

NBER WORKING PAPER SERIES

TESTING THE POSITIVE THEORY OF GOVERNMENT FINANCE

David S. Bizer

Steven N. Durlauf

Working Paper No. 3349

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
May 1990

Bizer's research was supported by the Lessing Rosenthal Research Fund, and Durlauf's by the Center for Economic Policy Research. We thank Ken Judd, Louis Maccini, Julio Rotemberg, Charles Stuart, Gregory Hess, an anonymous referee and two editors, and gratefully acknowledge Andrew Bernard, Suzanne Cooper and Scott Schuh for excellent research assistance. This paper is part of NBER's research programs in Taxation and Economic Fluctuations. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

NBER Working Paper #3349  
May 1990

TESTING THE POSITIVE THEORY OF GOVERNMENT FINANCE

ABSTRACT

Researchers characterizing optimal tax policies for dynamic economies have reasoned that optimally chosen tax rates should approximately follow a random walk. We conduct a frequency-domain examination of the properties of the tax rate series and conclude that while there is a substantial smoothing role for debt, one rejects the hypothesis that the first difference in the series is white noise. This conclusion follows both from an analysis of the entire spectral distribution function of tax changes as well as from the behavior of individual frequencies. The source of the rejection is pronounced activity of tax changes at an eight year cycle which is suggestive of an electoral component to tax changes. Regression analysis confirms the finding that there is a cyclical component to tax changes corresponding to changes in political party administration. The results suggest that the positive theory of government finance needs to be refined to incorporate features of political equilibrium.

David S. Bizer  
Department of Economics  
Johns Hopkins University  
Baltimore, MD 21218

Steven N. Durlauf  
Department of Economics  
Stanford University  
Stanford, CA 94305

## *1. Introduction*

Principally due to Barro [1979], the life-cycle model of consumption has been reinterpreted to model a positive theory of government finance. Whereas the life-cycle model of consumption considers a representative agent who chooses a sequence of consumption to maximize utility subject to exogenous income flows and a law of motion for individual wealth, a descriptive theory of tax rate determination conceives of an infinitely-lived government choosing a sequence of tax rates to minimize deadweight loss, subject to exogenous government spending flows and a law of motion for government bonds.

According to this theory of government finance, the tax rate prevailing at any point in time represents the solution to a dynamic programming problem. In the solution, minimizing the distortions imposed by a government's revenue requirements requires equating marginal inefficiencies across time. Furthermore, a quadratic loss function for the government permits a representation of the global optimum such that the tax rate approximately equals the marginal inefficiency and therefore possesses a martingale-like sample path.

The purpose of this paper is to more carefully address the adequacy of the positive theory of government finance by closely scrutinizing the testable implications of the martingale hypothesis. The guiding principle that facilitates our work is that the martingale hypothesis imposes restrictions on the entire range of autocorrelations of a differenced tax rate time series. These restrictions lead us to perform a set of frequency-domain econometric exercises that capture all implications of the null model. Our belief is that these investigations

shed light on the empirical significance of particular elaborations of the standard model, and thus serve as a useful guide for further research on tax rate determination.

The test statistics we employ allow us to discriminate among potential reasons why the basic martingale tax model may fail. This is important because the standard model that yields the martingale hypothesis for tax rates should be regarded as an approximation of a more sophisticated underlying model of dynamic taxation that might not have exactly the martingale property but that would still be consistent with tax smoothing, and thus should still conclude that optimal tax rates are well-predicted by current levels. Thus, a finding that taxes are not exactly a martingale need not imply that the data are inconsistent with the essential elements of the positive theory of government finance.

The most significant enhancement of the basic Barro [1979] model is due to Lucas and Stokey [1983]. Although their model shares central components with the more simple martingale model, the Lucas-Stokey environment is more general than Barro's in two ways. First, there is no linear-quadratic structure.<sup>1</sup> Second, Lucas-Stokey allow for fully state-contingent government debt. If bonds are permitted to play an insurance role, then in the optimum there is a single shadow price of government revenue across states of nature. Tax rates in any period are consequently governed by conventional optimal-tax rules concerning the current elasticity of the supply of labor. If this elasticity were constant

---

<sup>1</sup>Elsewhere, we question the logical correctness of the martingale property in the linear-quadratic framework. In particular, the martingale convergence theorem implies that a martingale in tax rates requires either that tax rates are constant or that the asymptotic variance of a tax rate series is infinite. Since the latter possibility entails a positive probability that either the private sector or public sector's budget constraints are violated, a researcher would reject the presence of a nondegenerate martingale almost surely with a sufficiently large sample. See Bizer and Durlauf [1990].

across all states, then the optimal tax rate would also be constant. Barro's martingale theory of the evolution of taxes, however, is derived in the absence of state-contingent debt. With perfectly safe bonds, the government cannot shift revenue across states without changing taxes, and so the shadow price of revenue is not constant. For this reason, according to the martingale hypothesis, tax rates might change even though the current elasticity of the supply of labor has not changed. Thus, while the Lucas-Stokey model does not imply that tax movements exactly obey a martingale, it does imply some smoothing of taxes.

A further assumption used to derive the martingale property is that either a single infinitely-lived government is able to tie its own hands and enforce a fully state-contingent tax policy rule set at the beginning of time, or that sequences of governments are able to bind their successors by means of an unspecified commitment technology. Time-consistent solutions typically do not imply an exact martingale in tax rates. Interestingly, Poterba and Rotemberg [1988] examine international aggregate tax data and find qualified evidence against the commitment solution and in favor of the time-consistent solution. However, Judd [1989] emphasizes that capital and labor should be taxed differently in the optimum. Upon disaggregating taxes appropriately, he finds support for optimality (and hence, commitment).

Given that these extensions of the Barro model entail substantial smoothing of taxes, it is not surprising that regression tests of the tax rate martingale hypothesis by Barro [1981], Kingston [1987], and Mankiw [1987] have found that the evolution of United States tax rates over time is roughly consistent with the martingale property. Important evidence contrary to the martingale hypothesis was documented by Sahasakul [1986], who found that movements in tax rates were predictably related to wars and recessions. However, it is not clear that Sahasakul's results constitute significant evidence against the idea that tax rates are

approximately dynamically optimal, since wars and large business cycle movements are precisely the circumstances under which the linear-quadratic specification of the martingale approximation is likely to be the poorest. In this vein, the labor supply elasticity might not be constant in wartime.

