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ABSTRACT

This paper investigates the precision of conventional and unconventional estimates of the natural rate of unemployment (the "NAIRU"). The main finding is that the NAIRU is imprecisely estimated: a typical 95% confidence interval for the NAIRU in 1990 is 5.1% to 7.7%. This imprecision obtains whether the natural rate is modeled as a constant, as a slowly changing function of time, as an unobserved random walk, or as a function of various labor market fundamentals; it obtains using other series for unemployment and inflation, including additional supply shift variables in the Phillips curve, using monthly or quarterly data, and using various measures for expected inflation. This imprecision suggests caution in using the NAIRU to guide monetary policy.

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1. Introduction

Debates on monetary policy in the U.S. often focus on the level of unemployment and, in particular, on whether the unemployment rate is approaching its natural rate. This is commonly taken to be the rate of unemployment at which inflation remains constant, the NAIRU (Non-Accelerating Inflation Rate of Unemployment). Unfortunately, the NAIRU is not directly observable, and so some combination of economic and statistical reasoning must be used to estimate it from observable data. The task of measuring the NAIRU is further complicated by the general recognition that, plausibly, the NAIRU has changed over the postwar period, perhaps as a consequence of changes in labor markets.

Although there is a long history of construction of empirical estimates of the NAIRU, measures of the precision of these estimates are strikingly absent from this literature; the only published estimates of standard errors of the NAIRU of which we are aware are the recent limited results reported by Fuhrer (1995) and King, Stock and Watson (1995). In this paper, we therefore undertake a systematic investigation of the precision of estimates of the NAIRU. This is done using both conventional models, in which the NAIRU is treated as constant over the sample period, and in models which explicitly allow the NAIRU to change over time. As a byproduct, we obtain formal evidence on whether the NAIRU has changed over the postwar period, and if so by how much. We also investigate whether these changes in the NAIRU are linked to labor market variables, such as demographic measures, which are suggested by search models of unemployment as plausible theoretical determinants of the natural rate.

To answer these questions, we consider two classes of models which implicitly or explicitly define the NAIRU. In the first class, the NAIRU is defined so that a stable Phillips-type relation exists between unexpected inflation and the deviation of unemployment from the

NAIRU. A variant of this approach introduces labor market variables as determinants of the NAIRU within the Phillips curve framework. These models for the NAIRU include those in the recent empirical literature (Congressional Budget Office (1994), Weiner (1993), Tootell (1994), Fuhrer (1995), Eisner (1995), King, Stock and Watson (1995), Gordon (1995)), along with other candidates. In the second class, the NAIRU is defined solely in terms of the univariate behavior of unemployment, with the assumption that over time unemployment returns to its natural rate.

Our main finding is that the natural rate is measured quite imprecisely. For example, we find that a typical value of the NAIRU in 1990 is 6.2%, with a 95% confidence interval for the NAIRU in 1990 being 5.1% to 7.7% (this is the "Gaussian" confidence interval for the quarterly specification with a constant NAIRU reported in section 2). This confidence interval incorporates uncertainty about the parameters, given a particular model of the NAIRU; because different models yield different point estimates and different confidence intervals, if one informally incorporates uncertainty over models then the imprecision with which the NAIRU is measured is arguably larger still. We find this substantial imprecision whether the natural rate is measured as a constant, as an unobserved random walk, or as a slowly changing function of time (implemented here alternatively as a cubic spline in time or as a constant with discrete jumps or breaks). This finding of imprecision is also robust to using alternative series for unemployment and inflation, to including additional supply shift variables in the Phillips curve (following Gordon (1982, 1990)), to using monthly or quarterly data, to using labor market variables to model the NAIRU, and to using various measures for expected inflation.

Because we find this imprecision for the models which are conventional in the literature for the measurement of the NAIRU (as well as for the unconventional models which we consider), these results raise serious questions about the role that estimates of the NAIRU should play in discussions of monetary policy.

The paper is organized as follows. Section 2 lays out our main findings in the context of a Phillips relation estimated with monthly data, with various specifications for the NAIRU. Section 3 provides details on the econometric methodology and describes additional statistical and economic models for the NAIRU. In the statistical models, the NAIRU is determined implicitly by the time series properties of the macroeconomic variables; in the economic models, labor market variables are investigated as possible empirical determinants of the NAIRU. Section 4 discusses some further econometric issues associated with computation of the confidence intervals, and includes a Monte Carlo comparison of two alternative approaches to the construction of confidence intervals in this problem. A full set of empirical results are given in section 5. Section 6 concludes.

2. The Phillips Relation and Conventional Estimates of the NAIRU

The leading framework for estimating the NAIRU arises from defining it to be the value of unemployment that is consistent with a stable expectations-augmented Phillips relation. Ignoring lagged effects for the moment, the expectations-augmented Phillips relation considered is,

$$\pi_t - \pi_t^e = \beta(u_{t-1} - \bar{u}) + \gamma X_t + v_t \qquad (1)$$

where u_t is the unemployment rate, π_t is the rate of inflation, π_t^e is expected inflation, $\bar{u}$ is the NAIRU, and v_t is an error term. The additional regressors X_t in (1) are included in some of the empirical specifications. These regressors are intended to control for supply shocks, in particular the Nixon-era price controls and shocks to the prices of food and energy, which some have argued would shift the intercept of the Phillips curve; cf. Gordon (1990).

Empirical implementation of (1) requires a series for inflationary expectations. Following Gordon (1990), the Congressional Budget Office (1994), Weiner (1993), Tootell (1994), Fuhrer (1995), and Eisner (1995), in this section we restrict attention to the "random walk" model for inflationary expectations, that is , $\pi_t^e = \pi_{t-1}$, so $\pi_t - \pi_t^e = \Delta\pi_t$; alternative measures of expected inflation are examined in section 5. (Note that, when lags of $\pi_t - \pi_t^e$ are included on the right hand side of (1), this is equivalent to specifying the Phillips relation in the levels of inflation and imposing the restriction that the sum of the lags add to one.) Accordingly, (1) becomes,

$$\Delta\pi_t = \beta(u_{t-1} - \bar{u}) + \gamma X_t + v_t \tag{2}$$

Empirical evidence on the expectations-augmented Phillips curve (2), excluding supply shocks, is presented in figure 1, in which the year-to-year change in CPI inflation is plotted against the lag of the annual unemployment rate, for annual U.S. data from 1955 to 1994. Two key features are apparent from this figure. First, there is clear evidence of negative relation: lower unemployment is associated with higher inflation. At least at this level of aggregation, the figure suggests that this relation holds in a more or less linear way throughout the range in which unemployment and inflation have fluctuated over the past four decades. Thus unemployment is a valuable predictor of changes in future inflation. Second, there appears to be considerable ambiguity about the precise value of the NAIRU, which in this bivariate relation would be the point at which a line drawn through these observations intersects the unemployment axis. Over these four decades, a value of unemployment in the range of five to seven is roughly equally likely to have been associated with a subsequent increase in inflation as with a subsequent decrease. For example, in the 13 years in which unemployment was between 5 and 6 percent, 8 years subsequently had an increase in inflation, while in the 6 years in which unemployment was between 6 and 7 percent, 3 years saw as subsequent increase in inflation;

these percentages, 61% and 50%, respectively, are qualitatively close and do not differ at any conventional level of statistical significance.

Although this graphical analysis suggests that the NAIRU will be difficult to measure precisely, this approach omits important subtleties, such as the effects of additional lags and supply shocks. Importantly, it does not provide rigorous statements of statistical precision. To address these concerns, it is conventional to perform regression analysis of the Phillips relation. The model (1) neglects lagged effects and plausible serially correlation in the error term, which might arise for example from serially correlated measurement error in inflation. Accordingly, in this section we consider regressions estimates of,

$$(3) \qquad \Delta\pi_t = \beta(L)(u_{t-1}-\bar{u}) + \delta(L)\Delta\pi_{t-1} + \gamma(L)X_t + \epsilon_t$$

where L is the lag operator, $\beta(L)$, $\delta(L)$, and $\gamma(L)$ are lag polynomials, and ϵ_t is a serially uncorrelated error term.

Table 1 reports estimated Phillips relations of the form (3) using data on the CPI and total unemployment for the U.S., 1955 - 1994. The regressions include two variables controlling for supply shocks. NIXON is a step function taken from Gordon (1990) designed to capture effects of imposing and eliminating Nixon era price controls. PFE_CPI is a measure of the contribution of food and energy supply shocks constructed according to King and Watson (1994, footnote 18), specifically, the difference between food and energy inflation and overall CPI inflation; here it is deviated from its mean over the regression period so that by construction it has zero net effect on the measurement of the NAIRU, and it enters the specifications with one quarter's worth of lags. Each regression in table 1 includes one years worth of lags of unemployment and changes in inflation. The first three regressions were performed on monthly data, and the final regression is based on quarterly data.

These regressions are consistent with others in the literature. The sum of coefficients on lagged unemployment are negative and statistically significant. The additional lags of unemployment and the change in inflation both enter significantly, and the food and energy supply shock variable is significant (although NIXON is not).

When the NAIRU is treated as constant over the sample, as it is in regression (a) in table 1, it can be estimated directly from the coefficients of the unrestricted regression including an intercept. Specifically, because $\beta(L)(u_{t-1}-\bar{u}) = \beta(L)u_{t-1}-\beta(1)\bar{u}$, where $\beta(1)=\sum_{i=0}^{p}\beta_i$ (where p is the order of the lag polynomial $\beta(L)$), $\bar{u}$ can be estimated as $\hat{\bar{u}} = -\hat{\mu}/\hat{\beta}(1)$, where $\hat{\mu}$ is the estimated intercept from the unrestricted regression,

$$(4) \qquad \Delta\pi_t = \mu + \beta(L)u_{t-1} + \delta(L)\Delta\pi_{t-1} + \gamma(L)X_t + \epsilon_t, \quad \mu=-\beta(1)\bar{u}.$$

For specification (a) in table 1, this yields an estimate of the NAIRU of 6.20%, a value within the range of plausible values based on the discussion of figure 1.

The fact that the NAIRU is computed as a nonlinear function of the regression coefficients introduces a bit of a complication into the computation of a confidence interval for the NAIRU. However, such a confidence interval is readily constructed by considering the related problem of testing the hypothesis that the NAIRU takes on a specific value, say $\bar{u}_0$. Suppose that the null hypothesis is correct, and further suppose that the errors ϵ_t are i.i.d. normal and that the regressors in (4) are strictly exogenous. Because under the null hypothesis $\bar{u}=\bar{u}_0$ the intercept in (4) is nonzero, an exact test of the null hypothesis against the two-sided alternative can be obtained by comparing the sum of squared residuals under the null ($SSR(\bar{u}_0)$) computed from (3), with $u_t-\bar{u}_0$ as a regressor, to the unrestricted sum of squared residuals from (4) ($SSR(\hat{\bar{u}})$), using the F-statistic,

(5) $$F_{\bar{u}_0} = [SSR(\bar{u}_0)\text{-}SSR(\hat{\bar{u}})]/[SSR(\hat{\bar{u}})/d.f.]$$

where d.f. is degrees of freedom of the unrestricted specification (4). Under the stated assumptions, this statistic has an exact $F_{1,d.f.}$ distribution.

Figure 2 plots $F_{\bar{u}_0}$ against $\bar{u}_0$ for various values of $\bar{u}_0$, along with the 5% critical value. For example, for $\bar{u}_0=7$, the F-statistic is not significant, so the hypothesis that the NAIRU is 7 percent cannot be rejected using this specification. On the other hand, the hypothesis that the NAIRU is 10 percent can be rejected at the 5% level.

The duality between confidence intervals and hypothesis testing permits us to use figure 2 to construct a 95% confidence interval for $\bar{u}$. A 95% confidence set for $\bar{u}$ is the set of values of $\bar{u}$ which, when treated as the null, cannot be rejected at the 5% level. Thus, a 95% confidence interval is the set of $\bar{u}$ for which $F_{\bar{u}_0}$ is less than the 5% critical value. Under the classical assumptions of exogenous regressors and Gaussian errors, the hypothesis test based on $\bar{F}_0$ is exact (its finite sample rejection rate under the null is exactly the specified significance level). Because of these properties, we will refer to confidence intervals constructed using this approach as "Gaussian."

For figure 2, this approach yields a 95% confidence interval of (4.7%, 8.3%) for the NAIRU in 1990. The confidence interval is wide, but this is perhaps unsurprising in light of the wide range of plausible estimates of the NAIRU in figure 1. Indeed, there is striking agreement between the plausible range based on informal inspection of figure 1, and the interval estimated using the formal techniques embodied in figure 2. Although there is a statistically significant negative relationship between unemployment and future changes in inflation, the observed data do not fall tightly along this relationship and the data simply do not contain enough information to provide precise estimates of the point around which this relationship is centered, the NAIRU.

Another approach to the construction of confidence intervals is to use the so-called "delta method", which involves making a first order Taylor series approximation to the nonlinear function $-\hat{\mu}/\hat{\beta}(1)$ and then using the formula for the asymptotic variance of this linearized function. In section 4 below, we compare the Gaussian confidence intervals and the delta method confidence intervals in a Monte Carlo experiment, with a design based on a typical empirical Phillips relation. We find that the Gaussian intervals both have better finite-sample coverage rates (that is, their coverage rates are closer to the desired 95%) and have better finite-sample accuracy. For this reason, we place primary weight on the Gaussian intervals. However, because the delta method is the usual textbook approach for constructing asymptotic standard errors, for completeness in table 1 we also present delta method confidence intervals (in square brackets). Generally speaking, the delta method confidence intervals are tighter than the Gaussian confidence intervals. For example, in specification (a), the spread of the Gaussian interval is 3.6 percentage points, while the spread of the delta method interval is 2.1 percentage points. Based on the Monte Carlo results, a plausible explanation for these shorter intervals is that their finite-sample coverage rates are less than the purported 95%. Indeed, 90% Gaussian confidence intervals for the specifications in table 1 are similar to the 95% delta method intervals. For example, the 90% Gaussian interval for table 1, column (a) is (5.14, 7.57), while the 95% delta method interval is (5.16, 7.24). Despite the differences between the Gaussian and delta method confidence intervals, the main qualitative conclusion, that the confidence intervals are quite wide, obtains using either approach.

Quite plausibly, the NAIRU has not been constant over time, and specifications (b) and (c) in table 1 investigate two models for a time-varying NAIRU. In specification (b), NAIRU is modeled using a cubic spline with three knot points, while in specification (c) it is allowed to take on three constant values over the sample, that is, to be a constant with two break points. (The econometric details of these specifications and the computation of associated confidence intervals for the NAIRU are discussed in the next section.) Interestingly, the point estimate of the NAIRU for 90:1 based on these three approaches is quite similar, approximately 6.2 percentage points. Although the confidence intervals differ, they all provide the same qualitative conclusion that the NAIRU is imprecisely estimated. The tightest of the three Gaussian confidence intervals for 90:1 is based on the 2-break model and is (4.3, 7.2), a spread of 2.9 percentage points of unemployment.

The unemployment rate, the estimated NAIRU, and the 95% confidence interval for the NAIRU are plotted in figures 3, 4 and 5 for specifications (a), (b), and (c) in table 1. Although the point estimates and confidence intervals produced by the spline and break models differ for some dates, the two sets of estimates are generally similar and yield the same qualitative conclusions. Both models estimate the NAIRU to have been higher during the late 1970's and early 1980's than before or after, and suggest that the NAIRU in the 1990's is slightly higher than it was in the 1960's. Throughout the historical period, the NAIRU is imprecisely estimated using either model, although the precision during the 1960's appears to be somewhat better than the precision during later periods.

Recent work using Canadian data has demonstrated that point estimates of the NAIRU (or, similarly, potential output) can be sensitive to seemingly modest changes in specification of the estimating equations (Setterfield, Gordon and Osberg (1992), van Norden (1995)). Therefore, a critical question is whether the main conclusion of this analysis, that the NAIRU is imprecisely estimated, is sensitive to changes in the specifications in table 1.

One such alternative specification is given in column (d) in table 1, which reports the constant NAIRU model estimated using quarterly data. In general, the monthly and quarterly models are quite similar, and the estimated NAIRU is 6.20 in both models. The Gaussian confidence intervals are somewhat tighter for the quarterly model, with a spread of 2.6 percentage points of unemployment compared with 3.1 percentage points for the monthly model. Looking ahead to the empirical results in section 5, this somewhat lower spread is perhaps more typical of the confidence intervals which obtain from other specifications. As was the case using monthly data, the main qualitative conclusion from this quarterly specification is that the NAIRU is imprecisely estimated.

