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THE DYNAMICS OF HIGH INFLATION

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ABSTRACT

This paper presents a model of a high-inflation economy. The model includes the government budget constraint and money demand equation of Cagan's 1956 model; an accelerationist Phillips curve that captures inflation inertia; and an aggregate-spending equation that accounts for the effects of the inflation tax. The paper derives the dynamic effects of fiscal policy, incomes policies, and supply shocks, and uses the results to interpret high-inflation episodes of the 1970s and 1980s.

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I. INTRODUCTION

In the 1980s, fourteen countries experienced annual inflation rates above 100%. A large literature discusses the sources of inflation in these countries and the many stabilization programs (for example, the papers in Bruno et al. [1988] and Bruno et al. [1991]). This literature addresses a dizzying array of phenomena: budget deficits, supply shocks, external debt, exchange rate crises, the effects of inflation on tax revenue, incomes policies, the choice of nominal anchors, the frequency of price adjustment and inflation inertia, real wage behavior, real interest rates and bankruptcies, distributional conflicts.... Different authors focus on different subsets of these phenomena, and so discussions of high-inflation experiences often bear little relation to each other. Some discussions, for example, focus on shifts in fiscal policy, whereas others emphasize supply shocks and inflation inertia. Exchange rates and external debt are central to some discussions, whereas others ignore open-economy issues.

What phenomena are essential for understanding high-inflation economies? Is there a unified explanation for various high-inflation episodes? The answers to these questions are unclear, because discussions of high inflation are not based on a common theoretical framework. The closest thing to a consensus theory -- the one that textbooks present -- is Cagan's 1956 model. This model is too limited, however, to capture central issues in applied discussions. It cannot explain the real effects of inflation and stabilization, because it assumes constant output and real interest rates, and it has no role for incomes policies or supply shocks. This paper presents a model that is similar to Cagan's in style and complexity but captures more of the high-inflation experience. I focus on episodes in which annual inflation peaks at three or four digits, as in Israel and many South American countries during the 1980s. (The relevance to even higher inflation is discussed at the end.)[1]

Sections II and III present the model and derive its basic dynamics. The model consists of four equations. Two are taken from Cagan: a government budget constraint that ties seignorage to the deficit, and a money demand equation. The third equation is borrowed from moderate-inflation macro: a Phillips curve that relates the change in inflation to the level of output. Similar equations are suggested for high-inflation countries by Bruno and Fischer (1986) and Cardoso (1991). The equation captures the inertia frequently ascribed to high inflation: inflation remains constant as long as output is at its natural level. In combining this assumption with Cagan's equations, the model synthesizes the "inertia" and "fiscal" approaches to inflation that are sometimes presented as alternatives (e.g. Kiguel and Liviatan, 1991).

The model's final assumption is an equation for aggregate output. It is similar to a textbook IS curve, except that it accounts for the inflation tax. The inflation tax reduces private spending in the same way as an income tax in the usual IS equation. This assumption introduces a new channel through which inflation affects the economy. For example, incomes policies reduce the output loss from stabilization because lower government spending is offset by a lower inflation tax.

Sections IV-VII derive the effects of macroeconomic shocks on output, inflation, interest rates, and real money balances. Throughout, I compare the results to actual experiences in high-inflation countries. Section IV considers

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[1]Cagan intended his model to apply to hyperinflation, defined as monthly inflation rates above 50%. Since Cagan wrote, however, the model has been used routinely to interpret three-digit annual inflation.

the two basic shocks in the model: shifts in government spending, and exogenous jumps in inflation. As detailed below, the latter can be interpreted as incomes policies or supply shocks. I use the results to interpret orthodox stabilizations (such as Chile 1975-77) and incomes policies without deficit reduction (such as Brazil 1986). One result concerning stabilization is that, as inflation falls, real balances initially fall before rising to a higher steady-state level. This result fits the facts, which the Cagan model cannot explain (Dornbusch et al., 1990).

Section V considers a heterodox stabilization -- a combination of a fiscal contraction and incomes policies that directly shift inflation. The results for this case fit many features of the 1985 Israeli stabilization. Section VI then turns to increases in inflation. I consider three sources of high inflation: fiscal expansions (as in Chile in the 1970s), cutoffs of foreign loans (as in Brazil in the 1980s), and monetary accomodation of supply shocks (as in Brazil and Israel in the 1970s).

As a final application of the model, Section VII considers the relationships among deficits, seignorage, and inflation. The failure of these variables to move together over time is often cited as a puzzle (e.g. Blanchard and Fischer, 1989, Ch. 10). My model can generate a variety of comovements among these variables, including opposite movements of seignorage and inflation. My theory of inflation and deficits is an alternative to those of Drazen and Helpman (1990) and Eckstein and Leiderman (1992).

Section VIII compares my model to other models of high inflation. I emphasize the Cagan model and Cardoso's 1991 model. The latter is closely related to mine, as it also synthesizes the inertia and fiscal approaches to inflation. However, Cardoso's assumptions about aggregate spending differ from mine (she does not account for the inflation tax), and there are important differences in results.

Section IX concludes the paper. As suggested by the first paragraph of this introduction, there are many possible extensions of the model.

II. THE MODEL

This section presents the four equations of the basic model, and a generalization in which the government raises revenue through foreign borrowing as well as seignorage.

A. The Government Budget Constraint

The first equation is taken from the Cagan model (Blanchard and Fischer, 1989, Ch. 4). A flow of real government spending G is financed with seignorage -- by printing money. Seignorage equals the flow of money, $\dot{M}$, divided by the price level P. This ratio equals the growth rate of money, $\dot{M}/M$, times the real money stock M/P. Thus

$$(1) \qquad G = \sigma m ,$$

where $\sigma \equiv \dot{M}/M$ and $m \equiv M/P$. In the applications below, G is usually treated as exogenous. Shifts in G are one of the driving forces of the model.

B. Money Demand

The model's second equation is similar to Cagan's money demand function. Demand for real balances depends inversely on the nominal interest rate:

$$(2) \qquad m = h(r + \pi) , \qquad h' < 0 ,$$

where r is the real interest rate and π is inflation. I assume that the function $\pi h(r+\pi)$ (which equals seignorage in steady state) is a Laffer curve in π: it first rises and then falls as π increases, and it approaches zero as $\pi \to \infty$. These assumptions hold, for example, for the constant-semi-elasticity form of h(•) that Cagan uses.[2]

In principle, money demand should depend on output as well as the nominal interest rate, because output varies in my model. In high-inflation economies, however, shifts in output are a minor source of money-demand movements relative to shifts in interest rates. Thus (2) is a good approximation to reality.

C. Inflation

I assume that inflation is inertial: it remains constant if output is at its natural level. Inflation rises in booms and falls in recessions; specifically,

$$(3) \qquad \dot{\pi} = \alpha y , \qquad \alpha > 0 ,$$

where y is the percentage deviation of output from its natural level. Equation (3) is a continuous-time version of an accelerationist Phillips curve. Appendix A to this paper derives (3) from a microeconomic model of price setting; the key assumptions are that prices are adjusted at discrete intervals, and that expected inflation equals current inflation. The parameter α is increasing in the frequency of price adjustment.

