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INTEREST RATE TERM PREMIUMS AND THE FAILURE OF THE SPECULATIVE EFFICIENCY HYPOTHESIS: A THEORETICAL INVESTIGATION

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ABSTRACT

This paper develops a new parity condition for international financial markets which relates differences between the forward exchange rate and the expected future exchange rate to interest rate term premiums. It begins with the general proposition that UIP cannot hold for all maturity horizons if interest rate term premiums are imperfectly correlated across countries and expectations are rational. The conditions under which UIP could hold for multiple horizons, under these two assumptions, are found to be very restrictive. It is argued that if UIP holds at all under these circumstances, it is only likely to hold at a very short time horizon. Finally, it is shown that under these assumptions, if UIP holds at the shortest time horizon then the difference between forward exchange rates and expected future spot rates at all other horizons will be the difference in expected term premiums at each maturity.

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Economists have puzzled over whether forward exchange rates are unbiased and efficient predictors of future spot exchange rates since the advent of floating exchange rates in 1973. Early tests during tended to support the hypothesis of speculative efficiency (Frenkel 1981), but more recent research has concluded that there are better ways to forecast exchange rates (Hansen and Hodrick 1980; Thomas 1986), and a substantial portion of the variance in the forecast error can be traced to predictable differences between these two exchange rates (Fama, 1984; Hodrick and Srivastava, 1986).

This paper develops a new parity condition for international financial markets which relates differences between the forward exchange rate and the expected future exchange rate to interest rate term premiums. It begins by showing that UIP cannot hold for all maturity horizons if (i) interest rate term premiums are not identically the same across countries and (ii) expectations are rational. Under the the two assumptions above, conditions required for UIP to hold for multiple horizons are are found to be quite restrictive, and if UIP holds at all it is only likely to hold at a very short time horizon. If it holds at the shortest time horizon, then the difference between the forward exchange rate and the expected future spot exchange rate will be the difference in expected term premiums at that maturity. This is the parity condition.

It is commonly supposed that during the floating rate period systematic differences between forward rates and future spot rates have been premiums for bearing the risk associated with exchange rate volatility. This hypothesis is supported by evidence in Cumby and Obstfeld (1981), who note that "[d]eviations from Fisher parity appear to be highly autocorrelated and so do not behave like expectational errors. The observed behavior of the series of deviations may be interpreted as evidence favoring the existence of a foreign exchange risk premium for most major currencies." However, attempts to relate forward premiums explicitly to this type of risk have met with only limited success (Geweke and Feige, 1978; Hodrick and Srivastava, 1984; Mark, 1985), and the effort to explain the systematic component of forecast errors continues.[1]

The evidence presented here indicates that under two assumptions -- imperfect correlation of term premiums across countries and rational expectations -- the price of the risk associated with interest rate term premiums will cause forward rates to differ systematically from expected future spot rates. In this sense the forward forecast error can be seen to reflect interest rate risk, even if exchange risk is ignored. Though there has long been an understanding of the fact that term premiums in interest rates are likely to be interdependent across countries, the contribution of this paper is to show how they will be jointly determined with exchange risk premiums.

Interest rate term premiums can exist even if agents are risk neutral, as Cox, Ingersoll, and Ross have shown (1981). In light of this the results of this paper paper indicate the theoretical possibility that "risk premiums" in the forward exchange market need not imply risk aversion on the part of market participants.

Following this brief Introduction, Part II presents the these theoretical results, while Part III discusses the assumptions underlying them. The literature on interest rate term premiums suggests strongly that they exist and that they are imperfectly correlated across countries, but that expectations may not be rational. The implications of the parity condition relating forward exchange rate forecast errors to interest rate term premiums are developed in Part IV. Part V concludes.

PART II: EXPECTATIONS OF FUTURE SPOT EXCHANGE RATES

The hypothesis of uncovered interest rate parity can be expressed as follows:

$$E_t ln\left[\frac{s(t+k)}{s(t)}\right] = [r(t,k) - r^*(t,k)]k$$

Here, $s(t)$ represents the spot exchange rate at time t, denominated in local currency per foreign currency. $r(t,k)$ is the continuously compounded yield on (euromarket) deposits in the domestic currency with maturity k, observed at time t, and $r^*(t,k)$ is the foreign currency counterpart to $r(t,k)$.term premiums in interest rates. Let $\Psi_t(\tau,k)$ be the instantaneous interest rate term premium expected at time t for instruments available at time τ with maturity k , calculated in terms of a continuously compounded excess return, and $R_t(\tau)$ be the expected instantaneous interest rate for instruments maturing instantly at time τ. Assume that $R_t(\tau)$ is continuous and integrable, in which case we define

$$\Psi_t(t,k)k = r(t,k)k - \int_0^k R_t(t+\tau)d\tau \qquad (1)$$

If the expectations hypothesis about the terms structure holds, then

$$r(t,k)k = \int_0^k R_t(t+\tau)d\tau .$$

and $\Psi_t(t,k) = 0$ for all $k \geq 0$. If, as we are assuming, there exist term premiums for some or all maturities longer than the shortest, one-period, horizon, then $r(t,k)k \neq \int_0^k R_t(t+\tau)d\tau$ for all $k > 0$, and therefore $\Psi_t(t,k) \neq 0$ for all $k > 0$ and $t \geq 0$.[2] For future reference, define $R_t(\tau,k) \equiv E_t\{r(\tau,k)\}$, and assume $R_t(\tau,k)$ is continuous over both τ and k, and that it has a finite number of critical points. Further, assume that $R_t(\tau,k)$ converges uniformly to $R_t(\tau)$ as $k \to 0$.

