NBER WORKING PAPER SERIES

NON-LEAKY BUCKETS: OPTIMAL REDISTRIBUTIVE TAXATION AND AGENCY COSTS

Karla Hoff
Andrew B. Lyon

Working Paper No. 4652

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 1994

We thank Bill Evans for bringing to our attention several studies of schooling, ability, and parental wealth. We are also indebted to Richard Bird, John Wallis, and seminar participants at the University of Michigan for valuable comments. This paper is part of NBER's research program in Public Economics. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

NBER Working Paper #4652
February 1994

NON-LEAKY BUCKETS: OPTIMAL REDISTRIBUTIVE TAXATION AND AGENCY COSTS

ABSTRACT

Economists have generally argued that income redistribution comes at a cost in aggregate incomes. We provide a counter-example in a model where private information gives rise to incentive constraints. In the model, a wage tax creates the usual distortion in labor-leisure choices, but the grants that it finances reduce a distortion in investment in human capital. We prove that simple redistributive policies can yield Pareto improvements and increase aggregate incomes. Where higher education is beyond the reach of the poor, the wage tax-transfer policy is under most circumstances more effective than targeted credit taxes or subsidies in increasing over-all efficiency.

Karla Hoff
Department of Economics
University of Maryland
College Park, MD 20742

Andrew B. Lyon
Department of Economics
University of Maryland
College Park, MD 20742
and NBER

1. Introduction

The standard view of income redistribution is that it comes at a cost: income is lost because the policies required to redistribute it misallocate resources. In a famous passage from his book, *Equality and Efficiency: The Big Tradeoff*, Arthur Okun compared the loss to a leak in a bucket. Under any tax and transfer program,

> the money must be carried from the rich to the poor in a leaky bucket. Some of it will simply disappear in transit, so the poor will not receive all the money that is taken from the rich. (Okun, 1975, p. 91)

One argument for redistribution based purely on efficiency has been made for the case when risk markets are incomplete and individuals are risk averse. Tax-transfer policies have the potential to provide a partial substitute for risk markets that are missing because of problems of information or because not all the parties that would benefit from a risk-sharing contract are alive at the same time.[1]

Our concern in this paper is with an entirely different efficiency-based argument for redistribution. The motivation of the tax-transfer policies considered in this paper is not to smooth utility across states of nature, but to reduce agency costs. We reach two central conclusions. First, Pareto improvements can be achieved from simple, redistributive tax-transfer policies in spite of the distortion in labor supply that they create, and in spite of our assumption of risk neutrality. Second, simple tax-transfer policies are in some circumstances more effective than direct interventions

[1]See, for example, Eaton and Rosen (1980), Varian (1980), Gordon and Varian (1988), and Hoff (1991).

through corrective taxes and subsidies.

Simple intuition supports these results. The use of collateral often efficiently resolves moral hazard and adverse selection problems. In fact, if individuals are risk neutral, a sufficiently high amount of wealth or collateral will always resolve a problem of moral hazard, since it makes it possible to structure transactions so that the party taking the "hidden action" bears fully the consequences of his action; the link between performance and rewards can be made perfect. In this paper, the tax-transfer scheme creates a bootstrap form of collateral. It does so by transforming future labor earnings subject to idiosyncratic risk into a risk-free form of income--and hence an ideal source of collateral.[2]

There is another way to explain the intuition behind our results. When there is private information in an economy, some mutually beneficial exchanges between transactors are not compatible with the incentives of the participants; an *incentive constraint* binds. A general property of incentive constraints is that they shift with even marginal redistributions of income, as illustrated in Hoff (1994). In the present paper, the tax-financed redistribution relaxes incentive constraints, and the resulting expansion in individuals' opportunities more than offsets the loss from the distortionary tax finance.

We demonstrate these points in a model of investment in higher education that is of some independent interest. The model addresses concerns raised by Arthur Okun. He speculated that one

---

[2]A critical factor in both this paper and the literature cited in footnote 1 is the incompleteness of markets. In this paper, there is an incomplete set of markets in state-contingent labor; see section 8.

of the most serious economic inefficiencies in the U.S. was under-investment in the human capital of the children of poor families (Okun, especially pp. 80-81). It remains true in the U.S. today that parental income is an important determinant in children's educational attainment.[3] Card (1993a, 1993b) finds that the marginal return to schooling among the population of less-educated individuals is *higher* than the return to the educated population. This evidence supports the contention that many children with high academic ability stop their schooling too soon, perhaps due to an inability to afford higher education. At the same time, there is other evidence of over-investment in the aggregate in college education, at least if we abstract from the non-economic ends of education. There are far more college graduates than traditionally college-level jobs. In each year between 1980 and 1990, the percentage of college graduates who were in jobs that did not require a degree or who were unemployed stood at roughly one in five (Hecker, 1992, table 1). The model provides a framework within which one can interpret both under-investment by the poor and over-investment in the

---

[3]Okun (p. 81) cites a study indicating that among male high school graduates with *equal* academic ability, the proportion going to college averages 25 percent lower in the bottom socioeconomic quarter of the population than in the top quarter; the corresponding figure for female high school graduates is 35 percent. Data from the National Opinion Research Center (1986) show a similar gap: Among high school seniors of *equal* academic ability, the proportion going to college in 1982 was 32 percent lower for the bottom third of families, ranked by annual family income, than for the top third. Taubman (1989) cites elasticity estimates of educational attainment with respect to parental income ranging from 3 to 80 percent. McPherson and Schapiro (1991) find that college enrollment of children from low-income families is particularly sensitive to the net cost of college tuition.

aggregate. The model is closely related to the two-state investment model used by de Meza and Webb (1987) and Bernanke and Gertler (1990).

Individuals in the model make choices over labor supply and occupation. Some occupations require higher education. Competitive equilibrium results in an efficient allocation of time between labor and leisure conditional on occupational choice, but an inefficient sorting of individuals across occupations. Inefficiencies arise because low-wealth individuals require outside finance to obtain higher education, and individuals have private information about their ability to repay the debt. Some of those who borrow to finance higher education will rationally undertake investments with negative expected present value because part of the cost of failure is shifted, through default, to others. This is the problem of *over-investment*. This may lead to the problem of *under-investment*, however: so many bad risks may enter the market that they raise interest rates for educational loans to prohibitive levels. The result may be that higher education is beyond the reach of low-wealth individuals, not because poorer persons are on average less able, but because bad risks drive out good.

Within the model, we show that there exist simple tax-transfer policies that yield a Pareto improvement. We suppose that government can tax future labor income, which is subject to idiosyncratic risk, and give each individual the expected value of his tax payment. The tax reduces the labor supply of all individuals. But the transfers reduce agency costs in the credit market (even though everyone is risk neutral). The net effect is to increase the expected income and utility of every individual.[4] In simulations, we illustrate the Pareto-efficient tax rates and show that the welfare gain from the tax-transfer scheme is quantitatively significant. In other simulations, we explore the tax rates that would maximize aggregate real income under a redistribution scheme where transfers are given only to low-wealth individuals. In these simulations, the income gains to the poor exceed the loss in income of the rich.

Thus, the redistributional buckets of Okun's image need not be leaky ones. We do not eliminate the adverse impact of taxation on hours worked, but we show that the beneficial net impact on investment in human capital may be more important than the impact on labor hours. In contrast, the traditional literature on redistributive taxation, because it abstracts from market imperfections, finds that a labor tax-transfer policy always reduces aggregate real incomes (see, for example, Browning and Johnson, 1984).

We start in section 2 by describing the basic model. Sections 3 through 5 present the information structure in which over-investment in education occurs, and describe our main results on Pareto-improving and income-increasing redistributions. Up to this point information is exogenous; in section 6 we assume that an individual has to exert effort to obtain information about his ability, and we analyze the problem of under-investment in education by low-wealth groups. Section 7 compares the effectiveness of targeted taxes, subsidies, loan guarantees, and transfers. Section 8 discusses the robustness of our results, and some concluding remarks on

---

[4]The expected utility is computed as of the date of the enactment of the policy, a point that we make precise below.

education policy follow.

2. Basic Model

There are two endowment goods, labor and a nonconsumable input good. The nonconsumable input good can be invested in higher education, which has a risky return, or in an asset that yields a riskless gross return r. Any amount of the input good can be invested in the safe asset, but a higher education program is indivisible: it requires one unit of the input good. The payoff if the individual successfully completes a program of higher education is an increase in his labor productivity from w to αw ($\alpha > 1$). The probability of successfully completing the program of higher education is given by p for each individual.

A natural way to interpret the model is that an individual's labor productivity is w in a low-skill occupation, while it is αw in a high-skill occupation. Successful completion of a program of higher education is a prerequisite to entry into a high-skill occupation. In this model α and w are exogenous; we focus on the choice of labor supply and occupation given α and w.