Equation (3) is a major departure from the Cagan model. In Cagan, inflation is determined by money demand: given nominal money growth, prices adjust passively to ensure that real balances equal the desired level. Money demand influences inflation in my model, but only indirectly. As in textbook macro, money demand affects interest rates: since π is inertial, r adjusts to equate money supply and money demand. Interest rates influence aggregate spending (as

[2]Note that money demand depends on the current level of inflation. As discussed below, Cagan assumes that money demand depends on expected inflation, which adjusts over time towards actual inflation. My specification is the limiting case of Cagan's in which the adjustment of expectations is instantaneous.

described below), and spending influences inflation by (3). My model makes this transmission mechanism explicit, and does not assume that the process occurs instantly. In particular, since (3) is based on microeconomic assumptions about price adjustment, it reflects the reality that "prices are set, rather than determined by an invisible money market-clearing hand" (Dornbusch et al., 1990, p. 8).

Although inflation is usually determined by (3), I allow occasional exogenous jumps in inflation. These jumps and shifts in G are the two basic shocks in the model. An inflation jump can be interpreted as any shock that breaks inflation inertia, such as an incomes policy or a supply shock. Appendix A describes how these interpretations of inflation jumps can be formalized.

D. Aggregate Spending

My only novel assumption concerns aggregate output, which is determined by aggregate demand. As in the textbook IS equation, demand depends on government spending, taxes, and investment, which is a function of the real interest rate. The departure from the usual IS equation is that taxes are inflation taxes. Thus

$$(4) \qquad y = G - \pi m + I(r), \qquad I'<0,$$

where I is the deviation of investment from its long-run level (which, as shown below, is a constant). πm is the inflation tax: the instantaneous decline in the value of money balances. By making output a function of $G-\pi m$, I ignore the balanced budget multiplier. (This is justified exactly if G is interpreted as subsidies rather than government purchases.) The assumptions that $G-\pi m$ and $I(r)$ have coefficients of one are normalizations.

Recall from equation (1) that G equals seignorage, σm. Aggregate spending therefore depends on the gap between seignorage and the inflation tax. This gap is zero in steady state, but generally non-zero out of steady state.

It appears theoretically correct to include the inflation tax in the IS curve. An inflation tax reduces disposable income just as much as an income tax: it does not matter whether real income is withheld from paychecks or vanishes in the process of spending money. Thus the inflation tax reduces private consumption. The tax can safely be ignored in moderate-inflation countries, where it is small, but not in high-inflation countries. Most discussions of high inflation do, however, ignore the effect of the inflation tax on spending. One exception is Blanchard et al. (1991), who account for this effect in arguing that stabilization need not reduce output:[3]

> All that is required [for stabilization] is a change in the structure of taxation and spending, typically the elimination of the inflation tax balanced by a decrease in subsidies. We do not ordinarily think of changes in taxation as major contractionary factors.... If the decrease in subsidies and the inflation tax are of equal magnitudes to start with, real [disposable] income -- that is, income including losses on money balances -- will be unchanged [pp. 9-10].

E. An Extension: Foreign Loans

A simple extension of the model allows us to interpret it more broadly. Assume that the government finances its spending through foreign loans as well as seignorage. These loans are the only international capital flows. In this case, the government budget constraint is

$$(1)' \qquad G = \sigma m + B ,$$

where B is foreign borrowing (net of interest and repayments). The aggregate-

[3]Foundations for equation (4) might be developed by combining a simple consumption function with a Baumol-Tobin model of money demand. Suppose, for example, that consumers receive a constant flow of real income and desire a constant flow of consumption. Income flows to bank accounts, where it is protected from inflation, and consumers withdraw cash at discrete intervals to finance consumption. In this case, the real value of consumer spending decreases one-for-one with the inflation tax on cash.

spending equation also changes, because there are net exports of -B. The equation becomes an open-economy IS equation:

$$(4)' \qquad y = G - \pi m + I(r) - B .$$

Equations (1)' and (4)' are the same as (1) and (4), except that G is replaced by G-B. Thus either higher spending or lower borrowing produces the effects of higher spending in the basic model.

Note that B <u>cannot</u> be interpreted as internal borrowing. Foreign and domestic loans have the same effect on the government budget constraint, but not on aggregate spending: domestic borrowing does not reduce net exports. The implications of internal debt are left for future research.

III. STEADY STATES AND DYNAMICS

A. Steady States

The model has two state variables, π and m. Differentiating m, the ratio of money to prices, yields $\dot{m}=(\sigma-\pi)m$. Combining this result with the government budget constraint yields

$$(5) \qquad \dot{m} = G - \pi m .$$

Real balances rise when government spending (and hence seignorage) exceeds the inflation tax.

To determine the dynamics of π, note that the money demand equation (2) defines r as a function of m and π. Substituting this function into I(r), we can write investment as $I(\pi,m)$, $I_\pi>0$, $I_m>0$. Combining this result with (3) and (4) yields

$$(6) \qquad \dot{\pi} = \alpha[G - \pi m + I(\pi,m)] .$$

Inflation rises when aggregate spending, given by the expression in brackets, is high.

A steady state is defined by $\dot{m}=0$ and $\dot{\pi}=0$. Together, these conditions imply $I=0$. Thus the steady-state real interest rate $\bar{r}$ is defined by $I(\bar{r})=0$. Substituting this fact and equation (2) into $\dot{m}=0$ yields

$$(7) \qquad G = \pi h(\bar{r}+\pi) \ .$$

This condition defines steady-state inflation.

Equation (7) is the same as Cagan's condition for steady-state inflation (assuming that $\bar{r}$ equals Cagan's fixed interest rate). As assumed above, the right side of (7) first increases and then decreases as π rises. If G is less than the maximum value of this expression, there are two steady states; if G exceeds this value, there is none. I focus on the case of two steady states.

B. Stability and Dynamics

When there are two steady states, the one with higher inflation is unstable (see Appendix B). I therefore focus on dynamics around the lower-inflation steady state. The $\dot{m}=0$ equation defines an inverse relation between π and m. $\dot{\pi}=0$ defines a relation with slope

$$(8) \qquad \frac{d\pi}{dm}\Big|_{\dot{\pi}=0} = \frac{\pi - I_m}{I_\pi - m} \ .$$

Both the numerator and denominator of this expression have ambiguous signs. To determine the dynamics of the model, one must put restrictions on these expressions.

