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INFLATION, INDEX-LINKED BONDS, AND ASSET ALLOCATION

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ABSTRACT

The recent introduction of CPI-linked bonds by several financial institutions is a milestone in the history of the U.S. financial system. It has potentially far-reaching effects on individual and institutional asset allocation decisions because these securities represent the only true long-run hedge against inflation risk.

CPI-linked bonds make possible the creation of additional financial innovations that would use them as the asset base. One such innovation that seems likely is inflation-protected retirement annuities. The introduction of index-linked bonds eliminates one of the main obstacles to the indexation of benefits in private pension plans. A firm could hedge the risk associated with a long-term indexed liability by investing in index-linked bonds with the same duration as the indexed liabilities.

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## 1. Introduction

A dramatic new development motivates this paper: the emergence of virtually risk-free securities linked to the U.S. consumer price level. The new securities were issued first by the Franklin Savings Association of Ottawa, Kansas, in January 1988 in two different forms. The first is certificates of deposit, called Inflation-Plus CDs, insured by the Federal Savings and Loan Insurance Corporation (FSLIC), and paying an interest rate tied to the Bureau of Labor Statistics' Consumer Price Index (CPI). Interest is paid monthly and is equal to a stated real rate plus the proportional increase in the CPI during the previous month. As of this writing (November 1988), the real rate ranges from 3% per year for a one-year maturity CD to 3.3% per year for a ten-year maturity.

The second form is twenty-year noncallable collateralized bonds, called Real Yield Securities, or REALs. These offer a floating coupon rate of 3% per year plus the previous year's proportional change in the CPI, adjusted and payable quarterly. A recent issue of similar bonds includes a put option.

Two other financial institutions have recently followed the lead of Franklin Savings.<sup>1</sup> If the trend continues, we have reached a milestone in the history of this country's financial

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<sup>1</sup>In August 1988 Anchor Savings Bank became the second U.S. institution to issue REALs, and in September 1988 JHM Acceptance Corporation issued modified index-linked bonds subject to a nominal interest rate cap of 14% per annum. The investment banking firm of Morgan Stanley and Company is the underwriter and market maker for REALs.

markets. Consider that for years prominent economists at all points of the ideological spectrum have argued that the U.S. Treasury should issue such securities, and scholars have speculated why private markets for them have not hitherto developed.<sup>2</sup> The current innovative environment in the U.S. financial markets appears to finally have put an end to this speculation by producing private indexed bonds in several forms.

This paper analyzes the gain to investors of this new investment alternative, considers likely changes in portfolio behavior that it might induce, and explores ways that it may be used in the future, principally to guarantee a safe stream of real benefits in retirement. The analytical framework is the familiar mean-variance model of portfolio selection of Markowitz and Tobin and the Capital Asset Pricing Model.<sup>3</sup>

The paper is organized as follows. Section 2 analyzes the difference between portfolio optimization in nominal and in real terms and shows how introduction of bonds offering a real risk-free rate of interest can improve portfolio efficiency. Section 3 discusses the difference between hedging against inflation in the short-run and in the long-run and shows why long-term index-linked bonds are the only true hedge against long-run inflation risk. Section 4 explains how index-linked bonds can be the basis

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<sup>2</sup>See, for example, the analysis in Fischer (1986).

<sup>3</sup>The reader who is unfamiliar with the mean-variance model and the CAPM, may refer to Bodie, Kane, and Marcus (1989), chapter 8.

for providing inflation-protected retirement benefits. In an appendix we derive the set of equilibrium expected real rates of return on stocks, bonds, and bills that are consistent with the 3% per year real risk-free rate now offered by the index-linked securities.

## 2. Asset Allocation in Real Terms

The individual investor is concerned ultimately with lifetime consumption. The appropriate focus in investment decision-making therefore should be real as opposed to nominal rates of return. A portfolio is therefore efficient if it offers the minimum variance of real rate of return for any given mean real rate of return.

