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ABSTRACT

This paper argues that the home, or nonmarket, sector is empirically large, whether measured in terms of the time devoted to household production activities or in terms of the value of home produced output. We also argue that there may be a good deal of substitutability between the market and nonmarket sectors, and that this may be an important missing element in existing macroeconomic models. We pursue this within a framework that labor economists have studied for some time. Symmetrically with the market, household production uses labor and capital to produce a nonmarket consumption good according to a possibly stochastic technology. We show any model with home production is observationally equivalent to another model without home production, but with different preferences. However, for a given set of preferences, incorporating household production can dramatically change the nature and the interpretation of several macroeconomic phenomena. As an example, we show that it is possible to have involuntary unemployment and normal leisure at the same time in models with home production, something that cannot arise in models without it. As another example, we discuss how home production affects the interpretation of models with consumer durables.

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I. Introduction

A standard assumption in many models of the labor market, and especially aggregate (macroeconomic) models, is that time has exactly two uses: market work and leisure.[1] This implies that individuals who do not work in the market, for whatever reason, must be enjoying leisure - which is patently false, as any homemaker could attest. The figures in Table 1, derived from Hill's (1985) analysis of the Michigan Time Use Survey, indicate that an average household consisting of a married couple spends about 57 hours per week in market work and 49 hours working in the home. As a fraction of "discretionary time" (market work plus homework plus leisure), market work amounts to 33 percent, while homework is only slightly less, at 28 percent. Notice also that leisure is roughly the same for married males and females, despite large differences in amount of market work.

Table 1: Time Use

Activity (hrs/wk)	Married Male	Married Female	Married Couple
Market work	40.18	16.73	56.91
Home work	14.25	34.85	49.10
Leisure	33.37	34.48	67.85
Sleep and other	80.20	81.94	162.14

Complementary to these data are studies that have attempted to measure the value of home produced output. Hawrylyshyn (1976), for example, estimates that the output of the household sector corresponding only to married women amounts to approximately 35% of measured GNP. Gronau (1980)

[1] An obvious exception is the extremely useful literature on job search; we simply have nothing to say about the search model in this paper.

estimates that the value of home production associated only with married women in 1973 can exceed 70% of a family's market income after taxes. These figures do not include the contributions of unmarried individuals or married males. Several researchers, including Nordhaus and Tobin (1972), Zolotas (1981), Fraumeni and Jorgenson (1987), and Eisner (1988, 1989), have studied the issue of modifying the existing National Income and Product Accounts to include household production (as well as a variety of other factors). Eisner (1988) provides an excellent summary of this literature, and reports a range of estimates for the value of home production relative to measured GNP of 20 - 50 percent.

Although the exact size of the household sector is difficult to measure, even a conservative estimate of 35% of measured GNP is a large amount of economic activity to ignore. By way of comparison, manufacturing output is only about 40% of GNP, and yet the manufacturing sector is heavily studied by macroeconomists. As an alternative comparison, consumption in the standard National Income and Product Accounts is approximately 70% of output (this figure counts expenditures on durables as investment rather than consumption, and excludes the public and foreign sectors). Therefore, by ignoring the consumption of home produced output, macroeconomists are excluding a category of consumption that is half as big as the one that they are including!

These facts lead us to conclude that home production is an empirically significant entity at the aggregate level, whether we measure it in terms of its labor input or its output.[2] In light of this, why is it conspicuously absent from existing macroeconomic models? One possible conjecture is that, although the home sector is large, its behavior is approximately independent

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[2] Greenwood and Herkovitz (1990) argue that the home sector also uses a large amount of physical capital.

of the market sector. The data in Table 2, also from Hill (1985), suggest this conjecture is mistaken. The fact that individuals employed in the market sector spend much less time working in the home leads us to believe that there is, in fact, substantial substitutability between market and nonmarket activity. Notice, in particular, that individuals who are not employed in the market sector do enjoy more leisure, on average, but the difference in leisure is much less than the difference in time spent in market work.

Table 2: Time Use and Employment

Activity (hrs/wk)	Married Male Full Time Employed	Married Male Not Employed	Married Female Full Time Employed	Married Female Not Employed
Market work	48.62	6.60	39.08	3.22
Home work	12.70	20.01	24.58	40.90
Leisure	29.23	51.24	27.95	38.27
Sleep and other	77.45	89.34	76.39	85.61

Additional evidence on the substitutability between market and home production is provided by Rios-Rull's (1988) analysis of the Panel Study of Income Dynamics. He calculates hours of market and home work for a subsample of individuals in five wage groups, and some results from his study are shown in Table 3.[3] The important feature of these data is the way that individuals substitute between time in the market and in home production as the wage varies; for example, notice that especially for the

[3] The wage groups (in 1969 dollars) were:

$$1\text{–}[0,2),\ 2\text{–}[2,2.8),\ 3\text{–}[2.8,3.8),\ 4\text{–}[3.8,5.3),\ 5\text{–}[5.3,\infty).$$

Note that because his subsample excluded individuals who did not report positive hours in the market for at least four years, these numbers are not comparable to those in the previous tables.

upper wage groups, total work is roughly the same despite significant differences in the allocation of time to the two types of work. All of this taken together indicates that the home sector is not only large, but that there is a good deal of substitutability between it and the market.

Table 3: Time Use and Wages

Group Averages (work = hrs/wk)	Wage Group 1	2	3	4	5
Hourly wage	1.48	2.37	3.28	4.46	7.24
Years of education	11.18	11.97	12.73	13.00	14.30
Market work	21.38	29.92	34.52	36.92	38.63
Home work	12.46	11.19	8.94	6.73	5.02
Total work	33.85	41.12	43.46	43.65	43.65

The above evidence suggests that household production could be an important missing element in existing models of the aggregate economy. Our goal is to explore this possibility. Following Gronau (1977, 1985), we adopt a version of Becker's (1965) model in which each household has a home production function with time and (possibly) capital as inputs, and a nonmarket consumption good as output. Introducing this simple additional element into standard models will turn out to have fairly dramatic implications, in a variety of contexts.[4]

In this paper, we start by exploring some basic theoretical issues. We prove that any model with household production has a reduced form that is

[4] There has been some previous analyses of the implications of home production for macroeconomics, including Benhabib, Rogerson and Wright (1988), Rios-Rull (1988), and Greenwood and Herkovitz (1990). Becker's (1988) address to the American Economic Association also argues that home production is an important missing element in macroeconomics, although he stresses *family behavior*, while the focus in this paper is on the implications for standard macroeconomic variables, and we do not look at issues such as marriage, divorce, fertility, etc.

observationally equivalent to another model, without home production, but with agents having different preferences. Thus, it is always possible to replicate the behavior generated by the home production economy for market employment, market consumption, and so on, with an economy that has no home production sector, *if* preferences can be chosen arbitrarily. However, for a *given* set of preferences, the addition of household production can matter a lot. As an example, home production is incorporated into an economy with random layoffs resulting from nonconvexities, and we show how this affects the nature and interpretation of unemployment. One result is that we can have involuntary unemployment in reasonable specifications of this model, in contrast to models without home production where involuntary unemployment arises if and only if leisure is an inferior good (in a particular sense). We also discuss how home production affects the interpretation of recent empirical models that include consumer durables.

The project is organized as follows. In the next section, we introduce the basic assumptions and notation, discuss the mapping described above between models with and without home production, and work out some illustrative examples. In Section III, we show the basic points in these examples are fairly general. In Section IV, we discuss the economy with involuntary unemployment, while in Section V, we pursue dynamic issues, including consumer durables. Some conclusions are contained in Section VI. The basic message is that explicitly recognizing nonmarket economic activity changes qualitatively the way we think about a number of topics related to market activity. In a companion paper — Benhabib, Rogerson and Wright (1990) — we introduce home production into the stochastic growth model, or real business cycle model, in order to study how it affects the nature of aggregate fluctuations quantitatively.

II. The Basic Framework

We start with an underlying von Neumann - Morgenstern utility function, $U = U(c_m, c_n, h_m, h_n)$, defined over four objects: consumption of a market good (c_m), consumption of a home produced or nonmarket good (c_n), hours of work in the market sector (h_m), and hours of work in the home or nonmarket sector (h_n).[5] What makes this a model with home production is the assumption that c_n and h_n are nontradable. In particular, we impose the *home production constraint*, $c_n \le g(h_n)$, where $g(\cdot)$ is the home production function. This leads to the following decision problem

$$\begin{aligned} &\max U(c_m, c_n, h_m, h_n) \\ &\text{st } c_m \le x + wh_m,\ c_n \le g(h_n),\ \text{and } h_n + h_m \le H \end{aligned} \tag{2.1}$$

(ignoring nonnegativity constraints on c_j and h_j), where w is the real wage, x is exogenous endowment income, and H is the total endowment of time.[6]

Assume $U(\cdot)$ is strictly monotonically increasing in consumption and decreasing in labor, and that $U(\cdot)$ and $g(\cdot)$ are continuous. Then we can substitute the home production constraint into the utility function and maximize with respect to homework, taking as given the values of the market variables, to define the following function:

$$V(c_m, h_m) = \max_{h_n} U[c_m, g(h_n), h_m, h_n] \text{ st } h_n \in [0, H-h_m]. \tag{2.2}$$

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[5] A special case is when market and home variables are perfect substitutes, say $U = u(c_m + c_n, h_m + h_n)$; we would not want to assume this in general.

[6] According to Pollack and Wachter (1975), the "fundamental assumption" of the home production model is the imposition of the home production constraint, in addition to the standard budget constraint and the constraint that total time use cannot exceed the time endowment.

Additionally, as long as either $U(\cdot)$ or $g(\cdot)$ is strictly concave, we can define the *homework function* $h_n = h(c_m, h_m)$ to be the unique solution to the maximization problem in (2.2), and the *home consumption function* by $c_n = c(c_m, h_m) = g \circ h(c_m, h_m)$.

Substituting this into the underlying utility function, we have

$$V(c_m, h_m) = U[c_m, c(c_m, h_m), h_m, h(c_m, h_m)]. \tag{2.3}$$

One can think of $V(\cdot)$ as a *reduced form utility function*, defined over market quantities only. The following result demonstrates that $V(\cdot)$ inherits some basic properties of $U(\cdot)$, so that it in fact describes a well behaved preference ordering over c_m and h_m.[7]

Theorem 1: If $U(\cdot)$ and $g(\cdot)$ are continuous, strictly monotonic, and concave, then $V(\cdot)$ is continuous, strictly monotonic, and concave. If either $U(\cdot)$ or $g(\cdot)$ is strictly concave, then $V(\cdot)$ is strictly concave.

Proof: First, if $U(\cdot)$ and $g(\cdot)$ are continuous then so are $V(\cdot)$ and $h(\cdot)$, by the Theorem of the Maximum. Now choose $\hat{c}_m > \tilde{c}_m$ and $\hat{h}_m < \tilde{h}_m$, and define $\hat{h}_n = h(\hat{c}_m, \hat{h}_m)$ and $\tilde{h}_n = h(\tilde{c}_m, \tilde{h}_m)$. Then we have

$$\begin{aligned} V(\hat{c}_m, \hat{h}_m) &= U[\hat{c}_m, g(\hat{h}_n), \hat{h}_m, \hat{h}_n] \geq U[\hat{c}_m, g(\tilde{h}_n), \hat{h}_m, \tilde{h}_n] \\ &> U[\tilde{c}_m, g(\tilde{h}_n), \tilde{h}_m, \tilde{h}_n] = V(\tilde{c}_m, \tilde{h}_m). \end{aligned}$$

Hence, V is monotonic. To check concavity, choose $\lambda \in (0,1)$, and let $\bar{c}_m = \lambda \hat{c}_m + (1-\lambda)\tilde{c}_m$, $\bar{h}_m = \lambda \hat{h}_m + (1-\lambda)\tilde{h}_m$, and $\bar{h}_n = \lambda \hat{h}_n + (1-\lambda)\tilde{h}_n$. Then we have

[7] Given differentiability, some of the results in this theorem can be derived in an alternative way. For example, using the Envelope Theorem, we have $V_1 = U_1$ and $V_2 = U_3$, establishing monotonicity immediately.

$$
\begin{aligned}
V(\bar{c}_m,\bar{h}_m) &= U[\bar{c}_m,g(\bar{h}_n),\bar{h}_m,\bar{h}_n] \\
&\geq U[\bar{c}_m,\lambda g(\hat{h}_n)+(1-\lambda)g(\tilde{h}_n),\bar{h}_m,\bar{h}_n] \\
&\geq \lambda U[\hat{c}_m,g(\hat{h}_n),\hat{h}_m,\hat{h}_n] + (1-\lambda)U[\tilde{c}_m,g(\tilde{h}_n),\tilde{h}_m,\tilde{h}_n] \\
&= \lambda V(\hat{c}_m,\hat{h}_m) + (1-\lambda)V(\tilde{c}_m,\tilde{h}_m).
\end{aligned}
$$

Hence, V is concave. Furthermore, if either $U(\cdot)$ or $g(\cdot)$ is strictly concave, then one of the inequalities will be strict, so $V(\cdot)$ will be strictly concave. ■

The above discussion implies that decision problem (2.1) generates the same values of c_m and h_m as the problem without home production,

$$\max V(c_m,h_m) \text{ st } c_m \leq x + wh_m,\ h_m \leq H. \tag{2.4}$$

This is an important point. For example, consider a representative agent economy with home production function $g(h_n)$ and aggregate market production function $f(h_m)$. Its competitive equilibrium is characterized as the unique solution to the social planning problem

$$\max W = U[x+f(h_m),g(h_n),h_m,h_n] \text{ st } h_m+h_n \leq H \tag{2.5}$$

(again ignoring non-negativity constraints). The solution to (2.5) yields the same values for h_m and c_m as the solution to

$$\max W = V[x+f(h_m),h_m] \text{ st } h_m \leq H. \tag{2.6}$$

The economy with home production is therefore *observationally equivalent* to another economy, with no home production, but with different preferences.

Hence, there is a sense in which adding a home sector does not add to the set of outcomes that were possible without it. One might conclude, therefore, that the practice of ignoring nonmarket activity involves no loss in generality. Yet it is precisely because *preferences would have to be different if home production was excluded* that it turns out to be such a useful concept for understanding and interpreting economic phenomena. An obvious point is that it would be a mistake to interpret leisure as $H-h_m$, as specified in the reduced form, since in fact $H-h_m-h_n$ is the correct measure of leisure (and h_n may not be constant). A more subtle point is that the reduced form utility function is actually the offspring of an underlying utility function combined with a home production function, and this can lead to agents acting as if they had preferences quite different from their true preferences.

One important example of this principle is the following: the fact that leisure defined by $H-h_m-h_n$ is a normal good according to the underlying preference structure does not imply that leisure defined by $H-h_m$ is normal according to the reduced form structure. In other words, in contrast to the properties discussed in Theorem 1, a property of $U(\cdot)$ that does not carry over to $V(\cdot)$ is the wealth effect. Several interesting economic issues are known to hinge on this wealth effect.[8] By including home production, we are able to account for agents acting as if leisure is inferior, without violating the reasonable intuition or the long run evidence that it is normal. Even if the sign of the wealth effect is not necessarily reversed, we demonstrate in the next section that, under reasonable conditions, it is necessarily reduced. Hence, a model with home production can display a labor supply elasticity that would be difficult to generate using an empirically reasonable model in which home production is absent.

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[8] Examples include some perhaps surprising results, such as the effect of asymmetric information in implicit contract theory (see, e.g., Cooper 1987), or the issue of whether unemployment is voluntary or involuntary in a large class of models (see, e.g., Rogerson and Wright 1988, or Section IV below).

A second example of the principle is this: once we recognize that home production is important, we are forced to conclude that preferences defined over market variables should not be stationary, non-stochastic, functions. Consider the home production function $g(h_n) = s_n G(h_n)$, where s_n is a stochastic innovation to the household technology. The reduced form utility function then becomes

$$V(c_m,h_m,s_n) = \max_{h_n} U[c_m,s_n G(h_n),h_m,h_n] \text{ st } h_m+h_n \le H. \tag{2.7}$$

Preferences over c_m and h_m as represented by $V(\cdot)$ now depend on s_n. Hence, it can appear in the reduced form economy as if there is a stochastic shock in the utility function, even though true preferences are stable. Similarly, to the extent that innovations to the home technology are accumulating over time, it will appear in the reduced form that there is trend drift in preferences, even if $U(\cdot)$ is stationary.

