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SPECIALIZATION, TRANSACTIONS TECHNOLOGIES, AND MONEY GROWTH

Harold Cole

Alan C. Stockman

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ABSTRACT

With some models of money and a representative-agent there is no reason for monetary trade because identical individuals can consume their own production. Lucas proposed a parable involving differentiated products in a cash-in-advance model to avoid this problem. This paper studies Lucas's suggestion by developing a differentiated product model with money, a cash-in-advance constraint for market purchases, and endogenous specialization. Individuals who are identical ex ante choose to differ ex post in equilibrium. Monetary exchange involves differentiated goods at a point in time, so a nonzero balance of trade is not a prerequisite for a monetary equilibrium. In contrast to results in some other models, we find that consumption of goods that are not purchased with money (analogous to leisure services or credit goods) can either rise or fall with a rise in the money growth rate. Finally, we allow for costly barter and examine individuals' choices of the method of payment. We discuss the implied nominal-interest elasticities of the (real) demand for money in the general equilibrium.

Harold Cole
Department of Economics
University of Pennsylvania
Philadelphia, PA 19104

Alan C. Stockman
Department of Economics
University of Rochester
Rochester, NY 14627

1. Introduction

There is no standard model of money in an individual optimization problem. Money-in-utility specifications, overlapping generations models, and cash-in-advance models are perhaps the three most popular optimizing models of money that are tractable in a general equilibrium context. One difficulty with the cash-in-advance setup in a representative-agent model is that there is no explicit reason for trade. If all individuals are alike, they can consume their own production, and money would be worthless. Lucas (1980) tried to get around this problem with a parable about differentiated products of different colors: each individual produces only one color but consumes many colors, so if the number of colors is large then he would consume almost entirely goods purchased on the market. While this parable has great appeal, it was not developed formally in the model. In particular, the choice decisions of individuals regarding which color(s) to produce and consume were not developed.

Subsequently, Lucas and Stokey (1983, 1987) developed a model with two types of goods, "cash goods" and "credit goods." The former require money for purchase, but the latter do not: they may be purchased on credit, i.e. they are analogous to goods that are both produced and consumed by a particular individual. A positive nominal interest rate is shown to distort decisions in their model, as it does investment decisions in Stockman (1981) and Abel (1985) and labor supply decisions in Aschauer and Greenwood (1983). But Lucas and Stokey took the identification of goods with the transactions technology (cash versus credit, or alternatively market purchase versus consumption out of one's own production) as exogenous.

This paper incorporates Lucas's "color parable" into an explicit model that permits specialization to be chosen optimally by individuals. We develop a differentiated product model with money, a cash-in-advance constraint (motivated by technology) for market purchases, and an endogenous specialization decision. Consequently, the choices of which goods to produce, which to consume, and which goods to buy on the market (using money) are endogenous.

In the overlapping generations (OG) model of money, individuals are differentiated by generation, and money is used for transactions between generations. Our model shares with OG monetary models the feature that individuals can consume their own endowments (or, in our model, the goods they produce), and money is used for transactions involving other goods. In our model, individuals are differentiated by which goods they choose to produce, and they choose to differentiate themselves in equilibrium. Unlike the OG model, these differences are not assumed exogenously. King and Plosser have recently developed a cash-in-advance model in which, as in our model, individuals who are alike *ex ante* choose to specialize to achieve gains in production. In the King-Plosser model, individuals choose human capital that gives them a comparative advantage either in goods in even-number periods or goods in odd-number periods, so individuals trade to smooth consumption over time.[1] Money is used for trades in the OG models and the King-Plosser model, but all trade is intertemporal. In contrast, our trade" each period, because our model generates endogenous differences among individuals that create nontrivial *intra*temporal trade that uses money.