I assume that the denominator of (8), $I_\pi - m$, is negative at the lower-inflation steady state. To interpret this assumption, note that $I_\pi - m$ equals $\partial y/\partial\pi$, the effect of inflation on aggregate spending for given real balances. The effect is ambiguous because higher inflation raises the inflation tax, which is contractionary, but also raises investment, which is expansionary. We can calibrate the net effect by examining episodes of incomes policies, which cause downward jumps in inflation without immediately shifting real balances. As described below, the effect of incomes policies in actual experience is to raise output (or reduce the output loss from an accompanying fiscal contraction). Thus it appears realistic to assume a negative effect of inflation on output.

The numerator of (8) equals $-\partial y/\partial m$, the effect of real balances on output. It is ambiguous because a higher m, like a higher π, raises the inflation tax but also raises investment. In this case it is difficult to measure the net effect, because there is no analogue to incomes policies: we do not observe discrete shifts in m. I therefore allow $\partial y/\partial m$ to be either positive or negative. (The assumption that $\partial y/\partial \pi$ is negative does not determine the sign of $\partial y/\partial m$).

For the two possible cases, Figure 1 shows the dynamics of π and m around the lower-inflation steady state. When $\pi - I_m$ is positive, both $\dot{m}=0$ and $\dot{\pi}=0$ slope down. $\dot{\pi}=0$ cuts $\dot{m}=0$ from below (see Appendix B). When $\pi - I_m$ is negative, $\dot{\pi}=0$ slopes up. The arrows in the Figure indicate the behavior of π and m out of steady state. The steady state is stable in both cases: it is a stable node if $\dot{\pi}=0$ slopes down, and either a stable node or a stable spiral if $\dot{\pi}=0$ slopes up.

IV. FISCAL AND INFLATION SHOCKS

The next four sections derive the dynamic effects of macroeconomic shocks. This section considers a cut in government spending and a downward jump in inflation.

A. A Decrease in G

A downward shift in G can be interpreted as an orthodox stabilization: a fiscal contraction without direct controls on inflation. Equation (7) and my restrictions on h(•) imply that the decrease in G reduces inflation in the stable steady state. The steady-state m is higher, and y and r are unchanged. In graphical terms, the decrease in G shifts both the $\dot{m}=0$ and $\dot{\pi}=0$ curves down. As shown in Figure 2, the shifts can have offsetting effects but the net effect is to raise m and lower π.

Figure 3 shows the transition from the old to the new steady state. The results are similar when $\dot{\pi}=0$ slopes down and when it slopes up, except possibly at the end of the transition: when $\dot{\pi}=0$ slopes up, convergence may occur through a spiral. For simplicity, I focus on the cases without a spiral in Panels A and B of the Figure. (When $\dot{\pi}=0$ slopes up, a spiral is ruled out if α is sufficiently small, i.e. if there is sufficient inflation inertia.)[4]

In the no-spiral cases, inflation falls smoothly from the old to the new steady state. Real balances first decrease and then increase. The initial decrease occurs because money growth jumps down when G is cut, whereas inflation falls gradually. The result that inflation falls during the transition means that output is below the natural level: stabilization causes a recession. Finally, since both m and π fall initially, the real interest rate exceeds its long-run level.[5]

The transitional recession has both fiscal and monetary causes. Initially, output jumps down because government spending is cut. However, after the transition path crosses the $\dot{m}=0$ line, $G-\pi m$ is positive: the decrease in the inflation tax more than offsets the cut in spending. In this part of the

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[4]Another possibility when $\dot{\pi}=0$ slopes up is that the transition path overshoots the steady-state π a single time. This case is intermediate between the ones in Panels B and C. See Appendix B for details.

[5]r exceeds $\bar{r}$ even during the later part of the transition, when m rises. During this period $G-\pi m$ is positive and y is negative. Thus, by (4), I(r) must be negative and r must exceed $\bar{r}$.

transition, output is depressed by a monetary crunch: the lower m and lower π raise the real interest rate and reduce investment.

These results fit the facts. The cleanest example of an orthodox stabilization from three-digit inflation -- a genuine fiscal consolidation with no incomes policies -- is the Chilean stabilization of 1974-77 (Bruno, 1991). As predicted by the model, inflation fell steadily over this period (CPI inflation was 498% in 1974, 235% in 1976, and 50% in 1978). The fall in inflation was accompanied by a deep recession, and real interest rates rose above 50% (Corbo and Solimano, 1991, Table 3.1). Finally, M2 fell from 12% of GDP in 1973 to 7% in 1974 and 1975, and then rose for the rest of the 1970s (Harberger, 1982, Table 3).

B. A Downward Jump in Inflation

As discussed in Appendix A, a downward jump in inflation can be interpreted as an incomes policy: a direct intervention in price setting that breaks inflation inertia. After the one-time jump, inflation is again determined by equation (3). Here, I consider an incomes policy with government spending held constant -- a "heterodox" stabilization without the orthodox component of a fiscal contraction. Such a policy is a good description of many failed stabilizations. In the 1986 Brazilian episode, for example, wage-price controls reduced annual inflation from 420% to 9% over two quarters, and the government deficit stayed almost constant (Vegh, 1992, Table 12).

Figure 4 shows the dynamic effects of an inflation jump. Since the shock does not affect the fundamentals of the model, the economy returns to its initial steady state. Once again, the cases in the Figure are similar, except that one includes a spiral at the end of the transition. In the no-spiral cases, inflation rises steadily back to its steady-state level. Real balances rise temporarily, because inflation drops below money growth. Output exceeds its long-run level throughout the transition, because of the temporary reduction in the inflation tax. Finally, the real interest rate follows a complex path: it jumps up when π jumps down, falls as m and π rise, eventually drops below its long-run level, and then rises again.[6]

These predictions fit the 1986 Brazilian stabilization. Inflation fell and then rose again over 1986-87. The money stock as a percent of GNP doubled in 1986 and returned to roughly its initial level in 1987. The economy boomed, with annual growth of 13% in the two quarters of falling inflation. Finally, the real interest rate on savings jumped to 213% in 1986:1 and then fell to -51% in 1986:4 (Vegh, 1992, Table 12; Kiguel and Liviatan, 1991, Table 6.6).

Although incomes policies are not a long-term cure for inflation, my model suggests that they are benign. In Figures 4A and 4B, incomes policies just reduce inflation temporarily and raise output. Thus my model provides counterexamples to Cardoso's (1991) argument that incomes policies produce a period of <u>higher</u> than steady-state inflation after an initial decline. (In actual experience, including Brazil's, inflation often overshoots its initial level after a failed stabilization. However, this can be explained by an upward drift in steady-state inflation due to worsening fiscal fundamentals.)

V. HETERODOX STABILIZATION

A. Basic Results

This section considers a heterodox stabilization: a combination of a decrease in G and a downward jump in π caused by incomes policies. A leading

[6] r lies below its long-run level while m is falling. During this period, $G-\pi m<0$ and $y>0$, so investment must exceed its long-run level.

example is the 1985 Israeli stabilization, in which wage-price controls quickly reduced inflation from 400% to 20% and the government eliminated a deficit of ten percent of GNP.