The main task of the remainder of this paper is to investigate more thoroughly the robustness of the conclusion that the NAIRU is imprecisely measured by examining alternative specifications. These include alternative measures of inflation and unemployment, alternative supply shock variables, different frequencies of observation, the use of other measures of inflationary expectations (including survey measures of expected inflation), and other statistical and economic models for the NAIRU. Before presenting those results, however, we first discuss econometric issues involved in these extensions.

3. Alternative Models and Econometric Issues

This section provides more precise descriptions of the various models of the NAIRU considered in the empirical analysis and the associated econometric issues. In addition to models based on Phillips type relations, we also consider models based on univariate properties of the unemployment rate.

3.1 Estimates of the NAIRU based on the Phillips curve

The first set of models are based on the generalized Phillips relation,

$$\pi_t - \pi_t^e = \beta(L)(u_{t-1} - \bar{u}_{t-1}) + \delta(L)(\pi_{t-1} - \pi_{t-1}^e) + \gamma(L)X_t + \epsilon_t \qquad (6)$$

To estimate (6), an auxiliary model or data source is needed to construct a proxy of inflationary expectations. In addition, statistical and/or economic assumptions are needed to identify the NAIRU when it is permitted to vary over time; these assumptions are discussed in subsequent subsections.

Three alternative approaches are used to model inflationary expectations:

(7a) $\pi_t^e = \mu + \alpha\pi_{t-1}$ ("AR(1) expectations")

(7b) $\pi_t^e = \mu + \alpha(L)\pi_{t-1}$ ("Recursive AR(p) expectations")

(7c) π_t^e = consensus or median forecast survey

where the survey forecasts refer to real time forecasts as collected by contemporaneous surveys of economists and forecasters. Two surveys of forecasters are used, the Survey of Professional Forecasters (SPF) now maintained by the Federal Reserve Bank of Philadelphia (previously collected as the ASA-NBER survey), and the Livingston survey, also now maintained by the Federal Reserve Bank of Philadelphia.

The premise of the AR(1) expectations model is that inflation is a highly persistent series: a unit root in the monthly consumer price index (CPI) cannot be rejected at the 10% level using the augmented Dickey-Fuller (1979) test. Thus inflationary expectations might plausibly be set to capture the long-run movements in inflation. Because the unit root cannot be rejected, a simple approach is to set $\alpha=1$. However, other values for the largest autoregressive root cannot be rejected, and in the empirical implementation we consider the endpoints of a 90% equal-tailed confidence interval for the largest autoregressive root in inflation and the value of the median-unbiased estimator of this largest root following the method of Stock (1991). Three methods of determining μ are used: setting $\mu=0$; estimating μ over the full sample for fixed α; and estimating μ recursively for fixed α to simulate real-time expectations formation.

The recursive AR(p) expectations are formed by first estimating a p-th order autoregression for inflation and using the predicted values as π^e_{t-1}. This is implemented by recursive least squares estimation of the AR(p), which simulates the real-time forecasts that would be produced under the autoregressive assumption.

The SPF forecast is the median value of forecasts from a panel of professional forecasters, which were originally collected in real time as a joint project of the American Statistical Association and the National Bureau of Economic Research. These data are available quarterly from 1968:I for the GNP (subsequently GDP) deflator and constitute a true real time forecast of inflation. The data used here are the forecast of GDP inflation over the quarter following the survey date. The SPF/ASA-NBER survey is described in more detail in Zarnowitz and Braun (1994).

The Livingston survey forecast is the mean from a semiannual forecast of the CPI. The specific forecast series used here is the median forecast of the inflation rate over the six months following the survey date.

3.2 Statistical models of the NAIRU

Four alternative statistical models for the NAIRU are investigated:

(8a) $\bar{u}_t = \bar{u}$ for all t ("Constant NAIRU")

(8b) $\bar{u}_t = \bar{\phi}' S_t$ ("Spline NAIRU")

(8c) $\bar{u}_t = \bar{u}_i$ if $t_{i-1} < t \leq t_i$, $i=1,\ldots,I$ ("Break NAIRU")

(8d) $\bar{u}_t = \bar{u}_{t-1} + \eta_t$, η_t i.i.d. $N(0,\lambda\sigma^2_\epsilon)$, $E\eta_t\epsilon_\tau=0_\tau$, all t,τ ("TVP NAIRU").

The constant NAIRU model assumes that the NAIRU does not change over the sample period. The remaining models permit NAIRU to vary over time. These models use no additional economic variables to identify NAIRU (models that do this are introduced in the next section), and so additional statistical assumptions are required to determine NAIRU. The spline, break and TVP models represent different sets of statistical assumptions with a similar motivation, specifically, that the NAIRU potentially varies over time, but that this variation is smooth and in particular these movements are unrelated to the errors ϵ_t in the Phillips relation (3).

In the spline model, NAIRU is approximated by a cubic spline in time, written as $\bar{\phi}'S_t$, where S_t is a vector of deterministic functions of time. (Including the constant, the dimension of S_t is the number of knots plus 4). The knot points of the spline are determined so that each spline segment is equidistant up to integer constraints. Accordingly, (6) can be rewritten,

$$(9) \qquad \pi_t-\pi_t^e = -\beta(1)\bar{\phi}'S_{t-1} + \beta(L)u_{t-1} + \gamma(L)X_t + \delta(L)(\pi_{t-1}-\pi_{t-1}^e) - \beta^*(L)\bar{\phi}'\Delta S_{t-1} + \epsilon_t,$$

where $\beta^*(L) = \sum_{i=1}^{p}\beta_i^* L^i$, with $\beta_i^* = -\sum_{j=i+1}^{p}\beta_j$, and where $\beta(L)$ and $\gamma(L)$ are as defined above. If the NAIRU changes slowly, then the term $\beta^*(L)\bar{\phi}'\Delta S_{t-1}$ will be small ($\beta^*(L)$ has finite order), and so to avoid nonlinear optimization over the parameters, it is convenient to treat this term as negligible. This approximation yields the estimation equation,

$$(10) \qquad \pi_t-\pi_t^e = \phi'S_{t-1} + \beta(L)u_{t-1} + \gamma(L)X_t + \delta(L)(\pi_{t-1}-\pi_{t-1}^e) + \epsilon_t,$$

where $\phi=-\beta(1)\bar{\phi}$. Equation (10) is estimated by ordinary least squares (OLS) and NAIRU is estimated as $-\hat{\phi}'S_t/\hat{\beta}(1)$.

In the break model, NAIRU is treated as taking on one of several discrete values, depending on the date. Given the break dates $\{t_i\}$, the estimation of the break model is similar to that of the spline model. Let $B_t = (B_{1t},\ldots,B_{It})$ be a set of dummy variables, where $B_{it}=1$ if $t_{i-1} < t \le t_i$

and $B_{it}=0$ otherwise. Then under the break model, NAIRU can be written as $\bar{u}_t=\lambda'B_t$, where λ is a I-vector of unknown coefficients. Given the break dates $\{t_i\}$, the coefficients are estimated using the specification (10) with $\phi'S_{t-1}$ replaced by $\lambda'B_{t-1}$ (so $\lambda=-\beta(1)\bar{\lambda}$). The breaks $\{t_i\}$ may either be fixed *a-priori* or estimated. In specifications in which they are fixed, we choose the breaks to divide the sample equally. In specifications in which they are estimated, they are chosen to minimize the sum of squared residuals from the regression (10) with $\lambda'B_{t-1}$ replacing $\phi'S_{t-1}$, subject to the restriction that no break occur within a fraction τ of another break or the start or end of the regression period. In the empirical work, τ is set to 7%, corresponding to approximately three years in our full data set. When there is more than one break, the computation of the exact minimizer of this sum of squares becomes burdensome, so we adopt a sequential estimation algorithm in which one break is estimated, then this break date is fixed and a second break is estimated, etc. Recently, Bai (1995) has shown that this algorithm yields consistent estimators of the break dates.

The time-varying-parameter (TVP) model is of the type proposed by Cooley and Prescott (1973a, 1973b, 1976), Rosenberg (1972, 1973), and Sarris (1973), although here the time variation is restricted to a single parameter, whereas in the standard TVP model all coefficients are permitted to vary over time. Estimation of the TVP model parameters and the NAIRU proceeds by maximum likelihood using the Kalman filter. (A related exercise is contained in Kuttner (1994), where the time-varying parameter framework is used to estimate potential output.) Standard errors of coefficients in the TVP model are computed assuming that ($u_t-\bar{u}_t$, $\pi_t-\pi_t^e$) are jointly stationary, the same assumption as for the spline model. The standard errors reported for the NAIRU are the square root of the sum of the Kalman smoother estimate of the variance of the state and the delta method estimate of the variance of the estimate of the state (Ansley and Kohn (1986)). Gordon (1995) estimates the NAIRU using the TVP model in specifications similar to those examined here, but does not provide confidence intervals for those estimates.

3.3. Models of the NAIRU Based on Theories of the Labor Market

An alternative to these statistical models is to model NAIRU as a function of observable labor market variables. Search models of the labor market have proved useful in explaining the cyclical components of unemployment and provide a reasonable basis for the existence of a short-run Phillips curve (see for example Bertola and Caballero (1993), Blanchard and Diamond (1989, 1990), Davis and Haltiwanger (1992), and Layard, Nickell and Jackman (1991)). While most of the work with search models focuses on understanding cyclical variation, these models also provide a conceptual framework for modelling NAIRU, which can be viewed as the model's steady-state unemployment rate.

For our purposes, the key theoretical and empirical insight of the recent search literature is that cyclical variation in unemployment is largely driven by variation in inflow rates (job destruction) while longer term trends in unemployment are largely driven by changes in exit hazards from unemployment (or equivalently, unemployment duration). Thus, unemployment exit hazards and the underlying factors that theoretically should influence these hazards may provide useful information for explaining the NAIRU.

We calculate the fraction of those recently unemployed who remain unemployed (one minus the exit hazard) as the number of persons unemployed 5-14 weeks in a given month divided by the number of new entrants into unemployment over the prior two months. To proxy for changes in search intensity and reservation wages among the unemployed, we calculate the fraction of the civilian labor force that are teen, female, and non-white. We also consider three institutional features of the labor market that have been hypothesized to affect search intensity and reservation wages: the nominal minimum wage, the unemployment insurance replacement rate (e.g. the ratio of average weekly benefits to average weekly wage), and the percentage of the civilian labor force that are union members.

This leads to modeling the NAIRU as,

$$\bar{u}_t = \Psi(L)Z_t \quad \text{("Labor Market NAIRU")} \tag{8e}$$

where Z_t is a vector of labor market variables. With the assumption that the variance of ΔZ_t is small, the derivation of (10) applies here as well with Z_t replacing S_t. Under the assumption that Z_t is uncorrelated with ϵ_t in a suitably redefined version of (10), then $\Psi(L)$ can be estimated by ordinary least squares.

3.4 Estimates of the NAIRU based solely on unemployment

If expectations of inflation are unbiased and if the supply shock variables X_t have mean zero or are absent, then the mean unemployment rate will equal the NAIRU. Alternatively, one can simply posit without reference to a Phillips curve that, over medium to long horizons, the unemployment rate reverts to its natural rate. In either case, the implication is that univariate data on unemployment can be used to extract an estimate of the NAIRU as a local mean of the series. For example, this view is implicit in estimates of the NAIRU based on linear interpolation of the unemployment rate between comparable points of the business cycle.

Our empirical implementation of the univariate approach starts with the autoregressive model, $u_t-\bar{u}_t = \beta(L)(u_{t-1}-\bar{u}_{t-1}) + \epsilon_t$, where $\bar{u}_t$ follows one of the models (8a)-(8c). For the spline model (8b), applying the derivation of (10) to the univariate model then yields,

$$u_t = \phi' S_{t-1} + \beta(L)u_{t-1} + \epsilon_t, \tag{11}$$

where $\phi=-(1+\beta(1))\bar{\phi}$. Estimation of (11) is by OLS, and the NAIRU is estimated as

$-\hat{\phi}'S_{t-1}/(1+\hat{\beta}(1))$. Estimation of the constant NAIRU model is a special case with $S_{t-1}=1$. Estimation of the break model proceeds by replacing $\phi'S_{t-1}$ with $\lambda'B_{t-1}$, as described following (10) with the modification that here $\lambda=-(1+\beta(1))\bar{\lambda}$.

4. Confidence Intervals for the NAIRU: Econometric Issues

We briefly digress to discuss additional issues in the computation of confidence intervals based on the models of the NAIRU other than the TVP model. The approach described in section 2 for computing confidence intervals must be modified when the NAIRU is allowed to vary over time. To be concrete, consider the spline NAIRU model (10), rewritten as,

$$(12) \qquad \pi_t-\pi_t^e = \beta(1)(u_{t-1}-\bar{\phi}'S_{t-1}) + \beta^*(L)\Delta u_{t-1} + \gamma(L)X_t + \delta(L)(\pi_{t-1}-\pi_{t-1}^e) + \epsilon_t,$$

where $\beta_j^*=-\sum_{i=j+1}^{p}\beta_i$. Suppose interest is in testing the null hypothesis relating to NAIRU at a fixed time τ-1, $\bar{u}_{\tau-1}=\bar{u}_{\tau-1,0}$. Without loss of generality, suppose that the constant appears first as the first spline regressor, so that $S_{t-1} = (1, S_{2,t-1})$, where $S_{2,t-1}$ denotes the additional spline regressors. Then the space spanned by regressors $\{S_t\}$ is equivalent to the space spanned by $\{\tilde{S}_t\}$, where $\tilde{S}_{t-1} = (1, S_{2,t-1}-S_{2,\tau-1})$, so in particular there is a unique $\tilde{\phi}$ such that $\bar{\phi}'S_{t-1}=\tilde{\phi}'\tilde{S}_{t-1}$. Let $\tilde{\phi}$ be partitioned as $(\tilde{\phi}_1, \tilde{\phi}_2)$ conformably with $\tilde{S}_{t-1}$. By construction, $\tilde{S}_{\tau-1} = (1, 0)$, so $\bar{u}_{\tau-1} = \tilde{\phi}'\tilde{S}_{\tau-1} = \tilde{\phi}_1$. Then (12) can be rewritten,

$$(13) \quad \pi_t-\pi_t^e = \beta(1)(u_{t-1}-\bar{u}_{\tau-1}) + \phi_2'\tilde{S}_{2,t-1} + \beta^*(L)\Delta u_{t-1} + \gamma(L)X_t + \delta(L)(\pi_{t-1}-\pi_{t-1}^e) + \epsilon_t,$$

where $\phi_2=-\beta(1)\tilde{\phi}_2$.

Because the hypothesis $\bar{u}_{T-1} = \bar{u}_{T-1,0}$ imposes no restrictions on $\tilde{\phi}_2$, $\beta(1)$, or the other coefficients, (13) can be used to construct an F statistic testing $\bar{u}_{T-1} = \bar{u}_{T-1,0}$ by comparing the restricted sum of squared residuals from (13) to the unrestricted sum of squared residuals, obtained by estimating (13) including an intercept. Evidently, confidence intervals for $\bar{u}_{T-1}$ can be constructed by inverting this test statistic, as discussed in section 2.

This procedure requires constructing separate regressors $\{\tilde{S}_t\}$ for each date of interest. However, the special structure of the linear transformation used to construct $\{\tilde{S}_t\}$ and standard regression matrix algebra deliver expressions which make this computationally efficient.

As mentioned in section 2, under the classical assumptions of exogenous regressors and Gaussian errors, the Gaussian confidence intervals have exact coverage rates. In the application at hand, however, the errors are presumably not normally distributed and the regressors, while predetermined, are not strictly exogenous (for example they include lagged dependent variables). Thus the formal justification for using these confidence intervals here relies on the asymptotic rather than the finite sample theory.

An alternative, more conventional approach is to compute confidence intervals based on the delta method, which is an asymptotic normal approximation. However, $\hat{\bar{u}} = -\hat{\mu}/\hat{\beta}(1)$ is the ratio of random variables, and such ratios are well known to have skewed and heavy-tailed distributions in finite samples. To the extent that the estimated coefficients have a distribution that is well approximated as jointly normal, then this ratio will have a doubly noncentral Cauchy distribution with dependent numerator and denominator. When $\beta(1)$ is imprecisely estimated, normality can provide a poor approximation to the distribution of this ratio. In this event, confidence intervals computed using the delta method may have coverage rates which are substantially different than the nominal asymptotic coverage rate.

The Gaussian and delta method tests of the hypothesis $\bar{u}_t = \bar{u}_{t,0}$ have the same local asymptotic power against the alternative, $\bar{u}_t = \bar{u}_{t,0} + d/\sqrt{T}$, where d is a constant. Which test to use for the construction of confidence intervals therefore depends on their finite sample properties. With fixed regressors and i.i.d. normal errors, the Gaussian test is uniformaly most powerful invariant. However, the regressors include lagged endogenous variables and the errors are plausibly nonnormally distributed, at least because of truncation error in the estimation of inflation. Thus, while the finite sample theory supporting the Gaussian intervals and the questionable nature of the first order linearization which underlies the delta method intervals both point towards prefering the Gaussian test, the exact distribution theory does not strictly apply in this application. Consequently, neither the asymptotic nor the exact finite sample theory provides a formal basis for selecting between the two intervals.