We show that the effects of an increase in the rate of money growth with endogenous specialization may differ from the results predicted by the Lucas-Stokey model with exogenously fixed specialization. An increase in the nominal interest rate raises the relative price of "cash goods" in terms of "credit goods" in their model, because the former (but not the latter) involve holding cash and paying the opportunity cost of the nominal interest rate. In our formalization of Lucas's color parable, this translates into the proposition that a higher nominal interest rate raises the cost of consuming goods purchased on the market relative to the cost of consuming goods produced by the individual himself, "home-produced goods" (which he can simply eat, and need not buy with cash). With a fixed degree of specialization, an increase in the nominal interest rate reduces consumption of market goods and raises the consumption of home-produced goods. Similarly, a higher nominal interest rate raises the quantity of leisure (a home-produced good) consumed as in the Aschauer-Greenwood model. However, we show that when the degree of specialization is endogenous, an increase in the nominal interest rate can reduce the consumption of each type of home-produced good, reduce rather than raise total leisure, and either reduce or raise the degree of specialization. We discuss properties of our model using simulations, and discuss the implied nominal-interest elasticities of the (real) demand for money in the general equilibrium.

Finally, we extend our model to include an alternative transactions method (ATM). Innovations in financial markets have created and will continue to create new methods of transacting, such as credit cards, debit cards, etc. We introduce into our model not only the distinction between purchasing goods on the market and producing them at home (for one's own consumption or for sale), but also the distinction between alternative payment methods for purchased goods. As before, we do not impose any exogenous requirement that certain goods must be purchased with money while others may be purchased with the ATM. Instead, we allow individuals to choose the method of payment for each good.[2] In particular, we assume that individuals may either pay with cash or may barter. Paying with cash involves an opportunity cost related to the nominal interest rate. Barter involves other real expenditures to complete a transaction: search costs due to the "double coincidence of wants" problem would be the most natural cost here, but for simplicity we model the costs of barter as a labor cost of transporting goods. The key distinction between he costs of using money and the costs of barter is that while the former are related to the nominal interest rate and so involve nominal variables directly, the latter are purely real. The ATM will be introduced in Section 4; we first turn to the basic model in Sections 2 and 3.

2. The Basic Model

We examine a simple differentiated product model with money. We assume there is a continuum of types of goods on the interval [0,1]. There is also continuum of individuals on a circle with unit circumference. Goods and individuals are each indexed by i on this unit interval (or circle). Individuals have identical preferences given by

$$V = E_o \Sigma \beta^t [\int U(C_t(i))di - h(\int L_t(i)di)], \qquad 0<\beta<1, \tag{1}$$

where U() is strictly concave and satisfies $U'(0)=\infty$, h() is strictly convex, $C_t(i)$ is consumption of the good of type i at date t, and $L_t(i)$ is labor effort used in producing the good of type i at date t.

We assume that technology developed by the society has resulted in the invention of vending machines, which are able to protect goods from being stolen until a payment is made for the goods. In addition, the vending machine has the name of the company that made it on the front, and the company's reputation would suffer if the machine failed to deliver goods (or a refund) after money was inserted. Finally, the machine is able to recognize money so that people are not able to put counterfeit coins or bills in the machine. Technology has not developed yet that would allow the machine to evaluate whether a consumption good is "real" or "counterfeit," so only money can be used to buy goods from vending machines: they cannot be made to sell goods in exchange for other goods. Each individual owns a vending machine that he uses to sell goods. The machine is located on the circle.

At the beginning of each period t, each individual receives a transfer of money τ_t from the government. The individual also observes the current state of the economy, to be specified below. He then chooses his desired labor effort on each of the continuum of goods on [0,1]. Labor effort is nonnegative, so in making this decision the individual is also choosing his degree of specialization by allocating positive labor effort to a certain set of goods. Define

$$e_t(i) = \begin{cases} 1 \text{ if } L_t(i) > 0 \\ 0 \text{ otherwise.} \end{cases} \tag{2}$$

Then define the degree of specialization by $1-\alpha_t$ where α_t measures the fraction of goods that the individual produces at date t, i.e.

$$\alpha_t = \int e_t(i)\, di. \tag{3}$$

The individual's production of each good i is denoted

$$y_t(i) = f(\alpha_t)L_t(i) \tag{4}$$

where f() is positive, decreasing, and concave. Equation (4) shows that there are gains to specialization, but the gains may be subject to diminishing returns. If there were no differential costs of buying goods on the market rather than consuming home-produced goods, then it would be optimal for individuals to specialize completely by setting $\alpha_t=0$. However, there are costs of buying goods on the market as a result of the monetary nature of exchange.[3] These costs will work against specialization and lead to an interior solution for α_t.