Most real world applications of portfolio theory, however, are cast in nominal terms. Typically, Treasury bills are taken as the risk-free asset, and the optimal combination of risky assets, or the tangency portfolio, is constructed on the basis of the covariance matrix of nominal returns. All efficient portfolios are combinations of bills and the tangency portfolio.

Now let us consider the portfolio optimization in real terms. When there is no risk-free asset in real terms, there is no tangency portfolio, that is, no single optimal combination of risky assets that can be combined with the risk-free asset to generate the efficient frontier.

Let us give a specific numerical illustration, using the probability distribution of real rates of return presented in Table 1. The standard deviations and correlations in the table

were estimated from monthly real rate of return data for the period 1983 through 1987, and the expected returns were computed according to the CAPM assuming market weights of 60% for stocks, 15% for bonds, and 25% for bills (as explained in the Appendix).

When we add a real risk-free asset offering a real interest rate of 3% per year to the other assets, the efficient portfolio frontier becomes a straight line that is tangent to the original efficient frontier of risky assets only. Table 2 and Figure 1 compare the efficient frontier with and without REALs.

It is clear that there is virtually no gain in efficiency from adding REALs to the set of other assets, stocks, bonds, and bills. The efficient frontier of risky assets only is almost a straight line and indistinguishable from the frontier obtained when we add REALs. This is because despite the fact that bills do not offer a completely risk-free real rate of return, their standard deviation is so small that for practical purposes they are virtually risk-free in the short-run. The main impact of adding REALs is to substitute for bills in the low risk and low expected return end of the frontier.

Table 1. Probability Distribution of Real Rates of Return on Stocks, Bonds, and Bills

	<u>Stocks</u>	<u>Bonds</u>	<u>Bills</u>
Expected return E(r)	11.12%	5.46%	3.04%
Standard deviation ( $\sigma$ )	17.99%	12.89%	0.83%

Correlation coefficients:

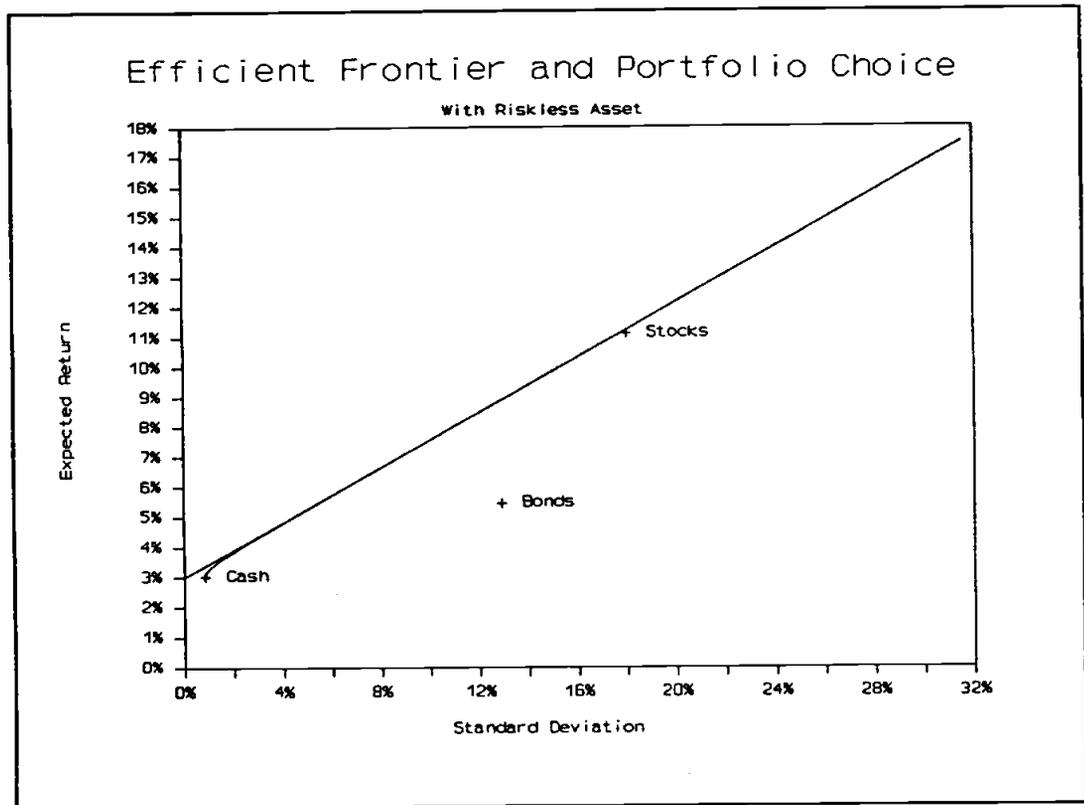
Stocks		.256	.016
Bonds			.394

Notes: The standard deviations and correlation coefficients were estimated using monthly data for the period January 1983 through December 1987. The series for stocks is the Standard & Poor's 500, for bonds Shearson Lehman's long-term government bond index, for bills one-month Treasury bills, and for inflation the CPI. The expected returns were computed according to the CAPM as explained below assuming market weights of 60% for stocks, 15% for bonds, and 25% for bills; relative risk aversion of 4; and a riskless real rate of 3% per year.

Table 2. Real Efficient Portfolios with and without a Real Risk-Free Asset (No short selling and no borrowing)

<u>Without Risk-Free Asset</u>					<u>With Risk-Free Asset</u>				
		<u>Portfolio Weights</u>					<u>Portfolio Weights</u>		
<u>Mean</u>	<u><math>\sigma</math></u>	<u>Stocks</u>	<u>Bonds</u>	<u>Bills</u>	<u><math>\sigma</math></u>	<u>Stocks</u>	<u>Bonds</u>	<u>Bills</u>	<u>REALs</u>
3.0	0.83	0	0	100.0	0	0	0	0	100.0
4.0	2.27	11.6	0.8	87.5	2.18	11.5	2.9	4.8	80.8
5.0	4.39	23.0	4.2	72.8	4.39	23.1	5.8	9.6	61.5
6.0	6.55	34.4	7.5	58.1	6.55	34.6	8.7	14.4	42.3
7.0	8.73	45.7	10.8	43.4	8.73	46.2	11.5	19.2	23.1
8.0	10.90	57.1	14.1	28.8	10.90	57.7	14.4	24.1	3.8
8.23	11.40	60.0	15.0	25.0	11.40	60.0	15.0	25.0	0
9.0	13.09	68.5	17.5	14.0	13.09	68.5	17.5	14.0	0
10.0	15.27	79.8	20.2	0	15.27	79.8	20.2	0	0
11.1	17.99	100.0	0	0	17.99	100.0	0	0	0

