Scarring Body and Mind: The Long-Term Belief-Scarring Effects of COVID-19
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ABSTRACT

The largest economic cost of the COVID-19 pandemic could arise from changes in behavior long after the immediate health crisis is resolved. A potential source of such a long-lived change is scarring of beliefs, a persistent change in the perceived probability of an extreme, negative shock in the future. We show how to quantify the extent of such belief changes and determine their impact on future economic outcomes. We find that the long-run costs for the U.S. economy from this channel is many times higher than the estimates of the short-run losses in output. This suggests that, even if a vaccine cures everyone in a year, the Covid-19 crisis will leave its mark on the US economy for many years to come.

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One of the most pressing questions of the day is the economic costs of the COVID-19 pandemic. While the virus will eventually pass, vaccines will be developed, and workers will return to work, an event of this magnitude could leave lasting effects on the nature of economic activity. Economists are actively debating whether the recovery will be V-shaped, U-shaped or L-shaped.\footnote{See e.g., Summers (FT, 2020), Krugman (2020), Reinhart and Rogoff (2020) and Cochrane (2020).} Much of this discussion revolves around confidence, fear and the ability of firms and consumers to rebound to their old investment and spending patterns. Our goal is to formalize this discussion and quantify these effects, both in the short- and long-run. To explore these conjectures about the extent to which the economy will rebound from this COVID-induced downturn, we use a standard economic and epidemiology framework, with one novel channel: a “scarring effect.” Scarring is a persistent change in beliefs about the probability of an extreme, negative shock to the economy. We use a version of Kozlowski et al. (2020), to formalize this scarring effect and quantify its the long-run economic consequences, under different scenarios for the dynamics of the crisis.

We start from a simple premise: No one knows the true distribution of shocks in the economy. Consciously or not, we all estimate the distribution using past events, like an econometrician would. Tail events are those for which we have little data. Scarce data makes new tail event observations particularly informative. Therefore, tail events trigger larger belief revisions. Furthermore, because it will take many more observations of non-tail events to convince someone that the tail event really is unlikely, changes in tail risk beliefs are particularly persistent.

We have seen the scarring effect in action before. Before 2008, few people entertained the possibility of a financial crisis in the US. Today, more than a decade after the Global Financial Crisis, the possibility of another run on the financial sector is raised frequently, even though the system today is probably much safer. Likewise, businesses will make future decisions with the risk of another pandemic in mind. Observing the pandemic has taught us that the risks were greater than we thought. It is this new-found knowledge that has long-lived effects on economic choices.

To explore tail risk in a meaningful way, we need to use an estimation procedure that does not constrain the shape of the distribution’s tail. Therefore, we allow our agents to learn about the distribution of aggregate shocks non-parametrically. Each period, agents observe one more piece of data and update their estimates of the distribution. Section 1 shows how this process leads to long-lived responses of beliefs to transitory events, especially extreme, unlikely ones. The mathematical foundation for such persistence is the martingale property of beliefs. The logic is that once observed, the event remains in agents’ data set. Long after the direct effect of the shock has passed, the knowledge of that tail event affects beliefs and therefore, continues to restrain economic activity.
To illustrate the economic importance of these belief dynamics, Section 2 embeds our belief updating tool in a macroeconomic model with an epidemiology event that erodes the value of capital. This framework is designed to link tail events like the current crisis to macro outcomes in a quantitatively plausible way and has been used – e.g. by Gourio (2012) and Kozlowski et al. (2020) – to study the 2008-09 Great Recession. It features, among other elements, bankruptcy risk and elevated capital depreciation from social distancing, which separates labor from capital. Section 3 describes the data we feed into the model to discipline our belief estimates. Section 4 combines model and data and uses the resulting predictions to show how belief updating can generate large, persistent losses. We compare our results to those from the same economic model, but with agents who have full knowledge of the distribution, to pinpoint belief updating as the source of the persistence.

We model the economic effects of the Covid-19 crisis as a combination of a productivity decline and accelerated capital obsolescence. We use the well-known SIR framework from the epidemiology literature to model the disease spread. But, it is the response to the disease that is the source of the adverse economic shock in our model. Our structure is capable of generating large asset price fluctuations, of the order observed at the onset of the pandemic, and provides a simple mapping from social distancing policies and other mitigation behavior to economic costs. It also allows us to connect to existing studies on tail risk in macroeconomics and finance. In our analysis, we present results for different scenarios, reflecting the considerable uncertainty about outcomes even in the short-run. Our point is not that these are the right forecast of the coming year’s events. The point is that whatever you think will happen over the next year, the costs of this pandemic are much larger than your short-run calculations suggest.

In the first scenario, GDP drops by about 10% in 2020, recovers gradually but does not go back to its previous trajectory. It persistently remains about 5% below the previous pre-Covid steady state. The discounted value of the lost output is almost 10 times the 2020 drop and belief revisions account for bulk of the losses (almost 6 times the short-run effect). Greater tail risk makes investing less attractive, reducing the stock of productive capital and (and therefore, labor input demand) persistently. In the second scenario, which captures a milder mitigation response to the spread of the disease, both short- and long-run economic costs are longer, but the relative importance of belief revisions remains the same.

The model also makes a number of predictions about asset prices. Interestingly, credit spreads and equity valuations are predicted to change very little in response to the rise in tail risk. This is because firms respond to this increase in riskiness by cutting back on debt. The effects of scarring are more clearly noticeable in riskless rates and in options prices. In scenario 1, for example, riskless rates are predicted to fall by almost 1%, while the implied third moment in the risk-neutral distribution of equity returns becomes significantly more negative.
These results also imply that a policy that prevents capital depreciation or obsolescence, even if it has only modest immediate effects on output, can have substantial long-run benefits, several times larger than the short-run considerations that often dominate policy discussion. Obviously, no policy can prevent people from believing that future pandemics are more likely than they originally thought. Policy can however affect how the ongoing crisis affects capital returns. By changing that mapping, the costs of belief scarring can be mitigated. For example, widespread bankruptcies can lead to destruction of specific investments and a permanent erosion in the value of capital. Policy interventions which prevent widespread bankruptcies can thus limit the adverse effects of the crisis on returns and yield substantial long-run benefits. While the short-run gains from limiting bankruptcies is well-understood, our analysis shows that neglecting the effect on beliefs leads one to drastically underestimate the benefits of such policies.

Comparison to the literature  There are many new studies of the impact of the COVID-19 pandemic on the U.S. economy, both model-based and empirical. Alvarez et al. (2020), Eichenbaum et al. (2020) and Farboodi et al. (2020) use simple economic frameworks to analyze the costs of the disease and the associated mitigation strategies. Leibovici et al. (2020) use an input-output structure to investigate the extent to which a shock to contact-intensive industries can propagate to the rest of the economy. Koren and Pető (2020) build a detailed theory-based measures of the reliance of U.S. businesses on human interaction. On the empirical side, Ludvigson et al. (2020) use VARs to estimate the cost of the pandemic over the next few months, while Carvalho et al. (2020) use high-frequency transaction data to track expenditure and behavior changes in real-time. We add to this discussion by focusing on the long-term effects from changes in behavior that persist long after the disease is gone.

Other papers share our focus on long-run effects. Jorda et al. (2020) study rates of return on assets using a data-set stretching back to the 14th century, focusing on 15 major pandemics (with more than 100,000 deaths). Their evidence suggests a sustained downward pressure on interest rates, decades after the pandemic, consistent with long-lasting macroeconomic after-effects. Correia et al. (1918) find evidence of persistent declines in economic activity following the 1918 influenza pandemic. A few papers also use beliefs but rely on other mechanisms, such as financial frictions, for propagation. Elenev et al. (2020) and Krishnamurthy and Li (2020) use propagate the shock primarily through financial balance sheet effects. In a more informal discussion, Cochrane (2020) explores whether the recovery from the COVID-shock will be V, U or L shaped. This work formalizes many of the ideas in that discussion.

Outside of economics, biologists and socio-biologists have noted long ago that epidemics change the behavior of both humans and animals. Loehle (1995) explore the social barriers to transmission in animals as a mode of defense against pathogen attack. Past disease events have
effects on mating strategies, social avoidance, group size, group isolation, and other behaviors for generations. Gangestad and Buss (1993) find evidence of similar behavior among human communities.

In the economics realm, a small number of uncertainty-based theories of business cycles also deliver persistent effects from other sorts of transitory shocks. In Straub and Ulbricht (2013) and Van Nieuwerburgh and Veldkamp (2006), a negative shock to output raises uncertainty, which feeds back to lower output, which in turn creates more uncertainty. To get even more persistence, Fajgelbaum et al. (2014) combine this mechanism with an irreversible investment cost, a combination which can generate multiple steady-state investment levels. These uncertainty-based explanations are difficult to embed in quantitative DSGE models and to discipline with macro and financial data.

Our belief formation process is similar to the parameter learning models by Johannes et al. (2015), Cogley and Sargent (2005) and Kozeniauskas et al. (2014) and is similar to what is advocated by Hansen (2007). However, these papers focus on endowment economies and do not analyze the potential for persistent effects in a setting with production. The most important difference is that our non-parametric approach allows us to incorporate beliefs about tail risk.

