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ON THE NUMBER AND SIZE OF NATIONS

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ABSTRACT

This paper studies the equilibrium determination of the number of political jurisdictions in different political regimes, democratic or not, and in different economic environments, with more or less economic integration. We focus on the trade off between the benefits of large jurisdictions in terms of economies of scale and the costs of heterogeneity of large and diverse populations. Our model implies that: i) democratization leads to secessions; ii) without an appropriate redistributive scheme (which we characterize) in equilibrium one observes an inefficiently large number of countries; iii) the equilibrium number of countries is increasing in the amount of economic integration. We also study the welfare effects of economic integration and free trade when the number of countries is endogenous.

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1. Introduction

In the last few years national borders have been redrawn to an extent which is rather exceptional for modern peacetime history. On the one hand, several countries[1] have disintegrated (Yugoslavia, the former Soviet Union, Czechoslovakia) and in other countries movements for regional autonomy or even independence have gained larger support (Canada, Spain, and Italy for example). On the other hand, Germany has reunified, and the European Union is moving toward economic integration and, to a much lesser extent, to some form of political integration. On balance, one can detect a tendency toward political separatism with economic integration. Recent border changes have been often accompanied (and caused by) the democratization process which has swept the world.

These changes have been so remarkable that some observers begin to wonder whether the current nation states in Europe are becoming obsolete, threatened by regional movements from below and supernational economic integration from above. For instance, Drèze (1993) argues for a "Europe of regions," namely an economically fully integrated area with politically independent regions within the framework of the European Union. Similar issues can be raised with respect to Quebec separatism in the context of NAFTA.

This paper sets up a simple politico-economic model to address both normative and positive questions concerning the number and size of nations, issues to which economists have devoted relatively little attention. Specifically, we ask the following questions:

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[1] Throughout this paper we use the words "country", "nation" and "political jurisdiction" interchangeably.

a) What are the size and number of nations that would result from a democratic process in which secessions and formations of larger political unities are allowed?

b) What are the welfare properties of the equilibrium size and number of nations?

c) What is the relationship between size and number of nations and economic integration? Does higher economic integration increase or reduce the equilibrium size of nations?

d) What are the welfare implications of higher economic integration when the number of nations is endogenous?

e) What are the size and number of nations that would be determined by the actions of rent-maximizing dictators? How do they differ from the democratic outcome?

Needless to say, the process of country formation and secessions and, therefore, the answers to all these questions depend upon an intricate web of geographical, historical, cultural, ethnic, ideological, military, political and economic forces. No single model can come even close to a complete and exhaustive treatment of all the relevant variables. Our aim is to address a specific trade off between the benefits of large political jurisdictions and the costs of heterogeneity in large populations. By modeling this trade off explicitly, we can provide some answers, which, although model specific, may serve as a stepping stone for further research. Our main results can be summarized in four points:

1) Democratization leads to secessions; one should observe fewer countries in a non democratic world than in a democratic one.

2) Without an appropriate system of redistribution within each country, the democratic process leads to an inefficiently large number of countries; namely, in the voting equilibrium,

more countries are created than with a benevolent world "social planner."

3) The equilibrium number of countries is increasing with the amount of economic integration.

4) In the absence of appropriate redistribution schemes that keep the number of countries at the efficient level, greater economic integration can reduce average welfare, when the number of countries is endogenous.

Relatively few economists have provided formal models of country formation. Friedman (1977) argues that countries are shaped to maximize their joint potential tax revenues, net of collection costs. Economies of scale and the need to limit international labor mobility imply large nations. Buchanan and Faith (1987) study how the option of secession places an upper limit on the tax burden that a ruling majority can impose on the minority. Casella and Feinstein (1990), and Casella (1992) study the relationship between economic and political integration. In their model, perfectly mobile individuals with heterogeneous endowments can choose what market to enter and which political institution to join. In equilibrium, the number of markets and the number of political institutions change with the level of development. Wei (1992a,b) develops modified versions of the Casella-Feinstein model in order to study secessions. In his model, the incentive to secede is higher for less developed countries, while more developed regions are more likely to unify into one nation with a federal system. Bolton and Roland (1993) present a model in which secessions are costly, but a majority might vote for a secession in a regional referendum because the median voter's benefits from the expected change in redistribution policy after the secession outweighs the efficiency loss.

What distinguishes our paper from these previous contributions is our emphasis on questions of optimality and stability of the equilibrium number of countries in different politico-economic regimes.

This paper is organized as follows: Section 2 presents the basic model and discusses the assumptions. Section 3 derives the solution chosen by a social planner who maximizes the sum of individual utilities. In Section 4 we define and characterize the equilibrium outcome in a democratic world with no social planners. In Section 5 we extend the model to include the effects of economic integration. Section 6 presents the solution chosen by Leviathans interested in maximizing their rents. The last section discusses extensions for future research.

2. The Model: Assumptions and Interpretation

We focus on a trade off between the benefits of large countries and the costs of heterogeneity in large populations. Larger political jurisdictions bring about several benefits. For example, the per capita cost of any non rival public good decreases with the number of people who finance it.[2] Clearly, beyond a certain point, these economies of scale may be counterbalanced by congestion, coordination problems, etc., but, up to a point, economies of scale prevail. Taking advantage of the economies of scale in relatively large countries, may, however, come at a "political cost." A larger population is likely to be less homogeneous. In other words, the average cultural or preference distance between individuals is likely to be positively correlated with the size of the country. In small, relatively homogeneous countries, public choices are closer to the preferences of the average individual than in larger, more heteroge-

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[2] Easterly and Rebelo (1993) provide empirical evidence of this effect.

neous countries. Barro (1991) succinctly captures this trade off:

> "We can think of a country's optimal size as emerging from a trade off: A large country can spread the cost of public goods, such as defining a legal and monetary system and maintaining national security, over many taxpayers, but a large country is also likely to have a diverse population that is difficult for the central government to satisfy."

To be concrete and keep the model as simple as possible, we consider only one public good, which identifies each nation. We call this public good the "government." With this term we identify a bundle of administrative, judicial, economic services and public policies associated with a particular government. While, in reality, the functions of a government are clearly multidimensional, we collapse them into a single dimension to avoid dealing with voting on more than one issue, a complication which is not our focus. The range of possible "governments" is normalized in the segment [0.1]. The world population has mass 1. Individuals have ideal "points," (i.e., ideal "governments") uniformly distributed on the segment [0,1], and their utility is decreasing with the distance from their government to their ideal.[3]

A "government" is a nonrival public good. Thus, every country needs a single government, and the citizens of each country have to finance and can take advantage of the only government of their country. The world needs at least one government, thus, $N \geq 1$ where N is the number of countries in the world, thus, the number of governments. Each government

[3]We could have the world modeled as a circle, rather than a segment, and a more general distribution of individual preferences. We make these assumptions to keep the algebra as simple as possible.

costs k, regardless of the size of the country.[4] Every individual has the same, exogenous income, Y, and pays the lump-sum tax t_i.[5] Thus, the utility of individual I is:

$$U_i = g(1 - al_i) + y - t_i \tag{1}$$

where g and a are two positive parameters and l_i is the preference distance from individual I to his government. The utility function is thus linear in consumption. The parameter g measures the maximum utility of the public good, when $l_i = 0$. The parameter a measures the loss in utility which an individual faces when the type of government is *far* from her preferred type. A sufficient condition to ensure that a higher g increases utility for every type of government is $a < 1$. This assumption is not necessary for the results of this paper, but is a natural assumption if we interpret g as a measure of "government services" and a as the 'marginal utility' of government services when the government is located at a distance l_i from the individuals preferred type.