We therefore performed a Monte Carlo experiment to compare the finite sample coverage rates and accuracy of the two confidence intervals, which is equivalent to comparing the size and power of the tests upon which the confidence intervals are based. The design is empirically based and is intended to be representative of, if simpler than, the empirical models considered here. A VAR(1) in u_t and $\Delta\pi_t$ (total unemployment and the CPI) was estimated using 80 biannual observations from 1955:I-1994:II. In both equations, u_{t-1} enters significantly using the standard t-test at the 5% significance level, but the coefficient $\Delta\pi_{t-1}$ is insignificant at the 10% level. To simplify the experiment, we therefore imposed these two zero restrictions. Upon reestimation under these restrictions, we obtained,

(14a) $$u_t = .566 + .906u_{t-1} + \epsilon_{1t}$$

(14b) $$\Delta\pi_t = \mu + \beta(1)u_{t-1} + \epsilon_{2t}$$

where $(\hat{\mu}, \hat{\beta}(1)) = (1.608,-0.260)$.

The data for the Monte Carlo experiment were generated according to (14) for various values of $(\mu, \beta(1))$. Two methods were used to generate the pseudorandom errors. In the first, the bivariate errors from the 1955-1994 regression were randomly sampled with replacement and used to generate the artificial draws. When μ and $\beta(1)$ take on the values estimated using the 1955-1994 regression, this corresponds to the bootstrap. In the second, $\{\epsilon_t\}$ was drawn from an i.i.d. bivariate normal with covariance matrix set to the the sample covariance matrix of the restricted VAR residuals.

The values of (μ,β) for which the performance of the procedures is investigated are the point estimates for the biannual 1955-1994 sample, (1.608,-0.260), which corresponds to an estimate of the NAIRU of 6.18, and three selected values which lie on the boundary of the usual 80% confidence ellipse for (μ,β) estimated from these 80 observations, specifically, (0.261,-0.026), (0.394,-0.070), and (2.202,-0.404), which correspond to values of the NAIRU of 10.04, 5.63, and 5.45.

Monte Carlo coverage rates of the two procedures are summarized in appendix table A.1. The Monte Carlo coverage rate of the Gaussian interval is generally close to its nominal confidence level. In contrast, the coverage rate of the 95% delta method confidence interval ranges from 64% to 99%, depending on μ and $\beta(1)$. Generally speaking, the deviations from normality of the delta method t-statistic are, unsurprisingly, greatest when $\beta(1)$ is smallest in absolute value. Evidently the coverage rate of the delta method confidence interval is poorly controlled over empirically relevant portions of the parameter space.

In finite samples one of the intervals might be tighter in some sense than the other, and if the delta method intervals were substantially tighter in finite samples then some researchers might prefer the delta method intervals to the Gaussian intervals despite the poor coverage rates in some regions of the parameter space. We therefore investigated the tightness of the confidence intervals, or more precisely, their accuracy. The accuracy of a confidence interval is one minus its probability of covering the true parameter, so it suffices to compare the power of tests upon which the delta method and Gaussian confidence intervals are based. Because the tests do not have the same rejection rates under the null, we compare size-adjusted as well as size-unadjusted (raw) powers of the tests. The size unadjusted power is computed using asymptotic critical values; the size adjusted power is computed using the finite-sample critical value for which, for this data generating process, the test has rejection rate 5% under the null. The power was assessed by holding $\beta(1)$ constant at -0.26 and varying $\bar{u}$ (equivalently, μ). The results are summarized in appendix table A.2. In brief, for alternatives near the null, the delta method and Gaussian tests have comparable size-adjusted power. However, for more distant alternatives, the Gaussian test has substantially greater power than the delta method test.

In summary, in this experiment the Gaussian intervals were found to have both less distortions in coverage rates and greater accuracy than the delta method confidence intervals. For this reason, when interpreting the empirical results we place primary emphasis on the Gaussian intervals.

5. Empirical Results for the Postwar U.S.

This section examines a variety of alternative specifications of the Phillips curve in an attempt to assess the robustness of the main finding in section 2, the imprecision of estimates of the NAIRU. As in section 2, the base specifications use monthly data for the United States, and regressions are run over the period 1955:1-1994:12, with earlier observations as initial conditions. Unless explicitly stated otherwise, all regressions control for the Nixon price controls and one quarter's worth of lags of shocks to food and energy prices (PFE_CPI). Throughout, inflation is measured as period-to-period growth at an annual rate.

Results for several baseline monthly models, using the all-items CPI for urban consumers and the total unemployment rate, are presented in table 2. The table provides results from each of the five models of the NAIRU given in (8a)-(8e). The first column of the table provides information on any changes from the base specification. The next column describes the model for inflation expectations; in table 2, estimates are reported for models in which inflationary expectations are equal to lagged inflation or, alternatively, equal to a recursive AR(12) forecast. The third column gives the number of lags of inflation and unemployment used in the models (12 of each for these baseline specifications), and the fourth column describes the NAIRU specification. The final five columns of the table summarize the estimation results. The column labeled $\beta(1)$ shows the estimated sum of coefficients for the lags of unemployment entering the Phillips relation. The next three columns present estimates of the NAIRU in 1970:1, 1980:1, and 1990:1 with 95% Gaussian confidence intervals and delta method standard errors. The final column of the table presents the F-statistic testing the null hypothesis that the NAIRU is constant. (This was computed for the spline, break, and labor market models only. Evidence on time variation in the TVP model is discussed below.)

The confidence intervals in table 2 are comparable to those discussed in section 2. For example, the tightest estimate of the NAIRU in 90:1 among the models reported in table 2 is 5.93 with a 95% Gaussian confidence interval of (4.98, 6.91). In this case, NAIRU is modeled as a cubic spline and inflationary expectations come from a recursive AR(12) forecast. The NAIRU estimates are fairly similar across the specifications, and the point estimates across the different specifications fall within each confidence interval in the table. The models that allow for a time varying NAIRU generally suggest that the NAIRU was approximately one to two percentage points higher in 1980 than it was in 1970 or 1990. However, due to the imprecision in estimating the NAIRU, typically only the models with recursive AR(12) forecasts of inflation reject the null of a constant NAIRU. (P-values for the F-tests are not reported for the break model with estimated breaks because these statistics do not have standard F distributions under the null of no breaks.)

An important factor contributing to the imprecision in the estimates of the NAIRU is that $\beta(1)$ is generally estimated to be small. If $\beta(1)=0$, then unemployment enters the Phillips relation only in first differences; the level of the unemployment rate does not enter the equation. In this case, the NAIRU is not identified from the Phillips relations. Although the hypothesis that $\beta(1)=0$ can be rejected at conventional levels for most of the models reported in table 2, the rejection is not overwhelming for many of the specifications. In other words, the estimates for most specifications are consistent with small values of $\beta(1)$ which would lead to imprecise estimates of the NAIRU. It is noteworthy that the specifications with the largest estimates of $\beta(1)$ also report the smallest confidence intervals for the NAIRU. This is a general property of the alternative specifications reported in the subsequent tables.

We investigate the robustness of the estimates to alternative inflation and unemployment series in table 3. In this table we consider models using inflation computed using the CPI excluding food and energy, and the unemployment rate for prime-aged males (age 25-54) or, alternatively, the married male unemployment rate. For simplicity, only results for constant NAIRU and spline NAIRU models are reported, and models in which inflationary expectations are either $\pi_t^e=\pi_{t-1}$ or are derived from a recursive AR(12) forecast. Once again, the most striking fact seen in these specifications is the large confidence intervals for all estimates of the NAIRU. In fact, the basic findings do not appear to be particularly sensitive to the choice of the inflation or unemployment series -- except, of course, NAIRU is estimated to be lower in models using prime-aged male and especially married male unemployment. As in table 2, models using the recursive AR(12) inflation forecast tend to estimate the largest values of $\beta(1)$ and the tightest confidence intervals for the NAIRU.

The sensitivity of the estimates to the specification of inflationary expectations is investigated in table 4. Again, only constant NAIRU and spline NAIRU models are considered. The various specifications report alternative methods of forming inflationary expectations. In forming AR(1) expectations, we used a median unbiased estimate of 0.984 for the largest autoregressive root of inflation, and the endpoints of the 90% confidence interval of (0.965,1.003). In addition, table 4 also reports estimates based on levels of inflation and estimates based on the univariate (unemployment-only) approach of section 3.4. As in the earlier tables, there is a striking similarity in the estimates and standard errors across models. For example, the univariate estimates of the NAIRU based only on unemployment are not very different (and no more precise) than the Phillips curve estimates with spline NAIRU from table 2. Similarly, the NAIRU results are not much affected by alternative methods of forming inflationary expectations. The one exception is when the model is estimated in levels of inflation, rather than deviations from expectations. However, the spline estimates of the NAIRU with inflation in levels are implausibly large: nearly 11 percent in 80:1 and well over 7 percent in 90:1. The estimates from this specification are, we suspect, biased by the near unit root in inflation.

The sensitivity of the results to the choice of lag length is investigated in table 5. The first three rows present models which include contemporaneous unemployment in three baseline specifications. For these baseline specifications, we also report alternative estimates when lags are chosen by BIC. The results are not sensitive to these changes. It is worth noting that the lag lengths selected by BIC are generally shorter than a year, occasionally much shorter.

Table 6 investigates the sensitivity of the results to a variety of other specification changes. As in tables 3 and 5, we focus on baseline specifications for the NAIRU and inflationary expectations. The first eight rows of the table report results for models with more and less flexible specifications of spline NAIRU and break NAIRU. The next three rows report models that do not control for supply shocks. The final three rows report results for models that use the log of the unemployment rate in place of unemployment in levels (although NAIRU is reported in levels in the table). This final alteration permits considering a log-linear Phillips relation. Comparing these results to those of table 2, it is apparent that the results are not particularly sensitive to any of these specification changes. For example, the specifications in table 6 that use spline NAIRU and recursive AR(12) forecasts of inflation give estimates and confidence intervals for the NAIRU that are all quite similar to each other and also to the comparable results in table 2.

One possibility is that the imprecision in the NAIRU estimates are a consequence of using noisy monthly data, and that the estimates will be more precise when temporally aggregated data are used. Table 7 therefore reports selected models using quarterly data, and documents that the lack of precision in the NAIRU estimates is not a consequence of using monthly data. The first eight specifications in table 7 correspond to baseline specifications reported in table 2 using monthly data, and the estimates of the NAIRU and its confidence interval are little changed (although confidence intervals are slightly smaller using quarterly data). The next three specifications present models using inflation constructed from the GDP deflator (which is not available at the monthly level). These models yield similar estimates of the NAIRU but confidence intervals that are noticeably larger. The final three specifications use inflation constructed from the fixed-weight personal consumption expenditure deflator (one of the series used by the Congressional Budget Office (1994) and by Eisner (1995) in their estimation of the NAIRU). These specifications also yield results that are quite similar to the baseline models.

Table 8 investigates the sensitivity of the estimates to specifying inflationary expectations as either Livingston or SPF forecasts. Models using the Livingston forecast are estimated using semi-annual observations that conform with the timing of the Livingston forecasts (taken in June and December), while models using the SPF forecasts use the GDP deflator and limit the sample to 71:I-94:IV (or in some cases 73:I-94:IV) because the SPF forecasts only began in 68:IV. For each forecast, we present both constant NAIRU and spline NAIRU models for baseline specifications (with one year of lags) and also models in which lags are chosen by BIC.

The estimates of the NAIRU over the entire sample for both these series are notably higher than for other methods of expectations formation. This is a consequence of the survey participants underestimating inflation on average over the history of the surveys. Otherwise the estimates are generally similar to earlier tables. The exception is the rather tight confidence intervals based on the SPF forecast in the spline model with one year of lags.

Table 9 further investigates the performance of models of the NAIRU based on labor market variables. For our base specifications, we report results when the NAIRU is modeled using various subsets of the labor market variables discussed in section 3.3. It is apparent that no combination of these labor market variables yield precise estimates of the NAIRU. The most precise Gaussian confidence interval for the NAIRU in 90:1 is (4.26, 6.38), which is for a specification which uses all of the labor market variables. In the models using monthly data, the only determinant of the NAIRU that is individually significant is the unemployment exit hazard, and it has the expected negative relationship with the NAIRU. In the models using quarterly data, the only determinant of the NAIRU that is individually significant is the fraction of the labor force in their teens. A larger fraction of teens is associated with a higher NAIRU, as would be expected. As a group, the demographic variables tend to be the most significant predictors of the NAIRU, primarily in models with recursive forecasts of inflation. On balance, the labor market variables appear to enter the model as expected, but fail to provide estimates of the NAIRU any more precise than do the statistical models.

The one set of specifications in which it is possible to obtain tight confidence intervals are those which include long lags of inflation. Several such specifications are reported in table 10. To facilitate a comparison with delta method standard errors reported by Fuhrer (1995) and King, Stock and Watson (1995), in this table the delta method standard error is reported in square brackets. The first specification is essentially the specification in Fuhrer (1995) and Tootell (1994) (they use only one quarterly lag of unemployment); the delta method standard error of .37 in table 10 is similar to the delta method standard error reported by Fuhrer (1995) of .33. (The specifications in table 10 are for quarterly data, but tight confidence intervals can also be obtained using 36 lags of $\Delta\pi_t$ with monthly data.) However, the more reliable Gaussian confidence intervals remain relatively large. Furthermore, AIC and BIC choose the substantially shorter lags (2,3), for which the delta method standard error is .84. Moreover, a conventional F-test of the significance of the additional 9 lags of inflation in the first specification has a p-value of .49. Thus the statistical support for the long-lag specification appears to us to be thin.

Similar or tighter confidence intervals obtain when three years of lags are used with the spline NAIRU models. For example, when π_t^e is constructed by recursive AR(4) for the spline model, the delta method standard error for the NAIRU in 90:I is less than .3, although once again the Gaussian confidence interval remains relatively large. However, the additional lags in the (2,12) and AIC specifications are statistically insignificant at the 5% level, relative to the BIC-chosen lags of (2,1), for which the delta method standard error is .53.

The tightest confidence intervals occur for long-lag specifications using the SPF forecast for π_t^e. (Because these models are estimated over a shorter time span, the maximum number of lags is set to two years for the AIC and BIC specifications with the SPF forecast.) The AIC specification with spline NAIRU has a delta method standard error of 0.13 in 90:I, and the Gaussian confidence interval is similarly tight. Unlike the other long-lag specifications, these additional lags are significant at the 5% (but not 1%) significance level, relative to the BIC-chosen lags. Note that the point estimate of $\beta(1)$ in these long-lag specifications with SPF inflation expectations are substantially larger than for the other specifications. In our view, the apparently tight estimates for the NAIRU in these specifications reflect overfitting the model, given the relatively short time span.

Our main conclusion from these long-lag results is that, for selected combinations of unemployment series and inflationary expectations, it is possible to estimate apparently tight confidence intervals for the NAIRU when long lags of inflation and a flexible NAIRU model are used. However, the additional lags necessary to obtain these tight intervals are not selected by the BIC and indeed are not statistically significant, with a single exception. The statistical evidence for using these long lags is therefore lacking, and the associated tight intervals therefore are most plausibly statistical artifacts which are a consequence of overfitting.

Time series of estimates of the NAIRU and associated (pointwise) confidence intervals are presented in figures 6-10 for selected alternative specifications. The TVP estimate of the NAIRU and its confidence interval are plotted in figures 6 for the case $\lambda=.15$, with inflationary expectations formed from a recursive AR(12) forecast. For the TVP model, the highest value of the likelihood occurs at $\lambda=0$, corresponding to a constant NAIRU. However, this estimation problem is similar to the problem of estimating a moving average root when the root is close to one, and the MLE can have a mass point at zero when the true value is small but nonzero.

Figures 3-10 provide an opportunity to compare the delta method and Gaussian confidence intervals. The delta method confidence intervals are typically tighter. Generally, however, the two sets of confidence intervals have similar qualitative features. In many cases, the confidence intervals contain most observed values of unemployment. An exception to this is the confidence intervals based on the Livingston and SPF forecast. For example, according to the Livingston estimates, unemployment was outside the 95% confidence band, and indeed far (over 2 percentage points) below the point estimate of the NAIRU, for most of the fifteen years from 1965 to 1980. Mechanically, the explanation for this is that during this period the Livingston forecast systematically underpredicted inflation. This consistent misestimation of even the average level of inflation raises questions about the reliability of this forecast as a basis for the NAIRU calculations. In particular, this casts further doubt on the relatively precise estimates found in table 10 using the SPF survey.

