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## FINANCIAL INTEGRATION, LIQUIDITY AND EXCHANGE RATES

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## FINANCIAL INTEGRATION, LIQUIDITY AND EXCHANGE RATES

#### ABSTRACT

This paper presents a two-country extension of Lucas' (1988) work on the effects of cash-in-advance constraints in asset markets on the pricing of financial assets. The model is one where there exists some degree of separation between the goods markets and the asset markets and money is used for transactions in both markets. The main results of the paper are the following. First, the equilibrium level of the exchange rate depends on the share of money used for asset transactions: a greater share will correspond to a more appreciated exchange rate. Second, under uncertainty, liquidity effects deriving from stochastic shocks to bond creation lead to an "excess" volatility of nominal and real exchange rates even when the "fundamental" value of the exchange rate is constant. Third, capital controls in the form of taxes on foreign asset acquisitions tend to appreciate the exchange rate. Fourth, the maturity structure of the public debt affects the equilibrium exchange rate. In particular, a move towards a longer maturity structure will tend to depreciate the exchange rate.

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#### 1. Introduction.

The objective of this paper is to analyze the open economy implications of models in which money is used both for transactions in the goods markets and for transactions in the asset markets. The model we present here is a two-country open economy extension of Lucas (1988) work on the effects of liquidity constraints in asset markets on the pricing of financial assets. Introducing cash-in-advance constraints in financial market transactions may be important for a number of reasons. As suggested by Lucas, a significant proportion of money holdings is managed by financial intermediaries rather than households. It is, therefore, important to consider the potential effects on asset prices of the demand for money by financial intermediaries. Specifically, as shown by Lucas, in models where money is required for asset transactions, asset prices will depend not only on their Fisherian fundamentals but also on the liquidity available in asset markets. In particular interest rates will show "excess volatility" that is not related to the volatility of the "fundamental" determinants of asset prices.<sup>1</sup>

As will be shown in our paper, in an open economy setting cash-in-advance constraints in the asset market have important additional effects. First, the equilibrium exchange rate will depend on the money demand in the asset markets and the share of money used in asset transactions. Second, the "excess" volatility of interest rates

 $<sup>^1</sup>$  As discussed by Lucas (1988), similar liquidity effects on assets prices are found in the work of Grossman and Weiss (1983) and Rotemberg (1984).

will spill over to the exchange rate market and lead to "excess" volatility of exchange rates as well. Third, the maturity structure of public debt will affect the equilibrium exchange rate by changing the equilibrium share of money used in asset transactions.

A important predecessor of our paper is the work of Helpman and Razin (1985) which first introduced the idea of a cash-in-advance constraint for asset transactions. However, both the specific nature of the cash-in-advance constraint in the asset market and the analytical objectives of Helpman and Razin are different from ours. In particular, Helpman and Razin analyze the case of a debtor country running a current account deficit which has to accumulate foreign currency in advance in order to repay its foreign debt. This generates an increase in the demand for foreign currency and thus a depreciation of the exchange rate. While in Helpman and Razin, it is the borrower that faces a cash in advance constraint (at the time of repayment), in our model it is the lender that faces a cash in advance constraint (at the time she purchases assets). Moreover, in our model, the current account is always balanced and thus there is no relation between the current account and the exchange rate.

The objectives of our paper are also different from those of Helpman and Razin. In particular, while we think that modeling the effects on the equilibrium exchange rate (and its volatility) of cash in advance constraints in asset market has an interest in its own, the model also allows us to analyze a number of important policy issues generated by the recent European Community decision to liberalize capital controls by 1990. In particular, this framework is well suited to address two important questions:

1) What will be the effect of this liberalization on the inter-EMS exchange rates ? Will the capital liberalization make it harder to maintain fixed exchange rate parities in the EMS area ?

2) Given the existence of large budget deficits and high public debt to GDP ratios in a number of EMS countries, will the capital liberalization make more difficult the financing and management of public debt ?

The first set of questions is important because, according to a popular argument<sup>2</sup>, capital controls have been important for the viability of the EMS. It is argued that capital controls in countries like France, Italy and Ireland have limited the potentially disruptive effects on the EMS fixed parities of disturbances external to the system (like movements of the dollar) or internal to the EMS (such as policy divergences and/or structural asymmetries among the member countries). This view suggests that the capital liberalization may render more difficult the goal of exchange rate stability in the EMS area.

The second set of questions is crucially linked to the first one because divergences in fiscal policy (and their ensuing monetary and/or bond financing requirements) are considered by many as one of the most important sources of potential exchange rate tensions in the EMS. In this view, restrictions to international capital movements in many EMS countries have helped the smooth financing of large budget deficits and have prevented (or limited) fiscal policy divergences in the EMS from disrupting the inter-EMS exchange rate parities. This view also suggests

 $<sup>^2</sup> See$  for example Giavazzi and Pagano (1986), Giavazzi and Giovannini (1986, 1989).

that the liberalization of capital movements might be incompatible with maintaining stable exchange rates in the EMS area in the absence of a process of fiscal policy convergence among the member countries.

Introducing liquidity constraints in asset transactions allows us to analyze capital controls in a more satisfactory way than existing studies. Traditionally, in the literature on the effects of foreign exchange controls (see for example Greenwood and Kimbrough (1987) and Stockman and Hernandez (1988)), restrictions on foreign exchange rate transactions are introduced in a way that makes them equivalent to restrictions on current account transactions rather than capital account transactions.

Given the substantial absence of restrictions on current account transactions in the OECD area and the widespread use of restrictions on capital account transactions, a study of the effects of capital controls on exchange rates requires the use of an analytical tool that permits discrimination between tariffs on goods and taxes on capital movements. The model that will be presented below has this property.

The formulation of a model with liquidity constraints on asset transactions allows us also to develop a more rich and realistic analysis of the second set of questions advanced above: the effects of structure of public debt on exchange rates and asset prices. Traditional cash-in-advance models with liquidity constraints on goods markets are usually formulated with one-period bonds only: therefore they do not have any implication for the term structure of interest rates. Even when multiperiod assets are introduced in these models, the perfect substitutability among these assets implies that the structure of public debt and the forms of financing of the budget deficit have no effect on

equilibrium interest rates and exchange rates. This theoretical irrelevance of the structure of the debt for the pricing of financial assets is at odds with the policy arguments that have been advanced in favor of a long maturity structure of the debt. In fact, in many EMS countries (Italy for example) it is argued that a public debt with a maturity structure biased towards short term bonds may have potentially disruptive effects on exchange rates. Compared to the above models characterized by the irrelevance of debt, the theoretical model presented in this paper has the attractive feature that the equilibrium level of the exchange rate depends, in a way to be qualified below, on the maturity structure of the debt.