A third example is this: to the extent that *relative* productivity changes in the market and nonmarket sectors matter for the short run allocation of time, the observed relation between measured productivity and employment hours can be severely affected. Let s_m and s_n be shocks to the market and home technologies.[9] When s_m is relatively high, labor will flow into the market so that productivity and real wages (correctly measured) will rise along with market hours; thus, s_m shocks trace out a "labor supply" curve for the economy. On the other hand, when s_n is relatively high, labor will flow into the nonmarket sector, raising productivity and real wages as market employment falls; thus, s_n shocks trace out a "labor demand" curve for the economy. As long as both shocks are important at

[9] It is certainly the case that innovations to home and market technologies are not perfectly synchronized (think of the introductions of micro computers and microwave ovens).

different points in time, a scatter plot between market hours and productivity (or real wages) need not show any discernible pattern. By incorporating nonmarket activity, it evidently becomes possible in principle to reconcile the lack of empirical correlation between employment and productivity (or real wages) with theories based on technology shocks.[10]

To be clear, our intention is *not* to show that adding home production generates outcomes that were not possible without it; that would be a futile task. Our intention is to show that adding home production allows us to organize and intepret observations in a useful way. Macroeconomics is an empirical discipline, with the goals of accounting for existing regularities and helping to predict the consequences of changes in the underlying environment. As Becker (19xx, p. 5) writes, "The assumption of stable preferences provides a stable foundation for generating predictions about responses to various changes, and prevents the analyst from succumbing to the temptation of simply postulating the required shift in preferences to 'explain' all apparent contradictions to his predictions." However, as he also points out, "The preferences that are assumed to be stable do not refer to market goods and services, like oranges, automobiles, or medical care, but to the underlying objects of choice that are produced by each household

[10] The lack of a strong correlation in the data between hours and productivity or wages over the cycle is, of course, a classic conundrum for macroeconomists; see Geary and Kennan (198x) or Christiano and Eichenbaum,(1988) for up to date discussions and references. Of course, another way to reconcile theory and evidence is to include shocks to the marginal rate of substitution between consumption and leisure (which is exactly what Bencivenga (1988) does, and is also pretty close to the solution suggested by Christiano and Eichenbaum, who attempt to measure these shocks using government spending under the assumption that an increase in public consumption raises marginal utility of private consumption. This is another example of the principle that the equilibrium of a home production economy can always be replicated by a model in which home production is absent, if we are given enough latitude to play with preferences. We discuss this further in Benhabib, Rogerson and Wright (1990).

using market goods and services, their own time, and other inputs. These underlying preferences are defined over fundamental aspects of life ... that do not always bear a stable relation to market goods and services."

Before proceeding to general issues, we close this section with some illustrative examples.[11] We begin by defining preferences by

$$U = \ln(C) + A\cdot\ln(H-h_m-h_n) + Bh_m, \tag{2.8}$$

where $A > B \geq 0$, and C is a composite consumption good given by

$$C = \left(a_m c_m^e + a_n c_n^e\right)^{1/e}. \tag{2.9}$$

The composite good is defined by means of a fairly flexible CES aggregator, with constant elasticity of substitution $1/(1-e)$. On the other hand, market and nonmarket work are perfect substitutes if $B = 0$, whereas if $B > 0$, then for a given amount of total work and consumption the agent would rather work in the market than at home. One can show that leisure, given by $L = H-h_m-h_n$, is necessarily a normal good for this class of preferences.[12] For now we also assume home production is linear, $g(h_n) = s_n h_n$, which will allow us to easily derive closed form solutions.

First, consider the case of $e = 0$, so that (2.9) in fact defines a Cobb-Douglas function, and the elasticity of substitution between c_m and c_n is unity. Assuming an interior solution, the homework function is

$$h_n = h(c_m,h_m) = \left[\frac{a_n}{A+a_n}\right](H-h_m).$$

[11] These examples are of particular interest, given the specifications in our quantitative analysis in Benhabib, Rogerson and Wright (1990).

[12] In the next section we prove that if the utility function is separable in consumption and hours, as (2.8) is, then leisure must be normal.

Substituting this into (2.8), the reduced form utility function becomes (after a linear transformation)

$$V = a_m \ln(c_m) + (A+a_n)\ln(H-h_m) + Bh_m. \tag{2.10}$$

In this case, home production adds nothing to the model, in the sense that if we had ignored it, or simply set c_n and h_n equal to constants in (2.8), then except for the constants in (2.10), nothing would have changed! To get any real effects with this specification, we therefore need to assume an elasticity of substitution different from unity.

Consider the case where c_m and c_n are perfect substitutes, $e = 1$, and assume for ease of notation that $a_n = a_m = H = 1$. Assuming an interior solution, the homework function in this case is

$$h_n = h(c_m,h_m) = \frac{s_n(1-h_m)-Ac_m}{s_n(1+A)} .$$

Substituting this into (2.8), the reduced form utility function now becomes (after a linear transformation)

$$V(c_m,h_m) = (1+A)\cdot \ln[c_m+s_n(1-h_m)] + Bh_m. \tag{2.11}$$

If $B = 0$, then (2.11) is of the special "zero wealth effect" class, $V(c_m,h_m) = v_1[c_m+v_2(1-h_m)]$, where v_1 and v_2 are increasing, concave functions; if $B > 0$, on the other hand, then leisure is actually inferior according to the reduced form utility function, even though it is normal according to the underlying utility function.[13]

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[13] Let h(x) solve the labor supply problem, maximize V(c,h) subject to $c = x + wh$. The standard result is that $h'(x)$ is proportional to $\eta = wV_{11} + V_{12}$, so that leisure is normal if and only if $\eta < 0$. If $V = v_1[c_m+v_2(1-h_m)] + Bh_m$, we have $\eta = -Bv_1''/v_1'$; hence, the wealth effect for this class of utility

To see how this might be important for macroeconomics, consider the following "pseudo-dynamic" representative agent problem

$$\max E \sum \beta^t U[c_{mt}, c_{nt}, h_{mt}, h_{nt}]$$

$$\text{st } c_{mt} = \Gamma^t s_{mt} F(h_{mt})$$

$$c_{nt} = \Gamma^t s_{nt} G(h_{nt})$$

where $\beta \in (0,1)$, and $\Gamma > 1$ represents exogenous technological growth common to the two sectors.[14] In order to capture a long run stylized fact of actual economies, we impose the condition that market hours do not grow or shrink on average along a balanced growth path. This means that if the s_{jt} are constant over time then h_{mt} will be constant, too, which means wealth and substitution effects must cancel each other. In economies without home production, this means the utility function must be from either the class

$$u(c_m, h_m) = \left[\frac{c_m^{1-\rho}}{1-\rho}\right] \cdot v(h_m)$$

with $\rho > 0$ and $\rho \neq 1$, or

$$u(c_m, h_m) = \ln(c_m) + v(h_m),$$

where in either case $v(\cdot)$ is concave (see King, Plosser and Rebello 1987 for a proof).

Suppose that we also insist that h_m be constant along a balanced growth

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functions depends exclusively on the sign of B.

[14] The model is "pseudo-dynamic" in that there is no capital formulation, and so it really reduces to a sequence of static economies (we introduce capital in Section V, but for present purposes, the simpler structure will suffice).

path in the home production economy. One specification that satisfies this criterion is easily seen to be:

$$U = \frac{\left[C^b L^{1-b}\right]^{1-r} - 1}{1-r}, \quad C = \left(a_m c_m^e + a_n c_n^e\right)^{1/e} \text{ and } L = H - h_m - h_n$$

As a special case, consider $U = \ln(c_m + c_n) + A \cdot \ln(H - h_m - h_n)$ and linear home production, for which we have already derived the reduced form in (2.11). Thus, this home production model is observationally equivalent to a model without home production and the zero wealth effect utility function

$$V(c_{mt}, h_{mt}) = \ln\left[c_{mt} + \Gamma^t s_{nt}(H - h_{mt})\right]. \tag{2.12}$$

These preferences imply potentially large intratemporal substitution effects (in addition to the intertemporal substitution effects that would be present if we included capital) despite of the fact that h_{mt} does not change on average along the growth path.[15]

To illustrate things further, consider the market technology $f(\cdot) = s_{mt} h_{mt}^{\theta}$. Then it is straightforward to check that the equilibrium allocation involves:

$$h_{mt} = \left(\frac{\theta s_{mt}}{s_{nt}}\right)^{\frac{1}{1-\theta}} \qquad h_{mt} + h_{nt} = \frac{1}{1+A}$$

Total work is constant, while the mix of hours between the home and market fluctuates according to the ratio (s_{mt}/s_{nt}), with an elasticity $1/(1-\theta)$. It is also straightforward to show that the marginal product of labor in the

[15] One way to interpret this result is to note that the term Γ^t inside of the square brackets in (2.12) acts to increase the reduced form's marginal utility of leisure at the same rate as the marginal product of market labor, keeping the value of h_{mt} that equates the two constant on average.

market is proportional in equilibrium to s_{nt}, the nonmarket shock, and so productivity is necessarily negatively related to employment. Also, instantaneous utility in equilibrium is (a linear function of) $\ln(s_n)$, and hence it too is negatively related to employment. While this example is obviously simplistic in its functional form as well as its neglect for important factors such as capital, it does demonstrate how introducing home production can have some rather dramatic effects.

III. More General Results[16]

In this section, we derive some more general results concerning the way home production affects the mapping between the underlying utility function $U(\cdot)$ and the reduced form utility function $V(\cdot)$. We concentrate on the case of perfect substitutes up to a linear perturbation,

$$U = u(c_m+c_n, h_m+h_n) + Ac_m + Bh_m$$

where A and B are constants. This is not the most general case, of course, but it does deliver some sharp predictions. The interpretation of the linear terms is that $A > 0$ ($B > 0$) means that market consumption (market employment) is superior to its nonmarket alternative. We allow for general technology specifications, $g(h_n) = s_n G(h_n)$ and $f(h_m) = s_m F(h_m)$.

As a special case of problem (2.5), the unique competitive equilibrium in the representative agent version of this model has first order conditions

$$s_m F'(h_m)[u_1(\cdot)+A] + u_2(\cdot) + B = 0$$

$$s_n G'(h_n)u_1(\cdot) + u_2(\cdot) = 0.$$

Notice A or $B > 0$ implies $f' = s_m F' < s_n G' = g'$, and the marginal product is lower in the home than the market. Differentiating, we have

$$D \begin{bmatrix} dh_m \\ dh_n \end{bmatrix} = - \begin{bmatrix} (u_1+A)F' + F\eta_m \\ F\eta_m \end{bmatrix} ds_m - \begin{bmatrix} G\eta_m \\ u_1 G' + G\eta_n \end{bmatrix} ds_n - \begin{bmatrix} \eta_m \\ \eta_n \end{bmatrix} dx$$

where $\eta_m = f'u_{11}+u_{12}$, $\eta_n = g'u_{11}+u_{12}$, and D is a matrix given by:

[16] This section contains some messy derivations, designed to show our earlier discussion is fairly general; readers not interested in these details can skip to the section on unemployment with no loss in continuity.

$$D = \begin{bmatrix} (u_1+A)f''+f'^2u_{11}+2f'u_{12}+u_{22} & f'g'u_{11}+(f'+g')u_{12}+u_{22} \\ f'g'u_{11}+(f'+g')u_{12}+u_{22} & u_1g''+g'^2u_{11}+2g'u_{12}+u_{22} \end{bmatrix}$$

The determinant of D is positive:

$$|D| = (A+u_1)f''(g'^2u_{11}^2+2g'u_{12}+u_{22}) + (u_{11}u_{22}-u_{12})(f'-g')^2$$
$$+ u_1g''(f'^2u_{11}^2+2f'u_{12}+u_{22}) > 0.$$

Using Cramer's rule and simplifying, the pure wealth effect in general equilibrium on market hours is

$$|D|\partial h_m/\partial x = - u_1g''\eta_m + (g'-f')(u_{11}u_{22}-u_{12}^2).$$

With perfect substitutes ($A = B = 0$), we have $g' = f'$, and the second term vanishes; in this case, the condition for h_m to decrease with x is the standard (from models without home production) normal leisure condition, $\eta_m = f'u_{11} + u_{12} < 0$. If $A, B > 0$, however, then $g' > f'$, and the second term is positive; in this case, hours worked in the market may actually increase with x, even if $\eta_m < 0$.[17] A symmetric result holds for homework,

$$|D|\partial h_n/\partial x = - (u_1+A)f''\eta_n - (g'-f')(u_{11}u_{22}-u_{12}^2),$$

and these can be combined to yield the effect on total leisure, $L = 1-h_m-h_n$,

$$|D|\partial L/\partial x = u_1g''\eta_m - (u_1+A)f''\eta_n.$$

If η_m, $\eta_n < 0$ then leisure is unambiguously normal, even though h_m can

[17]Note that linear home production implies the first term vanishes; therefore h_m increases with x *if and only if* $g' > f'$.

increase with x. In particular, $u_{12} = 0$ always implies $\partial L/\partial x < 0$.

We can also derive the effects of changes in the productivity parameters, s_m and s_n. For example,

$$\partial h_m/\partial s_m = - Q|D|^{-1}(u_1+A)F' + F\cdot\partial h_m/\partial x$$

where $Q = u_1 g'' + g'^2 u_{11} + 2g' u_{12} + u_{22} < 0$. The first term in this expression is the unambiguously positive substitution effect, while the second term is the wealth effect derived above. Finally, we can consider balanced technical progress by setting $s_m = s_n = s$. Then, assuming $A = u_{12} = 0$ to reduce the notation, it turns out that

$$\partial h_m/\partial s = |D|^{-1}u_1^2 sF' G'' + (F+G)\partial h_m/\partial x + |D|^{-1}(G' - F')u_1 u_{22}.$$

The key point here is that the third term is negative. Thus, in order to have h_m constant in response to balanced changes in technology, we do not need the wealth and substitution effects to cancel out, and we could easily have $\partial h_m/\partial x > 0$.

IV. Unemployment

In the previous sections, the competitive equilibrium involves everyone receiving exactly the same allocation. All agents spend the same amount of time in market work, and all agents spend the same amount of time in home work. There are a number of ways to amend the basic model to account for the fact that not all agents work in the market, and still maintain the tractability of a representative agent framework (e.g., any of a variety of fixed costs or other nonconvexities associated with market work could be modeled). For simplicity, we will assume directly that time allocated to the market can take on only two values, 0 or $\bar{h}$, where without loss in generality we set $\bar{h} = 1$ (renormalizing the total time endowment, H, if necessary). This is the indivisible labor assumption studied in Rogerson (1984, 1988), and subsequently employed in equilibrium macroeconomics by Hansen (1985), Greenwood and Huffman (1987), Hansen and Sargent (1988), Cho and Rogerson (1988), Christiano and Eichenbaum (1988), Cooley and Hansen (1989), and others.[18]

In nonconvex economies like this, it can be efficient to randomize the allocation. The relevant social planning problem is to choose a probability of employment for the representative agent, φ, and a consumption - homework package for both the employed and unemployed. Let c_m^j and h_n^j be consumption of the market good and hours of nonmarket work by an agent working j units of time in the market, j = 0 or 1. Then the planning problem is

$$\begin{aligned} &\max \varphi U[c_m^1, g(h_n^1), 1, h_n^1] + (1-\varphi)U[c_m^0, g(h_n^0), 0, h_n^0] \\ &\text{st } \varphi c_m^1 + (1-\varphi)c_m^0 \le x + f(\varphi) \text{ and } 0 \le \varphi \le 1, \end{aligned} \tag{4.1}$$

---

[18] See Prescott (1986) and Lucas (1987) for general discussions of the indivisible labor model in macroeconomics.

also subject to nonnegativity.[19] As always, we have substituted the home production constraints directly into the objective function. However, since individuals not employed in the market can still enjoy consumption of c_m, there is a single constraint concerning the market good. For robust specifications we can have $\varphi < 1$, and we assume this is the case in what follows. The fraction $(1-\varphi)$ of agents will be called *unemployed* (although they may well be working at home).

Let λ be the multiplier on the resource constraint in (4.1). Then the first order conditions are as follows:

$$U[c_m^1, g(h_n^1), 1, h_n^1] - U[c_m^0, g(h_n^0), 0, h_n^0] + \lambda[f'(\varphi) - c_m^1 + c_n^0] = 0 \tag{4.2}$$

$$U_2[c_m^1, g(h_n^1), 1, h_n^1] g'(h_n) + U_4[c_m^1, g(h_n^1), 1, h_n^1] = 0 \tag{4.3}$$

$$U_2[c_m^0, g(h_n^0), 0, h_n^0] g'(h_n) + U_4[c_m^0, g(h_n^0), 0, h_n^0] = 0 \tag{4.4}$$

$$U_1[c_m^1, g(h_n^1), 1, h_n^1] - \lambda = 0 \tag{4.5}$$

$$U_1[c_m^0, g(h_n^0), 0, h_n^0] - \lambda = 0 \tag{4.6}$$

$$x + f(\varphi) - \varphi c_m^1 - (1-\varphi) c_m^0 = 0. \tag{4.7}$$

These have straightforward interpretations. For example, let $g_j(\cdot)$ indicate that the home production function is being evaluated at h_n^j, and let $U^j(\cdot)$ indicate that the utility function is being evaluated at $[c_m^j, g(h_n^j), j, h_n^j]$, $j = 0$ or 1. Then (4.3) and (4.4) imply $g_j' = U_4^j/U_2^j$, so that the marginal product in home production is equated to the marginal rate of substitution

[19] The solution to problem (4.1) is the optimal randomized allocation, which can be decentralized in a variety of ways. For example, Shell and Wright (1989) show formally how to support the the planner's randomized allocation as a standard competitive equilibrium with extrinsic uncertainty represented by "sunspots" (there is no home production in that construction, but it is clear how to extend the results to include it).

for both unemployed and employed workers.