We will see below that in equilibrium, the relative price of goods i and j is unity for all i,j on [0,1]. This fact is useful for defining the individual's gross output,

$$\int y_t(i)\ di = f(\alpha_t) \int L_t(i) \equiv f(\alpha_t)\alpha_t L_t \tag{5}$$

where the last equality defines $\alpha_t L_t$ as total labor effort by the individual at date t. We assume perfect competition in product markets. (Even if α_t = 0, there may be an infinity of individuals producing each type of good.) It is harmless to treat each individual's labor effort for those goods he chooses to produce as equal for each good, i.e. $L_t(i)=L_t(j)=L_t$ for all i,j for which his labor effort is nonzero.

After producing goods, each individual consumes some of them and sells the rest on the market. The individual places the goods he wishes to sell in his vending machine, and (with full information about the current state of the economy) adjusts the machine to emit goods in response to a sum of money equal to the current equilibrium nominal price of the good. He then rides a tram (provided by nature) around the unit circle, using the money he had on hand in his vending machine at the beginning of period t (from sales at date t-1) and the transfer he received at the beginning of period t, to buy goods from other individual's vending machines. Goods are consumed on the spot as they are purchased. When the individual arrives home the period ends. The tram ride takes a full "period," so there is not time for another ride. The next period then begins with vending machines emptied of money and transfers from the government. There is no possibility of communication between individuals because all are at different spots on the tram. The exchange system does not require individuals to meet, but it does require that individuals allocate some wealth to "money" accepted by the vending machines. This induces a cash-in-advance constraint on goods acquired on the market.

Let $c_t(i)$ denote an individual's consumption ($C_t(i)$) out of his own production of good i. Let $c_t*(i)$ denote consumption of goods of type i purchased from other individuals' vending machines. Obviously

$$c_t(i) \leq y_t(i) \qquad \text{for all i.} \tag{6}$$

Purchasing from vending machines requires money, so

$$\int p_t(i)c*_t(i)\, di \leq M_{t-1} + \tau_t, \tag{7}$$

where M_{t-1} denotes money placed into the individual's vending machine at date t-1 and emptied at the beginning of period t, and $p_t(i)$ denotes the nominal price of good i at date t, which must be placed in vending machines at date t to buy the good.

The money supply growth rate is given by a stochastic process,

$$M_t = \mu_t M_{t-1} \tag{8}$$

where μt is a nonnegative random variable and is identically and independently distributed with density function $\Phi()$. Each individual gets at date t the transfer

$$(\mu_t - 1)M_{t-1} = \tau_t. \tag{9}$$

We can simplify the individual's optimization problem by making several observations. First, preferences and opportunities are symmetric with respect to all goods. This implies a unit equilibrium relative price of goods due to production arbitrage, i.e. $p_t(i)=p_t(j)$ for all i,j. So we can define a price level $p_t = p_t(i)$ for all i. Second, all individuals are identical in tastes and opportunities; they differ only by their names and the types of the goods they produce. But all will choose the same α_t and the same $L_t=L_t(i)$ in equilibrium. Diminishing marginal utility of consumption (and the absence of any increasing returns) guarantees that total output of each good will be identical in equilibrium. This observation also implies that no borrowing and lending, or other asset trade, will occur in equilibrium. However, in writing the budget constraint (10) below, we will include a term for nominal bond holdings because we will want to discuss the nominal interest rate on those bonds (at which zero trade is the equilibrium quantity).

The individual's budget constraint can then be written as

$$[B_t - (1+R_{t-1})B_{t-1} + M_t - (M_{t-1}+\tau_t)]/p_t \qquad (10)$$

$$\leq f(\alpha_t)\alpha_t L_t - \int c_t(i) + c_t*(i)\ di$$

where B_t denotes nominal bond holdings by the individual at date t, where a bond is an asset that costs one unit of money at date t and pays $1+R_t$ units of money at date t+1.