The structure of the paper is as follows. Section 2 presents a two-country version of the closed economy model of Lucas (1988) and solves it for the case of certainty and one period bonds. Section 3 introduces capital controls as taxes on foreign asset transactions and analyses the effects of these controls on equilibrium exchange rates. Section 4 considers the relation between debt management, maturity structure of the debt and exchange rates by extending the menu of assets to multiperiod bonds. Section 5 solves the model for the case of uncertainty about the size of the government bond issue and discusses the effects of liquidity constraints on the volatility of exchange rates. Section 6 presents some concluding remarks and suggestions for future research.

## 2. The Model.

In this section we present the basic structure of the model that is a two-country extension of Lucas (1988). In this section we will

consider the certainty version of the model by making the assumption of perfect foresight. In section 5 we will develop the analysis under the case of uncertainty.

We consider a two-country world and assume that the representative agents in both countries have an identical intertemporal utility function given by:

(1) 
$$U = \sum_{t=0}^{\infty} \beta^{t} U(c_{it}^{1}, c_{it}^{2})$$
 i= 1,2

where  $c_{lt}^{i}$  is the consumption by country 1 (Italy) of the good produced by country i (i-1,2) and  $c_{2t}^{i}$  is the consumption by country 2 (France) of the good produced by country i,  $\beta$  is a discount factor between zero and one and the usual assumption on U(.,.) hold.

The good produced by each country is non-storable and, in order to concentrate on the liquidity effects, we assume that production in the two countries is constant at  $y_1$  and  $y_2$ . This assumption allows us to abstract from potential dynamic effects deriving from consumption smoothing in the presence of non-constant endowments.

The transaction technology is the one of a cash-in-advance model complicated by the assumption that the representative agent faces not one but two liquidity constraints: one on the purchase of goods and the other on the purchase of assets. As in Lucas (1988) we assume the convenient artifact of a three member representative household : each member has a different task during a period and the household regroups at the end of the period to pool goods, assets and information. One member receives the endowment and sells it to other households. The cash

receipts from this period t endowment can be used only in the next period. The second member takes a fraction of the household's initial money holdings (domestic and foreign currency) and uses it to buy domestic and foreign goods from other households. Domestic (foreign) goods can be bought only with domestic (foreign) currency. The third member of the household carries the remaining domestic and foreign currency cash balances to the securities markets where she buys domestic and foreign securities. As with the purchase of goods, domestic (foreign) assets can be bought only with domestic (foreign) cash balances.

In this section we assume that there are only two securities, domestic and foreign one-period government discount bonds  $B_t^1$  and  $B_t^2$ . Since the bonds are one-period, net and gross asset transactions coincide. In section 4 we will extend the menu of available assets.

At the beginning of period t, therefore, the typical household will face the constraints:

- (2.a)  $Z_{it}^{1} \ge q_{t}^{1} B_{it}^{1}$  i-1,2
- (2.b)  $Z_{it}^2 \ge q_t^2 B_{it}^2$  i-1,2
- (2.c)  $M_{it}^{1} Z_{it}^{1} \ge P_{t}^{1} C_{it}^{1}$  i-1,2
- $(2.d) \qquad M_{it}^2 Z_{it}^2 \ge P_t^2 C_{it}^2 \qquad i=1,2$

where  $Z_{it}^{j}$  is the amount of money of country j held by the representative agent of country i for transactions in the asset market at time t;  $M_{it}^{j}$ 

is the total amount of money of country j held by the resident of country i at time t;  $q_t^1$  and  $q_t^2$  are the price of the domestic and foreign currency prices of the domestic and foreign discount bond;  $P_t^1$  and  $P_t^2$  are the domestic and foreign currency prices of good 1 and 2, respectively. Equations (2.a) and (2.b) are the cash-in-advance constraints for the asset markets, and (2.c) and (2.d) are the equivalent cash-in-advance constraints for the goods markets.

The agents in the home country also face the beginning of t+1 period budget constraint given by:  $^{3}$ 

(3)  $M_{1t+1}^1 + \tilde{e}_{t+1} M_{1t+1}^2 = [Z_{1t}^1 - q_t^1 B_t^1] + \tilde{e}_{t+1} [Z_{1t}^2 - q_t^2 B_{1t}^2] +$ 

+  $[M_{1t}^1 - Z_{1t}^1 - P_t^1 C_{1t}^1] + \tilde{e}_{t+1} [M_{1t}^2 - Z_{1t}^2 - P_t^2 C_{1t}^2]$ 

A note regarding the timing of the transactions in this case of certainty. At the beginning of the period, before the households separates, the initial money holdings may be reallocated in the exchange rate market to obtain the desired quantities of domestic and foreign currency: this is the sense of the budget constraint (3). After that, the money holdings are allocated in the shares that will be used in the goods market and the asset market. No further transactions are needed or made in the exchange rate market during the period: this is the meaning of the separate constraints (2.a) and (2.b). One could, of course, assume that the exchange rate market remains open while the financial markets are open so that foreign currency can be bought and sold while financial transactions occur. In this case, the relevant liquidity constraint in the asset  $Z_{it}^{1} + E_{t} Z_{it}^{2} \ge q_{t}^{1} B_{it}^{1} + E_{t} q_{t}^{2} B_{it}^{2}$ market will be given by: instead of the separate conditions (2.a) and (2.b). However, it turns out that in this case of certainty, the two ways of formulating the liquidity constraint give the same solution to the model. What is important, in the aggregate, is that foreign bonds can be bought only with foreign currency and domestic bonds with domestic currency. In the case of uncertainty, that will be discussed in section 5 below, the timing of the various transactions is more important so that we will be more specific about the exact timing of the opening and closing of the different markets.

+ 
$$P_t^1 y^1$$
 +  $B_{1t}^1$  +  $\tilde{e}_{t+1} B_{1t}^2$ 

where  $\tilde{e}_t$  is the nominal exchange rate at time t (defined as units of domestic currency per unit of foreign currency). If the interest rate is positive, i.e. q < 1 <sup>4</sup>, both cash-in-advance constraints will hold with equality, and thus, we can substitute them into (3) to obtain:

(3.a) 
$$M_{1t+1}^{1} + \tilde{e}_{t+1} M_{1t+1}^{2} - P_{t}^{1} y^{1} + \frac{z_{1t}^{1}}{q_{t}^{1}} + \frac{\tilde{e}_{t+1} z_{1t}^{2}}{q_{t}^{2}}$$

Analogously, for the foreign country:

(3.b) 
$$\frac{1}{\tilde{e}_{t+1}} M_{2t+1}^{1} + M_{2t+1}^{2} - P_{t}^{2} y^{2} + \frac{1}{\tilde{e}_{t+1}} \frac{z_{2t}^{1}}{q_{t}^{1}} + \frac{z_{2t}^{2}}{q_{t}^{2}}$$

The only function of the government in this model is to engage in open market operations with government bonds. In particular, each period the government of country i issues an amount of bonds equal to  $B_t^i - x_t^i \quad M_t^i$ . As a result of this operation the rate of growth of money between the the beginning of period t and the beginning of period t+1 is  $(1-q_t^i) \quad x_t^i$ .