Equations (4.5) and (4.6) imply the efficient risk sharing condition, equating the *marginal* utilities of the market good between employed and unemployed agents:

$$U_1^1 = U_1[c_m^1, g(h_n^1), 1, h_n^1] = U_1[c_m^0, g(h_n^0), 0, h_n^0] = U_1^0 \qquad (4.8)$$

In general, of course, (4.8) says nothing about total utility. Let z be the normalized difference between the total utilities of employed and unemployed agents: $z = (U^1 - U^0)/\lambda$. Then we define the case of $z > 0$ to be *involuntary unemployment*. In Rogerson and Wright (1988), in a model without home production, we found $z > 0$ if and only if $\partial\varphi/\partial x > 0$. Further, one can show that $\partial\varphi/\partial x > 0$ implies leisure is an inferior good, in the standard sense, over some region of commodity space, although not necessarily everywhere; see, e.g., Greenwood and Huffman (1988). Hence, it is impossible to have leisure everywhere normal and involuntary unemployment at the same time. We now show that in the model with home production, we still have $z > 0$ if and only if $\partial\varphi/\partial x > 0$, but the relation between this and normal leisure is broken.[20]

Begin by differentiating the first order conditions (4.2)-(4.7) and simplifying, to arrive at

---

[20]Note that from (4.2) that we also have

$$z = c_m^1 - c_m^0 - f'(\varphi).$$

This tells us that unemployment is involuntary if and only if the difference in market consumption between employed and unemployed workers exceeds the marginal product of an employed worker, which in a sense could be interpreted as saying that the employed are being paid "too much." This result is true in models without home production, too, by the way.

$$
\begin{bmatrix}
\lambda f'' & 0 & 0 & 0 & 0 & -z \\
0 & Q_1 & 0 & \eta_1 & 0 & 0 \\
0 & 0 & Q_0 & 0 & \eta_0 & 0 \\
0 & \eta_1 & 0 & U_{11}^1 & 0 & -1 \\
0 & 0 & \eta_0 & 0 & U_{11}^0 & -1 \\
-z & 0 & 0 & -\varphi & \varphi-1 & 0
\end{bmatrix}
\begin{bmatrix}
d\varphi \\ dh_n^1 \\ dh_n^0 \\ dc_m^1 \\ dc_m^0 \\ d\lambda
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -dx
\end{bmatrix}
\tag{4.9}
$$

where we have used the notation

$$Q_j = g_j'' U_2^j + g_j'^2 U_{22}^j + 2g_j' U_{24}^j + U_{44}^j$$

$$\eta_j = g_j' U_{21}^j + U_{41}^j.$$

Note that $Q_j < 0$, while η_j could be of either sign in general (one can show that $\eta_j < 0$ if and only if $\partial h_n^j/\partial x < 0$). In the Appendix, we prove the following result.

Lemma 1: $\Psi_j = Q_j U_{11}^j - \eta_j^2 > 0$, $j = 0, 1$.

This lemma is also used in the Appendix to verify that the second order conditions for problem (4.1) hold; thus, if we let Δ be the determinant of the square matrix in (4.9), $\Delta < 0$. The next result is then straightforward.

Theorem 2: Unemployment is involuntary if and only if $\partial\varphi/\partial x > 0$.

Proof: Solving (4.9) for $\partial\varphi/\partial x$ and simplifying yields

$$\partial\varphi/\partial x = -\Delta^{-1}\Psi_1\Psi_0 z.$$

Now $\partial\varphi/\partial x > 0$ if and only if $z < 0$, which by definition means if and only if $U^1 > U^0$. ∎

This extends the main result in Rogerson and Wright (1988) to the home production economy. However, we now argue that with home production there is no problem having involuntary unemployment and normal leisure at the same time.[21] We make this point by way of an example, using the utility function

$$U = v_1(c_m+c_n) + v_2(h_n+h_m) + Ac_m + Bh_m. \tag{4.10}$$

Again, the interpretation is that agents prefer market to home produced consumption if $A > 0$, and prefer market work to homework if $B > 0$. As shown in the previous section, such utility functions always entail normal leisure in the sense that $\partial L/\partial x > 0$, where $L = 1-h_m-h_n$, although at the same time, they potentially allow $\partial h_m/\partial x > 0$.

For utility function (4.10), the efficient risk sharing condition (4.8) implies that employed and unemployed agents enjoy the same total consumption, $c_m^1 + c_n^1 = c_m^0 + c_n^0 = c$, although the employed get more of the market good and the unemployed get more of the home produced good. Suppose g is linear, $c_n = B \cdot h_n$; then the efficient hours conditions, $U_4^j/U_2^j = B$ for j $= 0$ or 1, imply that the employed and unemployed also work the same number of total hours, $1 + h_n^1 = h_n^0$. Hence, we have

$$U^1 - U^0 = A(c_m^1-c_n^0) + B.$$

---

[21]In the special case of perfect substitutes and linear home production, one can show that employed and unemployed agents always get the same total consumption and hours,

$$c_m^1 + c_n^1 = c_m^0 + c_n^0 \quad \text{and} \quad 1 + h_n^1 = h_n^0.$$

In this case we can have normal leisure and $U^1 = U^0$. However, when the home technology is strictly concave, perfect substitutes, normal leisure and involuntary unemployment are not all possible at the same time. Thus, we need to assume less than perfect substitutes to get involuntary unemployment and normal leisure in the general case.

As long as either A or B is strictly positive, $U^1 > U^0$. Therefore it is possible to have involuntary unemployment simultaneously with normal leisure, at least if is linear. If $g'' < 0$, then one can show that the employed end up working fewer hours in the home but more hours in total, $1+h_n^1 > h_n^0 > h_n^1$. Nevertheless, simply by continuity, with $g'' < 0$ it is still possible to have involuntary unemployment simultaneously with normal leisure. In summary, we have:

Theorem 3: In the home production economy, involuntary unemployment does not imply inferior leisure.

We think that this result is important. It applies to not only the representative agent model with indivisible labor, but also to a variety of other models with random layoffs and efficient risk sharing. These include the standard Azariadis (1975) implicit contract model and versions of the Feldstein (1976) temporary layoff model of unemployment insurance (see Burdett and Wright 1989 for a discussion of these approaches). One reason such theories seem to have fallen into disfavor recently is that users were uncomfortable with the implication that laid off workers were happier than their employed colleagues, given normal leisure (see, e.g., the discussion in Rosen's 1985). Now it is obvious that these models should not be used to explain *all* types of unemployment, as they abstract from many relevant considerations for some types, such as frictional unemployment. Yet they do seem to be quite satisfactory for the analysis of other types, such as temporary layoff unemployment. It is perhaps comforting that versions that explicitly incorporate home production do allow the coexistence of efficient risk sharing, normal leisure, and involuntary unemployment.

To close this section, we briefly discuss how heterogeneity might enter the picture. Suppose individual types are indexed by i, and that

$$U^i = v_1(c_m+c_n) + v_2(h_m+h_n) + A^i c_m + B^i h_m.$$

Market labor is still indivisible, but now individuals also differ in terms of their productivities, say h_m and h_n hours of type i's time in the two sectors yield $h_m^i = \omega_m^i h_m$ efficiency units in the market and $h_n^i = \omega_n^i h_n$ in the home. The efficient allocation now determines a probability of employment in the market sector φ^i *for each type*. Any type with A^i, $B^i > 0$ and $h_m^i = 0$ is involuntarily unemployed, at least if g'' is close to 0, as shown above. However, if ω_m^i/ω_n^i is small, we expect there will be lots of type i workers unemployed. At the same time, A^i, $B^i < 0$ implies individual i would rather stay home, but if ω_m^i/ω_n^i is large, the efficient allocation will have him working in the market with positive probability. The point is that in a cross section it will be easy to find some agents not working in the market who wish they were, and at the same time, some who are employed but in a sense wish they were at home.

V. Dynamics

In this section we move to a genuinely dynamic formulation, in order to illustrate some other implications of home production. Consider the problem

$$
\begin{aligned}
\max\ & E \sum \beta^t U(c_{mt}, c_{nt}, h_{mt}, h_{nt}) \\
\text{s.t. } & c_{mt} = s_{mt} F(h_{mt}, k_{mt}) - i_t \\
& c_{nt} = s_{nt} G(h_{nt}, k_{nt}) \\
& h_{mt} + h_{nt} \le H \\
& k_{mt} + k_{nt} \le k_t \\
& k_{t+1} = (1-\delta) k_t + i_t
\end{aligned}
\tag{5.1}
$$

where k_{jt} is capital in sector j, k_t is total capital, i_t is investment, and $\delta \in (0,1)$ is the depreciation rate.[22] The constraints hold at every date t. The maximization is over time paths $\{c_{mt}, c_{nt}, h_{mt}, h_{nt}, k_{mt}, k_{nt}, i_t\}$, given processes for the shocks $\{s_{mt}, s_{nt}\}$ and initial conditions.

Suppose that we are given $\{c_{mt}, h_{mt}, k_{mt}, k_{nt}\}$, and we are asked to choose a path for homework $\{h_{nt}\}$ to solve

---

[22] We have set this up as an optimal growth problem, but of course the solution can be supported as a competitive equilibrium. We could have also illustrated essentially the same points with a single consumer decision problem. Also notice that although capital is an input into household production, we have assumed that it is produced exclusively in the market. Finally, we have assumed that capital can be freely moved between sectors. However, exactly the same message goes through if we alternatively assume two separate laws of motion,

$$k_{mt+1} = (1-\delta_m) k_{mt} + i_{mt} \quad \text{and} \quad k_{nt+1} = (1-\delta_n) k_{nt} + i_{nt},$$

where $i_{mt} + i_{nt} = i_t$.

$$\max E_0 \sum \beta^t U[c_{mt}, s_{nt}G(h_{nt}, k_{nt}), h_{mt}, h_{nt}]$$

$$\text{s.t. } h_{nt} \leq H - h_{mt} \text{ for all } t.$$

The first order conditions for this problem are $s_{nt}G_1 = -U_4/U_2$ for all t and for all realizations of $\{s_{mt}, s_{nt}\}$. This implies that, for all t, the *instantaneous homework function* is given by

$$h_{nt} = h(c_{mt}, h_{mt}, k_{nt}, s_{nt}).$$

Notice $h(\cdot)$ does not depend on k_{mt} or s_{mt}, or on t, or on variables at dates other than t. In the obvious way, we also have the *instantaneous home consumption function*,

$$c_{nt} = c(c_{mt}, h_{mt}, k_{nt}, s_{nt}),$$

and the *reduced form instantaneous utility function*,

$$V(c_m, h_m, k_n, s_n) = U[c_m, c(\cdot), h_m, h(\cdot)].$$

We now have an equivalent alternative formulation of (5.1), in which we choose $\{c_{mt}, h_{mt}, k_{mt}, k_{nt}, i_t\}$ to solve

$$\max E_0 \sum \beta^t V(c_{mt}, h_{mt}, k_{nt}, s_{nt})$$

$$\begin{aligned} \text{s.t. } c_{mt} &= s_{mt}F(h_{mt}, k_{mt}) - i_t \\ h_{mt} &\leq H \\ k_{mt} + k_{nt} &\leq k_t \\ k_{t+1} &= (1-\delta)k_t + i_t. \end{aligned}$$

In this problem, the home production variables c_n and h_n do not appear at all, although k_n and s_n do.[23] We interpret this by saying that the dynamic home production model is equivalent to a model without home production, but with different preferences, as well as a consumer durable good k_n. This is the natural extension of the static results in Section II.

For example, consider the utility function $U = \ln(C) + A\cdot\ln(L)$, where $C = C(c_m, c_n)$ and $L = 1-h_m-h_n$. Although it is obviously not possible to find an explicit solution for reduced form preferences, in general, it is possible for the following special cases.

Case i: $C = c_m + c_n$ (perfect substitutes) and $c_n = a_0k_n + a_1h_n$ (linear home production). In this case, the reduced form utility function is (after a linear transformation)

$$V = \ln\left[c_m + a_0k_n + a_1(1-h_m)\right]$$

Case ii: $C = c_m^a c_n^{1-a}$ (Cobb-Douglas) and $c_n = a_0k_n + a_1h_n$. In this case,

$$V = a\cdot\ln(c_m) + (1-a+A)\cdot\ln\left[a_0k_n + a_1(1-h_m)\right].$$

Case iii: $C = c_m^a c_n^{1-a}$ and $c_n = k_n^\eta h_n^{1-\eta}$. In this case,

$$V = a\cdot\ln(c_m) + (1-a)\eta\cdot\ln(k_n) + [(1-a)(1-\eta)+A]\cdot\ln(1-h_m).$$

The striking feature that emerges from the above examples is that one underlying utility function, $U = \ln(C) + A\cdot\ln(L)$, can give rise to such different reduced forms. In case i, the three commodities c_m, k_n, and $1-h_m$ are perfect substitutes, while in case iii, V is additively separable.

[23] Again, we are assuming capital can be moved freely between the two sectors at t, but a similar result follows if we assume the two capital stocks evolve separately, as in the previous footnote.

Applied researchers are typically concerned with the choice of functional forms, including such issues as separability between variables or groups of variables. As shown by these examples, however, the assumption of separability in the true underlying preferences may or may not carry over to the reduced form that one takes to the data. In particular, we note that with this structure it is apparently difficult to obtain a reduced form in which c_m and k_n enter as perfect substitutes, but separable from h_m, a specification that is often used in empirical studies of durable goods and intertemporal consumption decisions. More generally, we note that the traditional distinction in economics between preferences and technology has become somewhat blurred here.

Eichenbaum and Hansen (1990) provide an interesting recent contribution to the literature on durables and intertemporal consumption. They posit preferences over *consumption services*, defined as flows derived from stocks of durable goods. Our framework is a special case of theirs in that consumers here have exactly two choices: purchase consumption services c_m directly from the market, or receive services c_n from the stock of home capital. However, the salient element of our approach, which is missing from Eichenbaum and Hansen's, is the time allocation decision (i.e., the choice of h_m and h_n). The use of time intuitively seems essential to producing a service flow from home capital or durables. Furthermore, modeling the allocation of time explicitly has many other implications, as we have attempted to demonstrate. It is hoped that future work in the area may benefit from some of these results.

VI. Conclusion

In this paper we have explored some implications of introducing home production into simple economic models. One result is that there is a mapping between models with home production and those without home production but with different preferences, with the property that the implications of the two models for market variables are identical. However, for fixed preferences the model with home production can generate very different implications. Further, the model without home production might require properties, such as nonnormal leisure, time varying utility, etc., that *ex ante* we may not be willing to entertain. If macroeconomics (or any other applied field) is to be an empirical science, research must ultimately proceed to functional forms, or at least to restrictions that specify certain classes of functional forms and parameters. One way to interpret our claim for the usefulness of including a nonmarket sector in models of market activity is that recognizing home production leads us to examine functional form and parameter issues in a new light. We think that this will have important implications for our ability to understand and interpret empirical observations in macroeconomics, and in other areas.

Appendix

Proof of Lemma 1: We make use of the following inequality:

$$U_{11}(U_{22}U_{44}-U_{24}^2) - U_{12}(U_{12}U_{44}-U_{14}U_{42}) + U_{14}(U_{12}U_{24}-U_{14}U_{22}) < 0 \qquad (A.1)$$

This can be verified by noting expression in question equals the determinant of the Hessian matrix of $\bar{U} = U(c_m, c_n, \bar{h}_m, h_n)$, where $\bar{h}_m$ is fixed, which is a concave function. Now expanding Ψ_j (ignoring the subscript j), we have

$$\Psi = g''U_2U_{11} + g'^2(U_{11}U_{22}-U_{12}^2) + (U_{11}U_{44}-U_{14}^2) + 2g'(U_{11}U_{24}-U_{12}U_{14})$$

$$= g''U_2U_{11} + \left[g'(U_{11}U_{22}-U_{12}^2)^{.5} - (U_{11}U_{44}-U_{14}^2)^{.5}\right]^2$$

$$+ 2g'\left[(U_{11}U_{22}-U_{12}^2)^{.5}(U_{11}U_{44}-U_{14})^{.5} + (U_{11}U_{24}-U_{12}U_{14})\right]$$

after "completing the square." Since the first two terms are positive, it show the last term is, too. Suppose not; then

$$(U_{11}U_{22}-U_{12}^2)^{.5}(U_{11}U_{44}-U_{14})^{.5} < -(U_{11}U_{24}-U_{12}U_{14}).$$

But squaring both sides and simplifying contradicts (A.1). Hence, $\Psi > 0$, and this completes the proof. ■

Second order Conditions: We check the second order conditions for problem (4.1). Let H_k be the bordered Hessian matrix formed by deleting all but the last and the first k rows and columns the square matrix in (4.9). For a maximum, the determinants of these matrices must alternate in sign, starting with $|H_2| > 0$ (see, e.g., Takayama 1985). After a little algebra (rather a lot, actually), we find

$$|H_2| = - Q_1 z^2 > 0,$$

$$|H_3| = - Q_0 Q_1 z^2 < 0,$$

$$|H_4| = - Q_0(\varphi\lambda f'' Q_1 + z^2\Psi_1) > 0,$$

$$|H_5| = - z^2\Psi_0\Psi_1 - \lambda f''[\varphi Q_1\Psi_0 + (1-\varphi)Q_0\Psi_1] < 0$$

using Ψ_0, $\Psi_1 > 0$, as shown in Lemma 1. In particular, $\Delta = |H_5| < 0$, as used in Theorem 2. ■

References

Azariadis (1975) "Implicit Contracts and Underemployment Equilibria," *Journal of Political Economy* 83 (December), 1183-1202.

