OPTIMAL INVESTMENT STRATEGIES FOR UNIVERSITY ENDOWMENT FUNDS

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ABSTRACT

A common approach to the management of endowment is to treat it as if it were the only asset of the university. This approach leads to prescriptions for optimal investment and expenditure policies that are essentially the same across universities. Indeed, the resulting optimal portfolio strategies are focused almost exclusively on providing an efficient tradeoff between risk and expected return, a generic objective that is just as applicable to individuals and non-academic institutions as it is to universities. In contrast, the model developed here provides intertemporally optimal investment and expenditure rules for endowment that take account of the university's overall objectives and total resources. The explicit inclusion of other university assets in addition to endowment leads to optimal endowment portfolios that are not efficient in the sense of the risk-return tradeoff. Moreover, two universities with similar objectives and endowments can have very different optimal portfolios and expenditure patterns if their non-endowment sources of cash flow are different. The model also takes account of the uncertainty surrounding the costs of the various activities such as education, research, and knowledge storage that define the purpose of the university. As a result, the analysis reveals a perhaps somewhat latent role for endowment: namely, hedging against unanticipated changes in those costs.

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1. Introduction

To examine the question of optimal investment strategies for university endowment funds, one must, of course, address the issue of the objective function by which optimality is to be measured. My impression is that practicing money managers essentially side-step the issue by focusing on generically efficient risk-return objective functions for investment which are just as applicable to individuals or non-academic institutions as they are to universities. Perhaps the most common objective of this type is mean-variance efficiency for the portfolio's allocations. Black (1976) provides a deeper approach along those lines that takes account of tax and other institutional factors including certain types of non-endowment assets held by institutions. The Ford Foundation study of 1969 gave some early practical (if ex-post, somewhat untimely) guidance for investment allocations.

Much of the academic literature (which is not copious) seems to focus on appropriate spending policy for endowment, taking as given that the objective for endowment is to provide a perpetual level flow of expected real income (cf. Eisner (1974), Litvack, Malkiel, and Quandt (1974), Nichols (1974), and Tobin (1974)). Ennis and Williamson (1976) present a history of spending patterns by universities and a discussion of various spending rules adopted. They also discuss the interaction between spending and investment policies.

Fama and Jensen (1985) discuss the role of non-profit institutions as part of a general analysis of organizational forms and investment objective functions, but they do not address the functions of endowment in such institutions.

In contrast, Hansmann (1990) provides a focused and comprehensive review of the various possible roles for a university's endowment. Despite the broad coverage of possibilities ranging from tax incentives to promoting intergenerational equity, he is unable to find compelling empirical evidence to support any particular combination of objectives. Indeed, he concludes that "...prevailing endowment spending rules seem inconsistent with most of these objectives" (p.39). Hansmann goes on to assert (p.39-40):

> "It appears, however, that surprisingly little thought has been devoted to the purposes for which endowments are maintained and that, as a consequence, their rate of accumulation and the pattern of spending from their income have been managed without much attention to the ultimate objectives of the institutions that hold them."

The course taken here to address this question is in the middle range: It does not attempt to specify in detail the objective function for the university, but it does derive optimal investment and expenditure policy for endowment in a context which takes account of overall university objectives and the availability of other sources of revenue besides endowment. In that respect, it follows along lines similar to the discussion in Black (1976,26-8). In addition, our model takes explicit account of the uncertainties surrounding the costs of university activities. As a result, the analysis reveals another (perhaps somewhat latent) purpose for endowment: namely, hedging against unanticipated changes in those costs. Formal trading rules for implementing this hedging function are derived. However, the paper neither assesses which

costs, as an empirical matter, are more important to hedge nor does it examine the feasibility of hedging those costs using available traded securities. The interested reader should see Brinkman (1981,1990), Brovender (1974), Nordhaus (1989), and Snyder (1988) where the various costs of universities are described and modelled, both historically and prospectively.

Grinold, Hopkins, and Massy (1978) develop a budget-planning model which also integrates endowment returns with other revenue and expense flows of the university. However, their model differs significantly from the one presented here, perhaps because their focus is on developing policy guidelines for expenditures instead of optimal intertemporal management of endowment.

The mathematical model for optimal expenditures and investment is developed in Sections 2 and 3. It is based on a standard intertemporal consumption and portfolio-selection model. Hence, the formal structure of the optimal demand functions is already widely studied in the literature. It is the application of this model to the management of university endowment which is new. For analytical simplicity and clarity, the model is formulated in continuous time. However, it is evident from the work of Constantinides (1989), Long (1974), and Merton (1977) that a discrete-time version of the model would produce similar results. The reader who prefers to be exposed to the basic insights provided by the model before going through the detailed mathematical analysis should begin with the summary and conclusions discussion in Section 4.

2. The Model

The functions or purposes of the university are assumed to be a collection of activities or outputs such as education, training, research, and storage of knowledge. We further assume that the intensities of those activities can be quantified and there exists a preference ordering for ranking alternative intertemporal programs. In particular, the criterion function for this ranking can be written as:

$$\max \quad E_0 \left\{ \int_0^\infty \bar{U}[Q_1, \ldots, Q_m, t] dt \right\} \quad , \tag{1}$$

where $Q_j(t)$ denotes the quantity of activity j per unit time undertaken at time t, $j = 1, \ldots, m$; the preference function $\bar{U}$ is assumed to be strictly concave in $(Q_1, \ldots, Q_m)$; and E_t denotes the expectation operator, conditional on knowing all relevant information as of time t. This preference ordering satisfies the classic von Neumann-Morgenstern axioms of choice, exhibits positive risk aversion, and includes "survival" (of the institution) as a possible objective. The infinite time horizon structure in (1) implies only that there need not be a definite date when the university will liquidate. As shown in Merton (1990a, 149-51, 609-11), $\bar{U}$ can reflect the mortality characteristics of an uncertain liquidation date.

The intertemporally additive and independent preference structure in (1) can be generalized to include non-additivity, habit-formation and other path-dependent effects on preferences, along the lines of Bergman (1985), Constantinides (1990), Detemple and Zapatero (1989), Duffie and Epstein (1989), Hindy and Huang (1989), Sundaresan (1989), and Svensson (1989).

However, as shown in Merton (1990a, 207-9), those more-realistic preference functions do not materially affect the optimal portfolio demand functions. Moreover, just as Grossman and Laroque (1990) show for transactions costs in consumption, so it can be shown here that imposing adjustment costs for changing the levels of university activities does not alter the structure of the portfolio demand functions. Hence, because the focus of the paper is on optimal investment (rather than optimal expenditure) strategies, we assume no adjustment costs for activities and retain the additive independent preference specification to provide analytical simplicity.

Let $S_j(t)$ denote the (net) cost to the university of providing one unit of activity j at time t, $j = 1,\ldots,m$. For example, if $j = 1$ denotes the activity of having full-tuition-paying undergraduates, then S_1 would be the unit cost of providing the education minus the tuition received. If $j = 2$ denotes the activity of having undergraduates who receive financial aid, the unit cost S_2 would equal S_1 plus the financial aid given. In general, all costs and receipts such as tuition that are directly linked to the quantities of specific activities undertaken are put into the activity costs or prices, $\{S_j\}$. As will be described, fixed costs and sources of positive cash flows to the university that do not depend directly on the activity quantities are handled separately. As in Merton (1990a, 202, 499), we assume that the dynamics for these costs are described by the stochastic differential equations: for $S = (S_1,\ldots,S_m)$,

$$dS_j = f_j(S,t)S_j dt + g_j(S,t)S_j dq_j, \quad j = 1,\ldots,m \tag{2}$$

where f_j is the instantaneous expected rate of growth in S_j, g_j is the instantaneous standard deviation of the growth rate, and dq_j is a Wiener process with the instantaneous correlation coefficient between dq_i and dq_j given by v_{ij}, $i,j = 1,\ldots,m$. f_j and g_j are such that $dS_j \geq 0$ for $S_j = 0$ which ensures that $S_j(t) \geq 0$. Especially since $\{S\}$ has components that depend on tuition, financial aid and other variables over which the university has some control, one would expect that the dynamic path for those costs would be at least partially endogenous and controllable by the university, even though competition among universities would limit the degree of controllability. However, as specified, (2) is an exogenous process, not controlled by the university. Alternatively, it can be viewed as the "reduced-form" process for S after optimization over non-portfolio choice variables.