Thus far, we have been silent on the geographical distribution of individuals. Clearly, in our model, individuals who are close to each other in terms of preferences have an incentive to form a country together. If there were no relationship between location and preferences, then there would be no presumption that a country would be geographically connected. A country would be like a "club." From the point of view of realism, this problem can be addressed in two ways. One way is to impose costs on governments of countries which are not geographically connected.

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[4] This assumption can be easily relaxed. We could model the costs of government as: $k = \alpha + \beta s$, where $\alpha > 0$, $\beta > 0$ are parameters, and s is the size of the country. As long as $\alpha > 0$, our results generalize, without qualitative changes.

[5] Since income is exogenous, a lump-sum tax is equivalent to a proportional tax on income.

The second way is to assume that individuals who are close to each other in preferences are also close to each other geographically.

Both assumptions are reasonable. Here we adopt the simplest possible one by imposing that the *geographical and the preference dimensions coincide*. Therefore, the distance l_i, in (1) captures *both* the geographical and the preference distance. This interpretation implies that individuals who live far from the government, say, far from the political capital of the country, incur costs because of this distance *and* have preferences that are distant from those prevailing in the political capital where the government is located. What we really need is only that if two individuals live far from each other, they are also distant in preferences.[6] For example, a Spanish citizen living close to the border with France bears some costs because she is far from Madrid *and* because her preferred type of public policies are different from those chosen by the Spanish government.

We assume that individuals are *not* mobile. Country borders are endogenously determined in our model, but the geographical location of each individual is fixed. In order to analyze geographical mobility, we would need to break the identity between preferences and geographical location, and endogenously determine their equilibrium relationship; this is a task that we leave for future research.

Note that the literature on mobility across localities provides some theoretical underpinnings for our assumption. One of the most common results in this literature is that individuals with different preferences or characteristics (i.e. income) locate in the same community, namely stratification is achieved

[6]We conjecture, although we do not have a formal proof, that our results would hold qualitatively if the geographical and cultural distance were positively but not perfectly correlated.

in equilibrium.[7] That is, for a *given* number of jurisdictions, individuals sort themselves out in "homogenous," stratified groups. In a sense, we are assuming that this stratification has already taken place: individuals have already sorted themselves out, so that people with similar characteristics (preferences) live close to each other ("ideology" -, i.e., preferences - determines "geographic location"). Alternatively, one may argue that individuals who, historically, have lived close to each other for centuries have developed similar preferences.

3. The Social Planner Solution

We begin by finding the solution for the case of a world benevolent social planner, who can choose the number and sizes of countries and location of the public good within each country. We assume that the social planner maximizes the sum of individual utilities. Thus, the world social planner's problem is:

$$Max \int_0^1 U_i di \qquad (2)$$

$$s.t. \quad \int_0^1 t_i \, di = Nk$$

The following proposition characterizes the result:

__**Proposition 1.**__ *A social planner maximizing the sum of individual utilities would: I) locate the government in the middle of each country; ii) choose N^* countries of equal size, such that:*

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[7] The classic reference is Tiebout (1956). For a survey see Rubinfeld (1987). More recent contributions include Epple and Romer (1991), in which "sorting" is derived in a model with forward looking voters, and Benabou (1993), in which the overall distribution of types is determined in equilibrium, together with the composition of local communities.

$$N^* = \frac{1}{2}\sqrt{\frac{ga}{k}} \quad (3)$$

provided that $\frac{1}{2}\sqrt{\frac{ga}{k}}$ *is an integer. Otherwise, the efficient number of nations* N^* *is given by either the largest integer smaller than* $\frac{1}{2}\sqrt{\frac{ga}{k}}$ *or the smallest integer larger than* $\frac{1}{2}\sqrt{\frac{ga}{k}}$.

The formal proof is in Appendix, but the intuition is simple. Given a certain number of countries, the social planner locates the government in the middle of each one: this choice minimizes average distance for given costs of governments. Given the symmetry of the problem, and, in particular, the uniform distribution of preferences, average utility is maximized by choosing countries of equal size. The efficient number N^* optimizes over the trade off between economies of scale and average distance, i.e., between average taxes and average distance. Not surprisingly, the efficient number N^* is increasing in the costs of distance (g and a) and decreasing in the cost of the government (k).

Individual utility depends on the distribution of individual taxes t_i. If everybody pays the same tax, than $t=k/s$, where $s^* = \frac{1}{N^*}$ is the size of each country. In this case, not everyone achieves the same level of utility. Individuals close to the "borders" of each country are less happy that their compatriots located close to the middle. If the social planner wishes to achieve the same level of utility for everyone, he should compensate border citizens by redistributing away from those citizens in the middle of the country. The following tax-transfer scheme would ensure the same utility to every citizen:

$$t_i = ga\left(\frac{s^*}{4} - l_i\right) + \frac{k}{s^*} \quad (4)$$

Equation (4) shows that first-best taxes are decreasing with the distance from the government, and, beyond a certain distance, they become transfers. A perfectly informed social planner could always implement this type of scheme. In practice, however, this scheme is easily applicable only if the relevant concept of distance is the geographical one.[8] If the "distance" is along a preference-culture dimension, than the application of taxes as a function of preferences becomes much more problematic.[9]

An interesting empirical implication of the "equitable" social planner solution is those border regions receive a preferential fiscal treatment. Once again, "border region" has to be interpreted as a region at the geographical border <u>and</u> culturally distant from the political capital.[10]

4. The Stable Number of Nations

We now study the equilibrium number and size of nations without a social planner. We define and allow the following rules for border redrawing:

A. Each individual at the border between two countries can choose what country to join.

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[8] Note that in this case the price of land and housing may be the market solution which automatically enforces this tax/transfer scheme.

[9] There is a connection here in the literature on "revelation mechanisms"; a link that we do not develop.

[10] A fragment of evidence in this direction: the Italian regions with "special status" have received large transfers from the other Italian regions, even after controlling for their level of income. These "special status" regions are five regions located at the geographical border <u>and</u>, in some cases, with ethnic and linguistic minorities.

B. A new nation can be created, or an existing nation can be eliminated, if the modification is approved by majority rule in each of the existing countries affected by the border redrawing.

A configuration of *N* countries is:

1) an A-equilibrium if the borders of the *N* nations are not subject to change under rule A;
2) A-stable if it is an A-equilibrium and it is stable under rule A;
3) a B-equilibrium if it is A-stable and no new nation is created or no existing nation is eliminated under A-stable applications of rule B. That is, any A-stable proposed modification to create or eliminate a country is rejected by majority voting in at least one of the affected countries; and
4) B-stable if it a B-equilibrium and it is stable under rule B.

Throughout this paper, when we use the adjective "stable" without further qualifications we mean "B-stable."

Rule A captures the idea that each individual should not prefer to live in a different country from the one she belongs to. We are assuming that each citizen (or groups of citizens) living at the border between two countries cannot be prevented from joining his or her preferred county. Individuals are not geographically mobile; hence, borders change whenever a marginal individual (or groups) decides to move from a country to another.

Rule B allows for the direct creation of a new country, or the elimination of a given country, through majority-rule referenda held in each of the countries which are affected by the border change. Definition 3) implies that when groups of countries contemplate redrawing borders according to rule B, they consider only a new configuration of countries which is stable under rule A. In other words, proposals of border changes that are not A-stable are not admissible. Definition 4) focuses on equilibria that are globally stable under the dynamics implied by rules B: that is, if a configuration of N nations is B-stable, for any different configuration of N' nations, the (repeated) application of rule B will lead the system to converge to the B-stable configuration. In the Appendix we discuss an alternative rule B': a country can be created or eliminated if the border redrawing is approved by majority rule in each of regions which would constitute *new* countries under the proposed modification. We show that the B-stable configuration is also an equilibrium under this alternative rule.

