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MONETARY POLICY AND ASSET PRICE OVERSHOOTING:
A RATIONALE FOR THE WALL/MAIN STREET DISCONNECT

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Working Paper 27712
<http://www.nber.org/papers/w27712>

NATIONAL BUREAU OF ECONOMIC RESEARCH
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August 2020

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NBER Working Paper No. 27712
August 2020
JEL No. E21,E32,E43,E44,E52,G12

ABSTRACT

We analyze optimal monetary policy when asset prices influence aggregate demand with a lag (as is well documented). In this context, as long as the central bank's main objective is to minimize the output gap, the central bank optimally induces asset price overshooting in response to the emergence of a negative output gap. In fact, even if there is no output gap in the present but the central bank anticipates a weak recovery dragged down by insufficient demand, the optimal policy is to preemptively support asset prices today. This support is stronger if the acute phase of the recession is expected to be short lived. These dynamic aspects of optimal policy give rise to potentially large temporary gaps between the performance of financial markets and the real economy. One vivid example of this situation is the wide disconnect between the main stock market indices and the state of the real economy in the U.S. following the Fed's powerful response to the Covid-19 shock.

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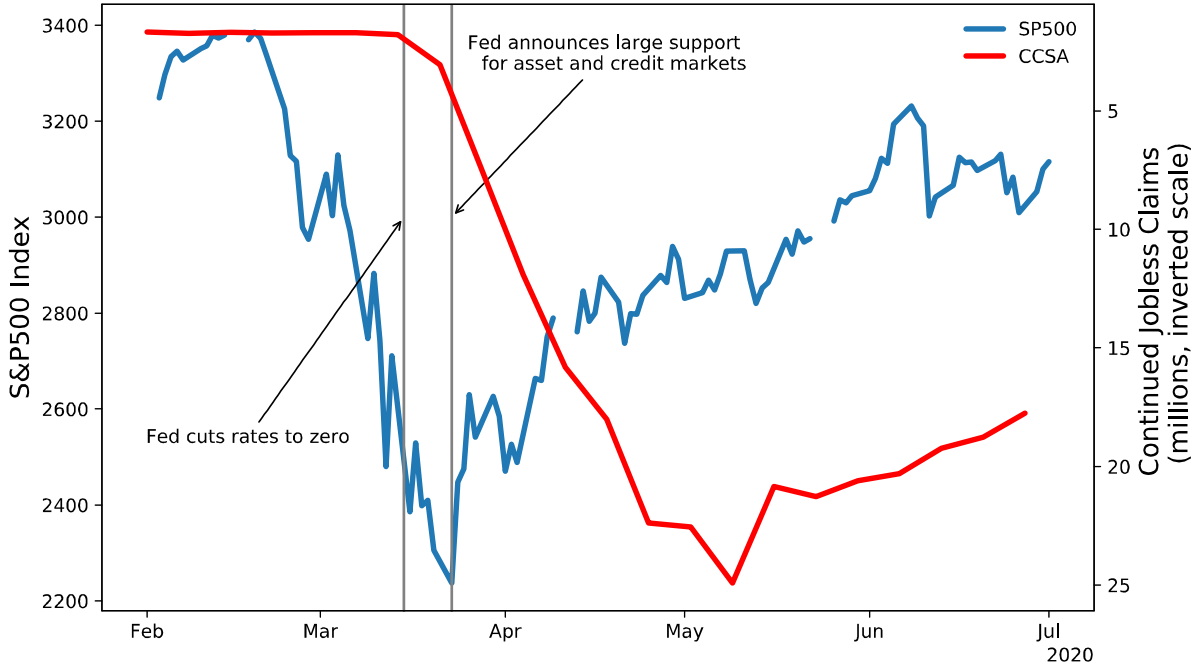


Figure 1: The disconnect between Wall Street and Main Street. Data: S&P500 index and continued jobless claims (seasonally adjusted). Source: FRED.

1. Introduction

The blue line in Figure 1 shows the acute drop in the S&P 500 once the economic impact of the Covid-19 shock became apparent. The figure also shows the equally dramatic recovery of the S&P 500 after the Fed announced its massive policy response (e.g., it cut the policy rate to zero and pledged close to 20% of GDP in asset purchase and credit support facilities). Other financial assets show a similar pattern. While the Fed was successful in reversing the financial meltdown, it did not prevent a dramatic collapse in the real economy. The red line in Figure 1 shows that the US continued claims (inverted scale) declined substantially and remained low. This disconnect between the quick recovery of financial markets and the sluggish response of the real economy has been the source of much debate, as highlighted by the cover page of *The Economist*, May 9th, 2020 (“A dangerous gap: The markets v the real economy”). To explain this discrepancy, some observers point at what they see as irrational exuberance in financial markets, while others point at the ineffectiveness of the Fed’s policies in helping main street.

In this paper we present a model where the facts depicted in Figure 1 are the natural result of *optimal* monetary policy—not (necessarily) a sign of irrationality or ineffective policy. As long as the central bank’s principal objective is to minimize the output gap,

and asset prices influence aggregate demand with a lag, the central bank’s optimal policy induces an asset price overshoot in response to an output gap. In fact, even if there is no output gap in the present but the central bank anticipates a weak recovery dragged down by insufficient demand, the optimal policy is to preemptively support asset prices today. These dynamic aspects of optimal policy give rise to potentially large temporary gaps between the performance of financial markets and the real economy.

Our model is set in continuous time. Output is determined by aggregate demand due to nominal rigidities. Aggregate demand depends on asset prices through a wealth effect on consumption. The key ingredient is that individual agents adjust their consumption infrequently, driven by a Poisson process that is independent across individuals. Therefore, asset prices affect consumption with a lag, consistent with empirical evidence (see, e.g., Chodorow-Reich et al. (2019)). Monetary policy affects aggregate demand through its impact on asset prices; thus, the policy also affects consumption and aggregate demand with a lag. The central bank sets interest rates to minimize output gaps, taking transmission lags into account.

We first analyze a “recovery” scenario in which potential output has just recovered from a large productivity shock (e.g., Covid-19). However, aggregate demand, dragged down by a recent history of poor economic conditions, is initially below potential output. Over time, demand gradually catches up with potential output and closes the negative output gap. In this context, our main result shows that the equilibrium features *asset price overshooting*: asset prices are initially high (above their levels consistent with potential output) even though output is low. Intuitively, a central bank that dislikes output gaps boosts asset prices to close the output gap as fast as possible. This boost creates a large temporary disconnect between financial markets and the real economy, but it also accelerates the recovery. In fact, the asset price boost (and its disconnect with the real economy) is greatest at the beginning of the recovery phase, when the output gap is widest.

We then consider the “recession” phase that precedes the “recovery” phase. In the recession, potential output has experienced a deep contraction but is expected to increase according to a Poisson event—in which case the economy transitions to the recovery. The overshooting result also applies in the recession: if output is below its (already low) potential level, then the central bank boosts asset prices to close the gap. In fact, the equilibrium in the recession also features *preemptive overshooting*: the central bank boosts asset prices *even if output is at its potential level*. This preemptive overshooting is stronger when the central bank expects a faster transition to recovery. Intuitively, the central bank anticipates that the “recovery” will start with a large negative output

gap due to the inertial behavior of aggregate demand. Therefore, the central bank acts preemptively to boost asset prices and aggregate demand during the recession—in order to ensure that aggregate demand is not too depressed during the early stages of the recovery phase.

Literature review. The concept of an overshooting asset price in response to monetary policy actions or shocks was introduced by Dornbusch (1976). That model and many others that followed it consider a world where goods’ prices and output adjust sluggishly and there is a well defined long run equilibrium level of the real exchange rate. Then, for the interest parity condition to hold across safe bonds, the nominal exchange rate must overshoot its long run response to a monetary shock. While our model also has sticky prices and sluggish aggregate demand, our overshooting result concerns asset prices more broadly—rather than just exchange rates, and it is a feature of optimal monetary policy—rather than the implication of an arbitrage condition.

Closer to our paper is Caballero and Lorenzoni (2014). Their small open economy model starts in an overvaluation phase (abnormally high demand for nontradables), and randomly transitions to a normal phase. Because the export sector faces financial constraints, the real exchange rate overshoots at the transition and gradually returns to its steady state level as the excess depreciation generates large profits for the financially constrained export sector. In this context, the optimal policy is to reduce the overvaluation in the first phase and exacerbate the overshooting at the transition, both with the purpose of relaxing the export sector’s financial constraint. The optimal policy also induces overshooting in our model, but for a different reason: to stimulate output in an environment with nominal rigidities and sluggish adjustment of aggregate demand.

Like in Caballero and Simsek (2020a,b,c), monetary policy in our model operates through financial markets. The central bank affects asset prices, which in turn affects aggregate demand. The distinctive feature of this paper is the delayed response of aggregate demand to asset prices, which is the main ingredient driving our overshooting results. In the context of the Covid-19 recession, Caballero and Simsek (2020a) provides an explanation for the large decline in asset prices and highlights the key role of large scale asset purchases (LSAPs) in reversing that decline, and this paper provides a rationale for the subsequent Wall/Main Street disconnect (see Figure 1).

In terms of the model’s ingredients, this paper is also related to and supported by an extensive empirical literature documenting that: (i) monetary policy affects asset prices (e.g., Thorbecke (1997), Jensen et al. (1996), Jensen and Mercer (2002), Rigobon and Sack (2004), Bernanke and Kuttner (2005), Ehrmann and Fratzscher (2004))—these papers find

that, on average, an unanticipated 100 bps increase in the federal funds rate is associated with a decrease in stock market returns in the range of 5.3% to 6.2%; (ii) asset prices affect aggregate demand and output (e.g., Davis and Palumbo (2001), Gilchrist and Zakrajšek (2012), Kyungmin et al. (2020), Dynan and Maki (2001), Di Maggio et al. (forthcoming), Chodorow-Reich et al. (2019), Mian and Sufi (2014), Guren et al. (2018))—these papers find wealth and balance sheet effects in the range of 5–10 cents on the dollar depending on the sample and the specific asset price; (iii) the effect of asset prices on aggregate demand and output is gradual (e.g., Davis and Palumbo (2001); Lettau and Ludvigson (2004); Carroll et al. (2011), Dynan and Maki (2001), Case et al. (2013), Chodorow-Reich et al. (2019))—these papers find that consumption typically takes between 8 and 9 quarters to fully adjust to stock price changes.