The expected utility of an individual is defined as

(1) $$U = E(y) - v(\ell)$$

where E(y) is expected income; $v(\ell)$ is the disutility of labor, with $v' > 0$ and $v'' > 0$. Abstracting from the fixed cost of education, the maximized surplus of an individual in the skilled occupation is

(2) $$R \equiv \max_{\ell_R} \{\alpha w \ell_R - v(\ell_R)\}.$$

("R" is for the return to the risky investment in higher education.) In the unskilled occupation, the maximized surplus is

(3) $$S \equiv \max_{\ell_S} \{w\ell_S - v(\ell_S)\}.$$

("S" is for the safe return to labor.) Hence $R > S$. Conditional on p, the expected social return to higher education is

(4) $$\Delta(p) \equiv p[R - S] - r,$$

since $p[R-S]$ is the expected increment to real income, and r is the opportunity cost (one unit of the endowment good times the gross interest factor r).

We assume that per capita wealth exceeds one, so that it is feasible for every individual in the economy to undertake higher education. Whether or not it is efficient to do so depends on the individual's probability, p, of successfully completing a program of higher education. We will assume, by choice of parameters, that it is efficient for some, but not all, individuals to undertake higher education.

To find the first-best allocation of individuals to higher education programs, a social planner would solve the following problem: Choose a cut-off value of p, denoted p^*_0, such that if, and only if, an individual's success probability is at least p^*_0, the social return to education is positive. For p above p^*_0, the individual undertakes higher education. For p below p^*_0, the individual does not undertake higher education. The first-best cut-off value, using (4), is

(5) $$p_0^* = \frac{r}{R - S}.$$

which can be rewritten as $\Delta(p^*_0) = 0$. The social planner expects zero social returns on the marginal student in a program of higher education. Since the marginal student has the lowest success probability of all students, expected social returns for all inframarginal students are positive.

3. Competitive Equilibrium with Private Information

This section derives an incentive constraint that leads to inefficient occupational choice in the competitive equilibrium with private information. We assume that lenders know only the probability distribution of abilities, but do not know which probability is applicable to a particular borrower. Until section 6, we will also assume that:

(A.1) An individual learns his probability of success costlessly before he undertakes higher education.

As before, we parameterize occupational choice as the choice of a cut-off value p^*, such that only if an individual has a success probability at least equal to his choice of p^* does he undertake the program of higher education. But now p^* is chosen by each individual, not by a social planner.

There is a perfectly competitive labor market and financial market. An individual whose endowment of the input good, W, is less than 1 will have to borrow in the financial market to undertake a program of higher education. If he succeeds in the program, he repays the loan, but if he fails at higher education, he defaults.[5] Because of the possibility of default, the lender's break-even interest factor, i, exceeds the opportunity cost of funds, r. One consequence of this and the assumption that endowments are observable is that an individual who undertakes higher education will always put up as

---

[5]This assumption is stronger than needed to obtain our qualitative results; we require only that an individual who fails in a higher education program has a probability of defaulting on his loan.

much finance himself as he can.[6] An individual for whom $W \geq 1$ does not borrow and solves the same problem as the social planner in making his occupational choice; thus, he invests in higher education if he learns that his success probability is at least p^*_0. An individual for whom $W < 1$ will choose a value of p^* that maximizes his expected utility, taking into account the possibility of default. He will therefore choose to undertake higher education for all p for which his expected income if he undertakes education is greater than or equal to his expected income if he does not:

$$p\{R - i[1-W]\} + [1-p]S \geq S + rW. \quad (6)$$

The Nash value of the cutoff p^* thus satisfies

$$p^*\{R - S - i[1-W]\} = rW \quad (7)$$

provided that

$$R - S - i[1-W] > 0, \quad (8)$$

i.e., that the interest rate is not so high that the individual does not gain from succeeding in the higher education program. At a sufficiently low value of W, (8) may not be satisfied, and in that case the outcome is zero investment in higher education regardless of p. We will discuss the case of zero-investment in a slightly richer model presented in section 6. Until then, we will assume that (8) is satisfied. (7) then says that the individual equates the expected gain from undertaking higher education at his reservation success

---

[6]A proof of this is in de Meza and Webb (1987, p. 289), who also show that the standard debt contract dominates an equity contract. A standard debt contract will be the optimal contract under a slight extension of the model where some individuals have a "distaste" for higher education and so will not pursue it regardless of their ability, and this taste parameter is private information. The proof follows the lines of Bernanke and Gertler (1990, Appendix).

probability with his opportunity cost of undertaking higher education.

Lenders lend based on an estimated probability $\hat{p} \equiv E(p \mid p \geq p^*)$ that they will be repaid. Because of maximum equity participation, p^* can be inferred from the loan amount, $1-W$. Perfect competition and the pooling of risk drive expected profits of lenders down to zero. To break even, a lender will require a gross finance charge per dollar lent of

$$(9) \qquad i(p^*) = \frac{r}{\hat{p}(p^*)},$$

Each person's p is a random variable drawn from a distribution function H (and density function h), so that

$$(10) \qquad \hat{p}(p^*) = \int_{p^*}^{1} p h(p)\, dp \;.$$

The interest rate reflects the average risk of individuals who are observationally equivalent to the lender: the good risks cross-subsidize the bad risks.[7] As a consequence, an individual's private expected return to higher education, $p\{R - S - i[1-W]\} - rW$, is more than (equal to, less than) the social expected return, $\Delta(p)$, as his success probability p is less than (equal to, more than) the average of his wealth class, $\hat{p}(p^*)$.

Substituting (9) into (7) yields the reduced form incentive-compatibility constraint governing p^*:

[7]Since p^* is a function of wealth (as we shortly show), interest rates charged by lenders will depend on wealth and the cross-subsidization will occur only *within* each wealth class.

(11) $$p^*[R - S] = r\left\{W + [1 - W]\frac{p^*}{\hat{p}(p^*)}\right\}.$$

Since $p^*/\hat{p}(p^*) < 1$ and $W < 1$ for any borrower, the term in braces is also strictly less than one, which implies that $p^*[R - S] < r$, so

(12) $$\Delta(p^*) < 0 .$$

The marginal borrower, who is just indifferent between undertaking and not undertaking higher education, makes a negative present value social investment. The reason is that individuals do not take account of the social cost borne by others when they default. They undervalue the true total costs and undertake education when it is inefficient to do so. Differentiating (11) yields $dp^*/dW > 0$. As simple intuition would suggest, the moral hazard problem is greater, the more of the individual's costs are borrowed funds.[8]

For any given wealth class that chooses $p^* < p^*_0$, the agency costs incurred in the competitive equilibrium allocation are equivalent to throwing away, on a per capita basis, resources with a value of

---

[8]With different assumptions about information and the choice set of borrowers, the marginal borrower (or marginal project) could be the *lowest* risk, not the *highest* risk, and therefore the most, not the least, profitable to the lender. In this case the problem of asymmetric information would normally lead to *under*-investment, not *over*-investment, in the competitive equilibrium, as de Meza and Webb (1987, Proposition 5) showed. Our central result in section 4 that there is scope for Pareto-improving redistributions holds, in general, in both cases. Our result turns on the fact that the transfer serves as a form of collateral, and an increase in collateral increases market efficiency. The latter result holds in a wide variety of models; see Chan and Thakor (1987) and Bester (1987). For limitations on that result that may arise when the borrower's wealth is not observable to the lender, see Stiglitz and Weiss (1981, Section III).

(13) $$L = -\int_{p^*}^{p_0^*} \Delta(p)h(p)dp.$$

We derive in Appendix A the relation between changes in L with respect to p* and changes in the equilibrium interest rate:

(14) $$\frac{dL}{dp^*} = \Delta(p^*)h^* = -\frac{\partial U}{\partial i}\frac{di}{dp^*} < 0.$$

(14) means that an intervention that increased the individual's choice of p* would, through its effect on the equilibrium interest rate, increase expected utility by exactly the marginal reduction in the loss, L.

4. Pareto-Improving Redistributions

This section will show that a labor tax whose revenues are returned in lump sum fashion can yield a Pareto improvement. We could think of the tax as financing lump-sum grants, G, of the consumption good at the end of the period or, alternatively, lump-sum transfers, G/r, of the endowment good at the beginning of the period.[9] All individuals with the same wealth face the same tax rate

[9]In the first case, the government would announce that it would make transfers at the end of the period. These transfers, being riskless and collateralizable, would enable individuals to issue bonds at the riskless rate r. In low-income countries, it is not uncommon for government transfer payments to be pledged as collateral for debts; see, e.g., Platteau et al. (1980, p. 1767) and Sanderatne (1986, p. 349).

In the second case, the government would use the receipts from a bond sale to finance grants of the endowment good at the t and receive the same transfer G. The tax-transfer program is ex-post redistributional because although everyone with equal wealth receives the same transfer payment, those who have succeeded in higher education enter the high-wage occupation and, given their higher labor income, pay more in taxes.

