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ABSTRACT

This paper evaluates the gains from international risk sharing in some simple general-equilibrium models with output uncertainty. Under empirically plausible calibration, the incremental loss from a ban on international portfolio diversification is estimated to be quite small--0.15 percent of output per year is a representative figure. Even the theoretical gains from asset trade may disappear under alternative sets of assumptions on preferences and technology. The paper argues that the small magnitude of potential trade gains, when coupled with small costs of cross-border financial transactions, may explain the apparently inconsistent findings of empirical studies on the degree of international capital mobility.

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1. Introduction

The expansion of global financial-market activity over the past two decades has inspired a large body of empirical research into the forces linking individual countries' financial markets. Findings about the nature and strength of these forces are important for their implications about public policy toward international capital movements. If international financial integration remains incomplete, measures to facilitate cross-border capital flows could enhance economic efficiency. An opposing position holds that the rapid pace of market integration has undermined the effectiveness of policies meant to safeguard economic stability. On this view, international financial integration has proceeded too far too soon, and should be curtailed through taxation or direct controls.

Empirical studies of financial interdependence among industrialized economies have proceeded along several lines and have uncovered three major facts that together constitute a puzzle. The first fact is that yields on comparable assets seem to be well arbitraged across borders (absent overt official restrictions), a finding consistent with a high degree of cross-border capital mobility. The second fact, which appears to contradict the previous one, is that the extent of international portfolio diversification is too low to be explained by standard models of financially integrated economies (even allowing for nontraded goods). A third fact, documented most prominently by Feldstein and Horioka (1980), is the generally low extent of foreign intertemporal trade, as measured by current-account balances. Some have argued that the small average size of current accounts, like the evidence on international portfolio positions, indicates high barriers to international asset trade.[1]

In this paper we propose a way to reconcile these seemingly contradictory findings. Our basic point is that the direct welfare gains from cross-border portfolio diversification--from the international pooling of national consumption risks--are likely to be quite small. That thesis leads to the following coherent interpretation of observed financial relationships among industrialized countries. When the gains from diversification abroad are small, even minor impediments to asset trade can wipe them out. Similarly, minor trade impediments can wipe out small gains from consumption-smoothing intertemporal exchanges. International interest-rate differentials would remain within narrow limits despite some transaction costs, and national capital markets might still be quite open to foreign linkages at the margin. A world with small barriers to foreign asset trade, but with limited gains to diversification, is therefore a world that could generate the empirical findings described above.[2]

[1] Some of the evidence on international financial integration is surveyed and evaluated by Obstfeld (1986, 1989a) and by Tesar (1988). Golub (1987) discusses the extent of international portfolio diversification by OECD countries. In the past decade the current-account imbalances of some major countries reached postwar-record levels, with the result that the original regression findings of Feldstein and Horioka (1980) have been weakened. See, for example, Frankel (1990).

[2] Any attempt to match this account with the data would be complicated by divergences between the social benefits we attempt to estimate below and the perceived private benefits that motivate private capital flows. For example, even when the social gains from diversification are small, tax evasion may result in a large volume of two-way international financial flows. This is just one of many issues that would have to be confronted to test our

We make our case that the gains from international risk sharing are small by examining the theoretical and empirical implications of some completely specified general-equilibrium models of rudimentary world economies.[3] In particular, simulation experiments based on an endowment model with exogenous stochastic growth show that the welfare loss from prohibiting international diversification is unlikely to exceed 0.15 percent of average national product per year, even at high levels of risk aversion. The models we use are, admittedly, very stylized; and a litany of important qualifications to our results is given in the conclusion of this paper. Nonetheless, we take the models seriously as parables that yield important lessons and, possibly, as springboards for further quantitative research. A major advantage of the general-equilibrium approach, one that in our view amply justifies the simplifications it requires, is that it gives a complete account of the mechanisms through which economic disturbances are transmitted among countries.

Given our underlying consumer-preference model and the stochastic properties of actual output growth rates, it is not surprising that we find small gains from international risk sharing. Using a related model, Lucas (1987) estimates the cost of postwar United States consumption variability to be quite small.[4]

hypothesis rigorously.

[3] This paper therefore follows the approach in Cole (1988), which develops alternative models for studying the aggregate implications of different international risk-sharing arrangements.

[4] While Lucas's empirical assumptions include a deterministic trend growth rate of consumption, we assume a stochastic trend. This difference prevents us from appealing to simple quadratic approximations, such as the one Lucas uses to estimate welfare

A crucial mechanism underlying our results, however, and absent from Lucas's (1987) framework, is the effect of output shocks on the relative prices at which international commodity trade occurs. We find that fluctuations in international terms of trade may play an important role in *automatically* pooling national economic risks, since they contribute to a negative correlation between a country's relative output growth rate and its terms of trade. Indeed, the models we work with below to highlight this effect, although quite standard, have the property that for certain parameter choices, terms-of-trade responses alone provide perfect insurance against output shocks. In such cases the gains from international portfolio diversification (and possibly the gains from intertemporal foreign trade as well) are nil. These are knife-edge results, of course, but the simulations mentioned above suggest that they are not dramatically wrong within realistic ranges of parameter variation.[5]

In the paper's next section we use Lucas's (1982) barter model of perfectly-pooled world financial markets to derive a first example of an economy in which international asset trade is redundant.[6] The model shows that for Cobb-Douglas commodity

losses. Our assumption of purely stochastic growth may also be the cause of larger estimated welfare losses than those Lucas reports. Another result similar in spirit to those reported below is Cochrane's (1989) estimate of minor costs of empirically plausible individual departures from intertemporal optimality.

[5] The result that international commodity trade can make international asset trade unnecessary is reminiscent of Mundell's (1957) celebrated insight into the substitutability between trade and factor movements.

[6] By characterizing asset trade as "redundant," we will mean only that the allocation reached without asset trade cannot be Pareto-improved by introducing asset markets.

demands and isoelastic utility, world equilibrium is Pareto-efficient *even when international asset trade is prohibited*. This specific equivalence result relies heavily, of course, on the assumed unit elasticity of relative demand with respect to price.

Lucas's economy is a pure exchange economy; our result is extremely sensitive to this, as extensions of the model that incorporate investment show. In section 3 we present two investment models--one a separable logarithmic model, one a model with a linear production technology subject to serially uncorrelated shocks--in which the earlier equivalence result carries through.[7] Despite these theoretical examples, considerations discussed in our concluding section suggest that future research along the lines of the present paper should focus on investment.

The mechanism driving the results of sections 2 and 3 is that *all* output shocks are transmitted *positively* between countries through the international price mechanism; in some very special circumstances, this transmission mechanism may provide complete insurance for domestic and foreign residents alike. When countries are not specialized in production, however, production shocks in

[7] The logarithmic model of this section draws heavily on Long and Plosser (1983). In interesting work closely related to ours, Cantor and Mark (1988) examine a logarithmic model with investment in which both countries produce the same good, rather than the distinct national outputs assumed by Lucas and by us. The nonspecialization assumption is a complication that allows Cantor and Mark to solve their model in closed form only when production shocks are serially independent. (Basically, the solution approach used by Long and Plosser becomes inapplicable.) As we observe below, nonspecialization in production is an important reason why our equivalence results do not hold in general.

common industries are transmitted *negatively* between countries. The price mechanism thus provides no automatic insurance in this case. Section 4 develops a model with nonspecialized economies to illustrate this negative transmission mechanism, and also studies the effects of an alternative kind of nonspecialization due to nontraded goods. The introduction of nontradables shows that Pareto efficiency does not necessarily imply a high correlation among national aggregate consumption levels.

Section 5 contains the paper's numerical results. In the simulations we conduct, preferences are varied to allow for various degrees of risk aversion and various elasticities of intratemporal substitution between national outputs. (Included is an infinite elasticity for the perfect-substitution case). Under all plausible parameter combinations, the gains from international portfolio diversification are quite small.

Section 6 summarizes and qualifies the paper's results.