Becker, G.S. (1965) "A Theory of the Allocation of Time," *Economic Journal* 75, 493-517.

Becker, G.S. (19xx) *The Economic Approach to Human Behavior.*

Becker, G.S. (1988) "Family Economics and Macro Behavior," *American Economic Review* 78 (March), 1-13.

Benhabib, J., R. Rogerson and R. Wright (1988) "Home Production and Aggregate Fluctuations," presented at the Northwestern University Summer Workshop in Macroeconomics, Evanston, Illinois, August 1988.

Benhabib, J., R. Rogerson and R. Wright (1990) "Homework in Macroeconomics II: Aggregate Fluctuations," manuscript, Stanford University.

Bencivenga, V. (1988) "An Econometric Study of Hours and Output Variation with Preference Shocks," manuscript, University of Western Ontario.

Burdett, K. and R. Wright (1989) Unemployment Insurance and Short-Time Compensation: The Effects on Layoffs, Wages and Hours per Worker," *Journal of Political Economy* (December).

Christiano, L. and Eichenbaum, M. (1988) "Is Theory Really Ahead of Measurement? Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations," Federal Reserve Bank of Minneapolis Working Paper 412 and NBER Working Paper 2700.

Cho, J. and R. Rogerson (1988), "Family Labor Supply and Aggregate Fluctuations," *Journal of Monetary Economics* 21, 233-245.

Cooley, T. and G. Hansen (1989) "The Inflation Tax in a Real Business Cycle Model," *American Economic Review* 79 (September), 733-748.

Cooper, R. (1987) *Wage and Employment Patterns in Labor Contracts: Microfoundations and Macroeconomic Implications*, Harwood Academic Publishers, New York.

Eichenbaum, M. and L.P. Hansen (1990) "Estimating Models with Intertemporal Substitution Using Aggregate Data," *Journal of Business and Economic Statistics* 8, 53-69.

Eisner, R. (1988) "Extended Accounts for National Income and Product," *Journal of Economic Literature* 26, 1611-1684.

Eisner, R. (1989) *The Total Income System of Accounts*, University of Chicago Press.

Feldstein (1976) "Temporary Layoffs in the Theory of Unemployment," *Journal of Political Economy* 84 (October), 937-957.

Fraumeni, B. and D. Jorgenson (1980) "The Role of Capital in U.S. Economic Growth 1948-1976," in *Capital, Efficiency and Growth*, edited by G. von Fustenberg, Cambridge.

Geary, P. and J. Kennan (198x) *Journal of Political Economy*.

Greenwood, J. and G. Huffman (1988) "A Dynamic Equilibrium Model of Inflation and Unemployment," *Journal of Monetary Economics* 19, 203-228.

Greenwood, J. and G. Huffman, (1989) "On Modeling the Natural Rate of Unemployment with Indivisible Hours," *Canadian Journal of Economics* 21 (August), 587-609.

Greenwood, J. and Herkovitz, Z. (1990) "Home Production," manuscript.

Gronau, R. (1977) "Leisure, Home Production and Work – The Theory of the Allocation of Time Revisited," *Journal of Political Economy* 85, 1099-1123.

Gronau, R. (1980) "Home Production – A Forgotten Industry," *Review of Economics and Statistics* 62, 408-415.

Gronau, R. (1985) "Home Production – A Survey," *Handbook of Labor Economics*.

Hansen, G. (1985) "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics* 16, 309-325.

Hansen, G., and T. Sargent (1988) "Straight Time and Overtime in Equilibrium," *Journal of Monetary Economics*.

Hawrylyshyn, O. (1976) "The Value of Household Services: A survey of Empirical Estimates," *The Review of Income and Wealth* 22, 101-131.

Hill, M. (1985) "Patterns of Time Use," in *Time, Goods and Well-Being*, edited by F.J. Juster and F.P. Stafford, University of Michigan.

King, R., C. Plosser and S. Rebello (1987) "Production, Growth and Cycles: Technical Appendix," manuscript, University of Rochester.

King, R., C. Plosser and S. Rebello (1988) "Production, Growth and Cycles I. The Basic Neoclassical Model," *Journal of Monetary Economics* 21, 195-232.

Lucas, R. *Models of Business Cycles*, Basil Blackwell, 1987.

Nordhaus, W. and J. Tobin (1972) *Economic Growth*, New York: NBER.

Pollack, R. and M. Wachter (1975) "Home Production," *Journal of Political Economy*.

Prescott, E. "Theory Ahead of Business Cycle Measurement," *Federal Reserve Bank of Minneaplois Quarterly Review* Fall 1986, 9-22.

Rios-Rull, J.-V. (1988) "Working in the Market, Home Production and the Acquisition of Skills: A General Equilibrium Approach," Federal Reserve Bank of Minneapolis Working Paper.

Rogerson, R. (1984) "Topics in the Theory of Labor Markets," unpublished Ph.D. dissertation, University of Minnesota.

Rogerson, R. (1988) "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics* 21 (January), 3-17.

Rogerson, R. and R. Wright, (1988) "Involuntary Unemployment in Economies with Efficient Risk Sharing," *Journal of Monetary Economics* 22 (November), 501-515.

Rosen, S. (1985) "Implicit Contract Theory: A Survey," *Journal of Economic Literature* 23 (September), 1144-1175.

Shell, K. and R. Wright (1989) "Indivisibilities, Lotteries, and Sunspot Equilibrium," manuscript, Cornell University and University of Pennsylvania.

Takayama, A., *Mathematical Economics*, The Dryden Press, Illinois, 1974.

Zolotas, X. (1981) *Economic Growth and Declining Social Welfare*, Bank of Greece, Athens.

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HOMEWORK IN MACROECONOMICS II: AGGREGATE FLUCTUATIONS

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HOMEWORK IN MACROECONOMICS II: AGGREGATE FLUCTUATIONS

ABSTRACT

This paper explores the implications of including home, or nonmarket, production in an otherwise standard model of cyclical fluctuations. In particular, we generalize the stochastic growth model, or the real business cycle model, to include a household sector using the basic framework that labor economists have studied for some time. Symmetrically with the market sector, the household sector uses labor and capital to produce output according to a stochastic technology. We calibrate the model based on microeconomic evidence and long run considerations, simulate it, and examine its statistical properties. Our finding is that introducing home production significantly improves the quantitative performance of the standard model along several dimensions simultaneously. It also implies a very different interpretation of the nature of aggregate fluctuations.

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I. Introduction

This project explores the implications of including home, or nonmarket, production in an otherwise standard model of aggregate fluctuations. In Benhabib, Rogerson and Wright (1990), we demonstrated that the household sector is large, whether measured in terms of the time allocated to home production activities or in terms of the estimated value of home produced output. We also argued that there may be a good deal of substitutability between the market and nonmarket sectors, and that this may be an important missing element in existing macroeconomic models. By way of some simple examples and comparative static analysis, we showed how home production can have important implications for the nature and interpretation of several macroeconomic phenomena. The goal in this paper is to pursue the argument further, qualitatively and quantitatively, by incorporating home production explicitly into the stochastic growth model, and comparing the results with both existing business cycle models and the actual data.

We use a framework that labor economists have studied for some time.[1] Symmetrically with market production, household production uses labor and capital as inputs to produce output according to a stochastic technology. We expect, *a priori*, that this would affect market activity for fairly obvious reasons. To the extent that individuals are willing to substitute between the market and household sectors, relative productivity differentials between the two induce volatility in market variables over time. When productivity in the market is relatively low, for example, we

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[1] The primary reference on household production is Becker (1965), and some of the ideas developed here are also discussed at a general level in Becker (1988). The particular formalization we use follows Gronau (1977, 1985); many details are explored at length in our companion paper (Benhabib, Rogerson and Wright 1990).

expect that the economy will allocate fewer hours to production in the market sector and concentrate instead on household activity. It follows that in a model with household production, fluctuations in aggregate market variables will depend on the occurrence of relative productivity shocks, and not just absolute shocks, as is typical in existing macroeconomic models.[2] Moreover, the size of fluctuations in market quantities will depend on the degree to which agents are willing to substitute between home produced and market produced goods, and not just the degree to which they are willing to substitute between time and goods at different dates, as in existing models.

In order to examine these effects quantitatively in a controlled setting, we introduce household production into what is currently the standard paradigm in macroeconomics – the stochastic growth model, or the *real business cycle* model.[3] To facilitate comparison with the existing literature, we stay as close to it as possible in our basic specification and functional forms. We also choose parameter values based on the same principles that earlier studies have adopted; when additional parameters are introduced, we appeal to additional microeconomic evidence and steady state considerations to tie them down. The essential departure in our economy from the standard model is that, instead of dividing time between leisure

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[2] Notable exceptions would include any models built around sectoral shifts.

[3] Diverse opinions from Prescott (1986) to Blanchard and Fisher (1989) agree that this is the standard model, although there is much disagreement as to just how much can be explained without adding various complications. There is also disagreement on many technical or methodological details, such as detrending, estimation, etc. Our general message, however, is meant to be independent of technical details, and independent of whether or not market failures, government policies, monetary factors, information processing problems, frictional unemployment, or other complications are empirically important. Our position is that introducing home production will likely have important effects in any reasonably specified model of macroeconomic activity.

and market labor, agents have to allocate total hours between leisure, work in the market, and homework. This simple elaboration turns out to have a significant quantitative impact, and also leads to a novel interpretation of several important macroeconomic phenomena.

It has already been established that even very simple versions of the real business cycle model, as described in Hansen (1985), Prescott (1986), Plosser (1989), or King and Plosser (1989), for example, do surprisingly well at accounting for certain salient aspects of the data. Using functional forms and parameter values that conform to microeconomic studies and long run observations, it accounts for a sizeable fraction of observed fluctuations in macroeconomic variables at cyclical frequencies given reasonable estimates of the actual process of technological change. Further, the model predicts phenomena such as the fact that consumption will be less volatile and investment more volatile than output, as observed not only in the postwar U.S. data, but also across many countries and time periods (see, e.g., Bacus and Kehoe 1989). Nevertheless, it is apparent that the standard model does not do as well along some dimensions as it does along others.

We identify the following problems with the standard real business cycle model:

1. output fluctuates too little;
2. relative to output, labor hours fluctuate too little;
3. relative to output, consumption fluctuates too little;
4. relative to output, investment fluctuates too much;
5. productivity's correlation with output or hours is far too high;
6. labor hours used to produce consumption goods are countercyclical.

We note that these have been recognized by practitioners of the model in the past, with the exception of problem 6; we will argue below, however, that it is central to understanding the nature of the other problems. Also, various extensions of the basic framework are known to ameliorate some of these problems when looked at in isolation. We demonstrate that introducing home production can improve the performance of the model along all of these dimensions simultaneously.[4]

In Section II we introduce the basic, one sector, stochastic growth model, and also present a simple way of disaggregating it into a two sector model. This allows us to keep track of the hours allocated to the production of consumption goods and the hours allocated to the production of investment goods separately, which provides much insight into the workings of the model. In Section III we discuss calibration, including some empirical issues that have typically not come up in this literature, such as the elasticity of substitution between home and market produced consumption goods as well as the correlation between innovations to the home and market technologies. In Section IV we analyze the results. Basically, we find that a model with home production is superior to one without it along every dimension that we consider. In Section V we discuss the sensitivity of our results to parameter choices, and present some general concluding remarks.

In view of the size of the houshold sector, it seems natural to investigate its impact in macroeconomic models. Our finding is that it definitely improves the quantitative performance of the standard real

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[4] There are, of course, some other problems with the standard model, such as its failure to account for the observed equity premium, or for certain nominal phenomena, about which we believe that home production will have little to contribute; therefore, we do not discuss them in this paper. One additional area where home production does seem to be important is in modeling investment in consumer durables over the cycle, as documented by the recent work of Greenwood and Herkovitz (1990). They do a good job of discussing that issue, so we basically ignore it, and concentrate instead on some of the other issues.

business cycle model. Furthermore, including home production does not require a radical departure from the basic framework, nor does it require a substantial increase in complexity, either conceptually or computationally. Therefore, based on several criteria, our conclusion is that home production should be part of the standard business cycle model.

II. The Basic Model

An appropriate starting point is the standard stochastic growth model.[5] There is a representative agent, with preferences over stochastic sequences of consumption and labor hours $\{c_t, h_t\}$ described by

$$E \sum \beta^t u(c_t, h_t),$$

where $u(\cdot)$ is increasing in c_t and decreasing in h_t, E denotes the expectation, and $\beta \in (0,1)$ is the discount factor. The agent has one unit of time to divide between leisure and labor each period. Labor and capital are used to produce output according to a (possibly time dependent) constant returns to scale technology, subject to a stochastic shock at each date, $y_t = s_t f_t(h_t, k_t)$. Capital evolves according to $k_{t+1} = (1-\delta)k_t + i_t$, where i_t is investment and $\delta \in (0,1)$ the depreciation rate, and the shock s_t evolves according to a law of motion to be described fully below. Feasibility requires $c_t + i_t \le y_t$, $h_t \le 1$, and nonnegativity, for all t. The initial conditions (k_0, s_0) are given.

Our immediate goal is to extend this model to include household, or nonmarket, variables. We begin by generalizing preferences as follows,

$$E \sum \beta^t U(c_{mt}, c_{nt}, h_{mt}, h_{nt}),$$

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[5] Although we will extend the standard real business cycle model to include home production, we ignore many of the interesting extensions that have already been studied elsewhere, including non-time-separable utility, time-to-build investment, variable capital utilization rates, inventories, indivisible labor, signal extraction problems, heterogeneity, government spending, taxation, imperfect competition, and a foreign sector. It may well be interesting to reconsider some of these issues in the context of models with an explicit home production sector.

where c_{mt} is consumption of the market good, c_{nt} is consumption of the nonmarket good, h_{mt} is time devoted to market work, and h_{nt} is time devoted to home work, at date t. The function $U(\cdot)$ is increasing in its first two arguments, and decreasing in the last two. The market technology is now written $y_t = s_{mt}f_t(h_{mt},k_{mt})$, where h_{mt} and k_{mt} are hours and capital in market production, while the nonmarket technology is $c_{nt} = s_{nt}g_t(h_{nt},k_{nt})$, where h_{nt} and k_{nt} are hours and capital in home production. Both are assumed to display constant returns to scale. Feasibility requires $k_{mt} + k_{nt} \leq k_t$, where k_t is now the total capital stock, plus $c_{mt} + i_t \leq y_t$, $h_{mt} + h_{nt} \leq 1$, and nonnegativity, for all t. Total capital evolves according to $k_{t+1} = (1-\delta)k_t + i_t$, the shocks s_{mt} and s_{nt} evolve according to a process to be described below, and the initial conditions are given.

Notice that in this specification capital is assumed to be freely mobile between the home and market sectors. By way of contrast, one could imagine a model where capital in a given sector cannot be transformed once it is in place. Theoretically these two cases are polar extremes. From a practical perspective, however, the difference is not substantial in the present context. Given depreciation, by choosing to not replace worn out capital in one sector and putting all new investment in the other, the economy can reallocate a considerable amount of capital across sectors without actually moving the stuff that is already in place. In the simulations conducted in this paper, only infrequently does any capital physically move between sectors, and even then, the amount that does move is quite small (rarely more than one half of one percent of the stock in the declining sector). Since at least a small amount of capital probably can be easily reallocated between the market and nonmarket sectors in the real world, we believe that the capital mobility issue is simply not of substance here.[6]

To close this section, we point out that although the models presented above ostensibly have only one market sector – i.e., they produce a single market output y_t that can be used either as consumption or capital – they can always be interpreted as special cases of more general two sector models. Given constant returns, there is a natural and very simple way to disaggregate.[7] Suppose there are separate technologies used to produce consumption and investment goods,

$$c_{mt} = s_{ct}\phi(h_{ct}, k_{ct})$$

$$i_t = s_{it}\psi(h_{it}, k_{it}),$$

where h_{jt}, k_{jt} and s_{jt} denote labor, capital and the technology shock in sector j, j = c or i. If the functional forms and shocks are identical, $\phi(\cdot) = \psi(\cdot)$ and $s_{ct} = s_{it}$, then efficiency dictates that the capital-labor ratios will be the same in the two sectors. Thus, in order to produce twice as much of the consumption good as the investment good, for example, the consumption sector will simply use twice as much of each input.