1 Belief Formation

Before laying out the underlying economic environment, we begin by explaining how we formalize the notion of belief scarring, the non-standard, but most crucial part of our analysis. We then embed it in an economic environment and quantify the effect of belief changes from the COVID-19 pandemic on the US economy.

No one knows the true distribution of shocks to the economy. All of us – whether in our capacity as economic agents or modelers or econometricians – estimate such distributions, updating our beliefs as new data arrives. Our goal is to model this process in a reasonable and tractable fashion. The first step is to choose a particular estimation procedure. A common approach is to assume a normal or other parametric distribution and estimate its parameters. The normal distribution, with its thin tails, is unsuited to thinking about changes in tail risk. Other distributions raise obvious concerns about the sensitivity of results to the specific distributional assumption used. To minimize such concerns, we take a non-parametric approach and let the data inform the shape of the distribution.

2 Other learning papers in this vein include papers on news shocks, such as, Beaudry and Portier (2004), Lorenzoni (2009), Veldkamp and Wolfers (2007), uncertainty shocks, such as Jaimovich and Rebelo (2006), Bloom et al. (2014), Nimark (2014) and higher-order belief shocks, such as Angeletos and La’O (2013) or Huo and Takayama (2015).
Specifically, we employ a kernel density estimation procedure, one of most common approaches in non-parametric estimation. Essentially, it approximates the true distribution function with a smoothed version of a histogram constructed from the observed data. By using the widely-used normal kernel, we impose a lot of discipline on our learning problem but also allow for considerable flexibility. We also experimented with a handful of other kernels.

Consider a shock \( \tilde{\phi}_t \) whose true density \( g \) is unknown to agents in the economy. The agents do know that the shock \( \tilde{\phi}_t \) is i.i.d. Their information set at time \( t \), denoted \( \mathcal{I}_t \), includes the history of all shocks \( \tilde{\phi}_t \) observed up to and including \( t \). They use this available data to construct an estimate \( \hat{g}_t \) of the true density \( g \). Formally, at every date, agents construct the following normal kernel density estimator of the pdf \( g \)

\[
\hat{g}_t (\tilde{\phi}) = \frac{1}{n_t \kappa_t} \sum_{s=0}^{n_t-1} \Omega \left( \frac{\tilde{\phi} - \tilde{\phi}_{t-s}}{\kappa_t} \right)
\]

where \( \Omega (\cdot) \) is the standard normal density function, \( \kappa_t \) is the smoothing or bandwidth parameter and \( n_t \) is the number of available observations of at date \( t \). As new data arrives, agents add the new observation to their data set and update their estimates, generating a sequence of beliefs \( \{\hat{g}_t\} \).

The key mechanism in the paper is the persistence of belief changes induced by transitory \( \tilde{\phi}_t \) shocks. This stems from the martingale property of beliefs - i.e. conditional on time-\( t \) information \( (\mathcal{I}_t) \), the estimated distribution is a martingale. Thus, on average, the agent expects her future belief to be the same as her current beliefs. This property holds exactly if the bandwidth parameter \( \kappa_t \) is set to zero and holds with tiny numerical error in our application.\(^3\)

In line with the literature on non-parametric assumption, we use the optimal bandwidth.\(^4\) As a result, any changes in beliefs induced by new information are expected to be approximately permanent. This property plays a central role in generating long-lived effects from transitory shocks.

\(^3\)As \( \kappa_t \to 0 \), the CDF of the kernel converges to \( G^0_t (\tilde{\phi}) = \frac{1}{n_t} \sum_{s=0}^{n_t-1} 1 \{ \tilde{\phi}_{t-s} \leq \tilde{\phi} \} \). Then, for any \( \tilde{\phi}, j \geq 1 \)

\[
E_t \left[ G^0_{t+j} (\tilde{\phi}) \mid \mathcal{I}_t \right] = E_t \left[ \frac{1}{n_t+j} \sum_{s=0}^{n_t+j-1} 1 \{ \tilde{\phi}_{t+s} \leq \tilde{\phi} \} \mid \mathcal{I}_t \right] = \frac{n_t}{n_t+j} G^0_t (\tilde{\phi}) + \frac{j}{n_t+j} E_t \left[ 1 \{ \tilde{\phi}_{t+1} \leq \tilde{\phi} \} \mid \mathcal{I}_t \right]
\]

Thus, future beliefs are, in expectation, a weighted average of two terms - the current belief and the distribution from which the new draws are made. Since our best estimate for the latter is the current belief, the two terms are exactly equal, implying \( E_t \left[ G^0_{t+j} (\tilde{\phi}) \mid \mathcal{I}_t \right] = G^0_t (\tilde{\phi}) \).

2 Economic and Epidemiological Model

To gauge the magnitude of the scarring effect of the COVID-19 pandemic on long-run economic outcomes, we need to embed it in an economic model in which tail risk and belief changes can have meaningful effects. For this, a model needs two key features. First, it should have the potential for ‘large’ shocks, that have both transitory and lasting effects. The former would include lost productivity from stay-at-home orders preventing services from reaching consumers. But, for this shock to look like the extreme event it is to investors, the model must also allow for the possibility of a more persistent loss of productive capital. The interior of the restaurant that went bankrupt, the unused capacity of the hotel that will not fill again for many years to come. When stay-at-home policies force consumers to find other ways to fulfilling their needs, tastes, habits, and consumption patterns may change permanently, rendering some capital obsolete. One might think this is hard-wiring persistence in the model. Yet, as we will show, this loss of capital by itself, has a short lived effect and typically triggers an investment boom, as the economy rebuilds capital better suited to the new consumption normal. We explore two possible scenarios that highlight the enormous importance of preventing capital obsolescence, because of the scarring of beliefs.

The second key feature is sufficient curvature in policy functions, which serves to make economic activity sensitive to the probability of extreme large shocks. Two ingredients – namely, Epstein-Zin preferences and costly bankruptcy – combine to generate significant non-linearity in policy functions.

It is important to note that none of these ingredients guarantee persistent effects. Absent belief revisions, shocks, no matter how large, do not change the long-run trajectory of the economy. Similarly, the non-linear responses induced by preferences and debt influence the size of the economic response, but by themselves do not generate any internal propagation. They simply govern the magnitude of the impact, both in the short and long run.

To this setting, we add belief scarring. We model beliefs using the non-parametric estimation described in the previous section and show how to discipline this procedure with observable macro data, avoiding free parameters. This belief updating piece is not there to generate the right size reaction to the initial shock. Instead, belief updating adds the persistence, which considerably inflates the cost.

2.1 The Disease Environment

This block of the model serves to generates a time path for disruption to economic activity, which will then be mapped into transitory productivity shock and capital obsolescence. Of course, we could have directly created scenarios for the shocks and arrived at the same predictions. The
explicit modeling of the spread of disease allows us to see how different social distancing policies map into shocks and ultimately into long-term economic costs from belief scarring. Given this motivation, we build on a very simple SEIR model, which is a discrete-time version of Atkeson (2020) or Stock (2020), who build on work in the spirit of Kermack and McKendrick (1927). To this model, we add two ingredients: 1) a behavioral / policy rule that imposes capital idling when the infection rate increase. This rule could represent optimal behavior or government policy; and 2) a higher depreciation rate of unused capital. While we normally think of capital utilization depreciating capital, this is a different circumstance where habits, technologies and norms are changing more rapidly than normal. Unused capital may be restaurants whose customers find new favorites, old conferencing technologies replaced with new online technology or office space that will be replaced with home offices. This higher depreciation rate represents a speeding up of capital obsolescence.

**Disease and shutdowns** On January 20 2020, the first case of COVID was documented in the U.S. Therefore, we start our model on that day, with one infected person. Because we are examining persistence mechanisms, we want to impose a clear end date to the COVID shock. Therefore, we assume that COVID-19 will be over by the end of 2020. The SEIR model predicts the evolution of the pandemic. Our policy shot-down rule, maps the infection rate series into a value for the aggregate shock to the US economy in 2020. From 2021 onwards, we assume that COVID-19 will be over. However, we explore scenarios where the economy may suffer other pandemics in the future.

Time is discrete and infinite. For the disease part of the model, we will count time in days, indexed by $\tilde{t}$. Later, to describe long-run effects, we will change the measure of time to $t$, which represents years. There are $N$ agents in the economy. At date 1, the first person gets infected. Let $S$ represent the number of people susceptible to the disease, but not currently exposed, infected, dead or recovered. At date 1, that susceptible number is $S(1) = N - 1$. Let $E$ be the number of exposed persons and $I$ be the number infected. We start with $E(1) = 0$ and $I(1) = 1$. Finally, $D$ represents the number who are either recovered or dead, where $D(1) = 0$.