The change in steady states is the same for a heterodox stabilization as for an orthodox one, but the transition begins with a downward jump in inflation. There are a number of possible transition paths, depending on parameters and on the relative sizes of the inflation jump and the fiscal contraction. Figure 5 presents several of the possibilities.

As shown in the Figure, inflation can either rise or fall after its initial jump, depending on whether it jumps below the $\dot{\pi}=0$ line. Rising inflation indicates a boom, and falling inflation indicates a recession. For the cases in the Figure, real balances rise monotonically (rather than falling initially as in an orthodox stabilization). Finally, the real interest rate jumps up when inflation jumps down and remains above its steady-state level throughout the transition.

In the first two years of the Israeli stabilization, output was at or perhaps above its natural level. (Output grew 6% in 1986 and 7% in 1987.) Thus we can interpret this episode as one in which inflation jumps down to or below the $\dot{\pi}=0$ line. As predicted by the model, real balances rose substantially after the stabilization. The real interest rate jumped to 100% in 1985 and remained over 30% in 1986-87 (Cukierman, 1988, Table 2.1; Bruno and Meridor, 1991, Table 7.1).

B. Boom-Recession Cycles?

Although the model explains many aspects of the Israeli experience, it probably does not capture all the dynamics. The Israeli stabilization was initially accompanied by strong growth, but a recession began in 1987. Such a "boom-recession cycle" is observed in many heterodox stabilizations (Kiguel and Liviatan, 1992), and authors such as Calvo and Vegh (1993) develop models to explain it. Can the current model generate such a cycle?

In principle, the answer is yes. Stabilization causes a boom and then a recession (indicated by rising and falling inflation) in Figure 5B. This result, however, is not robust: it cannot arise if $\dot{\pi}=0$ slopes up. In addition, the mechanism behind the lagged recession does not fit the Israeli experience. In the model, output falls after its initial increase because real balances rise, raising the inflation tax. The rising real balances imply that investment rises as the recession begins (although not enough to offset the higher inflation tax). In reality, the 1987 recession was driven by a <u>fall</u> in investment, which was apparently caused by the high interest rates of 1985-87 (Bruno and Meridor, 1991).

A sequel to this paper (Ball, 1993) modifies the current model to produce a better theory of boom-recession cycles. The crucial new assumption is that investment adjusts slowly to changes in interest rates; this sluggishness arises from costs of adjusting investment and from balance-sheet effects of high rates, which build over time. In the modified model, a heterodox stabilization produces a consumption-led boom followed by an investment-led recession.

<u>C. The Role of Incomes Policies</u>

In both this paper and Ball (1993), incomes policies raise output relative to the path it would follow without them. This result is conventional, but the mechanism is not. Proponents of incomes policies during disinflation usually argue that, by speeding the fall in inflation, these policies prevent a drop in real balances as money growth falls. Since real balances are higher with incomes policies, real interest rates are lower and investment is higher. In my model, incomes policies do raise real balances, but the effect on interest rates is more than offset, at least initially, by the direct effect of lower inflation. That is, in contrast to the usual story, incomes policies raise real interest rates and reduce investment. Incomes policies are expansionary overall because they reduce the inflation tax, which raises consumption. This effect of inflation on consumption is ignored in the conventional story.

My theory of incomes policies fits the Israeli experience. As noted above, real interest rates were very high between 1985 and 1987, and government spending was cut sharply. Output grew strongly despite these contractionary forces because lower inflation produced a consumption boom: consumption grew 14% in 1986 and 8% in 1987 (Bruno and Meridor, Table 7.1).

Although my model focuses on the inflation tax, economists have suggested a number of other channels through which lower inflation might stimulate consumption (see DeGregorio, Guidotti, and Vegh, 1993). For example, stabilization may increase consumer confidence or cause the consumer loan market to reopen. Future research could add such effects to the model: they would strengthen the consumption effects of incomes policies that I emphasize.

VI. INCREASES IN INFLATION

So far I have focused on episodes in which inflation falls temporarily or permanently. Can the model also explain why inflation rises in the first place?

A. An Increase in Government Spending

One source of a rise in inflation is an increase in G. The dynamics are the reverse of those for a decrease in G, shown in Figure 3. As inflation rises, output is high, the real interest rate is low, and real balances first rise and then fall.

It is difficult to test these predictions, because few actual increases in inflation follow purely from increases in G; most involve complications such as supply shocks or devaluations (see below). One episode that might be interpreted as a pure fiscal expansion is Chile under Allende, who increased the government deficit from 7% of GDP in 1970 to 31% in 1973 (Corbo and Solimano, 1991, Table 3.1). CPI inflation rose from 33% to 488% over this interval. As predicted by the model, the fiscal expansion caused a boom, with 9% real growth in 1971 and falling unemployment through 1972. Finally, the money stock as a proportion of GDP rose in 1971 and 1972 and then fell in 1973 (Harberger, 1982, Table 3).[7]

B. A Cutoff of Foreign Loans

Many increases in inflation arise not from higher government spending, but from shifts in deficit finance from debt to money. In particular, the debt crisis of the early 1980s forced countries such as Brazil and Argentina to rely more heavily on seignorage. The decrease in foreign borrowing required a decrease in the trade deficit, which was accomplished through sharp devaluations. I now investigate the effects of such a shock, using the open-economy model of Section IIE.

By itself, a decrease in borrowing has the same effect as an increase in government spending, since only the difference G-B enters the open-economy model. However, the accompanying devaluation complicates the matter: as described in Appendix A, a devaluation causes an upward jump in inflation. The combination of an increase in G-B and an upward inflation jump is the opposite of a heterodox stabilization. Thus, reversing the results from Section V, the shift in finance produces low real interest rates and monotonically decreasing real balances, and it reduces output if the inflation jump is large enough.

[7]Data on real interest rates during this period are not readily available.

These results fit the increase in Brazilian inflation before the 1986 stabilization (see Cardoso [1991] for a similar argument). During the debt crisis of 1982-83, a sharp devaluation created a large trade surplus. Output fell in 1983, inflation jumped from 6% per month to 10%, the real interest rate was near zero, and real balances fell (Modiano, 1988, Table 5A.1; Cardoso, Figure 5.9, Table 5.A1; Kiguel and Liviatan, 1991, Table 6.6).

C. Supply Shocks and Accomodating Policy

In moderate-inflation countries, increases in inflation are commonly blamed on supply shocks and accomodating government policies. The same mechanism accounts for some high-inflation experiences. In Brazil, for example, monthly inflation jumped from 1.5% to 3% after OPEC I, and from 3% to 6% after OPEC II. Similarly, inflation jumped in Israel after the two oil shocks; Bruno and Fischer (1986) describe the Israeli "inflationary process" as "shocks and accomodation." This view of inflation is sometimes presented as an alternative to Cagan's fiscal theory (e.g. Bole and Gaspari, 1991). It can, however, be captured easily in my model, despite Cagan's government budget constraint.