These results confirm the finding in table 1 that the NAIRU is measured quite imprecisely. This conclusion is insensitive to model specification. It is not solely a consequence of the

NAIRU being nearly unidentified when $\beta(1)$ is near zero, because comparable confidence intervals obtain when NAIRU is estimated using the univariate unemployment model. Because of the nonlinearity of the estimator of the NAIRU, delta method confidence intervals may have poor coverage rates, and we have therefore relied on Gaussian confidence intervals instead. Although the empirical Gaussian confidence intervals are typically wider than delta method confidence intervals, as can be seen from the figures, the general conclusions are little changed by using delta method intervals instead.

6. Discussion and Conclusions

There are at least three different types of uncertainty which produce imprecision of the estimates of the NAIRU. The first is the uncertainty arising from not knowing the parameters of the model at hand. All the confidence intervals presented in this paper incorporate this source of imprecision, and the Monte Carlo results in section 4 suggest that the Gaussian confidence intervals provide reliable and accurate measures of this imprecision.

A second source of uncertainty arises from the possibly stochastic nature of the NAIRU, and only the TVP confidence intervals include this additional source. Consider for example the break model of the NAIRU. In the implementation here, the breaks are treated as occuring nonrandomly and, once they have occurred, are treated as if they are known with certainty. An extension of this model, which is arguably more plausible on *a-priori* grounds, would be that the NAIRU switches stochastically among several regimes, and that at a given date it is unknown which regime the NAIRU is in. While the point estimates of the NAIRU in this regime-switching model might not be particularly different from those for the deterministic break model, the confidence intervals presumably would be, because the stochastic regime model intervals would incorporate the additional uncertainty of not knowing the current regime. The TVP model incorporates this additional source of uncertainty because the NAIRU is explicitly treated as unobserved and following a stochastic path. From our perspective, it is desirable to incorporate both sources of uncertainty in construction of confidence intervals. However, incorporating the second source of uncertainty increases the computational burden dramatically, so it would have been impractical to estimate the large number of models reported here using an explicitly stochastic model of the NAIRU. As a consequence, the confidence intervals for the NAIRU for the spline and break models arguably understate the actual imprecision that arises from unpredictable movements in the NAIRU itself.

A third source of uncertainty arises from the choice of specification (in textbook terminology, not knowing which of the models is "true"). To the extent that imprecision of estimates of the NAIRU has been mentioned in the literature, it has tended to be this type of uncertainty, as quantified by a range of point estimates from alternative, arguably equally plausible specifications. None of the confidence intervals presented in this paper formally incorporate this uncertainty. However, a comparison of the point estimates and confidence intervals in tables 3 through 10 for plausible alternative specifications indicates that informally incorporating this additional source further increases the uncertainty surrounding the actual value of the NAIRU.

A central conclusion from this analysis is that a wide range of values of the NAIRU are consistent with the empirical evidence. However, the unemployment rate and changes in the unemployment rate are useful predictors of future changes in inflation. While these two results might seem contradictory, they need not be; in principal, changes in unemployment could be strongly related to future changes of inflation, but the level of unemployment could enter with a negligibly small coefficient. In most of the specifications here, this slope, $\beta(1)$, is small (in the range -.25 to -.45) and imprecisely measured, although it is statistically significantly different from zero. This corresponds to the lesson from figure 1 that the value of unemployment corresponding to a stable rate of inflation is imprecisely measured, even though an increase in unemployment will on average be associated with a decline in future rates of inflation.

It should be cautioned that the conclusion of imprecision relates to conventional methods of estimating the NAIRU and to several time-varying extensions. Although we have examined a large range of specifications and found this conclusion robust, future research might produce new, more precise methods of estimating the NAIRU.

An obvious next step is the analysis of monetary policy rules in light of these findings. We do not undertake a thorough investigation here but offer some initial thoughts on the matter. Recent work on monetary policy in the presence of measurement error (for example Kuttner (1992) and Cecchetti (1995)) is consistent with placing less weight on poorly measured targets. In this spirit, a trigger strategy, in which monetary policy takes a neutral stance until unemployment hits the natural rate and then responds vigorously, is unlikely to produce the desired outcomes because the trigger point (the natural rate) is poorly estimated. Clearly, under a trigger strategy it matters whether the NAIRU is 5 or 7 percentage points. In contrast, a rule in which monetary policy responds not to the level of the unemployment rate but to recent changes in unemployment without reference to the NAIRU (and perhaps to a measure of the deviation of inflation from a target rate of inflation) is immune to the imprecision of measurement which is highlighted in this paper. An interesting question is the construction of formal policy rules which account for the imprecision of estimation of the NAIRU.

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Table 1

Estimated Models of the NAIRU

Regression: $\Delta\pi_t = \beta(L)(u_{t-1}-\bar{u}) + \delta(L)\Delta\pi_{t-1} + \gamma(L)X_t + \epsilon_t$

Inflation: CPI
Unemployment: Total civilian

	(a)	(b)	(c)	(d)
Frequency	Monthly 55:1-94:12	Monthly 55:1-94:12	Monthly 55:1-94:12	Quarterly 55:I-94:IV
#lags (u_t, $\Delta\pi_t$)	(12,12)	(12,12)	(12,12)	(4,4)
NAIRU Model	constant	spline, 3 knots	2 breaks, estimated at 73:8 and 80:4	constant
$\beta(1)$	-.217	-.413	-.384	-.242
(standard error)	(.085)	(.136)	(.127)	(.085)
P-values of Ftests of:				
Lags of unemployment	<.001	<.001	<.001	<.001
Lags of inflation	<.001	<.001	<.001	<.001
PFE_CPI	.002	.003	.003	.002
NIXON	>.1	>.1	>.1	>.1
$\bar{R}^2$	.431	.429	.443	.391
	Estimates of NAIRU and 95% confidence intervals			
70:1	6.20	5.36	5.12	6.20
	(4.74, 8.31)	(4.10, 8.05)	(4.07, 6.34)	(5.05, 7.70)
	[5.16, 7.24]	[4.26, 6.46]	[4.24, 6.00]	[5.28, 7.12]
80:1	6.20	7.32	8.81	6.20
	(4.74, 8.31)	(5.29, 8.77)	(7.22,12.80)	(5.05, 7.70)
	[5.16, 7.24]	[6.16, 8.48]	[6.85,10.77]	[5.28, 7.12]
90:1	6.20	6.22	6.18	6.20
	(4.74, 8.31)	(4.17, 8.91)	(4.25, 7.19)	(5.05, 7.70)
	[5.16, 7.24]	[4.87, 7.57]	[5.16, 7.20]	[5.28, 7.12]

Notes: Gaussian confidence intervals for the NAIRU are reported in parentheses. Delta method confidence intervals (based on heteroskedasticity-robust covariance matrix) are reported in square brackets. In all specifications, one quarter's worth of lags (and no contemporaneous value) of PFE_CPI were included, and NIXON enters contemporaneously. The spline and break models and the construction of the associated confidence intervals are described in section 3.

Table 2. Selected Estimates of NAIRU and ß(1) for Alternative Models of π^e and NAIRU.
Base Case: Monthly 55:01-94:12, π from All-Items Urban CPI, All-Worker Unemployment.

Differences from base case	Formation of π^e	# of lags $(U,\pi-\pi^e)$	Determinants of NAIRU	ß(1) (SE)	Selected Estimates of NAIRU (Gaussian 95% Confidence Interval) [Delta method s.e.] 70:1	 80:1	 90:1	F-test of constant NAIRU (p-value)
none	$\pi^e_t = \pi_{t-1}$	(12,12)	constant	-0.217 (0.085)	6.20 (4.74,8.31) [0.53]	6.20 (4.74,8.31) [0.53]	6.20 (4.74,8.31) [0.53]	na
none	rec AR(12) f'cast	(12,12)	constant	-0.241 (0.093)	6.41 (5.30,8.50) [0.50]	6.41 (5.30,8.50) [0.50]	6.41 (5.30,8.50) [0.50]	na
none	$\pi^e_t = \pi_{t-1}$	(12,12)	spline, 3 knots	-0.413 (0.136)	5.36 (4.10,8.05) [0.56]	7.32 (5.29,8.77) [0.59]	6.22 (4.17,8.91) [0.69]	0.96 (0.455)
none	rec AR(12) f'cast	(12,12)	spline, 3 knots	-0.751 (0.160)	5.76 (5.08,6.82) [0.34]	7.74 (7.07,8.47) [0.32]	5.93 (4.98,6.91) [0.37]	3.87 (0.001)
none	$\pi^e_t = \pi_{t-1}$	(12,12)	2 breaks, est'd	-0.384 (0.127)	5.12 (4.07,6.34) [0.45]	8.81 (7.22,12.80) [1.00]	6.18 (4.25,7.19) [0.52]	3.66
none	rec AR(12) f'cast	(12,12)	2 breaks, est'd	-0.324 (0.104)	8.40 (6.90,13.90) [1.01]	8.40 (6.90,13.90) [1.01]	6.02 (3.40,7.23) [0.59]	8.90
53:01-94:12 no supply shocks	$\pi^e_t = \pi_{t-1}$	(12,12)	TVP (λ=.05)	-0.195 (0.103)	6.15 (na) [0.72]	6.33 (na) [0.68]	6.18 (na) [0.73]	na
53:01-94:12 no supply shocks	$\pi^e_t = \pi_{t-1}$	(12,12)	TVP (λ=.15)	-0.148 (0.120)	6.30 (na) [1.27]	7.12 (na) [1.14]	6.03 (na) [1.20]	na
53:01-94:12 no supply shocks	rec AR(12) f'cast	(12,12)	TVP (λ=.05)	-0.237 (0.125)	6.57 (na) [0.66]	6.75 (na) [0.60]	6.48 (na) [0.65]	na
53:01-94:12 no supply shocks	rec AR(12) f'cast	(12,12)	TVP (λ=.15)	-0.288 (0.156)	6.94 (na) [0.94]	7.79 (na) [0.82]	6.14 (na) [0.82]	na
55:01-93:12	$\pi^e_t = \pi_{t-1}$	(12,12)	labor market variables	-0.889 (0.260)	4.96 (3.24,5.49) [0.34]	6.93 (5.63,8.02) [0.45]	5.43 (4.08,6.46) [0.50]	1.44 (0.186)
55:01-93:12	rec AR(12) f'cast	(12,12)	labor market variables	-0.973 (0.267)	5.52 (4.06,6.41) [0.40]	7.33 (6.28,8.45) [0.44]	5.46 (4.26,6.38) [0.45]	3.61 (0.001)

Table 3. Sensitivity of Estimates of NAIRU and ß(1) to Use of Alternative Data Series for π and U. Base Case: Monthly 55:01-94:12, π from All-Items Urban CPI, All-Worker Unemployment.

Differences from base case	Formation of π^e	# of lags $(U,\pi-\pi^e)$	Determinants of NAIRU	ß(1) (SE)	Selected Estimates of NAIRU (Gaussian 95% Confidence Interval) 70:1	80:1	90:1	F-test of constant NAIRU (p-value)
male 25-54 unemp.	$\pi^e_t = \pi_{t-1}$	(12,12)	constant	-0.188 (0.076)	4.50 (2.53,7.74)	4.50 (2.53,7.74)	4.50 (2.53,7.74)	na
male 25-54 unemp.	$\pi^e_t = \pi_{t-1}$	(12,12)	spline, 3 knots	-0.388 (0.133)	3.02 (1.60,5.94)	5.14 (2.94,6.84)	5.32 (3.12,8.62)	0.84 (0.536)
male 25-54 unemp.	rec AR(12) f'cast	(12,12)	spline, 3 knots	-0.609 (0.154)	3.58 (2.75,5.13)	5.52 (4.64,6.52)	4.97 (3.72,6.29)	1.85 (0.088)
married male unemp. 57:01-94:12	$\pi^e_t = \pi_{t-1}$	(12,12)	constant	-0.268 (0.107)	3.62 (2.20,5.15)	3.62 (2.20,5.15)	3.62 (2.20,5.15)	na
married male unemp. 57:01-94:12	$\pi^e_t = \pi_{t-1}$	(12,12)	spline, 3 knots	-0.472 (0.165)	2.52 (1.27,5.18)	4.26 (2.46,5.61)	4.00 (2.16,6.57)	0.63 (0.706)
married male unemp. 57:01-94:12	rec AR(12) f'cast	(12,12)	spline, 3 knots	-0.643 (0.185)	3.47 (2.58,6.01)	4.39 (3.43,5.32)	3.73 (2.43,5.06)	0.92 (0.481)
CPI less food/energy 62:01-94:12	$\pi^e_t = \pi_{t-1}$	(12,12)	constant	-0.195 (0.084)	6.17 (4.22,8.17)	6.17 (4.22,8.17)	6.17 (4.22,8.17)	na
CPI less food/energy 62:01-94:12	$\pi^e_t = \pi_{t-1}$	(12,12)	spline, 3 knots	-0.429 (0.137)	5.08 (3.69,7.58)	7.73 (6.23,9.40)	6.31 (4.67,8.49)	1.58 (0.151)
CPI less food/energy 62:01-94:12	rec AR(12) f'cast	(12,12)	spline, 3 knots	-0.545 (0.148)	4.69 (3.53,6.07)	8.63 (7.70,10.47)	5.88 (4.50,7.18)	4.30 (0.000)
CPI less food/energy male 25-54 unemp. 62:01-94:12	$\pi^e_t = \pi_{t-1}$	(12,12)	constant	-0.169 (0.072)	4.41 (1.90,7.30)	4.41 (1.90,7.30)	4.41 (1.90,7.30)	na
CPI less food/energy male 25-54 unemp. 62:01-94:12	$\pi^e_t = \pi_{t-1}$	(12,12)	spline, 3 knots	-0.357 (0.128)	2.81 (0.89,6.26)	5.53 (3.69,8.51)	5.45 (3.38,8.88)	1.20 (0.305)
CPI less food/energy male 25-54 unemp. 62:01-94:12	rec AR(12) f'cast	(12,12)	spline, 3 knots	-0.417 (0.137)	2.44 (0.59,4.48)	6.58 (5.34,10.77)	4.91 (2.75,6.99)	2.70 (0.014)
CPI less food/energy married male unemp. 62:01-94:12	$\pi^e_t = \pi_{t-1}$	(12,12)	constant	-0.293 (0.106)	3.54 (2.47,4.56)	3.54 (2.47,4.56)	3.54 (2.47,4.56)	na
CPI less food/energy married male unemp. 62:01-94:12	$\pi^e_t = \pi_{t-1}$	(12,12)	spline, 3 knots	-0.535 (0.155)	2.52 (1.38,4.06)	4.41 (3.30,5.69)	4.00 (2.76,5.61)	1.19 (0.312)
CPI less food/energy married male unemp. 62:01-94:12	rec AR(12) f'cast	(12,12)	spline, 3 knots	-0.590 (0.164)	2.25 (1.09,3.46)	5.19 (4.31,7.07)	3.65 (2.33,4.91)	2.87 (0.010)

Table 4. Sensitivity of Estimates of NAIRU and ß(1) to Use of Alternative Models of π^e.
Base Case: Monthly 55:01-94:12, π from All-Items Urban CPI, All-Worker Unemployment.