The following four equations describe the dynamics of the disease.

$$S(\tilde{t} + 1) = S(\tilde{t}) - \beta \frac{S(\tilde{t})I(\tilde{t})}{N}$$  \hspace{1cm} (2)

$$E(\tilde{t} + 1) = E(\tilde{t}) + \beta S(\tilde{t})I(\tilde{t}) - \sigma E(\tilde{t})$$  \hspace{1cm} (3)

$$I(\tilde{t} + 1) = I(\tilde{t}) + \sigma E(\tilde{t}) - \gamma I(\tilde{t})$$  \hspace{1cm} (4)

$$D(\tilde{t} + 1) = D(\tilde{t}) + \gamma I(\tilde{t})$$  \hspace{1cm} (5)
The parameter $\gamma_I$ is the rate at which people exit infection and become deceased or recovered. Thus, the expected duration of infection is approximately $1/\gamma_I$, and the number of contacts an infected person has with a susceptible person is $\tilde{\beta}$ times the fraction of the population that is susceptible $S(t)/N$. The initial reproduction rate, often referred to as $R_0$ is therefore $\tilde{\beta}/\gamma_I$.

We put a $t$ subscript on $\tilde{\beta}$ because behavior and policy can change it. When the infection rate rises, people reduce infection risk by staying home. This reduces the number of social contacts, reducing $\tilde{\beta}$. Lockdown policies also work by reducing $\tilde{\beta}$. We capture this relationship by assuming that $\tilde{\beta}$ can vary between a maximum of $\gamma_I R_0$ and a minimum of $R_{min}$. $R_{min}$ is the estimated U.S. reproduction rate for regions under lockdown. Where on the spectrum the contact rate lies depends on the last 30-day change in infection rates, measured with a 15-day lag.\footnote{This is consistent with the U.S. official policy on re-opening (CDC, 2020). Note that individual optimal choice to social distance are also included in this “policy.” These optimal choices look similar. See Kaplan et al. (2020).}

Let $\Delta I_t$ be the difference between the average 15-29 day past infections and the average of 30-44 day infections: $\Delta I_t = (1/15) \left( \sum_{\tau=15}^{29} I(t-\tau) - \sum_{\tau=30}^{44} I(t-\tau) \right)$. This captures the fact that most policy makers are basing policy on two-week changes in hospitalization rates, which are themselves observed with a 14-day lag. Then policy and individual behavior achieves a frequency of social contact:

$$\tilde{\beta}_t = \gamma_I \times \min(R_0, \max(R_{min}, R_0 - \zeta \ast \Delta I_t)) \tag{6}$$

The key part of the epidemic from a belief-scarring perspective is that reducing the contact rate requires separating labor from capital. In other words, capital is idle. No capital is idled (full capacity) when no mitigation efforts are underway, i.e. when $\tilde{\beta}_t = \gamma_I R_0$. But as $\tilde{\beta}_t$ falls, capital idling ($K^-$) rises. We formalize that relationship as

$$K^-_t = \tilde{\theta} \ast (R_0 - \gamma \tilde{\beta}_t) \tag{7}$$

Idle capital depreciates as a rate $\tilde{\delta}$. As mentioned before, this is not physical deterioration of the capital stock. Instead, it represent a loss of value from accelerated obsolescence due to changes in tastes, habits and technologies. It could also represent a loss in value because of persistent upstream or downstream supply chain constraints.

\subsection{2.2 The Economy}

**Preferences and technology:** To describe long-term economic consequences, we switch from the daily time index $\hat{t}$ to an annual time index $t$. An infinite horizon, discrete time economy
has a representative household, with preferences over consumption \((C_t)\) and labor supply \((L_t)\):

\[
U_t = \left(1 - \beta \right) \left( C_t^\gamma (1 - L_t)^{1 - \gamma} \right)^{1 - \psi} + \beta E_t \left( U_{t+1}^{1 - \eta} \right)^{1 - \psi} \tag{8}
\]

where \(\psi\) is the inverse of the inter-temporal elasticity of substitution, \(\eta\) indexes risk-aversion, \(\gamma\) indexes the share of consumption in the period utility function, and \(\beta\) represents time preference.

The economy is also populated by a unit measure of firms, indexed by \(i\) and owned by the representative household. Firms produce output with capital and labor, according to a standard Cobb-Douglas production function \(z_t = k_t^{\alpha} l_t^{1 - \alpha}\).

Aggregate uncertainty is captured by a single random variable, \(\phi_t\), which is i.i.d. over time and drawn from a distribution \(g(\cdot)\). The i.i.d. assumption is made in order to avoid an additional, exogenous, source of persistence.\(^6\) The effect of this shock on economic activity depends on the realized default rate \(Def_t\) (the fraction of firms who default in \(t\), characterized later in this section). Formally, it induces a capital obsolescence ‘shock’ \(\phi_t = \Phi(\phi_t, Def_t)\). The function \(\Phi(\cdot)\) will made explicit later. This composite shock has both permanent and transitory effects. The permanent component works as follows: a firm that enters the period \(t\) with capital \(\hat{k}_{it}\) has effective capital \(k_{it} = \phi_t \hat{k}_{it}\).

In addition to this permanent component, the shock \(\phi_t\) also has a temporary effect, through the TFP term \(z_t = \phi_t^{\nu}\). The parameter \(\nu\) governs the relative strength of the transitory component. This specification allows us to capture both permanent and transitory disruptions with only one source of uncertainty. By varying \(\nu\), we can capture a range of scenarios without having to introduce additional shocks.

Firms are also subject to an idiosyncratic shock \(v_{it}\). These shocks scale up and down the total resources available to each firm (after paying labor, but before paying debtholders’ claims)

\[
\Pi_{it} = v_{it} \left[ z_t k_{it}^{\alpha} l_{it}^{1 - \alpha} - W_t l_{it} + (1 - \delta) k_{it} \right] \tag{9}
\]

where \(\delta\) is the rate of capital depreciation. The shocks \(v_{it}\) are i.i.d. across time and firms and are drawn from a known distribution, \(F_7\). The mean of the idiosyncratic shock is normalized to be one: \(\int v_{it} \, di = 1\). The primary role of these shocks is to induce an interior default rate in equilibrium, allowing a more realistic calibration, particularly of credit spreads.

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\(^6\) The i.i.d. assumption also has empirical support. In the next section, we use macro data to construct a time series for \(\phi_t\). We estimate a (statistically insignificant) autocorrelation of 0.15.

\(^7\) This is a natural assumption: with a continuum of firms and a stationary shock process, firms can learn the complete distribution of any idiosyncratic shocks after one period.
What is capital obsolescence? Capital obsolescence shock reflects a long-lasting change in the economic value of the average unit of capital. A realization of $\phi < 1$ captures the loss of specific investments or other forms of lasting damage from a prolonged shut-down. This could come from the lost value of cruise ships that will never sail again, businesses that do not reopen, loss of customer capital or just less intensive use of commercial space due to a persistent preference for more distance between other diners, travelers, spectators or shoppers. It could also represent permanent changes in health and safety regulations that make transactions safer, but less efficient from an economic standpoint.

An important question is whether future investment could be made in ways or in sectors that avoid these costs. Of course, such substitution is likely to happen to some extent. But, the fact that the patterns of investment were not chosen previously suggests that these adjustments are costly or less profitable. More importantly, we learned that the world is riskier and more unpredictable than we thought. The shocks that hit one sector (or type of capital) today may hit another tomorrow, in ways that are impossible to foresee.

Credit markets and default: Firms have access to a competitive non-contingent debt market, where lenders offer bond price (or equivalently, interest rate) schedules as a function of aggregate and idiosyncratic states, in the spirit of Eaton and Gersovitz (1981). A firm enters period $t + 1$ with an obligation, $b_{it+1}$ to bondholders. The shocks are then realized and the firm’s shareholders decide whether to repay their obligations or default. Default is optimal for shareholders if and only if

$$\Pi_{it+1} - b_{it+1} + \Gamma_{t+1} < 0$$

where $\Gamma_{t+1}$ is the present value of continued operations. Thus, the default decision is a function of the resources available to the firm $\Pi_{it+1}$ (output plus undepreciated capital less wages) and the obligations to bondholders $b_{it+1}$. Let $r_{it+1} \in \{0, 1\}$ denote the default policy of the firm.