As in Lucas and Stokey (1987) and Lucas (1988) it is convenient to express the nominal variables of the model in terms of the money supply in which they are denominated. That is:

<sup>&</sup>lt;sup>4</sup> We will show below under what conditions this is true.

$$\begin{split} \mathbf{m}_{it}^{j} &= -\frac{\mathbf{M}_{it}^{j}}{\mathbf{M}_{t}^{j}} \qquad ; \qquad \mathbf{z}_{it}^{j} &= -\frac{\mathbf{z}_{it}^{j}}{\mathbf{M}_{t}^{j}} \\ \mathbf{b}_{it}^{j} &= -\frac{\mathbf{B}_{it}^{j}}{\mathbf{M}_{t}^{j}} \qquad \mathbf{p}_{t}^{j} &= -\frac{\mathbf{p}_{t}^{j}}{\mathbf{M}_{t}^{j}} \qquad ; \qquad \mathbf{e}_{t}^{j} &= \mathbf{e}_{t}^{j} &= \mathbf{M}_{t}^{2} \\ \mathbf{U}_{t}^{j} &= \mathbf{M}_{t}^{j} &= -\frac{\mathbf{p}_{t}^{j}}{\mathbf{M}_{t}^{j}} \qquad ; \qquad \mathbf{e}_{t}^{j} &= \mathbf{e}_{t}^{j} &= \mathbf{M}_{t}^{2} \\ \mathbf{U}_{t}^{j} &= \mathbf{M}_{t}^{j} &= -\frac{\mathbf{q}_{t}^{j}}{\mathbf{M}_{t}^{j}} \qquad ; \qquad \mathbf{e}_{t}^{j} &= \mathbf{e}_{t}^{j} &= -\mathbf{e}_{t}^{j} \\ \mathbf{U}_{t}^{j} &= -\mathbf{E}_{t}^{j} &= -\mathbf{E}_{t}^{j} &= -\mathbf{E}_{t}^{j} \\ \mathbf{U}_{t}^{j} &= -\mathbf{E}_{t}^{j} &= -\mathbf{E}_{t}^{j} &= -\mathbf{E}_{t}^{j} \\ \mathbf{U}_{t}^{j} &= -\mathbf{E}_{t}^{j} \\ \mathbf{U}_{t}^{j} &= -\mathbf{E}_{t}^{j} &= -\mathbf{E}_{t}^{j} \\ \mathbf{U}_{t}^{j} \\ \mathbf{U}_{t}^{j} &= -\mathbf{E}_{t}^{j} \\ \mathbf{U}_{t}^{j} \\ \mathbf{U}_{t}^{j} &= -\mathbf{E}_{t}^{j} \\ \mathbf{U}_{t}^{j} \\ \mathbf{U}_{t}^{j$$

We can now compute the stationary equilibrium of the model in terms of these normalized variables. Given the complete analogy between the domestic and the foreign agent maximization problem, we concentrate on the former:

P-1 
$$V(m_1^1 + e m_1^2) - \max_{\substack{z_1^1 \\ z_1^2}} U(\frac{m_1^1 - z_1^1}{p^1}, \frac{m_1^2 - z_1^2}{p^2}) + \beta V(m_1^{1'} + e' m_1^{2'})$$

where

$$\mathbf{m}_{1}^{1'} + \mathbf{e}' \ \mathbf{m}_{1}^{2'} - \frac{\mathbf{q}^{1} \ \mathbf{p}^{1} \ \mathbf{y}^{1} + \mathbf{z}_{1}^{1}}{\mathbf{q}^{1} [1 + (1 - \mathbf{q}^{1}) \ \mathbf{x}^{1}]} + \frac{\mathbf{e}' \ \mathbf{z}_{1}^{2}}{\mathbf{q}^{2} [1 + (1 - \mathbf{q}^{2}) \mathbf{x}^{2}]}$$

The first order conditions for the problem are:

(5.a) 
$$\frac{U_1(...)}{p^1} = \frac{\beta V_1(..)}{q^1[1 + (1 - q^1) x^1]}$$

(5.b) 
$$\frac{U_2(...)}{p^2} - \frac{\beta V_1(.) e'}{q^2[1 + (1 - q^2) x^2]}$$

and the envelope condition:

(6) 
$$V_1 (m_1^1 + e m_1^2) - \frac{U_1(...)}{p^1} - \frac{U_2(...)}{p^2 e}$$

Substituting the envelope condition (6) into (5.a) and (5.b) we then obtain:

(7) 
$$[1 + (1 - q^{i}) x^{i}] q^{i} - \beta$$
  $i = 1, 2$ 

Conditions identical to (7) would be obtained solving the foreign agent maximization problem.<sup>5</sup> Using the equilibrium conditions for the

<sup>&</sup>lt;sup>5</sup> It can be observed that conditions (7) imply that, in this certainty case, the real interest parity condition across the two countries is satisfied. In fact, the right hand side of (7) is equal to  $(1/(1+\delta))$  where  $\delta$  is common the rate of time preference in the two countries; while the left hand side is equal

security markets, that is:

(8) 
$$z_1^i + z_2^i - z^i - x^i q^i$$
  $i = 1, 2$ 

we get:

(9) 
$$z^{i} - \frac{\beta x^{i}}{1 + x^{i} - z^{i}}$$
  $i = 1, 2$ 

which are quadratic equations in  $z^1$  and  $z^2$ . Recalling that  $z^i$  must be less than one in equilibrium, we obtain:

(10) 
$$z^{i} = \frac{(1 + x^{i}) - [(1 + x^{i})^{2} - 4\beta x^{i}]^{1/2}}{2}$$
  $i = 1, 2$ 

We have so far assumed the existence of an interior equilibrium, i.e. one for which  $q^i < 1$ . Therefore, since  $q^i - (z^i / x^i)$ , we need that  $z^i < x^i$ . From (10) it is easy to see that this is always the case, given that  $\beta < 1$ . From (10) we also obtain that :

$$\frac{\partial z^{i}}{\partial x^{i}} - \frac{1}{2} \left( 1 - \frac{\left[ (1 + x^{i}) - 2\beta \right]}{\left[ (1 + x^{i})^{2} - 4\beta x^{i} \right]^{1/2}} \right) > 0$$

to the product of of  $(1+\pi)$  (where  $\pi$  is equal to the inflation rate) and q=(1/(1+i)) where i is the nominal interest rate. Since  $(1+\pi)/(1+i)$  is equal to (i+r) where r is the real interest rate, conditions (7) imply that  $r^1 = r^2 = \delta$ , or the equalization of the real interest rate in the two countries. Also, as will be shown below, the exchange rate depreciation is equal to the inflation rate differential between the two countries so that conditions (7) also imply that the nominal interest parity condition is satisfied.