Differences from base case	Formation of π^e	# of lags $(U,\pi-\pi^e)$	Determinants of NAIRU	ß(1) (SE)	Selected Estimates of NAIRU (Gaussian 95% Confidence Interval) 70:1	80:1	90:1	F-test of constant NAIRU (p-value)
full-sample demeaning of $\pi-\pi^e$	$\pi^e_t = \pi_{t-1}$	(12,12)	constant	-0.217 (0.085)	6.08 (4.46,7.95)	6.08 (4.46,7.95)	6.08 (4.46,7.95)	na
full-sample demeaning of $\pi-\pi^e$	$\pi^e_t = \pi_{t-1}$	(12,12)	spline, 3 knots	-0.413 (0.136)	5.29 (4.01,7.86)	7.25 (5.12,8.65)	6.15 (4.05,8.75)	0.96 (0.455)
none	full-sample AR(12) f'cast	(12,12)	constant	-0.134 (0.086)	6.06 (0.91,11.22)	6.06 (0.91,11.22)	6.06 (0.91,11.22)	na
none	full-sample AR(12) f'cast	(12,12)	spline, 3 knots	-0.745 (0.151)	5.16 (4.48,5.95)	8.09 (7.45,8.93)	5.87 (4.90,6.84)	5.76 (0.000)
recursive demeaning of $\pi-\pi^e$	$\pi^e_t = \pi_{t-1}$	(12,12)	constant	-0.190 (0.085)	5.55 (1.76,7.19)	5.55 (1.76,7.19)	5.55 (1.76,7.19)	na
recursive demeaning of $\pi-\pi^e$	$\pi^e_t = \pi_{t-1}$	(12,12)	spline, 3 knots	-0.372 (0.135)	5.10 (3.46,8.23)	6.90 (2.92,8.32)	6.12 (3.42,9.51)	0.75 (0.613)
recursive demeaning of $\pi-\pi^e$	$\pi^e_t = 0.965*\pi_{t-1}$	(12,12)	constant	-0.192 (0.086)	6.73 (5.36,10.81)	6.73 (5.36,10.81)	6.73 (5.36,10.81)	na
recursive demeaning of $\pi-\pi^e$	$\pi^e_t = 0.965*\pi_{t-1}$	(12,12)	spline, 3 knots	-0.636 (0.141)	5.75 (4.96,7.05)	8.27 (7.53,9.39)	5.81 (4.63,6.96)	4.42 (0.000)
recursive demeaning of $\pi-\pi^e$	$\pi^e_t = 0.984*\pi_{t-1}$	(12,12)	constant	-0.198 (0.085)	6.17 (4.25,9.07)	6.17 (4.25,9.07)	6.17 (4.25,9.07)	na
recursive demeaning of $\pi-\pi^e$	$\pi^e_t = 0.984*\pi_{t-1}$	(12,12)	spline, 3 knots	-0.501 (0.138)	5.49 (4.50,7.30)	7.72 (6.60,8.99)	5.93 (4.31,7.58)	2.11 (0.051)
recursive demeaning of $\pi-\pi^e$	$\pi^e_t = 1.003*\pi_{t-1}$	(12,12)	constant	-0.186 (0.085)	5.41 (1.43,6.95)	5.41 (1.43,6.95)	5.41 (1.43,6.95)	na
recursive demeaning of $\pi-\pi^e$	$\pi^e_t = 1.003*\pi_{t-1}$	(12,12)	spline, 3 knots	-0.347 (0.135)	4.99 (2.93,8.76)	6.67 (2.13,8.15)	6.18 (2.96,10.78)	0.60 (0.729)
π in levels	na	(12,12)	constant	-0.203 (0.086)	6.42 (3.88,13.43)	6.42 (3.88,13.43)	6.42 (3.88,13.43)	na
π in levels	na	(12,12)	spline, 3 knots	-0.882 (0.180)	7.01 (6.04,8.28)	10.78 (9.40,12.54)	7.60 (6.68,8.83)	3.76 (0.001)
univariate model	na	(12,na)	constant	-0.017 (0.006)	6.06 (4.72,7.53)	6.06 (4.72,7.53)	6.06 (4.72,7.53)	na
univariate model	na	(12,na)	spline, 3 knots	-0.045 (0.011)	4.78 (3.95,5.64)	7.63 (6.78,8.48)	6.15 (5.04,7.42)	2.46 (0.024)

Table 5. Sensitivity of Estimates of NAIRU and ß(1) to Contemporaneous Unemployment and BIC Lag Choice. Base Case: Monthly 55:01-94:12, π from All-Items Urban CPI, All-Worker Unemployment.

Differences from base case	Formation of π^e	# of lags $(U,\pi-\pi^e)$	Determinants of NAIRU	ß(1) (SE)	Selected Estimates of NAIRU (Gaussian 95% Confidence Interval) 70:1	80:1	90:1	F-test of constant NAIRU (p-value)
include contemporaneous U	$\pi^e_t = \pi_{t-1}$	(12,12)	constant	-0.220 (0.086)	6.20 (4.76,8.26)	6.20 (4.76,8.26)	6.20 (4.76,8.26)	na
include contemporaneous U	$\pi^e_t = \pi_{t-1}$	(12,12)	spline, 3 knots	-0.431 (0.138)	5.34 (4.14,7.77)	7.33 (5.47,8.69)	6.22 (4.30,8.70)	1.03 (0.405)
include comtemporaneous U	rec AR(12) f'cast	(12,12)	spline, 3 knots	-0.766 (0.160)	5.75 (5.09,6.78)	7.74 (7.08,8.45)	5.94 (5.01,6.89)	3.93 (0.001)
lags chosen by BIC	$\pi^e_t = \pi_{t-1}$	(5, 8)	constant	-0.203 (0.089)	6.17 (4.52,8.35)	6.17 (4.52,8.35)	6.17 (4.52,8.35)	na
lags chosen by BIC	$\pi^e_t = \pi_{t-1}$	(5, 8)	spline, 3 knots	-0.365 (0.123)	5.28 (3.81,7.90)	7.31 (5.09,8.93)	6.25 (3.95,9.17)	0.75 (0.612)
lags chosen by BIC	rec AR(12) f'cast	(2, 1)	spline, 3 knots	-0.508 (0.130)	5.64 (4.69,7.18)	7.71 (6.65,8.81)	5.91 (4.41,7.39)	1.75 (0.107)

Table 6. Sensitivity of Estimates of NAIRU and ß(1) to Other Changes in Specification.
Base Case: Monthly 55:01-94:12, π from All-Items Urban CPI, All-Worker Unemployment.

Differences from base case	Formation of π^e	# of lags $(U,\pi-\pi^e)$	Determinants of NAIRU	ß(1) (SE)	Selected Estimates of NAIRU (Gaussian 95% Confidence Interval) 70:1	80:1	90:1	F-test of constant NAIRU (p-value)
none	$\pi^e_t = \pi_{t-1}$	(12,12)	spline, 4 knots	-0.409 (0.135)	5.20 (3.62,8.65)	7.65 (5.40,9.59)	6.30 (4.13,9.07)	0.89 (0.511)
none	rec AR(12) f'cast	(12,12)	spline, 4 knots	-0.725 (0.157)	5.83 (4.95,7.27)	7.85 (6.99,8.73)	6.01 (4.99,7.04)	3.53 (0.001)
none	$\pi^e_t = \pi_{t-1}$	(12,12)	3 breaks, est'd	-0.334 (0.124)	5.13 (3.80,6.76)	9.23 (7.38,16.29)	6.67 (4.72,8.42)	3.33
none	rec AR(12) f'cast	(12,12)	3 breaks, est'd	-0.561 (0.150)	5.90 (4.76,7.73)	8.83 (7.69,10.92)	6.36 (5.38,7.03)	6.89
none	$\pi^e_t = \pi_{t-1}$	(12,12)	4 breaks, est'd	-0.441 (0.148)	5.08 (4.17,6.12)	8.64 (7.25,12.22)	6.04 (4.44,7.43)	2.72
none	rec AR(12) f'cast	(12,12)	4 breaks, est'd	-0.506 (0.148)	7.52 (5.67,11.93)	9.40 (8.05,12.61)	6.24 (4.99,6.98)	6.50
none	$\pi^e_t = \pi_{t-1}$	(12,12)	2 breaks, fixed	-0.236 (0.099)	7.09 (5.26,12.73)	7.09 (5.26,12.73)	6.02 (0.78,7.92)	1.02 (0.361)
none	rec AR(12) f'cast	(12,12)	2 breaks, fixed	-0.341 (0.110)	7.69 (6.41,11.22)	7.69 (6.41,11.22)	6.20 (3.94,7.45)	5.11 (0.006)
no supply shocks	$\pi^e_t = \pi_{t-1}$	(12,12)	constant	-0.235 (0.087)	6.17 (4.87,7.86)	6.17 (4.87,7.86)	6.17 (4.87,7.86)	na
no supply shocks	$\pi^e_t = \pi_{t-1}$	(12,12)	spline, 3 knots	-0.401 (0.140)	5.62 (4.37,9.34)	7.28 (4.94,8.81)	6.20 (3.96,9.17)	1.07 (0.377)
no supply shocks	rec AR(12) f'cast	(12,12)	spline, 3 knots	-0.733 (0.161)	5.93 (5.21,7.19)	7.72 (7.01,8.49)	5.92 (4.91,6.94)	3.95 (0.001)
log unemployment	$\pi^e_t = \pi_{t-1}$	(12,12)	constant	-1.151 (0.490)	6.05 (4.35,10.80)	6.05 (4.35,10.80)	6.05 (4.35,10.80)	na
log unemployment	$\pi^e_t = \pi_{t-1}$	(12,12)	spline, 3 knots	-2.338 (0.797)	5.10 (4.06,8.67)	7.17 (4.85,9.39)	6.23 (4.31,10.70)	1.01 (0.419)
log unemployment	rec AR(12) f'cast	(12,12)	spline, 3 knots	-4.913 (0.930)	5.42 (4.90,6.30)	7.69 (6.96,8.58)	5.93 (5.16,6.82)	4.44 (0.000)

Table 7. Selected Estimates of NAIRU and ß(1) Using Quarterly Data.
Base Case: Quarterly 55:I-94:IV, π from All-Items Urban CPI, All-Worker Unemployment.

Differences from base case	Formation of π^e	# of lags $(U,\pi-\pi^e)$	Determinants of NAIRU	ß(1) (SE)	Selected Estimates of NAIRU (Gaussian 95% Confidence Interval)			F-test of constant NAIRU (p-value)
					70:1	80:1	90:1	
none	$\pi^e_t = \pi_{t-1}$	(4, 4)	constant	-0.242 (0.085)	6.20 (5.05,7.70)	6.20 (5.05,7.70)	6.20 (5.05,7.70)	na
none	rec AR(4) f'cast	(4, 4)	constant	-0.244 (0.088)	6.35 (5.23,8.17)	6.35 (5.23,8.17)	6.35 (5.23,8.17)	na
none	$\pi^e_t = \pi_{t-1}$	(4, 4)	spline, 3 knots	-0.448 (0.143)	5.51 (4.38,7.66)	7.26 (5.54,8.47)	6.15 (4.42,8.29)	1.23 (0.293)
none	rec AR(4) f'cast	(4, 4)	spline, 3 knots	-0.769 (0.161)	5.91 (5.20,6.84)	7.78 (7.15,8.47)	5.83 (4.96,6.74)	5.94 (0.000)
none	$\pi^e_t = \pi_{t-1}$	(4, 4)	2 breaks, est'd	-0.431 (0.117)	5.18 (4.37,6.15)	8.34 (7.10,10.83)	6.15 (4.72,7.00)	7.59
none	rec AR(4) f'cast	(4, 4)	2 breaks, est'd	-0.308 (0.099)	8.58 (7.02,14.49)	5.84 (<-10,10.19)	5.84 (2.91,7.05)	10.46
quarterly 55:I-93:IV	$\pi^e_t = \pi_{t-1}$	(4, 4)	labor market variables	-0.691 (0.312)	4.91 (2.91,7.00)	7.06 (5.26,9.65)	5.85 (4.66,8.97)	1.06 (0.389)
quarterly 55:I-93:IV	rec AR(4) f'cast	(4, 4)	labor market variables	-0.821 (0.326)	5.76 (4.22,8.62)	7.63 (6.31,10.12)	5.96 (4.83,7.99)	3.79 (0.001)
GDP deflator	$\pi^e_t = \pi_{t-1}$	(4, 4)	constant	-0.168 (0.093)	5.97 (1.90,10.03)	5.97 (1.90,10.03)	5.97 (1.90,10.03)	na
GDP deflator	$\pi^e_t = \pi_{t-1}$	(4, 4)	spline, 3 knots	-0.195 (0.145)	6.40 (-5.06,17.85)	6.65 (-1.08,14.37)	5.83 (0.08,11.59)	0.20 (0.977)
GDP deflator	rec AR(4) f'cast	(4, 4)	spline, 3 knots	-0.503 (0.183)	6.62 (5.53,10.70)	7.50 (6.07,8.75)	5.62 (3.58,7.24)	2.86 (0.012)
fixed-weight PCE deflator	$\pi^e_t = \pi_{t-1}$	(4, 4)	constant	-0.213 (0.066)	6.21 (5.12,7.63)	6.21 (5.12,7.63)	6.21 (5.12,7.63)	na
fixed-weight PCE deflator	$\pi^e_t = \pi_{t-1}$	(4, 4)	spline, 3 knots	-0.374 (0.122)	5.57 (4.44,7.97)	7.39 (5.68,8.67)	5.92 (3.98,7.96)	1.35 (0.241)
fixed-weight PCE deflator	rec AR(4) f'cast	(4, 4)	spline, 3 knots	-0.622 (0.142)	5.85 (5.11,6.81)	7.87 (7.22,8.63)	5.92 (5.01,6.91)	4.14 (0.001)

Table 8. Sensitivity of Estimates of NAIRU and β(1) to Alternative Models of π^e, Quarterly Data.
Base Case: Quarterly 55:I-94:IV, π from All-Items Urban CPI, All-Worker Unemployment.

Differences from base case	Formation of π^e	# of lags $(U,\pi-\pi^e)$	Determinants of NAIRU	β(1) (SE)	Selected Estimates of NAIRU (Gaussian 95% Confidence Interval) 70:1	80:1	90:1	F-test of constant NAIRU (p-value)
GDP deflator 71:I-94:IV	SPF f'cast	(4, 4)	constant	-0.223 (0.123)	na	7.20 (3.87,10.53)	7.20 (3.87,10.53)	na
GDP deflator 71:I-94:IV	SPF f'cast	(4, 4)	spline, 2 knots	-0.836 (0.178)	na	8.00 (7.41,8.86)	6.16 (5.50,6.92)	3.99 (0.003)
GDP deflator 73:I-94:IV lags chosen by BIC	SPF f'cast	(2, 2)	constant	-0.309 (0.122)	na	7.20 (6.04,9.17)	7.20 (6.04,9.17)	na
GDP deflator 73:I-94:IV lags chosen by BIC	SPF f'cast	(2, 1)	spline, 2 knots	-0.562 (0.118)	na	7.92 (7.07,9.10)	6.21 (5.30,7.23)	4.52 (0.001)
semi-annual	Livingston f'cast	(2, 2)	constant	-0.284 (0.153)	7.07 (5.27,12.27)	7.07 (5.27,12.27)	7.07 (5.27,12.27)	na
semi-annual	Livingston f'cast	(2, 2)	spline, 3 knots	-0.782 (0.232)	7.07 (5.75,9.69)	7.97 (7.00,9.45)	6.06 (4.58,7.76)	2.77 (0.018)
semi-annual lags chosen by BIC	Livingston f'cast	(2, 1)	constant	-0.308 (0.142)	7.11 (5.82,11.95)	7.11 (5.82,11.95)	7.11 (5.82,11.95)	na
semi-annual lags chosen by BIC	Livingston f'cast	(2, 1)	spline, 3 knots	-0.716 (0.227)	7.06 (5.69,10.11)	7.94 (6.89,9.57)	6.09 (4.46,7.94)	2.70 (0.021)

Table 9. Sensitivity of Estimates of NAIRU and β(1) to Alternative Labor Market Models of NAIRU. Base Case: Monthly 55:01-94:12, π from All-Items Urban CPI, All-Worker Unemployment.