The results we present may be given an interpretation that distinguishes this paper from previous work. The empirical evidence clearly rejects the martingale hypothesis and instead points to a political business cycle: deviations from the null hypothesis of a martingale occur along certain electoral frequencies. The behavior of the sample spectral density of first-differences at the  $\frac{\pi}{4}$  frequency (an eight-year cycle) significantly departs from the hypothesized spectral density of white noise at that frequency. The calendar dates of these deviations correspond to the two years prior to a successful re-election bid for the presidency, and regression analysis reveals that taxes are regularly lowered at these points in time. This departure from the martingale null appears to be inconsistent with the more sophisticated versions of the basic model. In particular, because the Lucas-Stokey model suggests that tax changes only occur when government spending requirements alter the current elasticity of the supply of labor, and because there seems to be little reason to believe that there is pronounced movement in the labor supply elasticity at a  $\frac{\pi}{4}$  frequency, our results are difficult to reconcile fully with their theory. Instead, we interpret the findings as evidence that the positive theory of government finance neglects an important determinant of tax rate changes; namely it abstracts from the political process that results in changes in government. Recent work by Cukierman and Meltzer [1986], Rogoff [1989], and Rogoff and Sibert [1988] suggesting that tax reductions occur prior to incumbent re-elections is correspondingly given substantial support by the evidence. Our conclusion is that attempts to model government policy as arising

out of the decisions of some unchanging, infinitely-lived social planner inherently neglect fundamental determinants of tax rate decisions. The evidence indicates a need to refine the positive theory of government finance to incorporate features of political equilibrium into the analysis.

The paper is organized as follows. Section 2 briefly describes the optimal tax model and discusses frequency-domain tests of the martingale theory of tax fluctuations. These tests largely reject the martingale hypothesis. In order to more closely pursue an interpretation for these rejections, Section 3 performs regression analysis on the tax rate time series. These tests again reject the model and strongly indicate that political factors have significant explanatory power for tax changes. Section 4 summarizes and concludes.

## 2. Testable Implications of the Martingale Hypothesis

The martingale property of optimal tax rates typically derives from an optimal control problem comparable to solving at each  $t$ :

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left[ -a_0 \tau_{t+j} - \frac{a_1}{2} \tau_{t+j}^2 \right], \quad 0 < \beta < 1 \quad (1)$$

by choosing  $\{\tau_t, B_{t+1}\}_{t=0}^{\infty}$  subject to the budget constraint

$$B_{t+1} = R[B_t + g_t - \tau_t], \quad R > 1 \quad (2)$$

$B_t$  bounded, for all  $t$ .

The expectations operator is present because the sequence  $\{g_t\}_{t=0}^{\infty}$  is an exogenous stochastic process describing the ratio of government spending to GNP.  $\tau_t$  is the tax collected as a percentage of GNP, and may be

interpreted as an average tax rate.  $B_t$  is the value of a one-period bond to be repaid in period  $t$ , also as a percentage of GNP, and  $R$  is the relative price of tomorrow's output. Notice that  $B_t$  is not indexed by states of nature, so that all debt is risk-free. This assumption seems reasonable in the context of a study of U.S. tax rates, because U.S. government bonds are similarly not fully state-contingent. The constraint that  $B_t$  is bounded is necessary to impose a requirement that the government cannot increase the debt to GNP ratio without limit into the infinite future. The objective function is identical to that employed by Barro [1981] and others, and may be justified by models in public finance where deadweight loss is proportional to the square of the tax rate. The Euler equation that follows from this maximization problem implies:

$$E_t(\tau_{t+1}) = \alpha(a_0, a_1, [\beta R]^{-1}) + [\beta R]^{-1} \tau_t \quad (3)$$

In this formulation, when  $[\beta R]^{-1}=1$ , it can be shown that  $\alpha(a_0, a_1, [\beta R]^{-1})=0$ , and hence that tax rates obey a martingale. Condition (3) can be regarded as the null hypothesis of the dynamic optimal taxation problem as it has been traditionally treated.

We now examine the properties of the tax rate series to see whether the martingale hypothesis is consistent with the data. An implication of our empirical exercises is that evidence in the empirical literature in favor of the hypothesis may have been generated by examining a narrow subset of the potential alternative hypotheses.

Our empirical work concentrates on frequency domain-based hypothesis tests applied to the history of tax changes. While there is a mapping from the time domain autocorrelations to the spectral density, frequency domain techniques permit a more straightforward decomposition of a time series into long versus short run components.



We find this decomposition particularly instructive because there are alternative hypotheses to the underlying model for which one-step ahead tax rates are likely to be well-predicted by current levels. Indeed, any model that implies substantial smoothing of taxes will have this result. The likely violations of the martingale properties might then be associated with longer-run mean reversion, and regression analysis that presents evidence that short-run autocorrelations are insignificantly different from zero will not be a powerful test of the null hypothesis. Moreover, the frequency domain analysis permits us to search for this reversion without asserting any prior knowledge of which autocorrelations are important, as is necessary in standard regression analyses. In searching for departures from the martingale specification over the entire range of frequencies, we are able to locate deviations without presuming to know the specific frequencies at which these deviations occur. While our conclusions do not uniformly indicate rejection, they do suggest that previous time-domain empirical tests have overlooked violations because of an exclusive focus on low-order autocorrelations.

We take as the null hypothesis that the series is a non-degenerate martingale, and then attempt to identify where the null hypothesis fails. An obvious location to search for a violation is the value of the spectral density of the differenced series at the zero frequency. Under the null hypothesis of a martingale, the value of the spectral density of the differenced series at the zero frequency should be one (appropriately normalized). Thus, a straightforward first check of the martingale hypothesis comes from an examination of the value of the spectral density at the zero frequency. Under the alternative hypothesis that the tax series is stationary, the spectral density of the differenced series should be zero at the zero frequency. More generally, since the martingale theory imposes restrictions at every frequency, we develop

tests that capture deviations of the entire spectral density from its hypothesized functional form. If the tax rate series is a martingale, then the value of the spectral density of first-differences should be a constant across all frequencies.

Second, since all of the autocorrelations of the differenced tax series must be zero for the martingale theory to be true, an appropriate diagnostic is whether the approximation is good over all frequencies. The cumulated periodogram tests we perform are well suited for this question because they explore the implications of deviations from the null hypothesis implicitly averaging over the deviations at all frequencies. These tests are general goodness-of-fit procedures which are consistent against *all* deviations from the null. Periodogram tests are thus more appropriate as a measure of the utility of the martingale approximation than a low-order autocorrelation test since all implications of the null are simultaneously explored. The periodogram tests we perform predominantly reject the martingale null.