Having discussed uncovered interest parity and the term structure of interest rates, we are ready to take the first step towards developing the parity condition:

Proposition 1: *UIP cannot hold at all horizons under the following assumptions,*

(i) $\Psi_t(t,k)$ *is not identically equal to* $\Psi^*_t(t,k)$ *for some* $k > 0$,[3]

(ii) rational expectations.

Proof: Assume that UIP holds for some horizon $N > 0$. It will be shown that UIP cannot hold for at least one other horizon.

The assumption of rational expectations implies

$$E_0\left\{ln\frac{s(N)}{s(0)}\right\} = [r(0,N) - r^*(0,N)]N$$

where for notational simplicity the period in which expectations are formed is taken to be period 0. Using the definition of the interest rate term premium at the N-period horizon, we can restate this as follows:

$$E_0\left\{ln\frac{s(N)}{s(0)}\right\} = \left[\Psi_0(0,N)N + \int_0^N R_0(\tau)d\tau\right] - \left[\Psi^*_0(0,N)N + \int_0^N R^*_0(\tau)d\tau\right] \qquad (2)$$

Suppose that UIP also holds for some horizon $M < N$ where $N/M = H$, $H \in$ {Positive Integers}. Then by the definition of UIP and rational expectations, it must be the case that

$$E_0\left\{ln\frac{s(M)}{s(0)}\right\} = [r(0,M) - r^*(0,M)]M$$

and also

$$E_0\left\{ln\frac{s(2M)}{s(M)}\right\} = [r(M,M) - r^*(M,M)]M$$

and more generally

$$E_0\left\{ln\frac{s[(i+1)M]}{s(iM)}\right\} = [r(iM,M) - r^*(iM,M)]M$$

for any integer i. Together, these imply

$$E_0\left\{ln\frac{s(N)}{s(0)}\right\} = \left[\sum_{i=0}^{H-1} R_0(iM,M) - \sum_{i=0}^{H-1} R^*_0(iM,M)\right]M$$

The next step is to show that in the limit, as M gets smaller and approaches the instantaneous horizon,

$$E_0\left\{ln\frac{s(N)}{s(0)}\right\} = \int_0^N R_0(\tau)d\tau - \int_0^N R^*_0(\tau)d\tau \qquad (3)$$

This is proved in the Appendix.

Equations (2) and (3) imply that UIP holds for the instantaneous horizon and for any other horizon N, if and only if

$$\Psi_0(0,N)N = \Psi^*_0(0,N)N$$

that is, domestic and foreign term premiums are identically be the same at the N-period horizon.

Suppose, then, that the horizon N at which UIP holds has the property that domestic and foreign term premiums are identically the same. In this case UIP must also hold for the instantaneous horizon. By assumption (ii) there must be some other horizon, call it k, where term premiums are not identically the same across countries, so for this horizon UIP cannot also hold.

Likewise, suppose that the horizon N at which UIP holds has the property that domestic and foreign term premiums are not identically the same. Then UIP cannot hold for the instantaneous horizon.

QED

Proposition 1 says that if at least one country has a term premium for at least one maturity horizon, and their term premiums are not identical to each other, then UIP cannot hold for all maturities. The evidence supporting these assumptions will be discussed in Part III.

The next step in developing the parity condition is to show that UIP is unlikely to hold for more than one horizon at any one time. It might seem likely that UIP would hold for two horizons M and N if term premiums for these two horizons were always equal to each other. This condition turns out to be neither necessary nor sufficient for UIP to hold at those horizons, and the conditions under which UIP holds for any two horizons are actually quite restrictive.

Proposition 2: *Under the following two assumptions,*

(i) $\Psi_t(t,k)$ *is not identically equal to* $\Psi^*_t(t,k)$ *for some* $k > 0$;

(ii) rational expectations;

UIP can hold for two horizons M and N, $M \neq N$, *only if for all time periods t*

$$\sum_{i=0}^{X-1} \Psi_t(iN,N)N = \sum_{i=0}^{Y-1} \Psi_t(iM,M)M$$

and (4)

$$\sum_{i=0}^{X-1} \Psi^*_t(iN,N)N = \sum_{i=0}^{Y-1} \Psi^*_t(iM,M)M$$

where X and Y are the smallest positive integers such that $XN = YM$.

Proof: Suppose that UIP holds for some non-zero horizon M as well as the horizon N, $M \neq N$, $M \gtrless N$. There will also exist some third horizon H and positive integers X and Y such that

$XN = YM = H$. Taking the current period to be period 0, once again, note that the proof of Proposition 1 implies

$$E_0\left\{ln\frac{s(XN)}{s(0)}\right\} = \sum_{i=0}^{X-1} R_0(iN,N)N - \sum_{i=0}^{X-1} R*_0(iN,N)N \tag{5a}$$

and likewise

$$E_0\left\{ln\frac{s(YM)}{s(0)}\right\} = \sum_{i=0}^{Y-1} R_0(iM,M)M - \sum_{i=0}^{Y-1} R*_0(iM,M)M \tag{5b}$$

We can use equation (1) to restate equalities (4) and (5) as follows:

$$E_0\left\{ln\frac{s(XN)}{s(0)}\right\} = \left[\int_0^{XN} R_0(\tau)d\tau + \sum_{i=0}^{X-1} \Psi_0(iN,N)N\right] - \left[\int_0^{XN} R*_0(\tau)d\tau + \sum_{i=0}^{X-1} \Psi*_0(iN,N)N\right] \tag{5a'}$$

$$E_0\left\{ln\frac{s(YM)}{s(0)}\right\} = \left[\int_0^{YM} R_0(\tau)d\tau + \sum_{\tau=0}^{Y-1} \Psi_0(iM,M)M\right] - \left[\int_0^{YM} R*_0(\tau)d\tau + \sum_{i=0}^{Y-1} \Psi*_0(iM,M)M\right] \tag{5b'}$$