A central feature of our model is the infrequent adjustment of individual consumption (aggregate demand). An extensive literature on durables’ consumption (and investment) uses fixed adjustment costs to explain this feature. This literature documents infrequent adjustment in microeconomic data, and explains the inertia it introduces for aggregate durables’ consumption and investment (see Bertola and Caballero (1990) for an early survey). There is also a literature that emphasizes infrequent re-optimization for broader consumption categories—due to behavioral or informational frictions—and uses this feature to explain the inertial behavior of aggregate consumption (e.g., Caballero (1995); Reis (2006)) as well as asset pricing puzzles (e.g., Lynch (1996); Marshall and Parekh (1999); Gabaix and Laibson (2001)). We take the infrequent adjustment of individual consumption as given (driven by a Poisson process for simplicity) and study its implications for optimal monetary policy.

Finally, our paper is related to the growing empirical literature that analyzes the forces that are driving asset prices during the Covid-19 recession (e.g., Gormsen and Koijen (2020); Ramelli and Wagner (forthcoming); Landier and Thesmar (2020)). This literature typically attributes the large stock market swing in Figure 1 to changes in the effective discount rate for stocks. Gormsen and Koijen (2020) argue that the discount rate initially spiked and then declined to its pre-crisis levels—in part because of the decline in expected interest rates. This explanation suggests, consistent with our model, that monetary policy explains part of the recovery in the stock market. Other studies find that unconventional monetary policies (which we discuss in Remark 1) also had a large positive impact on asset prices (e.g., Fed (2020); Cavallino and De Fiore (2020); Arslan et al. (2020); Haddad et al. (2020)).

The rest of the paper is organized as follows. Section 2 introduces our baseline environ-

ment and describes the equilibrium conditions with optimal monetary policy. Section 3 analyzes the recovery phase and shows that the equilibrium features asset price overshooting. Section 4 considers the recession phase and shows that it features both overshooting and preemptive asset price overshooting. Section 5 provides final remarks, and is followed by several appendices.

2. The baseline model

In this section, we describe our baseline model and define the equilibrium. We envision a scenario that captures *the aftermath of a recessionary shock* to the economy, such as Covid-19. Specifically, potential output has already recovered to its pre-shock level, but initial aggregate demand and output—which were brought down by the recent negative shock—are below potential output. We use this setup to study the optimal monetary policy during the recovery from a recessionary shock. In Section 4, we extend the model to introduce an ex-ante period with low potential output and study the optimal policy during the recession (with an anticipated recovery).

Time $t \geq 0$ is continuous. There is a single consumption good and a single factor (capital). Capital is normalized to one unit and there is no investment or depreciation. Thus, potential output is the productivity of capital. Until Section 4, potential output $Y^* = 1$ is constant and normalized to one. In particular, there is no aggregate risk.

Actual output, denoted by $Y(t)$, can be different than potential output due to nominal rigidities. Specifically, there are New-Keynesian firms that have permanently fixed prices and can change their production by adjusting capital utilization.¹ These firms operate with a markup and find it optimal to meet the demand for their good, even when this demand deviates from their potential output (for relatively small deviations, which we assume). This setup ensures that output is determined by aggregate demand. Since there is no investment, output is determined by aggregate consumption:

$$Y(t) = C(t). \tag{1}$$

There are two financial assets. First, there is a market portfolio that reflects a claim on all current and future output. We let $Q(t)$ denote the price of the market portfolio. Second, there is a risk-free asset in zero net supply. We let $R(t)$ denote the risk-free

¹Formally, $Y(t) = \eta Y^*$ where η denotes capital utilization. We normalize the utilization that would obtain absent nominal rigidities to one, $\eta^* = 1$. Operating capital with higher or lower utilization is feasible but inefficient. For simplicity, we do not explicitly model capital utilization costs in this paper (see Caballero and Simsek (2020b) for a version of the model with these costs).

interest rate (nominal and real rates are the same since prices are permanently fixed), and is set by the central bank.

There is a continuum of identical consumer-investors. These consumers make a consumption-savings choice as well as a portfolio choice. Consumers have time-separable, log utility with discount rate ρ . With these preferences, if there were no other frictions, consumers would spend a constant fraction of their wealth, $C(t) = \rho Q(t)$. Since there is no risk, they would also ensure the expected return on the market portfolio is exactly equal to the risk-free rate, $R(t)$. At this expected return, consumers would be indifferent between the market portfolio and the risk-free asset. Assuming the central bank sets the interest rate to ensure output is equal to its potential, the economy would *immediately* reach a steady state with potential outcomes:

$$\begin{aligned} Y(t) &= Y^* = 1, & C(t) &= C^* = 1, \\ R(t) &= R^* = \rho & \text{and} & \quad Q^* = \frac{1}{\rho}. \end{aligned} \tag{2}$$

Set against this potential benchmark, our key friction is that consumers do not continuously adjust their consumption and portfolio allocations. At every instant, a random fraction of consumers adjusts, with constant hazard θ . Their allocations remain unchanged until the next time they have a chance to adjust.

First consider the portfolio choice. Since there is no risk, the return on the market portfolio is equal to the risk-free rate as in the benchmark. That is, the equilibrium asset price satisfies

$$R(t) = \frac{Y(t) + \dot{Q}(t)}{Q(t)}. \tag{3}$$

At this price, the consumers that adjust are indifferent to changing their portfolios. For concreteness, we assume all consumers invest an identical fraction of their wealth in the market portfolio. This fraction is equal to one to ensure market clearing.

Next consider the consumption choice. There is a representative consumer whose wealth equals aggregate wealth, $Q(t)$. When adjusting, we assume this consumer chooses her new level of consumption according to the rule

$$C^{adjust}(t) = C^* + m(Q(t) - Q^*) + n(Y(t) - Y^*).$$

Here, C^*, Y^*, Q^* are the potential levels in (2) and $m > 0, n \in [0, 1)$ are parameters. Individual consumers follow a similar rule scaled by their wealth shares.² Equivalently,

²Formally, let $a^i(t)$ denote a consumer's wealth and $\alpha^i(t) = \frac{a^i(t)}{Q(t)}$ denote her wealth share. A consumer

letting lowercase variables denote gaps relative to the potential, $x(t) \equiv X(t) - X^*$, we assume

$$c^{adjust}(t) = mq(t) + ny(t). \quad (4)$$

Hence, the adjusting consumers' spending (measured in gaps from potential) can be written as a linear and increasing function of the current asset price and of current output.

Our formulation in (4) allows for a rich set of possibilities for consumption behavior. For $m = \rho$ and $n = 0$, adjusting consumers are “myopic” and follow the same rule as in the benchmark case of a continuously-adjusting consumer, $C(t) = \rho Q(t)$. In Appendix A.2, we also analyze the decisions of a “fully rational” consumer who incorporates the fact that she might not get to adjust in future periods. Along the equilibrium path, this consumer's optimal choice (up to linearization) can be written as in (4) with endogenous weights $m \in (0, \rho)$ and $n \in (0, 1)$. We discuss the intuition behind this rule and the implications for our analysis in Section 3.2. Beyond these cases, the rule in (4) can also stand in for unmodeled ingredients that make (even unconstrained) households' spending react to current income changes beyond wealth changes (see, e.g., Parker (2017); Kueng (2018) for empirical evidence). Our main result applies for arbitrary parameters as long as they satisfy the mild assumptions $m > 0$ and $n < 1$: that is, consumption responds *at least somewhat* to asset prices, and its response to output is less than one to one. For most of our analysis (except in Section 3.2), we take Eq. (4) as an exogenous rule with fixed parameters m, n .³

Assuming that the adjusting consumers are randomly selected, and using Eq. (1), we also have an equation that describes the dynamics of output:

$$\dot{Y}(t) = \theta (C^{adjust}(t) - Y(t)). \quad (5)$$

Hence, output increases if and only if the adjusted spending exceeds the current level of spending and output, $C(t) = Y(t)$. The initial level of spending and output, $C(0) =$

with wealth share $\alpha^i(t)$ follows the rule

$$C^{i,adjust}(t) = \alpha^i(t) (C^* + n(Y(t) - Y^*) + m(Q(t) - Q^*)).$$

Aggregating across all adjusting consumers, we obtain (4) because the wealth shares satisfy $\sum_i \alpha^i(t) = 1$.

With this rule, there might be paths in which some consumers' wealth becomes zero, e.g., if the consumer does not readjust for a long time. If this happens, the budget constraint binds and consumption falls to zero. We ignore these paths since we focus on relatively small shocks and linearized dynamics.

³For the fully rational case, the weights m, n are endogenous to the dynamic equilibrium path but exogenous to monetary policy decisions (as long as the central bank sets interest rates without commitment, which we assume). We do not focus on this case and instead treat m, n as exogenous parameters to avoid having to deal with complex fixed-point arguments that are unrelated to our main contributions.

$Y(0)$, is exogenous (determined by unmodeled history). Rewriting the expression in terms of the gaps from potential, and substituting (4), we also obtain

$$\dot{y}(t) = \theta(mq(t) + ny(t) - y(t)) \quad \text{given } y(0). \quad (6)$$

It remains to describe monetary policy. The central bank sets a path of interest rates, $[R(t)]_{t \in (0, \infty)}$, and implements a path of output and asset prices that satisfy Eqs. (3) and (6). Note that it is equivalent to think of the central bank as setting a path of asset price gaps, $[q(t)]_{t \in (0, \infty)}$. Given this path and the initial output gap $y(0)$, Eq. (6) describes the equilibrium path of output gaps. Eq. (3) then describes the equilibrium interest rate.

We assume the central bank's objective function is

$$V(0, y(0)) = \int_0^\infty e^{-\rho t} \left(-\frac{y(t)^2}{2} - \psi \frac{q(t)^2}{2} \right) dt. \quad (7)$$

As usual, the central bank dislikes output gaps, $y(t)$. We assume a quadratic cost function, which leads to closed-form solutions. In addition, the central bank also dislikes asset price gaps, $q(t)$. The parameter ψ is the relative weight on asset price gaps. In the limit $\psi \rightarrow 0$, we have the conventional setup in which the central bank does not (directly) pay attention to asset prices. Our main results hold in this conventional limit but optimal asset price behavior is extreme and unrealistic; in practice, central banks do not tolerate, let alone induce, very large fluctuations in asset prices (see, e.g., Cieslak and Vissing-Jorgensen (2020)). We capture this "aversion-to-overshooting" with the parameter $\psi > 0$. The reasons for this aversion range from distributional considerations to financial stability. The central bank's discount rate is the same as that of the consumers, ρ .