Figure 1. The Efficient Portfolio Frontier

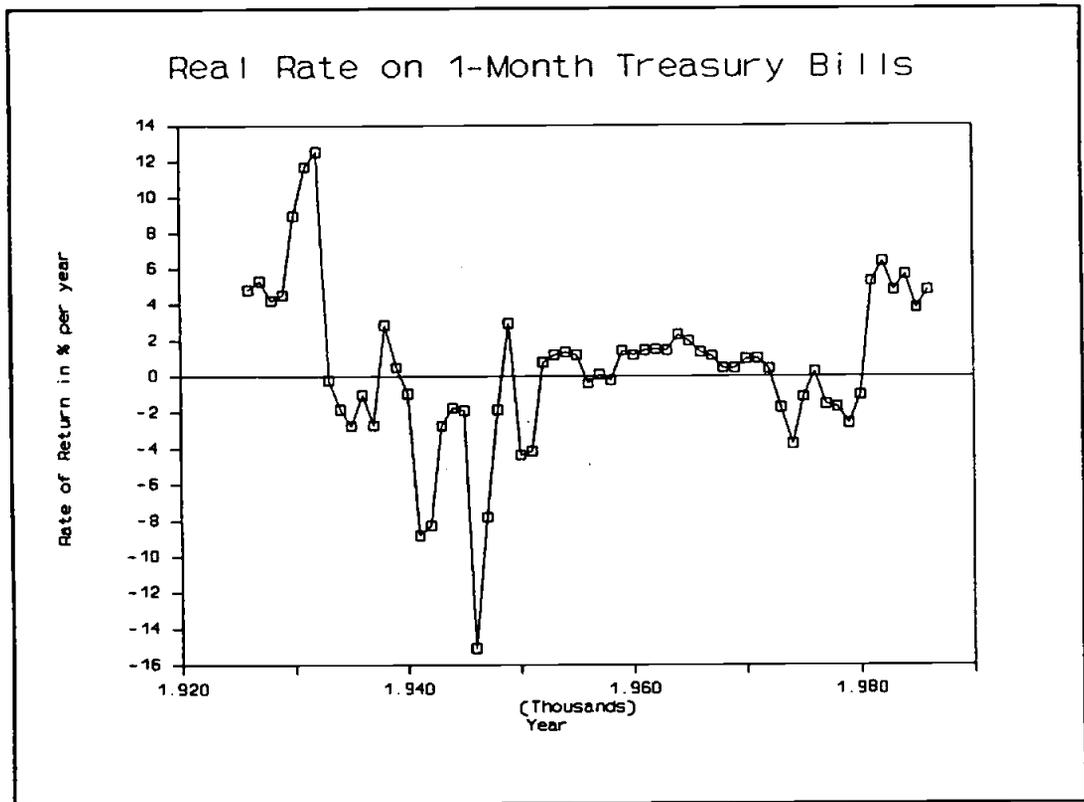


### 3. Short-Run vs Long-Run Inflation Hedging

While a policy of investing in bills is an effective hedge against inflation in the short run, it is not an effective way to lock in a real rate of interest for the longer run. For example, suppose you are an investor with an investment horizon of 10 years; let us say you are saving for a child's college education 10 years from now. While you can be reasonably sure of earning a real rate of return on bills of 3% per year for the next year or so, there is considerable uncertainty about the rate beyond that. Figure 2 plots the annual real rate of return on a policy of rolling over 1-month Treasury bills over the 60 year period 1926-1986.

It is clear from the graph that while the real rate on bills has been stable over periods as long as 10 or even 18 years (e.g., from 1953 to 1971), it has also exhibited considerable variability. Just in the most recent two decades its behavior has changed dramatically. In the 1970s the real rate on bills was substantially negative, averaging -1% per year for the 10 years from January 1970 to December 1979. In the 1980s, it has averaged 4% per year.

Figure 2. Real Rate of Return on 1-Month Treasury Bills



This instability of the real rate of return on money market instruments over long periods of time makes them unsuitable for investors who want to hedge long term liabilities that are fixed in real terms, such as the cost of a college education ten years from now. What you really need in order to hedge this liability is a 10 year zero coupon bond indexed to the cost of living, promising a fixed real interest rate like the Franklin Savings Association Inflation Plus CD described earlier in this paper.<sup>4</sup>

In this context it is worth pointing out a major disadvantage of REALs in the form of the coupon bonds issued by Franklin Savings Association. While the coupon interest rate on these bonds is fixed in real terms, their face value is fixed in nominal terms. Consequently, their duration is a function of the realized rate of inflation. Putting it in a slightly different way, the rate at which the real value of the principal declines is a function of the inflation rate. This is in contrast to bonds whose face value is fixed in real terms, such as the index-linked bonds issued by the government of the United Kingdom. In practical terms this means that REALs in their coupon bond form are of limited use in any duration-matching strategy that an investor would want to implement in order to hedge or "immunize" long-term indexed liabilities.

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<sup>4</sup>A better hedge would be a bond linked to the cost of college education. Such securities are currently available.