Since $XN = YM = H$,

$$E_0\left\{ln\frac{s(XN)}{s(0)}\right\} - E_0\left\{ln\frac{s(YM)}{s(0)}\right\} = 0,$$

which implies that

$$0 = \left[\int_0^{XN} R_0(\tau)d\tau + \sum_{i=0}^{X-1} \Psi_0(iN,N)N\right] - \left[\int_0^{XN} R*_0(\tau)d\tau + \sum_{i=0}^{X-1} \Psi*_0(iN,N)N\right]$$
$$- \left[\int_0^{YM} R_0(\tau)d\tau + \sum_{i=0}^{Y-1} \Psi_0(iM,M)M\right] + \left[\int_0^{YM} R*_0(\tau)d\tau + \sum_{i=0}^{Y-1} \Psi*_0(iM,M)M\right]$$

or

$$0 = \underbrace{\left[\sum_{i=0}^{X-1} \Psi_0(iN,N)N - \sum_{i=0}^{Y-1} \Psi_0(iM,M)M\right]}_{A} - \underbrace{\left[\sum_{i=0}^{X-1} \Psi*_0(iN,N)N - \sum_{i=0}^{Y-1} \Psi*_0(iM,M)M\right]}_{B} \tag{6}$$

Under assumption (ii) equation (6) cannot hold unless equations (4) also hold.

QED

It is easy to provide an example illustrating why identical expected term premiums for the N-period and M-period horizon is insufficient to ensure that equations (4) are satisfied. For example, suppose that N is one and M is two, that currently in the home country the one- and two-period term premiums per period are both "1%," and that both home country term premiums are expected to rise to "2%" next period. Then the term premiums one can expect to earn from rolling a one-period investment over once exceed the expected term premiums associated with investing once in a two-period investment, and

$$\sum_{i=0}^{X-1} \Psi_t(iN,N)N \;=\; 3\% \;\neq\; \sum_{i=0}^{Y-1} \Psi_t(iM,M)M \;=\; 2\%$$

The requirements on expected term premiums captured by equations (4) are actually very difficult to satisfy. The conditions will be violated in another very simple case: if expected term premiums rise monotonically with the maturity of the instrument, as most people think they do, and are expected to be constant over time, as is frequently assumed in economic research (Mishkin, 1981, is one example). In fact, rates of growth of term premiums over time will be constrained by the rates at which term premiums change with maturity. For example, suppose once again that $N = 1$ and $M = 2$, and that two-period premiums are always 10% higher than one-period premiums. In this case, term premiums must be expected to rise *indefinitely* by exactly 20% per period, in order to satisfy conditions (4). Likewise, for the simple case of $N = 1$ and $M = 2$, term premiums must decline with maturity if they are expected to fall over time, and they must rise with maturity if they are expected to rise over time.

It is clear that UIP will not just happen to hold for some two horizons, it must be made to hold for those two horizons by some effort on the part of agents which brings their expectations of future term premiums into line with the current term structure, or brings the term structure into line with their expectations. If agents are going to arrange this for some pair of horizons M and N there is no reason they would not arrange this for any other pair of horizons. Taken to its logical extreme, this implies that UIP would hold at all horizons if it holds at more than one -- which cannot hold, according to Proposition 1.

Our analysis so far indicates that if UIP holds at all it will hold for very few horizons, most likely only one. Of all the possible horizons at which UIP has been hypothesized to hold in the past, is there any one in particular horizon at which it is most likely to hold? In the very abstract world analysed so far, the shortest horizon -- the instantaneous horizon -- seems to be the most likely one. At whatever horizon UIP holds, the rewards to exchange rate speculation are expected to be zero, which is a minimum since these rewards cannot be negative. Therefore speculators are likely to assign the minimum reward to speculation to that horizon at which the risk of speculation is also minimized: the instantaneous horizon.

So far nothing has been said about forward exchange rates. It is known that covered interest parity holds for Euromarket interest rates, or

$$ln\left[\frac{f(t,k)}{s(t)}\right] = [r(t,k) - r^*(t,k)]k \quad \forall\, k$$

where $f(t,k)$ represents the forward rate observed in the market at time t applying to transactions which will take place at time $t+k$. Adding to this the assumption that UIP holds at the shortest practicable horizon and the two assumptions underlying Propositions 1 and 2 we arrive at the parity condition which is central to this paper:

Proposition 3: The difference between the forward exchange rate and the corresponding expected future spot rate will be the difference across countries in expected term premiums at that horizon under the following assumptions

(i) $\Psi_t(t,k)$ *is not identically equal to* $\Psi^*_t(t,k)$ *for some* $k > 0$,

(ii) rational expectations;

(iii) UIP holds for the instantaneous horizon.