By analogy to (2) and (3), define

$$S_t \equiv \max_{\ell_S} \{[1-t]w\ell_S - v(\ell_S)\}$$

$$R_t \equiv \max_{\ell_R} \{[1-t]\alpha w\ell_R - v(\ell_R)\} .$$

Recalling that in the Nash equilibrium, an individual finances his own education to the extent feasible, he now maximizes his expected utility by undertaking higher education for all p such that

$$(15) \qquad p\left\{R_t - i\left[1 - W - \frac{G}{r}\right]\right\} + [1-p]S_t \geq S_t + rW + G,$$

which defines p* by the relation

$$(16) \qquad p^*\left\{R_t - S_t - i\left[1 - W - \frac{G}{r}\right]\right\} = rW + G.$$

The individual equates the increase in his expected income if he undertakes higher education at his reservation probability of success, p*, to his opportunity cost of higher education, rW + G. Since G/r enters into (16) exactly as W does, we have

---

beginning of the period. At the end of the period, the government would pay off the bonds by taxing labor.

$$\frac{\partial p^*}{\partial\left(\frac{G}{r}\right)} = \frac{\partial p^*}{\partial W} > 0. \tag{17}$$

We call this effect the *collateral effect*. The transfer, G, allows the individual to put more of his own wealth at risk when he undertakes higher education. With greater equity in himself, the individual is more selective in his choice of whether or not to proceed with higher education; and this raises p^*.

The labor tax also affects p^* by affecting the payoffs to work, R_t and S_t. Differentiating (16) with respect to t yields what could be called the *relative price effect* of the tax policy:

$$\frac{\partial p^*}{\partial t} = \frac{p^*[\alpha w\ell_R - w\ell_S]}{R_t - S_t - i\left[1 - W - \frac{G}{r}\right]} \geq 0, \tag{18}$$

with strict inequality for $p^* > 0$. Thus, the tax tends to reduce the incentive to gamble on higher education, which raises p^*.

Fig. 1A illustrates the ability of the tax-transfer scheme to mitigate the incentive problem in the capital market. In the simulation, we assume that $v(\ell) = \ell^3$, corresponding to a constant (compensated and uncompensated) labor supply elasticity of 0.5. The wage rate w in the unskilled occupation is 3 and in the skilled occupation is 6. The gross riskless interest rate r over the period the educational loan is outstanding is 1.5. Success probabilities p for each individual are distributed according to the bell-shaped density function $h(p) = 6[p-p^2]$.

The shaded areas in the figure illustrate the deviation of p^* from p^*_0, for wealth endowments in the interval [0,1). The sum of these two areas represents the distortion in p^* from first-best under competitive equilibrium, and the hatched area represents that part of the distortion that is avoided under the Pareto optimal tax-transfer policy. For example, for a person with no endowment wealth, the competitive equilibrium yields $p^* = 0.0$, and the Pareto optimal tax-transfer policy raises p^* to .38. This is close to the first-best threshold, $p^*_0 = .41$, which is independent of endowment wealth.

Of course, the tax-transfer policy has to be judged by its effect on welfare. The effect of the tax-transfer policy on individual expected utility is the sum of three terms: the direct effect of the tax on income, the direct effect of the transfer on income, and the indirect effect of the tax-transfer policy via its influence on the equilibrium interest rate.[10] We have

$$(19)\quad dU = -N dt + dG + \frac{\partial U}{\partial i}\frac{di}{dp^*}\left[\frac{\partial p^*}{\partial W}\frac{dG}{r} + \frac{\partial p^*}{\partial t}dt\right]$$

$$= -N dt + dG - h^*\Delta_t(p^*)\left[\frac{\partial p^*}{\partial W}\frac{dG}{r} + \frac{\partial p^*}{\partial t}dt\right] \quad \text{(from (14))},$$

where N denotes an individual's expected pretax labor earnings,

$$(20)\qquad N \equiv w\left\{\ell_S + [1-H^*]\hat{p}[\alpha\ell_R - \ell_S]\right\}$$

The tax-transfer policy will be Pareto-improving if expected

[10]We apply the envelope theorem to the variables ℓ_S and ℓ_R, but not to p^*. The individual is not optimizing with respect to p^* since he treats i as parametric; thus the term $\partial U/\partial p^*|_{i \text{ fixed}} = 0$, whereas the total derivative, dU/dp^*, is positive (recalling (14)). The term $\Delta_t(p^*)$ is defined as $p^*[R_t-S_t]-r$ by analogy to (4). This term is not the expected social return to higher education, conditional on p^*, since it is net of tax collections. Instead, it is the combined after-tax expected return to the lender and marginal borrower.

utility increases when the net government cost of the policy is zero. The government budget associated with each person is, in expectation, $B = tN - G$. Diversification across taxpayers ensures that a transfer that is feasible in expected terms will also be feasible in realizations *ex post*. The budget constraint on the government means that

$$(21)\quad dB = Ndt - dG + t\left\{\frac{\partial N}{\partial t}dt + \frac{\partial N}{\partial p*}\left[\frac{\partial p*}{\partial G}dG + \frac{\partial p*}{\partial t}dt\right]\right\} = 0,$$

or that any incremental changes in revenues and transfers add up to 0. The difference, $Ndt - dG$, is the revenue effect of the policy at the initial tax base, while the terms inside the braces are the changes in the initial tax base resulting from changes in the tax-transfer policy.

These taxes and transfers show up in the model by changing the individual's endowments and his choices over labor hours and occupation. To check whether the tax-transfer policy is Pareto-improving, we substitute the balanced budget condition (21) into the welfare expression (19), which yields

$$(22)\quad dU|_{dB=0} = -h^*\Delta_t(p^*)\left[\frac{\partial p^*}{\partial W}\frac{dG}{r} + \frac{\partial p^*}{\partial t}dt\right] + t\left\{\frac{\partial N}{\partial t}dt + \frac{\partial N}{\partial p^*}\left[\frac{\partial p^*}{\partial W}\frac{dG}{r} + \frac{\partial p^*}{\partial t}dt\right]\right\}.$$

The tax-transfer policy raises the preferred cut-off point for investment in human capital, p^*, which reduces negative present value investments and so reduces the competitive interest rate charged. These effects are captured by the first of the two terms in

(22). As p^* rises, agency costs, given by (13), fall. The reduction will be larger (a) the greater the density of persons who would withdraw from the market if their collateral were increased, as measured by h^*, (b) the more negative the expected after-tax return to the marginal investor and his lender, $\Delta_t(p^*)$, (c) the greater the increase in p^* through the *collateral effect*, $\partial p^*/\partial(G/r) = \partial p^*/\partial W$, and (d) the greater the increase in p^* through the *relative price effect*, $\partial p^*/\partial t$.

The second term in (22) is the effect of the tax-transfer policy on the tax base. Starting from a zero tax rate, the government initially is collecting no money from labor taxes and so the reduction in labor earnings N does not affect the budget. The second term vanishes. Hence, $\frac{dU}{dt}\Big|_{\substack{dB=0\\t=0}} > 0$, which proves

Proposition 1. There exists a Pareto-improving redistributive policy consisting of a positive labor tax and a lump-sum grant to each individual in an amount equal to his expected tax payment.

Fig. 1B illustrates the welfare gain at the Pareto optimal tax rates. For each level of endowment wealth, the Pareto optimal wage tax rate occurs where the marginal deadweight loss from increasing the labor tax rate is just offset by the marginal gain from the shift in the incentive constraint (16). Given the parameter values used for fig. 1A, a tax rate on labor of 11.7 percent is Pareto optimal for individuals with zero wealth. The optimal tax rate declines approximately linearly as wealth increases from 0 to 1. (It is indicated for selected wealth values in fig. 1B.) For those with wealth of at least one unit, the incentive constraint (16) does not bind and the optimal tax rate is, of course, zero.

The figure expresses the Pareto improvement from the optimal labor tax rate as a percentage of the difference between expected utility obtained under a first-best allocation and the competitive equilibrium. The tax-transfer policy recaptures more than 70 percent of the agency costs for all groups with wealth between zero and 0.99 units.[11]

5. Redistributions to Maximize Aggregate Income

The previous section considered Pareto optimal tax-transfer schemes. Income redistribution was *ex post*, contingent on the resolution of uncertainty; each person expected to make tax payments precisely equal to the lump-sum transfer he received. This section considers instead a tax-transfer policy where transfers are paid only to the lowest wealth group. Tax rates are chosen to maximize the sum over all persons of expected real income, $E(y) - v(\ell)$. Parameter values for these simulations are the same as in the earlier simulation, but we now additionally need to specify a distribution of wealth. For simplicity, we assume there are only two wealth levels: zero wealth and two units of wealth.

---

[11]As noted above, the figure is based on an assumed compensated labor supply elasticity of 0.5. This exceeds most estimates of the compensated elasticity for primary workers, although it may understate the elasticity for secondary workers (see, e.g., Hausman, 1985). Sensitivity analysis was conducted with respect to the labor supply elasticity. Increases in the compensated labor supply elasticity increase the standard deadweight loss from taxation, but in this model they affect other parameters, such as p^* and p^*_0, which can also affect the welfare gains from the tax-transfer policy. We find significant efficiency gains from the tax-transfer policy at very low compensated labor supply elasticities (recovering as much as 99.8 percent of the loss in utility created by agency costs at a compensated elasticity of 0.01) and decreasing efficiency gains for a wide range of higher elasticities (e.g., ranging from 81 percent to 50 percent over the elasticity range of 0.75 to 1.5, given zero wealth).