Like a devaluation, a supply shock can be interpreted as an upward jump in inflation (see Appendix A). To capture accomodating policy, I assume that the government adjusts G (or G-B in the open-economy model) to keep output at its natural level. Thus G becomes endogenous. Since output is constant, inflation is constant except when it jumps exogenously. The only endogenous state variable is real balances m. Equations (4) and (5) and the assumption that $y=0$ imply $\dot{m}=-I(\pi,m)$.

In this version of the model, an upward jump in π causes an upward jump in G to keep output constant. G can then either rise or fall, but remains above its initial level. One can show that m declines monotonically to a lower steady-state level. r jumps down when inflation jumps up, and then returns to its long-run level.

This scenario fits the Israeli experience around the 1979 oil shock. An increase in the government deficit prevented a recession: despite the supply shock, output grew 5 percent in 1979 (Bruno and Fischer, 1986). Annual inflation rose from 40% before the shock to 130% in the early 1980s, and real balances fell rapidly throughout the period (Liviatan and Piterman, 1986, Figure 16.1, Table 16.4).

VII. DEFICITS, SEIGNORAGE AND INFLATION

Most theoretical models place the ultimate blame for high inflation on government budget deficits. In practice, however, deficits and inflation often do not move together over time. This fact is explained partly by shifts in deficit finance between borrowing and seignorage, which determines inflation. This is not the whole explanation, however, because seignorage and inflation often do not move together (see, for example, Liviatan and Piterman's [1986] discussion of Israel). This fact is commonly viewed as a puzzle, and authors such as Drazen and Helpman (1990) and Eckstein and Leiderman (1992) develop theories to explain it.

My model can easily explain the lack of comovement between seignorage and inflation. There is a tight relationship between these variables in steady state, but their short-run movements depend on the nature of shocks. An orthodox stabilization, for example, produces a downward jump in seignorage; then inflation falls slowly while seignorage remains constant. A pure incomes policy produces falling and then rising inflation with no change in seignorage. Various sequences of policies and inflation shocks can generate virtually any pattern of inflation and seignorage. (Bruno and Fischer [1986] make a similar point.)

In one empirically relevant case, inflation and seignorage move in opposite directions. Suppose that incomes policies are introduced and money growth is held constant. This experiment differs from the basic incomes-policy case in Section IV in that monetary rather than fiscal policy is held constant. With constant money growth and reduced inflation, real balances rise, and so seignorage rises. The higher seignorage implies higher government spending or less borrowing. This scenario fits the rise in seignorage and fall in inflation in Israel in 1985 and in Brazil in 1986 (Cukierman, 1988; Kiguel and Liviatan, 1991). (In the Israeli case, the deficit and inflation were cut in 1985, but money growth remained in three digits through 1986.)

VIII. COMPARISON TO PREVIOUS WORK

A. The Cagan Model

The classic Cagan model assumes that output and the real interest rate are constant. It includes the government budget constraint and money demand equation of my model, and one additional assumption: expected inflation (which enters money demand) adjusts slowly towards actual inflation.

As discussed in Section III, my model and Cagan's have the same steady states. The stability results and dynamics are quite different, however. In my model, the lower-inflation steady state is stable under my assumption that $\partial y/\partial \pi<0$. The higher-inflation steady state is unstable. In Cagan, the lower-inflation steady state is stable only if the adjustment speed of expectations is sufficiently slow. If expectations adjust quickly, the higher-inflation steady state is the stable one. This steady state has perverse properties; for example, inflation is decreasing in government spending.[8]

My model goes beyond Cagan's in explaining movements in y and r. In addition, it makes more realistic predictions about the dynamics of m and π, even if we focus on Cagan's lower-inflation steady state. My model predicts that a decrease in G causes a smooth fall in inflation and an initial decrease in real balances, which fit Chile's orthodox stabilization. In Cagan, by contrast, a decrease in G causes a downward jump in inflation and a monotonic increase in real balances. Similarly, Cagan incorrectly predicts that an <u>increase</u> in G, as in Chile before 1973, produces a monotonic fall in m. (This point is stressed by Dornbusch et al., 1990). Cagan's results follow from his assumption that money demand determines inflation. Inflation must jump when G falls because money growth jumps and real money demand has not yet changed; then real balances must rise because lower inflation raises money demand. My model makes more realistic predictions because it introduces inflation inertia. Real balances initially fall when G is reduced because inflation lags behind falling money growth.

<u>B. Forward-Looking Models</u>

Since Cagan, a number of authors have developed models of high-inflation dynamics. Starting with Sargent and Wallace (1981), most of this work has emphasized expectations of future policy shifts as a driving force behind inflation. One example is Drazen and Helpman (1990), who explain variation in inflation through changing expectations about government deficits, which shift money demand. Another is Calvo and Vegh (1993), who explain booms at the start

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[8]My model does not nest Cagan's. One can generate Cagan's assumption that y is constant by assuming $\alpha\rightarrow\infty$ (no inflation inertia), but r is still variable in this case. A constant r would require $I'(r)\rightarrow\infty$ (a flat IS curve), which violates the restriction that $\partial y/\partial\pi<0$. If $\alpha\rightarrow\infty$ and $I'(r)\rightarrow\infty$, my model has no stable steady state.

of stabilization through expectations that an exchange-rate peg will be abandoned. This expectation implies a future increase in the inflation tax, which induces intertemporal substitution towards current consumption. (See also the related work surveyed by Vegh [1992]).

In my view, expected policy shifts have been somewhat overemphasized in recent work relative to direct shocks to inflation such as incomes policies and supply shocks. In any case, my model attempts to explain as much as possible of inflation dynamics without invoking forward-looking behavior. Future research could integrate my approach with previous ones. For example, my model and Calvo-Vegh provide complementary explanations for the consumption booms that accompany sharp disinflations: a lower inflation tax can raise consumption both by raising disposable income and by inducing intertemporal substitution.[9]

C. Cardoso (1991)

The closest model to mine is Cardoso (1991) (see also Cardoso [1988]). Like my model, Cardoso's borrows the government budget constraint (1) and money demand equation (2) from Cagan. Instead of my (3) and (4), Cardoso assumes that the change in inflation depends on the gap between the real interest rate and the "full-employment" interest rate, which is a function of G. One can show that these assumptions are equivalent to mine, with one modification: the inflation tax term $-\pi m$ is omitted from the output equation. That is, Cardoso ignores the direct effect of inflation on aggregate spending.