These rules intend to provide the basic framework that regulates border redrawing in a democratic system, in which individuals can freely decide which country to belong to and borders can be redrawn through majority voting. The positive and normative properties of the equilibrium outcomes which these rules generate can then contribute to assess the positive and welfare implications of the restrictions on free border redrawing which have often been imposed by governments and international agreements.

One should note that rules A and B, *per se*, do not allow the *unilateral* creation of new countries by groups of individuals, an issue which we address below by showing that a B-stable configuration of countries is also stable with respect to unilateral secessions.

We make the following two assumptions about public policy and taxation:

(I) within each country, the location of the government is decided by majority rule. The vote on the type (i.e., location) of government is taken after the borders of the country have been established.

(ii) In each country taxes are proportional to income, with the same tax rate for every citizen.

The first assumption, realistic enough, implies that decisions about policy are taken after a country is formed. A straightforward application of the median voters' theorem implies that the government is always located in the middle of the country. The second assumption implies that, within each country, every citizen pays the same tax.[11] The extension to different taxes for different individuals would imply voting, within each country, on more than one dimension, namely the location of the government and the levels of individual taxation. This extension is not developed here.

We begin by considering the consequences of Rule A.

Proposition 2. *A configuration of countries is A-stable only if all countries have equal size. A configuration of N equally sized countries is A-stable if and only if:*

$$N < \sqrt{\frac{ga}{2k}} \tag{5}$$

Proof. See Appendix A.2.1.

First, Proposition 2 suggests that if all the countries do not have the same size, the equilibrium is not A-stable. In fact, the individual at the border between two countries of different sizes in general will not be indifferent between the two coun-

[11] The assumption can be viewed as an indirect restriction on side payments. Clearly, if side payments were completely unrestricted and transactions among individuals were costless, an efficient configuration of nations would result (Coase theorem).

tries.[12] Moreover, only countries which are not "too small" are A-stable. When countries are below a certain minimum size, any perturbation which increases the size of a country would induce even more people to join it. Thus, the size of the initial perturbation would be amplified.

Clearly, the simplifying assumption that individuals are uniformly distributed both geographically and ideologically is crucial for the result that countries of different size cannot constitute A-stable equilibria. Conceptually, it is easy to see how a nonuniform distribution of individuals would lead to equilibria with countries of different size.

We now proceed to study the consequences of rule B. Because of Proposition 2 and our definition of B-equilibrium, we can focus on countries of equal size. The main result is given by the following Propositions 3 and 4 (The proofs are in Appendix A.2).

Proposition 3. *A configuration of nations is a B-equilibrium if and only if a) all nations have equal size and b) their number $\tilde{N}$ is given by the largest integer smaller than:*

$$\sqrt{\frac{ga}{2k}} \tag{6}$$

Thus, Proposition 2 implies that any N below $\sqrt{\frac{ga}{2K}}$ is an A-equilibrium. Proposition 3 implies that only the largest N below that critical value is the unique B-equilibrium.

The proof of Proposition 3 is rather laborious, but the main structure can be briefly summarized as follows. First we focus on any configuration of N equally sized countries, and we derive the conditions under which, given N a majority in at least one

[12] There is only one case of "border indifference" with differently sized countries, but this case, is not A-stable. See the Appendix.

country would object to changes of borders which lead to the creation of $(N+1)$ or $(N-1)$ countries. We show that in each of the existing countries there exists a majority against the creation of a new country if and only if:

$$N\ (N+1) \geq \frac{ga}{2k} \tag{7}$$

This condition ensures that in each country the median reduction in the distance from the government does not compensate for the higher taxes that each citizen must pay when the number of nations increases. In other words, when the condition holds we have that less than fifty percent of the voters in each country gain less in terms of being close to the government, than it would cost to them to live in a smaller nation, in terms of higher taxes. On the other hand, there will always exist at least one country which will veto the shift from N to $N-1$ nations if and only if

$$N(N-1) \leq \frac{ga}{2k} \tag{8}$$

This condition ensures that in at least one country the decrease in taxation associated with the lower number of countries does not compensate a majority of voters for the increase in distance from the government. Interesting asymmetry is worth noting. In equilibrium, the change from N to $N+1$ is rejected in *each* country. On the other hand, the change from N to $N-1$ although vetoed by at least one country, will typically be approved in some other countries. It is immediate to notice that every A-stable configuration of nations satisfies condition (8). On the other hand, the only A-stable number of nations N that satisfies condition (7), for an A-stable number of nations $N+1$, is given by the largest integer smaller than $\sqrt{\frac{ga}{2k}}$. Therefore, this integer fully characterizes the unique B-equilibrium, as stated in Proposition 3.

The following proposition shows that this unique configuration is not only an equilibrium, but is also globally stable under applications of rule B:

Proposition 4. *Every configuration of nations which is a B-equilibrium is B-stable.*

Proposition 4 means that, for every other configuration of nations whose number is different from the number characterized in Proposition 3, there exists a finite sequence of applications of rule B which leads the system to the configuration characterized by Proposition 3. Therefore, Proposition 3 is necessary and sufficient to identify the B-stable number of countries.

An important implication of the above propositions is the following:

The stable number of countries is larger than the efficient one. The efficient number of countries is not stable.[13]

Consider the social planner solution, with $N = N^*$ and the same tax for everyone, $t = kN^*$. This configuration is efficient and A-stable, but not B-stable. In fact, there are too few countries. Individuals located far from the government have a low level of utility. As a result, in each country one can find a majority in favor of breaking down the existing number of countries in smaller but more numerous political jurisdictions.

[13] More precisely, $N^* \le \tilde{N}$ always, and $N^* < \tilde{N}$ for $\tilde{N} > 4$. One can also notice that efficient number and stable number tend to infinity for k tending to zero. This second limit case can be interpreted as an application of Tiebout's theorem (1956): when each individual , or group of individuals with *identical tastes*, can create their own nation, the stable outcome is also efficient.

The intuition that those who "break" the efficient equilibrium are the individuals close to the borders is confirmed by the observation that the stable number of countries $\tilde{N}$ solves the problem of a "Rawlsian" social planner, who maximizes the utility of the least well off individual but cannot use lump-sum transfers, as taxes must be equal across individuals.[14]

Conversely, the stable number of countries $\tilde{N}$, which is larger than N^*, is inefficient. Thus, with an appropriate scheme of lump-sum redistributions, a social planner could "move" the equilibrium from $\tilde{N}$ to N^* without making anybody worse off. This scheme would reward individuals who are located far from the borders. For the reasons discussed in the previous section, these transfer schemes are hard to implement since they should link taxes and transfers to individual preferences.

We conclude this section by discussing three important points. First, an interesting question is whether, leaving aside issues of revelation of preferences, one can find a majority in each country in support of the tax-transfer scheme which enforces the efficient number N^*. An answer to this question is not simple, because it implies considering the effects of three votes: on the location of the public good within a country; on the border of the country according to rule B; and on the tax-transfer scheme. Even though we have not solved this problem, we see no reason why the democratic outcome should generically reproduce the efficient outcome.