Proof: Taking 0 to be the current period once again, equation (3) shows that if UIP holds at the shortest horizon, then

$$E_0\left\{ln\frac{s(N)}{s(0)}\right\} = \int_0^N R_0(\tau)d\tau - \int_0^N R^*_0(\tau)d\tau$$

Taking the difference between the expected exchange rate change and the forward premium, we find that

$$E_0 ln\left[\frac{s(k)}{s(0)}\right] - ln\left[\frac{f(0,k)}{s(0)}\right] = \int_0^N R_0(\tau)d\tau - \int_0^N R^*_0(\tau)d\tau - [r(0,k) - r^*(0,k)]k$$

or

$$E_0 ln\left[\frac{s(k)}{s(0)}\right] - ln\left[\frac{f(0,k)}{s(0)}\right] = \Psi^*_0(0,k)k. - \Psi_0(0,k)k \qquad (8)$$

QED

This result can be interpreted as making rigorous the conditions under which the following conjecture by Logue and Sweeney (1984) would hold true:

> If the forward rate is appropriate to eliminate profitable interest arbitrage opportunities between, say, Eurodollars and Euro Swiss Francs and there are different term premiums in each, the forward rate would have to reflect this difference in term premiums. Under this circumstance...it is unlikely that the forward exchange rate is an unbiased predictor of future spot exchange rates.[4]

In Proposition 3 the analysis is taken a step further than this, and the bias in the forward exchange rate is characterized.

Thus far this paper has developed two implications of assuming (i) imperfect correlation of interest rate term premiums across countries and (ii) rational expectations. Specifically, UIP cannot hold at all horizons and is unlikely to hold for more than one. The next section examines the evidence concerning these two premises. The implications of parity condition represented by equation (8) are examined in Part IV.

PART III: EVIDENCE REGARDING INTEREST RATE TERM PREMIUMS AND RATIONAL EXPECTATIONS

All three propositions in Part II are based on the two fairly basic assumptions: interest rate term premiums are imperfectly correlated across countries, and expectations are rational. What is the evidence regarding term structures and expectations?

A good place to start is the question, Do term premiums exist? The theoretical literature on this topic has found many sets of conditions under which term premiums should exist. Particularly familiar is the approach of Vasicek (1977) and Cox,Ingersoll, and Ross (1981) who base their analyses on the assumption that the instantaneous riskless rate of interest follows a diffusion process and is the only state variable determining all bond prices. Some support for the Cox, Ingersoll, Ross approach is presented in Brown and Dybvig (1986), but the model has the unfortunate implications that all bond returns will be perfectly correlated and that long term interest rates are constant, neither of which seems likely to be supported empirically. Dothan (1978) also analyses a model in which risk-free instantaneous interest rates follow a diffusion process. He shows that even if instantaneous returns on all pure discount bonds are expected to equal the instantaneous risk free rate and utility is presumed to be logarithmic, "the explicit valuation formula for bonds, or, equivalently, the term structure, is quite complicated." This model is not very consistent with conventional views of the term structure, nor with most observed yield curves, however, since expected returns decline monotonically with maturity.

The empirical literature in this area, extensive as it is, has achieved a fairly strong consensus that term premiums do exist. Roll (1970) finds that these are "uniformly positive" over the range of maturities 1 - 25 weeks (p. 99). Startz (1982) also finds evidence of term premiums in returns to U.S. Treasury Bills when he concludes that at least 44 percent of the variance of the two-month-ahead variance of the one-month forward rate is due to changes in term premiums, and at least 36 percent of the variance of the three-month-ahead variance of the one-month forward rate is due to changes in term premiums.

Fama has documented the existence of term premiums in Treasury Bills (1984a), Treasury Bonds (1984b), and also in short-term private issuer securities (1986). With respect to bills and bonds, in his (1984b) paper he defines term premiums as "the difference between the one-month return on an instrument with a given maturity and the return on a one-month bill," an adjustment appropriate for tests of premiums in instruments with very long maturities. He finds that the tests "produce strong evidence against the hypothesis that all expected premiums are zero favor of the hypothesis that expected premiums are positive" (p. 531). For two- to twelve-month bills he finds that "the average premiums ... are all positive in all periods."[5]

In Fama's (1984a) paper, he analyses term premiums on Treasury Bills, Bankers' Acceptances, commercial paper, and "prime-quality" CDs. His analysis of CDs is especially important since euromarket time deposits are most closely related to these securities. He finds that term premiums do exist for CDs, that they "tend to increase with maturity when business activity is strong," (p. 176) and during recessions they "often decline monotonically with maturity" (p. 191).[6]

Assumption (i) says that term premiums will vary across countries. This assumption is supported by evidence presented in Logue and Sweeney, (1984). "We show term premiums exist and that they are variable. We further show that term premiums across Eurocurrency interest rates are uncorrelated." Campbell and Clarida (1987) are unable to reject the hypothesis that time-varying risk premia in different countries "move in proportion to a single latent variable." However, this latent variable leaves most of the variability in realized term premiums unexplained, even though the analysis is not cross-sectional, which implies that there may be country-specific factors affecting term premiums as well as this latent variable. More recent research by Kool and Tatom (1988) challenges the results of Campbell and Clarida. Kool and Tatom find that even though long-term interest rates are related across countries, as are short-term rates, long rates are not related to the short-term rates of other countries, so "the relationship between long-term nominal interest rates does not arise indirectly through an international term-structure transmission or through common short-term-rate movements that are transmitted through the domestic term structures" (p. 42).

In sum, the evidence indicates that term premiums exist in the major industrialized nations, and that while they may move in parallel sometimes in response to changes in some single variable, they are not perfectly correlated across countries. The evidence does not support the hypothesis that term premiums rise monotonically with the maturity of the instrument, or that they fall monotonically. In the U.S. term premiums do seem to change monotonically for maturities under 9 months, and tests of UIP have without exception involved horizons well below 9 months. Together, the evidence implies that if UIP holds for any single horizon below 9 months it could not be expected to hold for any other such horizon. Therefore, assumption (i) of Propositions 1 and 2 is likely to hold.