Finally, we assume the central bank sets the current policy *without* commitment: that is, it sets the current asset price gap $q(t)$, taking the path of future gaps as given. In this case, the central bank's policy problem can be formulated recursively as⁴

$$\begin{aligned} \rho V(y) &= \max_q -\frac{y^2}{2} - \psi \frac{q^2}{2} + V'(y) \dot{y}, \\ \dot{y} &= \theta(mq - (1-n)y), \\ V(y) &\leq 0 \text{ and } V(0) = 0. \end{aligned} \quad (8)$$

The constraints in the last line follow from the objective function in (7) and ensure that

⁴The lack of commitment does not restrict monetary policy as long as the consumption function in (4) is exogenous to the policy. That is, as long as m and n are exogenous parameters, the Principle of Optimality implies that maximizing (7) subject to (6) is equivalent to solving problem (8).

we pick the correct solution to the recursive problem.

For closed form solutions, we also linearize Eq. (3) around the potential levels in (2) to obtain

$$r(t) = \rho(y(t) - q(t)\rho + \dot{q}(t)), \quad (9)$$

where $r(t) \equiv R(t) - \rho$ denotes the interest rate gap. Thus, we define the equilibrium as follows.

Definition 1. *A (linearized) equilibrium with optimal monetary policy, $[y(t), q(t), r(t)]_{t=0}^{\infty}$, is such that the path of asset price and output gaps, $[q(t), y(t)]$, solve the recursive problem (8) and the interest rate gap is given by (9).*

Remark 1 (LSAPs). *In Caballero and Simsek (2020a) we demonstrated that Large Scale Asset Purchases (LSAPs) can be viewed as close substitutes to interest rate cuts in terms of their impact on asset prices and output when there is risk (and risk-aversion). Moreover, LSAPs are an ideal policy instrument to stabilize aggregate demand when conventional monetary policy is constrained. For these reasons, a broader interpretation of the interest rate policy in this paper (i.e., extended to an environment that includes risk considerations) includes not only the policy rate but also LSAPs. In fact, LSAPs were a crucial component of the Fed’s response to the Covid-19 shock.*

Remark 2 (Interpreting Infrequent Adjustment). *In the literature review section we cited a variety of reasons for infrequent adjustment of individual consumption, ranging from fixed costs for durable purchases to behavioral and informational reasons for broader categories of consumption. Yet another reason is habit formation: e.g., returning to restaurants and traveling after the Covid-19 threat is over may take a while.*

Remark 3 (Households without Financial Wealth). *For simplicity, we focus on relatively wealthy households that can adjust their spending in response to asset price changes. We can extend the model to accommodate households that do not hold financial wealth without changing anything substantive. For example, suppose we split consumers (and income) between our agents (share $1 - h$) and a group of hand-to-mouth consumers (share h) that spend all of their income, $C^h(t) = Y(t)$. Then, in discrete time, output satisfies*

$$\begin{aligned} Y(t) &= (1 - h)C(t) + hY(t), \\ \text{and } C(t) &= \theta C^{\text{adjust}}(t) + (1 - \theta)C(t - \Delta t). \end{aligned} \quad (10)$$

Here, we have used $C(t)$ to denote spending by consumers that are not hand-to-mouth. The first equation implies $Y(t) = C(t)$. Substituting this into the second equation and

taking the limit as $\Delta t \rightarrow 0$, we obtain Eq. (5): that is, output dynamics are exactly the same. Hence, our results are robust to allowing for households that do not have any financial wealth. These households reduce the direct impact of asset prices on output, but they also create a multiplier effect that leaves the total impact unchanged [cf. (10)].

3. Overshooting during the recovery

We next solve for the equilibrium and establish our main results about overshooting. To capture the recovery from a recessionary shock such as Covid-19, we focus on the case with a negative initial output gap, $y(0) < 0$. Specifically, potential output has already adjusted to its long-run level but actual output (driven by spending) has not, since it still reflects the recent decline in productivity and output. Our main result shows that the optimal policy features *asset-price overshooting*: the central bank optimally chooses a *positive asset price gap*. That is, even though output is below its potential level, the asset price is high and above its potential level.

The optimality condition for problem (8) is

$$q = \frac{\theta m}{\psi} V'(y). \quad (11)$$

This condition is intuitive. The asset price is the central bank's policy lever. Since the asset price increases the output gap, the central bank increases the price whenever *increasing* the output gap improves its value function. As we will see, this is the case for negative output gaps, $y < 0$, where raising the output gap toward zero is valuable.

We next differentiate Eq. (8) with respect to y and use the Envelope Theorem to obtain

$$(\rho + \theta(1 - n)) V'(y) = -y + V''(y) \theta(mq - (1 - n)y). \quad (12)$$

Eqs. (11) and (12) correspond to a second order ODE that characterizes the value function. We conjecture (and verify in the appendix) that the solution is a quadratic function,

$$V(y) = -\frac{1}{2v} y^2 \quad (13)$$

$$\text{where } 0 = v^2 - (\rho + 2\theta(1 - n))v - \frac{\theta^2 m^2}{\psi}. \quad (14)$$

Here, v ("value") denotes a normalized parameter that shifts the value function. Since $V(y) < 0$, we also have $v > 0$: the solution corresponds to the positive root of (14).

Combining this with (11), we solve for the optimal asset price as

$$q(t) = -\frac{\theta m}{\psi v} y(t). \quad (15)$$

This expression illustrates the overshooting of asset prices. Starting with a negative output gap, the optimal asset price is above its potential level.

Next consider the change in output gaps along the optimal path. Combining Eqs. (15) and (6), we obtain

$$\dot{y}(t) = -\gamma y(t) \quad (16)$$

$$\text{where } \gamma \equiv \theta \left(\frac{\theta m^2}{\psi v} + 1 - n \right) = v - (\rho + \theta(1 - n)) > 0. \quad (17)$$

The composite parameter, γ , captures the convergence rate. We used (14) to simplify the expression for this convergence rate. Starting with a negative output gap, and an associated positive asset price gap (in view of overshooting), both gaps monotonically converge to zero at rate γ .

Finally, consider the linearized interest rate gap. Using Eq. (9), we obtain

$$\begin{aligned} r(t) &= \rho q(t) \left(\frac{y(t)}{q(t)} - \rho + \frac{\dot{q}(t)}{q(t)} \right) \\ &= -\rho q(t) \left(\frac{\psi v}{\theta m} + \rho + \gamma \right), \end{aligned} \quad (18)$$

where the second line uses Eqs. (15) and (16). Hence, the interest rate gap is *inversely* proportional to the asset price gap. Starting with a negative output gap, and an associated positive asset price gap, the interest rate gap starts below zero and gradually increases, $r(0) < 0$ and $\dot{r}(t) > 0$. The following result summarizes this discussion.

Proposition 1. *The value function is given by (13), where $v > 0$ is the positive solution to the quadratic equation (14). The equilibrium path of the asset price gaps, the output gap, and the linearized interest rate gap $[q(t), y(t), r(t)]_{t=0}^{\infty}$, is characterized by Eqs. (15), (16), (18). Starting with a negative output gap, $y(0) < 0$, the equilibrium features **asset price overshooting**: the asset price is above its potential level, $q(0) > 0$, and the interest rate is below its potential level, $r(0) < 0$. Over time, output and asset price gaps monotonically converge to zero at the exponential rate $\gamma = v - (\rho + \theta(1 - n)) > 0$.*

Figure 2 illustrates the equilibrium dynamics for a particular parameterization starting with a negative output gap. The equilibrium (the solid lines) features overshooting: the

central bank sets a positive asset price gap and gradually improves the output gap. The central bank achieves this outcome by starting with a low interest rate, and then gradually increasing the interest rate and reducing the asset price gap.⁵

The figure also illustrates the effect of increasing the central bank’s aversion to overshooting, ψ (the dashed lines). In this case, there is less overshooting, hence output gaps are closed more slowly and the economy operates below its potential for a longer time. Also note that by overshooting less when ψ is high, the central bank might have to *overshoot for longer*. That is, asset prices can eventually be *higher* (and the interest rate *lower*) when the central bank has a greater aversion to overshooting asset prices.

3.1. Comparative statics of overshooting

We next characterize how different parameters in our model affect the overshooting result. In particular, the following result describes the extent of overshooting for a given output gap, $\left| \frac{q(t)}{y(t)} \right| = \frac{\theta m}{\psi v}$ (cf. (15)) as well as the convergence rate, γ (cf. (16)).

Proposition 2. *Consider the equilibrium path with optimal monetary policy. Then, the asset price overshoots by more per unit of output gap (greater $\left| \frac{q(t)}{y(t)} \right|$) and output gaps are closed faster (greater γ) in each of the following cases:*

- (i) *When the central bank has smaller aversion to overshooting (smaller ψ)*
- (ii) *When a larger fraction of consumers adjusts at any moment (larger θ)*
- (iii) *When adjusting consumers respond more to the asset price (larger m)*

In addition:

- (iv) *The asset price overshoots by more (larger $\left| \frac{q(t)}{y(t)} \right|$) but output gaps are closed more slowly (smaller γ), when adjusting consumers respond more to current income (larger n).*

The first part of Proposition 2 formalizes the comparative statics illustrated in Figure 2. The second and the third parts say that the central bank overshoots more (and induces a faster recovery) when the output-asset price relation is stronger—in the sense that the spending response to asset prices is either faster (with smaller lags) or larger. Strengthening the output-asset price relation generates two counteracting effects. On the one hand, it makes the asset price overshooting more effective—which induces more overshooting. On the other hand, it enables the central bank to achieve the same impact on

⁵In our model, the asset price *level* also declines over time (after an initial jump). This feature is not essential to the argument. The same result would still apply for the asset price *gap* in a variant with productivity growth. In this case, the potential asset price would also be increasing over time (at the growth rate), so overshooting would not necessarily imply a declining asset price level. It would only imply a frontloading of some of the future capital gains.