#### 4. Inflation-Proof Retirement Annuities

CPI-linked bonds make other innovations possible. Perhaps the most important of these is inflation-protected retirement annuities. Retired people have long been considered the most vulnerable to inflation risk, but proposals for private market solutions to this problem have been stymied by the lack of a real risk-free asset.<sup>5</sup>

In 1980, for example, Bodie proposed the idea of a variable annuity offering at least limited protection against inflation risk by hedging money market instruments with a small position in a diversified portfolio of commodity futures contracts.<sup>6</sup> Working against the proposal, however, was the low mean real rate of return available on money market instruments at that time (between 0 and 1% per year). The situation is markedly different now with the availability of virtually risk-free securities offering real rates in excess of 3% per year. Pension funds and other providers of retirement benefits, which currently offer only nominal annuities, could also offer attractive real annuity options to retirees.

To illustrate how such a real annuity option might work, assume at retirement that you are entitled to a benefit with a present value of \$100,000. Your retirement plan currently offers

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<sup>5</sup>Feldstein (1983) and Summers (1983) have both argued that the elderly may in fact be over-indexed already because of their claims to Social Security benefits and their ownership of real estate.

<sup>6</sup>See Bodie (1980).

you a conventional nominal annuity computed on the assumption of a nominal interest rate of 8% per year and a life expectancy of 15 years. Assuming the first payment is to be received immediately, the annual benefit is \$10,818. The plan hedges its liability to you by investing in risk-free nominal bonds paying a nominal rate of 8% per year.

From your perspective, the real value of this stream of benefits is uncertain. Consider the purchasing power of the final benefit payment to be received 14 years from now. If the rate of inflation turns out to be 5% per year, the real value of the final benefit will be \$5,464, about half the value of the first payment. If the rate of inflation turns out to be 10% per year, the real value of the final payment drops to \$2,849.

Contrast this with a hypothetical real annuity. Your plan can now invest your \$100,000 to earn a real risk-free rate of 3% per year, so it could offer you a real annuity computed on the assumption of 3% per year. Your annual benefit would be \$8,133 guaranteed in real terms. While the initial payment is lower than under the nominal option, the real value of the benefit is insured against inflation.

The real annuity, however, need not start at a lower value than the conventional nominal annuity. Bodie and Pesando (1983) have shown how real annuities can be designed with the same starting value as conventional nominal annuities. Such a real annuity would have to pay decreasing benefit amounts, just as the expected real value of the benefit stream from the nominal

annuity decreases. The essential difference is that the real annuity is insured against inflation, while the nominal annuity is not.

Indexing a retirement annuity after retirement is only one aspect of inflation-proofing private pension plans. Another is to index benefit accruals prior to retirement. Under private defined benefit (DB) plans the value of accrued benefits is extremely sensitive to inflation because once an employee stops working for the plan sponsor or once the sponsor terminates the plan, pension benefits are fixed in nominal terms.

For example, suppose you are 45 years old and have worked for the same employer for 20 years. Assume that your DB plan promises 1% of final salary per year of service; that your most recent salary was \$50,000; that normal retirement age is 65, and that your life expectancy is 80 years. Your claim on the pension fund is a deferred annuity of \$10,000 per year starting at age 65 and lasting for 15 years.

If you leave this employer, what do you have? The benefit is not indexed to any wage or price level the way Social Security is, so the benefit loses real value as prices rise. Assuming inflation of 5% per year, the value of a dollar will have fallen to \$.38 by the time you retire in another twenty years, and your first year benefit of \$10,000 will have a real value of only \$3,800. That value will continue to fall each year as inflation continues. If, however, you stay with your employer, your salary increases at the rate of inflation, and your employer indexes

your benefit to the cost of living after retirement, you will have an annuity worth \$10,000 of today's purchasing power per year for life.

Looking at the situation in terms of present values (assuming a nominal discount rate of 8% per year and a real discount rate of 3% per year), your accrued benefit if you switch jobs or if the plan is terminated has a present value of \$18,364. If you continue, with complete indexation both before and after retirement, the accrued benefit has a present value of \$66,097.

It is often said that DB plans lack portability. But this is not exactly correct. Once employees are vested in a DB plan they cannot lose the annuity they have earned. Rather, they lose value: because the annuity is not indexed to the cost of living or to wages, its worth is greatly diminished if the employee switches jobs or if the plan is terminated.