It can also be proved that:

$$\frac{\partial z^{i}}{\partial x^{i}} < 1$$

i.e. a permanent increase in the rate of bond creation  $x^{i}$  will increase the share of money used for asset transactions by less than  $x^{i}$ . This result also implies that an increase in the rate of bond creation  $x^{i}$ will lead to a reduction in the price of bonds and an increase in the nominal rate of interest.

From the cash-in-advance constraint in the goods markets, and observing that in equilibrium  $m^{i} - m^{i'} - 1$ , we get:

(11) 
$$m_1^i + m_1^i - (z_1^i + z_2^i) - 1 - z^i - p^i (c_1^i + c_2^i) \quad i = 1, 2$$

Also, the equilibrium conditions in the goods markets are:

(12) 
$$c_1^i + c_2^i - y^i$$
  $i = 1, 2$ 

Combining the conditions (11) and (12) we then derive the equilibrium price levels as:

(13) 
$$p^{i} - \frac{1 \cdot z^{i}}{y^{i}}$$
  $i = 1, 2$ 

i.e. the equilibrium price levels will depend not only on the money supply and the output level but also on the share of money used for asset transactions. In particular, since goods prices depend on the money used in goods transactions, an increase in the share of money used in asset transactions z<sup>1</sup>, will reduce the money available for good market transactions and will therefore reduce the equilibrium price of goods.

We have now all the elements necessary to solve for the equilibrium steady state exchange rate. By goods market arbitrage, it must be:

(14) 
$$p_{1}^{21} p_{1}^{1} = e p^{2}$$

where  $p^{21}$  is the relative price of good 2 in terms of good 1 which, by the first order conditions is equal to  $U_2(.,.)/U_1(.,.)$ . Therefore:

(15) 
$$e = \frac{p^1}{p^2} \frac{u_2}{u_1}$$

and, using (13):

(16) 
$$e = \frac{(1 - z^1)}{(1 - z^2)} \frac{y^2}{y^1} \frac{v_2}{v_1}$$

From (16) it is straightforward to derive the equilibrium nominal exchange rate as:

(17) 
$$\tilde{\mathbf{e}}_{t} = \frac{(1-z^{1})}{(1-z^{2})} \frac{M_{t}^{1}}{M_{t}^{2}} \frac{y^{2}}{y^{1}} \frac{U_{2}}{U_{1}}$$

If we compare (17) with the expression for the exchange rate of a

typical cash-in-advance model, we notice that the crucial difference is represented by the ratio  $\frac{(1-z^1)}{(1-z^2)}$ . The intuition for the effect of this ratio on the equilibrium exchange rate is clear: part of the money supply is held for use in the asset market, and thus, does not enter in the determination of the goods prices. Given the results above, it is also clear that:

(18) 
$$\frac{\partial e}{\partial x^1} < 0$$
;  $\frac{\partial e}{\partial x^2} > 0$ 

that is a larger steady state size of open market operations  $x^1$  will tend to appreciate the exchange rate.

# An Example.

As we have seen above, in order to trace the liquidity effects of the exchange rate, we do not need to specify the functional form of the utility function. In this section, however, we provide a complete solution to this type of model, and to do so we will assume that the utility function is given by:

(19) 
$$U(c_i^1, c_i^2) - \log c_i^1 + \alpha \log c_i^2$$

As seen above, the liquidity effect (i.e. cash-in-advance constraint in the security markets) enabled us to derive the total total shares of the two monies used in this market:  $z^1$  and  $z^2$ . However, in this perfect foresight model, domestic and foreign assets are perfect substitute and thus, we cannot derive the geographical composition of these shares, i.e. the values of  $z_1^1$ ,  $z_2^1$ ,  $z_1^2$  and  $z_2^2$ . Any combination, in fact, would be consistent with equilibrium. An intuitively appealing way to obtain these separate aggregates is to compute asset (cash) holdings in such a way that the individuals do not need to formally access the foreign exchange market, since their asset holdings exactly deliver the amount of cash that they need for goods and asset transactions. Formally this solution is obtained by imposing separate budget constraints, one for each currency, i.e.:

(20) 
$$m_{i}^{i'} = \frac{q^{i} p^{i} y^{i} + z_{i}^{i}}{q^{i} [1 + (1 - q^{i})x^{i}]}$$
 i= 1, 2

(21) 
$$m_{i}^{j'} - \frac{z_{i}^{j}}{q^{j}[1 + (1 - q^{j})x^{j}]}$$
 j-1, 2

Given the functional specification (19) of the utility function, it it easy to show that the consumption allocations are given by :

(22) 
$$c_{1}^{i} - \frac{y^{i}}{1 + \alpha}$$
;  $c_{2}^{i} - \frac{\alpha y^{i}}{1 + \alpha}$   $i = 1, 2$ 

Moreover, substituting condition (7) into (21), we obtain:

$$(23) z_i^j - \beta m_i^j$$

Substituting (22) and (23) into the cash-in-advance constraint (2.c) and (2.d) we get:

(24.a) 
$$m_1^2 - \beta m_1^2 = \frac{p^2 y^2}{1 + \alpha}$$

(24.b) 
$$m_2^1 - \beta m_2^1 - \frac{\alpha p^2 y^2}{1 + \alpha}$$

Then, using the equilibrium price levels (13) we get:

(25.a) 
$$m_1^2 - \frac{(1 - z^2)}{(1 + \alpha)(1 - \beta)}$$

(25.b) 
$$m_2^1 = \frac{\alpha (1 - z^1)}{(1 + \alpha)(1 - \beta)}$$

From (25.a) and (25.b) we can immediately derive  $m_1^1$  and  $m_2^2$  using the fact that  $m_1^j + m_2^j = 1$ . Finally, the  $z_1^j$ 's are obtained by using (23).

The effects of changes in the rate of bond creation on these money shares can be summarized as following. A permanent increase in the rate of bond creation in country i  $(x^i)$  will: a) increase the total share of currency i held by agents in country i  $(m_i^i)$  and reduce the share held by agents in the other country  $(m_j^i)$ ; b) increase the share of cash holdings for asset transactions in currency i held by agents in country i  $(z_i^i)$  and reduce the share held by agents in the other held by agents in the other country ( $z_i^i$ ).

# 3. Capital Controls and Exchange Rates.

As we mentioned in the introduction, the framework used in this paper allows us to model capital controls in a form that is closer to actual restrictions on asset transactions than standard cash-in-advance models. In reality, in most countries capital controls take the form of a tax on (net) transactions in foreign securities. This can be easily formalized in our model with the imposition of a tax ( $\tau$ ) on the cash-in-advance constraint relative to foreign asset transactions. Assuming that country 1 is imposing this tax on the foreign asset transactions of domestic agents, we get:

(26) 
$$z_{1t}^2 = (1+\tau) q_t^2 b_{1t}^2$$

Since we assume that only the home country imposes capital controls, the only expression in the model that is affected is the home country budget constraint which, in normalized terms, is given by:

(27) 
$$m_{lt+1}^{1} + e_{t+1} m_{lt+1}^{2} - \frac{q_{t}^{1} p_{t}^{1} y^{1} + z_{lt}^{1}}{q_{t}^{1} [1 + (1 - q_{t}^{1}) x_{t}^{1}]} +$$

$$+ \frac{e_{t+1}[(1+r) q_t^2 v_{1t}^2 + z_{1t}^2]}{(1+r) q_t^2 [1 + (1-q_t^2) x_t^2]}$$

where  $v_{lt}^2$  is a (foreign currency) transfer (used to return in lump sum form the revenues from the foreign asset taxation) which we will define in detail later on.