In the event of default, equity holders get nothing. The productive resources of a defaulting firm are sold to an identical new firm at a discounted price, equal to a fraction $\theta < 1$ of the value of the defaulting firm. The proceeds are distributed pro-rata among the bondholders.\(^8\)

Let $q_{it}$ denote the bond price schedule faced by firm $i$ in period $t$, i.e. the firm receives $q_{it}$ in exchange for a promise to pay one unit of output at date $t + 1$. Debt is assumed to carry a tax advantage, which creates incentives for firms to borrow. A firm which issues debt at price $q_{it}$ and promises to repay $b_{it+1}$ in the following period, receives a date-$t$ payment of $\chi q_{it} b_{it+1}$, where $\chi > 1$. This subsidy to debt issuance, along with the cost of default, introduces a trade-off in

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\(^8\)In our baseline specification, default does not destroy resources - the penalty is purely private. This is not crucial - it is straightforward to relax this assumption by assuming that all or part of the cost of the default represents physical destruction of resources.
the firm’s capital structure decision, breaking the Modigliani-Miller theorem.\footnote{The subsidy is assumed to be paid by a government that finances it through a lump-sum tax on the representative household.}

For a firm that does not default, the dividend payout is its total available resources, minus its payments to debt and labor, minus the cost of building next period’s capital stock (the undepreciated current capital stock is included in $\Pi_{it}$), plus the proceeds from issuing new debt, including its tax subsidy

$$d_{it} = \Pi_{it} - b_{it} - \hat{k}_{it+1} + \chi q_{it} b_{it+1}. \quad (10)$$

Importantly, we do not restrict dividends to be positive, with negative dividends interpreted as (costless) equity issuance. Thus, firms are not financially constrained, ruling out another potential source of persistence.

**Bankruptcy and obsolescence:** Next, we spell out the relationship between default and capital obsolescence, $\phi_t = \Phi(\tilde{\phi}_t, Def_t)$ where $Def_t \equiv \int r_{it} di$. This is meant to capture the idea that widespread bankruptcies can amplify the erosion in the economic value of capital arising from the primitive shock $\tilde{\phi}_t$. This might come from lost supply chain linkages, inter-firm relationships or other ways in which economic activity is inter-connected. For example, a retailer might ascribe a lower value to space in a mall if a number of other stores go out of business. Similarly, a manufacturer might need to undertake costly search or make adjustments to his factory in order to accommodate new suppliers. We capture these effects with a flexible functional form:

$$\ln \phi_t = \ln \Phi(\tilde{\phi}_t, Def_t) = \ln \tilde{\phi}_t - \mu Def_t^{1-\nu}. \quad (11)$$

**Timing and value functions:**

1. Firms enter the period with a capital stock $\hat{k}_{it}$ and outstanding debt $b_{it}$.

2. The aggregate capital obsolescence shocks are realized.\footnote{To simulate the Covid-19 pandemic, we run the epidemiology model from Section 2.1 for one year and use the predicted capital obsolescence as the realized shock for 2020. For more details, see Section 3.} Labor choice is made and production takes place.

3. Firm-specific shocks $v_{it}$ are realized. The firm decides whether to default or repay ($r_{it} \in \{0, 1\}$) its debt claims.

4. The firm makes capital $\hat{k}_{it+1}$ and debt $b_{it+1}$ choices for the following period.
In recursive form, the problem of the firm is

\[
V(\phi_t, \hat{k}_{it}, b_{it}, S_t) = \max \left[ 0, \max_{d_{it}, \hat{k}_{it+1}, b_{it+1}} d_{it} + E_t M_{t+1} V(\phi_{t+1}, \hat{k}_{it+1}, b_{it+1}, S_{t+1}) \right]
\]  

(12)

subject to

\begin{align*}
\text{Dividends:} & \quad d_{it} \leq \Pi_{it} - b_{it} - \hat{k}_{it+1} + \chi q_{it} b_{it+1} \\
\text{Resources:} & \quad \Pi_{it} = v_{it+1} \left[ \max_{l_{it}} z_t (\phi_t \hat{k}_t)^{\alpha \ln 1 - \alpha} - W_t l_{it} + (1 - \delta) \phi_t \hat{k}_t \right] \\
\text{Bond price:} & \quad q_{it} = E_t M_{t+1} \left[ r_{it+1} + (1 - r_{it+1}) \frac{\theta \hat{V}_{it+1}}{b_{it+1}} \right]
\end{align*}

(13) \hspace{1cm} (14) \hspace{1cm} (15)

Finally, firms hire labor in a competitive market at a wage $W_t$. We assume that this decision is made after observing the aggregate shock but before the idiosyncratic shocks are observed, i.e. labor choice is solves the following static problem:

\[
\max_{l_{it}} z_t (\phi_t \hat{k}_t)^{\alpha \ln 1 - \alpha} - W_t l_{it}
\]

The first max operator in (12) captures the firm’s option to default. The expectation $E_t$ is taken over the idiosyncratic and aggregate shocks, given beliefs about the aggregate shock distribution. The value of a defaulting firm is simply the value of a firm with no external obligations, i.e. $\hat{V}_{it} = V(\phi_t, \hat{k}_t, 0, S_t)$.

The aggregate state $S_t$ consists of $(\hat{K}_t, \hat{\phi}_t, I_t)$ where $I_t$ is the economy-wide information set. Equation (15) reveals that bond prices are a function of the firm’s capital $\hat{k}_{it+1}$ and debt $b_{it+1}$, as well as the aggregate state $S_t$. The firm takes the aggregate state and the function $q_{it} = q(\hat{k}_{it+1}, b_{it+1}, S_t)$ as given, while recognizing that its choices affect its bond price.

**Information, beliefs and equilibrium** The set $I_t$ includes the history of all shocks $\hat{\phi}_t$ observed up to and including time-$t$. The expectation operator $E_t$ is defined with respect to this information set. Expectations are probability-weighted integrals, where the probability density is $\hat{g}(\hat{\phi})$. The function $\hat{g}$ arises from using the kernel density estimation procedure in equation (1).

For a given belief $\hat{g}$, a recursive equilibrium is a set of functions for (i) aggregate consumption and labor that maximize (8) subject to a budget constraint, (ii) firm value and policies that solve (12), taking as given the bond price function (15) and the stochastic discount factor, (iii) aggregate consumption and labor are consistent with individual choices and (iv) capital obsolescence is consistent with default rates according to (11).
2.3 Characterization

The equilibrium of the economic model is a solution to the following set of non-linear equations. First, the fact that the constraint on dividends (13) will bind at the optimum can be used to substitute for $d_{i,t}$ in the firm’s problem (12). This leaves us with 2 inter-temporal choice variables ($\hat{k}_{i,t+1}, b_{i,t+1}$) and a default decision. The latter is described by a threshold rule in the idiosyncratic output shock $v_{i,t}$:

$$r_{i,t} = \begin{cases} 0 & \text{if } v_{i,t} < v_{i,t}^1 \\ 1 & \text{if } v_{i,t} \geq v_{i,t}^1 \end{cases}$$

which implies that the default rate $Def_t = F(v_{i,t})$. It turns out to be more convenient to redefine variables and cast the problem as a choice of $\hat{k}_{i,t+1}$ and leverage, $lev_{i,t+1} \equiv \frac{b_{i,t+1}}{\hat{k}_{i,t+1}}$. The full characterization to the Appendix. Since all firms make symmetric choices for these objects, in what follows, we suppress the $i$ subscript. The optimality condition for $\hat{k}_{i,t+1}$ is:

$$1 = \mathbb{E}[M_{t+1}R^k_{t+1}] + (\chi - 1)lev_{t+1}q_t - (1 - \theta)\mathbb{E}[M_{t+1}R^k_{t+1}h(\frac{lev_{t+1}}{R^k_{t+1}})]$$  \hspace{1cm} (16)

where

$$R^k_{t+1} = \frac{\phi_{t+1}^\alpha k_{t+1}^{1-\alpha}l_{t+1}^1 - W_{t+1}l_{t+1} + (1 - \delta) \phi_{t+1} \hat{k}_{t+1}}{\hat{k}_{t+1}}$$  \hspace{1cm} (17)

The object $R^k_{t+1}$ is the ex-post per-unit, post-wage return on capital, which is obviously a function of the obsolescence shock $\phi_t$. The default threshold is given by $v_{t+1} = \frac{lev_{t+1}}{R^k_{t+1}}$ while $h(v) \equiv \int_{-\infty}^v v f(v) dv$ is the default-weighted expected value of the idiosyncratic shock.

The first term on the right hand side of (16) is the usual expected direct return from investing, weighted by the stochastic discount factor. The other two terms are related to debt. The second term reflects the indirect benefit to investing arising from the tax advantage of debt - for each unit of capital, the firm raises $\frac{b_{i,t+1}}{\hat{k}_{i,t+1}}q_t$ from the bond market and earns a subsidy of $\chi - 1$ on the proceeds. The last term is the cost of this strategy - default-related losses, equal to a fraction $1 - \theta$ of available resources.

Note that the default threshold is a function of $\phi_t$, which in turn is affected by default, through (11). Therefore, the threshold equation $v_{t+1} = \frac{lev_{t+1}}{R^k_{t+1}}$ implicitly defines a fixed-point relationship:

$$v_{t+1} = \frac{lev_{t+1}}{R^k_{t+1}} = \frac{lev_{t+1}}{\phi_{t+1}^\alpha k_{t+1}^{1-\alpha}l_{t+1}^1 - W_{t+1}l_{t+1} + (1 - \delta) \phi_{t+1} \hat{k}_{t+1}}$$  \hspace{1cm} (18)

Next, the firm’s optimal choice of leverage, $lev_{t+1}$ is

$$(1 - \theta) \mathbb{E}_t \left[ M_{t+1} \frac{lev_{t+1}}{R^k_{t+1}} f \left( \frac{lev_{t+1}}{R^k_{t+1}} \right) \right] = \left( \frac{\chi - 1}{\chi} \right) \mathbb{E}_t \left[ M_{t+1} \left( 1 - F \left( \frac{lev_{t+1}}{R^k_{t+1}} \right) \right) \right].$$  \hspace{1cm} (19)
The left hand side is the marginal cost of increasing leverage - it raises the expected losses from the default penalty (a fraction $1 - \theta$ of the firm’s value). The right hand side is the marginal benefit - the tax advantage times the value of debt issued.