The experiment that we consider is differential proportional wage taxes on the zero-wealth and high-wealth individuals, with transfer payments made in lump sum fashion to those with no wealth. Recall that a wage tax has a *relative price effect* for all individuals, in addition to the traditional labor-leisure distortion. The relative price effect is strictly welfare-increasing for the low-wealth group, but it creates an additional distortion for the high-wealth group affecting occupational choice. As a result, the optimal labor tax rate on the low-wealth individuals will exceed that on high-wealth individuals.

Fig. 2A shows the real income-maximizing tax rate schedule for a range of populations at the two wealth levels. The horizontal axis measures the ratio of those with wealth of two units relative to those without wealth. If no high-wealth workers exist, the socially optimal tax rate is 11.7 percent, identical to the simulation examined in the last section. As the proportion of high-wealth workers increases, the taxes they pay make it desirable to reduce the tax rate on those without wealth. While tax rates fall, the size of the transfer payment increases. Because the larger grant to those without wealth induces them to choose a reservation success probability closer to the social optimum, and because the lower tax rates (but broader coverage) reduce the excess burden in the labor market, the *ex ante* redistributive tax-transfer policy can bring aggregate real income close to that of the first-best allocation.

Fig. 2B shows the gain that is achievable. The measure of the gain is the analog, in terms of aggregate real incomes, of that used in fig. 1B for individual real income. If there are as many as 10 individuals with wealth of two units for every individual with zero wealth, 95 percent of the loss in aggregate real income created by the incentive constraint can be recaptured, an increase relative to the 89 percent recaptured in the Pareto optimal tax-transfer policy.

6. A More General Model

The model of the preceding section showed how a tax-transfer scheme could be used to solve a problem of over-investment in higher education. But the more difficult and important problem of public policy is generally argued to be under-investment in higher education by the poor. The model above generated under-investment if (8) was violated. Under-investment is even more likely if we replace assumption (A.1) above by the more realistic assumption that knowledge of one's ability comes at a cost. For the remainder of this paper, we replace (A.1) by (A.2):

(A.2) An individual learns his probability of success in higher education by exerting effort at utility cost e. Without exerting such effort, he can never succeed at higher education.

It is natural to interpret the cost e as the effort cost of applying oneself in publicly provided primary and secondary school.

We assume that it is efficient for everyone to prepare for higher education so as to learn his ability to enter the high-wage occupation. Formally,

$$[1 - H^*_0]\Delta(\hat{p}(p^*_0)) > e. \tag{23}$$

This says that the expected return to preparation evaluated at p^*_0 exceeds the sunk cost e. But (23) does not ensure that everyone prepares for higher education in a competitive equilibrium.

Proposition 2. *For $W < 1$, each individual's private expected net return to preparation for higher education, $[1-H^*]\Delta(\hat{p}) - e$, is increasing in endowment wealth, W.*

Proof. We have[12]

$$\frac{d\{[1-H^*]\Delta(\hat{p})\}}{dW} = \left[-h^*\Delta(\hat{p}) + [1-H^*][R-S]\frac{d\hat{p}}{dp^*}\right]\frac{dp^*}{dW} \quad \text{(by using (4))}$$

$$= [1-H^*]\,i\,[1-W]\frac{d\hat{p}}{dp^*}\frac{dp^*}{dW} \quad \text{(by using (10) and (2-A) in the Appendix)}$$

which has the same sign as dp^*/dW and hence is strictly positive. See fig. 3.■

Proposition 2 immediately implies[13]

Proposition 3. *If e is sufficiently high, there will exist a nonnegative level of wealth, $\underline{W}$, such that individuals with endowments below $\underline{W}$ do not prepare for higher education.*

Thus, an individual will choose to prepare for higher education provided

(24) $$[1 - H^*]\Delta(\hat{p}(p^*(W)) > e;$$

Given the incentive constraints (11) and (24), every wealth class is in one of three cells illustrated in fig. 4. The fourth cell is empty, recalling (23).

7. The Scope for Price Policy

In economies with imperfect information, there generally exist corrective taxes (Pigouvian taxes or subsidies) that yield Pareto improvements (Greenwald and Stiglitz, 1986). This section will show that the scope for corrective taxes is limited in this model. Under

---

[12]We use the fact that $d\hat{p}/dp^* = h^*[\hat{p} - p^*]/[1-H^*]$.

[13]The intuition behind propositions 2 and 3 is the same as that behind Bernanke and Gertler's (1990) proposition 3.

some circumstances, credit taxes will not be effective at all; and under all circumstances, lump sum transfers will dominate credit subsidies.

In this model, corrective taxes operate through their effects on two incentive constraints, (24) and (11), interpretable as governing the decisions to invest effort in primary and secondary school, and to invest physical wealth in higher education. The first constraint is not binding on a borrower with sufficiently high wealth, i.e., for whom $W > \underline{W}$; the second constraint is always binding for a borrower. Barring one special condition, set out in proposition 4, a credit tax or subsidy has *opposite* effects on the two constraints: a price policy that loosens one constraint tightens the other.

First consider a tax on credit. An individual's choice of p^* is increasing in i; hence, a high enough tax on credit could always induce the efficient choice, p^*_0,[14] provided that the incentive constraint in (24) does not bind. Under one condition, stated as proposition 4 below, we obtain the novel result that a tax on credit can actually *raise* the borrower's expected after-tax private return to higher education.[15] It thereby simultaneously relaxes constraints (11) and (24). To see how this can occur, notice that a credit tax, by increasing an individual's preferred choice of p^*, increases the

---

[14]This result is verified by differentiating the expression defining p^* in (7) to obtain $dp^*/di = p^*[1-W]/\{R-S-i[1-W]\} > 0$.

[15]But a higher interest rate is not sustainable as a Nash equilibrium (by means other than government intervention) because every lender would have an incentive to shave his interest rate and thereby increase his market share and his profits.

expected probability of success and reduces the equilibrium interest rate. The condition under which these two effects more than offset the tax payment is that $\beta < 0$ in (25) below. This condition is more likely to hold if, for a given wealth group, the negative return on marginal investors in human capital is large relative to the average return for that group ($-\Delta(p^*)/\Delta(\hat{p})$ is large). It is also more likely if $d\ln\hat{p}/d\ln p^*$ is large; that is, a small percentage change in p^* induces a large percentage change in the average quality of the applicant pool.

Proposition 4. A tax on credit at rate T increases (decreases) the expected after-tax private return to preparation for higher education, $[1-H^*]\{\Delta(\hat{p}) - \hat{p}T[1-W]\}$, *if β is negative (positive), where*

$$\beta \equiv \hat{p}\Delta(\hat{p}) + p^*\Delta(p^*)\frac{d\hat{p}}{dp^*}. \tag{25}$$

Proof. See Appendix B.

In the "normal" case of $\beta > 0$, the credit tax *decreases* expected after-tax returns to preparation for higher education. As the credit tax is raised, it eventually depresses after-tax returns by enough to curtail any human capital formation by those with endowment wealth below one. Thus we have

Proposition 5. If $\beta > 0$, a tax on credit increases the range of wealth levels over which zero investment in education occurs.

Proof. Let $\phi(W,T) \equiv [1-H^*]\{\Delta(\hat{p})-\hat{p}T[1-W]\}$. After the imposition of a credit tax, $\underline{W}$ is implicitly defined by $\phi(\underline{W},T) = e$. Since $\partial\phi/\partial W > 0$ from proposition 2 and $\partial\phi/\partial T < 0$ from proposition 4 for $\beta > 0$, applying the implicit function theorem yields $d\underline{W}/dT > 0$, as was to be shown.■

In summary, there exists a credit tax that yields a first-best efficient outcome if either (24) does not bind or $\beta < 0$. But in the normal case where $\beta > 0$, the tax on credit tightens (24); in terms of fig. 4, a credit tax can push a wealth class from cell 1 into cell 2. With respect to individuals in cell 2, the objective of price intervention is to solve a problem of under-investment, not over-investment. The only price policy that can do this (given $\beta > 0$) is one that subsidizes the cost of education.

Because there is no credit rationing in this model, credit subsidies, direct loans, and loan guarantees have identical effects. For concreteness the next two propositions consider a credit subsidy to lenders at rate σ, where $0 \leq \sigma \leq r$. With the subsidy in place, the zero-profit condition for lenders is no longer given by (9) but by $r - \sigma = \hat{p}i$.[16]

Proposition 6. If $\beta > 0$, a credit subsidy reduces the range of wealth levels over which zero investment in education occurs.

Proof. The credit subsidy at rate σ is equivalent to a negative tax on credit at rate $T = -\sigma/\hat{p}$. It follows from proposition 5 that $d\underline{W}/d\sigma < 0$. Since this derivative is bounded away from zero for W below 1, there exists a subsidy rate σ^* that induces preparation for higher education, for any given endowment W.■

The remainder of this section shows that government would not wish to use a credit subsidy or loan guarantee; a lump-sum transfer would be more effective. This is established by showing

---

[16]Suppose that a loan guarantee ensured the lender of receiving an amount γ per dollar lent in the event of default. With the guarantee in place, the zero-profit condition for lenders is $r = \hat{p}i + [1-\hat{p}]\gamma$, so that the guarantee has the same effect in the competitive equilibrium as a subsidy where $\sigma = [1-\hat{p}]\gamma$.

that an increase in the subsidy rate σ, financed by a reduction in a lump-sum grant G, could never increase an individual's expected utility, and would generally lower it.