Let $r_t = c_t/y_t$ denote the fraction of output that goes to consumption, and suppose we have the path of total hours, h_t, in the standard one sector

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6 Also notice that in this specification capital is produced exclusively in the market sector, even though it is used as an input to both the home and market technologies. In the context of most physical capital and even much human capital, this is probably reasonable. However, for some other forms of capital, perhaps especially some forms of human capital, this seems to be a strong restriction, and it may be worth pursuing models in which it is relaxed.

7 We do not claim that there are no other multisector models that aggregate up to these one sector models, only that the disaggregation procedure to be presented here is a useful one.

economy. Then we can immediately disaggregate by setting $h_{ct} = \tau_t h_t$ and $h_{it} = (1-\tau_t)h_t$. Similarly, let $\mu_t = c_{mt}/y_t$, and suppose we have the path of market hours, h_{mt}, in the home production economy. Then we can disaggregate by setting $h_{ct} = \mu_t h_{mt}$ and $h_{it} = (1-\mu_t)h_{tm}$. The same procedures can be applied to capital. Disaggregating in this way does not affect aggregate market variables: keeping track of the inputs used to produce goods for consumption purposes and for investment purposes cannot change market output, consumption, investment, or total inputs. What it can do is generate paths for the sectoral utilization of labor and capital. Below we will use this to analyze how and why the model without home production has trouble accounting for some observations, including some observations that appear on the surface to be unrelated to sectoral phenomena.

III. Calibration

Deterministic steady states for the models described above are fairly easy to characterize (see Appendix A). However, with the exception of a few special cases (see, e.g., Long and Plosser 1983), stochastic growth models with or without home production cannot be solved analytically. In order to study the cyclical properties of the models we therefore follow an approach pioneered by Kydland and Prescott (1982), and since adopted by many others.[8] This approach consists of choosing functional forms and parameter values based on micro studies and long run observations, and solving the model numerically. The solution procedure used here employs a quadratic approximation to the planning problem around its (deterministic) steady state, which can then be solved analytically using standard techniques.[9] Statistics will then be computed using data generated by simulating the approximate model, and compared with the same statistics computed using actual data.

The functional forms typically employed in the previous literature (i.e., in real business cycle models without home production) are as follows. Preferences are described by a constant relative risk aversion utility function of a consumption-leisure composite,

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[8] Examples include Kydland (1984), Hansen (1985), Hansen and Sargent (1988), Kydland and Prescott (1988a) King, Plosser and Rebello (1988), Plosser (1989), Christiano (1988), Christiano and Eichenbaum (1988), Cho and Rogerson (1988), Cooley and Hansen (1989), Greenwood, Herkovitz and Huffman (1988), Bacus, Kehoe and Kydland (1989), McGratten (1988), and Rotemberg and Woodford (1990).

[9] An alternative procedure is to solve the original planning problem using numerical methods; the results of Christiano (1986) or Danthine, Donaldson and Mehra (1989) suggest that these two procedures will yield very similar results for the models studied in this paper.

$$u(c_t,h_t) = \frac{\left[c_t^b(1-h_t)^{1-b}\right]^{1-r} - 1}{1-r}$$

The market technology is given by a Cobb-Douglas production function

$$f_t(h_t,k_t) = \Gamma^t k_t^\theta h_t^{1-\theta}$$

where Γ, if greater than 1, yields exogenous technological growth. The shock to technology evolves according to $s_t = \rho s_{t-1} + \epsilon_t$, where $\rho \in (0,1)$ and ϵ_t is i.i.d. normal. Much has been written on these choices (see Prescott 1986 or King, Plosser and Rebello 1988, e.g.), and will not be repeated here. However, we note that this specification implies labor's share of aggregate income is constant and that hours devoted to market work are independent of the real wage along a balanced growth path, two properties that seem to characterize the data.[10]

We would like to preserve the above structure as much as possible, not only for the fundamental reasons that led previous researchers to adopt such a specification, but also to facilitate comparison with their results. We therefore assume preferences are described by

$$U(c_{mt},c_{nt},h_{mt},h_{nt}) = \frac{\left[C_t^b L_t^{1-b}\right]^{1-r} - 1}{1-r},$$

where $L_t = 1-h_{mt}-h_{nt}$ is leisure, and

$$C_t = \left[ac_{mt}^e + (1-a)c_{nt}^e\right]^{1/e}$$

[10] If $\Gamma = 1$ this economy does not grow, but settles down to a steady state, in the long run. If $\Gamma = 1$ this specification implies that hours devoted to market work are independent of the real wage in steady state.

is a CES aggregator over market and nonmarket consumption. The elasticity of substitution, which measures the degree to which agents are willing to substitute between c_m and c_n, is given by $1/(1-e)$. Thus, $e = 1$ implies perfect substitutes, while $e = 0$ implies that $C = c_m^a c_n^{1-a}$ is a Cobb-Douglas function.[11]

Some of the preference parameters are fairly easy to tie down. The discount factor β is set to .99. With the interpretation of a period in the model as corresponding to one quarter of a year, this implies a real annual interest rate in steady state of 4 percent. A review of the evidence concerning risk aversion leads to the conclusion that the preference parameter r is likely between 1 and 2 (see Prescott 1986). However, Hansen (1986) has found that within this and even a larger range, the value of r did not have a significant impact on the nature of cyclical fluctuations for models without home production. Thus, as in much of the literature we set $r = 1$, which implies that the momentary utility function can be written

$$U = b\cdot\ln(C_t) + (1-b)\cdot\ln(L_t).$$

However, like Hansen (1986), we did experiment with different values of risk aversion, and the results of changes in r as well as all of the other parameters are reported in Appendix B.

The parameters a and b are chosen so that the steady state of the model yields values for market work and homework that correspond to averages found in the data. Using the Michigan Time Use Survey, we compute market work and

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[11] In contrast to consumption, we assume that work in the market and work in the home are perfect substitutes. This simplifies some technical aspects of the solution procedure, and also gives rise to an easily interpreted notion of leisure, $L = 1-h_m-h_n$.

homework for an average household consisting of a married couple as fractions of discretionary time. Our definition of discretionary time includes market work plus homework plus leisure, all of which are measured directly by the time use survey (see Hill 1985). The main component not included in this definition is "personal care" which consists mainly of sleep. The results of these calculations are $h_m = .33$ and $h_n = .28$.[12] Although these numbers are probably quite accurate, the main results discussed here actually change little when the assumed steady state values of h_m and h_n are varied over a considerable range (see Appendix B). In any case, note that choosing h_m and h_n is equivalent to choosing a and b, as there is a unique choice of a and b that implies the model generates given values for h_m and h_n in steady state (see Appendix A for details).

The remaining preference parameter is the substitution elasticity. Although we are not aware of any direct estimate of e in the literature, there is some evidence that is suggestive. First, a recent paper by Eichenbaum and Hansen (1990) uses aggregate data to estimate a model in which individuals value both the services of market consumption goods and the flow of services from consumer durables, where the latter is akin to output from a home production process that uses capital (measured by durables) but no labor. Although their results are sensitive to various assumptions, for one set of findings they report there is "very little evidence against the hypothesis that the services from durable and non-durable goods are perfect substitutes" (p. 63). This would suggest

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12 We disagree with the assumption of Greenwood and Herkovitz (1990) that all nonmarket time should be interpreted as home production. This leads them to set hours of home work to .67 and leisure to zero, contradicting the direct measures in the time use data. In our opinion, the standard approach of dividing time into $L + h_m$ should be replaced by $L + h_m + h_n$ and not $h_m + h_n$.

setting e near 1, although again, because their framework does not explicitly include time as an input to home production, this estimate might be regarded tentatively.

Cross sectional data can also provide some information. Consider a static model, in which each individual i has preferences described by

$$\ln(C_i) + v_i(1-h_{mi}-h_{ni}),$$

where $C_i = \left[a_i c_{mi}^e + (1-a_i)c_{ni}^e\right]^{1/e}$ with a_i an individual specific constant distributed across the population. The function $v_i(\cdot)$ may also vary across agents. Suppose that all agents have the same home production technology, $c_{ni} = B \cdot h_{ni}$, but that agent i faces an individual specific market wage, w_i. Then it is straightforward to show that the solution to the utility maximization problem for i implies

$$\ln(h_{mi}/h_{ni}) = \frac{1}{e-1}\ln(B) - \frac{e}{e-1}\ln(w_i) + \frac{1}{e-1}\ln[(1-a_i)/a_i]$$

$$= \alpha_0 + \alpha_1 \ln(w_i) + \xi_i.$$

The interpretation of the above equation is that it represents the average time allocation decision as a function of the long run wage.

Using the pooled data from the Panel Study of Income Dynamics described in Rios-Rull (1988), we estimate the above equation and derive an implied value of $e = 0.6$, somewhat lower than the value implied by Eichenbaum and Hansen's (1990) results.[13] In some sense the two methods that were employed

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[13] We consider this estimate highly preliminary, for a variety of reasons. For one thing, his sample selection criterion severely under reports low wage workers (presumably with very low ratios of market to home hours). As a rough correction, we either adjusted home hours for the two lowest wage groups so that their total work is the same as the other groups, or we simply ignored the lowest wage groups. Either method results in a point

are polar extremes – the first uses aggregate data and abstracts from the time allocation decision, whereas the second uses micro data and abstracts from savings and capital. Both are obviously crude measures, although they are not uninformative. We emphasize the need for further empirical work along these lines, but for the time being we simply use the mean value of these two numbers, $e = .8$, as our base case for the simulations reported in the next section (Appendix B illustrates how variations in this important parameter matter).

We now describe the structure of technology. As in most of the studies with which we want to compare results, we set $\delta = .025$, implying an annual depreciation rate of 10 percent. We assume Cobb-Douglas production functions in both sectors,

$$f(h_m, k_m) = \Gamma^t k_m^{\theta} h_m^{1-\theta}$$

$$g(h_n, k_n) = \Gamma^t k_n^{\eta} h_n^{1-\eta}.$$

For the simulations reported in the next section, we abstract from exogenous growth by setting $\Gamma = 1$, so that the economy ends up fluctuating around some constant steady state level in the long run. As Hansen (1986) has shown for the standard model, incorporating a geometric trend by setting $\Gamma > 1$ does not affect the economy's cyclical properties, and since these are the properties that we focus on here, we simply set that trend to zero.

Since $1-\theta$ equals labor's share of total market income in equilibrium, we have a direct measure of this parameter from the national income

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estimate of about $e = .6$. Note that the data are pooled by wage interval, and we use the average of the logs of the interval endpoints as the independent variable. The lower endpoint is set at \$1.00 and the upper endpoint at \$8.00 (in 1969 dollars).

accounts, which leads us to set $\theta = .36$.[14] Unfortunately, there is no direct measure of labor's share in the nonmarket sector, and hence η cannot be found in an analogous manner. However, it can be determined indirectly by examining the deterministic steady state of the system. Given values for h_m, h_n, β, δ and θ, as discussed above, steady state depends only on η (as shown in Appendix A, the parameters e, a and b matter only inasmuch as they influence h_m and h_n, while the other parameters such as r do not matter at all for the steady state). Our strategy is to choose η to match certain aspects of the steady state to averages found in the postwar U.S. data.

Focusing first on consumption, it seems reasonable to insist that c_m/y lie between .70 and .75 in steady state (total consumption averages about 75 percent of GNP in the postwar data, excluding the foreign and government sectors; but including expenditures on consumer durables in investment rather consumption reduces this number to closer to 70 percent). As shown in Table 1, for this to be true η must be considerably less than θ, say $\eta \leq .10$ as compared to $\theta = .36$, consistent with the idea that much homework, like child care, is extremely labor intensive. We choose a value of $\eta = .08$ as our base case for the simulations in the next section, which implies $c_m/y = .71$, and at the same time, $c_n/y = .26$. The latter ratio is within, although at the low end of, the range of estimates reviewed by Eisner (1988), which puts it between .20 and .50.

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[14] The literature is not unanimous in this choice. Depending on how certain aspects of the data are interpreted, measurement of labor's share can come out to be anywhere from .57 to .75 (see Christiano 1988; note that no matter how it is measured, θ is approximately constant over time). Prescott (1986) argues for $1-\theta = .64$, greater than the conventional wisdom of .70, because he wants to include a measure of the service flow from consumer durables in GNP; but our model suggests that such a measure might better be included as home and not market production. We use $\theta = .36$ here in order to facilitate comparison with some existing studies, although this and potentially several other parameters may ultimately need to be to revised.

As table 1 shows, larger values of η increase c_n/y at the expense of decreasing c_m/y. It seems preferable to match the market consumption ratio, for at least three reasons: (1) we have less confidence in estimates of c_n/y than of c_m/y; (2) the model abstracts from some important considerations that would tend to increase the size of the home sector, such as taxation on market activity; and (3) since our goal is ultimately to demonstrate that including household production makes a difference, we do not want it to appear as if we have biased things in our favor by having too generous an amount of home production. Table 1 also indicates that the steady state ratio of market capital to (quarterly) output is about 10, consistent with the evidence, and that when $\eta = .08$, 12 percent of all capital is in the home sector in steady state, which is reasonable if we interpret home capital as household equipment and furniture.[15]

It remains to describe the stochastic structure. The technology shock in the market – the so-called the "Solow residual" – can be more or less accurately estimated from the aggregate data. For example, Prescott (1986) finds the process $s_{mt+1} = \rho_m s_{mt} + \epsilon_{mt}$ fits well with $\rho_m = .95$, and ϵ_{mt} i.i.d. normal with a standard deviation of approximately $\sigma_m = .007$ (the mean of ϵ_{mt} is normalized to $1-\rho_m$, so that the unconditional mean of s_{mt} is one). Obviously, less is known about the shock to the home technology. One natural starting point is to assume that it too follows a process of the form $s_{nt+1} = \rho_n s_{nt} + \epsilon_{nt}$, where ϵ_{nt} is i.i.d. normal with mean $1-\rho_n$ and a standard deviation σ_n. Thus, we need to determine ρ_n, σ_n, and the correlation between ϵ_{mt} and ϵ_{nt}. We simply set $\rho_m = \rho_n = \rho = .95$ and $\sigma_n =$

[15] Greenwood and Herkovitz (1990) choose to interpret home capital as all durable goods (including the housing stock and automobiles), which implies a greater fraction of total capital is in the home sector, and hence a larger value of η is required in order to match the data.

$\sigma_m = \sigma = .007$ for much of what follows. However, the basic message is affected little by variations in these parameters, with one important exception discussed at length below (the productivity statistics).

This leaves the correlation between the two shocks, which we denote by $\gamma = \text{corr}(\epsilon_m, \epsilon_n)$. We know of no independent estimate of this parameter. Our guess is that γ is certainly positive, but that it is also certainly less than unity (sometimes technological innovations affect productivity mainly in the market, like microcomputers, and sometimes they affect productivity mainly in the home, like microwave ovens). Smaller values of γ imply more frequent relative productivity differentials between the two sectors, and therefore more frequent opportunities for short run substitution between the market and the home. Intuitively, then, the smaller is γ the greater is the extent to which home production should affect the cyclical behavior of the system. We somewhat arbitrarily choose $\gamma = 2/3$ as the base case for the simulations reported below, although the basic results would not be affected very much if we were to choose $\gamma = 1/2$ or $3/4$, for example.

We end this section by pointing out that the standard model can always be nested in the home production model by forcing h_{nt} to be zero in steady state. This approach does not seem appealing, since the data indicate h_{nt} is nearly as large as h_{mt}, on average. However, the standard model can also be nested by setting $e = \eta = 0$, independent of the average size of h_{nt}. In Benhabib, Rogerson and Wright (1990), we prove the following result: the home production economy with $U = \ln(C) + A\cdot\ln(1-h_m-h_n)$, where C is the CES aggregator defined above and $e = \eta = 0$, generates *exactly* the same paths for all of the market variables as a model with no home production and preferences given by

$$V(c_m,h_m) = \ln(c_m) + B\cdot\ln(1-h_m).$$

This is, of course, precisely the specification used in the standard model without home production.

Thus, a value of e or η somewhat different from 0 is required to generate predictions that are different from those of the standard model. In fact, as η is increased from from $\eta = 0$ to the value of $\eta = .08$ discussed above, as long as e remains near 0 we found that simulations of the home production economy were still extremely close to the standard model. Hence, one way to interpret a model without an explicit description of household production is that it contains the home sector implicitly, but either assumes that h_{nt} is very small on average or that e is close to zero.