Cardoso's model is apparently the first to combine Cagan's equations with a $\dot{\pi}$ equation capturing inflation inertia. Part of my contribution is to develop

[9]In both the Calvo-Vegh model and mine, inflation dynamics are derived from underlying models of staggered price adjustment. However, the two models produce very different reduced forms for inflation: $\dot{\pi}$ is increasing in output in my model but _decreasing_ in output in Calvo-Vegh. This difference arises because Calvo-Vegh assume forward-looking expectations.

additional implications of Cardoso's approach. There are, however, important differences between my results and Cardoso's arising from the $-\pi m$ term in (3). Without this term, $\partial y/\partial \pi$ is positive: the only effect of inflation on output is the positive effect through investment. Consequently, incomes policies initially *reduce* output in Cardoso's model. In addition, with $\partial y/\partial \pi > 0$, the lower-inflation steady state can be unstable, because $\dot{\pi}$ is increasing in π. (The steady state is stable if α is sufficiently small.) Finally, if the steady state is stable, convergence always occurs through a spiral. This fact is the source of Cardoso's result that incomes policies produce a period of above-steady-state inflation.

IX. CONCLUSION

This paper presents a model of high inflation and uses it to interpret inflationary episodes of the 1970s and 1980s. The model is sufficiently simple and flexible to use in a variety of applied discussions. The paradigm that I seek to imitate is the ISLM / AD-AS model that economists use to interpret events in moderate-inflation countries. Like the ISLM model, my model makes heavy use of "shortcuts" (Blanchard and Fischer, 1989): reduced-form equations for money demand, aggregate spending, and so on that are not derived explicitly from optimization. Future work could attempt to provide microeconomic foundations for my assumptions. (Appendix A takes a step in this direction.)

The model could be extended to include many additional aspects of the high-inflation experience (recall the list of phenomena in the first paragraph of the paper). For example, one could introduce the Tanzi effect by making the government deficit a function of inflation. One could add explicit labor and foreign exchange markets that determine real wages and exchange rates. Such extended models would generate testable predictions about additional variables, and could address policy issues such as the choice between money- and exchange-rate-based stabilization.

One open question is the relevance of the model to hyperinflation. Can the model explain three-digit monthly inflation in Bolivia as well as three-digit annual inflation in Israel? Hyperinflation presumably eliminates inflation inertia, and thus might be interpreted as a special case in which $\alpha \to \infty$. However, hyperinflation is an unstable situation that is not well-modelled by shifts between steady states. Does hyperinflation arise when a shock pushes the economy into the unstable (π, m) region of the model? Or does it occur when no steady state exists, because G is too high? Or do we need a different model for hyperinflation?

APPENDIX A: THE BEHAVIOR OF INFLATION

Assumptions: To derive the equation for $\dot{\pi}$, assume the economy contains a continuum of monopolistically competitive firms with isoelastic cost and demand functions. As shown by Blanchard and Kiyotaki (1987), a firm's profit-maximizing relative price is

$$\text{(A1)} \qquad p^* - p = vy ,$$

where p^* is the log of the profit-maximizing nominal price and p is the log of the aggregate price level. Intuitively, an increase in aggregate spending raises a firm's desired price by shifting out the demand curve that it faces. The parameter v depends on the slopes of marginal cost and demand.

As in Calvo (1983), firms set actual prices at intervals of random length. There is a hazard γ that a given firm adjusts at each instant. As in Fischer (1977), a firm chooses a path of prices until the next adjustment rather than a single fixed price. (This assumption is natural under high inflation, because desired nominal prices change rapidly between adjustments.) I assume that a firm's price path must have a simple form: the price grows at a steady rate μ, starting from its level before the adjustment.

A firm chooses its rate of price growth μ to minimize the average squared deviations between its actual and desired prices. When the firm sets μ, its expectations of output and inflation are static: it assumes that these variables will remain constant at their current levels.

The two key features of the model are that firms adjust only at discrete intervals, and that expectations are not forward-looking. That is, as in traditional macro models, the Phillips curve is derived from nominal price stickiness and less-than-rational expectations. The other features of the specification are chosen for convenience.[10]

Derivation of (3): Consider a firm that sets μ at time t, and let the firm's current price be q(t). If the firm does not adjust again by t+s, s>0, its price at t+s is $q(t)+\mu s$. The firm's expectation of its desired price at t+s is $p(t)+vy(t)+\pi(t)s$, from (A1) and the assumption of static expectations. The firm minimizes the squared deviations between actual and desired prices for all s, weighted by $\exp[-\gamma s]$, the probability that it has not adjusted again by t+s. The first order condition for this problem is

$$\text{(A2)} \qquad \int_0^{\infty} se^{-\gamma s}[q(t)+\mu s-p(t)-vy(t)-\pi(t)s]ds = 0 .$$

Let U(t) be the average of μ across all firms that adjust at t. This variable is defined by (A2) with μ replaced by U(t) and q(t) replaced by p(t). (q(t) averages to the price level p(t) because adjusters are chosen randomly from all firms.) Integrating and solving for U(t) yields

$$\text{(A3)} \qquad U(t) = \frac{v\gamma}{2}y(t) + \pi(t) .$$

The inflation rate $\pi(t)$ is the average of $U(\tau)$ for all $\tau<t$, weighted by the proportion of adjusters at τ that have not adjusted again:

$$\text{(A4)} \qquad \pi(t) = \int_{-\infty}^{t} \gamma e^{\gamma(\tau-t)}U(\tau)d\tau .$$

Differentiating with respect to t yields

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[10]It is not clear whether non-rational expectations are essential, or whether sticky prices can generate inflation inertia even with rational expectations. Forward-looking expectations eliminate inertia in some sticky-price models, but research has not yet determined the robustness of this result. See Ball (1991) and Roberts (1993).

$$\text{(A5)} \qquad \dot{\pi}(t) = \gamma[\frac{v\gamma}{2}y(t) + \pi(t)] - \int_{-\infty}^{t}\gamma^2 e^{\gamma(\tau-t)}[\frac{v\gamma}{2}y(\tau)+\pi(\tau)]d\tau$$

$$= \frac{v\gamma^2}{2}y(t) \text{ .}$$

where the first line uses (A3) and the second line uses (A4). This equation is text equation (3) with $\alpha = v\gamma^2/2$. Note that α increases with γ, the frequency of price adjustment.

Inflation Jumps: In the text, I interpret a number of shocks as one-time jumps in inflation. I now sketch an approach to formalizing these interpretations.

Consider first the simple case of an incomes policy. At some instant, the government dictates that all firms adjust μ to a particular value. The inflation rate jumps to this value. After that instant, firms again make adjustments at stochastic intervals, and (3) again holds: inflation becomes inertial at its new level. Intuitively, the one-time policy has persistent effects because expected inflation jumps down with actual inflation.

To capture supply shocks, modify equation (A1) by adding a term θ that varies across sectors of the economy. An increase in a sector's θ is an increase in its profit-maximizing relative price. Following Ball and Mankiw (1992), I interpret a supply shock as an unusually large increase in θ for some sector (such as energy). When the shock occurs, firms can make special adjustments of μ by paying an extra adjustment cost. If the shock is large enough to induce special adjustments, and if the affected sector contains a positive fraction of all firms, then aggregate inflation jumps. After the jump, the dynamics of inflation are again given by (3).