This feature of DB plans may be a deterrent to employee turnover and as such could be an efficient long-term labor contracting device. But the strength of this deterrent depends mainly on inflation, which is not subject to anyone's control, except perhaps that of the fiscal and monetary policy authorities. It seems unlikely that it could be efficient to have the strength of the incentive be hostage to inflation. Furthermore, inflation so complicates the calculation of both the future real value of the stream of pension benefits and the present value of that stream that it is unlikely that workers can fully understand the set of incentives being offered.

Pension fund asset allocation could be profoundly affected were pension plans actually to offer indexed benefits to their employees. Many pension funds currently hedge their nominal pension liabilities by means of duration matching strategies that result in investments in long-term fixed-income securities. A switch to indexed pensions probably would result in hedging strategies involving investment in long-term index-linked securities.

#### 5. Summary and Conclusions

The introduction of CPI-linked bonds by several financial institutions is a milestone in the history of the U.S. financial system. It has potentially far-reaching effects on individual and institutional asset allocation decisions because they offer the only true long-run hedge against inflation risk. Risk-averse investors may achieve substantial efficiency gains by substituting these real risk-free securities for bills in their portfolios, especially since the real rate being offered by REALs seems quite high in light of the historical average returns on securities with low variance of real returns.

The existence of CPI-linked bonds may spur the creation of other financial innovations relying on such an asset base. The most likely innovation would appear to be inflation-protected retirement annuities.

Appendix: Equilibrium Real Rates of Return on Stocks, Bonds, and Bills

What set of expected rates of return on stocks, bonds, and bills is consistent with an observed real risk-free rate of 3% per year in equilibrium? By this we mean what set of expected real rates would make the average investor (that is, an investor with an average degree of risk aversion) willing to hold all assets in the proportions that they actually exist in the economy?

As Bodie, Kane, and McDonald (1985) have shown, under the assumptions of the Capital Asset Pricing Model the equilibrium risk premium on any asset is equal to the covariance of its real rate of return with the market portfolio times the average degree of relative risk aversion of market participants.<sup>7</sup>

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<sup>7</sup>The equilibrium risk premium on security  $i$  is given by the formula:  $E(r_i) - r_f = \delta \sigma_{im}$  where  $\sigma_{im}$  is the covariance between the real rate of return on security  $i$  and the market portfolio and  $\delta$  is the aggregate measure of relative risk aversion (a weighted harmonic mean).  $\sigma_{im}$ , in turn, is given by the formula:  $\sigma_{im} = \sum_j w_j \sigma_{ij}$  where  $w_j$  is the weight of security  $j$  in the market portfolio, and  $\sigma_{ij}$  is the covariance between the real rates of return on securities  $i$  and  $j$ .

Covariance Matrix

$$\begin{array}{ll} \sigma_{sb} = .00593641 & \sigma^2 = .03236401 \\ \sigma_{sc} = .00002389 & \sigma^{2s} = .01661521 \\ \sigma_{bc} = .00042153 & \sigma^{2b} = .00006889 \\ & \sigma_c^2 = .00006889 \end{array}$$

Assumed market weights: stocks .6, bonds .15, cash .25

$$\begin{array}{l} \sigma_{sm} = .02031484 \\ \sigma_{bm} = .00615951 \\ \sigma_{cm} = .00009479 \end{array}$$

Table 3 shows the equilibrium real rates of return corresponding to three sets of assumptions about the proportions of assets outstanding for an average degree of relative risk aversion of 4. Table 4 displays the full set of means and standard deviations, including a market portfolio that is 60% stocks, 15% bonds, and 25% bills.

Perhaps the most striking result is how small the risk premium on bills is: less than 10 basis points in every case. The last line in Table 3 probably represents the best estimate of the current asset proportions in the U.S. economy and therefore we chose it for our market portfolio in Table 4.<sup>8</sup>

One way to judge the "reasonableness" of these numbers is to make an assumption about the expected rate of inflation, add it to the equilibrium expected real rates of return, and then compare the resulting numbers to the observed nominal yields.<sup>9</sup> Of course, this will work only for bills and bonds, whose nominal yields are directly observable. If we assume an expected rate of inflation of 4% per year, then the implied nominal rates for outstanding asset proportions of 60% stocks, 15% bonds, and 25% bills are:  
stocks 15.12% per year, bonds 9.46%, and bills 7.04%.