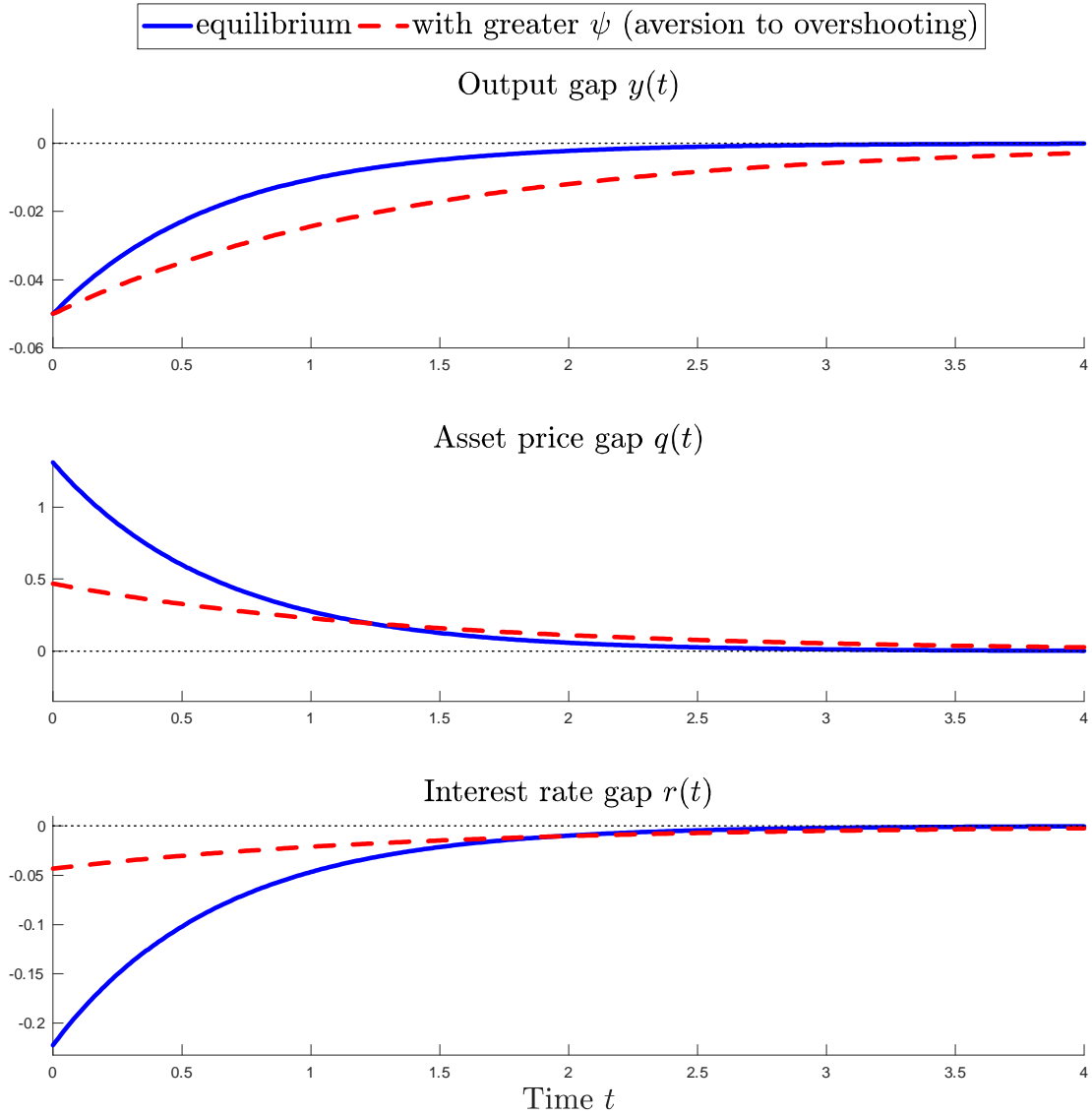


Figure 2: A simulation of the equilibrium for a parameterization with low ψ (solid lines) and high ψ (dashed lines). The dotted lines correspond to the potential levels for the corresponding variables.

the output gap with smaller (or shorter) overshooting—which induces less overshooting. With quadratic preferences (which are standard and commonly used in the literature), the first force dominates and the central bank induces more overshooting when it is more effective.

The final part of Proposition 2 considers the effect of increasing the consumers’ response to current income. This strengthens the *amplification* force by which current output gaps persist: for instance, negative output gaps tend to reinforce further negative gaps (cf. (6)). This induces the central bank to overshoot more in order to mitigate the amplification loop and speed up the convergence. Despite the central bank’s response, the amplification force prevails and implies that the output gaps are closed more slowly.

3.2. Overshooting with fully rational consumers

So far, we have assumed consumers exogenously follow the rule in (4). In Appendix A.2, we generalize our analysis to the case with fully rational consumers. Here, we briefly discuss the results.

The analysis proceeds in three steps. First, we characterize the general optimal (linearized) consumption rule for a rational consumer who *anticipates she will readjust in the future with Poisson probability θ* . In the benchmark with $\theta = \infty$ (continuous adjustment), the consumer has a constant marginal propensity to consume (MPC) out of wealth. Unlike this benchmark, the consumer with finite θ has an *endogenous* MPC out of wealth that depends on *the future path of interest rates*. As we explain below, keeping current wealth constant, a higher interest rate increases the MPC. That is, since the consumer cannot reoptimize in the future (in some states), the substitution effect becomes weaker. Therefore, despite log preferences, the substitution and income effects do not net out. Instead, the income effect dominates and implies that—keeping wealth constant—increasing the interest rate increases spending. Equivalently, higher interest rates reduce the cost of a fixed consumption stream until the next adjustment opportunity.

Second, we characterize the consumption rule further for the equilibrium path with optimal policy. Specifically, when the (linearized) output and the interest rate both converge to the steady state at a constant rate $\gamma > 0$, we show

$$c^{adjust}(0) = \rho \left((1 - \tilde{n}) q(0) + \tilde{n} \frac{y(0)}{\rho + \gamma} \right) \quad \text{where} \quad \tilde{n} = \frac{\theta + \rho}{\theta - 3\rho} \frac{\rho + \gamma}{\theta + \gamma}. \quad (19)$$

For intuition, note that $\frac{y(0)}{\rho + \gamma} = \int_0^\infty y(0) e^{-\gamma t} e^{-\rho t} dt$ corresponds to the consumer’s “fixed-rate wealth”: her expected lifetime budget evaluated at the steady-state interest rate ρ .

Hence, the consumer reacts to a weighted average of her actual wealth, $q(0)$, and her “fixed-rate wealth,” $\frac{y(0)}{\rho+\gamma}$. When asset prices increase because of an increase in current income ($y(0)$), the two wealth measures increase by the same amount and the consumer’s MPC out of wealth is as usual (ρ). However, when asset prices increase because of an interest rate cut, actual wealth increases while “fixed-rate wealth” remains constant. In this case, the consumer’s MPC out of wealth is smaller ($\rho(1-\tilde{n})$). This reflects the income effects associated with interest rate changes that we discussed earlier. Hence, Eq. (19) says that the consumer *does* react to wealth changes induced by interest rate changes, but *less so* than when she can adjust continuously. As expected, $\theta \rightarrow \infty$ implies $\tilde{n} \rightarrow 0$: when consumption adjustment is sufficiently rapid, we recover the usual consumption rule.

Finally, note that Eq. (19) is equivalent to the consumption rule in (4) with coefficients $m = \rho(1-\tilde{n})$ and $n = \rho\tilde{n}/(\rho+\gamma)$. Hence, our earlier analysis also applies in this case. However, the coefficients m and n are endogenous to the convergence rate γ . Recall that the convergence rate γ also depends on the coefficients m and n . Hence, with fully rational consumers, the equilibrium corresponds to a fixed point. In the appendix, we establish the existence of a fixed point with the desired properties and prove the following result.

Proposition 3. *Consider the equilibrium with fully rational consumers analyzed in Appendix A.2. As long as $\theta > 9\rho$, there exists an equilibrium in which consumption satisfies Eq. (19) with weight $\tilde{n}(\gamma) \in (0, 1)$ —and thus Eq. (4) with weights $m = \rho(1-\tilde{n}) \in (0, \rho)$ and $n = \rho\tilde{n}/(\rho+\gamma) \in (0, 1)$; and the equilibrium variables converge to the steady state at the rate $\gamma > 0$ characterized in Proposition 1. In particular, **the equilibrium with fully rational consumers also features asset price overshooting.***

Thus, as long as consumers’ adjustment is not too sluggish, our main result applies in this case. Intuitively, rational consumers *still respond* to asset price increases [cf. (19)], so the central bank also overshoots asset prices in this case.⁶

4. Preemptive asset price overshooting

So far, we have focused on an episode in which potential output has just recovered but aggregate demand and output are below potential output due to a recent recessionary

⁶Our analysis in Appendix A.2 also reveals that, with fully rational consumers, the convergence rate has a finite upper bound $\bar{\gamma}$ independent of aversion to overshooting ψ [cf. Lemma 2]. In particular, in the limit in which aversion approaches zero, $\psi \rightarrow 0$, the convergence rate does *not* approach infinity—as one might have expected from the analysis with exogenous m, n . As convergence becomes more rapid, the coefficient m (MPC out of actual wealth) endogenously declines toward zero. This in turn reduces the extent of overshooting and slows convergence. In the limit $\psi \rightarrow 0$, the coefficient m approaches zero and the convergence rate approaches its upper bound $\bar{\gamma}$.

shock. We next explicitly analyze the preceding “recession” episode and show that it features an even more extreme form of overshooting in order to accelerate the recovery once potential output improves.

Specifically, there are now two aggregate states denoted by subscript $s \in \{1, 2\}$. State $s = 2$ (the “recovery”) corresponds to our analysis up to now. In particular, potential output is constant and normalized to one, $Y_2^*(t) = 1$. State $s = 1$ (the “recession”) corresponds to an earlier period in which potential output is given by $Y_1^*(t) = 1 - \kappa < 1$ for some $\kappa > 0$. The economy starts in the recession state $s = 1$ and transitions to the recovery state $s = 2$ according to a Poisson process with intensity $\lambda > 0$. Once the economy transitions, it remains in state $s = 2$ forever.

In this section, we use the notation $y_s = Y_s(t) - Y^*$, $q_s = Q_s(t) - Q^*$, $r_s = R_s(t) - R^*$ to denote *normalized variables*: their difference from potential levels *in the recovery state* [cf. (2)]. We use \tilde{y}_s, \tilde{q}_s to denote output and asset price *gaps*: their difference from potential levels *within the corresponding state*. For the recovery state, the normalized and the gap variables are the same, $y_2 = \tilde{y}_2, q_2 = \tilde{q}_2$; but for the recession state they are different, $y_1 \neq \tilde{y}_1, q_1 \neq \tilde{q}_1$ (since the potential levels are lower).

We make three additional simplifying assumptions. First, consumers have Epstein-Zin preferences with discount rate ρ , EIS equal to one, and RRA equal to 0. Hence, consumers effectively have log utility with respect to consumption-savings decisions as before, but they are risk neutral with respect to portfolio decisions. This ensures that the asset pricing side of the model (which is not our focus) is the same as before. The risk-free rate is the same as the expected return on the market portfolio. Thus, the normalized interest rate is given by (9) in state $s = 2$ and by the following expression in state $s = 1$:⁷

$$r_1(t) = \rho(y_1(t) - \rho q_1(t) + \dot{q}_1(t) + \lambda(q_2(t) - q_1(t))). \quad (20)$$

The expected return on the market portfolio accounts for the change in the asset price if there is a transition.