The first set of results in this section are complementary to those of Sahasakul [1986] who found that there is some predictability of tax changes from wars and recessions. In admitting regressors beyond the univariate history of the time series under examination, one always worries about spurious correlations, since the size of each test is 5%. By looking only at autocorrelations, our research is disciplined against excessive rejections. Our nonparametric approach shows that the deviations from the null hypothesis found by Sahasakul are also embedded in the history of tax changes. However, the violations that we discover are different in kind than those located by Sahasakul, and are attributable to political factors that recur with regular periodicity.

The decomposition of a time series into long and short run components is best expressed in the frequency domain. If tax rates obey a martingale, then the first difference in tax rates,  $\Delta\tau_t$ , will be stationary.

Recall that any stationary time series such as  $\Delta\tau_t$ , will possess the Cramér Representation:

$$\Delta\tau_t = \int_{-\pi}^{+\pi} e^{-it\omega} dz(\omega) \quad (4)$$

where  $dz(\omega)$  is an orthogonal random variable such that the spectral density of  $\Delta\tau$  fulfills

$$f_{\Delta\tau}(\omega) = E(dz(\omega)dz(\omega)^H) \quad (5)$$

where superscript  $H$  denotes complex conjugate.

In this representation, the values of  $\omega$  near zero represent the long run characteristics of the series whereas the values of the spectral density near  $-\pi$  or  $\pi$  represent the short run components of the time series. To see this, notice that at low frequencies, the cosine wave  $\cos(t\omega)$  varies slowly with  $t$ . Thus, at low values of  $\omega$ , only large differences in  $t$  will yield changes in the series. Since by the Cramér Representation any time series can be expressed as the sum of random sinusoidal terms, the variances of the different components contribute differing amounts to the variance of the entire time series. To say that a time series is white noise is to say that the long and short run components of the time series contribute equally to the variance, i.e. that the spectral density is a rectangle over the interval  $[-\pi, \pi]$ . This interpretation is consistent with the representation of the spectral density as the Fourier transform of the autocovariance function,  $\sigma_{\Delta\tau}(j)$ ,

$$\begin{aligned} f_{\Delta\tau}(\omega) &= (2\pi)^{-1} \sum_{j=-\infty}^{\infty} \sigma_{\Delta\tau}(j) e^{-ij\omega} \\ &= \frac{\sigma_{\Delta\tau}^2}{(2\pi)} \text{ for white noise } \Delta\tau_t. \end{aligned} \quad (6)$$

Testing for the martingale property of taxes is equivalent to testing for the whiteness of the tax rate changes,  $\Delta\tau_t$ . We will examine the sample spectral density of the time series of tax rate changes. Sample spectral density estimates are normally based upon the periodogram,  $I_T(\omega)$ :

$$I_T(\omega) = (2\pi)^{-1} \sum_{j=-(T-1)}^{T-1} \hat{\sigma}_{\Delta\tau}(j) e^{-ij\omega} \quad (7)$$

where

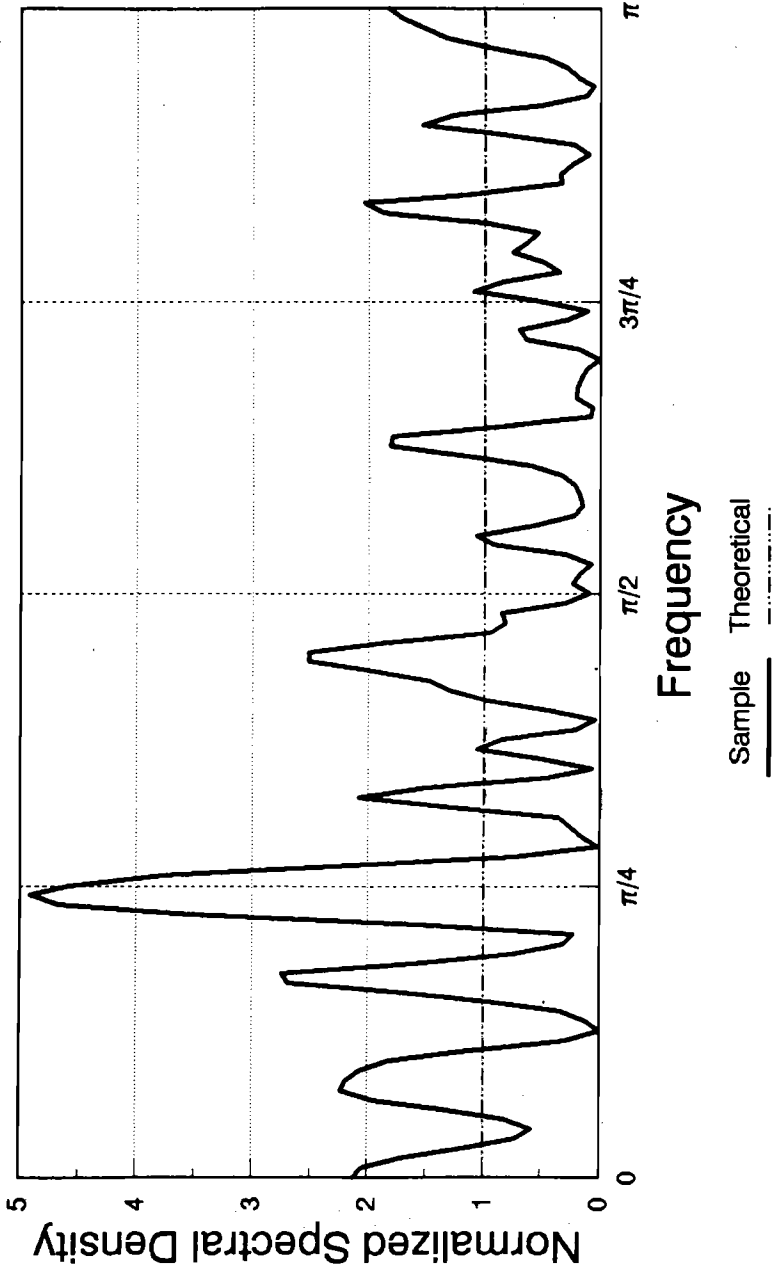
$$\hat{\sigma}_{\Delta\tau}(j) = T^{-1} \sum_{t=1}^{T-j} \Delta\tau_t \Delta\tau_{t+j} \quad (8)$$

Our empirical analysis is based upon an annual time series running from 1879-1986 on total taxes collected by the Federal government as a percent of GNP. The definition of taxes includes all revenues collected by the Federal government minus transfers from the Federal Reserve to the U.S. Treasury. This series is an extended version of that used in Barro [1981].