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<sup>8</sup>See Rouse (1988).

<sup>9</sup>As Fischer (1975) has shown, the relationship between the expected nominal and real rates of return is: expected nominal rate = expected real rate + expected rate of inflation + covariance between the real rate and the rate of inflation. We are ignoring the covariance term here because it is so small.

Table 3. Equilibrium Expected Real Rates of Return for Different Assumptions About Proportions of Assets Outstanding

<u>Equilibrium Expected Real Returns</u> % per year			<u>Asset Proportions Outstanding</u> %		
<u>Stocks</u>	<u>Bonds</u>	<u>Bills</u>	<u>Stocks</u>	<u>Bonds</u>	<u>Bills</u>
10.42%	6.86%	3.08%	50%	40%	10%
9.13	6.64	3.08	40	40	20
11.48	6.44	3.06	60	30	10
11.12	5.46	3.04	60	15	25

Assumptions: The variances and correlations are those assumed in Table 1, and the risk aversion parameter is 4. The risk-free real rate is 3% per year.

Table 4. Probability Distribution of the Real Rates of Return

		<u>Stocks</u>	<u>Bonds</u>	<u>Bills</u>	<u>Market Portfolio</u>	<u>Reals</u>
Expected return $E(r)$	11.12%	5.46%	3.04%	8.23%	3.00%	
Standard deviation ( $\sigma$ )	17.99%	12.89%	0.83%	11.40%	0	

Note: The market portfolio is assumed to consist of 60% stocks, 15% bonds, and 25% bills.

The 7% nominal yield on bills and the 9.46% yield on bonds are close to the observed market rates on Treasury securities as of this writing (November 1988). We therefore conclude that the set of assumptions underlying our calculations are not inconsistent with reality, even if they are not entirely correct.

The small risk premium on bills reported in Table 3 is explained by the fact that bills are such a good substitute for REALs in the short run. The long-run is quite a different story.

## References

- Bodie, Zvi, "An Innovation for Stable Real Retirement Income," The Journal of Portfolio Management, Fall 1980, pp. 5-13.
- , A. Kane, and R. MacDonald, "Why Haven't Nominal Rates Declined?" Financial Analysts Journal, March/April 1984.
- , A. Kane, and R. MacDonald, "Inflation, and the Role of Bonds in Investor Portfolios," in Corporate Capital Structures in the U.S., B. M. Friedman, ed., University of Chicago Press, 1985.
- , A. Kane, and R. MacDonald, "Risk and Required Returns on Debt and Equity," in Financing Corporate Capital Formation, Benjamin M. Friedman, ed., University of Chicago Press, 1986.
- , A. Kane, and A. Marcus, Investments, Richard B. Irwin, Homewood, Illinois, 1989.
- and J. Pesando, "Retirement Annuity Design in an Inflationary Climate," Chapter 11 in Bodie and Shoven, eds, Financial Aspects of the U.S. Pension System, University of Chicago Press, 1983.
- Feldstein, Martin, "Should Private Pensions Be Indexed," Chapter 8 in Financial Aspects of the U.S. Pension System, op. cit.
- Fischer, Stanley, "The Demand for Index Bonds," Journal of Political Economy, 1975, reprinted in Indexing, Inflation, and Economic Policy, MIT Press, 1986.
- , "On the Nonexistence of Privately Issued Index

Bonds in the U.S. Capital Market," chapter 10 in Indexing, Inflation, and Economic Policy, MIT Press, 1986.

Rouse, Michael, Unpublished Doctoral Dissertation, Boston University, 1988.

Summers, Lawrence, "Observations on the Indexation of Old Age Pensions," Chapter 9 in Financial Aspects of the U.S. Pension System, op. cit.