The cash-in-advance constraint (7) can be rewritten, using the observation that all relative prices are unity, as

$$\int c*_t(i)\ di \leq (M_{t-1} + \tau_t)/p_t. \qquad (11)$$

The state of the world at time t is (τ_t, M_{t-1}). The equilibrium nominal price and interest rates will be defined below as functions of the state $p(\tau_t, M_{t-1})$ and $R(\tau_t, M_{t-1})$ such that individuals choose $c_t(i)$ and $c_t*(i)$ for all i, and L_t, α_t, B_t, and M_t, for all t, to solve Problem One: maximize

$$V = E_0\ \Sigma\ \beta^t[\int U(c_t(i)+c_t*(i))\ di - h(\alpha_t L_t)], \qquad 0<\beta<1, \qquad (12)$$

subject to

$$c_t(i) \leq f(\alpha_t)L_t(i) \quad \text{for all i in } [0,\alpha_t), \qquad (13)$$

$$c_t(i) = 0 \text{ for all } i \text{ not in } [0,\alpha_t), \tag{14}$$

$$[B_t - (1+R_{t-1})B_{t-1} + M_t - (M_{t-1}+\tau_t)]/p_t \tag{15}$$

$$\leq f(\alpha_t)\alpha_t L_t - \int c_t(i) + c_t{*}(i)\, di,$$

$$\int c{*}_t(i)\, di \leq (M_{t-1} + \tau_t)/p_t, \quad \text{and} \tag{16}$$

$$c_t{*}(i) \geq 0 \text{ for all } i. \tag{17}$$

The equilibrium will also require that markets clear, i.e.

$$B_t = 0 \quad \text{and} \quad M_t = M_{t-1} + \tau_t. \tag{18}$$

Necessary conditions are sufficient in this convex maximization problem, and these conditions yield (13), (15) and (16) with equality, (17) with strict inequality, (18), and

$$U'(c_t(i)+c_t{*}(i)) = \Gamma_t \qquad \text{for all } i \text{ in } [0,\alpha_t), \tag{19}$$

$$U'(c_t(i)+c_t{*}(i)) = \Gamma_t + \delta_t \qquad \text{for all } i \text{ not in } [0,\alpha_t), \tag{20}$$

$$\Gamma_t/p_t = (1+R_t)\beta E_t(\Gamma_{t+1}/p_{t+1}), \tag{21}$$

$$\Gamma_t/p_t = \beta E_t[(\Gamma_{t+1} + \delta_{t+1})/p_{t+1}], \tag{22}$$

$$\Gamma_t f(\alpha_t) = h'(\alpha_t L_t), \qquad \text{and} \tag{23}$$

$$\Theta_t(\alpha_t)c_t*(\alpha_t) + \Gamma_t f'(\alpha_t)\alpha_t L_t = 0, \tag{24}$$

where $\Theta_t(i)$, Γ_t, and δ_t are the Lagrange multipliers on constraints (14), (15), and (16).

Equations (19) and (20) imply that the individual consumes the same amount of all goods that he produces, and the same amount of all goods that he does not produce. He may consume different amounts of these two classes of goods, however. We drop time subscripts when there is no confusion. Let c denote c(i) for all i in $[0,\alpha_t)$, and c* denote c*(i) for all i in $[\alpha_t,1]$. Then (13), (15), and (18) imply that $c(i) < f(\alpha_t)L(i)$ for all i in $[0,\alpha_t)$, so $c*(i)=0$ for all i in $[0,\alpha_t)$. This means that if an individual produces good i, he does not buy that good on the market. Instead, he sells some of each of the goods he produces in order to obtain money for future purchases of those goods that he does not expect to produce in the future. Consequently, (19) and (20) imply

$$U'(c) = U'(c*) - \delta. \tag{25}$$

Equations (21) and (22) imply

$$R_t = E_t[\delta_{t+1}/p_{t+1}] \;/\; E_t(\Gamma_{t+1}/p_{t+1}), \tag{26}$$

which shows that the shadow price of the cash-in-advance constraint is positive if and only if the nominal interest rate is positive. The return on nominal bonds dominates the pecuniary return on money (zero), so individuals choose to hold money only if its liquidity services, measured by δ, compensate for the difference in pecuniary returns.