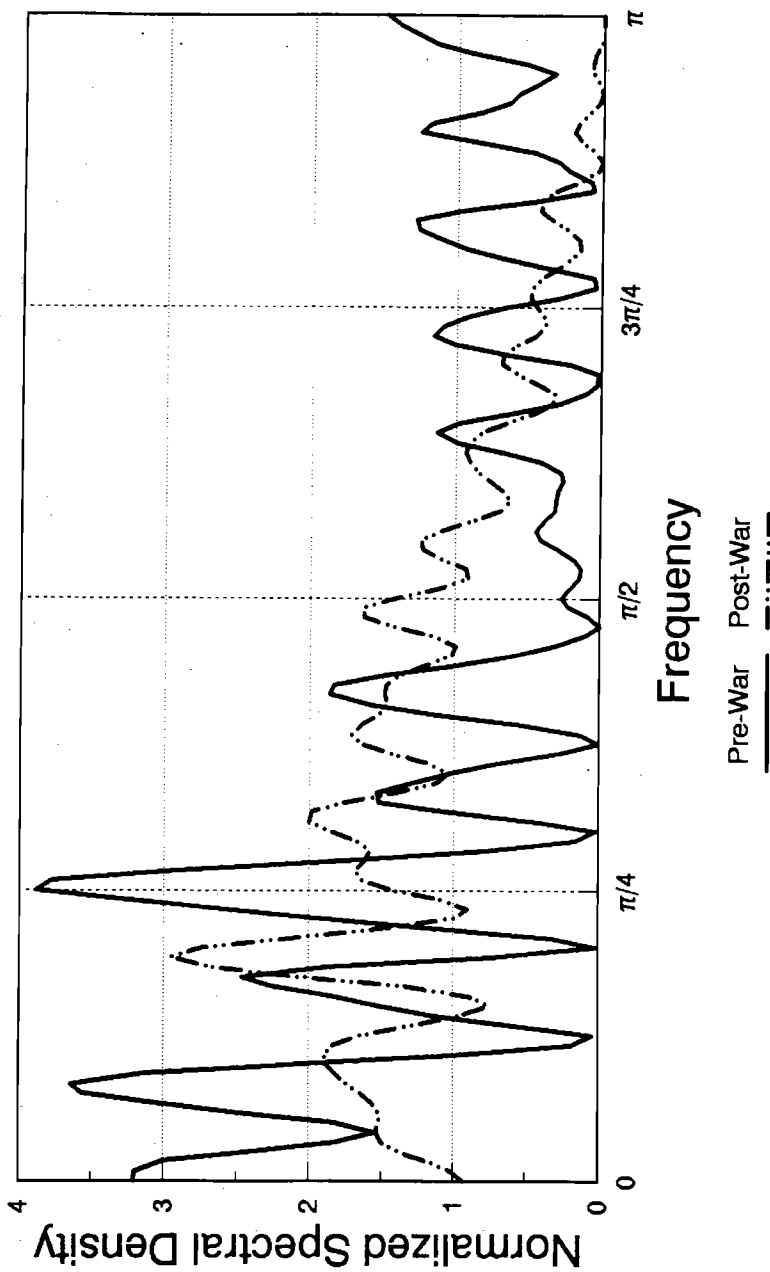
Figure 1 presents the periodogram of the tax rate changes. Superimposed on the periodogram is the theoretical spectral density under the null hypothesis. The periodograms reflect evidence of departures from the rectangular null, especially around the frequency  $\frac{\pi}{4}$ . In addition, there appears to be some concentration of power around the lower half of the frequencies. It is difficult to see any other significant departures from white noise.

Figure 2 contrasts the periodograms for tax changes pre- and post-World War II. The pre-war periodogram exhibits substantially more pronounced peaks and troughs. In addition, the pre-war periodogram is relatively more concentrated in the low frequencies. The casual evidence indicates that the pre-war data are primarily responsible for violations of the null.

**FIGURE 1**  
**Sample and Theoretical Periodograms**  
**Tax Rate Changes**



**FIGURE 2**  
**Periodogram Estimates**  
Pre-War and Post-War



In order to formalize the idea that the sample spectral density should be shaped like a rectangle, we need to develop some properties of the periodogram when treated as a random function. A standard result in spectral analysis shows that the periodogram is an inconsistent estimate of the spectral density at individual frequencies. However, our concern is not with individual frequencies but rather with the entire spectral density. Under the null hypothesis that the time series is white noise, the properties of the periodogram as a whole may be exploited even when the individual frequencies cannot be estimated consistently. The requirement leads us to consider behavior of the sample spectral distribution function. We consider the periodogram-based estimate of the spectral distribution function, when normalized so as to define a function on the interval  $[0,1]$ .

$$F_T(t) = \int_0^{\pi t} I_T(\mu) d\mu \quad t \in [0,1] \quad (9)$$

As the integral of a rectangle is a diagonal line, all testable implications of the null hypothesis may be reexpressed by the requirement that the sample spectral distribution function is approximately shaped as a diagonal.

Our analysis tests the shape of the complete spectral distribution function. A justification for our hypothesis tests of spectral shape is in Durlauf [1990a]. These tests examine the behavior of the deviations of the sample spectral distribution from the null hypothesis diagonal. To do this, we need to employ a Brownian Bridge, denoted as  $U(t)$  for  $t \in [0,1]$ .<sup>2</sup> The Brownian Bridge is a tied-down Brownian Motion process (i.e. its endpoints are zero). Its widest use comes in the analysis of empirical distribution functions. We employ the following Theorem

---

<sup>2</sup>See Shorack and Wellner [1987] for discussion of the Brownian Bridge.

which is valid for any process fulfilling the Hannan and Heyde [1972] conditions for asymptotic normality of sample autocorrelations.<sup>3</sup>

*Theorem.* If  $\Delta\tau_t$  is a uniformly bounded martingale difference sequence, then

$$(a) \quad U_T(t) = \sqrt{2} T^{1/2} \int_0^{\pi t} \left( \frac{I_T(\mu)}{\hat{\sigma}_{\Delta\tau}^2} - \frac{1}{2\pi} \right) d\mu \Rightarrow U(t) \quad \text{on } t \in [0,1]$$

$$(b) \quad CVM_T = \int_0^1 U_T^2(t) dt \Rightarrow \text{the Cramér-von Mises statistic}$$

$$KS_T = \sup_{[0,1]} |U_T(t)| \Rightarrow \text{the Kolmogorov-Smirnov statistic}$$

$$K_T = \sup_{[0 \leq s, t \leq 1]} |U_T(t) - U_T(s)| \Rightarrow \text{the Kuiper statistic}$$

$$R_T = \sup_{[0 < \alpha, 1]} \left| \frac{U_T(t)}{t} \right| \Rightarrow \text{the Renyi statistic}$$

(c)  $CVM_T$ ,  $KS_T$ ,  $K_T$ ,  $R_T$  all diverge if  $\Delta\tau_t$  is any other MA process satisfying the conditions of the Theorem.

