co-ordinate on the full budget constraint. Hence, for consumers initially at point A when a tax is applied, revenues raised at A are taken as given and returned to consumers at A, and utility is compared between points A and C on the constructed with tax budget line through A. If point C is welfare preferred to A, consumers at A try to move to C, revenues change, and movement to point B is what actually takes place. The opposite movement from B to A cannot occur due to a tax, since if a household is originally at point B, B must dominate A in welfare terms, and D is dominated by A. Only switches from A to B need be considered.

Figure 3

First Discrete Labour Supply Model; with Taxes and Revenue Recycled to Those Who Pay Taxes



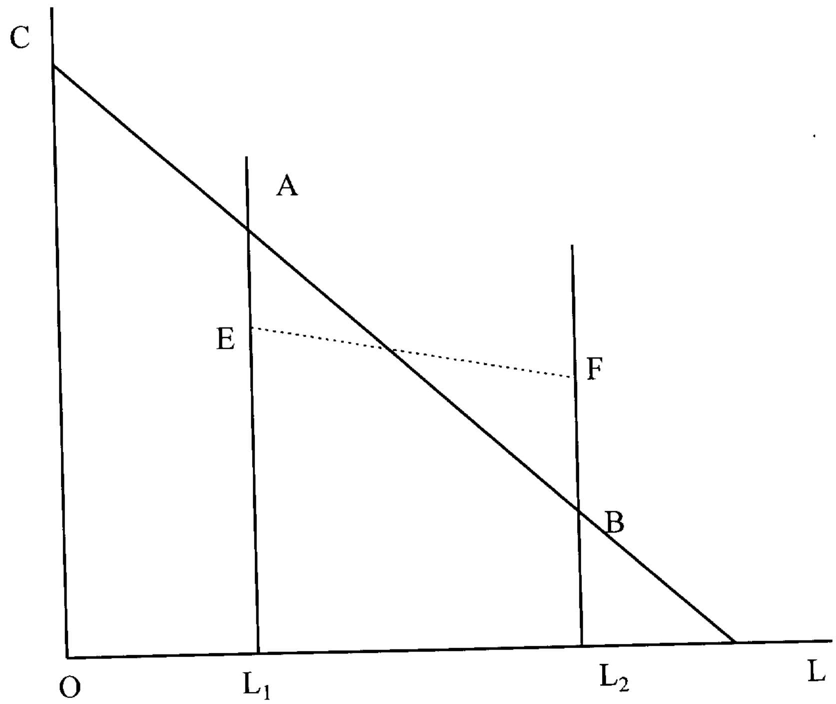
In the presence of discrete choice, using this formulation a tax on labour income can result in a move to a welfare inferior situation given by point B (i.e. a welfare loss from taxes results), in a manner similar to that which occurs in the continuous case. Without the use of such a construct to analyze the effects of taxes, the same difficulty would arise as in the continuous case. A comparison between A and B would be all that could be made, and one would dominate the other under all tax rates. Nothing in the structure would allow taxes to distort behaviour, and move consumption to an inferior point. The difference in the discrete case compared to the continuous case is that it is not possible to use tax distorted first order conditions to resolve the difficulty.

The second discrete formulation we use and is shown in Figure 4 involves a direct comparison of two points (E, and F) on an actual (rather than hypothetical) net of tax budget constraint. This removes the need for the comparison to hypothetical off budget comparison points used in the first formulation, but redistribution between households now enters the analysis. Under this formulation, taxes collected are not recycled to those paying the tax, but are instead redistributed on a fixed share basis. Taxes thus redistribute between those individuals who remain at their initial point (A or B) and those who reduce their labour supply (increase leisure consumption) and hence reduced their taxes paid.

In equilibrium under this formulation, individuals choose between E and F based on an assumed tax revenue distribution, and their choices between low and high level leisure consumption (labour supply) yields revenues consistent with those assumed. In effect, the slope of the budget constraint EF reflects tax rates, and its position reflects revenues raised, which, in turn, must be consistent with household utility rankings across E and F. An equilibrium condition then ensures government budget balance. In this formulation, introducing taxes induces some households to move from A to F; those who remain at either A or B experience a welfare loss due to the redistribution of lowered tax revenues. In deciding whether to move, individuals do not take into account the fiscal externality (reduced tax revenues) inflicted on others, and if the discrete choice is large (ie. the distance between L_1 and L_2) this effect can be pronounced.

Figure 4

Discrete Labour Supply Model with Tax Revenues Redistributed on an Equal Per Capita Basis



As the grid of points defining discrete labour supply options becomes everywhere dense, the outcome of each discrete model will asymptotically approach that of the continuous case for both of these formulations. Also, with multiple discrete points movement through the grid can be computed by a sequence of pair wise comparisons across points in the grid. The first formulation has the disadvantage that household utility comparisons involve off budget points, and switching moves households to a non-comparison point (B). It is, however, consistent with the usual public finance treatment that tax revenues are returned to those who pay the tax. The second formulation involves comparison across points on a true budget constraint, but significant redistribution between consumers can accompany the introduction of labour income taxes.

Also, in reality more continuity in labour supply exists in practice than in either of these two formulations, due to overtime, part-time work for secondary members of the household, and other margins of choice.[5] However, evidence that workers are discretely constrained in their labour supply behaviour is extensive. A recent piece by Stewart and Swaffield (1996) reports UK panel data in which 36 percent of male workers interviewed expressed a desire to work less than they currently did at the prevailing wage. Also, and as seems widely agreed, the conventional labour supply model with continuous choice and pay on an hourly basis does not apply to labour supply in the professions and public service, where the discrete choice is the participation decision rather than the number of paid hours.