Second, an empirically relevant issue concerns the possibility of unilateral secessions, in which groups of individuals from one or more countries break away to form a new country. We can

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[14] More precisely, the Rawlsian solution is the integer closest to the upper limit of the interval which characterizes the stable number of countries. In the continuum, the stable number tends to the Rawlsian solution. When we consider the integer constraint, the two solutions might differ at most by one.

show that our equilibrium $\tilde{N}$ is robust to unilateral secessions as long as rules A and B are allowed to operate, and individuals are forward looking. In fact, consider a group of individuals of size z, breaking away from a country (or two adjacent countries) in a stable equilibrium, where the number of countries is $\tilde{N}$. If the secession leads to the formation of $\tilde{N}+1$ countries of equal size, the application of rule B will bring the number of countries back to $\tilde{N}$. If, as a result of the secession, not all the countries are of equal size, applications of rule A would lead to an A-stable equilibrium.[15] If the A-rule dynamics eliminates one country, we are back to the original stable equilibrium. If the A-rule dynamics leads the system to a configuration of $\tilde{N}+1$ countries, the application of rule B to the new A-stable equilibrium will bring the system back to the original stable equilibrium $\tilde{N}$. If the group which considered the option of secession rationally forecasts the final outcome (i.e., that after the secession, the application of rules A and B lead to the original equilibrium), it would not break away. The argument can be easily generalized to the case of multiple, simultaneous secessions. Thus, in our model, secessions cannot be observed in equilibrium.

Finally, consider a generalization of rule B that allows voting on proposals for border redrawing leading to the formation or elimination of any number of countries. More precisely, one can ask the question whether the equilibrium $\tilde{N}$ is the preferred outcome when directly challenged by proposals of general border redrawing, which involve the complete reshaping of national borders and the direct shift to a configuration of $N' = \tilde{N} + X$ or $N' = \tilde{N} - Z$ nations of any size. One can show that a sufficient, but not necessary, condition for $\tilde{N}$ to be "robust" to general

[15] In fact, any A-equilibrium that is not A-stable can be perturbated by any (infinitesimal) unilateral secession.

border redrawing is the assumption that alternative configurations are admissible proposals for general border redrawing referenda only if they are B-stable. In other words, a sufficient restriction on proposals for $\tilde{N}$ being robust is that no change can be proposed if the alternative configuration is not stable under applications of rules A and B.

5. Economic Integration

We now extend our model in order to formalize the relationship between equilibrium size of nations and degree of economic integration. A larger country means a larger economy as long as there is imperfect economic integration among nations. In the extreme case of autarky, the size of the economy is identical to the size of the country. In the opposite extreme of complete free trade in goods and factors of production, the size of a country has no economic meaning. In other words, if there are increasing returns in the size of an economy, they translate in higher increasing returns in the size of the nation the lower the degree of international economic integration. Therefore, *ceteris paribus*, we should expect an inverse relationship between size of nations and degree of economic integration.

One among many reasons why there might be increasing returns in the size of an economy is given by the relationship between human capital and productivity.[16] Assume that human capital is uniformly distributed over the world population. Total human capital in the world is equal to h. Assume that an economy's per capita income y is a function of the aggregate human capital of the agents who interact in the economy. If the whole world is a

[16] Aggregate (human) capital externalities on total factor productivity are emphasized, for instance, by Romer (1986), Lucas (1988), and Grossman and Helpman (1991).

fully integrated economy, per capita income in each country will depend only on world aggregate human capital. If the N nations are economically autarkic, per capita income in each country will depend only on the national aggregate human capital. In general, per capita income in each country will be a function of a weighted average of national and foreign human capital, where the weight is the same only in the limit case of full economic integration. Formally:

$$y = bsh + (b-j)(1-s)h \tag{9}$$

where sh is the total amount of human capital in a country of size s, $(1-s)h$ is the amount of human capital in the rest of the world, b and j are parameters such that $b >$ and $0 \leq j \leq b$. If $j = 0$ we have complete economic integration, and income per capita depends on the world level of human capital; the size of the country does not affect income. This is the case analyzed so far. If $j = b$, the economy is closed (autarky), and only domestic human capital matters. Thus, *lower* levels of j represent higher levels of economic integration. Given our previous discussion, we confine ourselves to the case of countries of equal size, and of taxes identical for everybody. The utility of each individual is:

$$U_i = g(1 - al_i) + (b-j)h + sjh - \frac{k}{s} \tag{10}$$

The extension of the model to imperfect economic integration does not modify the results about median distance changes derived in the Appendix. Therefore, following the steps described in the previous sections and using the results derived in the Appendix, one obtains the efficient and stable number of countries, taking j as given. The efficient number N^* is the maximum between one and the integer closest to:

$$\frac{1}{2}\sqrt{\frac{ga - 4jh}{k}} \tag{11}$$

The stable number of countries $\tilde{N}$ is the maximum between one and the largest integer smaller than:

$$\sqrt{\frac{ga - 2jh}{2k}} \tag{12}$$

As before, we have that $\tilde{N}$ is larger than N^*. What is more interesting now, is that both the efficient and the stable numbers of nations are *increasing* in the degree of economic integration, since they are decreasing in j. A break up of nations is more costly if it implies more trade barriers and smaller markets. On the contrary, the benefits of large nations are less important, if small nations can freely trade with each other. Concretely, this result suggests that regional political separatism should be associated with increasing economic integration.

A second observation is that in the first-best solution (efficient number of nations), greater economic integration (smaller j) is always welfare-improving. But when the number of nations is endogenously determined, in the stable equilibrium, higher economic integration could lead to a *reduction* in average utility. Economic integration makes countries smaller. When countries are already "too small," the average welfare loss due to the shrinking size, can outweigh the income effects due to higher economic integration: this is an application of the second-best principle.

Formally, a change from $j' < j$ (greater economic integration) that increases the stable number of nations $(N' > N)$ will cause the following change in average utility:

$$u'-u=\frac{ag}{4}(s-s') + (j-j')(1-s')h-(s-s')jh-k(N'-N) \tag{13}$$

Average utility changes because of four effects, two positive and two negative. The smaller size reduces average distance: $ag/4(s-s') > 0$ and greater economic integration increases income through the bigger effect of foreign human capital on productivity: $(j-j')(1-s')h > 0$. However, as countries are now smaller, the average tax burden increases: $k(N'-N) > 0$, and aggregate domestic human capital is smaller in each country $(s-s')jh > 0$. As the stable number of nations does not maximize average utility, it is possible that the two negative effects dominate the two positive effects: that is, $u'-u < 0$. A numerical example can illustrate this point. Assume k=2, a=0.5, g= 320, h= 8, j= 1, j'=0.95. For j=1, the stable number of nations is 5. At j' = 0.95, , the stable number of nations is 6. It is easy to verify that $u'-u = -0.6$, that is, average utility is *reduced* by the higher level of economic integration.

In summary, this section has two empirical implications. First, political separatism should go hand in hand with economic integration. We feel that the current European experience, the idea of a "Europe of regions" (Drèze (1993)), and, perhaps, Quebec separatism in the context of NAFTA yields some support to this implication. Furthermore, the incentives for the regions of the former Soviet Union to break over would have been much lower had they expected to be economically isolated instead of economically integrated with the rest of the world (in particular, with Western Europe).

The second implication is that the benefit of country size on economic performance should decrease with the increase of economic integration and removal of trade barriers. Within the recent literature on growth, several authors have looked for the effects of country size, (for instance, see Barro and Sala-I-Martin (1994)). Our paper suggests that the effect of country size is mediated by the extent of trade barriers, and that the former variable, country size, is endogenous to the latter, the degree of economic integration, at least in the long run.

Finally, it is worth noting that, in principle, the causal relationship between degree of economic integration and national size can go both ways. Higher economic integration implies smaller countries, and smaller countries will need more economic integration. An interesting extension of our analysis would be to study the joint endogenous determination of j and N in equilibrium.