Finally, firm and household optimality implies that labor solves the intra-temporal condition:

$$\frac{(1 - \alpha)y_t}{l_t} = W_t = \frac{1 - \gamma}{\gamma} \frac{c_t}{1 - l_t}$$

(20)

The optimality conditions, (16) - (20), along with those from the household side, form the system of equations we solve numerically.

### 3 Measurement, Calibration and Solution Method

This section describes how we use macro data to estimate beliefs and parameterize the model, as well as our computational approach. A strength of our theory is that we can use observable data to estimate beliefs at each date.

**Measuring past shocks** Of course, we have not seen a health event like COVID in the last 95-100 years. However, from an economic point of view, COVID is one of many past shocks to returns that happens to be larger. When we think about COVID changing our beliefs, or our perceived probability distribution of outcomes, those outcomes are realized returns on capital. Therefore, to estimate the pre-COVID and post-COVID probability distributions, we first set out to measure past capital returns that map neatly into our model.

A helpful feature of capital obsolescence shocks, like the ones in our model, is that their mapping to available data is straightforward. A unit of capital installed in period $t - 1$ (i.e. as part of $\hat{K}_t$) is, in effective terms, worth $\phi_t$ units of consumption goods in period $t$. Thus, the change in its market value from $t - 1$ to $t$ is simply $\phi_t$.

We apply this measurement strategy to annual data on non-residential capital held by US corporates. Specifically, we use two time series Non-residential assets from the Flow of Funds, one evaluated at market value and the second, at historical cost.\textsuperscript{11} We denote the two series by $NFA_t^{MV}$ and $NFA_t^{HC}$ respectively. To see how these two series yield a time series for $\phi_t$, note that, in line with the reasoning above, $NFA_t^{MV}$ maps directly to effective capital in the model. Formally, letting $P_t^k$ the nominal price of capital goods in $t$, we have $P_t^k K_t = NFA_t^{MV}$. Investment $X_t$ can be recovered from the historical series, $P_{t-1}^k X_t = NFA_t^{HC} - (1 - \delta) NFA_{t-1}^{HC}$.

\textsuperscript{11}These are series FL102010005 and FL102010115 from Flow of Funds.
Combining, we can construct a series for $P_k^t\hat{K}_t$:

$$P_k^t\hat{K}_t = (1 - \delta) P_{t-1}^k K_{t-1} + P_k^t X_t$$

$$= (1 - \delta) NFA_{t-1}^{MV} + NFA_{t}^{HC} - (1 - \delta) NFA_{t-1}^{HC}$$

Finally, in order to obtain $\phi_t = \frac{K_t}{\hat{K}_t}$, we need to control for nominal price changes. To do this, we proxy changes in $P_k^t$ using the price index for non-residential investment from the National Income and Product Accounts (denoted PINDX$_t$).\footnote{Our results are robust to alternative measures of nominal price changes, e.g., computed from the price index for GDP or Personal Consumption Expenditure.} This yields:

$$\phi_t = \frac{K_t}{\hat{K}_t} = \left( \frac{P_k^t K_t}{P_{t-1}^k \hat{K}_t} \right) \left( \frac{PINDX_k^t}{PINDX_{t-1}^k} \right)$$

$$= \left[ \frac{NFA_{t}^{MV}}{(1 - \delta) NFA_{t-1}^{MV} + NFA_{t}^{HC} - (1 - \delta) NFA_{t-1}^{HC}} \right] \left( \frac{PINDX_k^t}{PINDX_{t-1}^k} \right) \quad (21)$$

Using the measurement equation (21), we construct an annual time series for capital depreciation shocks for the US economy since 1950. The mean and standard deviation of the series over the entire sample are 1 and 0.03 respectively. The autocorrelation is statistically insignificant at 0.15.

Next, we recover the primitive shock $\tilde{\phi}_t$ from the time series $\phi_t$. To do this, we use (11), along with data on historical default rates from Moody’s investors service (2015)\footnote{The Moody’s data are for rated firms and shows a historical average default rate of 1% (our calibration implies a default rate of 2%), probably reflecting selection. Accordingly, we scaled the Moody’s estimates by a factor of 2 while performing this calculation. We also used estimates of exit and bankruptcy rates from Corbae and D’Erasmo (2019) and found broadly similar results.} and values for the feedback parameters ($\mu, \varpi$) as described below. The first panel of Figure 2 shows the estimated $\tilde{\phi}$.

**Parameterization** A period $t$ is interpreted as a year. We choose the discount factor $\beta = 0.95$, depreciation $\delta = 0.06$, and the share of capital in the production, $\alpha$, is 0.40. The recovery rate upon default, $\theta$, is set to 0.70, following Gourio (2013). The distribution for the idiosyncratic shocks, $v_{it}$, is assumed to be lognormal, i.e., $v_{it} \sim N\left(-\frac{\hat{\sigma}^2}{2}, \hat{\sigma}^2\right)$ with $\hat{\sigma}^2$ chosen to target a default rate of 0.02.\footnote{This is in line with the target in Khan et al. (2014), though a bit higher than the one in Gourio (2013). We verified that our quantitative results are not sensitive to this target.} the share of consumption in the period utility function, $\gamma$, is set to 0.4.

For the parameters governing risk aversion and intertemporal elasticity of substitution, we use standard values from the asset pricing literature and set $\psi = 0.5$ (or equivalently, an IES
of 2) and $\eta = 10$. The tax advantage parameter $\chi$ is chosen to match a leverage target of 0.50, the ratio of external debt to capital in the US data – from Gourio (2013). Finally, we set the parameters of the default-obsolescence feedback function, namely $\mu$ and $\varpi$. Ideally, these parameters would be calibrated to match the variability of default and its covariance with the observed $\phi_t$ shock. Unfortunately, our one-shock model fails to generate enough volatility in default rates and therefore, struggles to match these moments. Fixing this would almost certainly require a richer model with multiple shocks and more involved financial frictions. We take a simpler way out here and target a relatively modest feedback with values of $\mu = 0.2$ and $\varpi = 0.5$. These values imply roughly an amplification 3% at a baseline default rate of 2%, rising to 5% for a 6% default.\footnote{Section 5.2 studies a version without default amplification and finds that it generates similar patterns, albeit with slightly smaller magnitudes, as our benchmark economy.}

**Epidemiology parameters.** A major hurdle to quantifying the long-run effects is the lack of data and uncertainty surrounding estimates of the short-run impact. While this will surely be sorted out in the months to come, for now, with the crisis still raging and policy still being set, the impact is uncertain. More importantly for us, the nature of the economic shock is uncertain. It may be a temporary closure with furloughs, or it could involve widespread bankruptcies and changes in habits that permanently separate workers from capital or make the existing stock of capital ill-suited to the new consumption demands. Since it is too early to know this, we present two possible scenarios, chosen to illustrate the interaction between learning and the type of shock we experience. All involve significant losses in the short term but their long-term effects on the economy are drastically different.

We begin by describing parameter choices that are fixed across the scenarios. Following Wang et al. (2020)’s study of infection in Hubei, China, we calibrate $\sigma_E = 1/5.2$ and $\gamma_I = 1/18$ to the average duration of exposure (5.2 days) and infection (18 days). We use an initial reproduction number of $R_0 = 3.5$, based on more recent estimates of higher antibody prevalence and more asymptomatic infection than originally thought and $R_{\text{min}} = 0$ based on the estimates of the spread in New York, at the peak of the lockdown (Center for Disease Control, 2020). This implies that the initial number of contacts per period must be $\tilde{\beta} = \gamma_I R_0$.

The extent to which capital idling reduces contact rates is set to $\tilde{\theta} = 1/3$. This implies that a lockdown which reduces the reproduction number to 0.8 is associated with 50% capital idling. This is broadly consistent with the 25% drop in output, estimated during the lockdown period in Hubei province, China. The rate of excess depreciation of idle capital at the rate of 6.5% per month or $\tilde{\delta} = 0.065/30$ daily. As we will see, this implies a 10% erosion of the value of capital in our first scenario, which lines up with the drop in commercial real estate prices
since the pandemic started – see CPPI (2020).

The two scenarios, which differ in the sensitivity of lockdown policy to observed infection increases, i.e. the parameter $\zeta$. In scenario 1, we set $\zeta = 300$, which generates an initial lockdown that lasts for 2 months. This version of the model predicts waves of re-infection and new lockdowns in the months to come, echoing predictions by the Center for Disease Control. Scenario 2, which considers a much less aggressive response by setting $\zeta = 50$, has only one lockdown episode.