The university is assumed to have N non-endowment sources of cash flows which we denote by $Y_k(t)dt$ for the kth source at time t. Examples of such sources are gifts, bequests, university business income, and public and private-sector grants. It can also be used to capture transfer-pricing for the use of buildings and other university-specific assets where Y_k is the rental rate and this rental fee appears as an offsetting charge in the $\{S_j\}$ for the appropriate university activities. The dynamics for these cash flows are modeled by: for $Y = (Y_1,\ldots,Y_N)$,

$$dY_k = \mu_k(Y,S,t)Y_k dt + \delta_k(Y,S,t)Y_k d\varepsilon_k \qquad (3)$$

where μ_k and δ_k depend at most on the current levels of the cash flows and the unit costs of university activities and $d\varepsilon_k$ is a Wiener process,

$k = 1, \ldots, N$. Equation (3) can also be used to take account of fixed costs or liabilities of the university such as faculty tenure commitments. by letting $Y_k < 0$ to reflect a cash outflow. However, the focus here is on assets only and therefore, we assume that μ_k and δ_k are such that $dY_k \geq 0$ for $Y_k = 0$ which implies that $Y_k(t) \geq 0$ for all t.

By inspection of (2) and (3), the dynamics for (Y, S) are jointly Markov. A more realistic model would have μ_k and δ_k depend on both current and historical values of $Q_1, \ldots, Q_m$. For example, if a university has undertaken large amounts of research activities in the past, it may attract more grants and gifts in the future. The university may also affect the future expected cash flows from non-endowment sources by investing now in building up those sources. Thus, the dynamic process for Y should be in part controllable by the university. However, again for analytical simplicity, the Y process is taken as exogenous, because that abstraction does not significantly alter the optimal portfolio demand functions.

If for $k = 1, \ldots, N$, $V_k(t)$ denotes the capitalized value at time t of the stream of future cash flows, $Y_k(\tau)$ for $\tau \geq t$, and if $K(t)$ denotes the value of the endowment at time t, then the net worth or wealth of the university, $W(t)$ is given by

$$W(t) = K(t) + \Sigma_1^N V_k(t) \tag{4}$$

A model for determining the $V_k(t)$ from the posited cash flow dynamics in (3) is developed in Section 3.

The endowment of the university is assumed to be invested in traded

assets. There are n risky assets and a riskless asset. If $P_j(t)$ denotes the price of the jth risky asset at time t , then the return dynamics for the risky assets are given by, for $j = 1, \ldots, n$,

$$dP_j = \alpha_j P_j dt + \sigma_j P_j dZ_j \tag{5}$$

where α_j is the instantaneous expected return on asset j; σ_j is the instantaneous standard deviation of the return; and dZ_j is a Wiener process. The instantaneous correlation coefficients $\{\rho_{ij}, \eta_{kj}, \zeta_{1j}\}$ are defined by, for $j = 1, \ldots, n$,

$$dZ_i dZ_j = \rho_{ij} dt \quad , \quad i = 1, \ldots, n$$

$$dq_k dZ_j = \eta_{kj} dt \quad , \quad k = 1, \ldots, m \tag{5a}$$

$$d\varepsilon_1 dZ_j = \zeta_{1j} dt \quad , \quad 1 = 1, \ldots, N$$

For computational simplicity and to better isolate the special characteristics of endowment management from general portfolio management, we simplify the return dynamics specification and assume that $\{\alpha_j, \sigma_j, \rho_{ij}\}$ are constants over time, $i, j = 1, \ldots, n$. As shown in Merton (1990a, Chs. 4, 5, and 6), this assumption of a constant investment opportunity set implies that $\{P_j(t + \tau)/P_j(t)\}$, $j = 1, \ldots, n$, for $\tau > 0$ are jointly lognormally distributed. The riskless asset earns the interest rate r which is also constant over time. Optimal portfolio selection for general return dynamics would follow along the lines of Merton (1990a, Chs. 5, 15, 16; 1990b, Sec. 7).

To analyze the optimal intertemporal expenditure and portfolio-selection problem for the university, we begin with a further simplified version of the

model in which the university's entire net worth is endowment [i.e., $Y_k(t) = V_k(t) = 0$, $k = 1,...,N$ and $W(t) = K(t)$]. The budget-equation dynamics for $W(t)$ is then given by,

$$dW = [(\Sigma^n_1 w_j(t)(\alpha_j - r) + r)W - \Sigma^m_1 Q_k S_k]dt + \Sigma^n_1 w_j(t)W\sigma_j dZ_j \qquad (6)$$

where $w_j(t) =$ the fraction of the university's wealth allocated to risky asset j at time t, $j = 1,...,n$; the fraction allocated to the riskless asset is thus $1 - \Sigma^n_1 w_j$. Trustees, donors, and the government are assumed not to impose explicit limitations on investment policy for the endowment, other than general considerations of prudence. In particular, borrowing and short-selling are permitted and so the choice for $\{w_j\}$ is unrestricted. We further posit that spending out of endowment is not restricted, either with respect to overall expenditure or with respect to the specific activities on which it is spent. However, we do impose the feasibility restrictions that total expenditure at time t, $\Sigma^m_1 Q_k S_k$, must be nonnegative and zero wealth is an absorbing state (i.e., $W(t) = 0$ implies $W(t + \tau) = 0$ for $\tau > 0$).

At each time t, the university chooses a quantity of activities $\{Q_1,...,Q_m\}$ and a portfolio allocation of its wealth so as to maximize lifetime utility of the university as specified in (1). Just as for the case of multiple consumption goods analyzed in Breeden (1979), Fischer (1975), and Merton (1990a, 205), so the solution for the optimal program here can be decomposed into two parts. First, at each t, solve for the utility-maximizing quantities of individual activities, $\{Q_1,...,Q_m\}$, subject to an overall expenditure constraint, $C(t) = \Sigma^m_1 Q_k(t)S_k(t)$. Second, solve for the optimal

level of overall expenditures at time t and the optimal portfolio allocation of endowment.

The first part is essentially the static activity-choice problem under uncertainty

$$\max_{\{Q_1,\ldots,Q_m\}} \quad \bar{U}[Q_1,\ldots,Q_m,t] \tag{7}$$

subject to $C(t) = \Sigma_1^m Q_k S_k(t)$. The first-order conditions for the optimal activity bundle $(Q_1^*,\ldots,Q_m^*)$ are given by: for $S_k(t) = S_k$,

$$\bar{U}_k[Q_1^*,\ldots,Q_m^*,t]/S_k = \bar{U}_j[Q_1^*,\ldots,Q_m^*,t]/S_j \qquad k,j = 1,\ldots,m \tag{8}$$

with $C(t) = \Sigma_1^m Q_k^* S_k$, where subscripts on $\bar{U}$ denote partial derivatives (i.e., $\bar{U}_k \equiv \partial\bar{U}/\partial Q_k$). It follows from (8) that the optimal quantities can be written as $Q_k^* = Q_k^*[C(t),S(t),t]$, $k = 1,\ldots,m$.