IV. Results

As is standard in this literature, our approach is to compare certain statistics computed from simulations of the models with those computed from actual post war U.S. time series. We are primarily interested in fluctuations of the data around some smooth trend, and therefore, as in much (but not all) previous work in the area we detrend by taking logarithms and applying the Hodrick-Prescott (1980) filter to all series before computing any statistics. Table 2a summarizes the behavior of five key macroeconomic variables, c_m = market consumption, i = investment, h_m = market hours, k = capital, and p = average productivity ($p = y/h_m$), in terms of two statistics for each series x: the standard deviation of x relative to the standard deviation of output, and the correlation of x with output. The data are quarterly for the period 1954.1 – 1988.2, and the standard deviation of GNP over this period is 1.74 percent.[16]

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[16] Several comments are in order concerning these numbers, which have been taken from Kydland and Prescott (1989), unless otherwise noted. The consumption series corresponds to expenditures on nondurables and services only. We added consumer durables to the investment series, and the result is a standard deviation of i relative to y of 2.82, lower than the Kydland-Prescott number of 3.17. Prescott (1986) also reports the standard deviation of i for several disaggregated categories of investment; for i = fixed investment, nonresidential investment, equipment, structures, and inventories, we have std(i)/std(y) = 3.01, 2.95, 2.61, 3.41, and 5.09. Eichenbaum and Christiano (1988) use a comprehensive measure including government investment, which yields 2.38, a number even lower than ours. The hours series is from the household survey, which is probably better than the establishment survey in capturing hours worked (rather than hours paid for). Using the establishment survey produces a standard deviation of hours relative to output of .97 and a correlation of hours with output of .88; this also affects the productivity calculations, resulting in a standard deviation relative to output of .48 and a correlation with output of .31. In either case, there is reason to believe that if a more appropriate quality weighted measure of hours were available the standard deviation of hours would be somewhat lower and that of productivity somewhat higher (see Kydland and Prescott 1988b). The statistics on capital are from Cooley and Hansen (1989), and include nonresidential structures, equipment, residential structures, and government capital.

Table 2b provides a summary of the properties of the standard model without home production using the parameter values described in the previous section (these numbers are averages over 50 simulations of 143 periods each). The results are the same as those reported by Hansen (1985) for his base economy except that some new statistics have been added, corresponding to the disaggregated employment variables h_{ct} and h_{it} discussed in Section II. Many authors have commented on how well this simple and abstract model captures several important aspects of the actual data, and we concur. For instance, it replicates the stylized fact that investment is more volatile than output and consumption is less volatile than output, and at least for some of the variables, the correlations with output are quite reasonable. Nevertheless, as promised in the introduction, we wish to draw attention to several dimensions along which there appears to be significant room for improvement.

First, observe that the model economy is not as volatile as the actual economy: the model has a standard deviation of output equal to only 1.29 percent, compared to the actual 1.74 observed in the data. Of course, this can be improved by increasing the variance of the technology shock, but this raises the question, why measure the Solow residuals in the first place? In any case, independently of the overall volatility of the model, consumption is not volatile enough and investment is too volatile *relative to* output. Because output is the sum of consumption and investment and all three are highly correlated, the standard deviation of y is essentially a weighted average of the standard deviations of c and i; hence, insufficient volatility in consumption and excess volatility in investment relative to output tend to go together. A further difficulty is that total market hours do not fluctuate enough in the model, which predicts a standard deviation of h_t that is only half as great as that of output. Further still, observe that although the correlations between output and most of the other variables are reasonable, the correlation between output and productivity is significantly off target when compared to the actual data.

We note that these problems are fairly well known, and also, that various embellishments of the basic framework have been shown to help each of them in isolation. There is, however, a feature of the model that is not well understood, but which is intimately related. This is that the model implies an almost perfect negative correlation between output and the hours allocated to the production of consumption goods: $corr(y_t, h_{ct}) = -.98$. Some additional results (not shown in the table) are that h_{ct} is also highly negatively correlated with h_t, and *a fortiori*, h_{it}. These predictions fly in the face of the conventional wisdom concerning actual business cycles, which is that various sectors tend to move up and down together.[17] Furthermore, the following theorem indicates that these predictions are not only independent of parameter values, but that they are robust in the sense that they will hold for any specification of the standard model consistent with the growth observations discussed earlier.

Theorem 1: Consider the model without home production, where preferences and technology are from the class that implies labor's share is constant and h_t is independent of the real wage in steady state or along a balanced growth

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[17] Actually, economists often define the cycle as the recurrent comovement of the *outputs* of various market sectors (see Lucas 1976, e.g.); but we doubt if anyone would argue empirically that the inputs of the various sectors move out of phase. Without attempting to catalogue various sectors as consumption or investment, there is no major sector of the U.S. economy that is known to have countercyclical employment (see Murphy, Schleifer and Vishny 1989). Using data provided by Donna Costello from five countries each disaggregated into five sectors, we examined the correlations with output of each sector's hours, and the cross correlations between sectoral hours. Almost all of these correlations were positive, and *none* were significantly negative.

path. In this model, h_{ct} and h_t are negatively correlated.

Proof: The class of preferences that satisfy the hypothesis of the theorem is the following: either

$$u(c_t,h_t) = \left[c_t^{1-r}/(1-r)\right]\cdot v(h_t)$$

with $r \geq 0$ and $r \neq 1$, or

$$u(c_t,h_t) = \ln(c_t) + v(h_t),$$

where in either case $v(\cdot)$ is a concave function (see King, Plosser and Rebello 1987 for a proof). Consider the second case (the first case is similar). At each point in time, the standard efficiency condition equating the marginal rate of substitution with the marginal product of labor in the production of consumption goods reduces to $c_t v'(h_t) = MPL_t$. Multiplying both sides by h_{ct}/c_t, we arrive at:

$$h_{ct}v'(h_t) = \frac{MPL_t \cdot h_{ct}}{c_t}$$

The right hand side is labor's share of output in the consumption sector, which will be constant by assumption. Hence, as long as $v(\cdot)$ is (strictly) concave, an increase in total hours h_t must be accompanied by a (strict) decline in h_{ct}. ∎

Corollary: Since h_{ct} and h_t are negatively correlated, so are h_{ct} and h_{it}.

We point out that the above results will only be reinforced if labor's share is countercyclical rather than constant (in the actual data it is approximately constant over the cycle, as well as in the long run, but perhaps slightly countercyclical). We would also like to emphasize that our proof does not restrict the technologies in the investment and consumption sectors to be the same or subject to the same shock. Hence, moving to a more general two sector model will not overturn it. The best one can do within the standard model is to make $v(\cdot)$ linear, in which case the theorem clearly implies that hours in the consumption sector will be *constant* over the cycle. The indivisible labor economy studied by Rogerson (1984, 1988) and Hansen (1985) is observationally equivalent to an economy with a linear utility of leisure. In that economy, although the two labor inputs h_{ct} and h_{it} do not move together, at least they do not move in opposite directions over the cycle.[18]

The intuition for these results is quite simple. Basically, a specification that implies hours do not change over time along a balanced growth path also implies individuals never supply more labor in order to produce more output for immediate consumption (a result of wealth and substitution effects that cancel). In particular, in a model that is otherwise standard except that it ignores capital (i.e., it sets $\theta = 0$), employment is *constant*, and consumption fluctuates one-for-one with the technology shock. Even though agents have the opportunity to work harder when productivity is high in order to increase consumption even further, they choose not to; they simply never work more if the only reward is increased contemporaneous consumption. When capital accumulation is re-introduced, labor does vary with productivity due to intertemporal substitution opportunities; but it is still the case that individuals do not work more to increase current consumption. In fact, since consumption now

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[18] In a different but similar structure, Long and Plosser (1983) obtain the result that employment hours in each sector of a multisector model is constant.

moves less than one-for-one with output, individuals spend less time in the production of consumption goods when productivity is high, and the increase in total hours of employment all goes to the production of capital goods.

We therefore have the following (somewhat bizarre) characterization of business cycles in the standard model: good times are periods when resources flow from the production of consumption goods to the production of investment goods. But even if one discounts this prediction - say, by arguing that the standard growth model is "really" only a one sector model - we think that it is useful to focus on the behavior of h_{ct} because it provides considerable insight into other, perhaps less controversial, issues. For instance, the fact that consumption is too smooth and investment too volatile relative to output, in the standard model, is easy to understand given that labor is being moved out of the production of consumption goods and into the production of investment goods as the cycle moves from trough to peak. Similarly, the fact that total hours are too smooth relative to output is easy to understand given that h_{ct} is countercyclical; if h_{ct} did not decrease whenever h_{it} rose, the sum h_t would be more volatile. Furthermore, if h_{ct} could be increased during upswings without decreasing h_{it} total output would also be more volatile.

What is needed is a mechanism that leads to hours in the consumption sector responding positively to an increase in market productivity. The addition of a home production sector provides exactly this mechanism. In addition to the standard motive for increasing labor hours when market productivity is high (i.e., the motive to accumulate capital), in the home production economy there is an additional motive to simultaneously substitute market for home produced consumption. The latter effect involves the transfer of hours from the home into the market consumption sector during upswings in the business cycle, and thereby could produce a procyclical pattern to h_{ct}. The addition of a household sector implies that upswings in aggregate market activity may turn out to correspond to periods when labor flows from the home into all market sectors, rather than periods when labor flows from the consumption into the investment sector. This intuition provides us with the qualitative impact of adding home production; the question is now one of quantitative importance.

Table 2c reports the results for simulations of the home production economy using the parameter values discussed in Section III. Compared to the standard model, the volatility of investment relative to output has decreased, while that of consumption has increased. Additionally, the variability of market hours relative to output is greater than in the standard model (and probably about as great as we would want it, given the data may be biased towards volatility due to the fact that hours are not quality weighted; see Kydland and Prescott 1988b). Output is also more volatile in the home production economy, with a standard deviation virtually the same as the U.S. data.[19] These improvements can be interpreted in light of the fact that h_{ct} is procyclical in the home production economy, although not overwhelmingly so. As shown in Appendix B, the correlation between h_{ct} and y_t is somewhat sensitive to the choices of parameters. However, we see that as long as it is even slightly positive, the model improves along several dimensions at once.

The only prediction of the standard model that seems to be closer to the actual data is the volatility of productivity; but it misses so badly on

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[19] There are some subtle points to be noted here. For example, although market consumption is more volatile here than in the standard model, it is really the composite good C that consumers care about, and that is actually quite smooth. Similarly, hours in home production act something like a buffer against volatility in market labor, so that leisure L is quite smooth, too. Hence, although market activity in the home production economy is apparently more volatile, agents in the model actually don't mind.

the correlation between p_t and y_t that getting the standard deviation right seems to be of little consolation. Further, not only does the standard model predict $corr(p_t, y_t) = .99$, it also predicts $corr(p_t, h_{mt}) = .99$ (not shown in the table). Christiano and Eichenbaum (1988), McCallum (1989), and others harshly criticize this prediction. In the aggregate U.S. data, h_t and p_t are in fact negatively correlated, as shown in Figure 1a, which plots percentage deviations (after detrending) in h_{mt} versus p_t.[20] For comparison, the data generated by the standard model are plotted in Figure 1b. To say that these pictures are different would be a serious understatement. Of course, there are several problems with the data, and correcting for measurement error suggests the true correlation may actually be positive, perhaps even as high as .44 (see Christiano and Eichenbaum's Table A.3). But even under the most favorable assumptions, it is certainly not .99.

The feature of the standard model responsible for this inconsistency with the data is that it is driven by a single shock to technology (i.e., it is a "one index" model), which implies a very tight relation between productivity and output or productivity and hours. Loosely speaking, shocks to technology shift labor demand and trace out a stable labor supply curve.[21] The home production economy with only a single shock to the market technology – i.e., with $var(\epsilon_n) = 0$ – also traces out a stable labor supply

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[20] The true relation between employment hours and productivity, or hours and the real wage, is an issue with a long history, and we will not attempt to provide references here. We do point out that in our model p is the average product, but this is proportional to the marginal product, which equals the real wage, given a Cobb-Douglas specification. It is generally a bad idea to make inferences about the marginal product from wage data constructed by dividing compensation by hours worked, due to well known biases resulting from long term employment contracts (see Wright 1988 for a recent discussion).

[21] This is only approximately true, since capital is also varying somewhat over time.

curve, as shown in Figure 1c. Notice, however, that this curve is much more elastic than the one in Figure 1b. In contrast to the standard model, which relies exclusively on intertemporal substitution between work at different dates, the home production model also includes intratemporal substitution between market work and homework. By including innovations to the home technology that are less than perfectly correlated with those in the market we add a different shock. In fact, the home production economy with only a shock to the home sector – i.e., with $var(\epsilon_m) = 0$ – traces out a stable labor demand curve (again loosely speaking), as shown in Figure 1d.[22]

When both shocks are present, the net effect is as depicted in Figure 1e. The correlation between h_{mt} and p_t in this case is .49, which is much better than the standard model, although perhaps still high. However, for obvious reasons this statistic is going to be sensitive to the relative size of the two shocks. Increasing $var(\epsilon_n)$ from .007 to .01 but keeping $var(\epsilon_m)$ as well as all of the other parameters constant, the correlation between h_{mt} and p_t is reduced to .08, which is well within the acceptable range. Other statistics for this parameterization are shown in Table 2d. Notice that $std(p_t)/std(y_t)$ and $corr(y_t,p_t)$ are also quite close to the data in this case, although market consumption has become somewhat too volatile.

Counter to the conclusion of Christiano and Eichenbaum (1988), we conclude that there is no problem, *in theory*, accounting for productivity or

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22 Clearly, similar shocks to labor supply could be generated by assuming that preferences vary over time, which is exactly what is done in Bencivenga (1988), and essentially the solution proposed by Christiano and Eichenbaum (1988), where it is suggested that changes in government spending be used to measure shifts in the marginal rate of substitution between consumption and leisure. This is, in fact, a reflection of the general result proved in Benhabib, Rogerson and Wright (1990), that any economy with home production is observationally equivalent to another economy without home production but with different preferences (and in this case, time varying preferences). The point is that for *given* preferences the introduction of home production can make a difference for market variables.

real wage observations using models driven exclusively by technology shocks. They argued that this would not be possible, and, therefore, that business cycle theories based on technological change were in need of a serious reconsideration. It is true, of course, that our explanation would be more complete if we had more precise measures of certain key parameters, but this only leads us to conclude that there is a great need for better measurement in this area. This is similar to the conclusions of Prescott (1986), and again counter to the conclusions of Christiano and Eichenbaum.

V. Discussion

Our reading of the results contained in the previous section is that the existence of a household sector can have a large effect on the behavior of aggregate market variables. In particular, home production improves the standard model's performance along each of the six dimensions outlined in the introduction. It is natural to inquire how sensitive these results are to the particular values of the parameters that we chose, especially since some of them are not especially well measured. Obviously parameter values do matter somewhat; for instance, as we alluded to earlier, a value for e near 0 simply reproduces the statistics of the standard model. In Appendix B we report the effects of changes in the parameters, in a region around our base case, on six statistics corresponding to the six dimensions discussed above (the standard deviation of output, and of consumption, investment, and hours relative to output, plus the correlations with output of p and of h_c). These results are intuitive and easily interpreted; hence, we leave their analysis to the reader.[23]

Basically, our finding is that home production matters a lot. The fact that the household sector is large is incontrovertible. In light of this, we view models without home production as having made the implicit assumption that the willingness and/or the incentive of individuals to substitute between market and nonmarket activity is small, which is not necessarily consistent with the evidence. The fact that available evidence on some important variables is imperfect only leads us to conclude that

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[23] For example, increasing e and reducing γ respectively raises agents' willingness and incentive to substitute between the market and nonmarket sectors, which leads to a greater impact of home production. If e gets too low or γ too high, we approach the standard model; if e gets too high or γ too low, the effects discussed above become exaggerated.

future research ought to subject parameters such as the elasticity of substitution and technological progress in the nonmarket sector to the same level of analysis that has been afforded other variables, such as the coefficient of risk aversion and the Solow residual for the market sector. If theory predicted that the choice of these parameters was of minor importance then their values would not be of much interest to macroeconomists; but this is not what theory predicts.

Appendix A

Here we analyze the deterministic steady state, and demonstrate how the parameters a and b are chosen. Begin by setting the shocks to their unconditional means, $s_m = s_n = 1$, and substituting the constraints into the utility function to yield the objective function

$$\sum \beta^t U\left[f(h_{mt},k_{mt})-k_{mt+1}-k_{nt+1}+(1-\delta)(k_{mt}+k_{nt}),g(h_{nt},k_{nt}),h_{mt},h_{nt}\right].$$

The first order conditions for maximizing this objective are

$$U_1(t)f_1(t) + U_3(t) = 0 \tag{A.1}$$

$$U_2(t)g_1(t) + U_4(t) = 0 \tag{A.2}$$

$$U_1(t)f_2(t) + (1-\delta)U_1(t) = \beta^{-1}U_1(t+1) \tag{A.3}$$

$$U_2(t)g_2(t) + (1-\delta)U_1(t) = \beta^{-1}U_1(t+1) \tag{A.4}$$

where the notation F(t) indicates that a function $F(\cdot)$ is being evaluated at arguments as of date t. In the steady state, of course, these arguments do not depend on time.