Before commencing our analysis, it is useful to place the equivalence propositions derived below in the context of related research. Diamond (1967), Hart (1975), Newbery and Stiglitz (1982, 1984), and Stiglitz (1982), among others, examine the allocational efficiency of economies in which some markets are missing. Our theoretical investigation can be viewed as an extension, based on a different class of models, of the program pursued by these authors. Stiglitz (1982), for example, considers the constrained Pareto optimality (in Diamond's 1967 sense) of a multigood one-shot stock-market economy equilibrium. He finds a result akin to ours, that constrained optimality holds when price elasticities of demand equal unity (the Cobb-Douglas assumption), so that there is no real-income risk. As is shown below, however, unitary price elasticities do not always suffice to ensure full Pareto optimality. Newbery and Stiglitz (1984) study the implications of Cobb-Douglas/isoelastic preferences for risk sharing between producers and consumers in a trade model with missing insurance markets. They focus on the possibility that in the absence of insurance markets, opening a country to commodity trade may make both producers and consumers worse off. Helpman and Razin (1978), in an international context, earlier noted the special implications of Cobb-Douglas preferences for the substitutability of equities in different industries.[8]

2. A Pure Exchange Economy

This section describes a two-country pure exchange economy and analyzes its behavior under two polar assumptions: perfect international integration of asset markets, and complete *absence* of international asset trade. The main conclusion is that for a popular class of utility functions, the portfolio-autarky equilibrium entails a Pareto-optimal allocation. Since any Pareto optimum can be decentralized as the competitive equilibrium of an economy with complete, integrated asset markets, financial integration has no observable implications in our example.

All the models we will use assume a single representative

[8] Newbery and Stiglitz (1982) present some approximate welfare-cost calculations whose message is the same as that of the simulations in section 5 below: for empirically plausible cases, the efficiency losses caused by missing risk markets may be small.

resident within each country. By making this abstraction, we do not mean to imply a belief that onshore financial markets are literally perfect. Rather, our goal is to evaluate the *incremental* welfare gain that *international* diversification opportunities offer. A finding that these incremental gains are small does not imply that financial markets in general are unimportant. It does imply that the reduction in individual consumption variability attained through the diversification opportunities available at home leaves little scope for further reduction through additional diversification abroad.

Equilibrium with financial integration. The basic setup of the model comes from Lucas (1982). There are two countries, the "home" and the "foreign" country, with stochastic endowments of distinct national outputs, denoted by X and Y respectively. Home-country residents maximize expected utility,

$$U(0) = E_0\left\{ \sum_{t=0}^{\infty} \beta^t u[x(t), y(t)] \right\} \qquad (\beta < 1),$$

where x(t) and y(t) are their consumptions of the home and foreign outputs. Foreign-country residents maximize the same function of their own consumptions, x*(t) and y*(t).

Lucas (1982) considered a perfectly-pooled equilibrium of the model in which the residents of each country own a fifty percent share of the other country's endowment process. In that equilibrium, portfolios are perfectly diversified across countries and the two countries always have equal wealth and consumption levels. Other equilibriums of the model can be generated, however, by considering a global central planning problem in which outputs are allocated optimally by command. Solutions to this problem are Pareto optimums whose counterpart equilibriums generally involve differing national wealth levels.

Because the pure-exchange case has no intertemporal dimension, the planning problem takes the (timeless) form:

$$\text{Maximize} \quad \mu \cdot u(x,y) + (1-\mu) \cdot u(x^*,y^*)$$

subject to

$$(1) \quad x + x^* = X, \qquad y + y^* = Y.$$

Above, μ is a planner weight that determines relative national wealth levels in the counterpart market equilibrium. Optimal allocations are determined by (1) and the first-order conditions

$$(2) \quad u_j(x,y)/u_j(x^*,y^*) = (1-\mu)/\mu \qquad (j = x,\ y).$$

This fundamental condition states that the international ratio of marginal utilities from consuming any good must be constant across states of nature. In statistical terms, condition (2) states that national marginal utilities from consuming any good are *perfectly* positively correlated across countries. We use condition (2) repeatedly to test whether allocations are Pareto optimal.

Specialize now to the Cobb-Douglas/isoelastic-preference case,

$$u(x,y) = (x^{\theta}y^{1-\theta})^{1-R}/(1-R).$$

Given this utility function, the planning solution is

$$(3) \qquad x = \omega X, \qquad x^* = (1-\omega)X,$$

$$y = \omega Y, \qquad y^* = (1-\omega)Y,$$

where, defining $\sigma = 1/R$,

$$(4) \qquad \omega = \frac{1}{1 + [(1-\mu)/\mu]^{\sigma}} .$$

Notice that when $\mu = 1/2$, the solution is the one studied by Lucas (1982), in which national wealths are equal ($\omega = 1/2 = 1-\omega$).

Different μ values, however, correspond to differing national wealth levels and different (efficient) market outcomes. In all cases, national consumptions of the two goods are perfectly correlated. By pooling portfolios, countries effectively insure each other, to the maximum extent possible, against country-specific output shocks.

Equilibrium under portfolio autarky. Now consider a model with *no* opportunity for international asset exchanges. In this case the home country's income is its endowment X, the foreign country's its endowment Y. If p is the price of good y in terms of good x, portfolio autarky forces the countries to face the respective budget constraints

$$x + py = X, \qquad x^*/p + y^* = Y,$$

in each period.

In a one-good model, a ban on asset trade would reduce both countries to complete autarky. That does not happen here because balanced trade in the two goods remains possible despite the unavailability of trade across time or across states of nature.

Endowment disturbances thus continue to be transmitted abroad.

Desired consumptions under portfolio autarky are

$$(5) \qquad x = \theta X, \qquad x^* = \theta pY,$$
$$y = (1-\theta)X/p, \qquad y^* = (1-\theta)Y,$$

demand functions valid for any utility function that depends on a Cobb-Douglas index. Market clearing requires that the total demand for good x equal the global endowment,

$$x + x^* = \theta X + \theta pY = X,$$

that is, that

$$p = \frac{(1-\theta)X}{\theta Y}.$$

This price solution, when combined with the consumption demands listed in (5), implies that equilibrium consumptions are:

$$(6) \qquad x = \theta X, \qquad x^* = (1-\theta)X,$$
$$y = \theta Y, \qquad y^* = (1-\theta)Y.$$

An equivalence proposition. Now compare (6) with (3). The market solution under portfolio autarky is the member of the Pareto-efficient family of planning solutions corresponding to $\theta = \omega$. Thus, a planner weight of

$$\mu = \frac{1}{1 + [(1-\theta)/\theta]^R}$$

for the home country leads to the same allocation as the market would if cross-border asset trades were prohibited.[9]

An example showing a Pareto-optimal market allocation under portfolio autarky has some potential implications for empirical assessments of the extent to which capital is mobile internationally. First, commodity real interest rates and other real asset returns can be identical across countries even with no capital mobility. Second, under capital mobility there may be no departures from current-account balance, even if the menu of assets traded internationally is quite limited. (In an exchange model, current-account imbalances merely substitute for diversification, and thus have no role to play if diversification is redundant.) Finally, quite small costs of international investment transactions could give rise to complete portfolio non-diversification even in an unrestricted market setting.

Counter-examples. The foregoing theoretical results are quite special, and disappear under slight generalization. Simulations we report later examine the empirical importance of deviations from some of the assumptions made above. For now, however, we simply present two counter-examples that show how the equivalence proposition can break down even in a pure endowment world with unitary intratemporal elasticities of substitution in consumption.

Counter-example 1: Country-specific preferences. Suppose that different nationalities attach different geometric weights to their consumptions of the two goods; thus, $u(x,y) = (x^{\theta}y^{1-\theta})^{1-R}/(1-R)$ but $u^*(x^*,y^*) = (x^{*\theta^*}y^{*1-\theta^*})^{1-R}/(1-R)$, $\theta \neq \theta^*$.