Define the indirect utility function U by $U[C(t),S(t),t] \equiv \bar{U}[Q_1^*,\ldots,Q_m^*,t]$. By substituting U for $\bar{U}$, we can rewrite (1) as:

$$\max E_0\left\{ \int_0^\infty U[C(t),S(t),t]dt\right\} \tag{9}$$

where the "max" in (9) is over the intertemporal expenditure path {C(t)} and portfolio allocations {$w_j(t)$}. Thus, the original optimization problem is transformed into a single-expenditure choice problem with "state-dependent" utility (where the "states" are the relative costs or prices of the various

activities). Once the optimal total expenditure rules, {C*(t)}, are determined, the optimal expenditures on individual activities are determined by (8) with $C*(t) = \Sigma_1^m Q_k^* S_k$.

The solution of (9) follows by applying stochastic dynamic programming as in Merton (1990a, Chs. 4, 5, and 6). Define the Bellman or derived-utility function J by

$$J[W,S,t] = \max E_t\{\int_t^\infty U[C(\tau),S(\tau),\tau]d\tau\}$$

conditional on $W(t) = W$ and $S(t) = S$. From Merton (1990a, 181, 202; 1990b, 555), J will satisfy:

$$0 = \max_{\{C,w\}} \left[U[C,S,t] + \lambda C + J_t + J_W\left((\Sigma_1^n w_j(\alpha_j - r) + r)W - C\right)\right.$$

$$+ \Sigma_1^m J_i f_i S_i + \frac{1}{2} J_{WW}\Sigma_1^n\Sigma_1^n w_i w_j \sigma_{ij} W^2 \qquad (10)$$

$$\left. + \Sigma_1^m\Sigma_1^n J_{iW} w_j W g_i S_i \sigma_j \eta_{ij} + \frac{1}{2} \Sigma_1^m\Sigma_1^m J_{ij} g_i S_i g_j S_j v_{ij}\right)$$

subject to $J(0,S,t) = \int_t^\infty \bar{U}[0,\ldots,0,\tau]d\tau$, where subscripts on J denote partial derivatives with respect to W, t, and S_i, $i = 1,\ldots,m$ and $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$, the instantaneous covariance between the return on security i and j. λ is a Kuhn-Tucker multiplier reflecting the non-negativity constraint on C and at the optimum, it will satisfy $\lambda^* C^* = 0$. The first-order conditions derived from (10) are:

$$0 = U_C[C^*,S,t] + \lambda^* - J_W(W,S,t) \qquad (11a)$$

and

$$0 = J_W(\alpha_i - r) + J_{WW}\Sigma_1^n w_j^*W\sigma_{ij} + \Sigma_1^m J_{kW}g_kS_k\sigma_i\eta_{ki}, \quad i = 1,\ldots,n \qquad (11b)$$

where $C^* = C^*(W,S,t)$ and $w_i^* = w_i^*(W,S,t)$ are the optimal expenditure and portfolio rules expressed as functions of the state variables and subscripts on U denote partial derivatives.

From (11a), the optimal expenditure rule is given by:

$$U_C[C^*,S,t] = J_W(W,S,t) \quad \text{for} \quad C^* > 0$$
$$\lambda^* = \max\{0,J_W(W,S,t) - U_C[0,S,t]\} \qquad . \qquad (12)$$

From (11b), the optimal portfolio allocation can be written as:

$$w_i^*W = A\, b_i + \Sigma_1^m H_k h_{ki}, \quad i = 1,\ldots,n \qquad (13)$$

where $b_i = \Sigma_1^n v_{ij}(\alpha_j - r)$; $h_{ki} = \Sigma_1^n \sigma_j g_k S_k \eta_{kj} v_{ij}$; v_{ij} is the ij-element of the inverse of the instantaneous variance-covariance matrix of returns $[\sigma_{ij}]$; $A = -J_W/J_{WW}$ (the reciprocal of absolute risk aversion of the derived-utility function); and $H_k = -J_{kW}/J_{WW}$, $k = 1,\ldots,m$. A and H_k depend on the individual university's intertemporal preferences for expenditures and its current net worth. However, b_i and h_{ki} are determined entirely by the dynamic structures for the asset price returns and the unit costs of the various activities undertaken by universities. Hence, those parameters are the same for all universities, independent of their preferences or endowment size.

To provide some economic intuition about the optimal allocation of endowment in (13), consider as a frame of reference the "standard" intertemporal portfolio selection problem with state-independent utility, $U = U[C(t),$

t]. As shown in Merton (1990a, 131-6), given the posited return dynamics in (5), all such investors will hold instantaneously mean-variance efficient portfolios as their optimal portfolios. For $\partial U/\partial S_k = U_k = 0$, $H_k = 0$, $k = 1,\ldots,m$. Hence, in this case, (13) becomes $w_i^*W = Ab_i$, and $w_i^*W/w_j^*W = b_i/b_j$, the same for all investors. This is the well-known result that the relative holdings of risky assets are the same for all mean-variance efficient portfolios. However, the state-dependent preferences for universities induced by the uncertainty surrounding the relative costs of undertaking different desired activities causes the more complex demand structure in (13).

To better understand this differential demand, $w_i^*W - Ab_i = \Sigma_1^m H_k h_{ki}$, it is useful to examine the special case where, for each cost S_k, there exists an asset whose instantaneous return is perfectly correlated with changes in S_k. By renumbering securities if necessary, choose the convention that $\eta_{kk} = 1$ in (5a), $k = 1,\ldots,m$ ($m < n$). As shown in Merton (1990a, 203-4), it follows that in this case, $h_{kk} = g_k S_k/\sigma_k$ for $k = 1,\ldots,m$ and $h_{kj} = 0$ for $k \neq j$. Hence, we can rewrite (13) as:

$$w_i^*W = Ab_i + H_i g_i S_i/\sigma_i \qquad i = 1,\ldots,m$$
$$= Ab_i \qquad\qquad\quad i = m+1,\ldots,n \quad . \tag{14}$$

By the strict concavity of U with respect to C, J is strictly concave in W. Hence, $J_{WW} < 0$ and $H_i = -J_{iW}/J_{WW}$ is positively proportional to J_{iW}. Thus, relative to a "normal" investor with state-independent preferences (i.e., $H_i = 0$, $i = 1,\ldots,m$), but the same current level of absolute risk aversion (i.e., $-J_{WW}/J_W$), the university will optimally hold more of asset i

if $J_{iW} > 0$ and less if $J_{iW} < 0$, $i = 1, \ldots, m$.

If $J_{iW} > 0$, then, at least locally, the university's marginal utility (or "need") for wealth or endowment becomes larger if the cost of undertaking activity i increases and it becomes smaller if this cost decreases. Because the return on asset i is perfectly positively correlated with the cost of activity i, a greater-than-expected increase in S_i will coincide with a greater-than-expected return on asset i. By holding more of asset i than a "normal" investor, the university thus assures itself of a relatively larger endowment in the event that S_i increases and the need for wealth becomes more important. The university, of course, pays for this by accepting a relatively smaller endowment in the event that S_i decreases and wealth is less important. The behavioral description for $J_{iW} < 0$ is just the reverse, because the need for endowment decreases if the cost of activity i increases.