Table 1 summarizes the resulting parameter choices.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.95</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\eta$</td>
<td>10</td>
<td>Risk aversion</td>
</tr>
<tr>
<td>$\psi$</td>
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<td>1/Intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>$\gamma$</td>
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<td>share of consumption in the period utility function</td>
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<td>Technology:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>Capital share</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.06</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>0.28</td>
<td>Idiosyncratic volatility</td>
</tr>
<tr>
<td>Debt:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi$</td>
<td>1.06</td>
<td>Tax advantage of debt</td>
</tr>
<tr>
<td>$\bar{\theta}$</td>
<td>0.70</td>
<td>Recovery rate</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.2</td>
<td>Default-obsolescence feedback</td>
</tr>
<tr>
<td>$\varpi$</td>
<td>0.5</td>
<td>Default-obsolescence elasticity</td>
</tr>
<tr>
<td>Disease / Policy:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_0$</td>
<td>3.5</td>
<td>Initial disease reproduction rate</td>
</tr>
<tr>
<td>$R_{min}$</td>
<td>0.8</td>
<td>Minimum U.S. disease reproduction rate</td>
</tr>
<tr>
<td>$\sigma_E$</td>
<td>1/52</td>
<td>Exposure to infection transition rate</td>
</tr>
<tr>
<td>$\gamma_I$</td>
<td>1/18</td>
<td>Recovery / death rate</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>300 (50)</td>
<td>Lockdown policy sensitive to past infections</td>
</tr>
<tr>
<td>$\bar{\theta}$</td>
<td>0.19</td>
<td>Capital idling required to reduce transmission</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.002</td>
<td>Excess depreciation (daily) of idle capital</td>
</tr>
<tr>
<td>$\rho_I$</td>
<td>0.95</td>
<td>Persistence of policy rule</td>
</tr>
</tbody>
</table>

Table 1: **Parameters** Number in parentheses is used in scenario 2.

Numerical solution method Because curvature in policy functions is an important feature of the economic environment, our algorithm solves equations (20) – (19) with a non-linear collocation method. Appendix A.2 describes the iterative procedure. In order to keep the computation tractable, we need one more approximation. The reason is that date-$t$ decisions (policy functions) depend on the current estimated distribution ($\hat{g}_t(\hat{\phi})$) and the probability
distribution \( h \) over next-period estimates, \( \hat{g}_{t+1}(\hat{\phi}) \). Keeping track of \( h(\hat{g}_{t+1}(\hat{\phi})) \), (a compound lottery) makes a function a state variable, which renders the analysis intractable. However, the approximate martingale property of \( \hat{g}_t \) discussed in Section 1 offers an accurate and computationally efficient approximation to this problem. The martingale property implies that the average of the compound lottery is \( E_t[\hat{g}_{t+1}(\hat{\phi})] \approx \hat{g}_t(\hat{\phi}), \forall \hat{\phi} \). Therefore, when computing policy functions, we approximate the compound distribution \( h(\hat{g}_{t+1}(\hat{\phi})) \) with the simple lottery \( \hat{g}_t(\hat{\phi}) \), which is today’s estimate of the probability distribution. We use a numerical experiment to show that this approximation is quite accurate. The reason for the small approximation error is that \( h(\hat{g}_{t+1}) \) results in distributions centered around \( \hat{g}_t(\hat{\phi}) \), with a small standard deviation. Even 30 periods out, \( \hat{g}_{t+30}(\hat{\phi}) \) is still quite close to its mean \( \hat{g}_t(\hat{\phi}) \). For 1-10 quarters ahead, where most of the utility weight is, this standard error is tiny.

To compute our benchmark results, we begin by estimating \( \hat{g}_{2019} \) using the data on \( \hat{\phi}_t \) described above. Given this \( \hat{g}_{2019} \), we compute the stochastic steady state by simulating the model for 5000 periods, discarding the first 500 observations and time-averaging across the remaining periods. This steady state forms the starting point for our results. Subsequent results are in log deviations from this steady state level. Then, we subject the model economy to two possible additional adverse realizations for 2020, one at a time. Using the one additional data point for each scenario, we re-estimate the distribution, to get \( \hat{g}_{2020} \). To see how persistent economic responses are, we need a long future time series. We don’t know what distribution future shocks will be drawn from. Given all the data available to us, our best estimate is also \( \hat{g}_{2020} \). Therefore, we simulate future paths by drawing many sequences of future \( \hat{\phi} \) shocks from the \( \hat{g}_{2020} \) distribution. In the results that follow, we plot the mean future path of various aggregate variables.

4 Main Results

Our goal in this paper is to quantify the long run effect of the COVID crises, stemming from the belief scarring effect, i.e. from learning that pandemics are more likely than we thought. We formalize and quantify the effect on beliefs, using the assumption that people do not know the true distribution of aggregate economic shocks and learn about it statistically. This is the source of the long-run economic effects. Comparing the resulting outcomes to ones from the same model under the assumption of full knowledge of the distribution (no learning) reveals the extent to which beliefs matter.

But first, we briefly describe the disease spread, the policy reaction and the economic shocks these policies generate.
Infection Dynamics
Scenario 1

Reproduction Rate and Capital Use
Scenario 1

Scenario 2 (less social distance)

Figure 1: Disease spread and capital dynamics.
Parameters listed in Table 1. Scenario 1 uses an aggressive lockdown policy $\zeta_I = 300$, while scenario 2 uses a more relaxed policy of $\zeta_I = 50$.

Epidemiology and economic shutdown. Figure 1 illustrates the spread of disease, in both scenarios, as well as the response, which results in capital idling. Recall that Scenario 2 has $\zeta = 50$, i.e., a policy that is six times less responsive to changes in the infection rate than the $\zeta = 300$ policy in scenario 1. As a result, it also has significantly less idle capital and a faster spike in infection rates.

For our purposes, the sufficient statistic in each scenario is the realization for $\tilde{\phi}_{2020}$. In scenario 1, the Covid-19 shock implies $\tilde{\phi}_t = 0.9$, i.e., the loss of value due to obsolescence is equal to 10% of the capital stock. In scenario 2, only 5% of capital is lost to obsolescence: $\tilde{\phi}_t = 0.95$. The target for the initial, transitory impact is line with most forecasts for 2020: an 10% (or 6.5%) annual decline in GDP. This is likely a conservative estimate for Q2 2020, but more extreme than some forecasts for the entire year.
How much belief scarring? We apply our kernel density estimation procedure to the capital return time series and our two scenarios to construct a sequence of beliefs. In other words, for each $t$, we construct $\{\tilde{g}_t\}$ using the available time series until that point. The resulting estimates for 2019 and 2020 are shown in Figure 2. The differences are subtle. Spotting them requires close inspection where the dotted and solid lines diverge, around 0.90 and 0.95, in scenarios 1, and 2 respectively. They show that the COVID-19 pandemic induces an increase in the perceived likelihood of extreme negative shocks. In scenario 1, the estimated density for 2019 implies near zero (less than $10^{-5}\%$) chance of a $\tilde{\phi} = 0.90$ shock; the 2020 density attaches a 1-in-70 or 1.4% probability to a similar event recurring.

![Figure 2: Beliefs about the probability distribution of outcomes, plotted before and during the COVID-19 crisis.](image)

The first panel shows the realizations of $\tilde{\phi}$. The second and third panels show the estimated kernel densities for 2019 (solid line) and 2020 (dashed line) for the two scenarios. The subtle changes in the left tail represent the scarring effect of COVID-19.

As the graph shows, for most of the sample period, the shock realizations are in a relatively tight range around 1, but we saw a large adverse realizations during the Great Recession of 0.93 in 2009. This reflects the large drops in the market value of non-residential capital stock. The COVID shock is now a second extreme realization of negative capital returns in the last 20 years. It makes such an event appear much more likely.

Effect on GDP Observing a tail event like the COVID-19 pandemic changes output in a persistent way. Figure 3 compares the predictions of our model for total output (GDP) to an identical model without learning. The units are log changes, relative to the pre-crisis steady-state. In the model without learning, agents are assumed to know the true probability of pandemics. As a result, when they see the COVID crisis, they do not update the distribution. This corresponds to the canonical “rational expectations” assumption in macroeconomics. The model with learning, which uses our real-time kernel density estimation to inform beliefs, generates similar short-term reactions, but worse long-term effects. The post-2020 paths are
Figure 3: **Output with scarring of beliefs (solid line) and without (dashed line).**

Units are percentage changes, relative to the pre-crisis steady-state, with 0 being equal to steady state and \(-0.1\) meaning 10% below steady state. Common parameters listed in Table 1. Scenario-specific parameters are: Scenario 1: \(\hat{\phi}_{2020} = 0.90\) Scenario 2: \(\hat{\phi}_{2020} = 0.95\).

simulated as follows: each economy is assumed to be at its stochastic steady state in 2019 and is subjected to the same 2020 \(\hat{\phi}\) shock; subsequently, sequences of shocks drawn from the estimated 2020 distribution.