[9] It is easy to show that if $\theta = 1/2$, the equivalence result holds, not only for isoelastic preferences, but for any monotonic concave function of a Cobb-Douglas index.

Equilibrium allocations without asset trade turn out to be

$$x = \theta X, \qquad x^* = (1-\theta)X,$$

$$y = \theta^* Y, \qquad y^* = (1-\theta^*)Y,$$

[cf. (6)]. So once again consumptions of each good are perfectly positively correlated across countries. Nonetheless, the allocation above is inefficient (except in one case to be discussed in a moment) because *marginal utilities* are not perfectly correlated across countries. To see this, just plug the above market-generated consumptions into the planner first-order conditions (2); the result is

$$\Theta[X(t)/Y(t)]^{(R-1)(\theta^* - \theta)} = (1-\mu)/\mu,$$

where Θ is a constant that depends on θ and θ^*. Clearly, the condition above cannot hold in all states of nature unless X and Y are perfectly correlated or unless R = 1 (the logarithmic case).

It may be surprising, even in the log case, that price responses alone ensure efficient risk sharing when country tastes differ.[10] One can check directly that the planner weight replicating the incomplete-markets allocation for R = 1 is

[10] To see why the log case "works," observe that the equilibrium relative price of good y, p, must equal a common marginal rate of substitution between the two goods, $p = (\partial u/\partial y)/(\partial u/\partial x) = (\partial u^*/\partial y^*)/(\partial u^*/\partial x^*)$. Consider a fall in y output, which causes p to rise. If preferences are isoelastic x and x* do not change, and if they are logarithmic (and thus separable in contemporaneous consumptions), $\partial u/\partial x$ and $\partial u^*/\partial x^*$ do not change either. So $\partial u/\partial y$ and $\partial u^*/\partial y^*$ must rise in proportion to p, and to each other.

Notice that in the symmetric case $(1-\theta) = \theta^*$, the equilibrium price is $p = X/Y$, so national incomes are the same in every state of nature. This equality is not always enough to ensure efficient risk sharing, as the discussion in the text shows.

$$\mu = \frac{1}{1 + [(1-\theta)/\theta^*]} = \frac{\theta^*}{1 - \theta + \theta^*}.$$

Counter-example 2: Nonisoelastic preferences. Let both countries share the same utility function,

$$u(x,y) = -exp[-x^{\theta}y^{(1-\theta)}].$$

This form leads to the same incomplete-markets equilibrium [given by (6)] as the Cobb-Douglas but isoelastic cases we've considered.

That equilibrium is now inefficient, however, as can be seen from looking once again at (2). Plugging the equilibrium outcomes (6) into the planner's first order condition yields

$$exp[(1-2\theta)X(t)^{\theta}Y(t)^{1-\theta}] = (1-\mu)/\mu,$$

which cannot hold for all realizations of X and Y except in the special symmetric case $\theta = 1/2$.

Thus, the constant expenditure share assumption, which leads to perfect correlation of consumptions across countries, is not generally sufficient to guarantee perfect correlation of *marginal utilities* across countries, as a first-best allocation requires.

3. The logarithmic and linear i.i.d. investment models

Investment is now introduced to add an intertemporal dimension to the inquiry. The basic setup of the first model analyzed in this section comes from the closed-economy analysis of Long and Plosser (1983) (which is easily reinterpreted as an open-economy analysis in the complete-markets case). As before, the model is worked out twice, once under free international trade in a complete set of state-contingent claims, once under the assumption of portfolio autarky. The second model discussed below, which is treated only briefly, is a two-country adaptation of Levhari and Srinivasan (1969). In both models, the underlying uncertainty comes from shocks to production technologies.

The assumptions under which the equivalence propositions of this section are derived are, once again, quite stringent. In an economy with investment, one can think of shocks to production functions as having two distinct effects. First, they affect current output, and second, they provide information about the future return on current investment. By itself, the first effect calls for no radical modification of our previous conclusions: insofar as domestic investment opportunities merely provide a channel for smoothing incipient consumption fluctuations, they reduce the incremental benefits from international risk sharing. The second effect is more consequential. Information about future investment productivity contained in current output shocks might cause a level of consumption variability that could be reduced through access to foreign capital markets.

The preference/production assumptions of this section's models ensure that current output shocks have no effect on expected investment profitability. This is one reason why portfolio autarky turns out to be efficient in the cases studied below. An appendix presents another special investment model in which international asset trade is redundant. In section 6 we discuss some empirical reasons for believing that the models of this section do not capture fully the role of investment.

Setting up the logarithmic model. The home and foreign countries are now specialized in their *production* of the two goods, rather than in endowments. Output of a good in period t+1, say, depends on a random period-t+1 productivity disturbance and period-t inputs of both goods. Let $k_{XX}(t)$ $[k_{YX}(t)]$ and $k_{XY}(t)$ $[k_{YY}(t)]$ be the home (foreign) inputs of goods x and y into the production of period-t+1 outputs. As in Long and Plosser (1983), this investment depreciates completely in the production process.

Home and foreign production functions are written in logarithmic form as

$$lnX(t+1) = ln\zeta^{X}(t+1) + \gamma_{XX}lnk_{XX}(t) + \gamma_{XY}lnk_{XY}(t),$$

$$lnY(t+1) = ln\zeta^{Y}(t+1) + \gamma_{YX}lnk_{YX}(t) + \gamma_{YY}lnk_{YY}(t).$$

Above, $\gamma_{XY} = 1 - \gamma_{XX}$ and $\gamma_{YX} = 1 - \gamma_{YY}$ (the constant-returns assumption). No specific distributional assumptions need to be made at this point about the multiplicative productivity disturbance $\zeta = (\zeta^{X},\zeta^{Y})$, other than the standard ones: ζ is Markovian and and its elements have positive support.

Let x(t) [x*(t)] and y(t) [y*(t)] again stand for home (foreign) consumptions of the two goods. Home residents maximize

$$U(0) = E_0\left\{ \sum_{t=0}^{\infty} \beta^{t}[\theta lnx(t) + (1-\theta)lny(t)] \right\}$$

and foreign residents maximize

$$U^*(0) = E_0\left\{ \sum_{t=0}^{\infty} \beta^t [\theta^* \ln x^*(t) + (1-\theta^*) \ln y^*(t)] \right\}.$$

Notice that the countries are allowed to attach different geometric weights to their consumptions of the two goods.

Efficient allocations. Consider again the set of Pareto-optimal solutions to the planning problem of maximizing

$$\mu \cdot U(t) + (1-\mu) \cdot U^*(t)$$

subject to constraints. As before, these can be decentralized as market equilibriums corresponding to various international distributions of wealth. Linearity allows one to decompose the planning problem into two stages. The first of these is to allocate given *world aggregate* consumption levels, $C_X(t)$ and $C_Y(t)$, between residents of the two countries. (This stage corresponds to the market process of sharing risks optimally given the international distribution of wealth.) The second stage of the planning problem is to choose the aggregate consumption levels $C_X(t)$ and $C_Y(t)$ optimally at each point in time.