6. A World of Leviathans

We now compare the stable number of countries in a democratic world with the number of countries in a world of dictatorships. A dictator is a Leviathan, who maximizes rents, i.e., tax revenues net of expenditures.[17] For simplicity, we return to a world with fixed individual income, equivalent to the case of $j = 0$ in the previous section.

We begin with the simplest case of a world Leviathan; who has to supply at least one government? In the absence of additional constraints, his problem is:

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[17] A classical analysis of governments as malevolent revenue maximizers is Brennan and Buchanan (1980).

$$\underset{t,N}{Max}\ t-NK$$
$$s.t.\quad t \le y \text{ and } N \ge 1.$$

This simple problem has an obvious (corner) solution at $t = y$ and $N = 1$. A Leviathan who does not care about individuals' utility would supply only the minimum possible amount of the public service (that is, one), and tax at the maximum feasible level. In our model, taxes are non distortionary. If they were -- for instance, on the labor supply -- the dictator would choose the tax rate that maximizes revenues.

More realistically, it is unlikely that even dictatorial governments can be completely insensitive to the welfare of their citizens. Even in nondemocratic societies, rulers might have to guarantee some minimal level of utility to at least part of the population, to ensure a minimum of popular support and avoid revolutions and turmoil. We model this constraint by posing that a Leviathan has to guarantee at least utility u_0 to a fraction δ of its citizens. The fraction δ can be interpreted as the limit to the degree of "totalitarianism" of the Leviathan. If $\delta = 0$ the degree of totalitarianism is maximum: the Leviathan can ignore the utility of his subjects. As δ increases, the "popular responsiveness" of the Leviathan increases.

Assume that we have a "class" of Leviathans, i.e., a group of individuals that can become country rulers. We now find their cooperative solution, namely, the number of countries that maximize the Leviathans' total rent.[18] We assume that redistributions within the group of potential Leviathan's enforces the

[18] This problem is similar to the questions posed by Friedman (1977).

cooperative solution. We continue to assume that taxes have to be the same for everyone.[19]

Proposition 5. *The Leviathans choose countries of equal size and locate the government in the middle. The number of countries which insure a minimum utility* u_0 *to a fraction* δ *of the population in each country is* $\max[1, N_\delta]$ *where:*

$$N_\delta = \sqrt{\frac{ag\delta}{2k}} \tag{14}$$

provided that $\max\ [1, N_\delta]$ *is an integer.*[20]
The tax rate is:

$$t_\delta = g\left(1 - \frac{a\delta}{2N_\delta}\right) + y - u_0 \tag{15}$$

Proof. The first part of the proposition is immediate. The number is found by solving:

$$\begin{array}{ll} \max\limits_{t,N} & t - Nk \\ s.t. & g(1 - a_\delta) + y - t = u_o \end{array} \tag{16}$$

where:

$$l_\delta = \frac{\delta}{2N} \tag{17}$$

Equation (17) follows form the observation that the utility constraint will be binding for the two individuals at a distance

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[19] If the Leviathans could discriminate amongst citizens and charge different taxes, they would. We do not develop this issue here.

[20] Otherwise, as usual, the solution is given by the integer that is closer to $\sqrt{\frac{ag\delta}{2k}}$.

$l_\delta = \dfrac{\delta}{2N} = \dfrac{\delta s}{2}$ from the government. This will ensure that the s_δ individuals at a distance smaller than l_δ have utility higher than u_0 Finally, note that for feasibility, we need that $t_\delta \leq y$, thus, from (15), we need $u_0 \geq g\left(1 - \dfrac{a\delta}{2N_\delta}\right)$.
Q.E.D.

Proposition 5 implies the following:

$$\text{if } \delta < \frac{1}{2} \qquad N_\delta < N^* \tag{18a}$$

$$\text{if } \delta = \frac{1}{2} \qquad N_\delta = N^* \tag{18b}$$

$$\text{if } \frac{1}{2} < \delta < 1 \qquad N^* < N_\delta < \tilde{N} \tag{18c}$$

$$\text{if } \delta = 1 \qquad N_\delta = \tilde{N} \tag{18d}$$

Thus, for any value of δ strictly less than one, *there are fewer countries with Leviathans than in a democratic world.* For $\delta < 1/2$ the number of countries with Leviathans is below the efficient number. Realistically, in a dictatorship, δ should be much less than ½. In fact, the distinctive feature of a dictatorship is that it can rule *without* the consensus of the majority of the population. Thus, the implication is that one has too few countries in a world of Leviathans. *Hence, democratization leads to the creation of more countries.*

How can we interpret this "cooperative solution" of a world class of Leviathans? The first interpretation is that of a supranational ruling class: a supranational coalition of rulers might choose the number of nations in order to maximize the rents of national governments under their joint control. Historical examples of this case might be: a) the relatively homogeneous and supranational aristocracy ruling Europe in the Eighteenth century, b) the European nations that divided amongst themselves (more or less cooperatively) large parts of the developing world in the colonial age; c) the relatively homogeneous communist nomenclature ruling in the Soviet area of influence in this century. Clearly, in all these historical examples, elements of conflict coexisted with elements of "collusion" amongst rulers. This cooperative solution of Leviathans can be the benchmark upon which to build a study of non-cooperative behavior of dictators, including models of military confrontation and empire building. We leave this task for future research.

Finally, it is useful to point out an analytical connection between our model of Leviathans and models of product differentiation. Consider the case of free entry of Leviathans in the "market" for governments. That is, any Leviathan can enter by paying the fixed cost k. Assume that citizens at the border between two jurisdictions have to be indifferent (that is, they can freely decide which Leviathan to join). Under these assumptions one can derive results that closely mirror models of industrial organization of spacial competition and product differentiation.[21] In particular, one can show that the equilibrium number of countries with free entry N^f is given by:[22]

$$N^f = \sqrt{\frac{ga}{k}} \tag{19}$$

Clearly N^f is larger than the collusive solution. It is also larger than the efficient equilibrium and the stable voting equilibrium. Therefore, the free entry case illuminates the difference between our stable voting equilibrium and the outcome of strategic interactions modeled in analogy with industrial

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[21] See Salop (1978) and Tirole (1988), chapter 8.

[22] The proof is available upon request.

organization models. In terms of realism "free entry" is not seemed to be a particularly appealing assumption for the "market" of Leviathans, except perhaps in highly anarchic and primitive phases of historical development.

7. Discussion

This paper is our first step toward applying economic analysis to the study of the number and size of countries. In many respects, we view it as a stepping stone. Therefore, rather than reviewing our results, in this conclusive section we highlight some of the many questions which we have left open.[23]

First, the coincidence of the geographical and cultural dimensions precluded the consideration of ethnic or cultural minorities. The latter are, instead, clearly crucial in the process of border redrawing. By removing the coincidence between the two dimensions, one can begin with an arbitrary distribution of preference and geographical location and study the adjustment process and the formation of countries. This development would require a study of geographical mobility, and may connect with the literature on migration.

Second, we have not modeled the role of military threats and of defense spending. The optimal size of a country and the optimal amount of its public good "defense" clearly depends upon the size of other countries, and their aggressive military potential. Empirically, one may argue that the emergence of regionalism in Europe (East and West) is related not only to democratization and economic integration, as we argue in this paper, but, also, to the disappearance of the Soviet military

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[23] Alesina, Perotti and Spolaore (1995) relate the results of this paper to other analytical contributions, and discuss the insights that this analysis offers on the costs and benefits of political and fiscal unions.

threat. We are currently at work toward extending our model toward incorporating a role for security considerations as determinants of country size.