[5]See also the discussion in Rosen (1976).

3. NUMERICAL EXAMPLE OF A DISCRETE LABOUR SUPPLY MODEL AND A COMPARISON TO A CONTINUOUS ANALOGUE MODEL

In this section, we present algebraic formulations of the discrete choice models in the presence of taxes discussed above in more detail, and explore their behaviour compared to analogue continuous models using simple numerical examples. For space reasons full algebraic detail on the second formulation is not provided, but its results are discussed. All models are solved using GAMS optimization code.[6]

We assume CES preferences, a linear technology, and, for now, N single worker households (this assumption is relaxed in the next section). The time endowment of each household is taken to be 70 hours per week. We calibrate each parametric specification of a discrete labour supply model so that the aggregate uncompensated labour supply elasticity is the same as that in an analogue continuous model, along with the base case aggregate wage bill. To both implement this calibration procedure and to generate approximately continuous aggregate behaviour from the discrete model, we assume that there are a number of heterogeneous households who follow a uniform distribution over some range of either share or elasticity parameters in preferences. The response to a given tax change in the aggregate is thus numerically close to being continuous in the discrete models we use.

Under the first formulation, given CES preferences, the utility evaluations for each household, h, of points A, B, C, and D in figure 3 are given by:

$$U_A^h = \left[\delta^h (C_{S,FC}^h)^{\frac{\sigma^h - 1}{\sigma^h}} + (1 - \delta^h)(L_S^h)^{\frac{\sigma^h - 1}{\sigma^h}} \right]^{\frac{\sigma^h}{\sigma^h - 1}} \qquad (5)$$

[6]GAMS denotes the Generalized Algebraic Modelling System (see Brooke, Kendrick, and Meeraus (1988)).

$$U_B^h = \left[\delta^h (C_{B,FC}^h)^{\frac{\sigma^h - 1}{\sigma^h}} + (1 - \delta^h)(L_B^h)^{\frac{\sigma^h - 1}{\sigma^h}} \right]^{\frac{\sigma^h}{\sigma^h - 1}} \tag{6}$$

$$U_C^h = \left[\delta^h (C_{S,OC}^h)^{\frac{\sigma^h - 1}{\sigma^h}} + (1 - \delta^h)(L_S^h)^{\frac{\sigma^h - 1}{\sigma^h}} \right]^{\frac{\sigma^h}{\sigma^h - 1}} \tag{7}$$

$$U_D^h = \left[\delta^h (C_{B,OC}^h)^{\frac{\sigma^h - 1}{\sigma^h}} + (1 - \delta^h)(L_B^h)^{\frac{\sigma^h - 1}{\sigma^h}} \right]^{\frac{\sigma^h}{\sigma^h - 1}} \tag{8}$$

where superscript h refers to household h, and the subscripts B (big) and S (small) refer to high and low (discrete) leisure consumption (labour supply) values, and C and L refer to goods and leisure. OC and FC refer to off budget constraint (C, D) and on budget constraint points (A,B). δ^h are preference share parameters, and σ^h are preference substitution parameters. U_A^h, U_B^h, U_C^h, and U_D^h refer to the utility evaluations by household h at points A, B, C, D in figure 3.

To determine whether a household, h, will change its goods-leisure allocation across the admissible discrete leisure (labour supply) values in response to a tax, we use the ratio criteria UR^h.

$$UR^h = \frac{U_C^h}{U_A^h} \tag{9}$$

This ratio involves comparing a point (A) on the full budget constraint for household h, to a point (C) off the budget constraint with constant income at tax distorted prices and evaluated at the high discrete leisure value. As noted earlier, point D is welfare dominated by point A. If a household is initially at point B (the no tax case) B must also dominate A; thus, a tax cannot induce a move to point D from point B.

We consider both sales taxes on C at rate t, and labour income taxes at rate t_l, both of which, in this model, have the effect of distorting labour supply decisions, and compounding with each other. Assuming a linear transformation frontier, so that the gross of tax wage and net of tax consumer prices are both one, consumption at the high leisure point off the full budget constraint in the presence of taxes is

$$C^h_{B,OC} = C^h_{S,FC} + (L^h_S - L^h_B)\left(\frac{1-t_1}{1+t}\right) \tag{10}$$

Using the utility ratio comparison above, total consumption summed across households (given the linear technology) is

$$C = \sum_{h=1}^{N}(70 - L^h_S) \text{ if } (U^h_A \geq U^h_B \text{ and } U^h_c \leq U^h_A) + \sum_{h=1}^{N}(70 - L^h_B) (\text{if } U^h_B \geq U^h_A \text{ or } U^h_C \geq U^h_A) \tag{11}$$

and tax revenues, R, (given that p = w = 1) are

$$R = \sum_{h=1}^{N}[(p\,t + t_1 w)(70 - L^h_S)] \text{ if } (U^h_A \geq U^h_B \text{ and } U^h_C \leq U^h_A) + \sum_{h=1}^{N}[(p\,t + t_1 w)(70 - L^h_B)] \text{ if } (U^h_B \geq U^h_A \text{ or } U^h_C \geq U^h_A) \tag{12}$$

If UR_h is greater than 1, the household wishes to move from A to C (Low to High leisure consumption) in the presence of the tax, actually moving to point B.