The scenarios under learning correspond to what one might call a V-shaped or tilted-V recession: the recovery after the shock has passed is significant but not complete. Note that the drop in GDP on impact is a calibration target – what we are interested in its persistence, which arguably matters more for welfare. The graph shows that, in Scenario 1, learning induces a long-run drop in GDP of about 5%. The right panel shows a similar pattern but the magnitudes are smaller. Of course, agents also learn from smaller capital obsolescence shocks. These also scar their beliefs going forward. But the scarring is much less, producing only a 3% loss in long-run annual output.

Higher tail risk (i.e. greater likelihood of obsolescence going forwards) increases the risk premium required on capital investments, leading to lower capital accumulation. It is important that these shocks make capital obsolete, rather than just reduce productivity, because obsolescence has a much bigger effect on capital returns than lower productivity does. Labor also contracts, but that is a reaction to the loss of available capital that can be paired with labor. When a chunk of capital becomes mal-adapted and worthless, that is an order of magnitude more costly to the investor than the temporary decline in capital productivity. Since most of the economic effect works through capital risk deterring investment, that lower return is important to get the economic magnitudes right.
Turning off belief updating  When agents do not learn, both scenarios exhibit quick and complete recoveries, even with a large initial impact. Without the scarring of beliefs, facilities are re-fitted, workers find new jobs, and while the transition is painful, the economy returns to its pre-crisis trajectory relatively quickly. In other words, without belief revisions, the negative shock leads to an investment boom, as the economy replenishes the lost effective capital. While the curvature in utility moderates the speed of this transition to an extent, the overall pattern of a steady recovery back to the original steady state is clear. This is in sharp contrast to the version with learning. Note that since the no-learning economy is endowed with the same end-of-sample beliefs as the learning model, they both ultimately converge to the same levels. But, they start at different steady states (normalized to 0 for each series). This shows that learning is what generates long-lived reductions in economic activity.

Decomposing long-run losses.  Next, we perform a simple calculation to put the size of the long-run loss in perspective. Specifically, we use the stochastic discount factor implied by the model to calculate the expected discounted value of the reduction in GDP. These estimates, reported in Table 2, imply that the representative agent in this economy values the cumulative losses between 57% and 90% of the pre-Covid GDP. Most of this comes from the belief scarring mechanism.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>2020 GDP drop</th>
<th>NPV(Belief Scarring)</th>
<th>NPV(Obsolete capital)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Tough</td>
<td>-9%</td>
<td>-52%</td>
<td>-38%</td>
</tr>
<tr>
<td>II. Light</td>
<td>-6%</td>
<td>-33%</td>
<td>-24%</td>
</tr>
</tbody>
</table>

Table 2: New present value costs in percentages of 2019 GDP.

Note that the 1-year loss during the pandemic is 6-9% of GDP. The cost of belief scarring is five to six times as large, in both cases. The cost of obsolete capital is about four times as large as the damage done during the pandemic. Figure 4 illustrates the losses each year from the capital obsolescence and belief changes. The area of each of these regions, discounted as one moves to the right in time, is the NPV calculation in the table above. The one-year cost is a tiny fraction of this total area.

Of course, that calculation misses an important aspect of what we’ve learned – that pandemics will recur. Since our agents have 70 years of data, during which they’ve seen one pandemic, they assess the future risk of pandemics to be 1-in-70 initially. That probability declines slowly as time goes on and other pandemics are not observed. But there is also the risk there will be more pandemics, like this. This is not really a result of this pandemic. But that risk of future pandemics is what we should consider if we think about the benefits of public health investments. The pandemic cost going forward, in a world where a pandemic has a
Scenario 1

2020 2040 2060 2080 2100
-0.1
-0.08
-0.06
-0.04
-0.02
0
Capital obsolescence
Belief scarring

Scenario 2

2020 2040 2060 2080 2100
-0.1
-0.08
-0.06
-0.04
-0.02
0
Capital obsolescence
Belief scarring

Figure 4: Long-term costs of the pandemic.

Figure 5: Long-term costs of future pandemics.

1/70th probability of occurring each year, is given in Figure 5. Note that the risk of future pandemics costs the economy about 7-12% of GDP. This is similar to the one-year cost during the COVID crisis.

**Investment and Labor.** Figure 6 shows the effect of belief changes on investment. When agents do not learn, investment surges immediately (as the economy replenishes the obsolete capital). With learning, investment shows a much smaller surge (starting in 2021), but eventually falls below the pre-Covid levels.

In Figure 7, we see that the initial reaction of labor is milder than for investment, but the bigger differences arise from 2021 onwards. When the transitory shock passes, investment surges, to higher than its initial level, to compensate for the obsolescence shock. But labor
remains below the pre-Covid levels, reflecting the effect of the scarring effect on the stock of capital and through that on the demand for labor.

**Defaults, Riskless Rates and Credit Spreads.** The scenarios differ in their short-term implications for default as well. Default spikes only in 2020, the period of impact, returning to average from 2021 onwards. But, the higher default rate in scenario 1 (6% relative to 4% in Scenario 2) contributes to greater scarring (since default amplifies obsolescence). This result suggests a role for policy: preventing default/bankruptcy can lead to long-lasting benefits. In Section 5, we present a quantitative analysis of such a policy.

Nearly immediately, after the pandemic passes, default rates in both scenarios return to their original level. While defaults leave permanent scars on beliefs, the defaults themselves are not permanently higher. It is the memory of a transitory event that is persistent.

Because defaults were elevated, the pandemic had a large, immediate impact on credit spreads. However, these high spreads were quickly reversed. Some authors have argued that heightened tail risk should inflate risk premia, as well as credit spreads (Hall, 2015). While the argument is intuitive, it ignores any endogenous response of discounting, investment or borrowing. A surge in risk triggers disinvestment and de-leveraging. Because firms borrow less, this bring default rates back down, which offsets the increase in the credit spread. We can see this channel at work in the drop in debt and the lack of change in long-run defaults (Table 3). The credit spread is the implied interest on risky debt, $1/q_t$ less the risk-free rate $r^f$. The credit spread in the stochastic steady state under the 2019 belief is less than a basis point higher is the
post-pandemic long-run. Thus, belief revisions can have significant and long-lived real effects, even if the long-run change in credit spreads is very small.

Where we do see effects of the riskier underlying environment is in the heightened demand for riskless assets. Just as firms react to the increased tail risk by de-leveraging, investors would like to protect themselves by holding more riskless assets. They cannot all hold more. Therefore, the price increases and the return falls, to clear the market. Table 3 reports the riskless rate falls by as much as 100 (10) bps in Scenario 1 (Scenario 2).

**Equity markets and implied skewness.** One might think the recent recovery in equity prices appears inconsistent with a persistent rise in tail risk. The model teaches us why this logic is incomplete. When firms face higher tail risk, they also reduce debt, which pushes in the opposite direction as the rise in risk. In our model, the market value of a dividend claim associated with a unit of capital is nearly identical under the post-Covid beliefs than under the pre-Covid ones. In other words, the combined effect of the changes in tail risk and debt reduction is actually mildly positive. While the magnitudes are not directly interpretable, our point is simply that rising equity valuations are not evidence against tail risk.

If credit spreads and equity premia are not clear indicators of tail risk, what is? For that, we need to turn option prices, in particular out-of-the-money put options on the S&P 500, which can be used to isolate changes in perceived tail risk. A natural metric is the third moment of the distribution of equity returns. The last row of Table 3 reports this object (computed under the risk neutral measure). It shows that the perceived distribution after the shock is
more negatively skewed.\textsuperscript{16}

5 Policy and Additional Results

5.1 Financial Policy

The Covid-19 pandemic has led to unprecedented response from policy-makers. These responses fall into three broad categories: social distancing and other mobility restrictions, transfers to households and financial assistance to firms. We explored the consequences of more lax social distance policy in the previous results. Transfers to households has an important role as a poverty alleviation tool, but does not directly affect productive capacity, the object of interest in our analysis. Financial assistance to firms, on the other hand, can help the economy maintain productive capacity, for example by preventing widespread bankruptcies. In our setting, such a policy would have beneficial long-run effects as well, since they mitigate the consequences of belief scarring. In this subsection, we use our model to quantify these long-run effects.

In line with this bankruptcy-preventing interpretation, we model a policy as a transfer to firms through a reduction in their debt burden. For concreteness, we present results for a policy that reduces each firm’s debt burden by 10%, financed with lump-sum taxes.

\textsuperscript{16}It is straightforward to compute this from the SKEW and VIX indices reported by the CBOE. The 3rd central moment under the risk-neutral measure is $E \left( R^e - \bar{R}^e \right)^3 = \frac{100 - \text{SKEW}}{10} \cdot VIX^3$. This calculation reveals that between February and May 2020 the market implied third moment also became significantly more negative (from -0.04 to -0.09).
Table 3: Changes in financial market variables: Baseline, Scenarios 1 and 2.
Baseline is the steady state pre-pandemic, under 2019 beliefs. Columns labelled “change” are the raw difference between the long run average values under 2019 and 2020 beliefs in each scenario. They do not capture any changes that take place along the transition path or during the pandemic. The aggregate market capitalization in our model is the value of the dividend claim times the aggregate capital stock. Third moment is $E \left[ (R - \bar{R})^3 \right] \times 10^4$, where $R$ is the return on equity. The expectation is taken under the risk-neutral measure. For the no-learning model, all changes are zero.