Assumption (ii), rational expectations, is the most likely source of problems with Propositions 1 and 2. A number of papers in which survey data on foreign exchange forecasts are tested against actual exchange rate realizations indicate that expectations very well may not be rational. Dominguez (1986) found that during 1983 through 1985 "consensus forecasts ... failed consistently at predicting future changes in the spot rate." In particular, at longer horizons (such as 1 or 3 months) there was "a tendency for forecasters to over-predict the size of future spot depreciation" (p. 281). Froot and Frankel (1989) concur, finding that expectational errors in the foreign exchange market seem to be correlated with forward premiums, and "investors would do better if they reduced fractionally the magnitude of expected depreciation." Similar criticisms have been leveled at interest rate forecasts by, among others, Friedman (1980), who finds that survey data on interest rate forecasts do not fully incorporate all available information and are therefore not consistent with the hypothesis of rationality. Froot (1987) analyses interest rate forecasts and finds that their rationality depends on the horizon: forecasts for shorter rates are rational while forecasts for longer rates do not seem to be.

Studies such as these are open to a number of serious critiques, among which the most significant is the observation that not all agents must be rational for prices in a given market to be efficient in the sense that all publicly available information is used to its best advantage: it is the expectation of the "marginal" investor that determines whether a given market will perform this way.[7] Suvery data cannot capture these "marginal" expectations. With regard to interest rates, some authors have challenged the view that survey forecasts accurately represent expectations of marginal investors. For example, Mishkin (1981) tests rationality in interest rate forecasts indirectly, without the presumption that surveys represent actual market forecasts, by running constrained and unconstrained regressions relating short and long term interest rates. He concludes that "the bond market data provide no evidence that interest rate forecasts are irrational," and infers that Friedman's "evidence.which finds irrationality in the Goldsmith-Nagan survey of interest rate expectations can be interpreted as casting doubt on the accuracy of this survey measure in describing market expectations" (p. 305).

Challenges to the hypothesis of expectational rationality in financial markets come from equity markets, as well, where the presumption of efficiency common in the 1970s was challenged when Schiller (1981) compared the actual volatility of stock prices to an estimate of their volatility had the future actually been known beforehand. His finding that the actual volatility of stock prices exceeds the maximum consistent with rational expectations, and his associated conclusion that expectations over-react to news, is consistent with the findings for foreign exchange expectations cited above. However, Schiller's methodology has been improved upon by a number of other authors, who do not all support his conclusions: for example, Flavin (1983) supports the conclusion of excess volatility, while Kleidon (1986) finds that a careful examination of stock price volatility need not rule out market efficiency.

In sum, the evidence regarding expected interest rate term premiums across countries seems supportive of the assumption that they are imperfectly correlated across countries, while the evidence concerning rationality of expectations is mixed.

PART IV: IMPLICATIONS OF THE PARITY CONDITION

In Proposition 3 a very concrete answer was provided to the question: What causes forward rates to differ from expected future spot rates? Specifically, at any maturity $m > 0$ these two variables will differ by the difference between domestic and foreign expected term premiums corresponding to maturity m.

The practical implication of this result are clear: if domestic term premiums exceed foreign ones, then to make a profit one should sell forward the foreign currency, and *vice versa*. It is generally thought by those that trade in international bond markets that Germany typically has a steeper yield curve than the U.S., which would imply that it should on average be profitable to buy DM forward in exchange for dollars, and sell them for dollars in the spot market as the contracts mature. In fact, another strategy common among participants in the international financial markets, referred to as a "currency hedge investment," relies on this view of forward rates and interest rate term premiums. Specifically, a trader borrows dollars on a 1-year basis, say, buys DM in the spot market, and uses the proceeds to invest in a 1-year DM security. The security would be hedged for a shorter time horizon, however, such as 1 month, at the end of which the position is unwound or rolled over. If the expected term premium on 1-year DM securities relative to 1-month DM securities exceeds the corresponding expected term premium on 1-year dollars, then the trader can expect to make profits on the scheme.[8]

Does the existence of risk premiums in the foreign exchange market imply that agents must be risk averse? The implication of this paper is that the answer may be "No." There are two reasons for this, both of which are based on the analysis in Cox, Ingersoll, and Ross (1981). First, they show that the expectations hypothesis as defined in the present paper is inconsistent with continuous time general equilibrium in a rational expectations market, whether or not agents are risk neutral. This has to do with non-linearities in functions over which expectations are taken and Jensen's inequality, but it does imply that the "expectations hypothesis" in its one permissible form, the "local expectations hypothesis" (see note 2), might hold while term premiums of the sort defined here would exist. Back-of the envelope calculations with regard to observed term premiums in bond markets indicate that even the "local expectations hypothesis" is not satisfied, however. Cox, Ingersoll, and Ross also show that risk neutrality is consistent with violations of the local expectations hypothesis; in this case foreign exchange "risk" premiums could be consistent with interest rate term premiums that in turn were consistent with risk neutrality.

How does this relate to recent studies of exchange risk premiums in the forward market? Hansen and Hodrick (1983), Hodrick and Srivastava (1984), Mark (1985), and Lewis (1988c), all show that under certain circumstances the difference between the forward exchange rate and the expected future spot rate should be determined according to the following relation:

$$E_0\left\{\frac{U'(c_k)P_0}{U'(c_0)P_k}\left\langle ln\left[\frac{f(0,k)}{s(0)}\right] - ln\left[\frac{s(k)}{s(0)}\right]\right\rangle\right\} = 0 \tag{9}$$

where $U(.)$ is the utility function, and P_t. is the price level at time t In turn, this implies that the k-period exchange-risk premium must satisfy

$$E_0\left\{ln\left[\frac{f(0,k)}{s(k))}\right]\right\} = \pi(0,k) = \frac{\text{cov}\left[\frac{U'(c_k)P_0}{U'(c_0)P_k}; ln\left[\frac{f(0,k)}{s(k))}\right]\right]}{E_0\left[\frac{U'(c_k)P_0}{U'(c_0)P_k}\right]}$$

This states that the risk premium for foreign exchange speculation at the k-period horizon is proportional to the conditional covariance of the intertemporal marginal rate of substitution in dollars and the forward premium.