*Pf.* Durlauf [1990a].

The idea of the Theorem is as follows. Under the null hypothesis, the cumulated periodogram of the time series should approximate a diagonal. If one normalizes the periodogram by the estimated variance, the cumulated deviations must sum to zero since the integral of the periodogram must equal the sample variance. A particular normalization of the deviations, under the martingale null, converges to

<sup>3</sup>The specific conditions developed by Hannan and Heyde are technical and are omitted. The conditions permit a great deal of heterogeneity in the data generating process, and impose weak restrictions on the tax rate series.



a Brownian Bridge. The consistency of the test occurs because under the null, these deviations will be bounded almost surely, whereas under the alternative they will explode, because of the  $T^{1/2}$  term which blows up the deviations of the cumulated periodogram from the null hypothesis.

Testing the shape of a random function is less straightforward than testing the value of a scalar random variable. The four different statistics in the Theorem provide various metrics for this shape relative to the martingale null. One advantage of these tests is that asymptotically they possess power 1 against all fixed MA alternatives, unlike time domain tests which examine a subset of the autocorrelations and correspondingly may be inconsistent.

Table 1 gives the results of calculating the four statistics on the annual tax data from 1879 to 1986. We focus first on the entire sample period, and then include values for the test statistics applied to a pre- and post-war subsample.

By these metrics, one overall rejects the white noise null hypothesis. Three of the four statistics consistently reject for the entire sample period. For the two subsamples, the evidence is apparently stronger against the null for the pre-war rather than the post-war period. For the post-war data only one of the four statistics, the Cramér-von Mises statistic, is inconsistent with the null hypothesis at the asymptotic significance level.<sup>4</sup> From inspection of the periodogram in Figure 1, the source of the rejections for the entire sample appears to be the concentration of large values of the periodogram in the low frequencies. If we think of the cumulated periodogram deviations as the realization of a Brownian bridge, then the volatility and height of the periodogram should not differ between the intervals  $[0, \frac{\pi}{2}]$  and  $[\frac{\pi}{2}, \pi]$ . The estimates

---

<sup>4</sup>The finite sample properties of the periodogram shape tests are relatively unknown, but simulation evidence in Bernard [1989] and Durlauf [1990a,b] suggests that the statistics will not lead to excessive (relative to 5%) rejections of the null.

Table 1

Periodogram-Based Tests of Model Noise

		$CVM_T$	$KS_T$	$K_T$	$R_T^\psi$
Entire Sample	(1879-1986)	0.81*	1.59*	1.65	11.2*
Pre-War	(1879-1945)	0.95*	1.63*	1.65	11.3*
Post-War	(1946-1986)	0.65*	1.33	1.34	5.50
5% Crit. Value		0.46	1.36	1.75	7.0
1%		0.74	1.64	2.01	8.5
0.1%		1.16	1.96	2.32	10.5

<sup>$\psi$</sup>  computed for  $\alpha=0.1$ . \* denotes significant at asymptotic 5% level.  
(sources: Shorack and Wellner [1987] and Owen [1962]).

of  $CVM_T$  are especially sensitive to this imbalance.

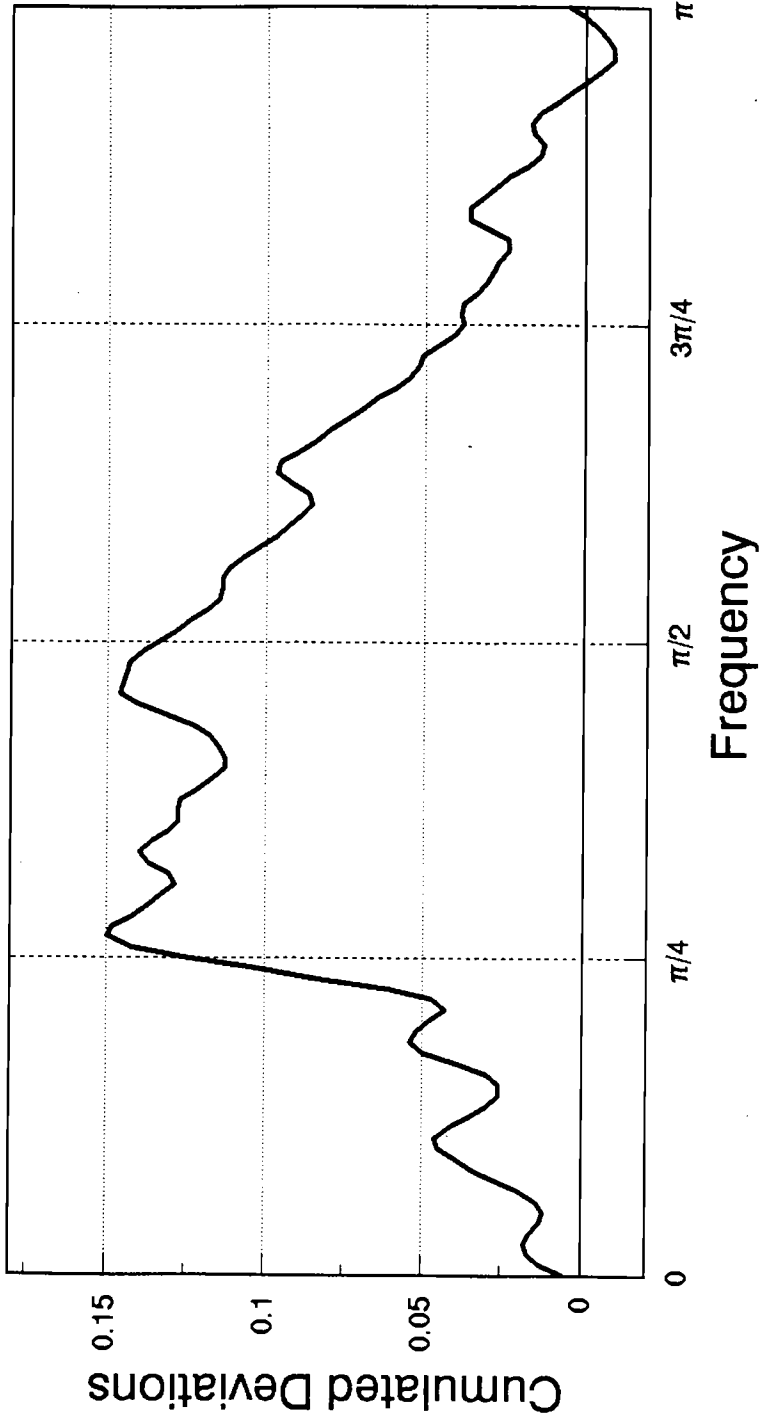
The acceptance of the null hypothesis by the Kuiper statistic is not surprising, given the nature of deviations of the cumulated periodogram from a diagonal line, as illustrated in Figure 3. The deviations are almost never negative, which means that the deviation between the maximum and minimum of the series is essentially equal to the maximum. Therefore,  $KS_T$  and  $K_T$  take on similar values with different significance levels. The Kuiper statistic has lower power in relation to the other statistics against the alternative of spectral density concentration in the low frequencies. As noted above, the periodogram reveals some concentration of power around the frequency  $\frac{\pi}{4}$ . Interestingly, this is the frequency which corresponds to a cyclical component to the time series with period of eight years. Eight years is the most common length of time for continuous political party control of the Presidency, and therefore is a natural frequency to examine the alternative hypothesis that political parties exert influence on the intertemporal tax smoothing calculus. Similarly,  $\frac{\pi}{2}$  corresponds to the frequency of Presidential elections, and thus is also of interest. Of course, we are also interested in mean reversion, so that the zero frequency is of interest. Our second set of tests therefore examine these "electoral" frequencies and the zero frequency.