To perhaps help in developing further insights, we use (12) to interpret the differential demand component in (14) in terms of the indirect utility and optimal expenditure functions. By differentiating (12), we have that for $C^*(W,S,t) > 0$,

$$J_{WW} = U_{CC}[C^*,S,t] \frac{\partial C^*}{\partial W}$$

$$J_{kW} = U_{CC}[C^*,S,t] \frac{\partial C^*}{\partial S_k} + U_{Ck}[C^*,S,t]$$

$$A = -U_C[C^*,S,t] / \left(U_{CC}[C^*,S,t] \frac{\partial C^*}{\partial W} \right)$$

$$H_k = -\left[\frac{\partial C^*}{\partial S_k} \middle/ \frac{\partial C^*}{\partial W} \right] + U_{Ck}[C^*,S,t] / \left(-U_{CC}[C^*,S,t] \frac{\partial C^*}{\partial W} \right)$$

(15)

for $k = 1,\ldots,m$. Because $U_{cc} < 0$ and $\partial C^*/\partial W > 0$ for $C^* > 0$, we see that the sign of H_k is determined by the impact of a change in the cost of activity k on two items: the optimal level of total current expenditure and the marginal utility of expenditure. So, for example, if an increase in S_k would cause both a decrease in optimal expenditure $(\partial C^*/\partial S_k < 0)$ and an increase in the marginal utility of expenditure $(U_{Ck} > 0)$, then from (15), $H_k > 0$ and the university will optimally hold more of asset k than the corresponding investor with a mean-variance efficient portfolio.

Following (14) causes the university's optimal portfolio to be mean-variance inefficient and therefore, the return on the endowment will have greater volatility than other feasible portfolios with the same expected return. However, the value of the endowment or net worth of the university is not the "end" objective. Instead, it is the "means" by which the ends of a preferred expenditure policy can be implemented. Viewed in terms of the volatility of the time path of <u>expenditure</u> (or more precisely, the <u>marginal utility of expenditure</u>), the optimal strategy given in (14) is mean-variance efficient [cf. Breeden (1979) and Merton (1990a, 487-8)]. That is, because $\partial C^*/\partial W > 0$, the additional increment in wealth that, by portfolio construction, occurs precisely when S_k increases will tend to offset the negative impact on C^* caused by that increase. There is thus a dampening of the unanticipated fluctuations in expenditure over time. In sum, we see that in addition to investing in assets to achieve an efficient risk-return tradeoff in wealth, universities should optimally use their endowment to hedge against unanticipated and unfavorable changes in the costs of the various activities

in wealth, universities should optimally use their endowment to hedge against unanticipated and unfavorable changes in the costs of the various activities that enter into their direct utility functions.

In closing this section, we note that the interpretation of the demand functions in the general case of (13) follows along the same lines as for the special case of perfect correlation leading to (14). As shown for the general case in Merton (1990a, 501-2; 1990b, 558-9), the differential demands for assets reflect attempts to create portfolios with the maximal feasible correlations between their returns and unanticipated changes in the S_k, $k = 1,...,m$. These maximally-correlated portfolios perform the same hedging function as assets $1,...,m$ in the limiting case of perfect correlation analyzed in (14). Furthermore, if other state variables besides the various activities' costs (e.g., changes in the investment opportunity set) enter a university's derived utility function, then a similar structure of differential asset demands to hedge against the unanticipated changes in these variables will also obtain.

3. Optimal Endowment Management with Other Sources of Income

In the previous section, we identified hedging of the costs of university activities as a reason for optimally deviating from "efficient" portfolio allocations when endowment is the only means for financing those activities. In this section, we extend the analysis to allow other sources of cash flow to support the activities. To simplify the analysis, we make two additional assumptions: first, we posit that μ_k and δ_k in (3) are constants, which implies that $Y_k(t)/Y_k(0)$ is log-normally distributed, $k = 1,\ldots,N$. Second, we assume that for each k, there exists a traded security whose return is instantaneously perfectly correlated with the unanticipated change in Y_k, $k = 1,\ldots,N$. By renumbering if necessary, we use the convention that traded security k is instantaneously perfectly correlated with Y_k. Hence, it follows that $\zeta_{kk} = 1$ in (5a) and,

$$d\varepsilon_k = dZ_k \ , \qquad k = 1,\ldots,N \qquad . \tag{16}$$

These two assumptions permit us to derive a closed-form solution for the capitalized values of the cash flows, $\{V_k(t)\}$, using contingent-claims analysis. As will be shown, those valuation functions are independent of the university's preferences or wealth level.

From (3), (5), and (16) with μ_k and δ_k constant, we have that the cash flows can be written as a function of the traded asset prices as follows, for $k = 1,\ldots,N$,

$$Y_k(t) = Y_k(0)\exp\left[-\phi_k t\right] \left(P_k(t)/P_k(0)\right)^{\beta_k} \tag{17}$$

where $\phi_k = \beta_k \left(\alpha_k - \sigma_k^2/2 \right) - \left(\mu_k - \delta_k^2/2 \right)$ and $\beta_k = \delta_k/\sigma_k$. That (17) obtains can be checked by applying Itô's Lemma. We now derive the capitalized value for Y_k following Merton (1990a, 415-9; 1990b, 562-3).

Let $F^k \left[P_k, t \right]$ be the solution to the partial differential equation, for $0 \leq t \leq T_k$,

$$0 = 1/2 \, \sigma_k^2 \, P_k^2 \, F_{11}^k + rP_k F_1^k - rF^k + F_2^k + Y_k \qquad (18)$$

subject to the boundary conditions

$$F^k[0,t] = 0 \qquad (19a)$$

$$F^k/(P_k)^{\beta_k} \quad \text{bounded as} \quad P_k \to \infty \qquad (19b)$$

$$F^k[P_k,T_k] = 0 \qquad (19c)$$

where subscripts on F^k in (18) denote partial derivatives with respect to its arguments P_k and t; Y_k is given by (17); and T_k is the last date at which the university receives the cash flows from source k, $k = 1,...,N$. It is a mathematical result that a solution exists to (18) - (19) and that it is unique. Moreover, for $Y_k \geq 0$, $F^k \geq 0$ for all P_k and t.

Consider a dynamic portfolio strategy in which $F_1^k[P_k(t),t]P_k(t)$ is allocated to traded asset k at time t and $V(t) - F_1^k[P_k(t),t]P_k(t)$ is allocated to the riskless asset, where $V(t)$ is the value of the portfolio at

time t. Furthermore, let the portfolio distribute cash (by selling securi-
ties if necessary) according to the flow-rate rule

$$D_2[P_k,t] = Y_k(t) \qquad (20)$$

as given by (17). Then the dynamics of the portfolio can be written as, for
$P_k(t) = P_k$ and $V(t) = V$,

$$dV = F_1^k[P_k,t]dP_k + (([V - F_1^k[P_k,t]P_k]r - D_2[P_k,t])dt \qquad . \qquad (21)$$

Since F^k satisfies (18), it is a twice continuously differentiable function
and therefore, by Itô's Lemma, we can write the dynamics for F^k as

$$dF^k = \left[1/2 \; \sigma_k^2 \; P_k^2 \; F_{11}^k + F_2^k \right] dt + F_1^k dP_k \qquad (22)$$

But, F^k satisfies (18) and hence, $1/2 \; \sigma_k^2 \; P_k^2 \; F_{11}^k + F_2^k = rF^k - rP_kF_1^k - Y_k$.
Substituting into (22), we can rewrite (22) as

$$dF^k = F_1^k \; dP_k + \left[rF^k - rP_kF_1^k - Y_k \right] dt \qquad (23)$$

From (21) and (23), we have that

$$dV\text{-}dF^k = \left[rV - rP_kF_1^k - D_2 - rF^k + rP_kF_1^k + Y_k \right] dt$$
$$= r\left[V - F^k \right] dt \qquad (24)$$

because $D_2 = Y_k$. By inspection, (24) is an ordinary differential equation with solution

$$V(t) - F^k[P_k(t),t] = (V(0) - F^k[P_k(0),0])\exp[rt] \qquad . \qquad (25)$$

Thus, if the initial investment in the portfolio is chosen so that $V(0) = F^k[P_k(0),0]$, then for all t and $P_k(t)$, we have that

$$V(t) = F^k[P_k(t),t] \qquad . \qquad (26)$$

To ensure that the proposed portfolio strategy is feasible, we must show that its value is always nonnegative for every possible sample path for the price P_k and all t, $0 \le t \le T_k$. Because F^k is the solution to (18) and $Y_k \ge 0$, $F^k \ge 0$ for all P_k and t. It follows from (26) that $V(t) \ge 0$ for all P_k and t. We have therefore constructed a feasible dynamic portfolio strategy in traded asset k and the riskless asset that produces the stream of cash flows $Y_k(t)dt$ for $0 \le t \le T_k$ and has zero residual value $(V(T_k) = 0)$ at T_k.