Consider stage one first, the same static problem examined in section 2. In this stage the planner maximizes

$$(7) \quad \mu \cdot [\theta \ln x(t) + (1-\theta) \ln y(t)] + (1-\mu) \cdot [\theta^* \ln x^*(t) + (1-\theta^*) \ln y^*(t)]$$

subject to the constraints

$$(8) \quad x(t) + x^*(t) \le C_X(t), \qquad y(t) + y^*(t) \le C_Y(t).$$

The resulting allocation rules are:

$$(9) \quad x(t) = \mu\theta C_X(t)/\phi,$$

$$x^*(t) = (1-\mu)\theta^* C_X(t)/\phi,$$

$$y(t) = \mu(1-\theta)C_Y(t)/(1-\phi),$$

$$y^*(t) = (1-\mu)(1-\theta^*)C_Y(t)/(1-\phi),$$

where

$$(10) \quad \phi = \mu\theta + (1-\mu)\theta^*, \qquad 1-\phi = \mu(1-\theta) + (1-\mu)(1-\theta^*).$$

The result of maximizing (7) subject to (8) defines an *indirect* planner utility function that depends on the world aggregates $C_X(t)$ and $C_Y(t)$. Direct substitution of solutions (9) and (10) into (7) shows that this indirect utility function equals

$$(11) \quad W[C_X(t), C_Y(t)] = \phi \ln C_X(t) + (1-\phi) \ln C_Y(t)$$

up to an additive constant. Equation (11) brings one back to stage two of the planning problem, choosing the optimal aggregate consumptions of the goods. This stage-two problem takes the form:

$$(12) \quad \text{Maximize} \quad E_0\left\{ \sum_{t=0}^{\infty} \beta^t W[C_X(t), C_Y(t)] \right\}$$

subject to the constraints

$$C_X(t) + k_{XX}(t) + k_{YX}(t) \leq X(t),$$

$$C_Y(t) + k_{YX}(t) + k_{YY}(t) \leq Y(t),$$

and given the intertemporal tradeoffs defined by the national investment functions.

When the aggregate planning problem is posed as in (12), it is clear that it is *exactly* the same problem as the one studied by Long and Plosser (1983). The solution to stage two therefore can be lifted from their paper with no modification. That solution is:

$$(13)\quad C_X(t) = (\phi/\kappa_X)X(t),$$

$$C_Y(t) = [(1-\phi)/\kappa_Y]Y(t),$$

$$k_{jX}(t) = (\beta\kappa_j\gamma_{jX}/\kappa_X)X(t) \qquad (j = X,Y),$$

$$k_{jY}(t) = (\beta\kappa_j\gamma_{jY}/\kappa_Y)Y(t) \qquad (j = X,Y),$$

where

$$(14)\quad \kappa_X = [\phi(1-\beta\gamma_{YY}) + (1-\phi)\beta\gamma_{YX}]/\Delta,$$

$$\kappa_Y = [(1-\phi)(1-\beta\gamma_{XX}) + \phi\beta\gamma_{XY}]/\Delta,$$

$$\Delta = (1-\beta\gamma_{XX})(1-\beta\gamma_{YY}) - \beta^2\gamma_{YX}\gamma_{XY} > 0.$$

The parameters κ_X and κ_Y are the coefficients of $ln X(t)$ and $ln Y(t)$ in the planner's period-t value function.

Allocation rules (9) and (13) have several important implications about world equilibrium with complete markets (given the classes of utility and production functions under study here). Specifically, consumption of every good is perfectly correlated across countries, as in Lucas's endowment model. Thus, each country's consumption of a good is perfectly correlated with world consumption of that good, which is proportional in turn to world output. Investment of every good is also perfectly correlated across countries, and proportional to current output.

In comparing the foregoing equilibrium with the one that results when there is no asset trade, it is useful to derive a key shadow price associated with the optimal allocation, the shadow price of good y in terms of good x, p. At any time t, p(t) is the marginal rate of substitution in consumption of good x for good y,

$$(15)\quad p(t) = (1-\phi)C_X(t)/\phi C_Y(t) = \kappa_Y X(t)/\kappa_X Y(t).$$

This price depends on μ only if θ and θ^* differ [see (10) and (14)]. In that case, an increase in the welfare weight of the country with a relative preference for y, say, raises the shadow relative price of y. The change lowers the amount of y invested in producing future units of good x and raises the amount of x invested in y.

Equilibrium under portfolio autarky. Market equilibrium is found in two steps. First, we solve the maximization problem of home and foreign social planners who take the path of the terms of trade, {p(t)}, as exogenously given. This step yields price-dependent consumption and investment demands for the two goods. Second, we solve for the terms of trade that clear world goods markets. This step yields reduced-form consumption and investment demands that can be compared with (9) and (13).[11]

[11] Dellas (1986) studies a model similar to this one and draws on the Long-Plosser solution to describe its equilibrium, even though his model assumes balanced international trade. We have been unable to find any direct justification for the solution procedure Dellas proposes.

Consider first the problem of a home social planner. Our conjecture is that this planner's value function takes the form

$$(16)\quad V[X(t),\varsigma^{X}(t),p(t)] = \kappa \ln X(t) + J[\varsigma^{X}(t)] + H[p(t)]$$

(up to an additive constant), where p(t), the price of good y in terms of good x, is given. Bellman's principle states that $V[X(t),\varsigma^{X}(t),p(t)]$ solves the problem of maximizing

$$\theta \ln x(t) + (1-\theta)\ln y(t) + \beta E_t\{V[X(t+1),\varsigma^{X}(t+1),p(t+1)]\}$$

subject to the X(t+1) production function and the constraint of balanced international trade,

$$x(t) + k_{XX}(t) + p(t)[y(t) + k_{XY}(t)] = X(t).$$

Given conjecture (16), first-order conditions for the problem are

$$(17)\quad \theta/x(t) = \lambda(t),$$

$$(1-\theta)/y(t) = \lambda(t)p(t),$$

$$\beta\kappa\gamma_{XX}/k_{XX}(t) = \lambda(t),$$

$$\beta\kappa\gamma_{XY}/k_{XY}(t) = \lambda(t)p(t),$$

where $\lambda(t)$ is the Lagrange multiplier associated with the trade-balance constraint. The parameter κ can be derived from the envelope condition,

$$\partial V[X(t),\varsigma^{X}(t),p(t)]/\partial X(t) = \kappa/X(t) = \lambda(t)$$

and (17); it is given by

(18) $\kappa = \lambda(t)X(t)$.

Combination of (17) and (18) with the economy's budget constraint leads to the solution

$$\kappa = 1/(1-\beta)$$

and to the following demand functions:

(19) $x(t) = \theta(1-\beta)X(t)$,

$y(t) = (1-\theta)(1-\beta)X(t)/p(t)$,

$k_{XX}(t) = \beta\gamma_{XX}X(t)$,

$k_{XY}(t) = \beta\gamma_{XY}X(t)/p(t)$.

The corresponding demand functions for the foreign country are the analogues of those listed in (19):

(20) $x^*(t) = \theta^*(1-\beta)p(t)Y(t)$,

$y^*(t) = (1-\theta^*)(1-\beta)Y(t)$,

$k_{YX}(t) = \beta\gamma_{YX}p(t)Y(t)$,

$k_{YY}(t) = \beta\gamma_{YY}Y(t)$.

Note that the investment demands in (19) and (20) do not depend on $\zeta^X(t)$ or $\zeta^Y(t)$: here as in the complete-markets case, production-function shocks can be thought of as pure endowment shocks.

Equilibrium in the market for the x good determines $p(t)$:

$$x(t) + x^*(t) + k_{XX}(t) + k_{YX}(t) = X(t).$$

Equations (19) and (20) imply that this condition holds when

$$(21)\quad p(t) = \frac{X(t)}{Y(t)} \times \frac{(1-\theta)(1-\beta) + \beta\gamma_{XY}}{\theta^*(1-\beta) + \beta\gamma_{YX}}\,.$$

An equivalence proposition. When combined with the price function (21), (19) and (20) show that the portfolio-autarky case bears many empirical similarities to the complete-markets case. For example, consumptions of every good are perfectly correlated across countries, as are investment levels. Aggregate consumptions and investments are again proportional to total supply. In fact, the equilibrium with restricted asset trade implies the same resource allocation as a particular optimal plan. We demonstrate this equivalence, once again, by constructing an optimal plan that calls for the incomplete-markets allocation.

Equation (15) giving the set of shadow prices p(t) generated by efficient allocations can be written as

$$(22)\quad p(t) = \frac{X(t)}{Y(t)} \times \frac{(1-\phi)(1-\beta) + \beta\gamma_{XY}}{\phi(1-\beta) + \beta\gamma_{YX}}\,.$$

Compare (21) and (22). They are the same for a planner weight of

$$(23)\quad \mu = \frac{\theta^*(1-\beta) + \beta\gamma_{YX}}{(1 - \theta + \theta^*)(1-\beta) + \beta(\gamma_{YX} + \gamma_{XY})}\,.$$

In the special case $\phi = \theta = \theta^*$, it is easy to verify directly that the planner weights producing the autarkic market allocation are μ

$= \kappa_X(1-\beta)$ and $1-\mu = \kappa_Y(1-\beta)$, where κ_X and κ_Y are given by (14).