In order to construct test statistics based upon the spectral density at particular values, it is customary to use a window estimator in place of the (inconsistent) periodogram estimate. For this purpose, we employ a Bartlett window, which modifies the periodogram by weighting high-order autocovariances towards zero. The procedure, described in Priestley [1981], is to define weights of the form

$$\lambda(j) = 1 - \frac{|j|}{M} \quad |j| \leq M, \quad 0 \quad \textit{otherwise} \quad (10)$$

**FIGURE 3**  
**Cumulated Periodogram Deviations**

Sample Minus Theoretical



where  $M$  is an increasing function of  $T$ . Multiplying the autocovariances by  $\lambda(j)$  produces a spectral density estimate  $B_T(M, \omega)$ :

$$B_T(M, \omega) = (2\pi)^{-1} \sum_{j=-M}^M \left(1 - \frac{|j|}{M}\right) \hat{\sigma}_{\Delta\tau}(j) e^{-ij\omega} \quad (11)$$

If  $\frac{M}{T} \Rightarrow 0$ , these modified estimates will be consistent under the null hypothesis, and when normalized are asymptotically normally distributed. Figure 4 illustrates various Bartlett estimates of the spectral density.

If taxes are stationary about some mean, then the spectral density of the first difference equals zero at the zero frequency. Under the martingale null,  $2\pi$  times the value of the spectral density at the zero frequency will equal one. Cochrane [1988] and Lo and MacKinlay [1987a] among others have interpreted zero frequency values below one as evidence of long run mean reversion. From the perspective of the utility of the martingale approximation, one would expect this test to reject the null.

Table 2 surprisingly reveals little evidence of mean reversion.<sup>5</sup> Even at 60 lags, there is no evidence against the null hypothesis of a martingale. The basic difficulty apparently is that the data series is too short to yield useful information on long run mean reversion. This problem is endemic to many empirical questions, as discussed in Cochrane [1988]. Unfortunately, the confidence intervals of the test statistic at the zero frequency are so large that we cannot reject the martingale null at this frequency.

Our next set of tests explore the behavior of the spectral density at electoral cycles. Table 3 reports the estimates of the spectral density

---

<sup>5</sup>In reporting estimates of the spectral density at selected frequencies, we use the normalized Bartlett estimates (i.e.  $B_T(M, \omega)$  is calculated using sample autocorrelations). We have also multiplied the estimates by  $2\pi$  for ease of interpretation.