Because the derived strategy exactly replicates the stream of cash flows generated by source k, it is economically equivalent to owning the cash flows $Y_k(t)$ for $t \le T_k$. It follows that the capitalized value of these cash flows satisfies

$$V_k(t) = F^k[P_k(t),t] \qquad (27)$$

for k = 1,...,N. Note that by inspection of (18)-(19), F^k, and hence $V_k(t)$,
does not depend on either the university's preferences or its net worth. The
valuation for source k is thus the same for all universities.

Armed with (27), we now turn to the optimal policy for managing endow-
ment when the university has N non-endowment sources of cash flows. To
derive the optimal policy, note first that even if those non-endowment sources
cannot actually be sold by the university for legal, ethical, moral hazard, or
asymmetric information reasons, the university can achieve the economic
equivalent of a sale by following the "mirror-image" or reverse of the
replicating strategy. That is, by (short-selling or) taking a $-F^k_1[P_k(t),t]P_k$
position in asset k and borrowing $\left[F^k - F^k_1 P_k\right]$ of the riskless asset at
each t, the portfolio will generate a positive amount of cash, $F^k[P_k,t]$,
available for investment in other assets at time t. The entire liability
generated by shorting this portfolio is exactly the negative cash flows,
$\{-Y_k dt\}$, for $t \le T_k$, because $V_k(T_k) = F^k[P_k, T_k] = 0$. But, since the univer-
sity receives $Y_k dt$ for $t \le T_k$ from source k, this short-portfolio liabili-
ty is entirely offset. Hence, to undertake this strategy beginning at time t
is the economic equivalent of selling cash flow source k for a price of
$V_k(t) = F^k[P_k(t),t]$.

As discussed more generally in Merton [1990a, Sec. 14.5, especially
465-7], the optimal portfolio strategy will be as if all N non-endowment
assets were sold and the proceeds, together with endowment, invested in the n
risky traded assets and the riskless asset. This result obtains because it is
feasible to sell (in the economic sense) the non-endowment assets and because

all the economic benefits from those assets can be replicated by dynamic
trading strategies in the traded assets. Hence, there is neither an economic
advantage nor a disadvantage to retaining the non-endowment assets. It
follows that the optimal demand for the traded risky assets is given by (13)
and the demand for the riskless asset is given by $(1-\Sigma_1^n w_i^*)W(t)$, where from
(4) and (27),

$$W(t) = K(t) + \Sigma_1^N F^k[P_k(t),t] \quad . \tag{28}$$

Because, however, the university has not actually sold the non-endowment
assets, the optimal demands given by (13) and (28) include both <u>implicit</u> and
<u>explicit</u> holdings of the traded assets. That is, the university's ownership
of non-endowment cash flow source k at time t is equivalent to having an
additional net worth of $F^k[P_k(t),t]$ (as reflected in (28)) <u>and</u> to having
$F_1^k[P_k(t),t]P_k(t)$ invested in traded asset k and $\left[F^k[P_k(t),t]\right.$
$\left. - F_1^k[P_k(t),t]P_k(t)\right]$ invested in the riskless asset. Thus, ownership of source
k causes implicit investments in traded asset k and the riskless asset.
Optimal explicit investment in each traded asset is the position actually
observed in the endowment portfolio and it is equal to the optimal demand
given by (13) and (28) minus the implicit investment in that asset resulting
from ownership of non-endowment assets. Let $D_i^*(t)$ denote the optimal
<u>explicit</u> investment in traded asset i by the university at time t. It
follows from (13) that

$$D_i^*(t) = Ab_i + \Sigma_1^m H_k h_{ki} - F_i^i[P_i(t),t]P_i(t), \quad i = 1,\ldots,N$$

$$= Ab_i + \Sigma_1^m H_k h_{ki} \quad , \quad i = N + 1,\ldots,n \tag{29}$$

where $W(t)$ used in the evaluation of A and H_k is given by (28). If we number the riskless asset by "n+1," then explicit investment in the riskless asset can be written as

$$D_{n+1}^*(t) = \left[1 - \Sigma_1^n w_j^*(t)\right] W(t) - \Sigma_1^N F^i[P_i(t),t] - F_i^i[P_i(t),t]P_i(t)\right]$$

$$- K(t) - \Sigma_1^n D_j^*(t) \quad . \tag{30}$$

By inspection of (29), it is apparent that in addition to the hedging of activity costs, the existence of non-endowment sources of cash flow will cause further differences between the observed holdings of assets in the optimal endowment portfolio and the mean-variance-efficient portfolio of a "standard" investor. Similarly, from (30), the observed mix between risky assets and the riskless asset will differ from the true economic mix.

To explore further the effects of those non-endowment sources of cash flows, we solve the optimal expenditure and portfolio-selection problem for a specific utility function, $\bar{U}$. However, in preparation for that analysis, we first derive explicit formulas for the capitalized values of those sources when $Y_k(t)$ is given by (17). As already noted, there exists a unique solution to (18) and (19). Hence, it is sufficient to simply find a solution. As can be verified by direct substitution into (18), the value of cash flow source k is given by, for $k=1,\ldots,N,$

$$F^k[P_k(t),t] = Y_k(0)\exp[-\phi_k t](1 - \exp[-\theta_k(T_k - t)])[P_k(t)/P_k(0)]^{\beta_k}/\theta_k \quad (31)$$

where β_k, ϕ_k are as defined in (17) and

$$\theta_k = r + \beta_k(\alpha_k - r) - \mu_k \quad . \quad (31a)$$

It follows from (31) that, for $k = 1,\ldots,N$,

$$F_1^k[P_k(t),t]P_k(t) = \beta_k F^k[P_k(t),t] \quad (32)$$

which implies that the capitalized value of source k has a constant elasticity with respect to the price of traded asset k. Equation (32) also implies that the replicating portfolio strategy is a constant-proportion or rebalancing strategy which allocates fraction β_k of the portfolio to traded asset k and fraction $(1 - \beta_k)$ to the riskless asset. In the case when positive fractions are allocated to both assets (i.e., $(1 - \beta_k) > 0$ and $\beta_k > 0$), then F^k is a strictly concave function of P_k. If $\beta_k > 1$, then F^k is a strictly convex function of P_k and the replicating portfolio holds traded asset k leveraged by borrowing. In the watershed case of $\beta_k = 1$, F^k is a linear function of P_k and the replicating portfolio holds traded asset k only.