If the relative price of the two goods under portfolio autarky equals the price generated by the plan, however, equations (15), (19), (20), and (23) can be used to show directly that both sets of arrangements lead to the same allocation. Efficiency is therefore assured even without asset trade.

The linear i.i.d. investment model. The next model extends the assumptions of Levhari and Srinivasan (1969) to a two-country trading world. Results are described only briefly. More detailed derivations would parallel the arguments in the appendix.

Preferences are again Cobb-Douglas/isoelastic with risk-aversion parameter R, and are the same everywhere. Production functions are, however, linear, such that available outputs are

$$\ln X(t+1) = \ln \zeta^X(t+1) + \ln k_{XX}(t),$$

$$\ln Y(t+1) = \ln \zeta^Y(t+1) + \ln k_{YY}(t).$$

The crucial assumption, which ensures that production shocks communicate no information about future productivity, is that $\zeta^X(t)$ and $\zeta^Y(t)$ are identically and independently distributed (i.i.d.) over time.

It can be verified that the aggregate planning solution is

$$(24) \quad C_X(t) = \alpha X(t),$$

$$C_Y(t) = \alpha Y(t),$$

$$k_{XX}(t) = (1-\alpha)X(t),$$

$$k_{YY}(t) = (1-\alpha)Y(t).$$

The constant consumption share α, which is the same for both goods, generalizes the formula of Levhari and Srinivasan (1969):[12]

$$(25)\quad \alpha = 1 - \left\{\beta E\left[(\zeta^X)^{\theta(1-R)}(\zeta^Y)^{(1-\theta)(1-R)}\right]\right\}^{1/R}.$$

Under portfolio autarky, however, the investment rules given by (24) and (25) remain optimal for price-taking residents of the two countries, given the commodity demands implied by their Cobb-Douglas preferences. The equilibrium terms of trade, the same as the "shadow" terms of trade associated with the plan, are $p(t) = (1-\theta)X(t)/\theta Y(t)$, and the balanced-trade equilibrium is in fact efficient. The planner weight leading to the portfolio-autarky allocation is

$$\mu = \frac{1}{1 + [(1-\theta)/\theta]^R},$$

just as in the equivalence proposition of section 2.

4. Nonspecialization

So far we have concentrated on models in which each of two countries is specialized in producing a single output. To illustrate that portfolio-autarky equilibria are generally inefficient when this assumption is relaxed, we examine in this section two types of nonspecialization in pure exchange economies. First we assume that in addition to their respective endowments of

[12] See also equation (A6) in the appendix.

goods x and y, *both* countries have a stochastic endowment of a third good, z, which is tradable. Our second example endows both countries with nontradable outputs in addition to their country-specific tradable endowments.

Nonspecialization in tradables. Assume that the home country now has a stochastic endowment of a second good, z, along with x. The foreign country has a stochastic endowment of *the same* good, z, along with y. Denote by Z and Z* the realizations of the home and foreign endowments of z.

Both countries' residents have Cobb-Douglas/isoelastic preferences with expenditure shares θ_x, θ_y, and θ_z. If p_x is the price of x in terms of z and p_y the price of y in terms of z, then it is easy to show, as in section 2, that equilibrium prices are

$$p_x = \frac{\theta_x(Z + Z^*)}{\theta_z X}, \qquad p_y = \frac{\theta_y(Z + Z^*)}{\theta_z Y}.$$

To check that condition (2) is generally not satisfied, consider consumptions of the x good, which are

$$(26) \quad x = \left\{\theta_z\left(\frac{Z}{Z + Z^*}\right) + \theta_x\right\} X, \qquad x^* = \left\{\theta_z\left(\frac{Z^*}{Z + Z^*}\right) + \theta_y\right\} X .$$

For log preferences, the international ratio of marginal utilities from consuming good x is proportional to x^*/x, which, as (26) shows, is not constant across states of the world unless Z and Z* are perfectly correlated. So the equilibrium with no asset trade is generally inefficient.

The discussion brings out the important distinction between *country-specific* output shocks, which simultaneously affect all sectors within a country, and *industry-specific* shocks (for example, technological advances disseminated quickly across national borders), which affect sectors producing the same good regardless of location. If most shocks to z output are industry-specific, it is plausible that Z and Z* are highly correlated and that international asset trade yields little in the way of efficiency gains. If shocks tend to be country specific, however, countries can gain by exchanging shares in common risky industries. Notice the implication of (26) that z-shocks are transmitted *negatively* to foreign consumption. This is the basic source of gains from asset trade in the present model.[13]

Nontradable goods. Another type of nonspecialization is due to the presence of nontradable goods. It is easy to incorporate into Lucas's (1982) model stochastic home and foreign endowments, N and N*, of nontradables.[14] Preferences are still assumed to be Cobb-Douglas/isoelastic, with expenditure shares θ_x, θ_y, and θ_n and a risk-aversion coefficient R common to both countries.

The balanced-trade equilibrium can be found by calculating demands as above and assuming that the two nontradable-good markets and the world market for the x good clear. The resulting

[13] Stockman (1988) concludes from evidence on seven European countries that country-specific shocks have had a substantial influence on output growth in the period since the mid-1960s. His findings are therefore consistent with the existence of efficiency benefits from international portfolio diversification.

[14] Stockman and Dellas (1984) extend Lucas's model in this way. Their focus, however, is on exchange-rate determination, and they assume that nontradables and tradables affect utility separably. Our example makes clear that separability has very special implications for optimal international portfolio diversification. Separability is rather implausible in any case, because tradables often come "bundled" together with such nontradables as marketing and distribution services.

demands for tradables are

$$(27)\quad x = [\theta_x/(1-\theta_n)]X, \qquad x^* = [\theta_y/(1-\theta_n)]X,$$

$$y = [\theta_x/(1-\theta_n)]Y, \qquad y^* = [\theta_y/(1-\theta_n)]Y,$$

expressions similar to those in (6) and which imply that consumptions of the same tradable good are perfectly correlated internationally. Of course, consumptions of nontradables are N and N* in the home and foreign countries, and thus can have an arbitrary correlation structure.

If the planner optimizes over tradable consumptions only, condition (2) still characterizes optimal allocations, except that marginal utilities of tradables may now depend on N and N*. This dependence turns out to be critical in assessing the efficiency of the incomplete-markets allocation.

To see this, note that the marginal-utility ratio in (2) is

$$\Omega[N(t)/N^*(t)]^{\theta_n(1-R)},$$

where Ω is a constant that depends on θ_x and θ_y. But unless N and N* are perfectly correlated, or unless the utility function is separable in consumptions (R = 1), the expression above cannot be constant across states of nature. The presence of nontradables may therefore lead to additional efficiency benefits from international risk sharing.

Nontradables also may influence the statistical correlation between broad national consumption measures. The models examined before imply that innovations in national consumption levels are much more highly correlated than is in fact true. But when a significant fraction of each country's consumption falls on nontradables, innovations in *aggregate* national consumptions need not be highly correlated, despite financial integration.[15]

5. How big are the gains from risk sharing? Some numerical results

In the previous sections we examined several very special models in which international portfolio diversification yields no welfare benefits at all. We now ask how far one must deviate from the specific parameter settings assumed in those models before the prohibition of asset trade causes a significant welfare loss. The results reported below are limited to the case of pure exchange. Future work will need to incorporate the additional gains from intertemporal trade between investment economies (see section 6).