We construct an index by household, S^h, which denotes whether or not household h is induced to discretely change its labour supply behaviour as a result of any given tax i.e. where

$$S^h = 1 \text{ if } (U^h_A \geq U^h_B) \text{ and } (U^h_C \geq U^h_A)\text{; and} = 0 \text{ otherwise.} \tag{13}$$

The total number of households who change their behaviour as a result of a tax change is then given as

$$T = \sum_{h=1}^{N} S^h, \tag{14}$$

and the aggregate labour supply of the economy (given unit prices, wage rates, and constant marginal product of labour) must be consistent with household consumption decisions, i.e.

$$LS = \sum_{h=1}^{N} C^h \ . \tag{15}$$

If the discrete values L_B^h and L_S^h are common to all households (and denoted by L_B and L_S), the change in aggregate labour supply as result of any tax is

$$\Delta LS = (L_B - L_S)T \ . \tag{16}$$

In the first of our discrete formulations, household welfare only changes when taxes induce households to alter their choice among the discrete labour supply values. We therefore restrict aggregate measures of the welfare cost of taxes to these households in the discrete case, and calculate their Hicksian equivalent and compensating variations. In the linear homogenous utility case, which corresponds to the CES functions we use above, these are given by[7]

$$EV^h = \left(\frac{U_B^h - U_A^h}{U_A^h} \right) I_n^h \quad \text{for those households, h, who switch} \tag{17}$$

and

$$CV^h = \left(\frac{U_A^h - U_B^h}{U_B^h} \right) I_W^h \quad \text{for those households, h, who switch} \tag{18}$$

and, for non switching households EV^h and CV^h are zero.

We can express these sums as a fraction (or percentage) of economy wide income to yield aggregate measures of the welfare cost of taxes comparable to those used elsewhere in the literature,

[7]See Shoven and Whalley (1992), Chapter 4.

$$AEV = \frac{\sum_{h=1}^{N} EV^h}{\sum_{h=1}^{N} I_o^h} \tag{19}$$

and

$$ACV = \frac{\sum_{h=1}^{N} CV^h}{\sum_{h=1}^{N} I_n^h} \quad . \tag{20}$$

These household welfare measures provide the basis for an economy wide welfare calculation of the distorting cost of any given sales and/or labour income tax.

Implementation of our second discrete choice formulation proceeds in analogous fashion. The difference is that an alternative two point comparison is involved in the choice of high or low consumption regimes; and revenues (redistributed on a fixed proportion basis) are endogenously determined to be consistent with those assumed distributed, and reflective of actual choices made by households between high and low discrete labour supply values. As noted above, in the second formulation redistribution between high and low leisure consumption households occurs in the presence of taxes, but unlike the first formulation above choice points lie on a true budget constraint.

We also construct continuous analogues to each of the discrete labour supply models described above, using a representative consumer model in which the aggregate labour endowment (and labour income) is the same as in the corresponding discrete model. We then choose parameters in preferences in model implementation such that the uncompensated aggregate labour supply elasticities are the same in corresponding discrete and analogue continuous models.

More specifically, and as in the discrete case, we assume CES preferences for the representative consumer to be

$$U = \left[\delta\, C^{\frac{\sigma-1}{\sigma}} + (1-\delta)\, L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{21}$$

These we maximize, subject to the with tax budget constraint

$$P(1+t)C = w(1-t_1)(\bar{L} - L) + R \tag{22}$$

where revenue collected by the government is

$$R = t_1\, w(\bar{L} - L) + tPC \tag{23}$$

The first order condition for utility maximisation is

$$\frac{L}{C} = \left[\frac{(1-\delta)\, P(1+t)}{\delta \quad w(1-t_l)} \right]^{\sigma} \tag{24}$$

We again assume a linear technology with constant marginal product of labour

$$Y = \bar{L} - L, \tag{25}$$

where household labour supply is given by

$$LS = \bar{L} - L. \tag{26}$$

We numerically calculate arc (uncompensated) labour supply elasticities for both discrete and continuous models, using a small perturbation in the wage rate around base model values. This gives the model labour supply elasticity (evaluated at the base (or initial) household labour supply values) as

$$e = \frac{\Delta LS}{LS} \frac{w}{\Delta w} \tag{27}$$

We iterate on model parameters to achieve aggregate model comparability of the form discussed above. This involves comparable aggregate (uncompensated) labour supply elasticities in both discrete and continuous models, and equal aggregate labour income in the base (no tax) case. We implement this procedure for numerical examples by first specifying and solving the continuous single agent model, and then manipulating distributions of share and elasticity parameters in preferences in the two discrete models so as to yield identical uncompensated labour supply elasticities across all models. In subsequent empirical application of the approach, we jointly calibrate both discrete and continuous models using UK data to be consistent in the aggregate with literature based labour supply elasticity estimates.

The specifications used for numerical examples based on these model formulations are shown in Table 1. The continuous model embodies a representative single consumer, while the discrete models capture the behaviour of a number of households uniformly distributed by preference share parameters towards leisure (and hence goods) over an interval. Each of 100 households in the discrete model is endowed with 70 hours of time per week, while the single representative consumer in the continuous model has a comparable aggregate labour endowment of 7000.