Third, we have greatly simplified the treatment of the "public good," or "government" which identifies a country. In reality, a government provides many functions. Some of them can be decentralized to regional or local governments within the context of a decentralized country. In other words, an answer to the trade off between economies of scale and heterogeneity can be found in a decentralized structure of government. The central government would retain jurisdictions on those activities for which economies of scale are especially important, while local governments would retain jurisdictions on those activities where differences in individual preferences are more important. The line of argument would connect us to the literature on fiscal federalism, an avenue certainly worth exploring.[24] In fact, our model may have implications on the degree of federalism in dictatorships or democracies. Rather than thinking about the division of the world into different countries, think about the division of a country into autonomous regions. Then our result implies that dictatorships should be more centralized than democracies. Ades and Glaeser (1993) provide some evidence which is indirectly supportive of this argument. They find that countries with a history of dictatorial regimes have capital cities which are much larger, relative to the size of the nation, than in democratic countries.

Fourth, we have largely ignored the question of the redistributive role of governments, since we have not considered differences in individual income. Differences in income, in addition to differences in preferences, may be crucial determi-

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[24] See Oates (1972), and, amongst others, the recent study by Hughes and Smith (1993).

nants of the degree of heterogeneity in the population which determines the equilibrium size and number of countries.

APPENDIX

A.1 Derivation of the social planner solution

The social planner maximizes the sum of all individual utilities

$$\int_0^1 U_i di = \sum_{x=1}^{N} s_x[g(1-aE_x(l_i))+y-E_x(t_i)] \quad \textbf{(A.1)}$$

s.t.

$$\sum_{x=1}^{N} E_x(t_i) = Nk \quad \textbf{(A.2)}$$

where $AE_x(l_i)$ and $E_x(t_i)$ are, respectively, the average distance and the average tax in country x. In order to minimize $E_x(1_i)$, for given N, the social planner locates the public good in the middle of each country. Hence, the average distance in each country is $E_x(1_i) = s_x/4$. Therefore, the social planner's problem can be written as:

$$Min \quad \frac{ga}{4}\sum_{x=1}^{N} s_x^2 + Nk \quad \textbf{(A.3)}$$

$$s.t. \quad \sum_{1}^{N} s_x = 1 \quad \textbf{(A.4)}$$

The sum of squares of sizes is clearly minimized by choosing countries of equal size $s = 1/N$. Therefore, the number of nations N will be the (strictly positive) integer that solves

$$Min \quad \frac{ga}{4N} + kN \quad \textbf{(A.5)}$$

The first order condition with respect to N imply:

$$N^* = \frac{1}{2}\sqrt{\frac{ga}{k}} \qquad \textbf{(A.6)}$$

The above expression gives us the solution only if it is an integer. In general, the solution will be characterized as follows. Define $M \equiv \frac{1}{2}\sqrt{\frac{ga}{k}}$. Define N' as the integer in the interval $(M-1, M]$ and N'' as the integer in the interval $(M, M+1)$. Then the efficient number of nations is N' if and only if size $s' = 1/N'$ gives average utility not lower than size $s = 1/N''$; that is, if and only if:

$$g(1 - \frac{a}{4N'}) - kN' \geq g(1 - \frac{a}{4N''}) - kN'' \qquad \textbf{(A.7)}$$

which implies:

$$N'N'' \geq \frac{ga}{4k}$$

Otherwise, N'' is the efficient number. Therefore, the efficient number of nations is equal to M itself if M is a positive integer. Otherwise, it is equal to:

a) the integer N' immediately below M if $N'(N'+1)$ is larger than $ga/4k$;

b) the integer $N'' = N'+1$ immediately above M if $N'(N'+1)$ is smaller than $ga/4k$;

c) both integers N' and N'' if $N'N'' = ga/4k$.

As case c) has measure zero, the efficient number of nations is generically unique.

A.2 Derivation of the stable number of countries

A.2.1. Derivation of Proposition 2.

First, we will show that new configuration of differently-sized countries is A-stable. Secondly, we will derive the condition that N equally-sized countries must satisfy to be A-stable. An individual at the border between two countries of sizes s_1 and s_2 is indifferent if and only if:

$$g\left(1 - a\,\frac{s_1}{2}\right) - \frac{k}{s_1} = g\left(1 - a\,\frac{s_2}{2}\right) - \frac{k}{s_2} \qquad \textbf{(A.8)}$$

The above condition is satisfied if:

$$s_1 = s_2 \qquad \textbf{(A.9)}$$

or:

$$s_1\, s_2 = \frac{2g}{ga} \qquad \textbf{(A.10)}$$

Condition (A.10) identifies a situation in which the border citizen is indifferent between two countries of different sizes. However, this case is unstable. Assume that the equilibrium is perturbated by an amount ε, arbitrarily small, so that country 1's new size becomes $s_1 + \varepsilon$ and country 2's new size becomes $s_2 - \varepsilon$. It is easy to verify that, for any ε small enough, the individual who is now at the border between the two countries strictly prefers country 1 to country 2, that is:

$$g(1-a\frac{s_1+\varepsilon}{2}) - \frac{k}{s_1+\varepsilon} > g(1-a\frac{s_2-\varepsilon}{2}) - \frac{k}{s_2-\varepsilon} \qquad \textbf{(A.11)}$$

Hence, any arbitrarily small perturbation of the differently-sized equilibrium will induce the system to diverge. Therefore, any differently-sized equilibrium is unstable under rule A.

The conditions under which configurations of N equally-sized countries are A-stable is derived as follows. Take N countries of equal size. Perturbate the equilibrium so that two bordering countries (call them 1 and 2) have now different size, $s-\varepsilon_1$ and $s+\varepsilon_2$, respectively, where ε_1 and ε_2 are two arbitrarily small positive real numbers. The original equilibrium is A-stable if and only if the individual at the new border always strictly prefers the smaller country (1) to the larger country (2), that is:

$$g(1-a\frac{s-\varepsilon_1}{2})-\frac{k}{s-\varepsilon_1} > g(1-a\frac{s+\varepsilon_2}{2})-\frac{k}{s+\varepsilon_2} \qquad \textbf{(A.12)}$$

which implies:

$$(s-\varepsilon_1)\,(s+\varepsilon_2) > \frac{2k}{ga} \qquad \textbf{(A.13)}$$

At the limit, for ε_1 and ε_2 tending to zero, the condition becomes:

$$s^2 > \frac{2k}{ga} \qquad \textbf{(A.14)}$$

By substituting $s=1/N$, we obtain:

$$N < \frac{ga}{2k}$$

Q.E.D.

A.2.2. Derivation of Propositions 3 and 4.

As stated in Proposition 2, only equally sized nations are A-stable. Thus, in what follows, "number of nations" is implicitly defined as meaning "numbers of nations of equal size." Also, because of our assumptions on voting within a country, i.e., for given borders, we can use the result that for any N, the government is located in the middle.

We start by introducing some useful definitions and notation:

Let $l_i\{N\}$ denote the distance of individual I's preferred type from the median voter's preferred type when the total number of nations is N. The new distance is given by $l_i\{N'\}$. Let $d_i\{N,N'\}$ denote the change in the distance experienced by individual i when we move from N to N' nations:

$$d_i\{N,N'\} = l_i\{N'\} - l_i\{N\} \tag{A.15}$$

Each individual i will prefer N to N' as long as:

$$g(1-al_i\{N\}) + y - kN \geq g(1-al_i\{N'\}) + y - kN' \tag{A.16}$$

that is:

$$agd_i\{N,N'\} + k(N'-N) \geq 0 \tag{A.17}$$

Let $d_m^x\{N, N'\}$ denote the median distance change in nation x (where x 1, 2, ..., N).[25]

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[25] That is, if N' nations were to be formed, half of the citizens of nation x would experience distance changes larger than $d_m^x\{N,N'\}$, and the other half would experience distance changes smaller than $d_m^x\{N,N'\}$, where x=1,2,...,N.