Finally, a devaluation is just a special case of a supply shock. A major devaluation implies large increases in the profit-maximizing relative prices of imported goods. This shock triggers special adjustments of actual prices in the import sector.

APPENDIX B: STABILITY AND DYNAMICS

Stability: As discussed in Section III, the model has two steady states for G sufficiently low. To determine whether the steady states are stable, I first linearize the equations for $\dot{m}$ and $\dot{\pi}$, (5) and (6). Letting $z=[m,\pi]'$, the linearized system is $\dot{z}=Az$, where A is a matrix with elements $a_{11}=-\pi$, $a_{12}=-m$, $a_{21}=\alpha(I_m-\pi)$, and $a_{22}=\alpha(I_\pi-m)$, and all variables are evaluated at the steady state. Necessary and sufficient conditions for stability are trace(A)<0 and $|A|>0$. Given my assumption that $I_\pi-m<0$, these conditions hold if and only if $mI_m-\pi I_\pi>0$.

The money demand equation (2) and the investment function I(r) imply $I_m=I'/h'$ and $I_\pi=-I'$. Using these facts, the condition for stability can be written as $m+h'\pi>0$. Note that $m+h'\pi$ is the derivative of steady-state seignorage, $\pi h(\bar{r}+\pi)$, with respect to π. By the Laffer-curve properties of $\pi h(\bar{r}+\pi)$, this derivative is positive at the lower-inflation steady state and negative at the higher-inflation steady state. Thus the former is stable and the latter is unstable.

$\dot{m}=0$ and $\dot{\pi}=0$: When $I_m-\pi<0$, both $\dot{m}=0$ and $\dot{\pi}=0$ slope down. Section III claims that $\dot{\pi}=0$ cuts $\dot{m}=0$ from below. To establish this, note that the slope of $\dot{m}=0$ ($-a_{11}/a_{12}$) is greater in absolute value than the slope of $\dot{\pi}=0$ ($-a_{21}/a_{22}$) if $m+h'\pi>0$. As discussed above, this condition is satisfied at the stable steady state.

Nodes or Spirals?: The lower-inflation steady state is a stable node if the eigenvalues of A are real and a stable spiral otherwise. Algebra establishes that the eigenvalues are real if and only if $(\alpha I_\pi-\alpha m+\pi)^2 > 4m\alpha(I_m-\pi)$. This condition holds if $I_m - \pi < 0$ (that is, if $\dot{\pi}=0$ slopes down). If $I_m - \pi > 0$, the condition may or may not hold. It must hold for α sufficiently small.

Transition Paths: In the experiments in the text, the transition path is usually clear from the above results. An exception is the behavior of π when $\dot{\pi}=0$ slopes up and the steady state is a stable node. In this case, a fall in G can produce either a monotonic decline in π (as in Figure 3B) or a path that overshoots the steady state a single time (as discussed in footnote 4). The first case is guaranteed if α is sufficiently small. To see this point, consider the limiting case as $\alpha \to 0$. In this case, the transition path moves horizontally to the $\dot{m}=0$ curve and then converges to the steady state along this curve. π falls monotonically in this case.