Two discrete high and low values of leisure consumption (with implied labour supply) of 35 and 15 hours per week are assumed. We use a linear technology, and choose units such that both the price of the consumption good and the wage rate are unity. Through calibration, uncompensated labour supply elasticities evaluated as point estimates at the no tax base equilibrium are similar in related discrete and continuous models in these examples. We then make calculations of the welfare costs of labour income and sales tax distortions as Hicksian equivalent and compensating variations generated through comparisons between no tax and with tax equilibria.

Table 1

Specifications Used in No Tax Base Case for Simple Numerical Examples Showing the Welfare Cost of Taxes in Comparable Discrete and Continuous Models.

	Continuous Model	Discrete Model 1[1]	Discrete Model 2[2]
Treatment of tax revenues	Returned to single consumer	Returned to those who pay the tax	Redistributed equally per capita
Number of households	1	100	100
Labour Endowment in Hours per Week	7000	7000 in aggregate, 70 per household. High and low discrete leisure consumption values for each household are 35 and 15;	7000 in aggregate, 70 per household. High and low discrete leisure consumption values for each household are 35 and 15;
Share parameter on leisure in utility function, δ	0.5	Uniform distribution across households over the range 0.1 to 0.9	Uniform distribution across households over the range 0.1 to 0.9
Net of tax price of consumption goods, P	1.0	1.0	1.0
Gross of tax wage, W	1.0	1.0	1.0
Point estimate of aggregate uncompensated labour supply elasticity (evaluated at no tax equilibrium	0.302	0.3	0.305
Elasticity of substitution in consumption, σ	0.67	0.525	1.5

Notes:

1 This model returns tax revenues to those who pay the tax in lump sum form; as in Figure 3.

2 This model redistributes tax revenues among households using a fixed proportions distribution scheme; as in Figure 4.

3 These are obtained by repeated iteration on various model parameters, and hence the values are only approximately equal across the models rather than exactly so.

Table 2 reports welfare cost estimates for alternative income and sales tax rates for the central case parameterizations of the two discrete and one continuous models set out in Table 1. Table 3 reports results for the 10 percent tax rate case for alternative elasticity values used in model calibration.

These results clearly show that where comparable tax and elasticity specifications are used, welfare cost estimates differ substantially across discrete and continuous models, with the direction of bias depending upon the discrete model formulation used. Comparing the first discrete model and the continuous model, using similar elasticities and tax rates, welfare cost estimates are around five times larger in the continuous model. Results using the second discrete model formulation show welfare costs of taxes are larger than in comparable continuous models by factors of around 3, and by even larger orders of magnitude for smaller tax rates.

The reasons for these differences in results across discrete models lie in the alternative model formulations used, and emphasize both the role that discrete choice can play in affecting estimates of the welfare cost of taxes, and also the equally crucial role that the choice of formulation can play. In our first formulation, the fact that only a small number of households adjust, but when they do the adjustment is large, means that the majority of the population experience zero welfare impacts from a tax. If they have reasonably elastic preferences, those that move may incur small losses under large adjustment because of the discreteness of the choice. As a result, welfare effects are substantially smaller than in a comparable continuous model exhibiting the same aggregate labour supply elasticity.

In our second formulation, an individual moving lowers aggregate tax collections, imposing a loss on all other households because of the per capita distribution of revenues. If the individual who moved was close to indifferent in terms of their utility comparison across states, they would impose a cost on all other non-movers approximately equal to the taxes they would otherwise have paid if they had remained in the high labour supply state. This fiscal externality occurs where individuals do not receive back the taxes they pay as transfers, and if the discrete choice is large has a significant impact on results.

Table 3 reports results for both continuous and discrete models where the tax change is the same across cases, but the value of the elasticity of labour supply to which calibration takes place changes. In these cases, welfare costs increase in the elasticity (although less so with discrete model 2), while the ordering of size of effects across models remains much as in Table 1. The implication of all these results taken as a set is thus to suggest that discreteness is important for assessments of the welfare costs of taxes which affect labour supply.

Table 2

Welfare Cost of Estimates of Different Taxes in the Discrete and Continuous Models Specified in Table 1

Tax Rates		Continuous Model		Discrete Model 1		Discrete Model 2	
Income Tax	Sales Tax	Hicksian EV[1]	Hicksian CV[1]	Hicksian EV[1]	Hicksian CV[1]	Hicksian CV[1]	Hicksian CV[1]
0.05	0.05	-0.084	0.084	-0.035	0.036	-1.739	1.797
0.10	0.10	-0.336	0.337	-0.082	0.085	-3.413	3.647
0.15	0.15	-0.761	0.767	-0.147	0.155	-4.992	5.525
0.20	0.20	-1.364	1.383	-0.231	0.246	-6.462	7.412
0.30	0.30	-3.142	3.244	-0.585	0.649	-9.022	11.125

Note:
1. As a percent of base case aggregate income.

Table 3

Welfare Costs of 10 percent Income and Sales Taxes under Alternative Values of Labour Supply Elasticities to which both Models are Jointly Calibrated

Calibrated Labour Supply Elasticity	Continuous Model		Discrete Model 1[2]		Discrete Model 2[3]	
	Hicksian EV[1]	Hicksian CV[1]	Hicksian EV[1]	Hicksian CV[1]	Hicksian EV[1]	Hicksian CV[1]
0.2	-0.226	0.226	-0.053	0.056	-3.089	3.305
0.3	-0.336	0.337	-0.118	0.122	-3.413	3.647
0.5	-0.561	0.564	-0.139	0.143	-3.792	4.049
0.8	-0.894	0.902	-0.228	0.235	-4.050	4.322

Notes:
1. As a percent of base case aggregate income.
2. This model involves taxes being returned to those who pay the tax; as in Figure 3.
3. This model involves tax revenues being distributed among households using a fixes proportions distribution scheme; as in Figure 4.