Differences from base case	Formation of π^e	# of lags $(U,\pi-\pi^e)$	Determinants of NAIRU	β(1) (SE)	Selected Estimates of NAIRU (Gaussian 95% Confidence Interval) 70:1	80:1	90:1	F-test of constant NAIRU (p-value)
55:01-93:12	$\pi^e_t = \pi_{t-1}$	(12,12)	demographics, institutions, exit hazard	-0.889 (0.260)	4.96 (3.24,5.49)	6.93 (5.63,8.02)	5.43 (4.08,6.46)	1.44 (0.186)
55:01-93:12	rec AR(12) f'cast	(12,12)	demographics, institutions, exit hazard	-0.973 (0.267)	5.52 (4.06,6.41)	7.33 (6.28,8.45)	5.46 (4.26,6.38)	3.61 (0.001)
55:01-93:12	$\pi^e_t = \pi_{t-1}$	(12,12)	demographics, institutions	-0.435 (0.175)	5.44 (3.47,9.00)	7.68 (4.51,10.29)	6.25 (3.41,9.24)	0.49 (0.815)
55:01-93:12	rec AR(12) f'cast	(12,12)	demographics, institutions	-0.611 (0.195)	6.22 (5.16,8.66)	8.10 (6.75,9.81)	6.03 (4.34,7.48)	2.84 (0.010)
55:01-93:12	$\pi^e_t = \pi_{t-1}$	(12,12)	demographics	-0.264 (0.101)	6.30 (3.67,10.20)	6.91 (4.96,10.36)	6.43 (2.48,9.13)	0.44 (0.725)
55:01-93:12	rec AR(12) f'cast	(12,12)	demographics	-0.426 (0.112)	6.91 (5.76,8.90)	7.72 (6.81,9.60)	6.36 (4.74,7.66)	4.62 (0.003)
55:01-93:12	$\pi^e_t = \pi_{t-1}$	(12,12)	exit hazard	-0.456 (0.183)	5.15 (3.27,7.52)	6.08 (5.34,7.00)	5.53 (4.68,7.09)	2.62 (0.106)
55:01-93:12	rec AR(12) f'cast	(12,12)	exit hazard	-0.350 (0.181)	5.67 (3.53,10.39)	6.28 (5.45,8.64)	5.92 (4.87,9.53)	0.630 (0.428)
quarterly 55:I-93:IV	$\pi^e_t = \pi_{t-1}$	(4, 4)	demographics, institutions, exit hazard	-0.691 (0.312)	4.91 (2.91,7.00)	7.06 (5.26,9.65)	5.85 (4.66,8.97)	1.06 (0.389)
quarterly 55:I-93:IV	rec AR(4) f'cast	(4, 4)	demographics, institutions exit hazard	-0.821 (0.326)	5.76 (4.22,8.62)	7.63 (6.31,10.12)	5.96 (4.83,7.99)	3.79 (0.001)
quarterly 55:I-93:IV	$\pi^e_t = \pi_{t-1}$	(4, 4)	demographics, institutions	-0.417 (0.171)	4.93 (2.71,7.69)	7.34 (4.84,10.22)	6.60 (4.72,9.92)	1.04 (0.400)
quarterly 55:I-93:IV	rec AR(4) f'cast	(4, 4)	demographics, institutions	-0.619 (0.187)	6.07 (5.10,8.09)	7.99 (6.92,9.64)	6.38 (4.97,7.93)	4.30 (0.001)
quarterly 55:I-93:IV	$\pi^e_t = \pi_{t-1}$	(4, 4)	exit hazard	-0.334 (0.192)	5.73 (3.56,9.63)	6.26 (5.15,8.37)	5.93 (4.97,8.79)	0.38 (0.536)
quarterly 55:I-93:IV	rec AR(4) f'cast	(4, 4)	exit hazard	-0.143 (0.188)	7.89 (-17.13,32.91)	6.52 (0.93,12.10)	7.37 (-9.35,24.08)	0.44 (0.510)

Table 10. Sensitivity of Estimates of NAIRU and ß(1) to Long Lags.
Base Case: Quarterly 55:I-94:IV, π from GDP Deflator, All-Worker Unemployment.

Differences from base case	Formation of π^e	# of lags $(U,\pi-\pi^e)$	Determinants of NAIRU	ß(1) (SE)	Selected Estimates of NAIRU (Gaussian 95% Confidence Interval) [delta method s.e.] 70:1	80:1	90:1	F-test of constant NAIRU (p-value)
none	$\pi^e_t = \pi_{t-1}$	(2,12)	constant	-0.295 (0.123)	6.01 (4.76,7.20) [0.37]	6.01 (4.76,7.20) [0.37]	6.01 (4.76,7.20) [0.37]	na
lags chosen by BIC (same as AIC)	$\pi^e_t = \pi_{t-1}$	(2, 3)	constant	-0.136 (0.084)	6.00 (0.95,11.05) [0.84]	6.00 (0.95,11.05) [0.84]	6.00 (0.95,11.05) [0.84]	na
none	$\pi^e_t = \pi_{t-1}$	(2,12)	spline, 3 knots	-0.451 (0.179)	6.53 (5.31,10.99) [0.74]	6.68 (3.45,7.92) [0.56]	5.93 (3.65,8.21) [0.38]	1.06 (0.389)
lags chosen by BIC (same as AIC)	$\pi^e_t = \pi_{t-1}$	(2, 3)	spline, 3 knots	-0.084 (0.124)	9.35 (-35.06,53.76) [7.40]	5.25 (-20.69,31.18) [4.32]	5.71 (-7.36,18.79) [2.18]	0.31 (0.930)
none	rec AR(4) f'cast	(2,12)	constant	-0.200 (0.102)	6.16 (2.84,9.49) [0.55]	6.16 (2.84,9.49) [0.55]	6.16 (2.84,9.49) [0.55]	na
lags chosen by AIC	rec AR(4) f'cast	(2, 3)	constant	-0.208 (0.097)	6.15 (4.33,8.69) [0.54]	6.15 (4.33,8.69) [0.54]	6.15 (4.33,8.69) [0.54]	na
lags chosen by BIC	rec AR(4) f'cast	(2, 1)	constant	-0.257 (0.086)	6.11 (5.01,7.33) [0.44]	6.11 (5.01,7.33) [0.44]	6.11 (5.01,7.33) [0.44]	na
none	rec AR(4) f'cast	(2,12)	spline, 3 knots	-0.657 (0.202)	6.90 (5.92,9.30) [0.58]	7.58 (6.78,8.53) [0.32]	5.61 (4.31,6.71) [0.26]	3.60 (0.002)
lags chosen by AIC	rec AR(4) f'cast	(3, 6)	spline, 3 knots	-0.760 (0.203)	6.42 (5.67,7.79) [0.45]	7.56 (6.87,8.26) [0.28]	5.67 (4.68,6.59) [0.23]	4.94 (0.000)
lags chosen by BIC	rec AR(4) f'cast	(2, 1)	spline, 3 knots	-0.350 (0.119)	7.28 (5.72,13.53) [1.13]	7.43 (5.44,9.22) [0.65]	5.53 (2.62,7.81) [0.53]	2.74 (0.015)
73:I-94:IV	SPF f'cast	(2,8)	constant	-0.160 (0.117)	na	6.92 (2.75,11.09) [0.70]	6.92 (2.75,11.09) [0.70]	na
lags chosen by AIC 73:I-94:IV	SPF f'cast	(3, 4)	constant	-0.217 (0.115)	na	7.05 (3.90,10.21) [0.53]	7.05 (3.90,10.21) [0.53]	na
lags chosen by BIC 73:I-94:IV	SPF f'cast	(2, 2)	constant	-0.309 (0.122)	na	7.20 (6.04,9.17) [0.38]	7.20 (6.04,9.17) [0.38]	na
73:I-94:IV	SPF f'cast	(2,8)	spline, 2 knots	-1.067 (0.202)	na	8.45 (7.98,9.17) [0.24]	6.23 (5.74,6.69) [0.13]	8.60 (0.000)
lags chosen by AIC 73:I-94:IV	SPF f'cast	(3,8)	spline, 2 knots	-1.196 (0.204)	na	8.37 (7.98,8.99) [0.22]	6.19 (5.77,6.59) [0.13]	8.34 (0.000)
lags chosen by BIC 73:I-94:IV	SPF f'cast	(2, 1)	spline, 2 knots	-0.562 (0.118)	na	7.92 (7.07,9.10) [0.40]	6.21 (5.30,7.23) [0.28]	4.52 (0.001)

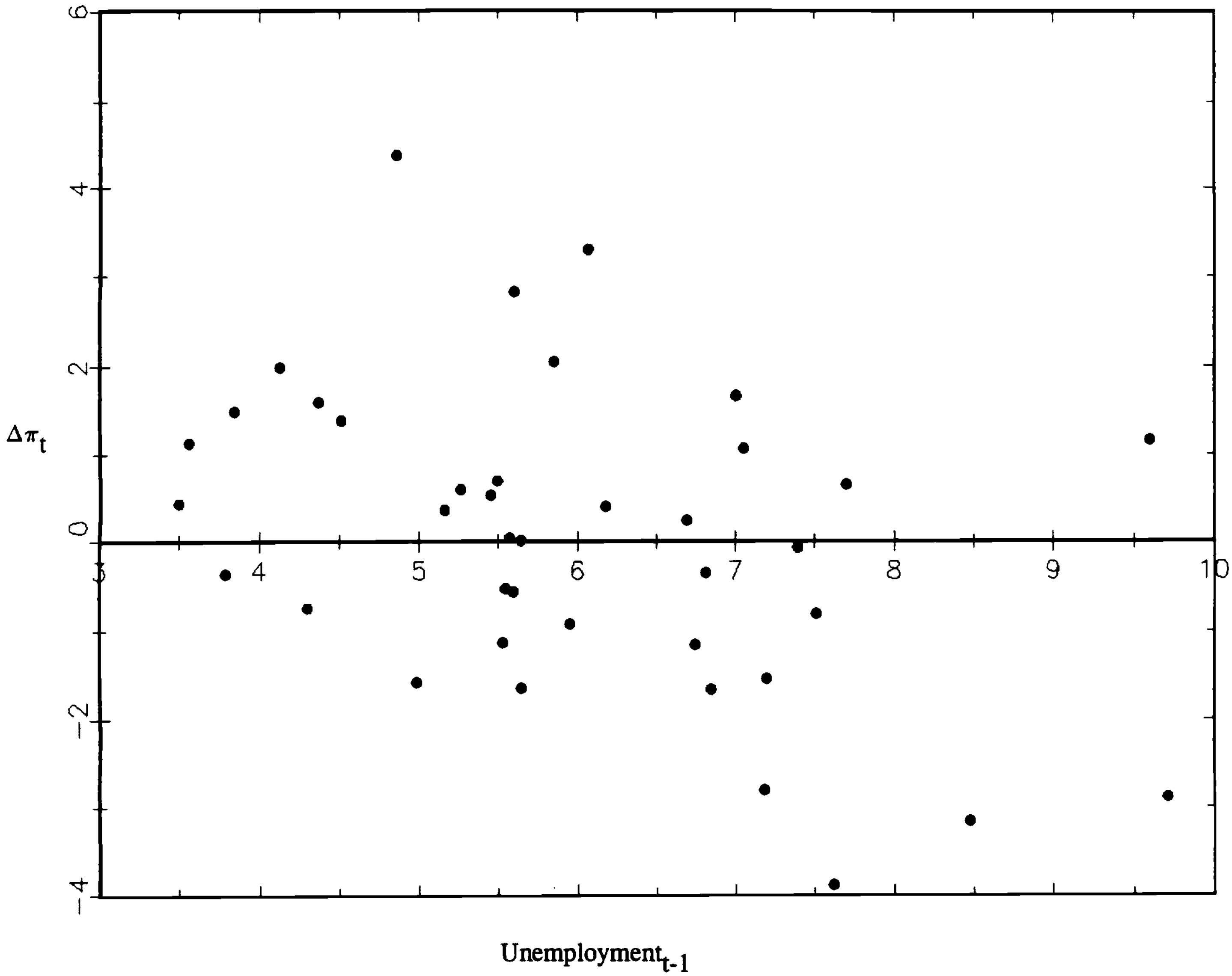

Figure 1. Year-to-year change in CPI inflation vs. total unemployment in the previous year, annual data for the United States, 1955 - 1994

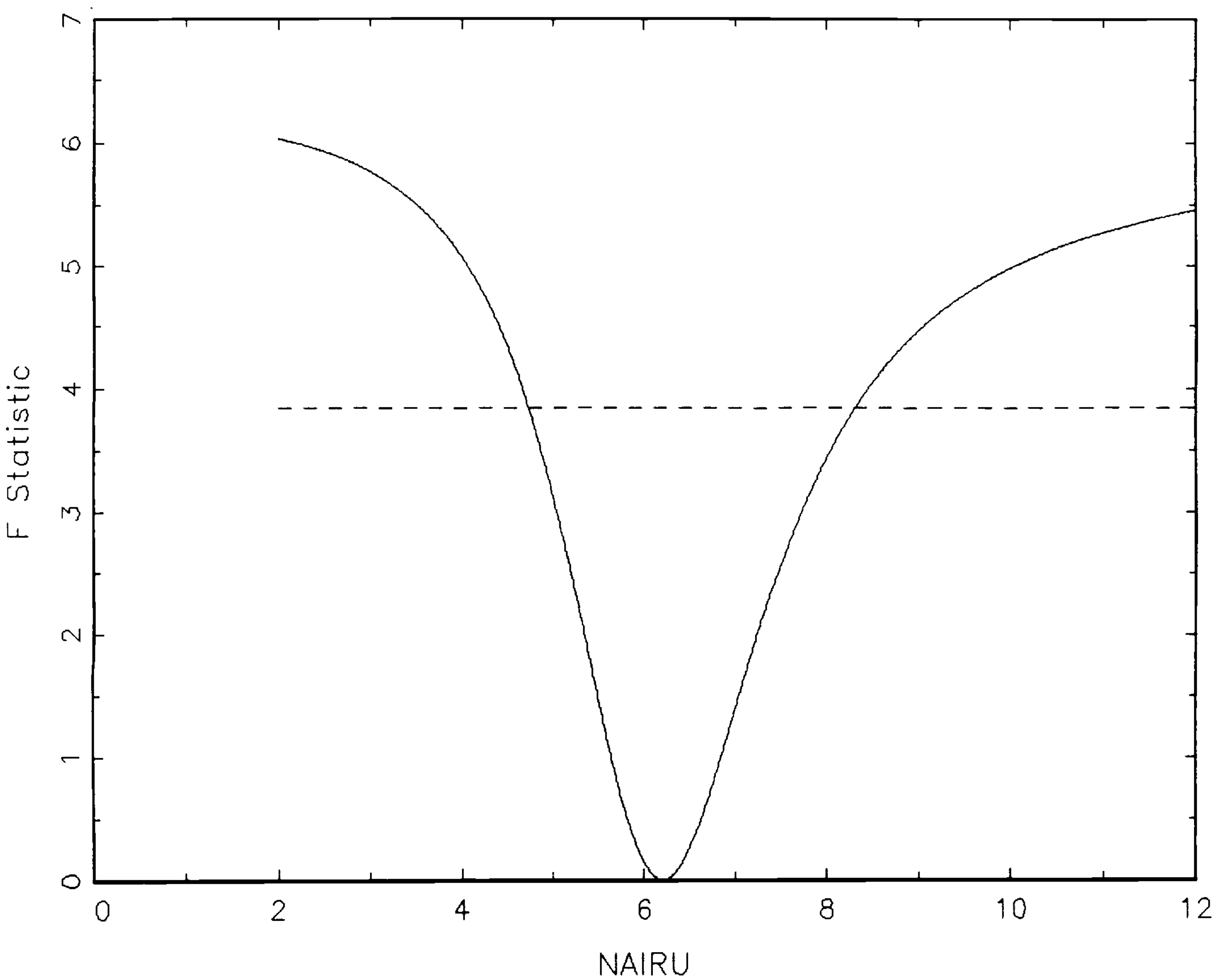

Figure 2. F-statitistic testing of the hypothesis $\bar{u} = \bar{u}_0$,
with $\bar{u}_0$ plotted on the horizontal axis, for specification (a) in table 1

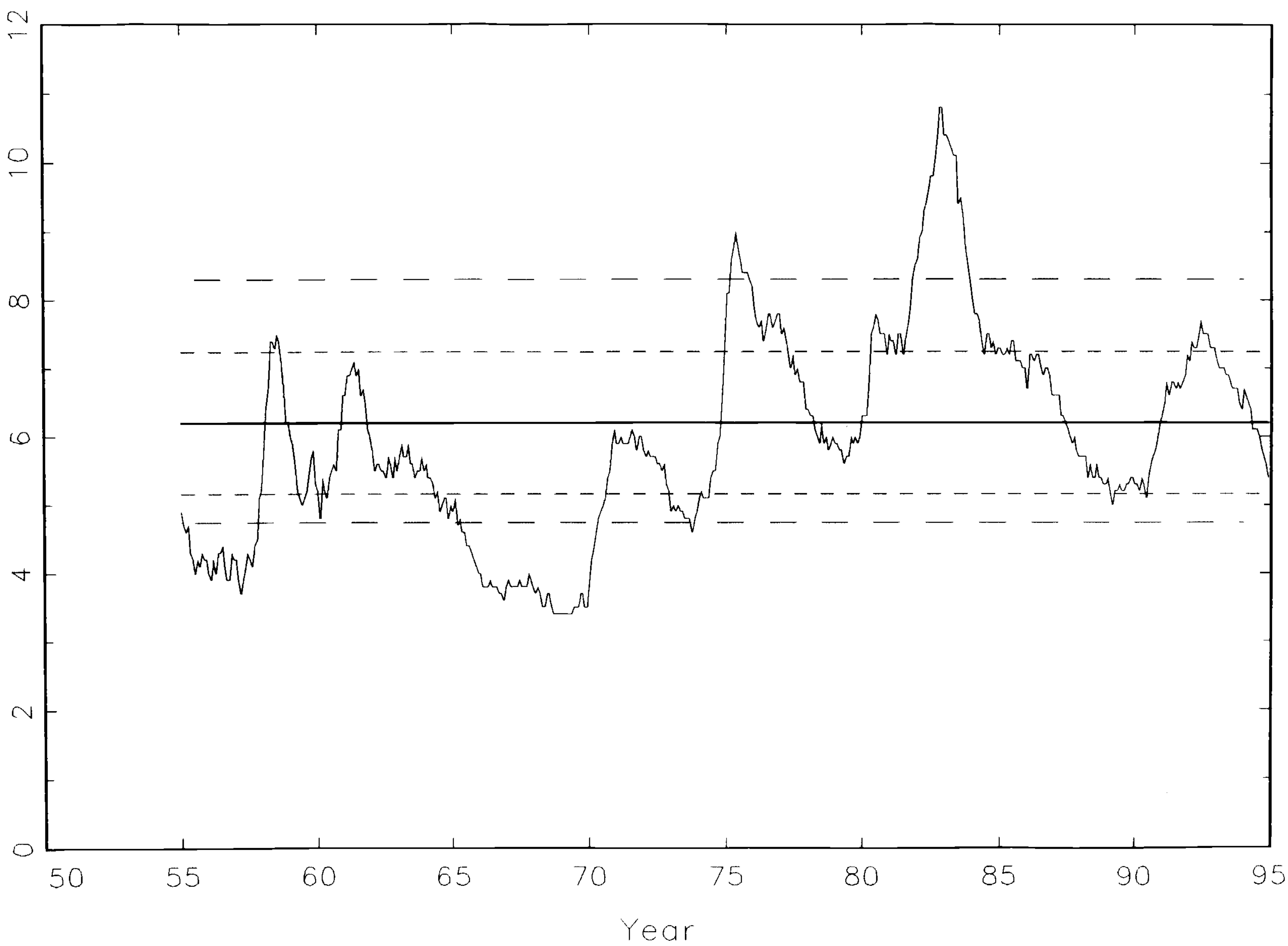

Figure 3. Constant estimate of NAIRU, 95% Gaussian confidence interval (long dashes), delta method confidence interval (short dashes), and unemployment. $\pi_t^e = \pi_{t-1}$, monthly, 55:1-94:12 (table 1, model (a))