Second, adjusting consumers follow the rule in (4) in both the recovery and the recession states. When consumers are fully rational, the optimal consumption rule would take this form with possibly state-dependent coefficients m_s, n_s . We assume the same rule for

⁷We obtain (20) by linearizing the following equation that describes the interest rate (around the potential levels in the recovery state (Y^*, Q^*, R^*)):

$$R_1(t) = \frac{Y_1(t) + \dot{Q}_1(t) + \lambda(Q_2(t) - Q_1(t))}{Q_1(t)}.$$

both states to keep the notation simple, although our results would qualitatively apply in the more general case (as long as $m_s > 0, n_s \in (0, 1)$ for both states). In view of this assumption, normalized output follows the same dynamics in the recession state [cf. (6)]:

$$\dot{y}_1 = \theta (mq_1 - (1 - n) y_1). \quad (21)$$

Finally, we let the central bank solve the following version of problem (8) in state 1:

$$\begin{aligned} \rho V_1(y_1) &= \max_{q_1} -\frac{\tilde{y}_1^2}{2} - \psi \frac{\tilde{q}_1^2}{2} + V_1'(y_1) \dot{y}_1 + \lambda (V_2(y_1) - V_1(y_1)), & (22) \\ \text{where } \tilde{y}_1 &= y_1 - y_1^* \text{ with } y_1^* = -\kappa < 0, \\ \text{and } \tilde{q}_1 &= q_1 - q_1^* \text{ with } q_1^* = -\kappa(1 - n)/m < 0. \end{aligned}$$

Given current normalized output y_1 , the central bank solves a gap-minimizing problem as before. However, the gaps in state $s = 1$ are no longer equal to the normalized variables. The potential output, y_1^* , is lower (recall that $Y_1^* = 1 - \kappa$). The potential asset price, q_1^* , is also lower and reflects the lower potential output. If consumers adjusted instantaneously ($\theta = \infty$), then keeping the asset price at its potential would ensure that output is also at its potential [cf. (21)]. The value function also accounts for the expected transition to the recovery state. In particular, $V_2(\cdot)$ denotes the recovery value function that we characterized earlier [cf. (13)].

We define the equilibrium with optimal monetary policy as before. In the recession state, output and asset prices solve problem (22) given (21), and the interest rate is given by (20). After transition to the recovery state, the equilibrium is given by Definition 1.

To analyze the equilibrium, note that Eq. (21) also holds in terms of the gaps:

$$\frac{d\tilde{y}_1}{dt} = \theta (m\tilde{q}_1 - (1 - n) \tilde{y}_1). \quad (23)$$

Thus, problem (22) has a similar structure to the earlier problem (8). In the appendix, we show that the solution also has a similar form [cf. (13)]:

$$V_1 = a + b\tilde{y}_1 - \frac{1}{2v}\tilde{y}_1^2, \quad (24)$$

$$\text{where } b = \frac{\lambda}{\lambda + \rho + \gamma} \frac{\kappa}{v} > 0. \quad (25)$$

Here, $v, \gamma > 0$ are the same as before. In this case, the value function features an additional linear component, $a + b\tilde{y}_1$, with $b > 0$. In particular, a positive output gap,

$\tilde{y}_1 > 0$, increases the value function relative to the recovery state, $b\tilde{y}_1 > 0$.

Intuitively, a positive gap is now less costly since it will reduce the size of the output gap once potential output recovers. This suggests that the central bank might *preemptively* induce positive gaps in the recession state. We next state our main result in this section, which completes the characterization and verifies that the equilibrium features *preemptive overshooting*.

Proposition 4. (i) *In the recession state $s = 1$, the value function is given by (24), where a is a constant and $b > 0$ is given by (25). Absent transition, the optimal asset price and output gaps follow the dynamics*

$$\begin{aligned}\tilde{q}_1(t) &= \frac{\theta m}{\psi} \left(b - \frac{1}{v} \tilde{y}_1(t) \right), \\ \frac{d\tilde{y}_1(t)}{dt} &= \frac{\theta^2 m^2}{\psi} b - \gamma \tilde{y}_1(t).\end{aligned}$$

These gaps monotonically converge to **strictly positive steady state levels** given by

$$\tilde{y}_1(\infty) \equiv \frac{\gamma - \theta}{\gamma} \frac{\lambda}{\lambda + \rho + \gamma} \kappa > 0 \quad \text{and} \quad \tilde{q}_1(\infty) = \frac{1 - n}{m} \tilde{y}_1(\infty) > 0. \quad (26)$$

The normalized output and asset price $y_1(t), q_1(t)$ converge to corresponding steady states $y_1(\infty) = \tilde{y}_1(\infty) + y_1^*$ and $q_1(\infty) = \tilde{q}_1(\infty) + q_1^*$. The steady state output exceeds its potential in the recession state but remains below its potential in the recovery state, $y_1(\infty) \in (-\kappa, 0)$ (equivalently, $\tilde{y}_1(\infty) \in (0, \kappa)$).

(ii) Suppose the normalized output is initially not too high, $y_1(0) < y_1(\infty)$ (e.g., it is at potential $y_1(0) = y_1^* = -\kappa$). Before transition, the equilibrium features **preemptive asset price overshooting**: The normalized asset price starts above its steady state level as well as its potential, $q_1(0) > q_1(\infty) > q_1^*$. Absent transition, the normalized output monotonically increases toward $y_1(\infty)$, and the normalized asset price monotonically decreases toward $q_1(\infty)$. At the moment of transition to recovery (time t'), **asset price overshooting increases**: the normalized output remains unchanged but the output gap becomes negative, $\tilde{y}_2(t') = y_2(t') = y_1(t') < 0$, and the normalized asset price level as well as its gap jump upward, $\tilde{q}_2(t') = q_2(t') > \tilde{q}_1(t') > q_1(t')$.

Figure 3 illustrates the equilibrium (the solid lines) for a particular parameterization in which the economy starts with a zero output gap and the transition takes place at its expected time, $t' = 1/\lambda$. The dotted lines illustrate the within-state potential levels (the gaps are the distance from these lines). The equilibrium features preemptive overshooting:

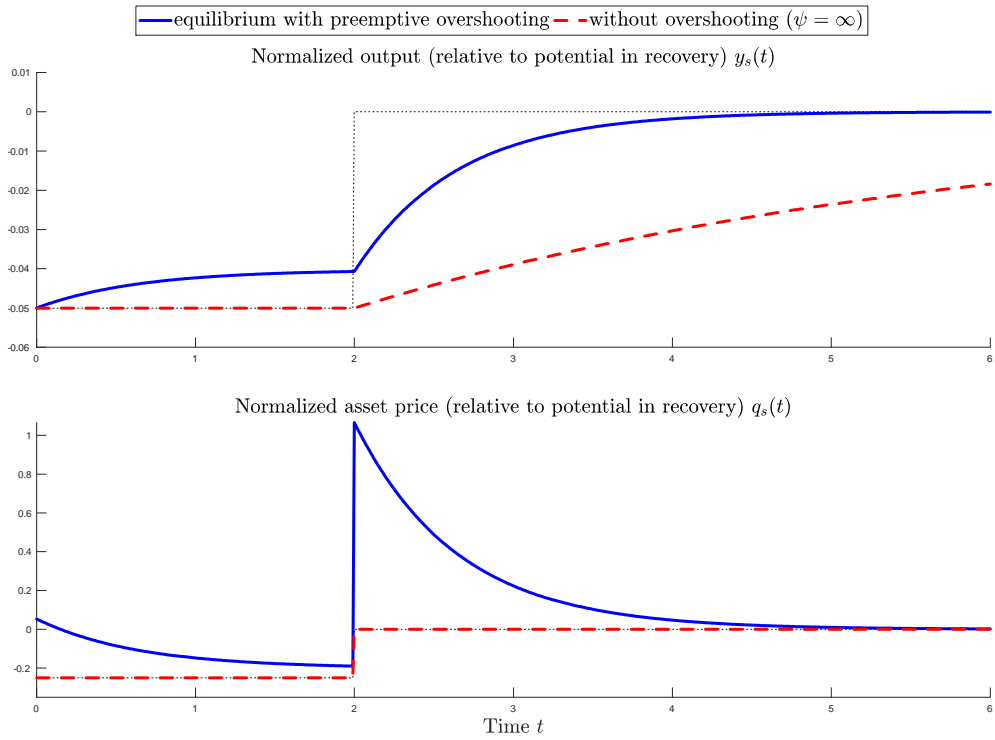


Figure 3: A simulation of the equilibrium variables starting in the shock state $s = 1$ with overshooting (solid lines) and with extreme aversion to overshooting ($\psi = \infty$, dashed lines). The dotted lines correspond to the within-state potential output and asset price.

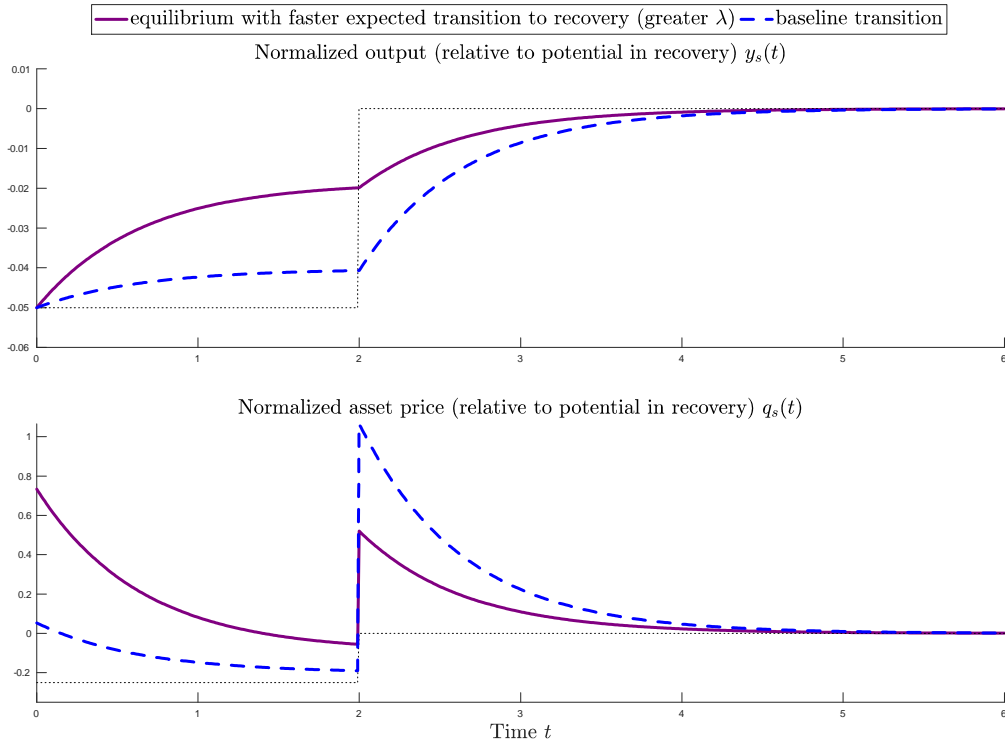


Figure 4: Equilibrium with a greater expected transition rate ($\tilde{\lambda} > \lambda$, solid lines) and with the baseline rate (λ , dashed lines).

the central bank sets a positive asset price gap, which gradually induces a positive output gap. After transition, the central bank overshoots the asset price even more, which helps close the negative output gap. For comparison, the figure also illustrates what happens when the aversion to overshooting is extreme ($\psi = \infty$) so that the central bank always keeps the asset price at its potential. In this case, output does not start to recover until productivity actually recovers. As a result, the economy enters the recovery with a greater aggregate demand gap and closes this gap more slowly [cf. Figure 2].