4. A TWO LABOUR TYPE ONE HOUSEHOLD DISCRETE CHOICE MODEL

We have deliberately kept the models used in section 3 simple so as to make our main point, namely, that discreteness matters in the measurement of the welfare costs of taxes affecting labour supply. But more elaborate and, hence more realistic, models are needed to underpin any general conclusions as to the role of discreteness in analyzing tax distortions of labour supply.

One particularly relevant issue is that in two person households with both primary and secondary workers, it is typically only the primary worker who faces discrete labour supply choices, since secondary workers take on part-time work with flexible hours. In such models, the likelihood is that discreteness will have less effect on the welfare costs of taxes, since substitution margins involving continuity of labour supply are present for the household as a whole. A household model with two labour types, in which only one is constrained by discrete choice, also raises the related issue of whether high labour supply responsiveness by unconstrained secondary workers under household utility maximization over goods and both leisure types largely reflects constraints on primary workers. Discreteness of choice for primary workers may thus generate data which suggests high labour supply elasticities for secondary workers, even though household preferences towards their leisure are no different from those of primary workers.

We have constructed a two labour type discrete choice household model to examine some of these issues, where we again assume a uniform distribution of households across either preference parameters or substitution elasticities. In this model, each household, h, has a preference function defined over goods and leisure of the two household members (primary and secondary, L_1^h and L_2^h).

$$U^h = U^h\left(C^h, L_1^h, L_2^h\right) \tag{28}$$

We again consider two regimes, which now correspond to high and low labour supply (leisure consumption) of the primary worker. Household incomes in the two high and low regimes IH^h and IL^h, are given by

$$IH^h = w_1\overline{L}_1^h + w_2\overline{L}_2^h + RH^h \tag{29}$$

$$IL^h = w_1\overline{L}_1^h + w_2\overline{L}_2^h + RL^h \tag{30}$$

where $\overline{L}_1^h$ and $\overline{L}_2^h$ are the endowments of the labour types by household. RH^h and RL^h are revenues received by household h in the high and low regimes; w_1 and w_2 are the wage rates of the labour types.

Again using a linear technology assumption, goods consumption CH^h and CL^h in the high and low regimes are given by

$$CH^h = w_1\left(\overline{L}_1^h - LH_1^h\right) + w_2\left(\overline{L}_2^h - LH_2^h\right) \tag{31}$$

$$CL^h = w_1\left(\overline{L}_1^h - LL_1^h\right) + w_2\left(\overline{L}_2^h - LL_2^h\right) \tag{32}$$

where LL_1^h and LH_1^h represent the leisure consumption by household h of labour type 1 in the low and high leisure consumption regimes; and LL_2^h and LH_2^h are similar variables for the continuously supplied labour of type 2.

Using our first formula from earlier, we again evaluate utility at the same four consumption points as in the single labour type discrete model above. For analytical convenience, we assume that preferences are additively separable in the discrete and continuous variables. This allows us to determine demands for goods, and the type 2 (continuous) leisure by each household given the values for the discrete labour supply by the first labour type. These conditional demand functions for household consumption and leisure of the secondary labour type are obtained by maximizing a sub-utility function defined over the continuous variables for each of the high and low regimes subject to the high and low regime sub budget constraints. We construct these by defining household income net of consumption of discrete leisure in the high and low regimes, INH^h and INL^h, as:

$$INH^h = IH^h - w_1 LH_1^h \tag{33}$$

and

$$INL^h = IL^h - w_1 LL_1^h \tag{34}$$

Off constraint consumption in the high leisure consumption regime is as in the one labour type model i.e.

$$CH_{OC}^h = CL^h + w_1\ (LH_1^h - LL_1^h)\left(\frac{1-t_1}{1+t}\right) \tag{35}$$

The two labour types are assumed to be perfect substitutes in production. Equilibrium is then given by market clearing in the goods market and for labour type 2, given the choice of regime for discrete labour type one, and the fixed prices from the linear technology.

In the analogue continuous model, the representative household derives utility from a CES function defined over consumption of goods and leisure of household members one and two, as

$$U = \left[\delta_c C^{\frac{\sigma-1}{\sigma}} + \delta_1 L_1^{\frac{\sigma-1}{\sigma}} + \delta_2 L_2^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{36}$$

Income is derived from the two labour endowments and revenue.

$$I = w_1\ (1-t_1)L_1 + w_2\ (1-t_2)L_2 + R \tag{37}$$

Demand functions for goods and leisure are

$$C = \left[\frac{\delta_C}{(1+t)}\right]^{\sigma}\left[\frac{I}{\delta_C^{\sigma}(P(1+t))^{1-\sigma}+\delta_1^{\sigma}(w_1(1-t_1))^{(1-\sigma)}+\delta_2^{\sigma}(w_2(1-t_2))^{1-\sigma}}\right] \tag{38}$$

$$L_1 = \left[\frac{\delta_1}{w_1\ (1+t)}\right]^{\sigma}\left[\frac{I}{\delta_C^{\sigma}(P(1+t))^{1-\sigma}+\delta_1^{\sigma}(w_1(1-t_1))^{(1-\sigma)}+\delta_2^{\sigma}(w_2(1-t_2))^{1-\sigma}}\right] \tag{39}$$

and,

$$L_2 = \left[\frac{\delta_2}{w_2\ (1+t)}\right]^{\sigma}\left[\frac{I}{\delta_C^{\sigma}(P(1+t))^{1-\sigma}+\delta_1^{\sigma}(w_1(1-t_1))^{(1-\sigma)}+\delta_2^{\sigma}(w_2(1-t_2))^{1-\sigma}}\right] \tag{40}$$