Is the derivation which gives equation (8) inconsistent with these analyses? No. The k-period exchange-risk premium determined in equation (9) should be determined jointly with the total premium described in equation (8). That is, the k-period term premiums at home and abroad and the sequence of expected one-period exchange risk premiums must satisfy

$$[\Psi_0(0,k) - \Psi^*{}_0(0,k)]k = \pi(0,k) = \frac{\text{cov}\left[\frac{U'(c_k)P_0}{U'(c_0)P_k}; ln\left[\frac{f(0,k)}{s(k))}\right]\right]}{E_0\left[\frac{U'(c_k)P_0}{U'(c_0)P_k}\right]}$$

This shows that k-period pure exchange risk premiums must be jointly determined with term premiums in both countries, if rational expectations holds, term premiums are imperfectly correlated across countries, and UIP holds at the instantaneous horizon.

If UIP holds instead for a slightly longer horizon such as one week the relation between forward and expected future spot exchange rates can still be characterized in terms of risk premiums. Let the generalized term premium notation, $\Psi(t,\lambda,k)$, refer to the expected premium for investing at the k-week horizon instead of investing at the λ-week horizon. Further, let "1 period" correspond to "1 week". Using reasoning similar to that of Propositions 1 through 3 it is straightforward to show that the difference between the expected future spot exchange rate and the forward rate is still the difference between foreign and domestic term premiums:[9]

$$E_0 ln\left[\frac{s(k)}{s(0)}\right] - ln\left[\frac{f(0,k)}{s(0)}\right] = \left[\Psi_0(0,1,k) - \Psi^*{}_0(0,1,k)\right]k. \tag{10}$$

Equation (10) implies that international differences in expected term premiums relative to one week returns should determine the predictable component of exchange rate forecast errors based on forward rates. This is consistent with many otherwise unrelated empirical observations concerning these errors and interest rates. For example, movements in the term structure seem to be autocorrelated, at least in the United States (see Roll 1970 or Oldfield-Rogalski 1987). If this were true for both countries, and the term structure movements represented autocorrelated movements in term premiums, at least in part, then the difference between domestic and foreign term premiums would also be autocorrelated. Cumby and Obstfeld (1981) find that in fact exchange risk premiums do move in an autocorrelated manner.

Buckles (1985) and Korajczyk (198?) have noted that forecasting errors seem to be correlated with real interest rate differentials across countries. This is also consistent with equation (10), since under rational expectations *ex post* real interest rate differentials will incorporate within them *ex post* term premiums, which will be correlated with *ex ante* term premiums. Likewise, Logue and Sweeney (1987) find that incorporating measures of the relative steepness of yield curves, specifically the difference between one-month and three-month Euromarket rates, adds significantly to the explanatory power of a regression of actual exchange rate changes on the forward premium.

It is possible to characterize the relationship between expected future spot rates and forward rates in terms of international differences in term premiums even if UIP does not hold at the one week horizon, or at any horizon at all. Suppose concretely that at the one week horizon there is a risk premium dividing the forward rate from the expected future spot rate. This particular risk premium would represent the pure foreign exchange risk associated with uncovered exchange rate speculation for one week. Mathematically, assume:

$$E_0 ln\left[\frac{s(1)}{s(0)}\right] = ln\left[\frac{f(0,1)}{s(0)}\right] + \pi(0,1) = [r(0,1) - r^*(0,1)] + \pi(0,1) \tag{11}$$

where $\pi(0,1)$ is the exchange-risk premium for the one-period horizon. Under rational expectations, it will be true that

$$E_0 ln\left[\frac{s(2)}{s(1)}\right] = [R_0(1,1) - R^*{}_0(1,1)] + \Pi_0(1,1) \tag{12}$$

Together (11) and (12) imply

$$E_0 ln\left[\frac{s(2)}{s(0)}\right] = [r(0,1) + R_0(1,1)] - [r^*(0,1) + R^*{}_0(1,1)] + [\pi(0,1) + \Pi_0(1,1)]$$

From this and covered interest parity we would conclude that

$$E_0 ln\left[\frac{s(1)}{s(0)}\right] - ln\left[\frac{f(0,1)}{s(0)}\right] = E_0 ln\left[\frac{s(1)}{f(0,1)}\right]$$

$$= [\pi(0,1) + \Pi_0(1,1)] + 2[\Psi_0(0,1,2) - \Psi^*{}_0(0,1,2)]$$

and, more generally,

$$E_0 ln\left[\frac{s(k)}{s(0)}\right] - ln\left[\frac{f(0,k)}{s(0)}\right] \equiv \pi(0,k)$$

$$= [\pi(0,1) + \sum_{i=1}^{k}\Pi_0(i,1)] + [\Psi_0(0,1,k) - \Psi^*{}_0(0,1,k)]k$$

This equation highlights neatly the fact that two types of risk can determine the difference between the forward rate and the expected future spot rate. The first term in square brackets captures the exchange rate risk inherent in uncovered speculation on exchange rates. The second term in square brackets captures the importance of price risk, or whatever it is that causes expected returns for securities with longer maturities to exceed expected returns to securities with shorter maturities.