**FIGURE 4**  
**Bartlett Window Estimates**  
10, 30, and 60 lags

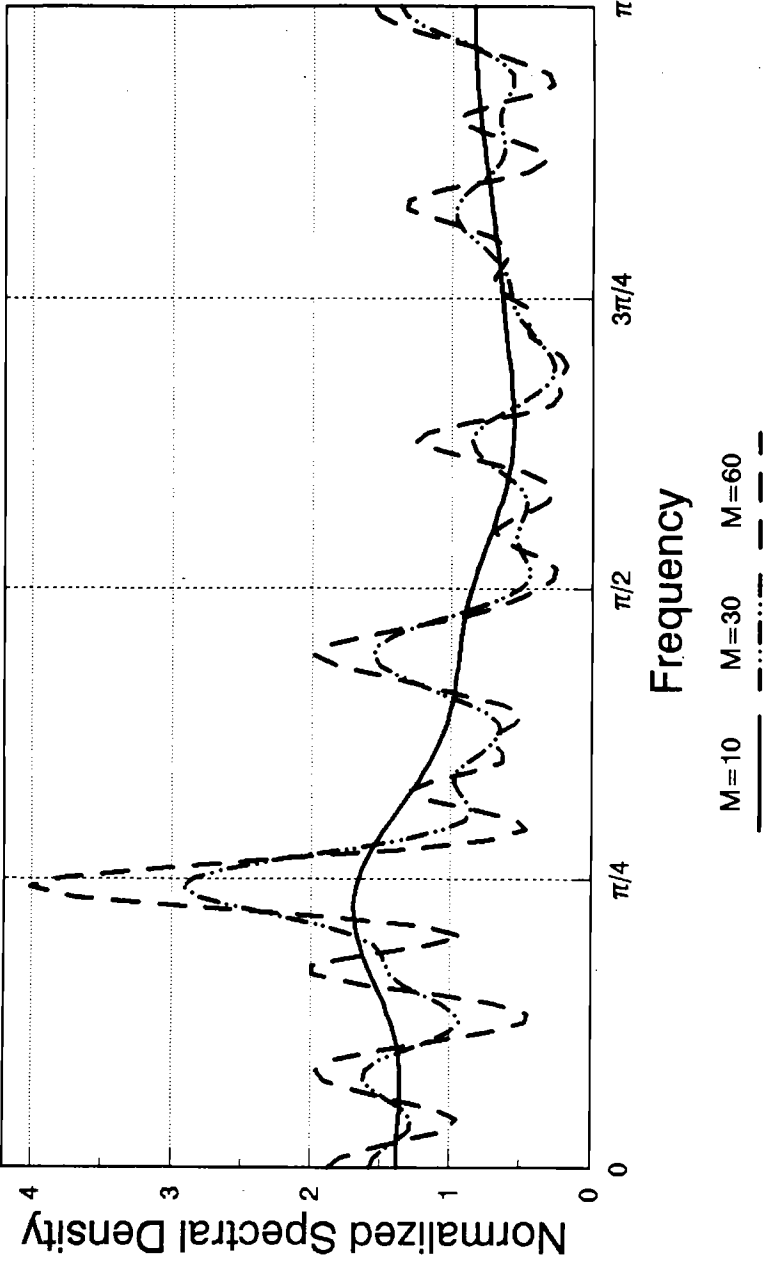


Table 2

Bartlett Estimates at Zero Frequency

Window Size( $M$ )	$2\pi B_{\Gamma}(M,0)$	$H_0: 2\pi f(0)=1$
10	1.38 (0.35)	1.09
20	1.47 (0.50)	0.94
30	1.57 (0.61)	0.93
40	1.72 (0.71)	1.01
50	1.83 (0.79)	1.05
60	1.87 (0.86)	1.01

Standard errors are in parentheses. Column 2 reports t-statistic associated with  $H_0$ .

Table 3

Bartlett Estimates at "Electoral" Frequencies

Window Size( $M$ )	$2\pi B_T(M, \frac{\pi}{4})$	$H_0: 2\pi f(\frac{\pi}{4})=1$	$2\pi B_T(M, \frac{\pi}{2})$	$H_0: 2\pi f(\frac{\pi}{2})=1$
10	1.66 (0.25)	2.64*	0.86 (0.25)	-0.56
20	2.25 (0.35)	3.57*	0.73 (0.35)	-0.77
30	2.85 (0.43)	4.30*	0.50 (0.43)	-1.16
40	3.34 (0.50)	4.68*	0.42 (0.50)	-1.16
50	3.60 (0.56)	4.64*	0.39 (0.56)	-1.09
60	3.78 (0.61)	4.56*	0.37 (0.61)	-1.03

Standard errors are in parentheses. Columns 2 and 4 report t-statistics associated with  $H_0$ . \* indicates significant at asymptotic 5% level.



at the  $\frac{\pi}{2}$  and  $\frac{\pi}{4}$  frequencies. The Table has two noteworthy features. First, we find no evidence that the spectral density at the  $\frac{\pi}{2}$  frequency deviates from a value of one. In the language of political business cycles, we do not find a systematic relationship between tax changes and elections. However, at the  $\frac{\pi}{4}$  frequency, we find strong evidence that the null hypothesis of a martingale is uniformly rejected. This result may be tentatively interpreted as saying that there is a tendency towards mean reversion across political party administrations. This result makes intuitive sense since historically, presidential administrations are most active in the early years of the first term and these parties typically remain in office for an eight year tenure. Since the beginning of the sample, there have been twenty-eight elections, but only twelve changes in party control, implying an average political party change roughly every other election. Indeed, with the exception of the Carter presidency, since World War II every administration has been a two-term presidency.

### *3. Regression Analysis*

The higher concentration of tax changes at a frequency of eight years provocatively suggests that political events account for significant variability in tax rates. To investigate this possibility more closely, we attempt to identify the source of these tax changes. Unfortunately, spectral analysis is not capable of identifying these linkages because, while it reveals that there is pronounced activity every eight years, it does not reveal *which* eight year intervals are critical. Regression analysis is more properly suited for this task. Thus, our final examination of the behavior of tax rates consists of identifying in

calendar time the eight year cycle responsible for the spike in the spectral density at  $\frac{\pi}{4}$ , and then ascertaining whether these dates have a political interpretation.

Table 4 presents the results of a regression of tax changes on different eight year cyclical dummy variables. Each dummy variable is given a value of one every eight years: the variables are distinguished by the consecutive calendar dates in which they turn "on". Thus,  $D1$  attains a value of one beginning at the start of the sample, 1879, 1887,...,1983.  $D2$  attains a value of one in 1880, 1888,..., 1984,..., and  $D8$  attains a value of one in 1886, 1894,..., 1982. Entering each of the eight dummy variables separately, we ran regressions of the form

$$\Delta\tau_t = C + \beta_1 GF_{t-1} + \beta_2 R_{t-1} + \beta_3 Di_t + \epsilon_t \quad i=1,\dots,8. \quad (12)$$

The regressors include a constant, a government spending variable ( $GF$  is the ratio of federal government spending to GNP, as used in Barro [1981]), and a recession dummy ( $R$  is one if there was a recession that year according to NBER business cycle dates).<sup>6</sup> These variables are designed to control for violations of the null not associated with the eight year cycle. According to the null hypothesis, the error term of this regression is a function of the contemporaneous shock to the government spending ratio variable. Thus, the error term can deviate from its mean from either a contemporaneous shock to the level of government spending or from a contemporaneous shock to GNP, so that both  $GF$  and  $R$  must be lagged in the regression.<sup>7</sup>

Table 4 reveals that only  $D1$  and  $D4$  are significant at

---

<sup>6</sup>Sahasakul [1986] found a temporary government spending variable and business cycle movements to be significant in a test of the optimal tax hypothesis.

<sup>7</sup>See Sargent [1987].