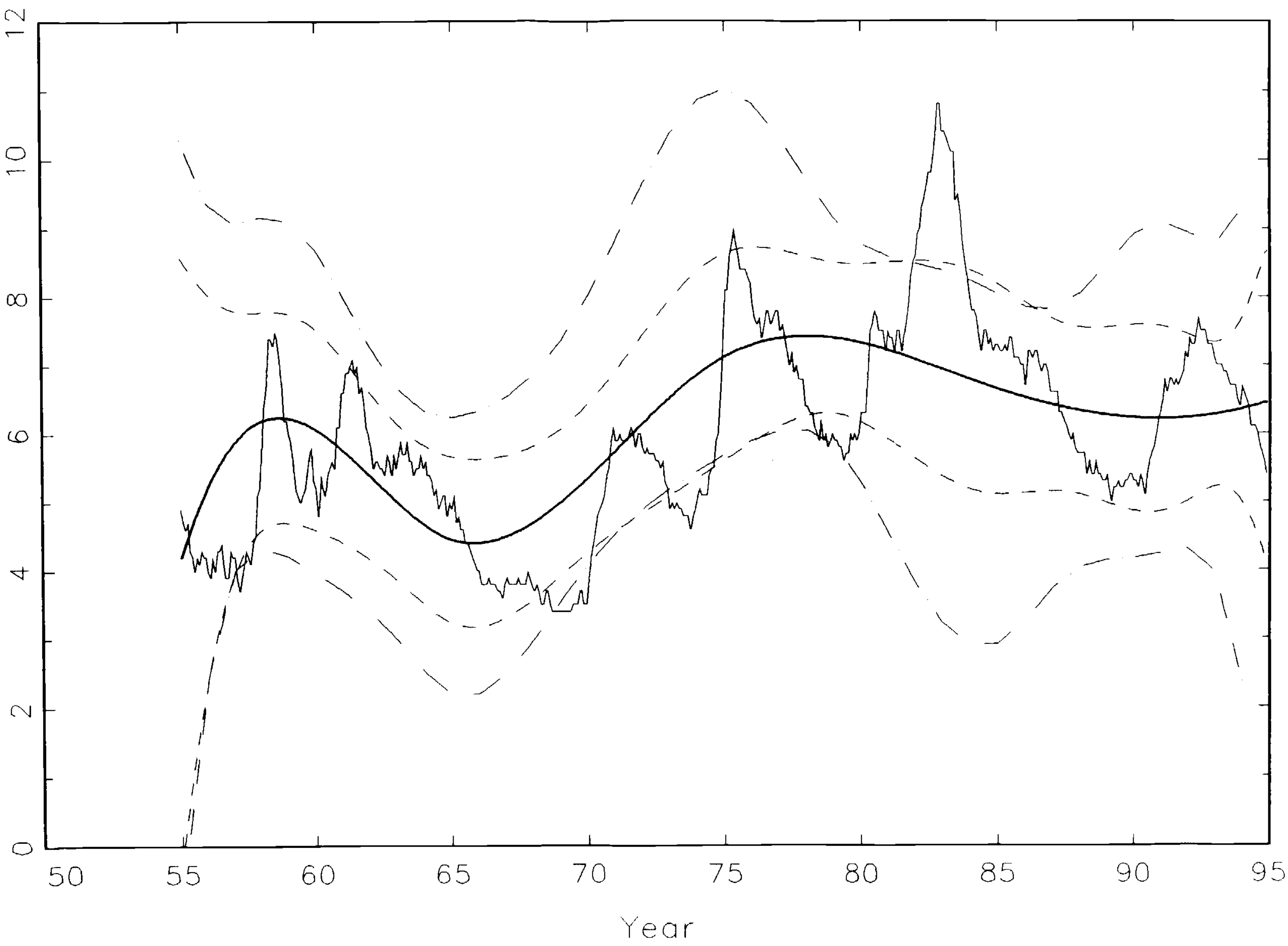

Figure 4. Spline estimate of NAIRU, 95% Gaussian confidence interval (long dashes), delta method confidence interval (short dashes), and unemployment. $\pi_t^e = \pi_{t-1}$, monthly, 55:1-94:12 (table 1, model (b))

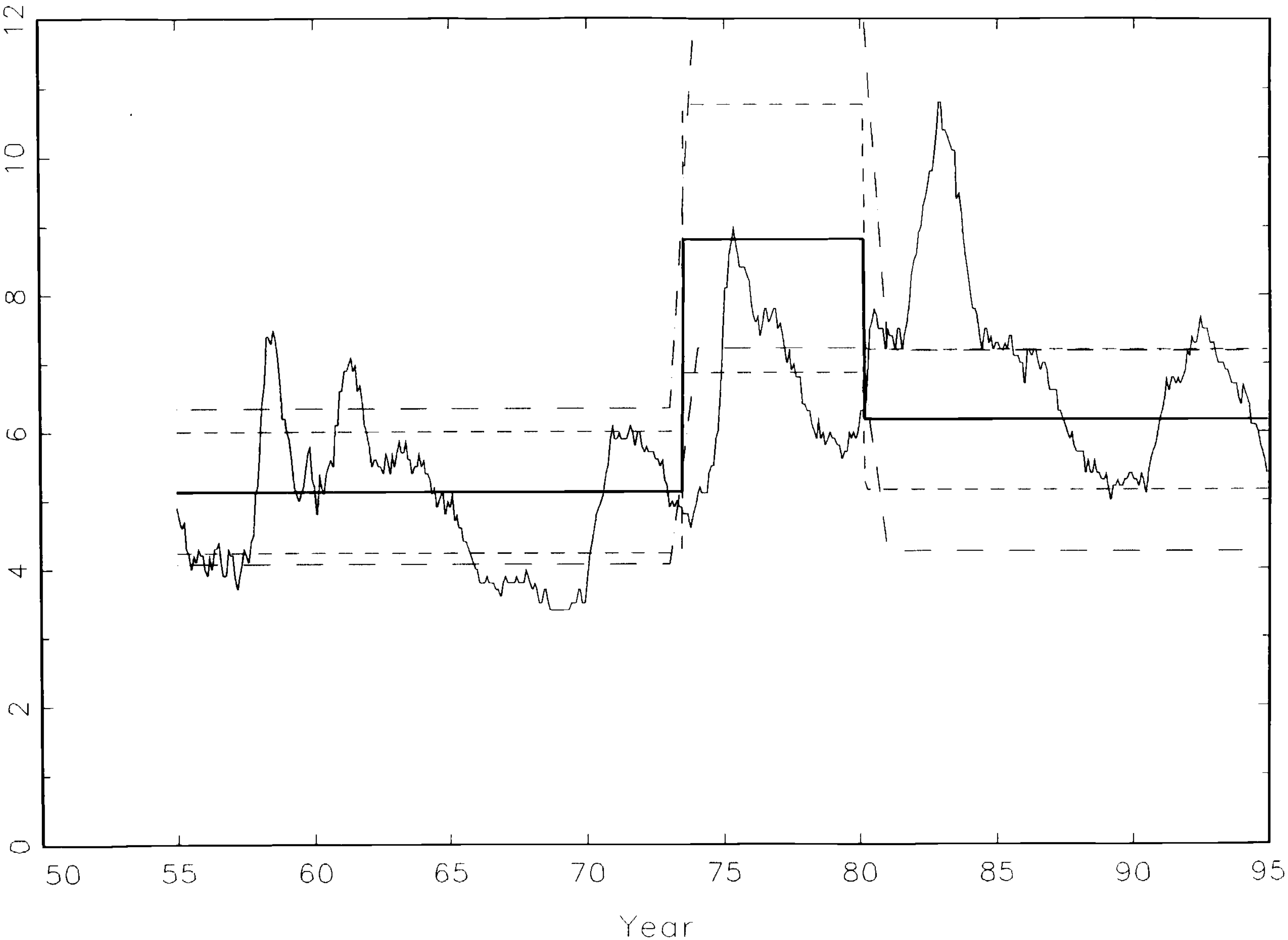

Figure 5. Two-break estimate of NAIRU, 95% Gaussian confidence interval (long dashes), delta method confidence interval (short dashes), and unemployment. $\pi_t^e = \pi_{t-1}$, monthly, 55:1-94:12 (table 1, model (c))

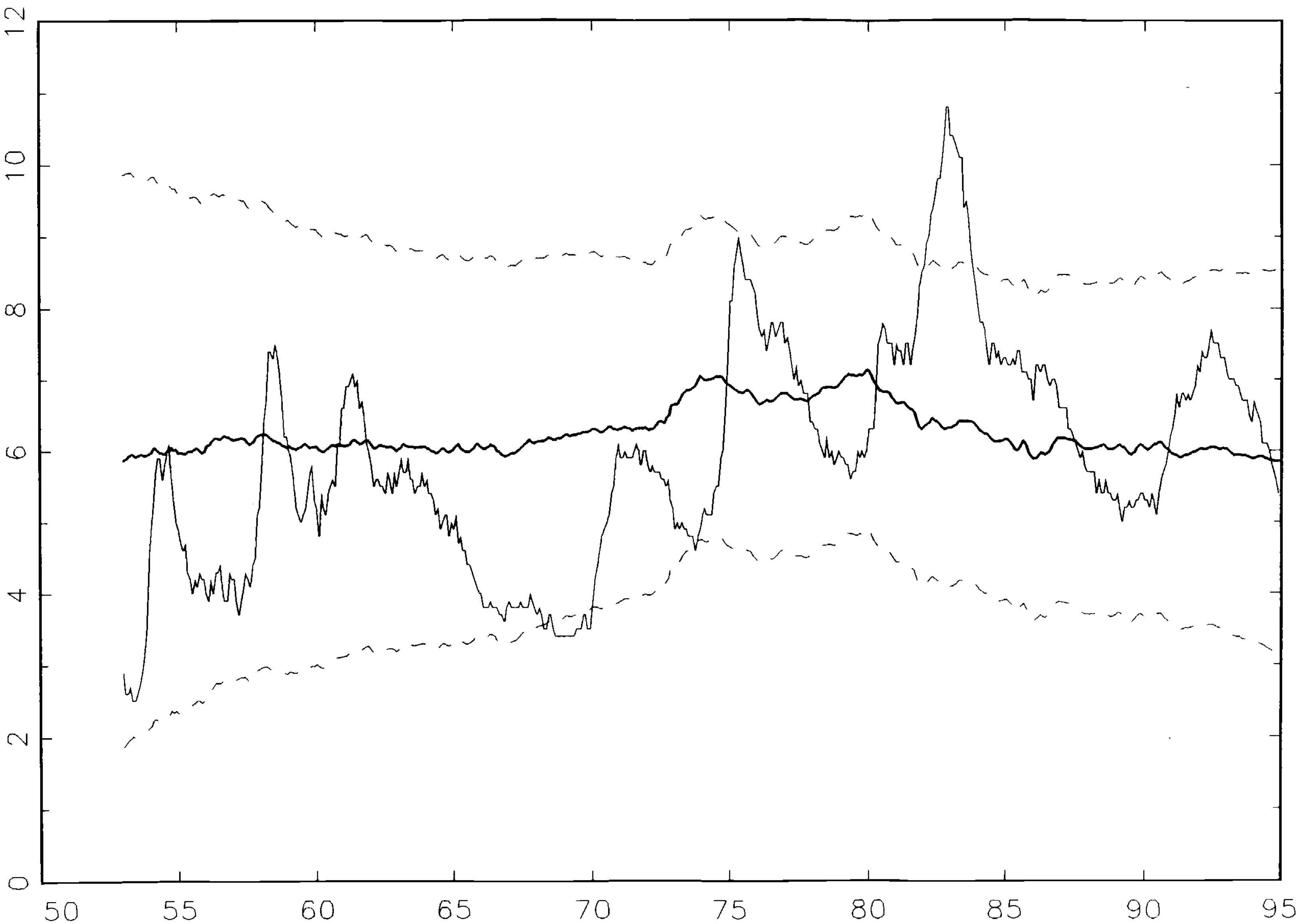

Figure 6. TVP estimate of NAIRU, 95% delta method confidence interval, and unemployment.
λ=0.15, $\pi_t^e=\pi_{t-1}$, monthly, 53:1-94:12

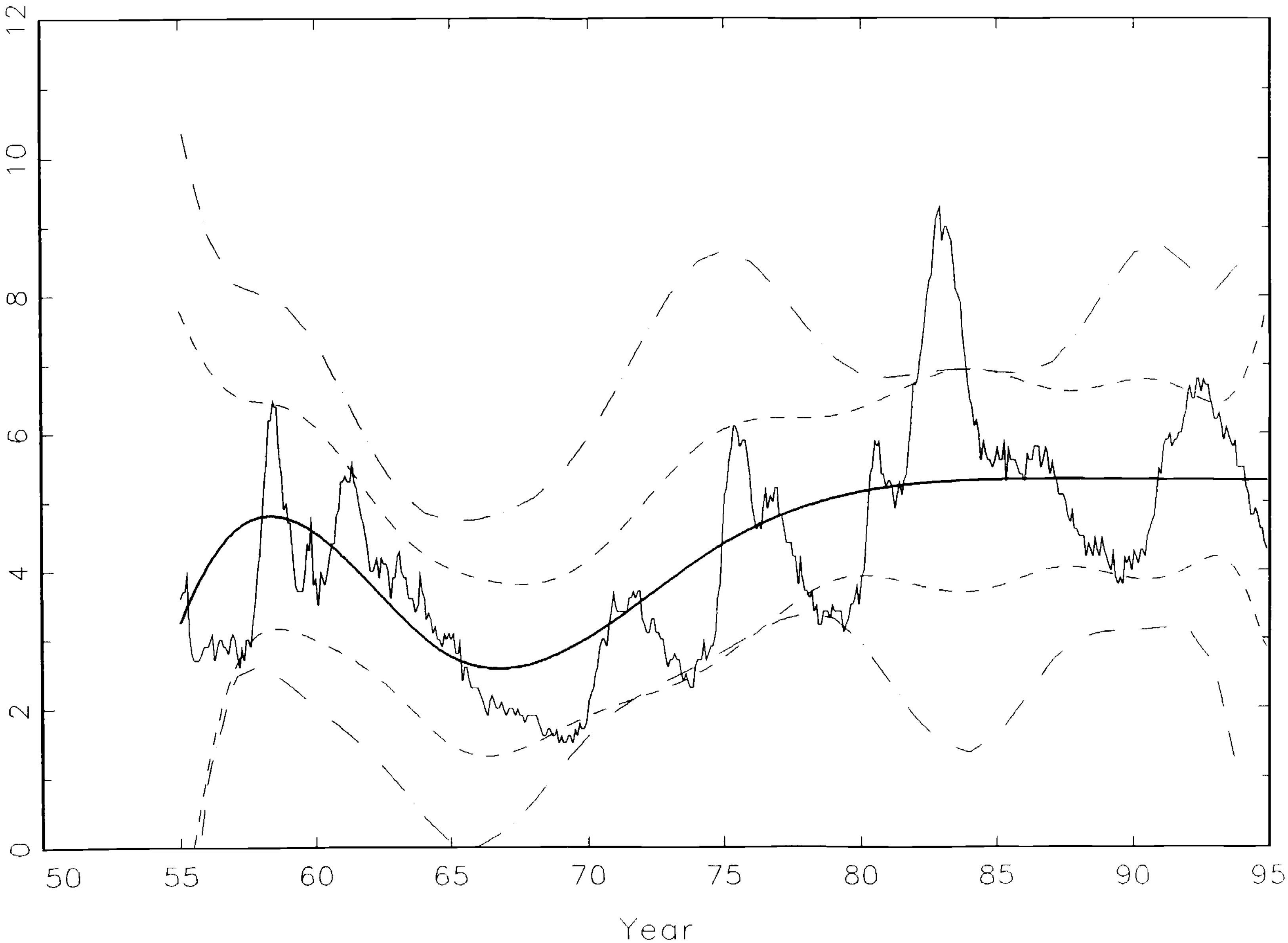

Figure 7. Spline estimate of NAIRU, 95% Gaussian confidence interval (long dashes), delta method confidence interval (short dashes), and unemployment. $\pi_t^e = \pi_{t-1}$, monthly, 55:1-94:12, (12,12) lags, CPI, prime-age male unemployment