Even within a pure exchange setting, there are several directions along which one could relax the extreme assumptions that lead to asset-trade redundancy above. As a first step, it seems most natural to continue assuming that both countries are specialized in their endowments, but to relax the Cobb-Douglas preference assumption, which makes the terms of trade unit-elastic with respect to relative output. One case covered is that in which the outputs of the two countries are perfect

[15] Consumption aggregates could still have a definite stochastic relationship, however, depending on the equilibrium price functions. In the example above, the equilibrium price of home-country nontradables in terms of x goods is $p_n = [\theta n/(1-\theta n)](X/N)$, so national consumptions measured in any numeraire are still perfectly correlated. Such simple correlations typically don't arise with non-Cobb-Douglas utility functions, not even with those separable in tradables and nontradables.

substitutes, so that terms-of-trade effects are altogether absent.

Preferences and equilibrium. To allow for non-unitary price elasticities, we change from the earlier Cobb-Douglas to a constant elasticity of substitution (CES) utility specification. Period utility is CES/isoelastic:

$$u(x,y) = [(x^{\rho} + y^{\rho})^{1/\rho}]^{1-R}/(1-R), \qquad (\rho \leq 1).$$

For $\rho = 1$, goods x and y are perfect substitute in consumption; $\rho = 0$ is the Cobb-Douglas case, with equal weights of 1/2 on the two goods.[16]

Absent asset trade, desired consumption levels are

$$(28) \quad x = X/[1 + p^{\rho/(\rho-1)}], \qquad x^{*} = pY/[1 + p^{\rho/(\rho-1)}],$$

$$y = p^{1/(\rho-1)}X/[1+p^{\rho/(\rho-1)}], \quad y^{*} = p^{\rho/(\rho-1)}Y/[1+p^{\rho/(\rho-1)}].$$

Market clearing yields the equilibrium price function

$$(29) \quad p = (X/Y)^{1-\rho}.$$

Equilibrium price is elastic with respect to relative supply if $\rho < 0$ and inelastic if $\rho > 0$; only for $\rho = 0$ are the two countries' incomes always equal. Cases in which $\rho < 0$ imply "immiserizing growth" and probably are unrealistic. Our simulations therefore look only at cases in which $\rho \in [0,1]$.

Numerical methodology. National outputs are assumed to grow according to

[16] The logarithmic perfect substitution case ($R = 1$, $\rho = 1$) is the one studied by Cantor and Mark (1988).

$$(30)\quad X(t+1) = [1 + \epsilon^X(t)]X(t), \quad Y(t+1) = [1 + \epsilon^Y(t)]Y(t),$$

where $\{\epsilon^X(t)\}$ and $\{\epsilon^Y(t)\}$ are exogenous stochastic processes. Each of these processes is a two-state Markov process, with the two states corresponding to "high growth" and "low growth." The state of the world economy as a whole on date t is given by the vector $[\epsilon^X(t),\epsilon^Y(t)]$, which can take four possible values.[17]

We calibrate our simulation model so that the mean, standard deviation, and first lagged autocorrelation of either country's output growth rate equal those of the United States annual output growth rate over the years 1968-1987. The numbers are 2.7 percent per year (mean growth), 2.5 percent per year (standard deviation of growth), and 0.156 (first lagged autocorrelation of growth). It is also assumed that the contemporaneous correlation coefficient between the two countries' growth rates equals that between U.S. and Japanese growth over the same two decades (that is, 0.334).[18]

Calibration amounts to choosing the two possible realizations of the Markov growth processes and the probability entries in the state transition matrix. The four possible states for the world economy are taken to be:

State 1: $\epsilon^X = 0.052$, $\epsilon^Y = 0.052$

State 2: $\epsilon^X = 0.052$, $\epsilon^Y = 0.002$

State 3: $\epsilon^X = 0.002$, $\epsilon^Y = 0.052$

State 4: $\epsilon^X = 0.002$, $\epsilon^Y = 0.002$.

[17] The simulation model is a two-country version of the model used by Mehra and Prescott (1985).

[18] Data come from *OECD Economic Outlook* 43 (June 1988), table R1, p. 170.

Let π_{ij} denote the probability of moving to state j next period when the current state is i. The transition matrix $\Pi = [\pi_{ij}]$ is

$$\Pi = \begin{bmatrix} 0.600 & 0.050 & 0.050 & 0.300 \\ 0.129 & 0.299 & 0.443 & 0.129 \\ 0.129 & 0.443 & 0.299 & 0.129 \\ 0.300 & 0.050 & 0.050 & 0.600 \end{bmatrix},$$

with an implied steady-state distribution described by the unconditional probabilities $[\pi_1 \ \pi_2 \ \pi_3 \ \pi_4] = [0.36 \quad 0.14 \quad 0.14 \quad 0.36]$.

The model has been set up so that the two countries are perfectly symmetric under portfolio autarky. In particular, the transition matrix Π implies a symmetric joint distribution for the two countries' growth rates. The symmetry assumption is deliberate: it implies that the opening of asset trade would move the world economy immediately to a perfectly pooled equilibrium in which both countries hold equal wealth. In the numerical results that follow, it is with the latter perfectly pooled equilibrium that the autarkic equilibrium is compared.

Results. We assume that the two countries always start out with predetermined base output levels $\bar{X}$ and $\bar{Y}$, where $\bar{X} = \bar{Y}$ is assumed to maintain symmetry. A single realized history for the world economy is generated as follows. In period $t = 0$, an initial pair of growth rates, $[\epsilon^X(0), \epsilon^Y(0)]$, is drawn from the steady-state distribution. Consumptions for $t = 0$ are then determined according to the assumptions about financial integration, given that output levels are $[1 + \epsilon^X(0)]\bar{X}$ and $[1 +$

$\epsilon^{Y}(0)]\bar{Y}$. Subsequent growth rates are draws from the conditional distribution defined by the transition matrix Π, and these generate subsequent output and consumption levels.

Estimates of home-country expected utilities under portfolio autarky and under perfect pooling, denoted $\hat{U}^{A}(\bar{X})$ and $\hat{U}^{P}(\bar{X})$, respectively, are obtained by averaging the lifetime utility levels achieved in 5,000 independent histories. (Expected utility is the same at home and abroad, under either set of arrangements.) In these calculations, it is assumed that $\beta = 0.98$ and that the economy's horizon is 50 periods.[19]

Our measure of welfare cost is the fraction δ by which the base outputs $\bar{X}$ and $\bar{Y}$ would have to be reduced in the perfectly pooled case to leave individuals with the expected utility attainable when balanced trade is imposed. The fraction δ is estimated as the solution to $\hat{U}^{A}(\bar{X}) = \hat{U}^{P}[(1 - \hat{\delta})\bar{X}]$. It is the *permanent* percentage reduction in average global product equivalent to the prohibition of international portfolio diversification.

Table 1 reports the estimates $\hat{\delta}$ for CES coefficients ranging from 0.25 to 1.0 and for risk aversion coefficients ranging from 2 to 10. (Approximate standard errors, which indicate that the output-loss estimates are quite precise, appear in parentheses.) The most striking fact revealed by the table is how small the gains from international asset trade are in this pure exchange model. The largest welfare loss reported, 0.29 percent of output

[19] We estimated expected utility under both regimes, rather than calculating it exactly with closed-form solutions such as those given by Mehra (1988), because those solutions are not readily applicable to the portfolio-autarky case.