If UIP does not hold at the one-week horizon, k-period exchange risk premiums, 1-week exchange risk premiums, and term premiums will all be jointly determined:

$$[\pi(0,1) + \sum_{i=1}^{k-1} \Pi_0(i,1)] + [\Psi_0(0,1,k) - \Psi^*_0(0,1,k)]k = \pi(0,k) = \frac{\mathrm{cov}\left[\frac{U'(c_k)P_0}{U'(c_0)P_k} ; ln\left[\frac{f(0,k)}{s(k))}\right]\right]}{E_0\left[\frac{U'(c_k)P_0}{U'(c_0)P_k}\right]}$$

PART V: CONCLUSIONS

In this paper a parity condition was developed which related the difference between the forward exchange rate and the expected future spot exchange rate to differences across countries in interest rate term premiums.

The parity condition is based on three hypotheses. The first of these, imperfect correlation of term premiums across countries, has substantial support from the term structure literature. The second hypothesis, rational expectations, is widely adopted in both the economics and finance literatures but finds only mixed support when tested against actual data. Together these assumptions imply Propositions 1 and 2 of this paper: UIP cannot hold at all horizons, and is very unlikely to hold at more than one horizon.

A direct implication of these two propositions is that if UIP holds at all it is most likely to hold at a very short horizon, in which case the parity condition and Proposition 3 follows directly.

UIP may not hold at any horizon, in which case the difference between the forward exchange rate and the expected future spot exchange rate at maturities M greater than m periods should still be positively related to the difference between domestic and foreign term premiums for that maturity M, but expected future m-period pure-exchange risk premiums will also be part of that difference. One avenue of future research is to test forward exchange forecast errors to see if they are related to international differences in term premiums.

This paper has not presented an integrated theoretical explanation of risk premiums in interest rates and exchange rates. It has shown at a theoretical level that exchange risk premiums will be closely tied to interest rate term premiums under the assumption of rational expectations, but it has not gotten to the bottom of the matter because the reason(s) for the existence of interest rate term premiums is (are) still unclear. An appropriate step for future research might be a two-country analysis of term premiums using the continuous time framework of Cox-Ingersoll-Ross (1981, 1985) augmented to incorporate forward exchange rates.

APPENDIX

In this appendix it is proved that in the limit, as $M \to 0$,

$$E_t\left\{ln\frac{s(t+N)}{s(t)}\right\} = \left[\sum_{i=0}^{H-1} R_t(iM,M) - \sum_{i=0}^{H-1} R*_t(iM,M)\right]M \tag{A1}$$

becomes

$$E_t\left\{ln\frac{s(t+N)}{s(t)}\right\} = \int_0^N R_t(t+\tau)d\tau - \int_0^N R*_t(t+\tau)d\tau \tag{A2}$$

We begin by proving that

$$\lim_{J\to\infty} \sum_{i=0}^{J-1} R_t(t+i\frac{N}{J},\frac{N}{J})\frac{N}{J} = \int_0^N R_t(t+\tau)d\tau$$

where M has been replaced with N/J .

(1) Note that since $R_t(t+\tau,k)$ converges uniformly to $R_t(t+\tau)$ as $k \to 0$,

$$\forall\ \tilde{\delta} > 0\ \exists\ \tilde{\varepsilon}\ s.t.\ \forall\varepsilon \le \tilde{\varepsilon},\ \left|R_t(t+\tau,\varepsilon) - R_t(t+\tau)\right| < \tilde{\delta}\ \forall\ \tau \ge 0$$

or

$$\forall\ \tilde{\delta} > 0 \text{ and any } N > 0\ \exists \tilde{J} \in \{Z^+\} \text{ such that } \forall J > \tilde{J}$$
$$\left|R_t(t+\tau,\frac{N}{J}) - R_t(t+\tau)\right| < \tilde{\delta}\ \forall\ \tau \ge 0$$

By the triangle inequality it must be the case that

$$\forall\ J \ge \tilde{J}\ \left|\sum_{i=0}^{J-1} R_t(t+i\frac{N}{J},\frac{N}{J}) - \sum_{i=0}^{J-1} R_t(t+i\frac{N}{J})\right|\frac{N}{J} < \frac{N}{J}\tilde{\delta}J = N\tilde{\delta}$$

(2) Similarly, since $R_t(t+\tau)$ is integrable, it must be the case that

$$\forall\ \bar{\delta} > 0 \text{ and } \forall N > 0\ \exists\ \bar{J} \in \{Z^+\} \text{ s.t. } \forall J \ge \bar{J}\ \left|\sum_{i=0}^{J-1} R_t(t+i\frac{N}{J})\frac{N}{J} - \int_0^N R_t(t+\tau)d\tau\right| < \bar{\delta}$$

(3) The claim that $\lim_{J\to\infty} \sum_{i=0}^{J-1} R_t(i\frac{N}{J},\frac{N}{J})\frac{N}{J} = \int_0^N R_t(t+\tau)d\tau$ is equivalent to the claim that

$$\forall\ \delta^* > 0\ \exists\ J^* \in \{Z^+\}\ \text{s.t.}\ \forall\ J \geq J^*\ \left| \sum_{i=0}^{J-1} R_t(t+i\frac{N}{J},\frac{N}{J})\frac{N}{J} - \int_0^N R_t(t+\tau)d\tau \right| < \delta^*$$

I will prove that such a J^* exists by construction. For a given $\delta^* > 0$, chose some arbitrary $\bar{\delta}$ such that $\delta^* > \bar{\delta} > 0$, and calculate $\tilde{\delta}$ such that $\delta^* - \bar{\delta} = N\tilde{\delta}$. Find $\tilde{J}$ such that $\forall\ J \geq \tilde{J}$,

$$\left| \sum_{i=0}^{J-1} R_t(i\frac{N}{J},\frac{N}{J}) - \sum_{i=0}^{J-1} R_t(i\frac{N}{J}) \right| \frac{N}{J} < N\tilde{\delta}$$