The first order and envelope conditions for the domestic agents now imply:

(28.a) 
$$\beta$$
 = [1+(1-q^1) x<sup>1</sup>] q<sup>1</sup>

(28.b) 
$$\frac{\beta}{(1+\tau)} = [1+(1-q^2)x^2]q^2$$

while the foreign individual optimization would imply:

(29.a) 
$$\beta = [1 + (1 - q^1) x^1] q^1$$

(29.b) 
$$\beta = [1 + (1 - q^2) x^2] q^2$$

As is known from the literature on capital taxation in an open economy (see for example Hansson and Stuart (1986)), in the presence of taxes the strict interest parity condition cannot hold simultaneously for both countries. In other terms, (28.b) and (29.b) cannot be satisfied both with equality. Here, as usual in the literature, we assume that the home country holds both assets, so that (28.b) is the relevant expression determining the foreign price of bonds. <sup>6</sup> The foreign country, on the other hand, will hold only foreign assets, since the return on the home country bond is lower than the return on a foreign bond for a foreign resident.

Next, we have to specify what happens in the home country to the tax revenue from foreign asset transactions. In order to concentrate on the price effects of capital controls, we prevent changes in the money supply from occurring as a result of the tax. We assume, therefore, that

<sup>&</sup>lt;sup>6</sup> The alternative assumption that the country without controls is holding both assets would make the model trivial: domestic agents would not hold foreign assets and therefore the capital controls would have no effects on the exchange rate.

the government reintroduces in the system the money collected with the tax. There are two obvious alternative ways to achieve this result. According to the first, the government uses the foreign currency to purchase foreign goods and transfer them to the domestic agents. Alternatively, the government could use the proceeds to purchase foreign assets (foreign reserves) and transfer the proceeds back to the agents in the form of a transfer  $v_1^2$ . Both alternatives also have the advantage of eliminating possible wealth effects associated with the tax. It turns out that the effect on the exchange rate is the same under both arrangements.

Consider the first case, in which the money collected is spent in the foreign goods market. Equilibrium in the securities market implies:

(30.a) 
$$z_1^1 + z_2^1 - z_1^1 - \tilde{z}^1 - q^1 x^1$$

(30.b) 
$$\frac{z_1^2}{(1+\tau)} + z_2^2 = \tilde{z}^2 = q^2 x^2$$

Using these conditions together with conditions (28.a)-(28.b), we can derive the solutions for  $\tilde{z}^1$  and  $\tilde{z}^2$  :

(31.a) 
$$\tilde{z}^1 - \frac{(1+x^1) - [(1+x^1)^2 - 4\beta x^1]^{1/2}}{2}$$

(31.b) 
$$\tilde{z}^2 - \frac{(1+x^2) - [(1+x^2)^2 - 4\beta x^2/(1+\tau)]^{1/2}}{2}$$

We can observe that  $\tilde{z}^2$  is a negative function of  $\tau$ , the degree of capital controls, i.e. an increase in the tax on foreign asset purchases will reduce the total share of foreign money used for foreign asset purchases. <sup>7</sup>.

The domestic and foreign price levels are now given by:

(32.a) 
$$p^{1} - \frac{m_{1}^{1} + m_{2}^{1} - z_{1}^{1} - z_{2}^{1}}{y^{1}} - \frac{1 - \tilde{z}^{1}}{y^{1}}$$

As is known from the literature on two-country infinite horizon models, divergences in the rate of time preference in the two countries from the net real interest rate would lead to unappealing long run properties. Since interest parity conditions imply that the return on the domestic asset should be equal to the net return on foreign assets (the foreign real rate minus the rate of capital taxation, i.e.  $r = r^* - r$ ), equality of the two countries rates of time preference would imply that either the domestic rate of return and/or the foreign one will diverge from their respective rate of time preference (i.e. either  $r = r^* + \delta$  or  $r^* \neq \delta$ ). Since our interest here is to concentrate on the exchange rate effects of capital controls, we want to abstract from current account movements driven by divergences between rates of time preference and real interest rates. We therefore assume that the foreign discount factor ( $\beta^{*}$  =  $1/(1+\delta^{*}))$  is equal to  $\beta/(1+r)$ . In other terms, we hypothize that the difference between the foreign and the domestic rate of time preference is equal to the tax rate on foreign asset transactions  $(\delta^* - \delta - \tau)$ . It then follows that:  $\delta = r = (r^* - \tau) < r^* = \delta^*$ . The above assumption guarantees the equality between each country's rate of time preference and the net rate of return on financial assets while preventing the divergence between the two countries rates of time preference from creating incentives to borrow or lend. Our approach is quite similar to the one followed by Svensson (1988, 1989) who, in a different two-country model, assumes that the rate of time preference differs across countries in order to prevent country differences in the degree of risk aversion from affecting the current account. One could, of course, take a different approach and move to a finite horizon model or to an overlapping generations model: these models, however, would introduce dynamic effects such as borrowing and lending from which we want to abstract. Finally, whenever we perfom comparative statics exercises in which the rate of capital taxation is changed, we adjust the value of the discount factor  $\beta^*$  so that the equality of  $\beta^*$  with  $\beta/(1+r)$  is maintained and potential current account effects are neutralized.

(32.b) 
$$p^2 = \frac{m_1^2 + m_2^2 - z_1^2 - z_2^2 + (\tau/(1+\tau))z_1^2}{y^2} = \frac{1 - \tilde{z}^2}{y^2}$$

where  $\frac{\tau}{1+\tau} z_1^2$  is the amount collected by the government and used on the foreign good market.

It then follows that the exchange rate is given by:

(33) 
$$e = \frac{(1 - \tilde{z}^1)}{(1 - \tilde{z}^2)} \frac{y^2}{y^1} \frac{U_2}{U_1}$$

Since the equilibrium  $\tilde{z}^2$  is negatively related to an increase in  $\tau$ , it follows that a higher level of capital controls (a higher  $\tau$ ) will correspond to a higher level (i.e an appreciation) of the exchange rate.