Using (17) and (27), we can rewrite (31) to express the capitalized value of source k in terms of the current cash flow it generates: namely,

$$V_k(t) = Y_k(t)(1-\exp[-\theta_k(T_k - t)])/\theta_k, \quad k = 1,\ldots,N \qquad . \qquad (33)$$

From (17), (31a), and (32), it is a straightforward application of Itô's Lemma to show that the total expected rate of return for holding source k from t to $t + dt$ is given by

$$E_t[Y_k(t)dt + dV_k]/V_k(t) = (\mu_k + \theta_k)dt$$
$$= [r + \beta_k(\alpha_k - r)]dt \qquad . \qquad (34)$$

Thus, if the rights to the cash flows Y_k between t and T_k were sold in the market place, the expected rate of return that would be required by investors to bear the risk of these flows is $r + \beta_k(\alpha_k - r)$. Therefore, θ_k equals the required expected rate of return ("the capitalization rate") minus the expected rate of growth of the cash flows, μ_k. By inspection of (33), $V_k(t)$ can be expressed by the classic present-value formula for assets with exponentially growing cash flows. For $\theta_k > 0$, the perpetual ($T_k = \infty$) value is $Y_k(t)/\theta_k$ and the limiting "Earnings-to-Price" ratio, $Y_k(t)/V_k(t)$, is θ_k, a constant. Applying the closed-form solution for F^k, we can by substitution from (27) and (32) into (29) and (30) rewrite the optimal demand functions as

$$D_i^*(t) = Ab_i + \Sigma_1^m H_k h_{ki} - \beta_i V_i(t), \qquad i = 1,\ldots,N$$
$$= Ab_i + \Sigma_1^m H_k h_{ki} \qquad , \qquad i = N+1,\ldots,n \qquad (35a)$$

and

$$D_{n+1}^*(t) = \left[1 - \Sigma_1^n w_j^*(t)\right] W(t) - \Sigma_1^N (1-\beta_i)V_i(t)$$
$$= K(t) - A\Sigma_1^n b_j - \Sigma_1^m \Sigma_1^n H_k h_{kj} + \Sigma_1^N \beta_i V_i(t) \qquad . \qquad (35b)$$

Having derived explicit formulas for the values of non-endowment assets, we turn now to the solution of the optimal portfolio and expenditure problem in the special case where the university's objective function is given by

$$\bar{U}[Q_1, \ldots, Q_m, t] = \exp[-\rho t] \Sigma_1^m \Gamma_j \log Q_j \tag{36}$$

with $\rho > 0$ and $\Gamma_j \geq 0$, $j = 1, \ldots, m$. Without loss of generality, we assume that $\Sigma_1^m \Gamma_i = 1$. From (8), the optimal Q_j satisfy

$$Q_j^*(t) = [\Gamma_j C(t)]/S_j(t) \quad , \ j = 1, \ldots, m \quad . \tag{37}$$

From (36) and (37), the indirect utility function can be written as

$$U[C, S, t] = \exp[-\rho t]\{\log C - \Sigma_1^m \Gamma_j[\log S_j - \log(\Gamma_j)]\} \quad . \tag{38}$$

It follows from (11a) that the optimal expenditure rule is

$$C^*(t) = \exp[-\rho t](1/J_W[W, S, t]) \quad . \tag{39}$$

It is straightforward to verify by substitution into (10), (11a), and (11b) that

$$J[W, S, t] = \frac{1}{\rho} \exp[-\rho t]\log W + I[S, t] \tag{40}$$

for some function $I[S, t]$. By the verification theorem of dynamic programming, satisfaction of (10), (11a), and (11b) is sufficient to ensure that J in (40) is the optimum.

It follows from (40) that $J_{kW} = 0$ and hence that $H_k = 0$ in (13) and (35), $k = 1, \ldots, m$. Therefore, for the log utility specified in (36), there

are no differential hedging demands for assets to protect against unanticipated changes in the costs of university activities. The optimal allocation of the university's total net worth is thus instantaneously mean-variance efficient. Noting that $A = -J_W/J_{WW} = W$, we have that (35) can be written in this special case as

$$D_i^*(t) = b_i W - \beta_i V_i(t) \quad , \quad i = 1,\ldots,N$$

$$= b_i W \quad , \quad i = N+1,\ldots,n \quad (41a)$$

and

$$D_{n+1}^*(t) = (1 - \Sigma_1^n b_j)W - \Sigma_1^N(1 - \beta_i)V_i(t) \quad . \quad (41b)$$

By inspection of (41), in the absence of non-endowment assets, the fraction of endowment allocated to risky asset i in the university's optimal portfolio is b_i, $i = 1,\ldots,n$ and the fraction allocated to the riskless asset is $(1 - \Sigma_1^n b_j)$, independent of the level of endowment. If $\chi_i^* = D_i^*(t)/K(t)$ is the optimal fraction of endowment invested in asset i, then from (41), the difference in fractional allocations caused by the non-endowment assets is

$$\chi_i^*(t) - b_i = R[b_i - \beta_i \lambda_i] \quad , \quad i = 1,\ldots,N$$

$$= Rb_i \quad , \quad i = N+1,\ldots,n \quad (42a)$$

and

$$\chi_{n+1}^*(t) - (1 - \Sigma_1^n b_j) = -R[\Sigma_1^n b_k - \Sigma_1^N \beta_k \lambda_k] \quad (42b)$$

where $\lambda_k = V_k(t)/\Sigma_1^N V_i(t)$ is the fraction of the capitalized value of the

university's total non-endowment assets contributed by cash flow source k at time t, $k = 1,\ldots,N$ and $R = \Sigma_1^N V_i(t)/K(t)$ is the ratio of the values of the university's non-endowment assets to its endowment assets at time t.

The differences in (42) are the result of two effects: (i) the "wealth" effect caused by the difference between the net worth and the endowment of the university and (ii) the "substitution" effect caused by the substitution of non-endowment asset holdings for traded asset holdings. Suppose, for concreteness, that the expected returns, variances, and covariances are such that a positive amount of each traded risky asset is held in mean-variance-efficient portfolios. Then, $b_i > 0$, $i = 1,\ldots,n$. It follows that the impact of the wealth effect in (42), $\{Rb_i\}$, is unambiguous: it causes a larger fraction of the optimal endowment portfolio to be allocated to each risky asset and therefore, a smaller percentage allocation to the riskless asset. Because $\beta_i \geq 0$ and $\lambda_i > 0$, $i = 1,\ldots,N$, we have that the impact of the substitution effect in (42), $\{R\beta_i\lambda_i\}$, is also unambiguous: for those traded assets $1,\ldots,N$ for which the non-endowment assets are substitutes, the fractional allocation is smaller; for the traded assets $N + 1,\ldots,n$, the fractional allocation is unchanged; and the allocation to the riskless asset thus increases.

Because the wealth and substitution effects are in opposite directions for $b_k > 0$, whether the optimal endowment portfolio allocates an incrementally larger or smaller fraction to traded asset k depends on whether $b_k > \beta_k\lambda_k$ or $b_k < \beta_k\lambda_k$. $\beta_k\lambda_k$ is the fraction of the total increment to net worth (from non-endowment assets) that is implicitly invested in asset k as

the result of owning cash flow source k. If that fraction exceeds the

optimal one for total wealth, b_k, then the optimal endowment portfolio will

hold less than the mean-variance-efficient allocation. Indeed, if $\lambda_k > (1 +$

$R)b_k/(R\beta_k)$, then $\chi_k^*(t) < 0$ and the university would optimally short-sell

traded asset k in its portfolio. This is more likely to occur when R is

large (i.e., non-endowment assets are a large part of university net worth)

and λ_k is large (i.e., cash flow source k is a large part of the value of

non-endowment assets).