Also, find $\bar{J}$ such that $\forall\ J \geq \bar{J}$,

$$\left| \sum_{i=0}^{J-1} R_t(t+i\frac{N}{J})\frac{N}{J} - \int_0^N R_t(t+\tau)d\tau \right| < \bar{\delta}$$

To construct J^*, we simply take the largest of J and J. In this case, we can invoke the triangle inequality once again to show that

$$\left| \sum_{i=0}^{J-1} R_t(t+i\frac{N}{J},\frac{N}{J})\frac{N}{J} - \int_0^N R_t(t+\tau)d\tau \right| < N\tilde{\delta} + \bar{\delta} = \delta^*$$

(4) Since $R^*_t(t+\tau,k)$ has the same mathematical properties as $R_t(t+\tau,k)$ and $R^*_t(t+\tau)$ has the same mathematical properties as $R_t(t+\tau)$, the reasoning of steps (1) through (3) can be used to prove that

$$\lim_{J\to\infty} \sum_{i=0}^{J-1} R^*_t(t+i\frac{N}{J},\frac{N}{J})\frac{N}{J} = \int_0^N R^*_t(t+\tau)d\tau$$

(5) Since limits have the property that

$$\lim_{J\to\infty} [a(J) + b(J)] = \lim_{J\to\infty} a(J) + \lim_{J\to\infty} b(J)$$

this completes the subproof.

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NOTES

[1] The literature on forward exchange rates was recently surveyed in Hodrick (1988).

[2] In the term structure literature, four different versions of the "expectations hypothesis" have been promoted by different sets of authors. The one employed here is referred to by Cox-Ingersoll-Ross as the "Return-to-Maturity Expectations Hypothesis." Another version, which they call the "Unbiased Expectations Hypothesis," asserts that the forward rate of interest equals the expected future spot rate of interest. The "Local Expectations Hypothesis" claims that bonds with differing maturities will have equal expected return over a certain holding period, usually the shortest possible one. Finally, the "Yield-to-Maturity Expectations Hypothesis" holds that expected yields should be the same whether a bond is held to maturity or a series of shorter bonds are rolled over. (In continuous time the second and third of these are equivalent.) Associated with each hypothesis is a particular definition of a term premium. Jarrow (1981) shows that all these approaches cannot hold at once, without claiming superiority for any single approach, while Cox-Ingersoll-Ross show that only one approach (Local Expectations) is consistent with sustainable rational expectations general equilibrium in a continuous time framework. This paper claims that the Return to Maturity Hypothesis is violated, which is consistent with sustainable equilibrium in a continuous time framework.

[3] Note that this implies that for some $k > 0$; either $\Psi_t(t,k) \neq 0$, or $\Psi^*_t(t,k) \neq 0$.

[4] These results were arrived at independently.

[5] Fama has used measured term premiums that are consistent with verifying the "local expectations hypothesis," rather than term premiums as they have been defined in the present paper, which are consistent with the "return-to-maturity expectations hypothesis." While it is possible for term premiums of one sort to exist while they do not exist for the other, the size of the premiums he finds is such that this is extremely unlikely to have occurred in reality.

[6] Roll (1970) also found that term premiums do not necessarily increase with maturity. Since the propositions in Part II do not rely on term premiums following any particular pattern at all, these results have no bearing on the validity of our conclusions.

[7] See Mishkin (1978) for greater discussion of this issue.

[8] This information comes from personal conversation with Vinay Pande, foreign bond trader for the World Bank.

[9] If UIP holds at the one-week horizon, then

$$E_0 ln\left[\frac{s(1)}{s(0)}\right] = ln\left[\frac{f(0,1)}{s(0)}\right] = r(0,1) - r^*(0,1)$$

It must also be the case that

$$E_0 ln\left[\frac{s(2)}{s(1)}\right] = R_0(1,1) - R^*_0(1,1)$$

which implies, in turn,

$$E_0 ln\left[\frac{s(2)}{s(0)}\right] = [R_0(1,1) - R^*_0(1,1)] + [r(0,1) - r^*(0,1)].$$

$$= [r(0,1) + R_0(1,1)] - [r^*(0,1) + R^*_0(1,1)]\,. \qquad (*)$$

By covered interest parity, we know that

$$ln\left[\frac{f(0,2)}{s(0)}\right] = 2[r(0,2) - r^*(0,2)] \qquad (**)$$

Together, equations (*) and (**) imply that

$$ln\left[\frac{f(0,2)}{s(0)}\right] - E_0 ln\left[\frac{s(2)}{s(0)}\right] = 2[r(0,2) - r^*(0,2)] -$$

$$\{[R_0(1,1) + r(0,1)] - [R_0^*(1,1) + r^*(0,1)]\}$$

or

$$E_0 ln\left[\frac{s(2)}{s(0)}\right] - ln\left[\frac{f(0,2)}{s(0)}\right] = 2[\Psi_0(0,2) - \Psi^*_0(0,2)].$$

Likewise, it must be the case that

$$E_0 ln\left[\frac{s(k)}{s(0)}\right] = \left[r(0,k)k + \sum_{i=1}^{k-1} R_0(i,1)\right] - \left[r^*(0,k)k + \sum_{i=1}^{k-1} R^*_0(i,1)\right]$$

Coupling this with the covered interest parity relation, assumption (v), we find that

$$E_0 ln\left[\frac{s(k)}{s(0)}\right] - ln\left[\frac{f(0,k)}{s(0)}\right] = \Psi_0(0,k)k - \Psi^*_0(0,k)k.$$