Table 1

Welfare Loss Due to a Ban on International Diversification

(fraction of national product per year)

R \ ρ	0.25	0.50	0.75	1.00
2	0.000045	0.000351	0.000916	0.001743
	(0.000000)	(0.000000)	(0.000000)	(0.000002)
4	0.000094	0.000518	0.001274	0.002369
	(0.000000)	(0.000001)	(0.000005)	(0.000016)
6	0.000114	0.000580	0.001403	0.002595
	(0.000000)	(0.000002)	(0.000012)	(0.000043)
8	0.000127	0.000618	0.001482	0.002738
	(0.000000)	(0.000004)	(0.000023)	(0.000082)
10	0.000136	0.000647	0.001545	0.002855
	(0.000000)	(0.000006)	(0.000036)	(0.000135)

Note: For a given CES utility function parameter ρ and risk aversion coefficient R, the reported number is the fraction by which base-year output must be reduced to yield a welfare loss equal to that caused by a ban on international asset trade. Expected utility levels are calculated as the average of utility realizations in 5,000 independent replications of a symmetric two-country world economy in which national output growth rates follow a two-state Markov process. (Approximate standard errors, rounded, appear in parentheses below the output-loss estimates.)

per year, occurs when $\rho = 1$ and $R = 10$. Recall that when $\rho = 1$, national outputs are perfect substitutes and endogenous terms-of-trade fluctuations therefore provide no insurance against endowment risk. Estimates based on $\rho = 1$ might be relevant for a small country producing an output that is also produced by many foreign producers; but welfare losses are certainly smaller for larger economies. In any case, even a yearly welfare loss equivalent to 0.3 percent of output is not crushingly large.[20]

Values of ρ somewhere between 0.25 and 0.75 are probably relevant for most industrial countries. The likely magnitude of R suggested by the empirical literature seems to be below 4 (see, e.g., Mehra and Prescott 1985, Obstfeld 1989a); most recently, Pindyck (1988) has reported point estimates that tend to lie between 3 and 4. For reasonable parameters, table 1 thus implies a yearly welfare loss unlikely to exceed 0.15 percent of output

If these low estimates of the welfare gains from international risk pooling are accurate, rather small impediments to asset trade could discourage a large volume of two-way capital flows. Since current-account movements can in some respects substitute for international portfolio diversification, small gains from diversification could help explain the small scale of current-account imbalances as well. Strong conclusions cannot be drawn from simulations that do not account for investment. A conservative inference, however, is that limited gains from asset

[20] This figure is less than half the probable loss to the U.S. due to a ceteris paribus move from a zero net external debt to a permanent debt/GNP ratio of 25 percent (Obstfeld 1989b). See Lucas (1987) for discussion of other pertinent welfare comparisons.

trade offer a possible clue to the puzzles surrounding empirical measures of capital mobility among industrial economies.

6. Conclusions and qualifications

This paper has evaluated the gains from international risk sharing in some simple models with output uncertainty. Under alternative sets of restrictive assumptions on preferences and technology, these gains may disappear. Numerical simulations show that empirically plausible deviations from the assumptions yielding risk-market redundancy still imply small welfare gains from international portfolio diversification. To find significant gains from international asset trade, one must go beyond the gains attainable through the pooling of national business-cycle risks.

For reasons spelled out in the introduction, our findings may help explain the seemingly contradictory evidence about international capital mobility among industrialized economies. More detailed empirical work is needed, however, before such an explanation can be accepted with any confidence. The key limitation of our simulation analysis is that it neglects investment. Although there are several investment models in which international asset trade does not enlarge the set of consumption opportunities, there are good reasons to believe that these models capture only part of the role investment plays in reality. They capture how investment can be used to smooth consumption in the face of transitory output shocks--essentially the "storage" role of investment--but not the possibility that output shocks contain information about *future* investment productivity, information that might induce welfare-enhancing intertemporal trade.

Time-series evidence supports the hypothesis that output shocks do contain such information. Three robust empirical regularities, valid across a broad sample of countries and time periods, are that net exports are countercyclical, that investment is procyclical, and that the percentage variance of investment exceeds that of output (Backus and Kehoe 1988). These patterns suggest a world in which positive output shocks contain favorable news about investment productivity, leading to higher investment financed in part out of foreign savings. In contrast, the variance of investment would not exceed that of output if investment were basically a storage activity: in the models of section 3, for example, the variance of investment and output are the same.

More realistic models of investment would also help account for the gains reaped in the past from foreign capital flows to developing countries. Our analysis has not addressed the possibility of substantial intertemporal trade gains between countries at very different stages of economic development. One reason for this omission is that the puzzle about international capital mobility described in the introduction is somewhat less relevant for developing countries than for developed. For example, the Feldstein-Horioka finding of tightly correlated saving and investment rates comes through more weakly in studies of developing-country experience prior to the early 1980s.[21]

[21] See Dooley, Frankel, and Mathieson (1987) and Summers (1988). A model suited to capture the role of capital flows in development would probably incorporate both capital installation costs and country-specific factors of production (such as land and raw labor). Mendoza (1989) presents an interesting study of the costs

There are some other possible gains from asset trade that the models we have worked with may overlook. Monetary and fiscal policy disturbances, for example, are absent. To the extent that such omitted factors affect welfare through their consumption effects, however, their explicit inclusion should not add much.[22]

A more interesting extension would allow for the possibility that international financial integration itself affects output *growth rates*, as suggested by the recent literature on endogenous long-run growth.[23] Opening national capital markets to foreign competition might improve the efficiency of domestic financial intermediation, with permanent effects on the level of investment and growth. Or, direct investment by foreigners could increase the speed with which technological innovations are disseminated between countries. The mechanisms through which financial integration promotes growth might well involve externalities not captured by private investors.

Our hunch is that growth effects such as these are likely to be a source of quantitatively important gains from financial integration. Even if this guess is right, however, our suggested resolution of the capital-mobility puzzle could still stand: while growth effects may alter our assessment of the gains from asset trade, they can influence private capital flows only to the extent

of capital controls in a stochastic small-country model with these features. His application to the Canadian economy produces estimates of welfare loss of the same order of magnitude as those reported above.

[22] Stockman and Svensson (1987) describe a model with a richer menu of disturbances.

[23] See Romer (1989) for a survey.

that they are reflected in the private returns that investors perceive. Further research into these questions is needed to resolve the riddle in the data and to evaluate the role of global financial markets more generally. More than intellectual curiosity is at stake. Ultimately, the policies governments adopt are likely to depend on the way economists interpret the empirical record.

Appendix: The symmetric investment model

If international symmetry conditions are imposed on the Cobb-Douglas/isoelastic utility functions of individuals and on production functions, another equivalence proposition, valid for all values of the risk-aversion parameter R and for serially correlated output shocks, can be derived. To this end, assume

$$\theta = \theta^* = 1/2$$

and

$$\gamma_{XX} = \gamma_{YY} = \gamma.$$

We return to the assumption that capital depreciates fully in use.

Characterizing a planning solution. The planning solution that will turn out to be relevant is one in which residents of the two countries have equal welfare weights ($\mu = 1/2$). In this case, the planner's period utility, in terms of consumption aggregates, can be taken to be

$$W[C_X(t), C_Y(t)] = [C_X(t)^{1/2} C_Y(t)^{1/2}]^{1-R}/(1-R);$$

these world consumption aggregates are allocated equally between the two countries.

We now characterize a solution but stop short of a formal existence/uniqueness proof. As the first step, write the Euler conditions for goods x and y:

$$\text{(A1)}\quad C_X(t)^{\frac{-(1+R)}{2}} C_Y(t)^{\frac{(1-R)}{2}} = \beta E_t\left\{\gamma\zeta^X(t+1)\left[\frac{k_{XY}(t)}{k_{XX}(t)}\right]^{1-\gamma} C_X(t+1)^{\frac{-(1+R)}{2}} C_Y(t+1)^{\frac{(1-R)}{2}}\right\},$$

$$C_Y(t)^{\frac{-(1+R)}{2}} C_X(t)^{\frac{(1-R)}{2}} = \beta E_t\left\{\gamma\zeta^Y(t+1)\left[\frac{k_{YX}(t)}{k_{YY}(t)}\right]^{1-\gamma} C_Y(t+1)^{\frac{-(1+R)}{2}} C_X(t+1)^{\frac{(1-R)}{2}}\right\}.$$