The implications of (42) for optimal endowment are intuitive. If, for

example, a significant amount of gift-giving to a particular university

depends on the performance of the general stock market, then in effect that

university has a "shadow" investment in that market. Hence, all else the

same, it should hold a smaller portion of its endowment in stocks than another

university with smaller amounts of such market-sensitive gift-giving. The

targeting can be more specific: If a school specializes in science and

engineering and if an important part of gifts comes from entrepreneur alums,

then the endowment should underweight (or even eliminate) investment in

venture capital and hi-tech companies. Indeed, if a single donor is expected

to give a large block of a particular stock, then the optimal explicit holding

of that stock in the endowment may be negative (although such short-sales

might offend some donors). Much the same story applies to concentrations in

other assets, including real estate. The same analysis also follows where

grants from firms or the government are likely to be strongly correlated with

the financial performance of stocks in the related industries.

It is perhaps a bit of a paradox that the university should under-invest the endowment in those sectors of assets for which it has special expertise (e.g., science and technology). But, the seeming paradox is resolved once the principle of diversification is invoked. However, the underweighting in those assets for diversification reasons can be offset by sufficiently strong demands to hedge against costs. For example, suppose that a specialized institute of biology believes that the cost of keeping the faculty will rise by considerably more than tuition in the event that there is a strong demand for such scientists outside academe. Then it may be optimal to invest a portion of its endowment in bio-tech stocks to hedge this cost even though those stocks' returns are highly correlated with alum gifts and industry grants.

The analysis leading to (29) and (30) requires that there exist traded securities which are instantaneously perfectly correlated with the changes in $Y_1,\ldots,Y_N$. If this assumption is relaxed, then the capitalized values of those non-endowment cash flow sources will no longer be independent of the university's preferences and endowment. However, the impact on endowment investments will be qualitatively similar. This more general case of non-replicable assets can be analyzed along the lines of Svensson (1988).

We can use our model to examine the impact of non-endowment cash flow sources on optimal expenditure policy. From (39) and (40), we have that the optimal expenditure rule is the constant-proportion-of-net-worth policy

$$C^*(t) = \rho W(t) \qquad . \tag{43}$$

However, current expenditure <u>from</u> <u>endowment</u> will not follow a constant

proportion strategy. Optimal expenditure from endowment at time t is

$[C^*(t) - \Sigma_1^N Y_k(t)]dt$, which can be either positive or negative (implying net

saving from non-endowment cash flow sources). If $s^*(t)$ denotes the optimal

expenditure rate as a fraction of endowment($= [C^*(t) - \Sigma_1^N Y_k(t)]/K(t)$), then

from (4) and (43)

$$s^*(t) = \rho + R(t)[\rho - y(t)] \tag{44}$$

where $R(t)$ is as defined in (42) and $y(t) = [\Sigma_1^N Y_k(t)]/[\Sigma_1^N V_k(t)]$ is the

current yield on the capitalized value of the non-endowment sources of cash

flow. In the special case of (33) where the cash flows are all perpetuities

(i.e., $T_k = \infty$ and $\theta_k > 0$, k = 1,...,N), $V_k(t) = Y_k(t)/\theta_k$, and the current

yield on source k is constant and equal to θ_k. In that case, y(t) =

$\Sigma_1^N \lambda_k \theta_k$, the value-weighted current yield. From (31a), θ_k will tend to be

smaller for assets with higher expected growth rates of cash flow, $\{\mu_k\}$. If

on average, the current yield on non-endowment assets is less than ρ, then

the current spending rate out of endowment will exceed ρ. If the current

yield is high so that $y(t) > \rho$, then $s^*(t) < \rho$. Indeed, if $y(t) > \rho(1 +$

$R)/R$, then $s^*(t) < 0$ and optimal total expenditure is less than current cash

flow generated by non-endowment sources. Because both $R(t)$ and $\lambda_k(t)$

change over time, we have from (44) that the optimal current expenditure rate

from endowment is not a constant, even when expected returns on assets, the

interest rate, and the expected rate of growth of non-endowment cash flows are

constants.

We can also analyze the dynamics of the mix of the university's net worth between endowment and non-endowment assets. If $\alpha = r + \Sigma_1^n b_i(\alpha_i - r)$ denotes the instantaneous expected rate of return on the growth-optimum, mean-variance-efficient portfolio, then as shown in Merton (1990a, 169-71), the resulting distribution for that portfolio is log-normal with instantaneous expected return $\alpha(> r)$ and instantaneous variance rate equal to $(\alpha - r)$. It follows from (6), (41), and (43) that the dynamics for the university's net worth are such that $W(t)/W(0)$ is log-normally distributed with

$$E_0[W(t)] = W(0)\exp[(\alpha - \rho)t]$$

$$E_0(\log[W(t)/W(0)]) = [(\alpha + r)/2 - \rho]t$$

$$\text{Var}(\log[W(t)/W(0)]) = (\alpha - r)t \qquad . \qquad (45)$$

If $X_k(t) = V_k(t)/W(t)$ denotes the fraction of net worth represented by non-endowment cash flow source k, then because V_k and W are each log-normally distributed, $X_k(t)$ is log-normally distributed and from (33) and (45),

$$E_0[X_k(t)] = X_k(0)\exp[(\rho - \theta_k)t]$$

$$E_0(\log[X_k(t)/X_k(0)]) = [\mu_k + \rho - (\alpha + r + \delta_k^2)/2]t$$

$$\text{Var}(\log[X_k(t)/X_k(0)]) = [\delta_k^2 + \alpha + r - 2(\mu_k + \theta_k)]t \qquad (46)$$

for $k = 1,\ldots,N$.

From (46), the fraction of total net worth represented by all sources of non-endowment cash flow, $X(t) = \Sigma_1^N X_k(t) = R(t)/[1 + R(t)]$, is expected to grow or decline depending on whether $\rho > \theta_{min}$ or $\rho < \theta_{min}$ where $\theta_{min} = \min[\theta_k]$, $k = 1,\ldots,N$. In effect, a university with either a high rate

of time preference or at least one (perpetual) high-growth non-endowment asset (i.e., with $\rho > \theta_{min}$) is expected to "eat" its endowment. Indeed, it may even go to a "negative" endowment by borrowing against the future cash flows of its non-endowment assets. Whether this expected growth in $X(t)$ is the result of declining expected net worth or rising asset values can be determined from (45). Because $\alpha > r$, if $\rho \leq r$, then both the arithmetic and geometric expected rates of growth for net worth are positive. For $\rho < \theta_{min}$, it follows that $E_0[X(t)] \to 0$ as $t \to \infty$. Hence, in the long run of this case, endowment is expected to become the dominant component of the university's net worth. Of course, these "razor's edge" results on growth or decline reflect the perpetual, constant-growth assumptions embedded in non-endowment cash flow behavior. However, this special case does capture the essential elements affecting optimal portfolio allocation and expenditure policies. [cf. Tobin (1974).]

4. Summary and Conclusions

As indicated at the outset, a common approach to the management of endowment is to treat it as if it were the only asset of the university. A consequence of this approach is that optimal portfolio strategies are focused exclusively on providing an efficient tradeoff between risk and expected return. The most common practice is to measure portfolio risk by the variance or standard deviation of its return. Because the returns on all mean-variance efficient portfolios are perfectly correlated, a further consequence of exclusive focus on efficiency is that the optimal endowment portfolios of different universities should have quite similar risky components, at least as measured by the correlations of their returns.

But, as we all know, universities have other assets, both tangible and intangible, many of which are important sources of cash flow. As noted in Section 2, examples of such sources include gifts, bequests, university business income, and public and private-sector grants. The analysis in the preceding sections shows that taking account of those assets can cause the composition of the optimal endowment portfolio to deviate significantly from mean-variance efficiency. It follows that two universities with similar objectives and endowments of the same size can nevertheless have very different optimal endowment portfolios if their non-endowment sources of cash flow are different. The effect on the composition of the optimal endowment portfolios induced by differences in the size of non-endowment assets can be decomposed into two parts: the wealth effect and the substitution effect.