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Figure 1

Dynamics Around the Stable Steady State

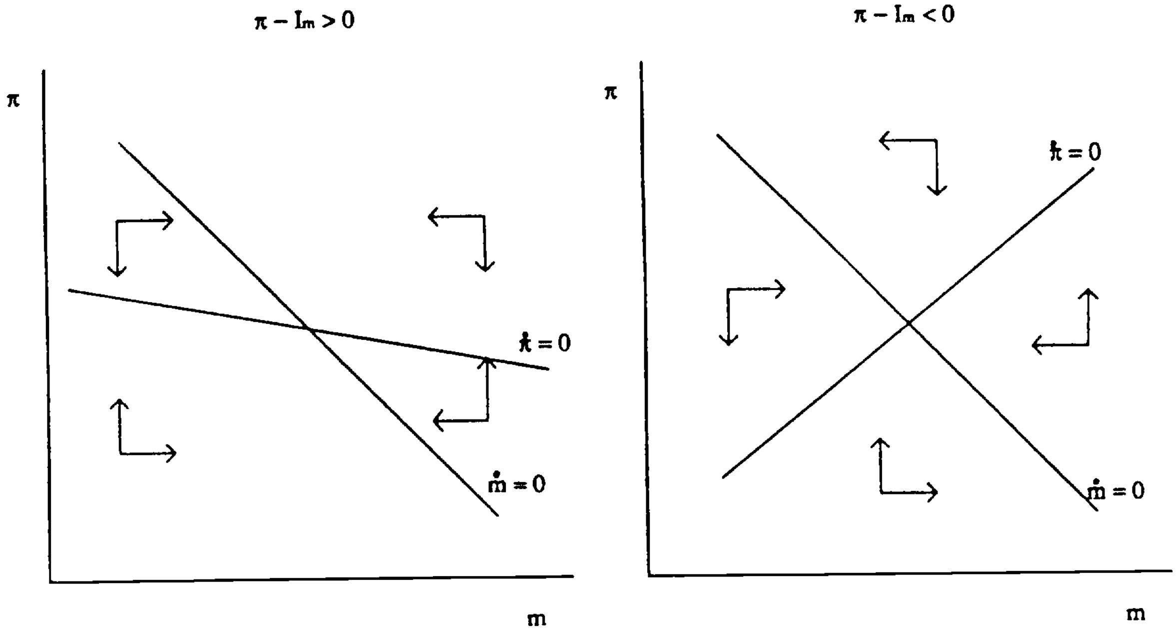

Figure 2

Effects of a Decrease in G

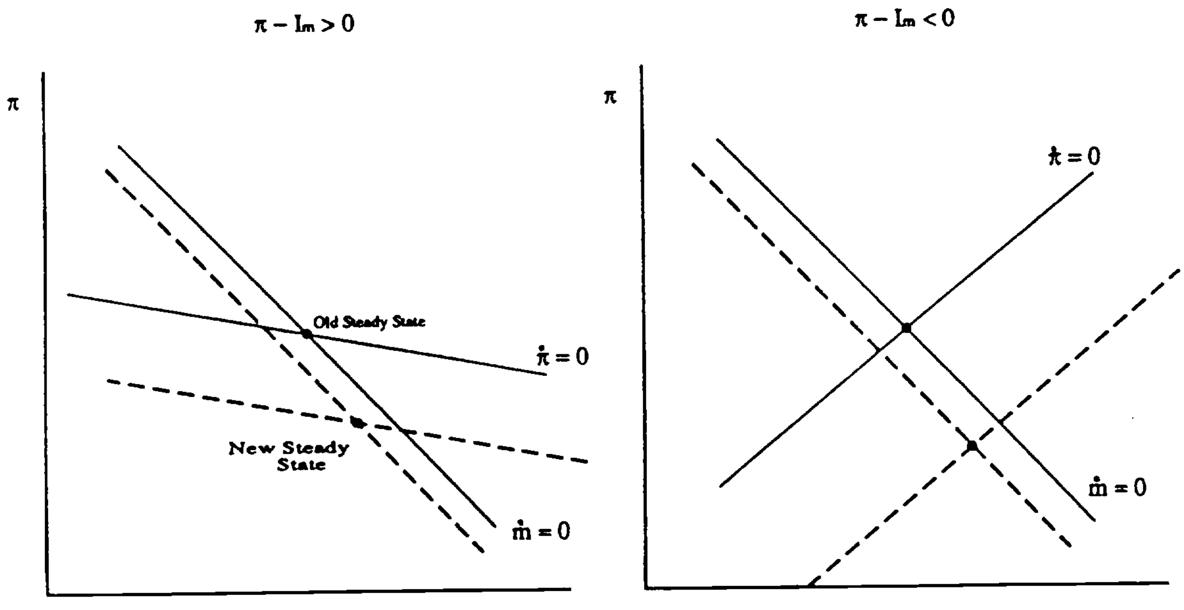

Figure 3

Transition Path Following a Decrease in G

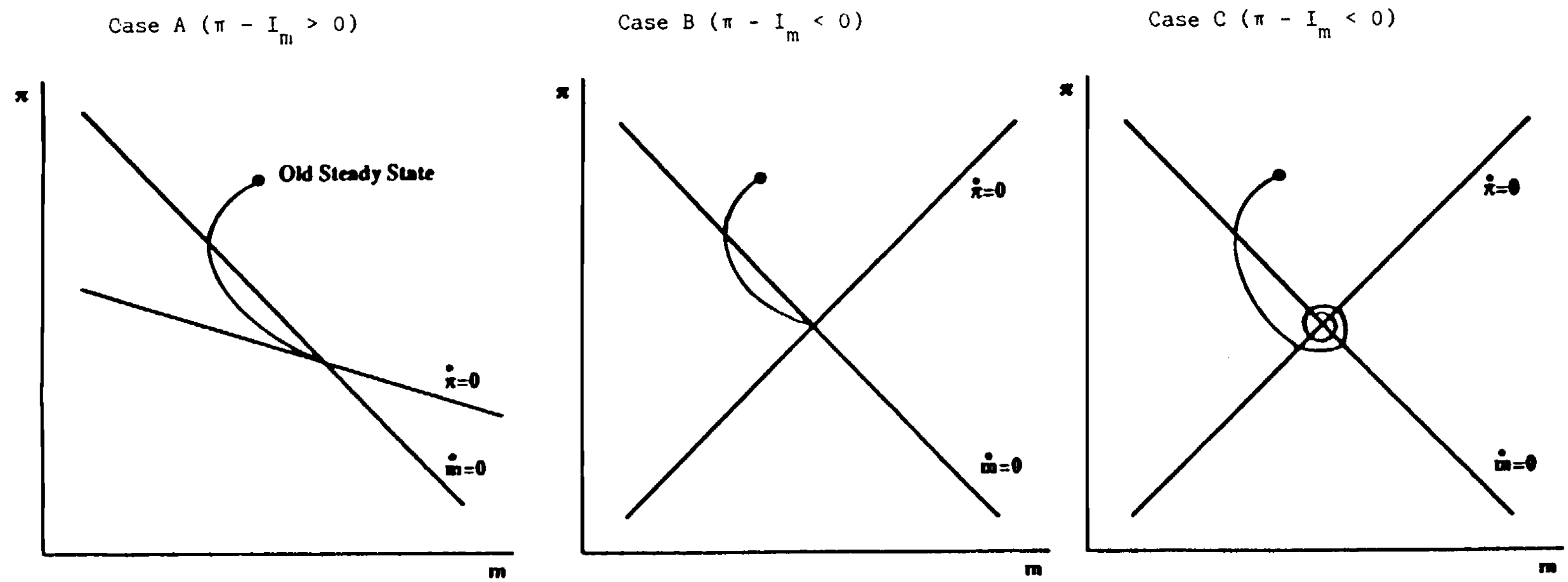

Figure 4

Transition Path Following a Downward Jump in Inflation

Case A ($\pi - I_m > 0$) Case B ($\pi - I_m < 0$) Case C ($\pi - I_m < 0$)

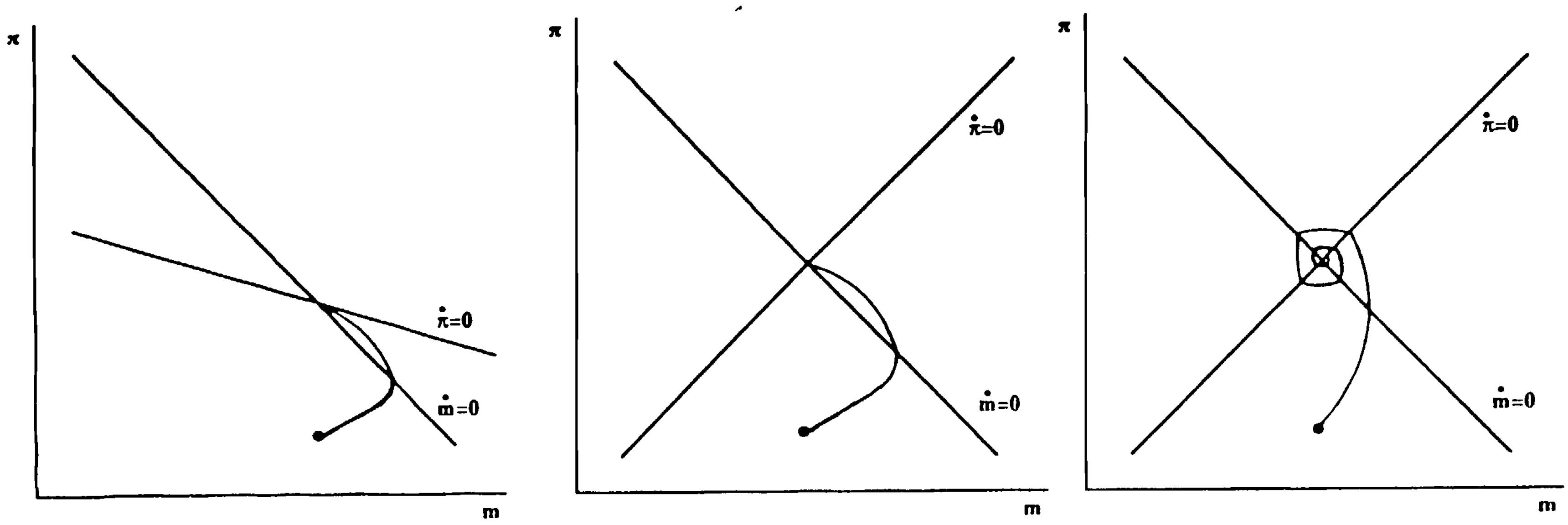

Figure 5

Heterodox Stabilization: Several Possibilities