The result that capital controls tend to appreciate the exchange rate is intuitive, and similar to results previously obtained by Greenwood and Kimbrough (1987) and Stockman and Hernandez (1988). However, it is important to notice that the reason for such an appreciation is quite different in our model and, we believe, more plausible. In the other models, foreign exchange controls act as a tariff on goods purchases (rather than a tax on asset purchases as a capital control should be); therefore, they alter the relative price of the goods and the equilibrium real allocation. Then, the effect on the exchange rate is through a change in terms of trade and a change in the marginal rate of substitution  $U_2/U_1$ . In our model, instead, there is no change in the relative price of the two goods or in their allocation across countries: capital controls are neutral from that point of view. Here, the introduction of capital controls reduces the demand for the

foreign security and the amount of foreign money held for transactions in the asset market. Consequently, there is an increase in the money holdings for transactions in the foreign goods market and this tends to increase the nominal price of foreign goods and thus appreciate the home country's exchange rate.

It is easy to show that the above results are unchanged under the alternative hypothesis that the revenues from the tax are used in the foreign security market. In this case the equilibrium in the foreign asset market would be modified to:

(34) 
$$\frac{z_1^2}{(1+\tau)} + z_2^2 + \frac{\tau}{(1+\tau)} z_1^2 - z_1^2 + z_2^2 - q^2 x^2$$

Using (34) together with (28.b), we obtain:

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(35) 
$$z^2 = \frac{(1+x^2) - [(1+x^2)^2 - 4\beta x^2/(1+r)]^{1/2}}{2}$$

The foreign price level is now given by:

(36) 
$$p^2 = \frac{m_1^2 + m_2^2 - z_1^2 - z_2^2}{y^2} = \frac{1 - z^2}{y^2}$$

From (31.b) and (35) it is also clear that  $\tilde{z}^2 - z^2$  and thus the exchange rate is the same as in (33).

The main result of this section is that capital controls have a positive effect on the exchange rate. This result suggests that a

liberalization of capital restrictions is likely to lead to a depreciation of the currency that is being liberalized. It also suggests that capital controls in a number of EMS countries might have been effective in limiting exchange rate depreciations resulting from divergent economic policies among the member countries.

## 4. Liberalization of Capital Movements and Debt Management.

Recently, we have observed considerable interest among academics and policy makers concerning the issue of the relation between debt maturity structure, sustainability of public debt and interest rates (see Giavazzi and Pagano (1989) and Alesina, Prati and Tabellini (1989)). This interest has been stimulated by the observation that in a number of EMS countries, increasing budget deficits have led to a sizable increase in the debt to GDP ratio .<sup>8</sup> However, not enough attention has been devoted to the implications for the exchange rate of the structure of public debt. In this section we would like to discuss some of these implications.

As will be seen below, we find in our framework a non-neutrality of the maturity structure that is reminiscent of the mechanism working in Giavazzi and Pagano (1989). However, here the mechanism is not driven by the probability of default (as in Giavazzi and Pagano) but rather by the relative "liquidity" of goods and asset markets.

In this and the later sections it is convenient to abstract from the growth in money supply: the model, in fact, does not have any particularly new implication regarding the effects of inflation.

<sup>&</sup>lt;sup>8</sup> As of 1989, the public debt is over 100% of GDP in three EMS countries: Italy, Belgium and Ireland.

Therefore, as in Lucas (1988), we assume that the government imposes a lump-sum tax (of size  $\pi^{i} M_{t}^{i}$ ) at the beginning of each period (prior to any trading), which is exactly sufficient to remove the increase in money generated by the previous period open market operation. Then, the tax is such that money growth is zero or:

(37) 
$$\frac{M_{t+1}^{i}}{M_{t}^{i}} = 1 + (1 - q^{i}) x^{i} - \pi^{i} = 1$$

Substituting (37) in (7), we obtain that the price of the one period bond now reduces to  $q^i - \beta$  and, therefore,  $z^i - \beta x^i$ . In other words, the real interest rate is equal to the rate of time preference  $(r - \delta)$ . In this type of model, this equality is independent of the maturity structure of the debt so that, an n period bond will be priced  $q_n^i - \beta^n$ . In an economy where only n-period bonds exist, the amount of money held for transactions on the asset market will therefore be  $z^i - \beta^i x_n^i$ . In the general case in which n different asset of maturity from 1 to n periods are contemporaneously issued the value of  $z^i$  would be:  $\circ$ 

$$(38) z^{i} - \sum_{j=1}^{n} \beta^{j} x_{j}^{i}$$

The important point of this exercise is to note that changes in the maturity structure of the debt alter  $z^{i}$  and, therefore, have effects on the level of the exchange rate. First, for equal face value of the

<sup>&</sup>lt;sup>9</sup> We refer the reader to Lucas (1988) for the formal, more general derivation of these type of results in a closed economy setting.

issue, i.e.  $x_m = x_n$  where m < n,  $z^i$  will be larger if the open market operation is conducted in the shorter maturity bond (m). Therefore, the steady state exchange rate will be more appreciated in this scenario.

In the case above, however, the total revenues from the bond issue are different. In fact, the shorter maturity bond, having a higher price, will deliver higher proceeds. Consider then the situation in which the government has a constant amount of debt outstanding and has to decide the steady state maturity of this debt. For simplicity, consider the two cases in which the government has all its debt in one period bonds  $(D_1)$  and the case in which the debt is in n-period bonds evenly spread so that each period  $D_n/n$  come to maturity. Then, in this case,  $x_1^i - D_1$  while  $x_n^i - D_n/n$ . Consequently, in the first case  $z^1 - \beta D_1$ while in the second  $z_n - \beta^n D_n/n$ . Since the present discounted value of the two forms of debt financing must be equal, it must be that:

(39) 
$$\sum_{j=0}^{n-1} \beta^j \frac{D_n}{n} = D_1$$

Since  $z_n = \beta^n D_n/n$  in the case of n-period bonds financing, using (39) we get:

(40) 
$$z_n - \beta^n \frac{(1 - \beta)}{(1 - \beta^n)} D_1 < \beta D_1 - z_1$$

This result implies that, the longer is the maturity structure of the debt, the smaller will be the equilibrium share (z) of money holdings used for asset transactions. The result also suggests that in the case of longer maturity bonds the exchange rate will be more depreciated:

since a longer maturity of the debt leads to a lower share of cash balances used for asset purchases (z), the equilibrium in the exchange rate market implies a more devalued currency.

This effect of the maturity structure on the exchange rate suggests two policy observations. First, it might explain why in countries with a short maturity structure of the debt (such as Italy), the exchange rate tends to be relatively strong in spite of the large money creation. If asset purchases are subject to a liquidity constraint, short term bond financing implies a larger share of money balances used for asset transactions and therefore a more appreciated exchange rate. Second, the result implies that a policy move towards a longer maturity structure of the debt will lead to a reduction of the share of money used for asset transactions that will tend to depreciate the exchange rate. It then follows that, if the objective of the policy makers to stabilize the exchange rate, a move toward a longer maturity of the debt should be accompanied by a reduction in the money supply so that the excess liquidity created by the fall in z is absorbed and the exchange rate does not depreciate.