Consider a government budget constraint in terms of per capita expenditures for a target low-wealth group, $B = [1-H^*][1-W-G/r]\sigma + G$, where B denotes revenues financed by taxes imposed on a high-wealth group,[17] and the right-hand side terms denote, respectively, expected outlays through the credit subsidy program and outlays through the grant. All amounts are in units of the end-of-period consumption good.

The individual chooses either to prepare for higher education, obtaining expected utility denoted U^P (for prepare), or not to prepare for higher education, obtaining expected utility denoted U^{NP} (for not prepare), where

$$U^P \equiv S + H^*[rW + G] + [1-H^*]\hat{p}\left\{R - S - \frac{r-\sigma}{\hat{p}}[1 - W - \frac{G}{r}]\right\} - e,$$

and

$$U^{NP} \equiv S + rW + G.$$

His willingness to give up transfers in exchange for an increase in the subsidy rate is of course zero when $U^{NP} \geq U^P$, and otherwise is

[17]This assumption simplifies the proof because it makes revenues independent of the policy mix between subsidies and transfer. But our result that a lump-sum transfer Pareto dominates the credit subsidy is independent of that assumption.

$$(26)\quad \frac{-dG}{d\sigma}\Big|_{\bar{U}^*} = \frac{[1-H^*]\left[1-W-\frac{G}{r}\right] + \frac{\partial U}{\partial i}\frac{di}{dp^*}\frac{\partial p^*}{\partial \sigma}}{1-[1-H^*]\frac{\sigma}{r} + \frac{\partial U}{\partial i}\frac{di}{dp^*}\frac{\partial p^*}{\partial G}} < \frac{[1-H^*]\left[1-W-\frac{G}{r}\right]}{1-[1-H^*]\frac{\sigma}{r}}$$

The far right-hand side fraction is what the marginal rate of substitution (MRS) *would be* if his choice over p^* were independent of σ and G: The numerator is the direct effect on expected income of an increase in σ, which depends on the probability that he borrows ($1-H^*$) and the amount he borrows ($1-W-G/r$). The denominator is the direct effect on expected income of an increase in G by one unit, which is equal to one less the credit subsidy foregone as a result of the fall in his loan size by $1/r$ units, times the probability that he borrows ($1-H^*$). This fraction exceeds the actual MRS, as shown. In the numerator of the MRS, there is an additional term, which is the loss in expected utility produced by the increase in the equilibrium interest rate as σ rises. In the denominator, there is an additional term, which is the gain produced by a fall in the equilibrium interest rate as G rises. Recalling (14), the additional term in the numerator and denominator, respectively, can be rewritten as $\partial L/\partial\sigma$ (a strictly positive term), and $\partial L/\partial G$ (a strictly negative term): *credit subsidies increase waste, government transfers reduce it.*

Now consider the tradeoff between σ and G along the government's budget constraint. A balanced budget change increasing σ requires no adjustment to G when $U^{NP} \geq U^{P}$, and otherwise entails

$$\frac{-dG}{d\sigma}\Big|_{dB=0} = \frac{[1-H^*]\left[1-W-\frac{G}{r}\right] - \sigma\left[1-W-\frac{G}{r}\right]h^*\frac{\partial p^*}{\partial \sigma}}{1-[1-H^*]\frac{\sigma}{r} - \sigma\left[1-W-\frac{G}{r}\right]h^*\frac{\partial p^*}{\partial G}}$$

(27)

$$> \frac{[1-H^*]\left[1-W-\frac{G}{r}\right]}{1-[1-H^*]\frac{\sigma}{r}}$$

The term shown to the right of the inequality is identical to the last term of (26); it is what the balanced-budget tradeoff *would be* if p^* were held fixed. But it is less than the government's actual tradeoff because as σ rises and G falls, the individual's preferred choice of p^* falls, increasing by h^*dp^* the probability with respect to that individual of government outlays of $\sigma[1-W-G/r]$.

Since, for each individual, the MRS is less than or equal to the tradeoff between σ and G along the government's budget constraint, we have:

Proposition 7. *A lump-sum grant dominates a credit subsidy for every wealth group.*

The proposition illustrates the general principle that in order to design effective policies to remedy a market failure, one has to understand its underlying source. The source of the market failure is that individuals who do not succeed in the higher education program default and shift part of the cost of education to others. An equilibrium with zero investment in education by a given wealth group occurs if the cost-shifting problem is sufficiently severe that, for that wealth group, the expected private return to preparation for higher education is negative. *Credit subsidies do not correct the externality*; instead, they mask its effects by lowering the interest rate charged to borrowers, and they tighten the incentive constraint (11) that is at the heart of the market failure. On efficiency grounds it is therefore better to distribute funds to low-wealth persons through grants, which relax the incentive constraint, than through a credit subsidy.

8. Robustness

A principal conclusion of this paper has been that redistribution financed with distortionary taxes can increase efficiency. It is natural to ask whether this conclusion can be laid to the special assumptions of the model. We already touched on the consequences of some alternatives to the informational assumptions used in the model, and argued that the answer to this question was "No." In this section we explore the consequences of adding a market in forward labor, and allowing for pre-existing labor taxes.

8.1 *Forward Labor Markets*

Suppose that there was a forward market in labor. Then one might expect that an individual who had decided to borrow would gain from forward labor sales. A competitive market would pay him $\hat{p}/r$ per dollar of future labor earnings in the high-skill occupation. Recall that if the individual fails in the higher education program, he defaults. Knowing this, and knowing that individuals who choose not to undertake higher education could never gain from forward labor sales, a buyer on the forward labor market would pay nothing for earnings in the unskilled occupation. Thus, for every dollar an individual could obtain through forward sales, he would forego $r/\hat{p}$

of future expected income. The forward transaction would allow him precisely the same intertemporal reallocations as the credit market. The opening of forward labor market would be redundant!

More generally, the redundancy result would obtain to the extent that bankruptcy laws treat in the same way creditors in the financial market and owners of contracts in the forward labor market. Some labor income will normally be protected from the claims of creditors, whether they be banks, employers, or any other claimants.[18] The fundamental causes of agency costs in this model are asymmetric information combined with limited liability and limited capacity to repay debt in the case of failure, not the absence of forward labor markets.

This model provides a rationale for the widespread use of one kind of forward labor market, where an employer finances the education of his employee, contingent on the employee's commitment to remain with the firm for a minimum term. Such tied labor-credit arrangements occur because the employer has better knowledge than a bank would have about the employee's abilities.

8.2 *Pre-existing taxes*

Suppose that there were pre-existing taxes. One might think that our result that a tax-transfer policy increases aggregate real incomes would be very much weakened when allowance was made for high, pre-existing labor taxes, since the marginal excess burden per additional dollar of tax revenue would in that case be high. This is a valid criticism of our results on the scope for Pareto-improving

---

[18]For example, most U.S. states permit a debtor to exempt a certain amount of personal property, including a portion of wages or earnings. See Epstein, Nickles, and White (1993, section 8-6).

taxes (section 4). But it does not apply persuasively to policies that would increase the tax rate on the primary earner in a household and redistribute the revenues as transfers to some other group. A result due to Stiglitz and Dasgupta (1971, pp. 158-59) and Atkinson and Stern (1974) is that, given a pre-existing labor tax, the marginal cost of transferring income from an individual through an increase in the labor tax rate depends on his *uncompensated*, not his *compensated* labor supply: the marginal cost of funds from an increase in a proportional wage tax is one if labor supply is perfectly inelastic (and less than one if backward bending).[19] Much empirical evidence suggests that the uncompensated labor supply elasticity is close to zero for prime-age male earners [see, e.g., Killingsworth (1983)]. This suggests that pre-existing labor taxes in themselves do not rule out the ability to increase aggregate real incomes through income transfers from high-wealth to low-wealth groups.

9. Conclusion

Most analytical work on the trade-off between efficiency and equality was implicitly based on the assumption of perfect markets. Once that assumption is abandoned, the conclusion that redistribution through distortionary taxes reduces aggregate incomes no longer need hold. We considered an example where asymmetric information about individuals' abilities creates inefficiencies in an assignment problem: some high-ability persons have no access to higher education and are thus assigned to low-skill jobs; some low-ability persons invest in higher education when the expected return

[19]For a recent discussion of this result, see Ballard and Fullerton (1992). This result is obtained in a model where labor hours, but not occupation, are endogenous.

is negative. We showed that redistribution through distortionary labor taxes, by creating a collateralizable asset, can improve on the competitive solution to the assignment problem and thereby yield Pareto improvements and increase aggregate incomes. We focused on one particular model of the credit market, but the basic principle applies to many other stories of the credit market and to other models with moral hazard and adverse selection, where an increase in an agent's wealth or "bond" leads to more efficient allocations.