Labour supply of each labour type is

$$LS_1 = \overline{L}_1 - L_1 \tag{41}$$

$$LS_2 = \overline{L}_2 - L_2 \tag{42}$$

Technology is again assumed to be linear,

$$Y = LS_1 + LS_2 \tag{43}$$

where Y is production of the consumption good. Tax revenue consists of sales taxes and income taxes applying to the two types of labour supply.

$$R = t\ PC + t_1\ w_1\ LS_1 + t_2\ w_2\ LS_2 \tag{44}$$

Equilibrium in this case is given by single agent household optimizing behaviour subject to budget and revenue constraints, and the appropriate tax distorted first order conditions. As with the single labour type case, we parameterize comparable discrete and continuous models where, in this case, comparability involves the two uncompensated labour supply elasticities for primary and secondary workers rather than, as before, one single elasticity value for aggregate labour supply. These we make comparable across discrete and continuous models.

Table 4

No Tax Base Case Specification Used in A Numerical Example Showing the Welfare Cost of Taxes in Comparable Discrete and Continuous Two Member Household Models.

	Continuous Model	Discrete Model Formulation
Number of households	1	100
Labour Endowment in hours per week.	14000	14000 in aggregate, 140 per household,
Leisure consumption of primary worker	Continuous variable	Constrained to high and low discrete values for each household ; $L^H = 35$; $L^L = 15$;
Leisure consumption of secondary worker	Continuous variable	Continuous variable
Share parameters, δ, in the utility function	$\delta_0 = 0.492$ $\delta_1 = 0.208$ $\delta_2 = 0.30$	share parameters at both levels of preferences are distributed across the 100 households
Net of tax price of consumption goods, P	1.0	1.0
Gross of tax wage, W	1.0	1.0
Point estimates of aggregate uncompensated labour supply elasticities (evaluated at no tax equilibrium)	0.144[1] 0.510[1]	0.146[1] 0.517[1]
Elasticity of substitution in consumption, σ	$\sigma = 0.5$ $\sigma_1 = 0.5$	$\sigma = 0.5$ σ, has a distribution of values across households, with mean value 1.025

Note:
[1] Numerical difficulties in calibrating the discrete two labour type model imply that elasticities across the two models are close, but not identical.

Table 4 sets out the numerical specifications we use for the two labour type models in a similar manner to that in Table 1. Lower level share parameters apply to consumption and leisure of type 2 (the continuous labour variable), with top level share parameters to the lower level composite, and leisure of type 1. We calibrate to labour supply elasticities of 0.15 (approximately) for primary workers, and 0.5 (approximately) for secondary workers. These specifications allow us to perform similar analyses to those reported on in Tables 2 and 3. We report on these in Table 5, and provide welfare cost estimates for different tax rates in the two models in a similar manner to the one labour type case.

As earlier for the single labour type model, for comparable elasticities and tax rates, welfare cost estimates are smaller for the discrete rather than the continuous model, although the differences are less between the two labour type and the continuous model than was true for the comparable one labour type model in which tax revenues are return to those who pay the tax. Welfare costs of similar taxes are comparable between one and two labour type continuous models. Compared to the continuous model, welfare costs of similar taxes are smaller by a factor of two in the discrete two labour type model, but are above those in the one labour type discrete model where no continuous labour variable enters.

Table 5

Welfare Costs of Alternative Tax Distortions of Labour Supply Using the Discrete and Continuous Models Specified in Table 4[1]

Tax Rates			Continuous Model		Discrete Model	
Income tax 1	Income tax 2	Sales tax	Hicksian EV[2]	Hicksian CV[2]	Hicksian EV[2]	Hicksian CV[2]
0.05	0.05	0.05	-0.099	0.099	-0.045	0.047
0.10	0.10	0.10	-0.398	0.400	-0.229	0.239
0.15	0.15	0.15	-0.898	0.907	-0.546	0.583
0.20	0.20	0.20	-1.605	1.632	-0.864	0.939
0.30	0.30	0.30	-3.680	3.820	-1.697	1.908

Note:
1. In the continuous model base case parameters are sigma = 0.5, no tax, alpha c = 3/6, alpha2 = 2/6, alpha1 = 1/6.
2. As a percent of base case aggregate income.

Implementing the two labour type discrete model is more complex than for the one labour type case due to the calibration to a pair of labour supply elasticities rather than to a single elasticity, and exact comparability is also harder to obtain. Because of this, the second formulation of discrete choice involving fixed proportions tax revenue redistribution to households has not been implemented in this two labour type case, and no results from it are reported although the expectation is that higher estimates than in the continuous case would again be obtained, although not as high as in the one labour type model.