Table 4: Firm Financial Assistance Policy: No Assistance vs. 10% Debt Reduction
Results are for scenario 1 ($\hat{\sigma}_{2020} = 0.90$). Numbers shown are in percentages of the pre-Covid steady-state GDP.
Figure 9: **Belief scarring lowers riskless rate in the long-run.**

Results show the return on a riskless asset, with scarring of beliefs (solid line) and without (dashed line). Common parameters listed in Table 1. Scenario-specific parameters are: Scenario 1: $\tilde{\phi}_{2020} = 0.90$ Scenario 2: $\tilde{\phi}_{2020} = 0.95$.

From Table 4, we see that firm financial assistance of this magnitude only saves 1% of GDP in 2020. From that metric alone, one might judge the cost of the policy to be too high. However, preventing bankruptcies in the short-run also helps reduce long-run output losses. The present discounted value of those savings are worth 11% of 2019 GDP. Of that 11%, 7% comes from ameliorating belief scarring and another 4% comes from the direct effects of limiting capital obsolescence. This exercise shows that considering the long-run consequences can significantly change the cost-benefit analysis for financial policies aimed at assisting firms.

### 5.2 Role of default-obsolescence feedback

In this subsection, we present results under the assumption that obsolescence is entirely exogenous, i.e. does not vary with default. This amounts to setting $\mu = 0$ in (11). Figure 10 shows the GDP impact of the Covid-19 shock under our benchmark specification (in the left panel) and without default feedback (in the right panel). The broad patterns are similar with belief revisions accounting for a significant portion of the impact, but the magnitudes are slightly smaller in the right panel (GDP falls by just under 4% in the long-run, relative to a 5% drop in the benchmark).

### 5.3 What if we had seen a pandemic like this before?

In our benchmark analysis, pre-Covid beliefs were formed using data that did not witness a pandemic (though it did have other tail events like the 2008-09 Great Recession). But,
Figure 10: Default feedback increases long-run effects.
Results show with scarring of beliefs (solid line) and without (dashed line), often with the two lines on top of each other. Common parameters listed in Table 1. Scenario-specific parameters are: Scenario 1: $\phi_{2020} = 0.90$
Scenario 2: $\phi_{2020} = 0.95$

Pandemics have occurred before – Jorda et al. (2020) identify 12 pandemics (with greater than 100,000 deaths) going back to the 14th century. This raises the possibility that economic agents in 2019 had some awareness of these past tail events and believed that they could happen again. To understand how this might change our results, we assume that the pre-Covid data sample includes the 1918 episode. Unfortunately, we do not have good data on capital utilization and obsolescence during that period,\(^\text{17}\) so we simply use the time series for the capital return shock $\tilde{\phi}_t$ from 1950-2020 as a proxy for the $\tilde{\phi}_t$ series from 1880-1949. In other words, we are asking: What if we had seen all of this unfold exactly the same way before?

The previous data does not change the short-term impact of the shock. But, it does cut the long-term effect of in half. Just before the pandemic of 2020 struck, our data tells us that there has been one pandemic in nearly 140 years. We assess the probability to be about 1-in-140. After 2020, we saw two pandemics in 141 years. Therefore, we revise our perceived probability from 1-in-140 to 2-in-141. That is about half the change in probability, relative to the original model where the probability rose from zero to 1-in-70.

But considering data from so long ago does raise the question of whether it is perceived as less relevant. There is a sense that the world has changed, institutions are stronger, science has advanced, in ways that alter the probability of such events. Such gradual change might logically lead one to discount old data.

In a second exercise, we assume that agents discount old data at the rate of 1% per year.

\(^{17}\)See Correia et al. (1918) and Velde (2020) for analysis of the economic effects of the 1918 over the short-to medium-term.
In such a world, two forces compete. Events from 1918 are there and reduce the surprise of the new pandemic, as before. Except, now this event is given tiny weight 102 years later. The countervailing force is that when old data is down-weighted, new data is given a larger weight in beliefs. The larger role of the recent pandemic in beliefs going forward makes belief scarring stronger for the next few decades. The net effect of these two forces is indistinguishable, in every respect, from the original results with data only from 1950.

Of course, more recently, we saw SARS, MERS and Ebola arise outside the U.S. Other countries may have learned from these episodes. But the lack of preparation and slow response to events unfolding in China suggests that U.S. residents and policy makers seem to have inferred only that diseases originating abroad stay outside the U.S. borders.

6 Conclusion

No one knows the true distribution of shocks to the economy. Macro经济学家 typically assume that agents in their models know this distribution, as a way to discipline beliefs. For many applications, assuming full knowledge has little effect on outcomes and offers tractability. But for unusually large events, like the current crisis, the difference between knowing these probabilities and estimating them with real-time data can be large. We argue that a more plausible assumption for these phenomena is to assume that agents do the same kind of real-time estimation along the lines of what an econometrician would do. This introduces new, persistent dynamics into a model with otherwise transitory shocks. The essence of the persistence mechanism is this: Once observed, a shock (a piece of data) stays in one’s data set forever and therefore persistently affects belief formation. The less frequently similar data is observed, the larger and more persistent the belief revision.

When we quantify this mechanism, our model’s predictions tell us that the ongoing crisis will have large, persistent adverse effects on the US economy, far greater than the immediate consequences. Preventing bankruptcies or permanent separation of labor and capital, could have enormous consequences for the value generated by the U.S. economy for decades to come.
References


Velde, F. R. (2020). What happened to the us economy during the 1918 influenza pandemic? a view through high-frequency data.


A Solution

A.1 Equilibrium Characterization

An equilibrium is the solution to the following system of equations:

\[ 1 = \mathbb{E} M_{t+1} \left[ R^k_{t+1} \right] J^k (\tilde{\nu}_t) \] (22)

\[ R^k_{t+1} = \frac{(1 - \alpha) \phi_{t+1}^{\alpha+\nu} K_{t+1}^{1-\alpha} + (1 - \delta) \phi_{t+1} \hat{K}_{t+1}}{K_{t+1}} \] (23)

\[ \frac{1 - \gamma}{\gamma} C_t = \frac{(1 - \alpha) Y_t}{L_t} \] (24)

\[ (1 - \theta) \mathbb{E}_t \left[ M_{t+1} \tilde{\nu}_t f (\tilde{\nu}_t) \right] = \left( \frac{\chi - 1}{\chi} \right) \mathbb{E}_t \left[ M_{t+1} (1 - F (\tilde{\nu}_t)) \right] \] (25)

\[ C_t = \phi_t^{\alpha+\nu} K_t^{1-\alpha} + (1 - \delta) \phi_t \hat{K}_t - \hat{K}_{t+1} \] (26)

\[ U_t = \left[ (1 - \beta) \left( u(C_t, L_t) \right)^{1-\psi} + \beta \mathbb{E} \left( U_{t+1}^{1-\eta} \right)^{\hat{\nu}} \right]^{\frac{1}{1-\psi}} \] (27)

where

\[ \ln \phi_t = \ln \tilde{\phi}_t - \mu F(\tilde{\nu}_t)^{1-\nu} \]

\[ \tilde{\nu}_t = \frac{lev_{t+1}}{R^k_{t+1}} \]

\[ J^k(\tilde{\nu}_t) = 1 + (\chi - 1) \tilde{\nu}_t (1 - F(\tilde{\nu}_t)) + (\chi \theta - 1) h(\tilde{\nu}_t) \]

\[ M_{t+1} = \left( \frac{dU_t}{dC_t} \right)^{-1} \frac{dU_t}{dC_{t+1}} = \beta \left[ \mathbb{E} \left( U_{t+1}^{1-\eta} \right) \right]^{\frac{\nu}{1-\eta}} U_{t+1}^{-\psi} \left( u(C_{t+1}, L_{t+1}) \right)^{-\psi} \]

A.2 Solution Algorithm

To solve the system described above at any given date \( t \) (i.e. after any observed history of \( \tilde{\phi}_t \)), we recast it in recursive form with grids for the aggregate state \( \hat{K} \) and the shocks \( \tilde{\phi} \). We then use an iterative procedure:

- Estimate \( \hat{g} \) on the available history using the kernel estimator.
- Start with a guess (in polynomial form) for \( U(\hat{K}, \tilde{\phi}), C(\hat{K}, \tilde{\phi}), L(\hat{K}, \tilde{\phi}) \).
- Solve (22)-(25) for \( \hat{K}', \hat{lev}'(\Pi, L) \), \( L \) using a non-linear solution procedure.
- Verify/update the guess for \( U, C, L \) using (26)-(27) and iterate until convergence.