We are given values for the parameters β, δ, θ and η, plus the steady state time allocation h_m^* and h_n^*. Using the functional forms described in the text, (A.3) immediately implies $\theta(k_m/h_m)^{\theta-1} = \beta^{-1}-1+\delta$, and this can be solved for k_m^*. The first order conditions also imply the following relation between the capital labor ratios in the market and household,

$$k_m/h_m = \frac{\eta(1-\theta)}{\theta(1-\eta)} k_n/h_n,$$

which can be solved for k_n^* given the solution for k_m^*. Now $c_n^* = g(h_n^*,k_n^*)$ and

$c_m^* = f(h_m^*,k_m^*) - i^*$, where $i^* = \delta(k_m^*+k_n^*)$. Notice that we have solved for the steady state allocation $(c_m^*,c_n^*,h_m^*,h_n^*)$ without using the instantaneous utility function at all. The strategy is to now determine the parameters a and b of this function so that this solution satisfies the marginal conditions (A.1) and (A.2). In fact, the ratio of these conditions is $f_1/g_1 = U_2/U_1$, since $U_3 = U_4$ for the preference structure assumed in the text. This condition is independent of b and can be solved for a unique value of a. Then (A.1) or (A.2) can be solved for a unique value of b. The elasticity parameter e affects the implied values of a and b, but none of the observable variables, while the risk aversion parameter r does not affect the steady state at all.

Appendix B

a) The effect of changing e on six key statistics

e	std(y)	std(x)/std(y)			corr(y,x)	
		x = i	x = c_m	x = h_m	x = h_c	x = p
0.6	1.52	2.82	.36	.60	-.39	.94
0.7	1.58	2.79	.42	.66	-.15	.88
0.8	1.70	2.68	.52	.77	.16	.74
0.9	2.02	2.64	.67	.91	.36	.42
1.0	2.89	3.10	.90	1.14	.43	-.33

b) The effect of changing γ on six key statistics

γ	std(y)	std(x)/std(y)			corr(y,x)	
		x = i	x = c_m	x = h_m	x = h_c	x = p
1/3	2.00	2.38	.59	.85	.50	.62
1/2	1.88	2.53	.53	.81	.34	.68
2/3	1.70	2.68	.52	.77	.16	.74
3/4	1.62	2.85	.48	.71	-.05	.79
1.0	1.27	3.41	.26	.47	-.90	1.00

c) The effect of changing h_m on six key statistics

h_m	std(y)	std(x)/std(y)			corr(y,x)	
		x = i	x = c_m	x = h_m	x = h_c	x = p
.23	1.90	2.67	.53	.82	.21	.69
.28	1.79	2.68	.53	.79	.19	.71
.33	1.70	2.68	.52	.77	.16	.74
.38	1.62	2.68	.52	.74	.13	.76
.43	1.55	2.67	.51	.72	.10	.78

d) The effect of changing h_n on six key statistics

h_n	std(y)	std(x)/std(y)			corr(y,x)	
		x = i	x = c_m	x = h_m	x = h_c	x = p
.13	1.52	2.92	.43	.68	-.08	.85
.18	1.58	2.84	.47	.72	.04	.81
.23	1.64	2.76	.50	.75	.11	.77
.28	1.70	2.68	.52	.77	.16	.74
.33	1.74	2.61	.54	.78	.20	.71

e) The effect of changing θ on six key statistics

θ	std(y)	std(x)/std(y)			corr(y,x)	
		x = i	x = c_m	x = h_m	x = h_c	x = p
.28	1.92	2.88	.57	.76	.36	.76
.32	1.80	2.77	.54	.77	.26	.75
.36	1.70	2.68	.52	.77	.16	.74
.40	1.60	2.60	.50	.77	.07	.73
.44	1.51	2.53	.49	.77	-.02	.72

f) The effect of changing η on six key statistics

η	std(y)	std(x)/std(y)			corr(y,x)	
		x = i	x = c_m	x = h_m	x = h_c	x = p
0	1.52	3.39	.41	.75	-.14	.73
.04	1.60	3.02	.46	.76	.02	.73
.08	1.70	2.68	.52	.77	.16	.74
.12	1.80	2.39	.58	.77	.28	.75
.16	1.90	2.14	.64	.77	.38	.77

g) The effect of changing r on six key statistics

r	std(y)	std(x)/std(y)			corr(y,x)	
		x = i	x = c_m	x = h_m	x = h_c	x = p
0.5	1.83	3.03	.42	.81	-.02	.68
1.0	1.70	2.68	.52	.77	.16	.74
1.5	1.63	2.52	.59	.74	.24	.76
2.0	1.60	2.43	.64	.73	.27	.78
2.5	1.58	2.37	.67	.72	.29	.79

h) The effect of changing σ on six key statistics

σ	std(y)	std(x)/std(y)			corr(y,x)	
		x = i	x = c_m	x = h_m	x = h_c	x = p
.005	1.21	2.68	.52	.77	.16	.74
.006	1.45	2.68	.52	.77	.16	.74
.007	1.70	2.68	.52	.77	.16	.74
.008	1.94	2.68	.52	.77	.16	.74
.009	2.18	2.68	.52	.77	.16	.74

i) The effect of changing σ_m on six key statistics

σ_m	std(y)	std(x)/std(y)			corr(y,x)	
		x = i	x = c_m	x = h_m	x = h_c	x = p
.005	1.16	2.73	.66	.85	.19	.54
.006	1.42	2.71	.57	.79	.16	.66
.007	1.70	2.68	.52	.77	.16	.74
.008	1.98	2.65	.49	.76	.19	.79
.009	2.27	2.62	.47	.75	.23	.83

j) The effect of changing σ_n on six key statistics

σ_n	std(y)	std(x)/std(y)			corr(y,x)	
		x = i	x = c_m	x = h_m	x = h_c	x = p
.005	1.76	2.60	.46	.75	.27	.85
.006	1.74	2.64	.49	.75	.20	.80
.007	1.70	2.68	.52	.77	.16	.74
.008	1.66	2.71	.56	.79	.16	.67
.009	1.64	2.73	.62	.82	.17	.60

k) The effect of changing ρ_m on six key statistics

ρ_m	std(y)	std(x)/std(y)			corr(y,x)	
		x = i	x = c_m	x = h_m	x = h_c	x = p
.00	1.53	2.32	.62	.92	.52	.41
.50	1.49	2.49	.57	.87	.37	.52
.90	1.67	2.68	.51	.77	.16	.72
.95	1.70	2.68	.52	.77	.16	.74
.99	1.70	2.71	.54	.79	.17	.73

l) The effect of changing ρ_n on six key statistics

ρ_n	std(y)	std(x)/std(y)			corr(y,x)	
		x = i	x = c_m	x = h_m	x = h_c	x = p
.00	2.00	2.53	.64	.81	.44	.84
.50	1.92	2.67	.62	.79	.31	.85
.90	1.73	2.80	.57	.76	.12	.78
.95	1.70	2.68	.52	.77	.16	.74
.99	1.68	2.44	.48	.78	.32	.67

References

Bacus, D. and P. Kehoe (1989) "International Evidence on the Historical Properties of Business Cycles," Federal Reserve Bank of Minneapolis Working Paper 402R.

Bacus, D. and P. Kehoe and F. Kydland (1989) "International Trade and the Business Cycle," Federal Reserve Bank of Minneapolis Working Paper 426.

Becker, G.S. (1965) "A Theory of the Allocation of Time," *Economic Journal* 75, 493-517.

Becker, G.S. (1988) "Family Economics and Macro Behavior," *American Economic Review* 78 (March), 1-13.

Bencivenga, V. (1988) "An Econometric Study of Hours and Output Variation with Preference Shocks," manuscript, University of Western Ontario.

Benhabib, J., R. Rogerson and R. Wright (1988) "Home Production and Aggregate Fluctuations," paper presented to the Northwestern University Summer Workshop on Macroeconomics, Evanston, Illinois, August 1988.

Benhabib, J., R. Rogerson and R. Wright (1990) "Homework in Macroeconomics I: Basic Theory," manuscript.

Blanchard, O. and S. Fisher (1989) *Lectures in Macroeconomics.*

Christiano, L. (1986) "On the Accuracy of Linear Quadratic Approximations: An Example," Federal Reserve Bank of Minneapolis Research Paper.

Christiano, L. (1988) "Why Does Inventory Investment Fluctuate So Much?" *Journal of Monetary Economics* 21, 247-280.

Christiano, L. and M. Eichenbaum (1988) "Is Theory Really Ahead of Measurement? Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations," Federal Reserve Bank of Minneapolis Working Paper 412 and NBER Working Paper 2700.

Cho, J. and R. Rogerson (1988) "Family Labor Supply and Aggregate Fluctuations," *Journal of Monetary Economics* 21, 233-245.

Cooley, T. and G. Hansen (1989) "The Inflation Tax in a Real Business Cycle Model," *American Economic Review* 79 (September), 733-748.

Danthine, J.-P., J. Donaldson and R. Mehra (1989) "On Some Computational Aspects of Equilibrium Business Cycles Theory," *Journal of Economic Dynamics and Control* 13 (July), 449-470.

Eichenbaum, M. and L.P. Hansen (1990) "Estimating Models with Intertemporal Substitution Using Aggregate Data," *Journal of Business and Economic Statistics* 8, 53-69.

Eisner, R. (1988) "Extended Accounts for National Income and Product," *Journal of Economic Literature* 26, 1611-1684.

Eisner, R. (1989) *The Total Income System of Accounts*, University of Chicago Press.

Greenwood, J. and Z. Herkovitz (1990) "Home Production," manuscript, Federal Reserve Bank of Minneapolis.

Greenwood, J., Z. Herkovitz and G. Huffman (1988) "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review* 78 (June), 402-417.

Gronau, R. (1977) "Leisure, Home Production and Work – The Theory of the Allocation of Time Revisited," *Journal of Political Economy* 85, 1099-1123.

Gronau, R. (1985) "Home Production – A Survey," in *Handbook of Labor Economics*, vol. II, edited by O. Ashenfelter and R. Layard, North-Holland, Amsterdam.

Hansen, G. (1985) "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics* 16, 309-325.

Hansen, G. (1986) "Growth and Fluctuations," manuscript, UCSB.

Hansen, G., and T. Sargent (1988) "Straight Time and Overtime in Equilibrium," *Journal of Monetary Economics*.

Hawrylyshyn, O. (1976) "The Value of Household Services: A survey of Empirical Estimates," *The Review of Income and Wealth* 22, 101-131.

Hill, M. (1985) "Patterns of Time Use," in *Time, Goods and Well-Being*, edited by F.T. Juster and F.P. Stafford, University of Michigan.

Hodrick, R. and E. Prescott (1980) "Post War U.S. Business Cycles: An Empirical Analysis," manuscript, Carnegie-Mellon University.

King, R. and C. Plosser (1989) "Real Business Cycles and the Test of the Addelmans," manuscript, University of Rochester.

King, R., C. Plosser and S. Rebello (1987) "Production, Growth and Business Cycles: Technical Appendix," manuscript, University of Rochester.

King, R., C. Plosser and S. Rebello (1988) "Production, Growth and Business Cycles I. The Basic Neoclassical Model," *Journal of Monetary Economics* 21, 195-232.

Kydland, F. (1984) "Labor Force Heterogeneity and the Business Cycle," *Carnegie-Rochester Conference Series in Public Policy* 21, 173-208.

Kydland, F. and E. Prescott (1982) "Time to Build and Aggregate Fluctuations," *Econometrica* 50 (November), 1345-1370.

Kydland, F. and E. Prescott (1988a) "The Workweek of Capital and its Cyclical Implications," *Journal of Monetary Economics* 21, 343-360.

Kydland, F. and E. Prescott (1988b) "Cyclical Movement of of the Labor Input and its Real Wage," Federal Reserve Bank of Minneapolis Working Paper 413.

Kydland, F. and E. Prescott (1989) "Hours and Employment Variation in Business Cycle Theory," Federal Reserve Bank of Minneapolis Institute for Empirical Macroeconomics Discussion Paper 17.

Long, J. and C. Plosser (1983) "Real Business Cycles," *Journal of Political Economy* 91 (February), 36-69.

Lucas, R. (1976) "Understanding Business Cycles" *Carnegie-Rochester Conference Series in Public Policy* 5, 7-29.

McCallum, B. (1989) "Real Business Cycle Models," in R. Barro, edito, *Modern Business Cycle Theory*, Harvard University Press, Cambridge.

McGratten, E. (1988) "The Macroeconomic Effects of Tax Policy in an Equilibrium Model," manuscript, Stanford University.

Murphy, K., A. Schliefer and R. Vishny (1989). "Building Blocks of Market Clearing Macroeconomic Models," in O. Blanchard and S. Fisher, editors, *NBER Macroeconomics Annual* 1989, MIT Press, Cambridge.

Plosser, C. (1989) "Understanding Real Business Cycles," *Journal of Economic Perspectives* 3, 51-77.

Prescott, E. (1986) "Theory Ahead of Business Cycle Measurement," *Federal Reserve Bank of Minneaplois Quarterly Review* Fall 1986, 9-22.

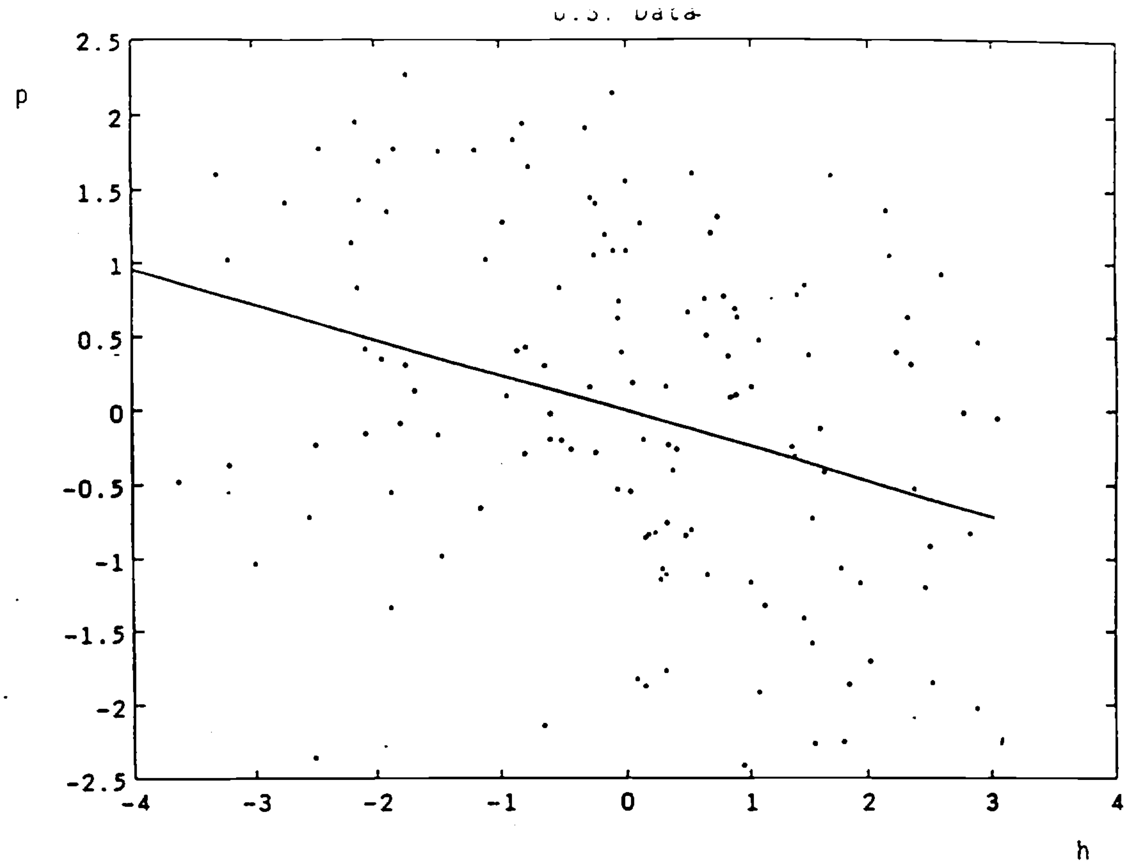
Rios-Rull, J.-V. (1988) "Working in the Market, Home Production and the Acquisition of Skills" A General Equilibrium Approach," Federal Reserve Bank of Minneapolis Working Paper.

Rogerson, R. (1984) *Topics in the Theory of Labor Markets*, unpublished Ph.D. dissertation, University of Minnesota.

Rogerson, R. (1988) "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics* 21 (January), 3-16

Rotemberg, J. and M. Woodford (1990), "Oligopolistic Pricing and the Sources of Business Cycle Fluctuations," manuscript.