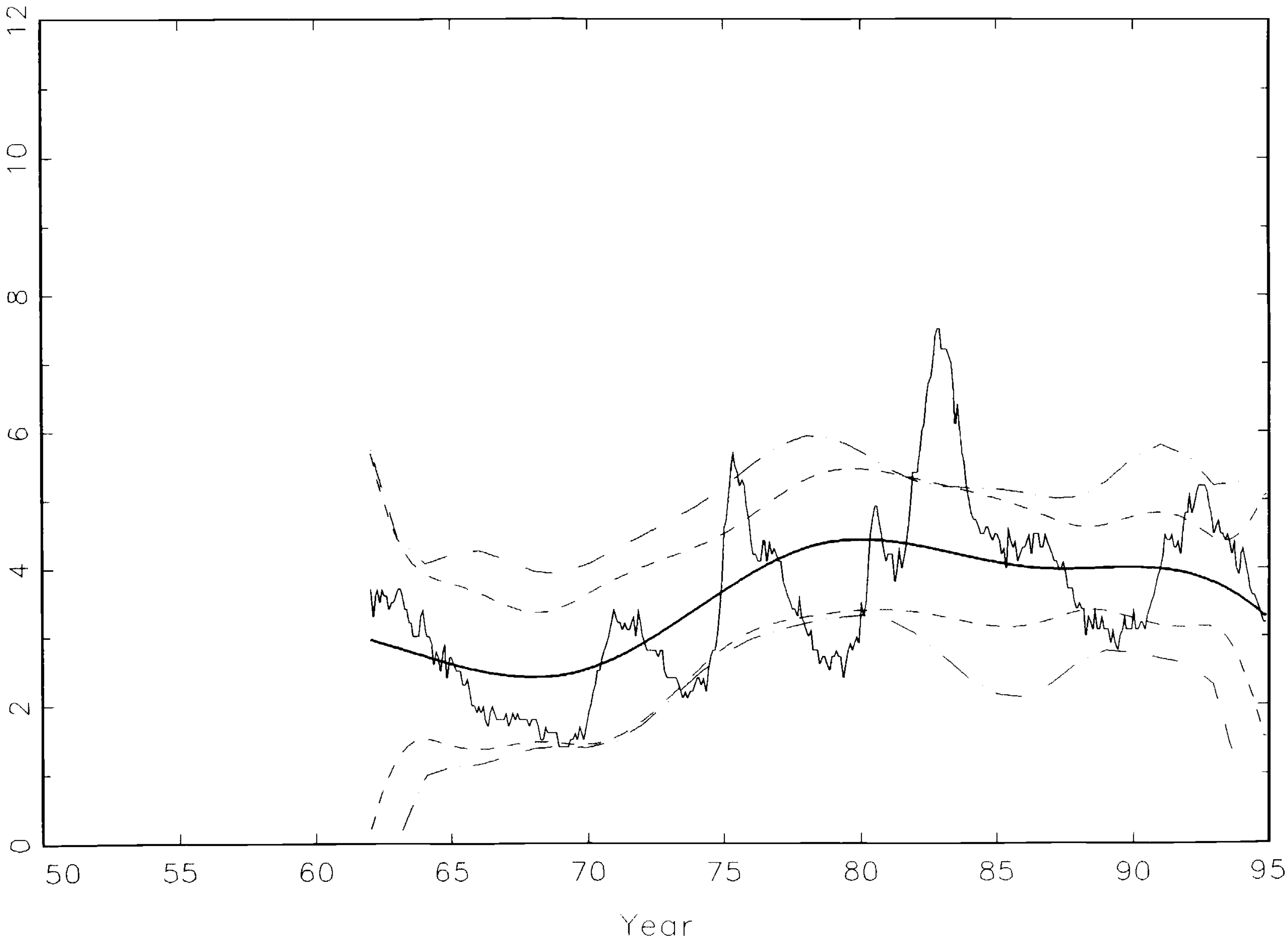

Figure 8. Spline estimate of NAIRU, 95% Gaussian confidence interval (long dashes), delta method confidence interval (short dashes), and unemployment. $\pi_t^e = \pi_{t-1}$, monthly, 62:1-94:12, (12,12) lags, CPI-ex food and energy, married male unemployment

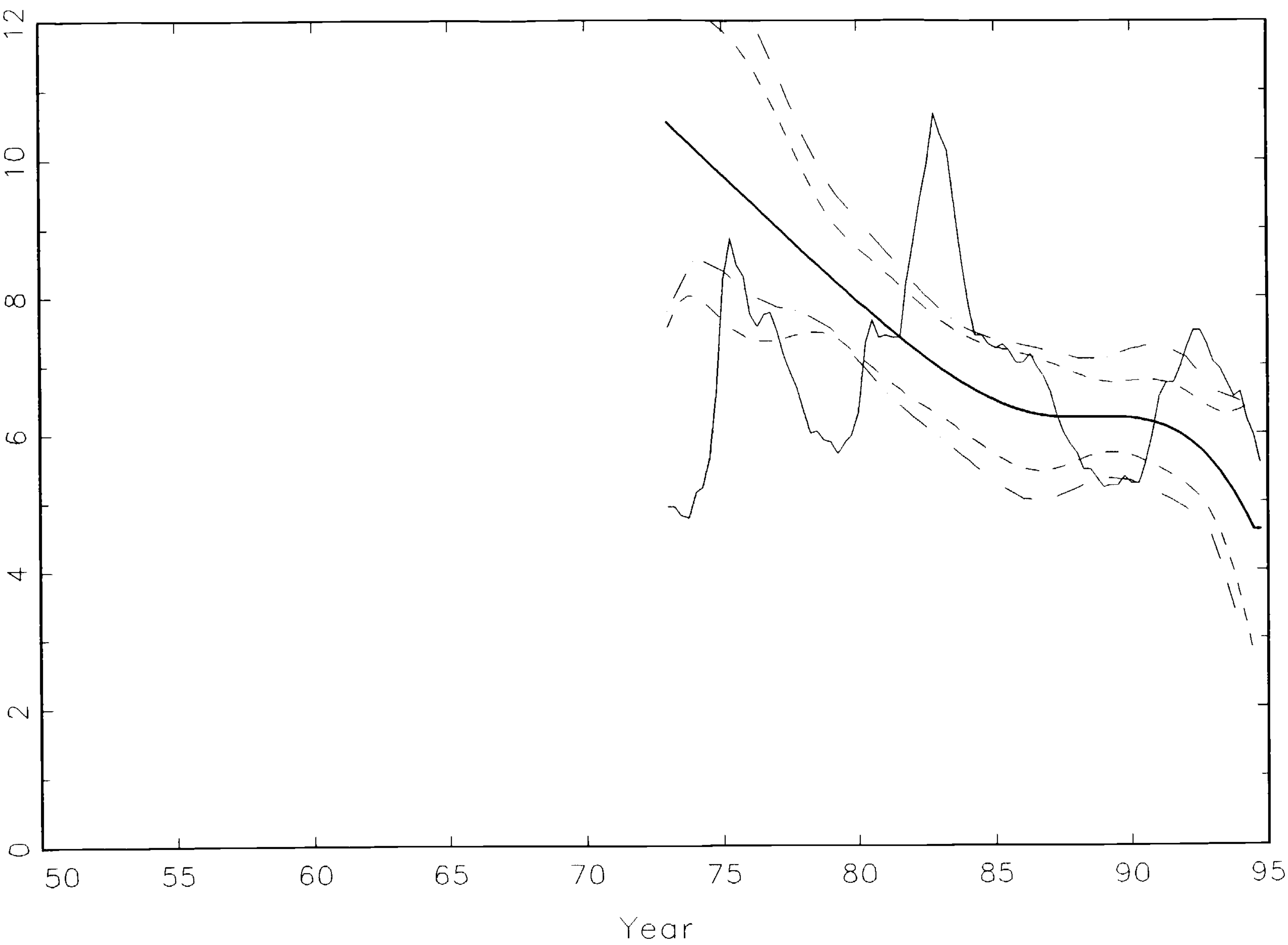

Figure 9. Spline estimate of NAIRU, 95% Gaussian confidence interval (long dashes), delta method confidence interval (short dashes), and unemployment. π_t^e=Survey of Professional Forecasters, quarterly, 73:I-94:IV, BIC lags, GDP deflator, total unemployment

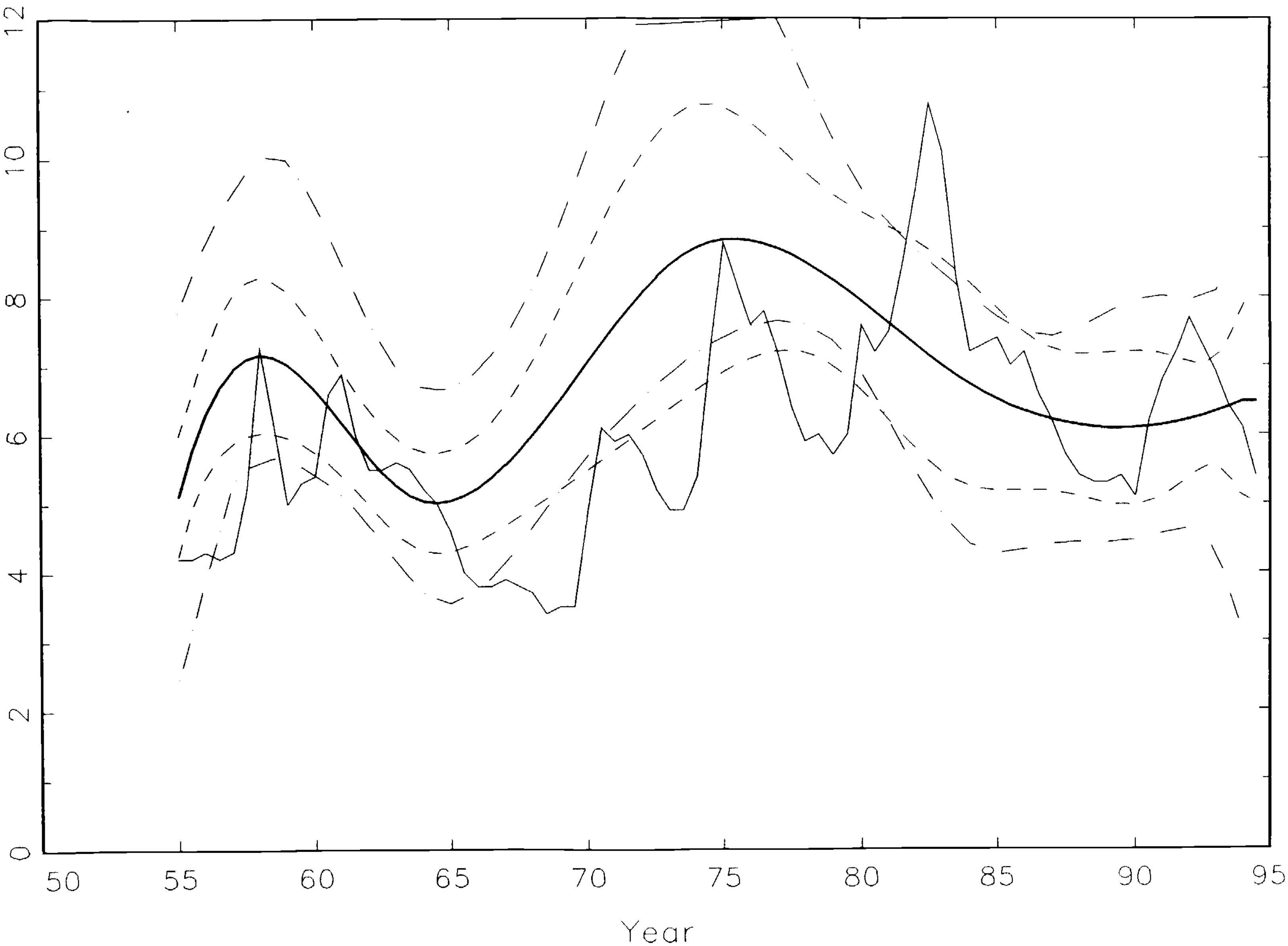

Figure 10. Spline estimate of NAIRU, 95% Gaussian confidence interval (long dashes), delta method confidence interval (short dashes), and unemployment. π_t^e=Livingston survey, semiannual, 55:I-94:II, BIC lags, CPI, total unemployment

Appendix

Results of Monte Carlo Experiment Comparing Delta Method and Gaussian Confidence Intervals

Table A.1

Finite Sample Coverage Rates of Delta Method and Gaussian Confidence Intervals

		- Quantiles of Delta - - Method t-statistic -			- - Monte Carlo Coverage Rates - - Delta Method		- Gaussian -	
$\beta(1)$	$\bar{u}$	0.10	0.50	0.90	90%	95%	90%	95%
		A. Errors drawn from the empirical distribution						
-0.26	6.18	-0.92	-0.01	0.82	0.98	0.99	0.89	0.94
-0.03	10.04	-4.96	-1.21	0.03	0.58	0.64	0.89	0.94
-0.07	5.63	-0.55	0.09	1.04	0.96	0.98	0.88	0.94
-0.40	5.45	-0.92	-0.04	1.16	0.96	0.98	0.88	0.94
		B. Gaussian errors						
-0.26	6.18	-0.92	0.00	0.84	0.98	0.99	0.88	0.94
-0.03	10.04	-4.75	-1.19	0.03	0.59	0.64	0.89	0.94
-0.07	5.63	-0.56	0.09	1.01	0.96	0.98	0.89	0.94
-0.40	5.45	-0.90	-0.05	1.13	0.96	0.99	0.89	0.94

Note: Data generated using a restricted VAR(1) as described in the text. Based on 10,000 Monte Carlo replications, with 80 observations (plus 60 startup draws).

Table A.2

Finite-Sample Power of Delta Method and Gaussian Confidence Tests

Entries are probability of rejecting the null hypothesis $\bar{u}_H$=6.18

	- - Size Unadjusted - - (asymptotic critical values)				- - - Size Adjusted - - - (adjusted critical values)			
	Delta Method		Gaussian		Delta Method		Gaussian	
$\bar{u}$	10%	5%	10%	5%	10%	5%	10%	5%
2.00	0.56	0.46	1.00	0.99	0.74	0.66	1.00	0.99
3.00	0.55	0.43	0.98	0.97	0.73	0.65	0.98	0.97
4.00	0.47	0.34	0.90	0.84	0.70	0.60	0.89	0.83
5.00	0.22	0.13	0.53	0.41	0.48	0.35	0.50	0.38
6.00	0.03	0.01	0.12	0.07	0.12	0.06	0.11	0.06
6.18	0.02	0.01	0.11	0.06	0.10	0.05	0.10	0.05
7.00	0.08	0.04	0.35	0.24	0.28	0.16	0.32	0.21
8.00	0.32	0.19	0.84	0.75	0.62	0.48	0.82	0.73
9.00	0.47	0.33	0.98	0.97	0.71	0.61	0.98	0.96
10.00	0.51	0.39	1.00	0.99	0.72	0.63	1.00	0.99

Note: Data generated using a restricted VAR(1) with $\beta(1)$=-0.26, as described in the text. The column headers 10% and 5% refer to the nominal level of the test (this is 100% minus the nominal confidence level of the associated confidence interval). The "size unadjusted" results are the rejection rates computed using the asymptotic critical value from the χ^2_1 distribution. The "size adjusted" results are computed using the finite-sample critical value taken from the Monte Carlo distribution of the test statistic computed under the null $\bar{u}$=6.18. Based on 10,000 Monte Carlo replications, with 80 observations (plus 60 startup draws).