To illustrate the wealth effect, consider two universities with identi-

cal preference functions and the same size endowments, but one has non-endowment assets and the other does not. If, as is perhaps reasonable to suppose, the preference function common to each, exhibits decreasing absolute risk aversion, then the university with the non-endowment assets (and hence larger net worth) will prefer to have a larger total investment in risky assets. Such behavior is consistent with the belief that wealthier universities can "afford" to take larger risks with their investments. Thus, if the average risk of the non-endowment assets is the same as the risk of the endowment-only university's portfolio, then the university with those assets will optimally invest more of its endowment in risky assets.

The substitution effect on the endowment portfolio is caused by the substitution of non-endowment asset holdings for endowment asset holdings. To illustrate, consider the two universities of the previous paragraph, but now increase the size of the endowment of the endowment-only university so that net worth or wealth is the same for both. The optimal deployment of total net worth is identical for each university. However, those assets held as non-endowment assets will not appear in the endowment portfolio. Thus, if the composition of the two endowment portfolios are compared, they will differ in both scale and fractional allocations among the various assets.

To illustrate the substitution effect more concretely, we draw on some examples analyzed in Section 3. Consider a university that on a regular basis receives donations from alums. Clearly, the cash flows from future contributions are an asset of the university, albeit an intangible one. Suppose that the actual amount of gift-giving is known to be quite sensitive to the

performance of the general stock market. That is, when the market does well, gifts are high and when it does poorly, gifts are low. Through this gift-giving process, the university thus has a "shadow" investment in the stock market. Hence, all else the same, it should hold a smaller portion of its endowment in stocks than another university with smaller amounts of such market-sensitive gift-giving.

As discussed in the previous section, the same principle applies to more specific asset classes. If an important part of gifts to a school that specializes in science and engineering comes from entrepreneur alums, then the school de facto has a large investment in venture capital and hi-tech companies and it should therefore invest less of its endowment funds in those areas. That the school should optimally invest less of its endowment in the science and technology areas where its faculty and students have special expertise may seem a bit paradoxical. But, the paradox is resolved by the principle of diversification once the endowment is recognized as representing only a part of the assets of the university. The same analysis and conclusion applies if we were to change the example by substituting either government and corporate grants for donations and gift-giving as the sources of cash flows or alum wealth concentrations in other assets such as real estate for concentrations in shares of stock. As also shown in the preceding section, the nature and size of a university's non-endowment assets should significantly influence optimal policy for expenditure of endowment. Hence, neglecting those other assets will bias the optimal expenditure policy.

In addition to taking account of non-endowment assets, our analysis

differs from the norm because it takes account of the uncertainty surrounding

the costs of the various activities such as education, research, and knowledge

storage that define the purpose of the university. As discussed in Section 2,

the breakdown of activities can be considerably more refined. For instance,

one activity could be the education of a full-tuition-paying undergraduate and

a second could be the education of an undergraduate who receives financial

aid. The unit (net) cost of the former is the unit cost of providing the

education less the tuition received and the unit cost of the latter is this

cost plus the financial aid given. As shown in Section 2, an important

function of endowment investments is to hedge against unanticipated changes in

the costs of university activities.

As an example, consider the decision as to how much (if any) of the

university's endowment to invest in local residential real estate. From a

standard mean-variance-efficiency analysis, it is unlikely that any material

portion of the endowment should be invested in this asset class. However,

consider the cost structure faced by the university for providing teaching and

research. Perhaps the single largest component is faculty salaries. Univer-

sities of the same type and quality compete for faculty from the same pools.

To be competitive, they must offer a similar standard of living. Probably the

largest part of the differences among universities in the cost of providing

this same standard of living is local housing costs. By investing in local

residential housing, the university hedges itself against this future cost

uncertainty by acquiring an asset whose value is higher than expected when the

differential cost of faculty salaries is higher than expected. This same

asset may also provide a hedge against unanticipated higher costs of off-campus housing for students which would in turn require more financial aid if the university is to compete for the best students. Note: This prescription of targeted investment in very specific real estate assets to hedge against an unanticipated rise in a particular university's costs of faculty salaries and student aid should not be confused with the often-stated (but empirically questionable) assertion that investments in real estate generally are a good hedge against inflation. See Bodie (1976;1982) for empirical analysis of the optimal assets for hedging against general inflation.

Similar arguments could be used to justify targeted investment of endowment in various commodities such as oil as hedges against unanticipated changes in energy costs. Uncertainty about those costs is especially significant for universities located in extreme climates and for universities with major laboratories and medical facilities that consume large quantities of energy.

As discussed at the end of Section 2, the hedging role for endowment derived here is formally valid as long as there are traded securities with returns that have non-zero correlations with unanticipated changes in the activity costs. However, the practical significance for this role turns on the magnitude of the correlations. As illustrated in Bodie's (1976;1982) work on hedging against inflation, it is often difficult to construct portfolios (using only standard types of traded securities) that are highly correlated with changes in the prices of specific goods and services. Nevertheless, the enormous strides in financial engineering over the last decade have greatly

expanded the opportunities for custom financial contracting at reasonable

costs. As we move into the twenty-first century, it will become increasingly

more common for the financial services industry to offer to its customers

private contracts or securities that allow efficient hedging when the return

properties of publicly-traded securities are inadequate. That is, implementa-

tion of the strategies prescribed in Sections 2 and 3 will become increasingly

more practical for universities and other endowment institutions. See Merton

(1990a, Ch. 14;1990c, 264-9) for a prospective view on financial innovation

and the development of custom financial contracting.

There are of course a variety of issues involving endowment management

that have not been addressed but could be within the context of our model.

One such issue is the decision whether to invest endowment in specific-purpose

real assets such as dormitories and laboratories instead of financial (or

general-purpose physical) assets. The returns on those real assets are likely

to be strongly correlated with the costs of particular university activities

and thereby the assets form a good hedge against unexpected rises in those

costs. However, because the real asset investments are specialized and

largely irreversible, shifting the asset mix toward such investments reduces

flexibility for the university. That is, with financial assets, the universi-

ty has more options as to what it can do in the future. In another paper, I

plan to analyze this choice problem more formally by using contingent-claims

analysis to value the tradeoff between greater flexibility in selecting future

activities and lower costs in producing a given set of activities.

Another issue not explicitly examined is the impact of long-term, fixed

liabilities such as faculty tenure contracts on the management of endowment. As noted early in Section 2, our model using contingent claims analysis (CCA) can handle this extension. See McDonald (1974) and Merton (1985) for CCA-type models for valuing tenure and other wage guarantee contracts.

The formal analysis here assumes that endowment is fungible for other assets and that neither spending nor investment policy are restricted. Such restrictions on endowment could be incorporated using the same Kuhn-Tucker type analysis used in Section 2 to take account of the constraint that total expenditure at each point in time is nonnegative. The magnitudes of the Kuhn-Tucker multipliers at the optimum would provide a quantitative assessment of the cost of each such restriction. However, including those restrictions is not likely to materially change the basic insights about hedging and diversification derived in the unrestricted case. The model can also be integrated into a broader one for overall university financial planning. Such integration would permit the evaluation of other non-endowment financial policies such as whether the university should sell forward contracts for tuition.

In summary, we have explored two classes of reasons why optimal endowment investment policy and expenditure policy can vary significantly among universities. The analysis suggests that trustees and others who judge the prudence and performance of policies by comparisons across institutions should take account of differences in both the mix of activities of the institutions and the capitalized values of their non-endowment sources of cash flows.

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