Wright, R. (1988) "The Observational Implications of Labor Contracts in a Dynamic General Equilibrium Model," *Journal of Labor Economics* 6 (October), 530-551.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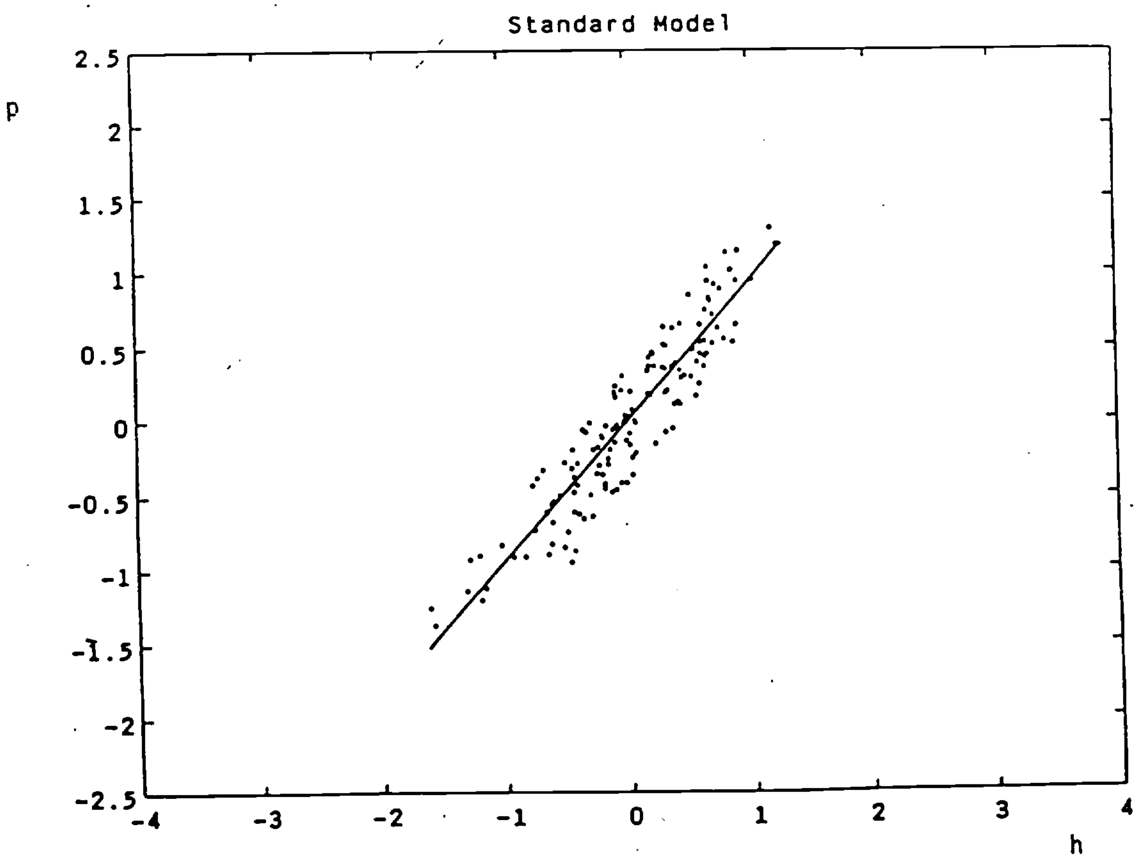
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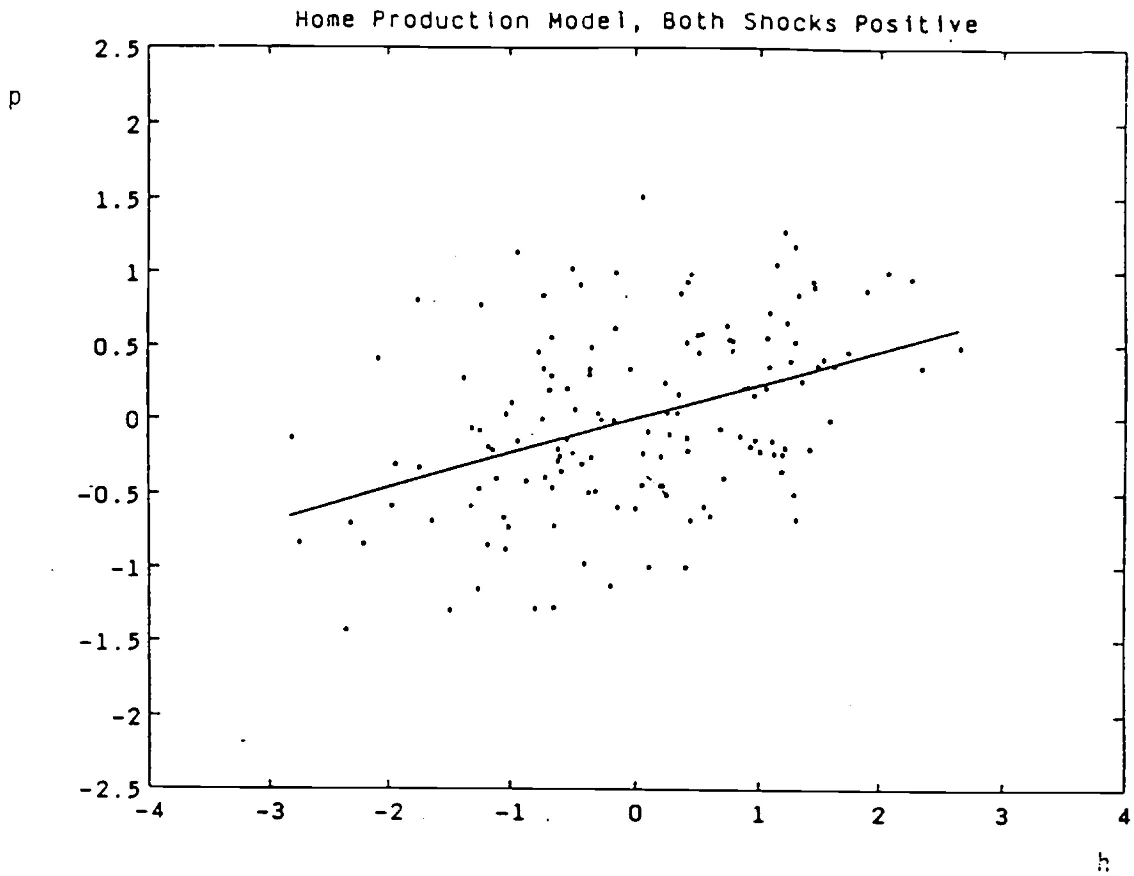
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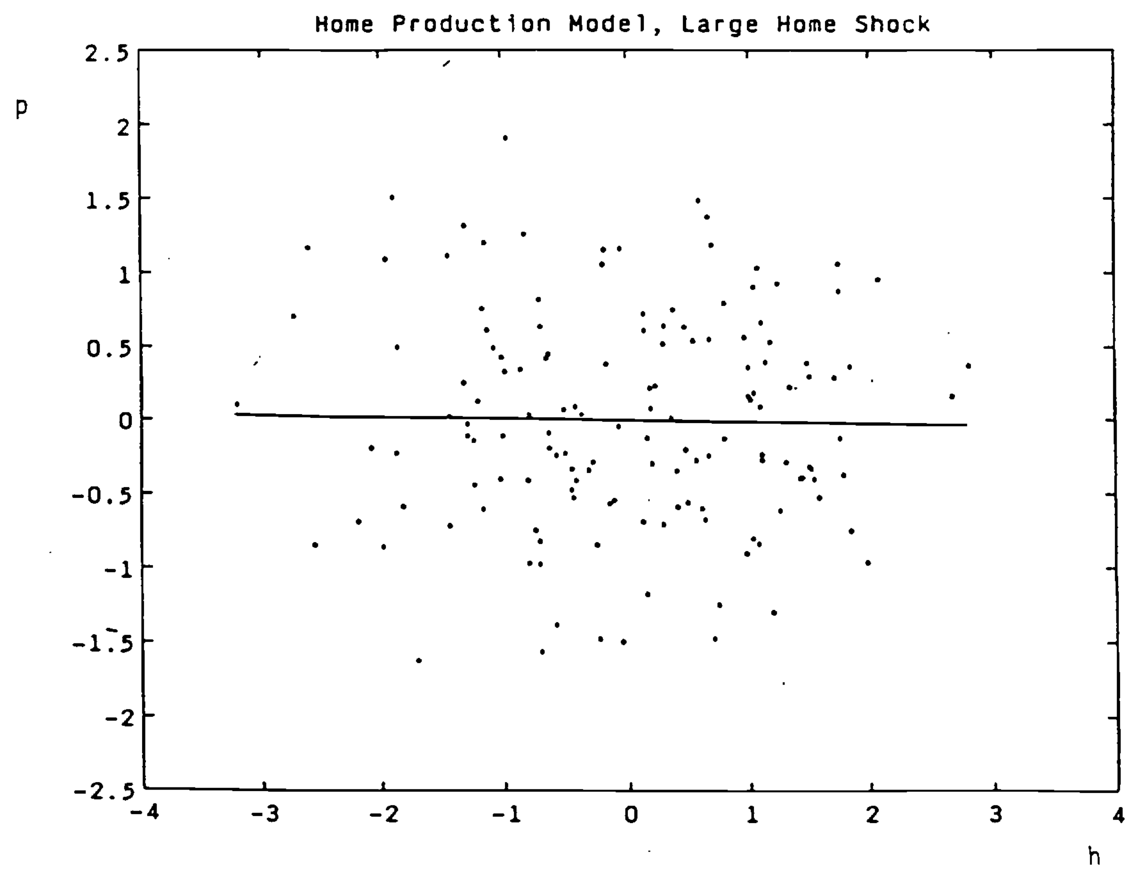
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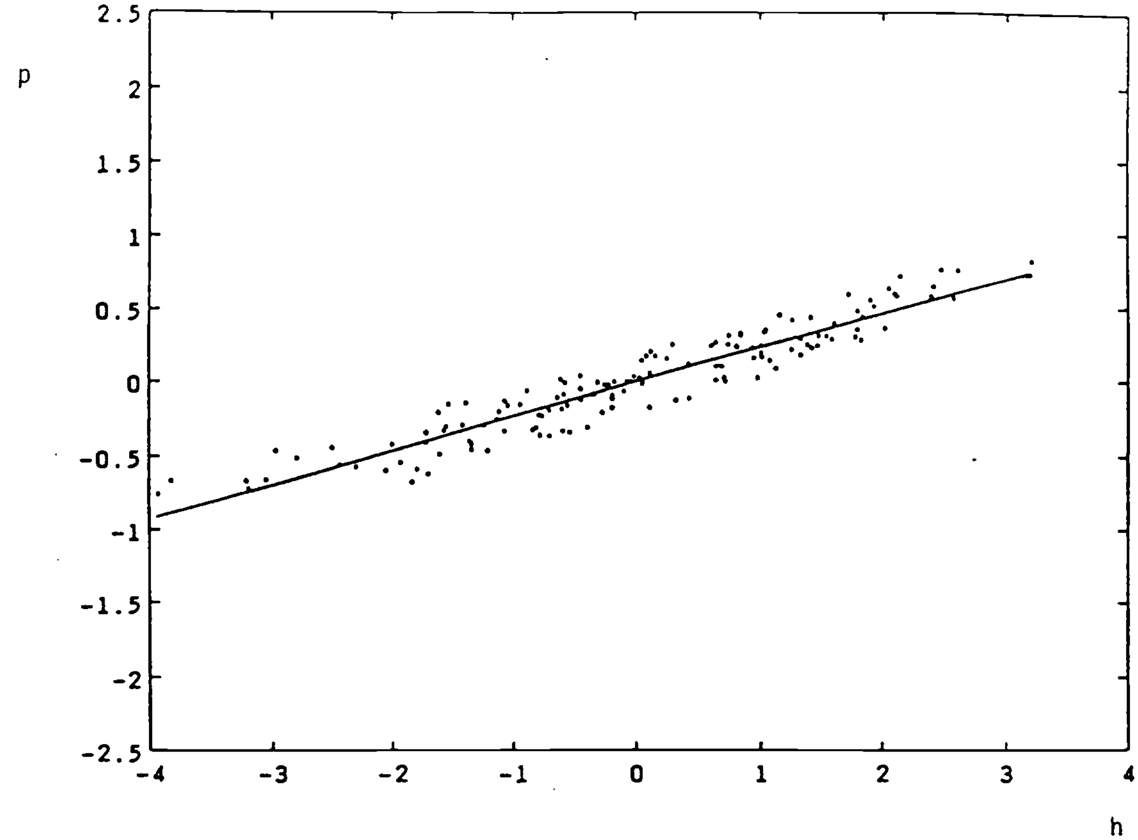
Home Production Model, Zero Home Shock



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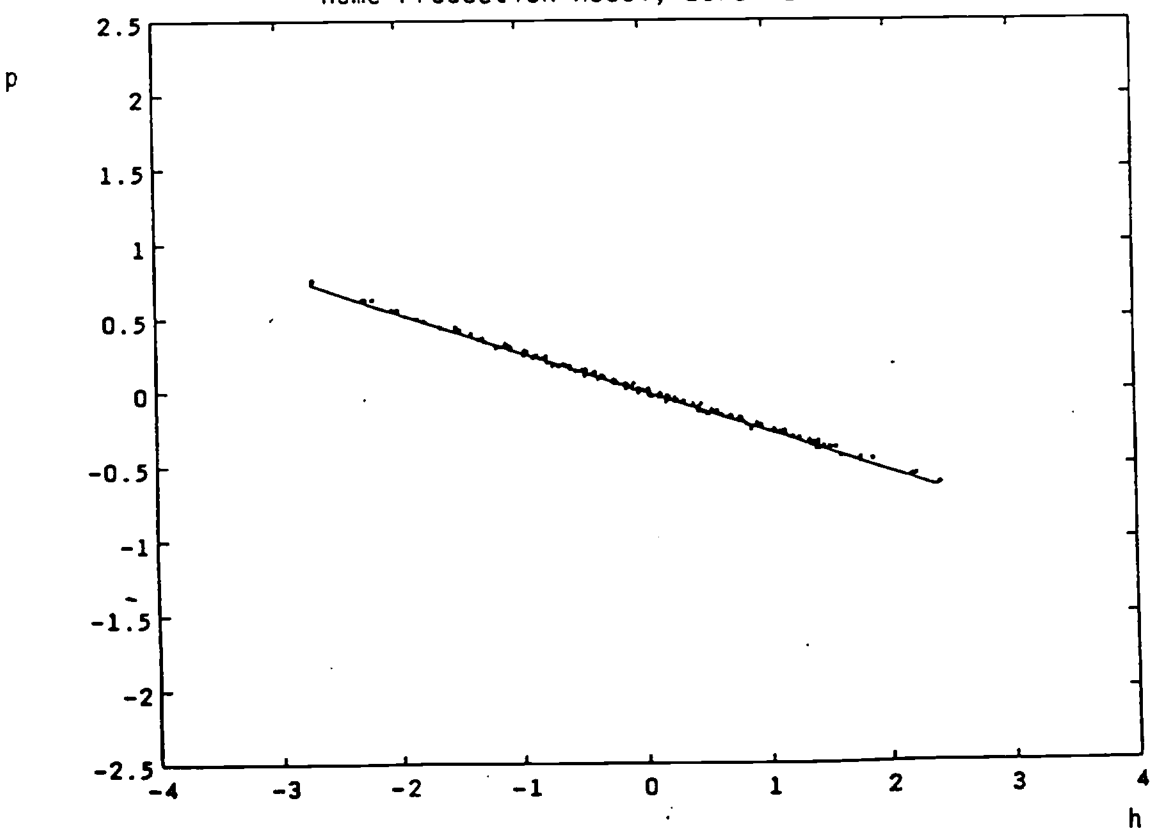

Table 1: The Effect of η on Steady State

η	c_m/y	c_n/y	k/y	k_m/y	k_m/k
.00	.74	.23	10.26	10.26	1.00
.02	.74	.23	10.57	10.26	.97
.04	.73	.24	10.90	10.26	.94
.06	.72	.25	11.24	10.26	.91
.08	.71	.26	11.60	10.26	.88
.10	.70	.28	11.98	10.26	.86
.12	.69	.30	12.37	10.26	.83
.14	.68	.32	12.77	10.26	.80
.16	.67	.34	13.20	10.26	.78
.18	.66	.37	13.65	10.26	.75
.20	.65	.40	14.12	10.26	.73

Table 2

a) U.S. Data: std(y) = 1.74

x =	c_m	i	p	k_m	h_m
$\frac{std(x)}{std(y)}$	.49	2.82	.52	.38	.86
cor(x,y)	.76	.96	.51	.28	.86

b) Standard Model: std(y) = 1.29

x =	c_m	i	p	k	h_m	h_i	h_c
$\frac{std(x)}{std(y)}$	.30	3.14	.52	.26	.50	2.66	.25
cor(x,y)	.90	.99	.99	.05	.98	.98	-.98

c) Home Production Model: std(y) = 1.71

x =	c_m	i	p	k	h_m	h_i	h_c	h_n
$\frac{std(x)}{std(y)}$	.51	2.73	.39	.23	.75	2.40	.59	.70
cor(x,y)	.69	.94	.75	.09	.94	.95	.10	-.76

d) Home Production Model With var(ϵ_n) = .01: std(y) = 1.61

x =	c_m	i	p	k	h_m	h_i	h_c	h_n
$\frac{std(x)}{std(y)}$	.68	2.82	.48	.24	.84	2.43	.93	.93
cor(x,y)	.68	.88	.54	.09	.89	.91	.14	-.63