5. DISCRETENESS AND THE WELFARE COSTS OF TAXES IN THE UK

In this final section we take our analysis of the welfare cost of taxes beyond the numerical examples of the previous sections, and evaluate how discreteness affects the estimation of the welfare cost of labour income and commodity taxes in the UK case. We employ the same analysis as in previous sections, but instead use actual data on tax rates, labour supply, discrete variables and literature based elasticity estimates to make welfare cost calculations. In these, the base case is an equilibrium in the presence of existing (1994) UK taxes; the counterfactual is the no tax case. We use the same four models as earlier (continuous and discrete (with and without revenue redistribution) and for 1 and 2 labour types in the household). We calibrate each to be consistent with aggregate income from employment in the UK in 1994 from National Accounts sources, leisure consumption (from time use data), tax rate data and elasticity estimates.

Wage Bill (Labour Supply)

We use total UK income from employment of £362.758 billions for 1994 (from Table 4.1 (p.45) of the UK National Income Accounts, 1995) as our reference point. This estimate is before income tax, and includes employers contributions to both social security funds and to private pensions funds. It excludes income from self employment. In using this data to parameterize the one household model, the implicit assumption is that all labour is homogeneous, and that a unit of labour sells for £1. The base case price of labour (the gross of income tax wage rate) is thus unity.

In the two labour type model, the wage bill accruing to primary and secondary labour from market labour supply is separately constructed using data from the UK Labour Force

Survey (March 1996) on average weekly earnings and numbers of full time employees (in Autumn 1994, in Tables 31 and 32, p. 42). This yields the wage bill of primary workers as £296.291 billion, and (by residual) the wage bill of secondary workers as £66.467 billion.

Discrete Leisure Consumption and Labour Supply Choices

We calibrate all four models to be consistent with available data on leisure consumption and labour supply for the U.K. For the UK, there are no official time use surveys which give the time allocation between work, leisure and other activities; although such surveys exist for other countries. Information from the ESRC British Household Panel Study (BHPS) reported in Dex et.al.(1995) (Tables 4.1 and 4.2:pp 12-13) indicates that for men performing the main job in the household, mean hours of work in 1991 were 40.3, which increased to 43.9 hours if overtime was added. For women performing the main job in the household, the mean hours were 28.9, increasing to 29.1 hours with overtime. For part time work, data from the Warwick Institute of Employment Research (1985, p.15) indicate average weekly hours for part time men across all industries (including services) of 17 hours, and 22 hours for part time non-manual women.

We thus use model specifications in which all one member households are assumed to have 70 hours of discretionary (non-sleep) time available each week, and two member households 140 hours. In the presence of 1994 taxes, single worker households work for 35 of these hours, while two worker households have primary workers who work for 40 hours and secondary workers who work for 20 hours. In the single labour type model, discrete labour supply values of 35 and 20 hours are used. In the two labour type model we use discrete values for primary workers only of 40 and 20 hours.

Tax Rates

We use a combined average marginal tax rate on consumption in all models which is constructed so as to capture the influence of both UK sales and income taxes. In the single labour type models we take payments of income tax on income from employment for 1994 from the 1995 UK National Income Accounts (Table 4.1, p. 45), along with combined employer and employee social security contributions (Table 9.6, p.83). This gives a combined average/marginal income tax rate of 30.4 percent (in both continuous and discrete models). In reality, because of the ceiling on social security contributions for many households the effective marginal tax rate from this source is zero. We have, however, ignored this feature in our calculations due to the difficulties of separating out household income groups above and below the ceiling.

For indirect (sales) taxes, we take total payments of VAT, specific excise taxes, and other small miscellaneous government charges, grouped as taxes on expenditure in the 1995 UK National Accounts. Using data on these payments for 1994 (Table 9.5, p.82) along with data on consumer expenditures, we obtain a total effective sales tax rate of 17.5 percent (also, by coincidence, equal to the current statutory VAT rate). This combines with the income tax rate to yield a combined model equivalent marginal tax rate on consumption.

For the two labour type tax model, we use separate marginal income tax rates for primary and secondary workers whose weighted average is the same as in the single labour type model. Setting the average marginal income tax rate for primary workers at 35 percent implies an average marginal income tax rate for secondary workers of 10.4 percent.

Elasticities

The key behaviourial parameters to which we calibrate all four of our models are uncompensated elasticities of labour supply, both aggregate and separately for primary and secondary workers in the two labour type model. Unfortunately, literature on labour supply elasticities is both confusing to read, and at times contradictory. Killingsworth (1983) remains the most comprehensive source for such estimates, and stresses how older (pre 1980) studies tended to produce low elasticity estimates, while more recent studies produce higher values taking into account such features as non-linear budget constraints. Estimates also vary by the segment of the labour market involved, with especially pronounced differences in labour supply elasticities occurring between primary and secondary workers. Appealing to these literature estimates, we use uncompensated labour supply elasticities in the one labour type model of 0.25 (towards the lower end of the range of second generation elasticities reported by Killingsworth), and 0.15 for primary workers and 0.5 for secondary workers in the two labour type model. Further discussion of these latter labour supply elasticity estimates can also be found in Piggott and Whalley (1996).

In the case of the single labour type continuous model, we fix share parameters in consumption to calibrate to base period data on labour supply and leisure consumption, and calibrate the implied elasticity of substitution in preferences to literature based estimates of the uncompensated aggregate labour supply elasticity. This is done using the point estimate of the labour supply elasticity at the base case equilibrium solution. For the two labour type model, we determine values of substitution elasticities as consistent as we can make them through iterative calculation with combinations of literature based labour supply elasticity estimates for both primary and secondary workers, also obtained as point estimates at the benchmark equilibrium.