The result that simple tax-transfer policies can increase aggregate incomes would be of limited policy relevance if superior instruments were available to government to reduce agency costs. For in that case once government had optimally used corrective policies, then no further gains could be achieved through redistribution. But we found that where higher education is beyond the reach of the poor, income transfers are in most circumstances more effective than targeted credit taxes, subsidies, or loan guarantees, because the latter do not address the source of the market imperfection. Moreover, as de Meza and Webb and other scholars have emphasized, using price policy to solve an information problem is perilous because the information requirements for successful intervention are extreme.[20] In contrast, the ability of an increase

[20]If the forces governing competitive equilibrium are as described in the model of section 3 of this paper, then a *tax* on interest income can achieve social efficiency. If the forces governing competitive equilibrium are those of the model of Stiglitz and Weiss, then a *subsidy* on interest income can increase social efficiency because it encourages borrowing by the marginal individuals (and these are the lowest risk type). If market forces lead to a separating equilibrium with rationing, then credit subsidies and direct provision of loans have distinct effects, and the latter

in collateralizable wealth to enhance efficiency appears to be fairly robust.

What does this analysis tell us about real world policy towards the financing of higher education? U.S. education policy is beset by many problems, including high default rates, a high rate of unprepared students entering higher education and then dropping out, and fraudulent training schools that become mills for obtaining tuition financed by government-guaranteed loans.[21] The model of this paper is too simple to provide a firm guide to public policy. But one might wish to examine the following proposal, which differs greatly from most current proposals: Every eligible child shall receive a grant, which would be available for school financing when the child reaches 18 years of age, or for use without restriction at some later age.[22]

The analysis of this paper suggests that such a proposal would bring the private return to investment in human capital closer to the social return; it would address the problem of both over- and under-investment in higher education. Where this proposal differs from most current educational grant proposals is that the grant can alternatively be used for non-educational purposes after attainment of some minimum age. By creating an opportunity cost to the use of

---

reduces efficiency (Smith and Stutzer, 1989 and Gale, 1990).

[21]A detailed account of one such school is given in a recent U.S. legal case, Williams v. National School of Health, Technology, Inc., DC EPa, No. 92 2536, 10/22/93.

[22]We do not discuss here the question of eligibility, but see Edlin's (1993) proposal for reforming eligibility under the current U.S. program of college financial aid.

funds for educational purposes, students would have a strong incentive to enter a program of higher education, if at all, only after having prepared for it. Preparation as modelled in this paper should be interpreted broadly, including, in particular, the acquisition of information about the quality of any higher education program in which an individual would consider investing--reducing the likelihood of fraudulent "education loan mills."

The basic policy question for education finance is quite simple: if a government department has a mandate to address a failure in the market for financing higher education, what should it do? We showed that the classical solution to a market failure through corrective taxes would, under some circumstances, be impossible to achieve even if government had perfect information about the source of the market failure. If government does not have perfect information, implementing corrective taxes on borrowing risks lowering welfare. We have argued that a better solution may exist simply through government grants to the targeted low-wealth group. Transfers can increase over-all economic efficiency and the targeted group's welfare at the same time; under many circumstances, corrective price policy and loan guarantees cannot.

Appendix

A. Proof of (14)

The choice of p* under individual expected utility maximization may be written as

$$U(i,W) = \underset{p^*}{\mathrm{Max}}[1-H^*]\left\{\hat{p}\left[R-S-i[1-W]\right]-rW\right\} + rW + S$$

(1-A)

$$= \underset{p^*}{\mathrm{Max}}[1-H^*]\Delta(\hat{p}) + rW + S \text{ (by using (9) and (4))}$$

so that the first-order condition is equivalent to

(2-A) $$h^*\Delta(\hat{p}) = [1-H^*]\left\{R-S-i[1-W]\right\}\frac{d\hat{p}}{dp^*}.$$

Differentiating the lenders' zero-profit condition, $i = r/\hat{p}$, we have

$$\frac{di}{dp^*} = -\frac{i}{\hat{p}}\frac{d\hat{p}}{dp^*}.$$

Since $\partial U/\partial i = -[1-H^*]\hat{p}[1-W]$, we have

$$\frac{\partial U}{\partial i}\frac{di}{dp*} = [1-H^*][1-W]\,i\,\frac{d\hat{p}}{dp^*}$$

$$= -[1-H^*]\left\{R-S-i[1-W]\right\}\frac{d\hat{p}}{dp^*} + [1-H^*][R-S]\frac{d\hat{p}}{dp^*}$$

$$\left(\text{by subtracting and adding } [1-H^*][R-S]\frac{d\hat{p}}{dp*}\right)$$

$$= -h^*\Delta(\hat{p}) + [R-S]h^*[\hat{p}-p^*] \quad \text{(by using (2-A))}$$

$$= -h^*\left\{[R-S]p^* - r\right\} \quad \text{(by using (4))}$$

$$= -h^*\Delta(p*) \quad \text{(by using (4) again)},$$

as was to be shown.

B. Proof of Proposition 4

The proof is in two parts. Part (i) signs $\frac{d\{[1-H^*]\Delta(\hat{p})\}}{dR}$, and part (ii) proves that a per unit tax on credit has the same effect on the expected private return to preparation, $[1-H^*]\Delta(\hat{p})$, as a reduction in R.

(i) Differentiating (7) yields

$$\text{(3-A)} \qquad \frac{dp^*}{dR} = -\frac{p^*}{R-S-i[1-W]} < 0.$$

Differentiating (1-A) yields

$$\frac{dU}{dR} = \frac{d\{[1-H^*]\Delta(\hat{p})\}}{dR}$$

$$\text{(4-A)} \qquad = [1-H^*]\hat{p} + h^*\Delta(p^*)\frac{p^*}{R-S-i[1-W]} \quad \text{(by using (3-A)}$$

$$= \frac{\partial U}{\partial R} + \frac{\partial U}{\partial i}\frac{\partial i}{\partial p^*}\frac{\partial p^*}{\partial R} \quad \text{(by using (14)).}$$

Rearranging (4-A), we have

$$= \frac{h^*}{R-S-i[1-W]}\left[\frac{\hat{p}[1-H^*]}{h^*}[R-S-i[1-W]] + p^*\Delta(p^*)\right]$$

$$= \frac{h^*}{\frac{d\hat{p}}{dp^*}\{R-S-i[1-W]\}}\left[\hat{p}\Delta(\hat{p}) + p^*\Delta(p^*)\frac{d\hat{p}}{dp^*}\right] \quad \text{(by using (2-A)).}$$

The denominator is positive from (8), so

$$\text{sign}\left(\frac{dU}{dR}\right) = \text{sign}\left(\hat{p}\Delta(\hat{p}) + p^{*}\Delta(p^{*})\frac{d\hat{p}}{dp^{*}}\right),$$

as was to be shown.

Using the definition of $\Delta(p)$ in (4), the above can be written as

$$\text{(5-A)} \qquad \frac{dU}{dR} \gtreqless 0 \quad \text{as} \quad \frac{r}{R-S} \lesseqgtr \frac{\hat{p}^2 + p^{*2}\frac{d\hat{p}}{dp^{*}}}{\hat{p} + p^{*}\frac{d\hat{p}}{dp^{*}}}.$$

We need to check if either of these inequalities is ruled out by other conditions on $\frac{r}{R-S}$ in the model. It follows from (7), (8), and the lender's break-even condition (9) that $p^*[R-S] - r < 0$ and that $\hat{p}[R-S] - r > 0$, so $\hat{p} > \frac{r}{R-S} > p^*$, which is consistent with both sets of inequalities in (5-A).

(ii) Given (5-A), it remains only to show that $\text{sign}\,\frac{dU}{dT} = -\,\text{sign}\,\frac{dU}{dR}$, where T is a positive per unit tax on credit.

Expected utility after the tax is imposed is

$$U = \underset{p^{*}}{\text{Max}}\left([1-H^{*}]\left\{\hat{p}\Big[R - [i+T][1-W]\Big] + [1-\hat{p}]S\right\} + H^{*}[S+rW]\right) - e,$$

so that the tax affects U in the same way as would a fall in R in the amount T[1-W].■

References

Atkinson, Anthony B. and Nicholas H. Stern. "Pigou, Taxation and Public Goods," *Review of Economic Studies* 41 (1974): 119-128.

Ballard, Charles L. and Don Fullerton. "Distortionary Taxes and the Provision of Public Goods," *Journal of Economic Perspectives* 6 (1992): 117-131.

Bernanke, Ben and Mark Gertler. "Financial Fragility and Economic Performance," *Quarterly Journal of Economics* 105 (1990): 87-114.

Bester, Helmut. "The Role of Collateral in Credit Markets with Imperfect Information," *European Economic Review* 31 (1987): 887-899.

Browning, Edgar K. and William R. Johnson. "The Trade-off between Equality and Efficiency." *Journal of Political Economy* 92 (1984): 175-203.

Card, David. "Using Geographic Variation in College Proximity to Estimate the Return to Schooling," mimeo, Princeton University (1993a).

Card, David. "Earnings, Schooling, and Ability Revisited," unpublished paper, Princeton University (1993b).

Chan, Yuk-Shee and Anjan V. Thakor. "Collateral and Competitive Equilibria with Moral Hazard and Private Information," *Journal of Finance* 42 (1987): 345-363.

de Meza, David and David Webb. "Too Much Investment: A Problem of Asymmetric Information," *Quarterly Journal of Economics* 102 (1987): 281-292.