As an immediate implication of the above definitions, nation x will have a majority that prefers N to N' if and only if

$$agd_m^x\{N,N'\} + k(N' - N) \geq 0 \qquad \textbf{(A.18)}$$

We define rules B1, and B2 as follows:

B1. A new country can be *created* when the change is approved by majority rule in each existing country whose territory will be affected by the border redrawing.

B2. An existing country can be *eliminated* when the change is approved by majority rule in each existing country whose territory will be affected by the border redrawing.

A configuration of N nations is:

- A B1-equilibrium if no nation is created under applications of rule B1. That is, any proposed modification to increase the number of countries by one is rejected by majority voting in at least one of the affected countries.
- A B2-equilibrium if no nation is eliminated under applications of rule B2. That is, any proposed modification to decrease the number of countries by one is rejected by majority voting in at least one of the affected countries. We will then derive the B-equilibrium configuration, i.e., the configuration that is a B1-equilibrium *and* a B2-equilibrium when *only A-stable* modifications can be proposed.

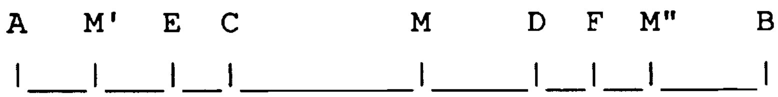
First, we state and prove the following four lemmata:

LEMMA 1. *The median distance change $d_m^x\{N,N+1\}$ is the same for all x (x=1,2,...,N), and is given by:*

$$d_m^x\{N, N+1\} = \frac{s' - s}{2} \tag{A.19}$$

where $s \equiv \frac{1}{N}$ *and* $s' \equiv \frac{1}{N+1}$

Proof. Consider a nation x of size $s = 1/N$, which goes from point A to point B, as in the following figure:



Let $s = |AB|$ denote the distance between A and B. Call M the point in the middle of nation x (*i.e.*, $|AM| = |MB| = s/2$). When $N+1$ nations are formed, the citizens of nation x are divided in two new nations of size $s' = 1/(N+1)$. Call them x' and x'' M' (M'') will denote the point in the middle of nation x' (x''). Call C the point located halfway between M' and M (i.e., $|M'C| = |CM|$), and D the point located halfway between M and M'' (i.e., $|MD| = |DM''|$). As $|M'M''| = s'/2$, we also have

$$|CD| = s'/2 \tag{A.20}$$

The distance change is positive for all individuals between C and D, null for the individuals located in points C and D, and negative for all individuals between A and C and between D and B.[26] That is:

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[26] Note that all individuals between C and D belong to country x. In fact, the maximum distance between M and either M' or M" is $m = 3s'/2 - s/2$ ($|MM''| = m$ for $x = 1$ and $|MM'| = m$ for $x = N$). Therefore, C and D are located at a distance from M that cannot exceed $3s'/4 - s/4$, which is strictly smaller than $s/2$.

$$d_i\{N,N+1\}\geq 0 \quad \text{for } i\in [C,D]$$
$$d_i\{N,N+1\}<0 \quad \text{for } i\in [A,C] \text{ and } i\in (D,B] \qquad \textbf{(A.21)}$$

In particular, for every point H' between M' and C which belongs to nation x, the distance change is given as follows:

$$d_{H'}\{N,N+1\} = |M'H'| - |MH'|$$
$$= (|M'C| - |H'C|) - (|CM| + |H'C|) = -2|H'C \qquad \textbf{(A.22)}$$

Analogously, for every point H'' between D and M'' which belongs to nation x we have a distance change equal to $-2|DH''|$. Call E the point between M' and C, and F the point between D and M'', chosen in order to satisfy the two following conditions:
From (A.22) and (A.23) we have:

$$|EC| = |DF| \qquad \textbf{(A.23)}$$

and:

$$|EF| = s/2 \qquad \textbf{(A.24)}$$

From (A.22) and (A.23) we have:

$$d_E\{N, N+1\} = d_F\{N, N+1\} = -2|EC| \qquad \textbf{(A.25)}$$

From (A.26) and (A.21) we get:

$$EF = |EC| + |CD| + |DF|$$
$$= 2|EC| + |CD| = 2|EC| + s'/2 + s/2$$

which implies:

$$|EC| + (s - s')/4 \qquad \textbf{(A.26)}$$

As the maximum distance between M and M' is $3s'/2 - s/2$ (which holds for $x = N$), then max $|EM| = (3s'/2 - s/2)/2 + (s-s')/2 = (s$

$+ s')/4 < s/2$. Therefore, point E belongs to nation x for every x. Analogously, max $|MM''| = 3s'/2 - s/2$ (which holds for $x = 1$), and max $|MF| = (s + s')/2 < s/2$. Hence, point F belongs to nation x for every x. By substituting (A.26) in (A.25) we obtain:

$$d_E\{N,N+1\} = d_F\{N,N+1\} = (s'-s)/2 \tag{A.27}$$

Because of (A.21) and (A.22), all individuals between E and F experience a distance change higher than the distance change experienced by the individuals located in point E and in point F, and all individuals between A and E and between F and B experience a smaller distance change experienced by the individuals at E and F. [27] As the individuals between E and F are half the citizens of country x by (A.26), the individuals at E and F experience the median distance change. Hence:

$$d_m^x\{N,N+1\} = d_E\{N,N+1\} = d_F\{N,N+1\} = (s'-s)'/2 \tag{A.28}$$

Q.E.D.

LEMMA 2. The maximum median distance change $d_m^x\{N, N-1\}$ is given by:

$$\max d_m^x\{N,N-1\} = \frac{s''-s}{2}$$

$$\text{where } s \equiv \frac{1}{N} \textit{ and } s'' \equiv \frac{1}{N-1}$$

[27] This is true as long as both E and F belong to nation x. In fact, we will show that this is indeed the case.

Proof. N odd. The maximum median distance will obtain in the middle nation $x = (N+1)/2$. In this nation, the median voter M is located exactly in the middle of the interval [0,1]. When $N-1$ nations are formed, M becomes the extreme voter -- i.e., the distance between M and the new median voters M' and M'' is $|M'M| = |MM''| = s''/2$. Call A and B the points at the borders of nation $x = (N+1)/2$, as in the following figure:

```
M'   A      E      M      F      B    M"
|...|______|______|______|______|...|
```

The distance change is increasing between A and M, and decreasing between M and B. The median distance change is experienced by the individuals E and F who are located at a distance $|EM| = |MF| = s/4$ from the former median voter (clearly, $|EF| = s/2$). Their distance change will be given by

$$\begin{aligned} d_m^x\{N,N-1\} &= |M'E| - |EM| = (|M'M| - |EM|) - |EM| \\ &= (s''/2 - x/4) - s/4 = (s'' - s)/2 \end{aligned} \qquad \textbf{(A.30)}$$

N even. The maximum median distance will obtain in the middle nations $x = N/2$ and $x+1 = N/2 + 1$. Consider nation $x = N/2$. Call A and B the points at its borders, as in the following figure:

```
   M'       A    E    A'   M    F         B
..|......|____|____|____|____|________|
```