Table 4

Regression Estimates of Eight Year Cycles

Dummy Variable	<i>GF</i> (-1)	<i>R</i> (-1)	<i>D</i>	<i>Sig. level</i>
<i>D1</i>	.007 (.012)	-.004 (.002)	.006 (.003)	.07
<i>D2</i>	.009 (.011)	-.004 (.002)	-.002 (.003)	.53
<i>D3</i>	.008 (.011)	-.004 (.003)	.001 (.005)	.87
<i>D4</i>	.009 (.013)	-.004 (.003)	-.008 (.003)	.02
<i>D5</i>	.008 (.011)	-.004 (.003)	-.002 (.003)	.55
<i>D6</i>	.008 (.011)	-.004 (.002)	-.003 (.003)	.40
<i>D7</i>	.009 (.011)	-.004 (.002)	.003 (.003)	.36
<i>D8</i>	.009 (.010)	-.003 (.002)	.005 (.003)	.11
<i>POLDUM</i>	.001 (.001)	-.004 (.003)	-.008 (.003)	.02

Standard errors in parentheses. The last column is the marginal significance level of the t-statistic for exclusion of the dummy variable.

approximately the five per cent level. These variables are prominent in two respects: they coincide with wartime build-ups and they often coincide with the year or two preceding a presidential re-election bid. The second possibility is consistent with models of electoral competition that conclude that taxes should be manipulated prior to re-election by incumbents in order to signal competence (Rogoff [1989], Rogoff and Sibert [1988], Cukierman and Meltzer [1986]). In order to investigate this link more directly, we create a final dummy variable, *POLDUM*, that takes a value of one two years prior to a successful re-election bid (where success is defined as maintaining party control of the presidency), and is zero otherwise. One's prior expectations for the political dummy must be low since the variable restricts attention to the two year lead on a re-election bid. In doing so, the variable imposes greater temporal structure on the political influence on tax rates than models of electoral competition generally assert. Put another way, models of electoral competition could be an important driving force in tax rate changes even if tax rate changes occur sometimes at one year leads to a re-election bid, and other times at two year leads. Nevertheless, the results for *POLDUM* are striking: the variable is significant at the 2 percent level and has the negative sign predicted by the electoral competition literature: tax burdens are reduced prior to successful re-election bids.

#### *4. Conclusions*

This paper has been designed to study the claim in the public finance literature that tax rates follow a random walk. We have attempted to reconcile our evidence against this theory of government finance with the mixed empirical support that the martingale hypothesis

has received in the literature. A reconciliation can be achieved by recognizing that previous tests restricted attention to short-run autocorrelations, whereas the null hypothesis imposes restrictions on all autocorrelations. Tests robust to violations at all frequencies reveal that the martingale approximation is not strong for the overall shape of the spectral distribution function (the presence of long run mean reversion cannot, however, be confirmed). The approximation is especially poor at one of the electoral seasonals. This evidence points compellingly to the hypothesis that tax rates contain a predictable component corresponding to electoral events. Specifically, there appears to be a cyclical component to tax changes with a period of 8 years, which roughly corresponds to changes in political party administration. Regression analysis further suggests that taxes are reduced two years prior to successful presidential re-election attempts.

In terms of future research, our evidence on behalf of an electoral seasonal in taxes suggests that the interaction of taxes with political considerations should receive greater scrutiny, and that the positive theory of tax rate determination should be modified to incorporate important elements of political equilibrium. Government financial decisions are made by political regimes, and these regimes change over time. The regularity of this phenomenon induces a distinctive regularity in the time series behavior of tax rates, despite the fact that tax rates appear otherwise quite smooth over time. A more careful theoretical delineation of the relationship between political competition, political change, and tax rate determination would therefore seem valuable, and would lead to a more accurate description of the actual evolution of tax rates.

## *Bibliography*

- Barro, R. [1979]. "On the Determination of Public Debt." *Journal of Political Economy*, 87, 940-71.
- \_\_\_\_\_ [1981]. "On the Predictability of Tax Rate Changes." *NBER Working Paper 636*.
- Bernard, A. B. [1989]. "Testing for Random Walks in Exchange Rates." *Working Paper, Stanford University*.
- Bizer, D. S. and S. N. Durlauf. [1990]. "Martingale Approximations and Statistical Inference." *Working Paper in progress, Stanford University*.
- Cochrane, J. [1988]. "How Big is the Random Walk in GNP?" *Journal of Political Economy*, 96, 893-920.
- Cukierman, A. and A. Meltzer. [1986]. "A Positive Theory of Discretionary Policy, the Cost of Democratic Government and the Benefits of a Constitution." *Economic Inquiry*, 101, 367-388.
- Durlauf, S. N. [1990a]. "Spectral Based Testing of the Martingale Hypothesis." *Working Paper, Stanford University*.
- \_\_\_\_\_ [1990b]. "Time Series Properties of Aggregate Output Fluctuations." *Working Paper, Stanford University*.
- Hall, R. E. [1978]. "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence." *Journal of Political Economy*, 86, 971-987.
- Hannan, E. J. and C. C. Heyde. [1972]. "On Limit Theorems of Quadratic Functions of a Discrete Time Series." *Annals of Mathematical Statistics*, 43, 2058-2066.
- Judd, K. L. [1989]. "Optimal Taxation in Dynamic Stochastic Economies: Theory and Evidence." *Working Paper, Hoover Institution*.
- Kingston, G. H. [1984]. "Efficient Timing of Taxes." *Journal of Public Economics*, 24, 271-80.
- Lucas, R. E. and N. Stokey. [1983]. "Optimal Fiscal and Monetary Policy in an Economy Without Capital." *Journal of Monetary*

*Economics*, 12, 55-93.

Mankiw, N. G. [1987]. "The Optimal Collection of Seignorage: Theory and Evidence." *Journal of Monetary Economics*, 20, 327-41.

Owen, D. B. [1962]. *Handbook of Statistical Tables*. Reading, Mass.: Addison-Wesley.

Phillips, P. C. B. [1987]. "Time Series Regression with a Unit Root." *Econometrica*, 55, 277-302.

Poterba, J. M. and J. J. Rotemberg. [1988]. "Inflation and Taxation with Optimizing Governments." *NBER Working Paper 2567*.

Priestley, M. B. [1981]. *Spectral Analysis and Time Series*. New York: Academic Press.

Rogoff, K. [1989]. "Elections and Political Budget Cycles." *American Economic Review*, forthcoming.

\_\_\_\_\_ and A. Sibert. [1988]. "Elections and Macroeconomic Policy Cycles." *Review of Economic Studies*, LV, 1-16.

Sahasakul, C. [1986]. "The U. S. Evidence on Optimal Taxation Over Time." *Journal of Monetary Economics*, 18, 251-75.

Sargent, T. J. [1987]. *Macroeconomic Theory*. New York: Academic Press.

Shorack, G. R. and J. A. Wellner. [1987]. *Empirical Processes with Applications to Statistics*. New York: John Wiley.