Eaton, Jonathan and Harvey Rosen. "Optimal Redistributive Taxation and Uncertainty," *Quarterly Journal of Economics* 95 (1980): 357-364.

Edlin, Aaron S. "Is College Financial Aid Equitable and Efficient?" *Journal of Economic Perspectives* 7 (1993): 143-158.

Epstein, David G., Steve H. Nickles and James J. White. *Bankruptcy*, St. Paul, MN: Western Publishing Company, 1993.

Gale, William G. "Collateral, Rationing, and Government Intervention in Credit Markets" in R. Glenn Hubbard (ed.), *Asymmetric Information, Corporate Finance, and Investment*, Chicago: University of Chicago Press (1990).

Gordon, Roger H. and Hal R. Varian. "Intergenerational Risk Sharing," *Journal of Public Economics* 37 (1988): 185-202.

Greenwald, Bruce and Joseph E. Stiglitz. "Externalities in Economies with Imperfect Information and Incomplete Markets," *Quarterly Journal of Economics*, 101 (1986): 229-264.

Hausman, Jerry A. "Taxes and Labor Supply" in Alan J. Auerbach and Martin Feldstein (eds.), *Handbook of Public Economics*, Vol. I, Amsterdam: North-Holland (1985): 213-263.

Hecker, Daniel E. "Reconciling Conflicting Data on Jobs for College Graduates," *Monthly Labor Review*, July 1992, 3-12.

Hoff, Karla. "Land Taxes, Output Taxes, and Sharecropping: Was Henry George Right?" *World Bank Economic Review* 5 (1991): 93-111.

Hoff, Karla. "The Second Theorem of the Second Best," *Journal of Public Economics* (1994), forthcoming.

Killingsworth, Mark R. *Labor Supply*. Cambridge: Cambridge University Press (1983).

McPherson, Michael S. and Morton Owen Schapiro. "Does Student Aid Affect College Enrollment? New Evidence on a Persistent Controversy," *American Economic Review* 83 (1991): 309-318.

National Opinion Research Center. *High School and Beyond, 1980 Sophomore Cohort Third Follow-Up (1986) Data File User's Manual*, 1987, U.S. Department of Education, Washington D.C.

Okun, Arthur M. *Equality and Efficiency: The Big Tradeoff*, Washington, DC: The Brookings Institution, 1975.

Platteau, J-Ph., J. Murickan, A. Palatty, and E. Delbar, "Rural Credit Market in a Backward Area: A Kerala Fishing Village," *Economic and Political Weekly* (1980): 1765-1780.

Sanderatne, Nimal, "The Political Economy of Small Farmer Loan Delinquency," *Savings and Development* 10 (1986): 343-353.

Smith, Bruce D. and Michael J. Stutzer. "Credit Rationing and Government Loan Programs: A Welfare Analysis," *AREUEA Journal* 17 (1989): 177-193.

Stiglitz, Joseph E. and Partha S. Dasgupta. "Differential Taxation, Public Goods, and Economic Efficiency, *Review of Economic Studies* 38 (1971): 151-74.

Stiglitz, Joseph E. and Andrew Weiss. "Credit Rationing in Markets with Imperfect Information," *American Economic Review* 73 (1981): 393-410.

Taubman, Paul. "Role of Parental Income in Educational Attainment," *American Economic Review Papers and Proceedings* 79 (1989): 57-61.

Varian, Hal R. "Redistributive Taxation as Social Insurance," *Journal of Public Economics* 14 (1980): 49-68.

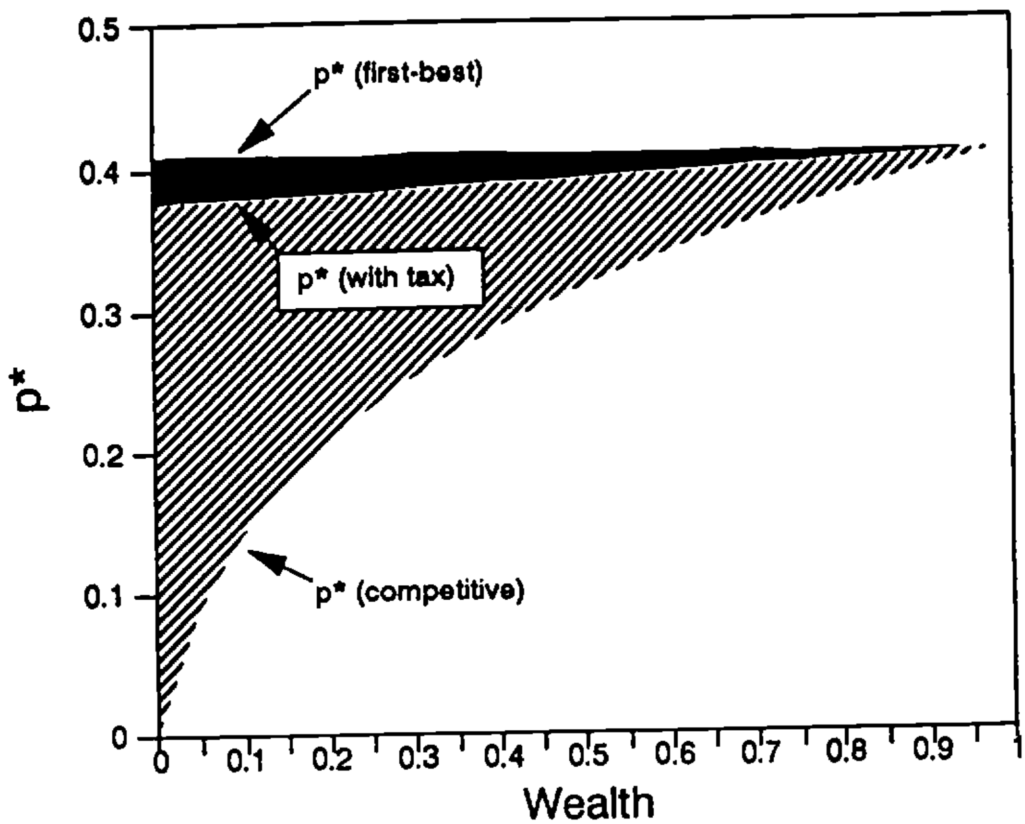

Figure 1A. The reservation success probability as a function of endowment wealth.

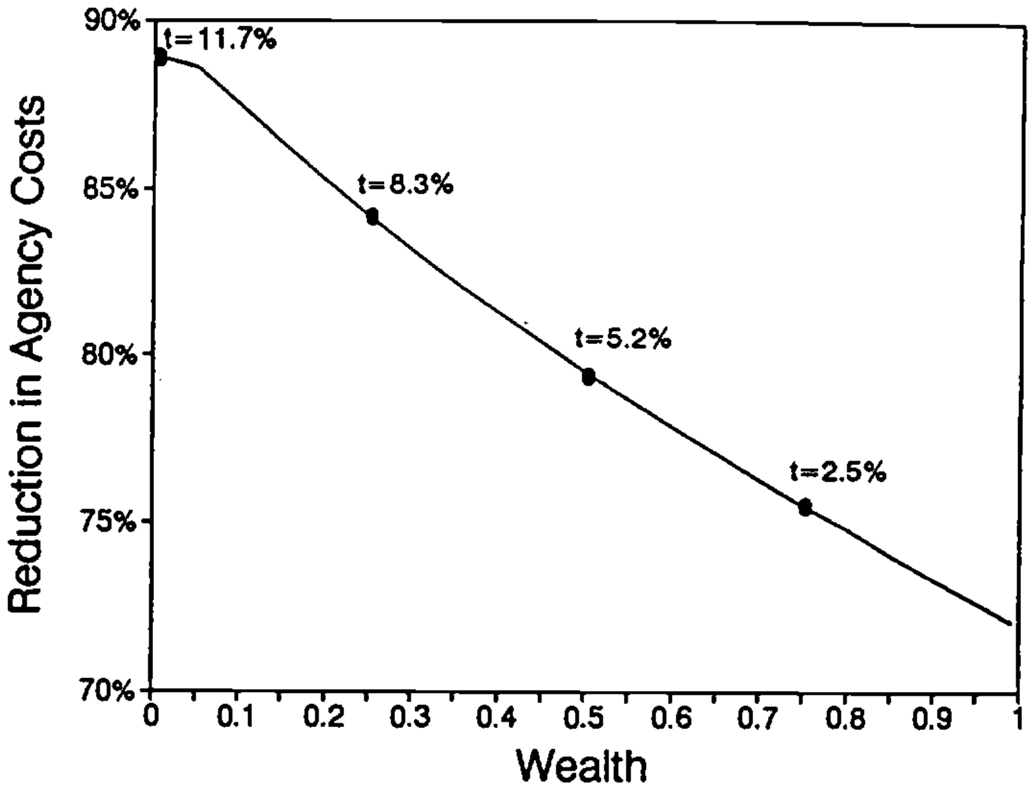

Figure 1B. Percentage reduction in agency costs with Pareto-optimal wage tax rates.