When $N-1$ nations are formed, B becomes the median voter of nation $x' = (N-1)/2$. Call A' the point at the left border of nation

x'.[28] As B is the median voter of nation x', we have $|A'B| = s''/2$. The distance between A' and M is:

$$|A'M| = |A'B| - |MB| = s''/2 - s/2 \tag{A.31}$$

The distance change is increasing between A and A', maximum between A' and M (where it is equal to $s/2$), and decreasing between M and B. In particular, every individual H' located at a distance $|H'A'|$ from A' will experience a distance change equal to:

$$\begin{aligned}|M'H'| - |H'M| &= (|M'A'| - |H'A'|) - (|H'A'| + |A'M|) \\ &= s''/2 - (s''/2 - s/2) - 2|H'A'| = s/2 - 2|H'A'|\end{aligned} \tag{A.32}$$

Analogously, every individual H'' located between M and B at a distance $|MH''|$ from M experiences a distance change equal to

$$s/2 - 2|MH''| \tag{A.33}$$

Call E the individual located between A and A', and F the individual located between M and B, such that

$$|EA'| = |MF| \tag{A.34}$$

and:

$$|EF| = s/2 \tag{A.35}$$

By construction,

$$|EF| = |EA'| + |A'M| + |MF| \tag{A.36}$$

Substituting (A.31), (A.33) and (A.36) in (A.36), we obtain:

[28] We will derive the median distance change for nation x. As the median voter of nation x' is located at the border between x and x+1, the distance changes in the two nations are perfectly symmetric, and the median distance change is the same.

$$s/2 = 2|EA'| + (s''-s)/2 \tag{A.37}$$

from which we get:

$$|EA'| = s/2 - s''/4 \tag{A.38}$$

Hence, for H'=E, we can substitute (A.38) in (A.32), and derive the median distance change:

$$d_m^x\{N, N-1\} = s/2 - 2|EA'| = s/2 - s + s''/2 = (s''-s)/2 \tag{A.39}$$

Q.E.D.

LEMMA 3. *A number of nations N is a B1-equilibrium if and only if the following condition holds:*

$$N(N+1) \geq \frac{ga}{2k} \tag{A.40}$$

for every $x = 1,2,...N$.

Proof. Consider (A.18) when $N' = N+1$. Then, every nation has a majority against the shift from N to $N+1$ if and only if:

$$agd_m^x\{N,N+1\} + k \geq 0 \tag{A.41}$$

Using Lemma 1, we can substitute $d_m^x\, N, N+1$ with $(s'-s)/2 = -1/[2N(N+1)]$ for every x and obtain the above condition (A.40).

Q.E.D.

LEMMA 4. *A number of nations N is a B2-equilibrium if and only if the following condition holds:*

$$(N-1)N \leq \frac{ga}{2k} \qquad \textbf{(A.42)}$$

Proof. Consider (A.18) when N' = N-1. Then, the nation x with the maximum median distance change will prefer N to N-1 if and only if:

$$agd_m^x\{N,N-1\} - k \geq 0 \qquad \textbf{(A.43)}$$

Using Lemma 2 we can substitute $d_m^x\{N, N-1\}$ with $(s''-s)/2 = 1/[2N(N-1)]$ and obtain the above condition (A.42), which is therefore sufficient for N to be a B2-equilibrium. On the other hand, if condition (A.42) did not hold, all nations would have majorities that prefer *N-1* to *N*, which implies that (A.42) is necessary for *N* to be a B2-equilibrium. Q.E.D.

__Proof of Proposition 3__

Because of Proposition 2 and Lemmata 3 and 4, a configuration of *N* countries is a B-equilibrium if and only if:

(a) All *N* countries have equal size.

(b) $N < \sqrt{\frac{ga}{2k}}$

(c) $N(N+1) \geq ga/2k$ (Lemma 3) for $N+1 < \sqrt{\frac{ga}{2k}}$

(d) $N(N-1) \leq ga/2k$ (Lemma 4) for $N-1 < \sqrt{\frac{ga}{2k}}$

Clearly, (b) implies (d). We now claim that there exists one and only one integer which simultaneously satisfies (b) and (c). Denote with $\tilde{N}$ the largest integer smaller than $\sqrt{\frac{ga}{2k}}$. It is immediate to check that $\tilde{N}$ satisfies (b) and (c). On the other

hand, any other integer larger than $\tilde{N}$ does not satisfy (b), and any other integer small than $\tilde{N}$ does not satisfy (c). Therefore, the unique B-equilibrium configuration of countries is given by the configuration of $\tilde{N}$ equally sized countries.
Q.E.D.

Proof of Proposition 4.

Consider an A-stable configuration of N' nations, with $N' < \tilde{N}$. Clearly, this configuration does not satisfy the above condition (c). Consequently, there exists a majority within each country in favor of shifting to the new A-stable configuration of $N' + 1$ equally sized countries (Lemma 3). By repeated application of rule B, the system will converge to $\tilde{N}$, which is therefore B-stable.
Q.E.D.

A.3. An alternative to rule B.

Rule B requires a majority in each of the N existing nations in order to modify their borders. In this section, we show that our characterization of the stable number of nations is the same under an alternative rule B' requiring the majority in each of the N' new nations. Assume that, given N nations, N' new nations can be formed if in the territory of each of the proposed N' nations there exists a majority in favor of the change. In analogy with the previous discussion, we can define the concepts of B'1-equilibrium, B'2-equilibrium, and B'-equilibrium as follows:

- a configuration of nations is a B'1-equilibrium (B'2-equilibrium) if no new nation is created (eliminated) under the alternative rule B': that is, any proposed modification to increase (decrease) the number of countries by one is rejected by majority voting in at least one of the new countries whose territory is affected by the change.

- a B'-equilibrium if it is A-stable and it is an equilibrium under A-stable applications of rule B'.

The following proposition shows that rule B' implies the same characterization of the stable number of countries as rule B:

Proposition 6.

A configuration of countries of equal size is a B'-equilibrium if and only if it is a B-equilibrium.

Proof. Consider the proposal of forming $N-1$ nations. It will be rejected by majority voting in nation x $(x-1,2,\ldots,N-1)$ as long as:

$$agd_m^x\{N-1,N\} + k[N-(N-1)] \leq 0 \qquad \textbf{(A.45)}$$

From Lemma 1, substituting N with $N-1$ and $N+1$ with N, we obtain:

$$d_m^x\{N-1,N\} = 1/[2N(N-1)] \qquad \textbf{(A.46)}$$

For every $x = 1,2,\ldots,N-1$.

Substituting (A.46) in (A.45), we obtain that the proposal of moving from N to $N-1$ will be rejected by a majority of voters in each of the potential $N-1$ will be rejected by a majority of voters in each of the potential $N-1$ countries if and only if

(A.47)

which is identical to condition (A.42).

Conversely, we will find that the proposal of moving from *N* to *N+1* countries will be rejected by majority voting in at least one of the potential N+1 nations if and only if:

$$ag\, maxd_m^x\, \{N+1, N\} + k[N - (N+1)] \leq 0 \tag{A.48}$$

Using Lemma 2 (substituting *N* with *N+1* and *N-1* with *N*), we obtain the following condition:

$$N(N+1) \geq ga\, /\, 2k \tag{A.49}$$

which is identical to condition (A.40).
Therefore, a configuration of N countries is a B'-equilibrium if and only if:

(a) All *N* countries have equal size.

(b) $N < \sqrt{\frac{ga}{2k}}$

© $N(N+1) \geq ga/2k$ for $N+1 < \sqrt{\frac{ga}{2k}}$.

(d) $N(N-1) \leq ga/2k$ for $N+1 < \sqrt{\frac{ga}{2k}}$.

Hence, the B-equilibrium and the B'-equilibrium configurations coincide.
Q.E.D.

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