Figure 4 illustrates the effect of increasing λ —the expected transition rate to recovery. The solid lines plot the equilibrium when the expected transition rate is higher ($\tilde{\lambda} > \lambda$) but the actual transition takes place at the same time as in the previous figure ($t' = 1/\lambda$). In this case, preemptive overshooting is stronger: asset prices increase by more and the output in the recession converges to a level closer to its potential in the recovery [cf. (26)]. Intuitively, since the central bank expects a fast recovery, it also engages in considerable overshooting in the recession. In fact, the figure highlights that, by frontloading much of the overshooting to the recession, the central bank might do less overshooting when the recovery finally arrives (compared to the early stages of the recession).

5. Final Remarks

We proposed a model to illustrate that large gaps between the performance of financial markets and the real economy need not reflect an unsustainable situation. In our model, such gaps result from optimal monetary policy when aggregate demand is below its potential and responds to asset prices with a lag. The central bank boosts asset prices to close the output gap as fast as possible. In fact, the central bank also boosts asset prices if aggregate demand is currently at potential but expected to fall below potential—due to an anticipated recovery of potential output. These optimal policy responses create large temporary gaps between financial markets and the real economy, but they also accelerate the recovery.

In practice, policymakers face effective lower bound constraints for a variety of reasons: e.g., a zero lower bound, an exchange rate stabilization concern, or a fear of destabilizing the banking sector or money markets. While LSAPs can partially alleviate these constraints (see Remark 1), they are likely to face their own limits. In our model, the impact of these limits can be captured in reduced form by increasing the parameter ψ during the recovery phase. The main implication of increasing this parameter, aside from reducing the size of the overshooting during the recovery, is that the central bank has an even larger incentive to preemptively boost asset prices before the recovery.

Similarly, if we were to introduce fiscal policy, the anticipation of a reversal of fiscal support in the future, and the corresponding contraction in aggregate demand, would also lead the central bank to preemptively boost asset prices today. Conversely, the anticipation of further fiscal stimulus reduces the need for the central bank to boost asset prices in the present.

Throughout our analysis we assume that all firms are equally affected by the recession. Adding heterogeneity (a central feature of the Covid-19 recession) does not change our main results, but it introduces a large dispersion in asset prices across firms during the recession phase. In particular, firms that are not adversely affected by the productivity shock see their shares' value rise above its pre-recession level since they benefit from the central bank's attempt to boost asset prices without suffering from the decline in potential output. In the Covid-19 recession, this provides a rationale for the extraordinary performance of indices such as the Nasdaq 100, whose main components consist of “Covid-sheltered” firms.

Finally, while we have demonstrated that the broad features of asset markets during the Covid-19 episode are consistent with a well managed monetary policy framework, we do not wish to imply that there are no anomalies or pockets of irrational exuberance in

some markets. Having said this, the logic of the model suggests that experiencing an episode of irrational exuberance during a deep recession with a difficult recovery ahead has a positive dimension, since it reduces the burden on the central bank to engineer an overshooting.

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A. Appendix: Omitted derivations

A.1. Omitted proofs

Proof of Proposition 1. Most of the analysis is provided in the main text. It remains to verify that the value function conjectured in (13) solves the HJB equation for appropriate $v > 0$. Substituting the conjecture together with $q = \frac{\theta m}{\psi} V'(y)$ into (12), we obtain:

$$(\rho + 2\theta(1 - n)) \frac{y}{v} = y - \frac{\theta^2 m^2}{\psi} \frac{y}{v^2}.$$

After canceling y 's from both sides and rearranging terms, we obtain the quadratic equation in (14):

$$P(v) \equiv v^2 - (\rho + 2\theta(1 - n))v - \frac{\theta^2 m^2}{\psi} = 0.$$

This quadratic equation has one positive root and one negative root. The solution corresponds to the positive root, $v > 0$, since we require $V(y) = -\frac{1}{2v}y^2 \leq 0$. This root has the following closed form solution:

$$v = \frac{\rho + 2\theta(1 - n) + \sqrt{(\rho + 2\theta(1 - n))^2 + 4\frac{\theta^2 m^2}{\psi}}}{2}. \quad (\text{A.1})$$

The rest of the analysis is in the main text. Using Eq. (A.1), we can also calculate the convergence rate in closed form as follows [cf. (17)]:

$$\gamma = v - (\rho + \theta(1 - n)) = \frac{\sqrt{(\rho + 2\theta(1 - n))^2 + 4\frac{\theta^2 m^2}{\psi}} - \rho}{2}. \quad (\text{A.2})$$

This completes the proof. □

We next prove Proposition 2 that describes the comparative statics of the equilibrium. The following lemma facilitates the proof.

Lemma 1. *Consider the normalized value v which corresponds to the positive root of the polynomial (14) given by (A.1).*

- (i) *Increasing ψ decreases v and increases $v\psi$.*
- (ii) *Increasing θ increases v and decreases v/θ .*
- (iii) *Increasing m increases v and decreases v/m .*
- (iv) *Increasing n decreases v and v/m .*

Proof of Lemma 1. Eq. (A.1) implies v is decreasing in ψ , but $v\psi$ is increasing in ψ because

$$v\psi = \frac{(\rho + 2\theta(1-n))\psi + \sqrt{\psi^2(\rho + 2\theta(1-n))^2 + 4\psi\theta^2 m^2}}{2}$$

This proves the first part.

The same equation also implies v is increasing in θ , but v/θ is decreasing in θ because:

$$\frac{v}{\theta} = \frac{\frac{\rho}{\theta} + 2(1-n) + \sqrt{\left(\frac{\rho}{\theta} + 2(1-n)\right)^2 + 4\frac{m^2}{\psi}}}{2}.$$

This proves the second part.

The same equation also implies v is increasing in m , but v/m is decreasing in m because:

$$\frac{v}{m} = \frac{\frac{\rho+2\theta(1-n)}{m} + \sqrt{\left(\frac{\rho+2\theta(1-n)}{m}\right)^2 + 4\frac{\theta^2}{\psi}}}{2}.$$

This proves the third part. Finally, this expression together with Eq. (A.1) imply that both v and v/m are decreasing in n . This proves the last part. \square

Proof of Proposition 2. Note that the overshooting and the convergence rate satisfy, $\left|\frac{q(t)}{y(t)}\right| = \frac{\theta m}{\psi v}$ and $\gamma = v - (\rho + \theta(1-n))$ [cf. (17)]. Thus, parts (i) and (iii) follow directly from the corresponding parts in Lemma 1.

Consider part (ii). Lemma 1 implies increasing θ increases θ/v . Thus, it also increases the overshooting, $\left|\frac{q(t)}{y(t)}\right| = \frac{\theta m}{\psi v}$ as well as the convergence rate, $\gamma \equiv \theta \left(\frac{\theta m^2}{v\psi} + 1 - n\right)$ [cf. (17)].

Consider part (iv). Lemma 1 implies increasing n increases m/v . Therefore, it increases overshooting, $\left|\frac{q(t)}{y(t)}\right| = \frac{\theta m}{\psi v}$. Eq. (A.2) illustrates that it also decreases the convergence rate, γ , completing the proof. \square

Proof of Proposition 4, Part (i). First consider the value function that solves problem (22). After changing the variables to gaps, and using (6), we can rewrite the problem as:

$$\rho V_1(\tilde{y}_1) = \max_{\tilde{q}_1} -\frac{\tilde{y}_1^2}{2} - \psi \frac{\tilde{q}_1^2}{2} + V_1'(\tilde{y}_1)\theta(m\tilde{q}_1 - (1-n)\tilde{y}_1) + \lambda(V_2(\tilde{y}_1 - \kappa) - V_1(\tilde{y}_1)). \quad (\text{A.3})$$

Here, with a slight abuse of notation, we continue to use $V_1(\cdot)$ to denote the value function defined over the output gap \tilde{y}_1 instead of the normalized output level y_1 . After transition to recovery, the output gap declines by κ even though the normalized output remains unchanged (since potential output increases by κ).

Using the optimality conditions, we obtain analogues of Eqs. (11) and (12):

$$\tilde{q}_1 = \frac{\theta m}{\psi} V_1'(\tilde{y}_1) \quad (\text{A.4})$$

$$(\rho + \theta(1-n) + \lambda) V_1'(\tilde{y}_1) = -\tilde{y}_1 - \lambda \frac{1}{v} (\tilde{y}_1 - \kappa) + V_1''(y_1) \theta (m\tilde{q}_1 - (1-n)y_1) \quad (\text{A.5})$$

Here, the second line uses (13) to substitute for $V_2'(\tilde{y}_1 - \kappa)$.

We conjecture that the solution has the form in (24):

$$V_1(\tilde{y}_1) = a + b\tilde{y}_1 - \frac{1}{2v}\tilde{y}_1^2,$$

Combining this conjecture with Eq. (A.5), and using $y = \tilde{y}_1$ to denote gaps, we obtain:

$$(\rho + \theta(1-n) + \lambda) \left(b - \frac{1}{v}y \right) = -y - \lambda \frac{1}{v} (y - \kappa) - \frac{1}{v} \frac{\theta^2 m^2}{\psi} \left(b - \frac{1}{v}y \right) + \frac{1}{v} \theta (1-n) y.$$

Collecting the terms with y , we obtain:

$$-\frac{1}{v} (\rho + \theta(1-n) + \lambda) y = -1 - \frac{1}{v} \lambda y + \frac{1}{v^2} \frac{\theta^2 m^2}{\psi} y + \frac{1}{v} \theta (1-n) y.$$

After canceling $-\frac{1}{v}\lambda y$ from both sides, and dropping y , we obtain the same quadratic (14). Hence, v is the same as before.