This measure is $\frac{U_t - U_c}{U_0 - U_c}$, where subscripts 0, c, and t indicate the first-best allocation, competitive equilibrium, and equilibrium under the tax-transfer scheme.

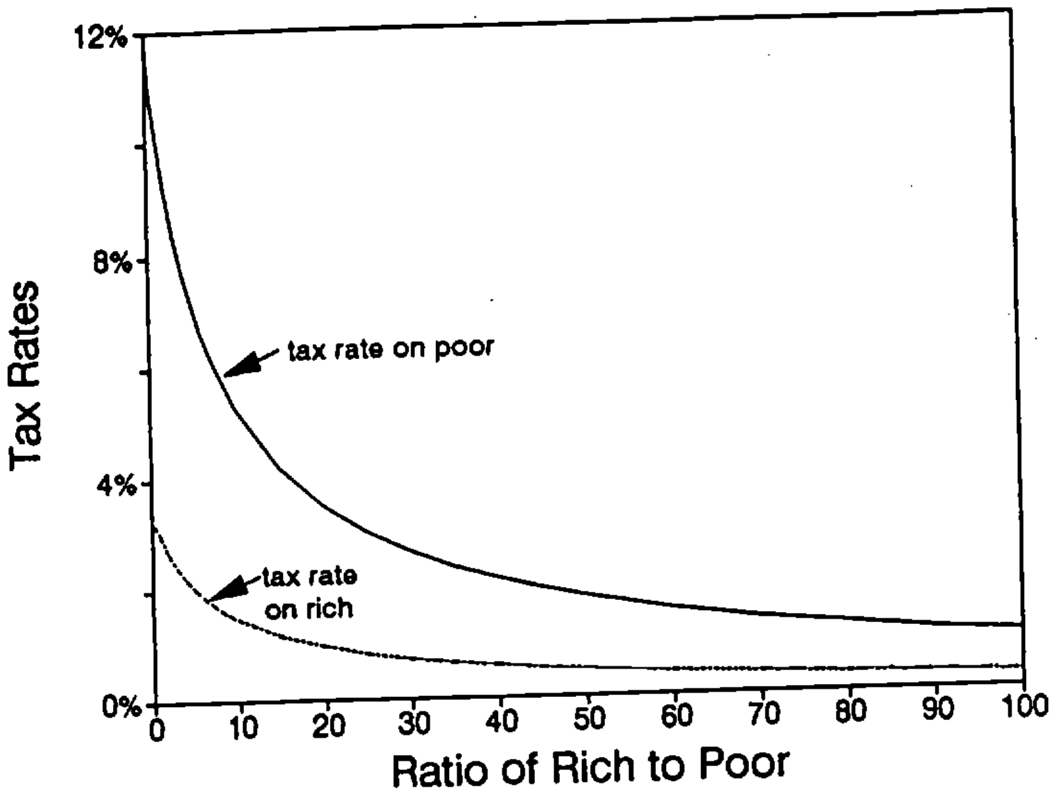

Figure 2A. Real-income maximizing wage tax rates.

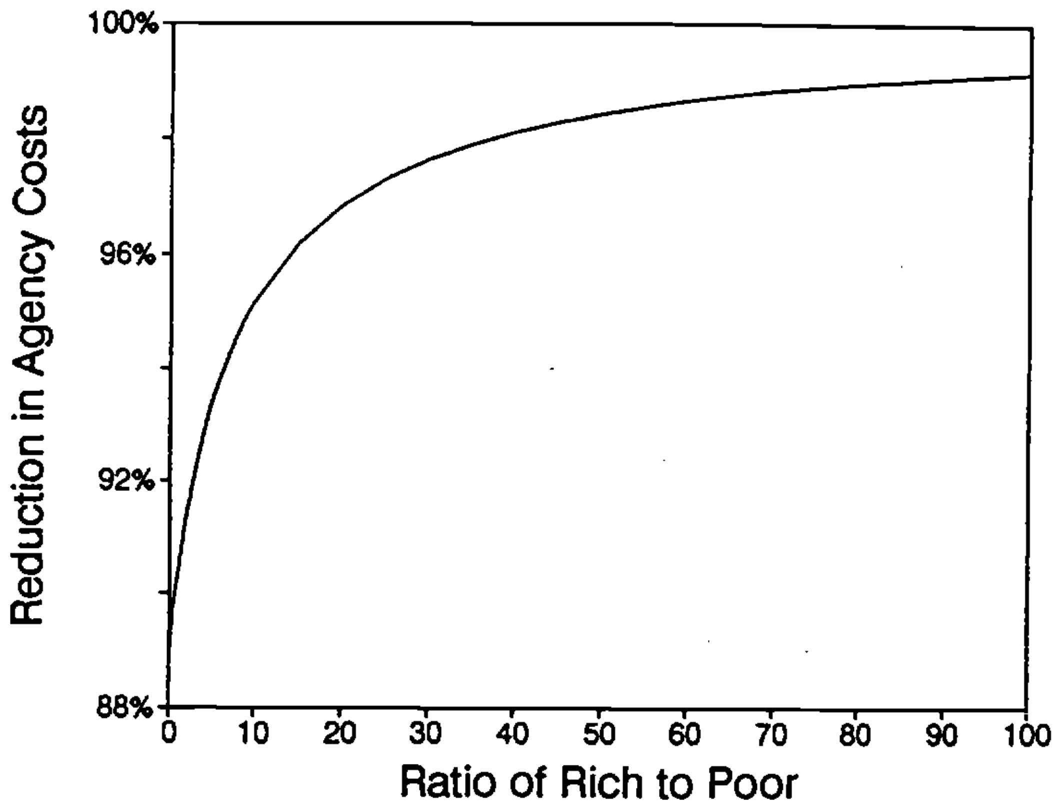

Figure 2B. Percentage reduction in agency costs with real-income maximizing wage tax rates. This measure is $\frac{W_t - W_c}{W_0 - W_c}$, where W is aggregate real income and subscripts 0, c, and t indicate the first-best allocation, competitive equilibrium, and equilibrium under the tax transfer scheme.

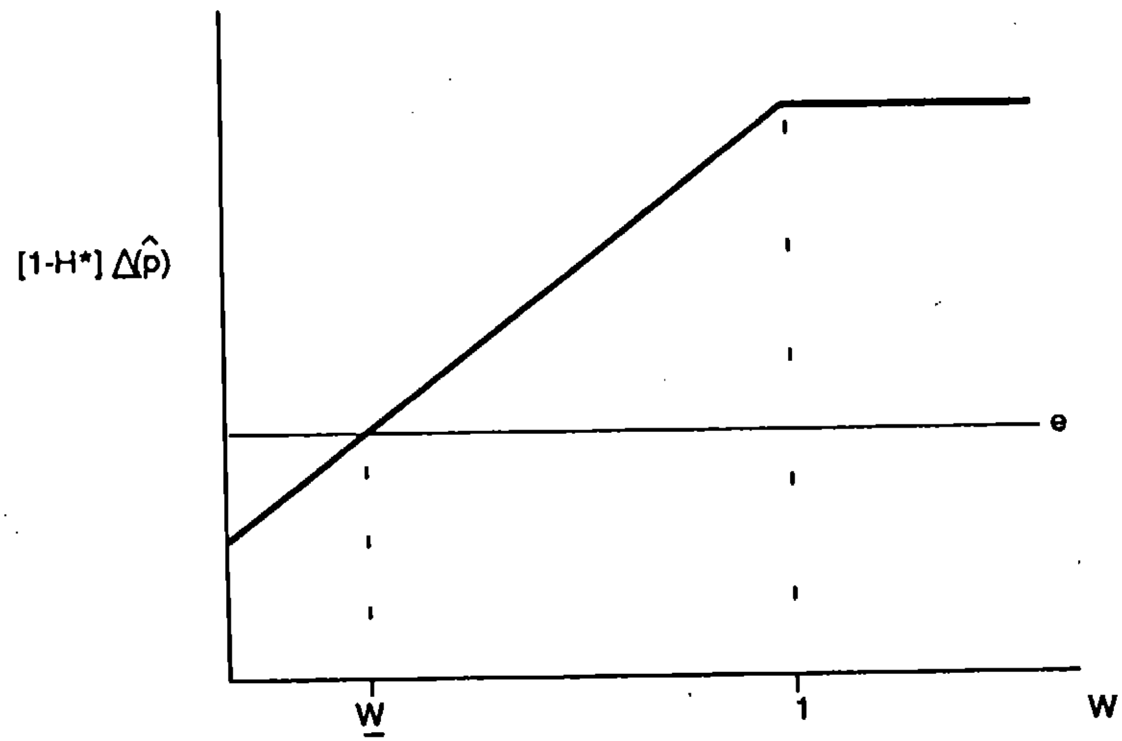

Figure 3. The expected return to preparation for higher education as a function of wealth.

	$W < 1$	$W \geq 1$
$[1 - H^*]\Delta(\hat{p}) > e$	Over-investment (1)	Efficient investment (3)
$[1 - H^*]\Delta(\hat{p}) \leq e$	Zero-investment (2)	(4)

Figure 4. Investment in education.