Finally, it is interesting to compare the implications of the maturity structure for the exchange rate in our model and the the one of Giavazzi and Pagano (1989). In Giavazzi and Pagano (1989), a longer maturity of the debt reduces the probability of a devaluation and therefore may prevent the occurrence of a devaluation crisis. Our model, however, suggest that, <u>if</u> liquidity constraints in the asset markets are important, the monetary authorities should also consider the effects of a longer maturity on the exchange rate through the reduction in the share of money used for security purchases.

#### 5. Uncertainty and exchange rate volatility.

In this section we extend the previous framework to allow for uncertainty. The objective is to analyze how the results of Lucas (1988) on the "excess" volatility of the interest rate in the presence of liquidity effects extend to the exchange rate in an open economy setting.

As in Lucas, we assume that the only source of uncertainty in the model is the size of  $x^1$  and  $x^2$  which are distributed with a joint density function  $f(x^1, x^2)$ . We also assume that  $x^1$  and  $x^2$  are i.i.d. random variables. In the uncertainty case it is important to specify the exact timing of the various transactions, and of the opening and closing of the various markets. We assume that at the beginning of the period, each household divides their initial money balances between cash holdings for purchases of goods  $(n_i^1, n_i^2)$  and assets  $(z_{i,}^1, z_{i}^2)$ . Next, two different members of the household access the two different markets. The agent which transacts in the good market does not have any further opportunity to make transactions in the foreign exchange market <sup>10</sup>, and thus faces the constraints:

- (41.a)  $p^{1}c_{i}^{1} = n_{i}^{1}$  i = 1, 2
- (41.b)  $p^2 c_i^2 = n_i^2$  i = 1, 2

The agent that transacts in the exchange market, instead, has access to

<sup>&</sup>lt;sup>10</sup> One can think of this assumption as implying that the transactions costs to return on the exchange rate market are too high.

the foreign exchange market. Therefore she faces a budget constraint of the form:  $^{11}$ 

(42) 
$$z_i^1 + e z_i^2 - (m_i^1 - n_i^1) + e (m_i^2 - n_i^2) - q^1 b_i^1 + e q^2 b_i^2$$
 i-1,2

As before, transactions in the markets occur with the seller currency so that it is still true that bond prices are determined by :

(43.a) 
$$z^1 - q^1 x^1$$

(43.b) 
$$z^2 - q^2 x^2$$

One can observe that while  $x^1$  and  $x^2$  are stochastic, the equilibrium values of  $z^1$  and  $z^2$  are constant since the the  $x^i$ 's are assumed to be i.i.d. variables. The equilibrium conditions (43.a)-(43.b) then imply that  $q^1$  and  $q^2$  will be stochastic as well.

Formally, the representative domestic household solves:

(44) 
$$V(m_1^1, e m_1^2) = \max_{\substack{n_1^1, n_1^2, b_1^1, b_1^2}} U(\frac{n_1^1}{p^1}, \frac{n_1^2}{p^2}) + \beta \int_{x^1} \int_{x^2} V(m_1^{1'}, e' m_1^{2'}) f(dx^1, dx^2)$$
  
s.t.  $(m_1^1 - n_1^1) + e(m_1^2 - n_1^2) = q^1 b_1^1 + e q^2 b_1^2$ 

<sup>11</sup> Again, we assume here the existence of an interior equilibrium.

where 
$$m_1^1$$
 and  $m_1^2$  are defined by:

(45.a) 
$$m_1^1 - \frac{p^1y^1 + b_1^1}{[1 + (1-q^1) x^1]}$$

(45.b) 
$$m_1^2 = -\frac{b_1^2}{[1+(1-q^2)x^2]}$$

The first order conditions (where  $\lambda$  is the multiplier associated with the budget constraint) are:

(46.a) 
$$\frac{U'_{1}(\frac{n_{1}^{1}}{p^{1}})}{p^{1}} - \int_{x^{1}}\int_{x^{2}}\lambda(x^{1}, x^{2}) f(dx^{1}, dx^{2})$$

(46.b) 
$$\frac{U_2'(\frac{n_1^2}{p^2})}{p^2} - \int_{x^1} \int_{x^2} e^{\lambda(x^1, x^2)} f(dx^1, dx^2)$$

(47.a) 
$$\lambda - \beta = \frac{V'_1 (m_1^{1'}, e' m_1^{2'})}{[1 + (1 - q^1) x^1] q^1}$$

(47.b) 
$$e \lambda = \beta = \frac{V'_2(m_1^{1'}, e' m_1^{2'}) e'}{[1 + (1 - q^2) x^2] q^2}$$

and the envelope conditions are:

(48.a) 
$$V'_1(.) = \frac{U'_1(.,.)}{p^1}$$

(48.b) 
$$V'_{2}(..) = \frac{U'_{2}(...)}{p^{2}e}$$

Recalling that prices and  $n_i^1$  are not stochastic (since the resolution of uncertainty occurs after the decision on  $n_1^1$  and  $n_1^2$ ), from the first order conditions, integrating conditions (47.a)-(47.b) and using the envelope conditions, we get:

$$(47.a)' \quad 1 \quad - \quad \beta \quad \int_{x^{1}} \int_{x^{2}} \frac{1}{[1 + (1 - q^{1}) x^{1}] q^{1}} f(dx^{1}, dx^{2})$$

$$(47.b)' \quad 1 - \beta \int_{x^{1}} \int_{x^{2}} \frac{1}{[1 + (1 - q^{2}) x^{2}] q^{2}} f(dx^{1}, dx^{2})$$

Finally, using the equilibrium conditions (43.a) and (43.b), we obtain:

$$(48.a)' \qquad z^{1} - \beta \int_{x^{1}} \int_{x^{2}} \frac{x^{1}}{[1 + (1 - q^{1}) x^{1}]} f(dx^{1}, dx^{2})$$

(48.b)' 
$$z^2 - \beta \int_{x^1} \int_{x^2} \frac{x^2}{[1 + (1 - q^2) x^2]} f(dx^1, dx^2)$$

Moreover, if we simplify the environment by assuming the existence of a lump-sum tax  $\pi(x^1)$  that keeps the money holdings constant, the expressions (48.a) and (48.b) reduce to:

(49.a) 
$$z^{1} - \beta \int_{x^{1}} \int_{x^{2}} x^{1} f(dx^{1}, dx^{2})$$

(49.b) 
$$z^2 - \beta \int_{x^1} \int_{x^2} x^2 f(dx^1, dx^2)$$

We can further simplify this solution by assuming that  $x^1$  and  $x^2$  are independent (  $f(dx^1, dx^2) - f_1(dx^1) f_2(dx^2)$  ) so that:

(50.a) 
$$z^{1} = \beta \int_{x^{1}} x^{1} f_{1}(dx^{1})$$

(50.b) 
$$z^2 - \beta \int_{x^2} x^2 f_2(dx^2)$$

One can observe that the equilibrium  $z^1$  and  $z^2$  are constant over time, independent of the shocks. Similarly,  $n^1$  and  $n^2$  are constant so that the price levels in the two countries are constant as well. What about the equilibrium level of the exchange rate ? Notice that, since the asset markets and the good market are separate, under conditions of uncertainty the exchange rate cannot equilibrate both markets simultaneously. Here the exchange rate equilibrates the asset market, a consequence of our previous assumption that the asset market and the exchange rate market are simultaneously open. In particular, from (47.a)-(47.b) we obtain:

(51) e 
$$-\frac{p^1}{p^2}\frac{U_2}{U_1}\frac{q^1}{q^2}$$

or, using the equilibrium price levels:

(52) e - 
$$\frac{1 - z^1}{1 - z^2} \frac{y^2}{y^1} \frac{U_2}{U_1} \frac{q^1}{q^2}$$

Notice that the equilibrium exchange rate level is composed of two parts. First, a "fundamental" component:

(52.a) 
$$\frac{1 \cdot z^1}{1 \cdot z^2} \frac{y^2}{y^1} \frac{U_2}{U_1}$$

which is the exchange rate that would prevail in the absence of liquidity effects, i.e. the exchange rate reflecting the law of one price. Second, a part that would be traditionally defined as "non-fundamental" but that is still an equilibrium component in the framework of this model:

(52.b) 
$$\frac{q^1}{q^2} = \frac{z^1}{x^1} \frac{x^2}{z^2}$$

which is due to the random bond shocks in the asset market.

It can also be observed that, while arbitrage in the goods market does not hold instantaneously, it still holds in an "expected" sense.<sup>12</sup>

<sup>&</sup>lt;sup>12</sup> If this were not the case there would be systematic profit opportunities deriving from arbitrage in the goods market. Then, the assumption that the agen in the goods market does not access the exchange rate market because it is too

In fact, (52) implies that the expected exchange rate is given by the "fundamental" exchange rate.  $^{13}$ 

The equilibrium exchange rate (52), therefore, can be written in logs as:

(53) 
$$\ln e - s - k + \ln(x^2) - \ln(x^1)$$

where k is a constant including the non-stochastic components of the exchange rate. It then follows that the variance of s is given by:

(54) 
$$Var(s) = Var[ln(x^{1})] + Var[ln(x^{2})]$$

We therefore find for the exchange rate the same excess volatility result as found by Lucas for the interest rate. In particular, since output, real allocation, money holdings and prices are constant, the model would imply that the fundamental value of the exchange rate should be constant as well. Then, the variance of the exchange rate based on fundamentals should be equal to zero. Here, instead, stochastic open market operations lead to interest rate shocks, liquidity effects and exchange rate volatility in spite of the fact that the fundamental value of the exchange rate should be constant. In other terms, the excess volatility of interest rates found by Lucas for a closed economy spills over in an open economy on the exchange rate.<sup>14</sup>

costly to do so would be untenable.

<sup>13</sup> For simplicity, we disregard Jensen's inequality.

<sup>14</sup>We have so far considered the extreme case where a lump sum tax maintains constant money holdings so that the variance of the price level is zero; as seen above, in spite of this zero variance of prices, We have considered above the case in which  $x^1$  and are  $x^2$  are independent. In the general case in which the two random variables are correlated we would get:

(55) 
$$Var(s) = Var[ln(x^{1})] + Var[ln(x^{2})] - 2 Cov(ln(x^{1}), ln(x^{2}))$$

It is then easy to see that, even in the presence of unexpected liquidity shocks, the variance of the exchange rate can be reduced to the variance of its fundamental value (i.e. zero) if the monetary policies of the two countries are coordinated so that  $x^1$  and  $x^2$  are perfectly correlated.

Finally, one can observe that we obtain a continuous variation of the real exchange rate (re) measured as:

(56) 
$$\ln(re) = s + \ln(p^2) - \ln(p^1)$$

with all the movements being driven by movements in the nominal exchange rate e. In particular we obtain that:

(59) Var(ln(re)) = Var(s)

the variance of the exchange rate is positive and equal to the sum of the variance of the logarithms of the  $x^{i}$ 's. In general, without the tax, the variance of the inflation differential is given by:

$$\operatorname{Var}(\frac{p^{-}}{p^{1}}-\frac{p^{-}}{p^{2}}) = \operatorname{Var}(x^{1}) + \operatorname{Var}(x^{2})$$

which is still much smaller than the variance of the exchange rate (remember that  $x^i$  is constrained to be less than one in equilibrium).

It can be observed that this result is consistent with the empirical evidence about the high correlation between nominal and real exchange rates (e.g. Mussa (1987)). One can also notice that this result is obtained in a model in which prices are not sticky but are free to adjust to changes in money supply growth.

These results depend on the assumption that goods markets and asset markets (inclusive of the foreign exchange market), while simultaneously open, are spatially separated. An alternative modeling strategy is followed by Stockman and Svensson (1987) who assume a sequential order in the opening and closing of the markets. In particular, goods market open first and only after they close do the asset markets open. More importantly, foreign exchange markets are open both during the goods and assets transactions. In this way, in every period we observe two different exchange rates, each equilibrating the goods and the asset market, respectively. Our analysis could be easily reformulated assuming a similar type of sequential opening of markets. In that case we would also obtain two exchange rates in each period. The first, equilibrating the good market, would be equal to the "fundamental component" (52.a), and the other, equilibrating the asset market, would equal to the one derived in (52).

# 6. Conclusions

The main results of the paper are the following. In models where there exists some degree of separation between the goods market and the financial market, the exchange rate is affected by the relative liquidity of the two markets. In particular, the equilibrium level of

the exchange rate depends on the share of money used for asset transactions: the greater this share the more appreciated the exchange rate. Also, the share of money used for bond transactions is positively related to the rate of bond creation in the the economy. Second, capital controls in the form of taxes on foreign asset acquisitions tend to appreciate the exchange rate in the domestic economy because they reduce the share of foreign money used for asset transactions. Conversely, a liberalization of capital controls tends to depreciate the exchange rate. Third, the maturity structure of public debt affects the equilibrium exchange rate. In particular, a move towards a longer maturity structure will tend to depreciate the exchange rate by reducing the share of domestic money used for asset transactions. It then follows that countries which are simultaneously liberalizing capital movements and trying to lengthen the maturity structure of their debt may face a tendency towards a depreciation of their exchange rate. Fourth, under uncertainty, liquidity effects lead to volatility of nominal and real exchange rates even when the fundamental value of the exchange rate is constant. In particular, the liquidity effects on interest rates of stochastic shocks to bond creation spill over to the foreign exchange market and lead to excess exchange rate volatility. Then, only a very tight coordination of the monetary policies of two countries can assure the stability of the exchange rate in the presence of these stochastic liquidity effects.

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