Results Using UK Data

We have calculated the welfare cost of UK taxes on labour supply using all these data and parameter estimates, in the four interrelated models presented earlier (continuous/discrete(of the two forms discussed earlier); one/two household labour type). Results are reported in Table 6. These show that in comparable continuous and discrete models with one labour type estimates of welfare costs of taxes again differ sharply; smaller by a factor of 3 for the single labour type discrete model with revenues returned to those who pay the tax; and larger by a factor of 5 in the discrete model with revenues redistributed on a per capita basis.

On average, these are somewhat smaller proportional differences than in the earlier numerical examples, in part because elasticities, data and the distributions used for individual preferences in the discrete model all differ from those in the numerical examples. Also the tax change involved in the model experiments in Table 6 is larger than that used in the numerical examples, and so more individuals (or households) adjust discretely, moving discrete model estimates of welfare changes closer to those of the continuous model. Differences are less extreme in the two labour type model, where one labour type is continuous and the other is discrete; and in the results reported welfare costs of taxes in the discrete model exceed those of the continuous model.

Taken as a set, however, these results reinforce the earlier finding that welfare costs of taxes are typically smaller in models with discrete than continuous choice where taxes are returned to those who pay the tax, and larger where fixed shares apply to the distribution of revenues (with the associated fiscal externality). Moreover, the differences remain large, suggesting that discrete choice should enter more centrally into evaluations of the welfare costs of taxes than it does currently.

Table 6

Estimates of the Welfare Costs of 1994 UK Income and Sales Tax Generated by Comparable[1] Continuous and Discrete Models

	Continuous Model		**Discrete Model**		
	Continuous 1 (1 labour type)	Continuous 2 (2 labour type)	Discrete 1: Revenue Returned to those who pay tax	Discrete 2: Endogenous Revenue Determination	Discrete 3: 2 labour type Model
A. Parametric Specification					
Labour endowments (based on wage bill from 1994 National Accounts)	£725.5 bill.	L_1 = £472.8 bill. L_2 = £232.6 bill.	£725.5 bill.	£725.5 bill.	L_1 = £472.8 bill. L_2 = £232.6 bill.
Uncompensated labour supply elasticities used for calibration	0.302	$e_1 = 0.194$ $e_2 = 0.299$	0.30	0.305	$e_1 = 0.192$ $e_2 = 0.302$
Base period tax rates, combined income, payroll, VAT, from 1994 National Accounts)	$t = 0.175$ $t_1 = 0.304$	$t = 0.175$ $t_1 = 0.35$ $t_2 = 0.104$	$t = 0.175$ $t_1 = 0.304$	$t = 0.175$ $t_1 = 0.304$	$t = 0.175$ $t_1 = 0.35$ $t_2 = 0.104$
B. Welfare Costs of UK Taxes Affecting Labour Supply (percent of base (no tax) national income)					
EV (Sum of EV in discrete cases)	-1.300%	-2.605%	-0.514%	-8.132%	-1.54%
CV (Sum of CV in discrete cases)	1.284%	2.675%	0.554%	8.322%	1.72%

Note: Comparability refers to similar aggregate uncompensated labour supply elasticities, calculated as point estimates at the benchmark equilibrium.

6. CONCLUSION

This paper discusses the influence of discrete labour supply (or leisure) choice for estimates of the welfare costs of taxes on labour supply, issues which appear to be little discussed in the literature.

We construct comparable discrete and continuous models of labour supply with similar point estimates of uncompensated elasticities of aggregate labour supply around a base case equilibrium in the presence of taxes. In the continuous model, a single representative consumer adjusts labour supply continuously as taxes vary. In discrete analogues, there is a distribution of individuals over preference parameters, and a subset of individuals experiences large adjustments between discrete variables as taxes change, while other individuals show unchanged behaviour. We highlight the importance of the treatment of tax revenues in these models, adopting one formulation where revenues are returned in lump sum form to those who pay the tax, and another where revenues are distributed using a fixed proportions scheme. In the latter case, households who adjust to taxes by discretely lowering labour supply inflict a fiscal externality on other households since transfers received by non-adjusting households fall.

We show numerically that aggregate observationally equivalent discrete and continuous models of labour supply behaviour can give substantially different estimates of the welfare cost of equivalent taxes. In numerical examples, differences of a factor of 5 can occur between estimates of the welfare cost of taxes across similar discrete and continuous models. Smaller differences occur when similar calculations are made using 1994 UK data and literature based parameter estimates, but the differences are still large (factors of 3 - 5). When household models are used with discrete choice only applying to one of two types of labour (primary workers) the differences in results further narrows, but remains.

The precise influence of discreteness on welfare cost estimates in particular cases will, in practice, depend on a larger number of variables than those captured in this paper. The number of discrete choices matters, as does the step size between each. However, the general intuition that large all or non adjustments concentrated in a few households will give sharply smaller welfare costs of policy interventions than in models with continuous behaviourial response seems borne out by our results for the case where revenues are redistributed to those who pay the tax. And the importance of fiscal externalities under discrete adjustment is highlighted by our fixed proportion revenue distribution case. The conclusion is that discrete choice matters, and needs further consideration in the evaluation of the costs of taxes.

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