Likewise, collecting the terms without y , we obtain:

$$(\rho + \theta(1-n) + \lambda) b = \frac{\lambda \kappa}{v} - \frac{1}{v} \frac{\theta^2 m^2}{\psi} b.$$

After rearranging terms, we solve:

$$b = \frac{\frac{\lambda \kappa}{v}}{\rho + \theta(1-n) + \lambda + \frac{1}{v} \frac{\theta^2 m^2}{\psi}} = \frac{\lambda}{\rho + \lambda + \gamma} \frac{\kappa}{v}$$

Here, the second equality substitutes for γ from (17). This proves the value function takes the form in (24) with the coefficient $b > 0$ given by (25).

Next consider the solution. Substituting the value function into Eq. (A.4), we obtain:

$$\tilde{q}_1(t) = \frac{\theta m}{\psi} \left(b - \frac{1}{v} \tilde{y}_1(t) \right). \quad (\text{A.6})$$

Combining this with (23), and using (17) to substitute γ , we obtain:

$$\frac{d\tilde{y}_1(t)}{dt} = \frac{\theta^2 m^2}{\psi} b - \gamma \tilde{y}_1(t). \quad (\text{A.7})$$

This equation implies that the output gap converges to a steady state given by:

$$\tilde{y}_1(\infty) = \frac{\theta^2 m^2 b}{\psi \gamma} = \frac{\gamma - \theta(1-n)}{\gamma} \frac{\lambda}{\rho + \lambda + \gamma} \kappa$$

Here, we have substituted for b [cf. (25)] as well as $\frac{\theta^2 m^2}{\psi v} = \gamma - \theta(1-n)$ [cf. (17)]. Note that this also implies $\tilde{y}_1(\infty) \in (0, \kappa)$. Then, Eq. (23) implies the asset price gap converges to a steady state given by:

$$\tilde{q}_1(\infty) = \frac{1-n}{m} \tilde{y}_1(\infty) > 0.$$

This establishes Eq. (26) completes the proof of the first part.

Part (ii). Suppose $y_1(0) < y_1(\infty)$, which implies the output gap starts below its steady state, $\tilde{y}_1(0) < \tilde{y}_1(\infty)$. Absent transition, Eq. (A.7) implies that the output gap monotonically increases toward its steady state. Combining this with Eq. (A.6), we also obtain that the asset price gap starts above its steady state, $\tilde{q}_1(0) > \tilde{q}_1(\infty)$, and monotonically declines toward its steady state. This also implies the normalized output $y_1(0)$ monotonically increases towards its steady state $y_1(\infty)$; and the normalized asset price monotonically decreases toward its steady state.

Next consider the time t' at which the economy transitions to the recovery (starting with $y_1(0) < y_1(\infty)$). Normalized output remains unchanged. It is below its new potential because, $y_2(t') = y_1(t') < y_1(\infty) < 0$. The asset price gap after transition is given by Eq. (15) whereas the asset price gap before transition is given by Eq. (A.6). That is:

$$\begin{aligned} \tilde{q}_2(t') = q_2(t') &= -\frac{\theta m}{\psi v} y_1(t') \\ \tilde{q}_1(t') &= \frac{\theta m}{\psi} \left(b - \frac{1}{v} (y_1(t') - y_1^*) \right) \end{aligned}$$

After taking the difference, and substituting $y_1^* = -\kappa$, we obtain:

$$\tilde{q}_2(t') - \tilde{q}_1(t') = \frac{\theta m}{\psi} \left(\frac{\kappa}{v} - b \right) = \frac{\theta m \kappa}{\psi v} \frac{\rho + \gamma}{\rho + \lambda + \gamma} > 0.$$

Here, the second equality substitutes b from (25). Since $q_1^* < 0$, we also have $\tilde{q}_1(t') >$

$q_1(t')$. Thus, we obtain $\tilde{q}_2(t') = q_2(t') > \tilde{q}_1(t') > q_1(t')$. At the time of transition, the normalized asset price as well as its gap increases. This completes the proof. \square

A.2. Overshooting with rational consumers

Consider the baseline model analyzed in Sections 2 and 3. In the main text, we assume adjusting consumers exogenously follow the rule in (4) with parameters $m \in (0, \rho)$, $n \in (0, 1)$. In this appendix, we generalize the analysis to the case with fully rational adjusting consumers that incorporate the fact that they will get to readjust in the future with Poisson probability θ .

The analysis proceeds in three steps. First, we characterize the (linearized) optimal consumption rule for an arbitrary path of (linearized) future income and interest rate gaps, $[y(t), r(t)]_{t=0}^{\infty}$. Second, we show that, along the equilibrium path in which $y(t)$ and $r(t)$ converge to zero at an exponential rate $\gamma > 0$, consumption rule is given by Eq. (19), which implies Eq. (4) for endogenous coefficients $m(\gamma)$, $n(\gamma)$. Third, we prove Proposition 3 that generalizes our main overshooting result to this setting.

Optimal consumption with Poisson adjustment. Consider a consumer with assets $a(0)$ that gets to adjust her consumption. For now, we take the initial asset level as exogenous (from the interest rate) and endogenize it subsequently. The consumer's recursive problem is given by:

$$\begin{aligned} V(a(0)) &= \max_c \int_0^{\infty} \theta e^{-\theta T} \left[\int_0^T e^{-\rho t} \log c dt + e^{-\rho T} V(a(T)) \right] dT \quad (\text{A.8}) \\ &= \max_c \int_0^{\infty} \theta e^{-\theta T} \left[\frac{1 - e^{-\rho T}}{\rho} \log c + e^{-\rho T} V(a(T)) \right] dT \end{aligned}$$

$$\text{where } \dot{a}(t) = R(t)a(t) - c$$

Here, we write the consumer's problem as an integral over adjustment times T . The probability that adjustment takes place at time T is $\theta e^{-\theta T}$. Conditional on adjustment at time T , the consumer receives the utility in the brackets.

Suppose consumption is strictly positive along the optimal path (this will be the case since we focus on small deviations from the steady state). Then, the value function has the homogeneity property:

$$V(a(t)) = \frac{\log a(t)}{\rho} + v(t).$$

Note also that we can integrate the budget constraint forward to solve for the value of assets at time T . In particular, the present value of $a(T)$ is equal to the initial assets net of the present value of the fixed consumption path until time T :

$$\begin{aligned} \frac{a(T)}{\exp\left(\int_0^T R(s) ds\right)} &= a(0) - c \int_0^T \exp\left(-\int_0^t R(s) ds\right) dt \\ &= a(0) - c \int_0^T \exp\left(-\int_0^t (r(s) + \rho) ds\right) dt \\ &= a(0) - c \int_0^T \exp(-\rho t) \exp\left(-\int_0^t r(s) ds\right) dt \end{aligned}$$

The last line writes the solution in terms of the interest rate gap, $r(s) = R(s) - \rho$.

Substituting these observations into (A.8), we obtain the following optimality condition:

$$\int_0^\infty \theta e^{-\theta T} \left[\frac{1 - e^{-\rho T}}{c\rho} - \frac{\exp(-\rho T)}{\rho} \frac{\int_0^T \exp(-\rho t) \exp\left(-\int_0^t r(s) ds\right) dt}{a(0) - c \int_0^T \exp(-\rho t) \exp\left(-\int_0^t r(s) ds\right) dt} \right] dT = 0.$$

After rearranging terms, we have:

$$\frac{1}{c} \frac{1}{\theta + \rho} = \int_0^\infty \frac{\theta}{\rho} e^{-(\theta+\rho)T} \frac{\int_0^T \exp(-\rho t) \exp\left(-\int_0^t r(s) ds\right) dt}{a(0) - c \int_0^T \exp(-\rho t) \exp\left(-\int_0^t r(s) ds\right) dt} dT.$$

The solution scales with initial wealth. Thus it can be written as

$$c = \rho(1 + \chi(0)) a(0) \tag{A.9}$$

for some $\chi(0)$. After substituting, we further obtain

$$\frac{1}{1 + \chi(0)} = \int_0^\infty \theta (\theta + \rho) e^{-(\theta+\rho)T} \frac{\int_0^T \exp(-\rho t) \exp\left(-\int_0^t r(s) ds\right) dt}{1 - \rho(1 + \chi(0)) \int_0^T \exp(-\rho t) \exp\left(-\int_0^t r(s) ds\right) dt} dT. \tag{A.10}$$

Consider the potential steady state, $R^*(t) = \rho$ for each t , which corresponds to $r^*(t) = 0$ for each t . In this case, it is easy to check $\chi^*(0) = 0$ and thus $c = \rho a(0)$. Consider also the special case $\theta \rightarrow \infty$. It can be checked that this also gives the same solution $\chi(0) = 0$ and $c = \rho a(0)$.

Beyond these special cases, the solution is complicated. In particular, $\chi(0)$ depends

on the whole future path of interest rates. To make progress, we view (A.10) as an implicit function of the variables $\chi(0)$, $[r(t)]_{t=0}^{\infty}$. We then linearize this function around the potential steady state $\chi^*(0) = 0$ and $r^*(t) = 0$ for each t . This gives:

$$\begin{aligned}\chi(0) &= \int_0^{\infty} \theta(\theta + \rho) e^{-(\theta + \rho)T} \left\{ \begin{aligned} &\rho\chi(0) \int_0^T e^{-\rho t} dt \frac{\int_0^T e^{-\rho t} dt}{(1 - \rho \int_0^T e^{-\rho t} dt)^2} + \\ & - \int_0^T e^{-\rho t} \left(- \int_0^t r(s) ds \right) dt \frac{1}{1 - \rho \int_0^T e^{-\rho t} dt} \\ & - \rho \int_0^T e^{-\rho t} \left(- \int_0^t r(s) ds \right) dt \frac{\int_0^T e^{-\rho t} dt}{(1 - \rho \int_0^T e^{-\rho t} dt)^2} \end{aligned} \right\} dT \\ &= \int_0^{\infty} \theta(\theta + \rho) e^{-(\theta + \rho)T} \left\{ \chi(0) \frac{(1 - e^{-\rho T})^2}{\rho(e^{-\rho T})^2} + \frac{\int_0^T e^{-\rho t} \int_0^t r(s) ds dt}{(e^{-\rho T})^2} \right\} dT.\end{aligned}$$

Here, the first line uses the chain rule to evaluate the derivatives and cancels the constant terms from both sides (since the equation holds for $\chi(0) = r(t) = 0$). The second line calculates the integrals. After rearranging terms, we further obtain

$$\chi(0) \left[1 - \int_0^{\infty} \frac{\theta(\theta + \rho)}{\rho} e^{-(\theta + \rho)T} (1 - e^{-\rho T})^2 dT \right] = \int_0^{\infty} \theta(\theta + \rho) e^{-(\theta + \rho)T} \int_0^T e^{-\rho t} \int_0^t r(s) ds dt dT$$

Calculating the integrals and simplifying further, we obtain:

$$\chi(0) \left(\frac{\theta - 3\rho}{\theta - \rho} \right) = \int_0^{\infty} \theta(\theta + \rho) e^{-(\theta + \rho)T} \int_0^T e^{-\rho t} \int_0^t r(s) ds dt dT. \quad (\text{A.11})$$

Recall that $c = (1 + \chi(0))\rho a(0)$. Hence, Eq. (A.11) illustrates that the slope of the consumption function deviates from the usual slope iff the interest rates are different from their steady state levels. As long as $\theta > 3\rho$ (which we assume), keeping current wealth constant, greater interest rate increases (resp. decreases) spending. Intuitively, since consumer cannot reoptimize in the future (in some states), the substitution effect becomes weaker. Therefore, despite log preferences, the substitution and income effects do not net out. Instead, the income effect dominates and implies—keeping the wealth constant—increasing the interest rate increases spending. Intuitively, a higher interest rate makes it cheaper to finance a steady consumption stream (c), which induces the consumer to spend more.