Let the marginal consumption propensities out of the two goods be $\alpha_X(t)$ and $\alpha_Y(t)$; corresponding to these fractions are the investment rules

$$\text{(A2)}\quad k_{XX}(t) = \gamma[1-\alpha_X(t)]X(t),$$

$$k_{XY}(t) = (1-\gamma)[1-\alpha_Y(t)]Y(t),$$

$$k_{YX}(t) = (1-\gamma)[1-\alpha_X(t)]X(t),$$

$$k_{YY}(t) = \gamma[1-\alpha_Y(t)]Y(t).$$

The assumed consumption and investment rules can now be used to write all quantities in (A1) in terms of X(t) and Y(t), e.g., $C_X(t+1) = \alpha_X(t+1)\zeta^X(t+1)\{\gamma[1-\alpha_X(t)]X(t)\}^{\gamma}\{(1-\gamma)[1-\alpha_Y(t)]Y(t)\}^{1-\gamma}$. After this step and tedious algebra, (A1) reduces to the equations

(A3) $$\left[\frac{\alpha_X(t)}{1-\alpha_X(t)}\right]^{\frac{-(1+R)}{2}}\left[\frac{\alpha_Y(t)}{1-\alpha_Y(t)}\right]^{\frac{(1-R)}{2}} =$$

$$\beta E_t\left\{\Gamma^{1-R}[\zeta^X(t+1)\zeta^Y(t+1)]^{\frac{(1-R)}{2}}\alpha_X(t+1)^{\frac{-(1+R)}{2}}\alpha_Y(t+1)^{\frac{(1-R)}{2}}\right\},$$

$$\left[\frac{\alpha_Y(t)}{1-\alpha_Y(t)}\right]^{\frac{-(1+R)}{2}}\left[\frac{\alpha_X(t)}{1-\alpha_X(t)}\right]^{\frac{(1-R)}{2}} =$$

$$\beta E_t\left\{\Gamma^{1-R}[\zeta^X(t+1)\zeta^Y(t+1)]^{\frac{(1-R)}{2}}\alpha_Y(t+1)^{\frac{-(1+R)}{2}}\alpha_X(t+1)^{\frac{(1-R)}{2}}\right\},$$

where $\Gamma = \gamma^\gamma(1-\gamma)^{1-\gamma}$. The equations in (A3) are necessary conditions for an optimal consumption/investment plan.

Conjecture now that the amounts of goods x and y consumed on date t under an efficient plan are

(A4) $$C_X(t) = \alpha(t)X(t), \qquad C_Y(t) = \alpha(t)Y(t)$$

(that is, $\alpha_X(t) = \alpha_Y(t) = \alpha(t)$ for all t). This conjecture is suggested by the symmetry of the model; under the parameters assumed in this section, $\kappa_X = \kappa_Y$ in the earlier logarithmic model, so (A4) holds there with $\alpha(t)$ constant over time. Our general strategy will be to argue that we can characterize a single random process $\{\alpha(t)\}$ such that when $\alpha_X(t) = \alpha_Y(t) = \alpha(t)$, the planner's intertemporal Euler conditions are satisfied.

Suppose that there exists a process $\{\alpha(t)\}$ that satisfies

$$\text{(A5)} \quad \left[\frac{\alpha(t)}{1-\alpha(t)}\right]^{-R} = \beta E_t\left\{\Gamma^{1-R}[\zeta^X(t+1)\zeta^Y(t+1)]^{\frac{(1-R)}{2}}\alpha(t+1)^{-R}\right\}.$$

It is straightforward to check that if one sets $\alpha_X(t) = \alpha_Y(t) = \alpha(t)$ for all t, then the processes $\{\alpha_X(t)\}$ and $\{\alpha_Y(t)\}$ satisfy *both* of the necessary conditions listed in (A3). Furthermore, the conditions of static efficiency and market clearing hold. Thus, assuming the existence of a solution to the stochastic difference equation (A5) with $0 < \alpha(t) < 1$ on all dates, and assuming uniqueness of the optimal plan, the marginal propensities $\alpha_X(t)$ and $\alpha_Y(t)$ must be the same, and equal to $\alpha(t)$, on all dates.

An analytical characterization of processes satisfying (A5) is obviously too much to hope for if $R \neq 1$. When productivity shocks are i.i.d. over time, however, $\alpha(t)$ is a constant, α, and (A5) implies that

$$\text{(A6)} \quad \alpha = 1 - \left\{\beta\Gamma^{1-R}E\left[(\zeta^X\zeta^Y)^{(1-R)/2}\right]\right\}^{1/R}.$$

Notice the resemblance between (A6) and equation (25) of section 3. These two equations coincide when $\gamma = 1$ and $\theta = 1/2$.

Portfolio autarky: Necessary conditions for equilibrium. Turn once again to the case in which all assets are nontraded. Let p(t) continue to denote the relative price of the y good, and introduce the following convenient definitions:

$$c = x + py,$$

$$c^* = x^*/p + y^*.$$

Thus, c is home consumption measured in home output, c* foreign consumption measured in foreign output.

In the above notation, the intertemporal Euler condition for the home country, taking the terms of trade as given, is

$$\text{(A7)}\quad c(t)^{-R}p(t)^{\frac{-(1-R)}{2}} = \beta E_t[\Gamma\zeta^{X}(t+1)p(t)^{\gamma-1}p(t+1)^{\frac{-(1-R)}{2}}c(t+1)^{-R}].$$

In essence, (A7) is an "indirect utility function" version of the standard intertemporal Euler condition.[24] The corresponding condition abroad is

$$\text{(A8)}\quad c*(t)^{-R}p(t)^{\frac{(1-R)}{2}} = \beta E_t[\Gamma\zeta^{Y}(t+1)p(t)^{1-\gamma}p(t+1)^{\frac{(1-R)}{2}}c*(t+1)^{-R}].$$

An equivalence proposition. The egalitarian command optimum examined earlier can be characterized by the marginal propensity to consume out of *either* output, $\alpha(t)$, which is a function of the state of the economy, $s(t) = [X(t),Y(t),\zeta(t)]$. The optimal plan also induces conditional distribution functions $F[s(t+1)|s(t)]$, and it is with respect to these c.d.f.s that the Euler condition (A5) must hold.

The basic idea of the argument is to show that when saving and investment decisions under balanced trade are governed by the optimal plan, and when market expectations about the transition between states are governed by the c.d.f.s $F[s(t+1)|s(t)]$, conditions (A7) and (A8) hold for the resulting equilibrium terms-of-trade (which coincide with the corresponding shadow price

[24] Indirect utility is $(1-R)^{-1}[(1/2)c(t)]^{(1-R)}p(t)^{-(1-R)/2}$.

in the plan). Since market expectations are then rational, this demonstration proves that the incomplete-markets equilibrium is the same as a particular complete-markets equilibrium and is thus efficient even without international asset trade.

The first step is to calculate the equilibrium terms of trade under portfolio autarky. Equality of supply and demand in the market for the x good means that

$$x(t) + x^*(t) + k_{XX}(t) + k_{YX}(t) = X(t).$$

Under the optimal plan, domestic spending is $\alpha(t)X(t)$ and foreign spending in terms of the x good is $\alpha(t)p(t)Y(t)$. The equilibrium condition above therefore implies (recalling the assumption of common expenditure shares of $\theta = 1-\theta = 1/2$) that $X(t) = (1/2)\alpha(t)[X(t) + p(t)Y(t)] + [1-\alpha(t)][\gamma X(t) + (1-\gamma)p(t)Y(t)]$.[25] Solving for $p(t)$ results in

(A9) $p(t) = X(t)/Y(t)$.

Substitute into (A7) and (A8) using (A9) and the saving and investment functions associated with the plan. After calculation, it turns out that both (A7) and (A8) hold if

$$\left\{\frac{\alpha(t)}{1-\alpha(t)}\right\}^{-R} = \beta E_t\left\{\Gamma^{1-R}\,[\zeta^X(t+1)\zeta^Y(t+1)]^{\frac{(1-R)}{2}}\,\alpha(t+1)^{-R}\right\}.$$

This equation is, however, the same as (A5), the equation

[25] Here is where the balanced-trade assumption comes in. Notice that the last term in the previous expression comes from the assumption that production functions are Cobb-Douglas.

characterizing the command optimum. It follows that the market allocation under portfolio autarky is efficient.

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