As before, there is also a wealth effect once we endogenize the initial wealth $a(0)$. In particular, note that the wealth of the representative adjusting consumer is equal to the

price of the market portfolio, $a(0) = Q(0)$. This implies:

$$C^{adjust}(0) = (1 + \chi(0)) \rho Q(0)$$

$$\text{where } Q(0) = \int_0^\infty e^{-\rho t} Y(t) e^{-\int_0^t r(s) ds} dt.$$

Linearizing these expressions around the steady state ($Y^* = C^* = 1, Q^* = 1/\rho$), we obtain:

$$c^{adjust}(0) = \chi(0) + \rho q(0) \tag{A.12}$$

$$q(0) = \int_0^\infty e^{-\rho t} \left[y(t) - \int_0^t r(s) ds \right] dt \tag{A.13}$$

As usual, increasing the interest rate reduces the value of the market portfolio. All else equal, this reduces spending.

Optimal consumption along the equilibrium path. We next calculate consumption along the equilibrium path and prove Eq. (4). In equilibrium, we have:

$$y(t) = y(0) e^{-\gamma t} \text{ and } r(t) = r(0) e^{-\gamma t}.$$

Substituting these expressions, we solve for the asset price along the equilibrium path:

$$q(0) = \frac{y(0)}{\gamma + \rho} - \frac{r(0)}{\rho(\rho + \gamma)}. \tag{A.14}$$

As expected a greater initial output increases the price and a greater initial interest rate decreases the price.

Next, we define \tilde{n} such that:

$$\chi(0) = \frac{r(0)}{\rho + \gamma} \tilde{n}. \tag{A.15}$$

This normalization is useful since it leads to a simple consumption function. Specifically, combining this with Eqs. (A.12) and (A.13), we obtain:

$$\begin{aligned} c^{adjust}(0) &= \frac{r(0)}{\rho + \gamma} \tilde{n} + \rho q(0) \\ &= \rho \left(\frac{y(0)}{\rho + \gamma} - q(0) \right) \tilde{n} + \rho q(0) \\ &= \rho \left((1 - \tilde{n}) q(0) + \tilde{n} \frac{y(0)}{\rho + \gamma} \right) \end{aligned} \tag{A.16}$$

Here, the second line substitutes $\frac{r(0)}{\rho+\gamma} = \rho \left(\frac{y(0)}{\rho+\gamma} - q(0) \right)$ from Eq. (A.13) to express the consumption function in terms of $y(0), q(0)$ (instead of $y(0), r(0)$).

It remains to solve for \tilde{n} along the equilibrium path. Combining Eqs. (A.11) and (A.15), and substituting $r(s) = r(0) e^{-\gamma s}$, we calculate:

$$\begin{aligned}
\tilde{n} &= \frac{(\rho + \gamma)(\theta - \rho)}{\theta - 3\rho} \int_0^\infty \theta(\theta + \rho) e^{-(\theta - \rho)T} \int_0^T e^{-\rho t} \frac{1 - e^{-\gamma t}}{\gamma} dt dT \\
&= \frac{(\rho + \gamma)(\theta - \rho)\theta(\theta + \rho)}{\gamma(\theta - 3\rho)} \int_0^\infty e^{-(\theta - \rho)T} \left[\frac{1 - e^{-\rho T}}{\rho} - \frac{1 - e^{-(\rho + \gamma)T}}{\rho + \gamma} \right] dT \\
&= \frac{(\rho + \gamma)(\theta - \rho)\theta(\theta + \rho)}{\gamma(\theta - 3\rho)} \left\{ \frac{1}{\rho} \left(\frac{1}{\theta - \rho} - \frac{1}{\theta} \right) - \frac{1}{\rho + \gamma} \left(\frac{1}{\theta - \rho} - \frac{1}{\theta + \gamma} \right) \right\} \\
&= \frac{(\rho + \gamma)(\theta - \rho)\theta(\theta + \rho)}{\gamma(\theta - 3\rho)} \left\{ \frac{1}{(\theta - \rho)\theta} - \frac{1}{(\theta - \rho)(\theta + \gamma)} \right\} \\
&= \frac{\theta + \rho}{\theta - 3\rho} \frac{\rho + \gamma}{\theta + \gamma}. \tag{A.17}
\end{aligned}$$

The final expression has intuitive comparative statics. For instance, as $\theta \rightarrow \infty$, we have $\tilde{n} \rightarrow 0$. That is, when the consumer adjusts very rapidly, we recover the standard rule.

Combining Eqs. (A.16) and (A.17), we prove the consumption rule in Eq. (19) (see the main text for intuition). This rule also implies Eq. (4) with coefficients

$$n = \rho \frac{\tilde{n}}{\rho + \gamma} = \frac{\theta + \rho}{\theta - 3\rho} \frac{\rho}{\theta + \gamma} \tag{A.18}$$

$$m = \rho(1 - \tilde{n}) = \rho \left(1 - \frac{\theta + \rho}{\theta - 3\rho} \frac{\rho + \gamma}{\theta + \gamma} \right). \tag{A.19}$$

Overshooting with rational consumers. Note that the slope coefficients depend on the equilibrium convergence rate, that is, $m = m(\gamma)$ and $n = n(\gamma)$. Conversely, our analysis in the main text shows the convergence rate, γ , depends on m, n . Specifically, Proposition 1 implies [cf. (A.2)]:

$$\gamma = F(\gamma) \equiv \frac{\sqrt{(\rho + 2\theta(1 - n(\gamma)))^2 + 4\frac{\theta^2 m(\gamma)^2}{\psi}} - \rho}{2}. \tag{A.20}$$

Therefore, the equilibrium with rational consumers corresponds to the fixed point of function $F(\cdot)$. We next establish the following lemma, which shows that (as long as θ is sufficiently large) there exists an interior fixed point that satisfies the properties we assumed in the main text. We then use the lemma to prove Proposition 3, which shows that our main overshooting result also applies with fully rational consumers.

Lemma 2. *Suppose $\theta > 9\rho$. Then, the function $F(\cdot)$ has an interior fixed point $\gamma \in (\rho, \bar{\gamma})$ that satisfies $\tilde{n}(\gamma) \in (0, 1)$ (and thus $m(\gamma) \in (0, \rho)$ and $n(\gamma) \in (0, 1)$). There is a finite upper bound on this fixed point given by:*

$$\bar{\gamma} = \frac{(\theta - \rho)(\theta + \rho)}{4\rho} - \theta > \rho. \quad (\text{A.21})$$

Proof. Rewriting Eqs. (A.18 – A.19), we have:

$$\begin{aligned} n(\gamma) &= \rho \frac{\theta + \rho}{\theta - 3\rho} \frac{1}{\theta + \gamma}, \\ m(\gamma) &= \rho \left(1 - \frac{\theta + \rho}{\theta - 3\rho} \frac{\rho + \gamma}{\theta + \gamma} \right) = \frac{\rho}{\theta - 3\rho} \left(\frac{(\theta - \rho)(\theta + \rho)}{\theta + \gamma} - 4\rho \right). \end{aligned}$$

Thus, as long as $\theta > 3\rho$, the functions $m(\cdot)$ and $n(\cdot)$ are both strictly decreasing in γ . Note also that the upper bound in (A.21) ensures $m(\bar{\gamma}) = 0$. As long as $\theta > 9\rho$, this upper bound also satisfies $\bar{\gamma} > \theta > 0$. We claim $F(\rho) > \rho$ and $F(\bar{\gamma}) < \bar{\gamma}$, which implies there exists a fixed point that satisfies, $\gamma \in (\rho, \bar{\gamma})$.

To prove the claim, first consider $F(\bar{\gamma})$. We have $m(\bar{\gamma}) = 0$ and $n(\bar{\gamma}) > 0$. Combining this with Eq. (A.20), we obtain:

$$F(\bar{\gamma}) = \theta(1 - n(\bar{\gamma})) < \theta < \bar{\gamma}.$$

Next consider $F(\rho)$. We have:

$$\begin{aligned} n(\rho) &= \frac{\rho}{\theta - 3\rho} \in (0, 1) \\ m(\rho) &= \frac{\rho}{\theta - 3\rho} (\theta - 5\rho) \in (0, \rho). \end{aligned}$$

Combining this with Eq. (A.20), we obtain:

$$F(\rho) \geq \theta(1 - n(\rho)) = \theta \frac{\theta - 4\rho}{\theta - 3\rho} > \rho.$$

Here, the last inequality follows since $\theta > 5\rho$. This proves the claim.

It remains to check that the fixed point satisfies $\tilde{n}(\gamma) \in (0, 1)$. Note that $\tilde{n}(\gamma) = 1 - m/\rho$. Since $m(\gamma)$ is decreasing in γ , $\tilde{n}(\gamma)$ is increasing in γ . In addition, Eq. (A.17) implies $\tilde{n}(\rho) = \frac{2\rho}{\theta - 3\rho} > 0$ and $m(\bar{\gamma}) = 1$ implies $\tilde{n}(\bar{\gamma}) = 1$. This establishes $\tilde{n}(\gamma) \in (0, 1)$, completing the proof. \square

Proof of Proposition 3. Let γ denote the fixed point characterized by Lemma 2. Our analysis in this section shows that, given the convergence rate γ , the optimal consumption rule is given by (4) with $m(\gamma) \in (0, \rho)$ and $n(\gamma) \in (0, 1)$. Proposition 1 implies that, given this consumption rule, the optimal policy induces the equilibrium variables to converge to zero at rate $\gamma > 0$. \square