Equation (21), with (19) substituted for Γ, is the standard first-order condition of the permanent income model or the consumption-based capital asset pricing model,

$$U'(c_t) = (1+R_t)\beta E_t\{U'(c_{t+1}) \, p_t/p_{t+1}\}. \tag{27}$$

There is no monetary wedge in this case because equation (19) applies only to goods produced by the individual. However, substitution of (20) into (11) yields an analogous equation, for goods purchased on the market, in which a monetary wedge appears (in the form of the multiplier δ). Alternatively, this wedge may be seen by substituting (20) into (22); the result is

$$U'(c_t) = \beta E\{U'(c*_t) \, p_t/p_{t+1}\}. \tag{28}$$

Equations (27) and (28) imply that the expected marginal utility per dollar of consumption of goods purchased on the market is less than the expected marginal utility per dollar of own-produced goods whenever the nominal interest rate is positive. This highlights the analogy between a positive nominal interest rate and a tax on market purchases of goods.

Equation (23) shows that a positive nominal interest rate also creates a wedge between the marginal utility of consumption of market goods and the ratio of the marginal disutility of labor to the marginal product of labor:

$$h'(\alpha_t L)/f(\alpha_t) = U'(c) < U'(c*). \tag{29}$$

This wedge has been discussed in previous papers by Aschauer and Greenwood (1983), and a similar wedge in a model with investment appears in Stockman (1981) and Abel (1985).

The optimal degree of specialization, i.e. the choice of the number of goods to which an individual should devote positive labor effort, is determined by equation (24). Given total labor effort $\alpha_t L$, a small increase in α_t, which corresponds to a fall in the degree of specialization, lowers total output by $Lf'(\alpha_t)$, with utility cost $\Gamma\alpha_t Lf'(\alpha_t)$. The marginal benefit of raising α_t must be equated to its marginal cost. That benefit is the ability to consume an extra type of good (type α_t) out of home production, without having to use money to buy it from other individuals' vending machines. Previously, c* units of the good of type α_t were purchased from these machines. The marginal utility of relaxing the constraint (14) - which prevented consumption out of home production when there is no home production of this good - is $\Theta(\alpha_t+)$. So the marginal benefit of increasing α_t is $\Theta(\alpha_t)c*$. Notice that $\Theta(\alpha_t) = \delta$, because (given total consumption of the good, c*) the inability to consume out of home production adds to purchases on the market, which require money. So

$$(\delta/\Gamma)c* \;=\; f'(\alpha_t)\alpha_t L. \tag{30}$$

The rate of monetary growth was assumed to be drawn independently over time from a fixed distribution function, and always strictly positive. There is no other source of randomness in the model. Define real money balances,

$$m_t = (M_{t-1} + \tau_t)/p_t. \tag{31}$$

3. Properties of Equilibrium in the Basic Model

An equilibrium is a set of functions m or p, R, c, c*, α_t, L, Γ, δ, that solve problem one and satisfy the market-clearing conditions (18). Clearly there is a time-invariant real equilibrium in this model with a strictly positive nominal interest rate. Note that (15) and (18), along with our results that consumption is the same for all goods produced at home and also is the same for all goods purchased on the market, imply

$$\alpha c + (1-\alpha)c* = f(\alpha)\alpha L. \tag{32}$$

The equilibrium of the model can now be summarized as (32) and the following equations, where we drop time subscripts for time-invariant real variables:

$$p_{t+1}/p_t = \mu_t, \tag{33}$$

$$m = (1-\alpha)c*, \tag{34}$$

$$1+R = 1/\beta E(\mu) > 0, \tag{35}$$

$$(1+R)U'(c) = U'(c*), \tag{36}$$

$$f'(\alpha)\alpha L = -Rc*, \quad \text{and} \tag{37}$$

$$h'(\alpha L) = f(\alpha)U'(c). \tag{38}$$

Equations (32) and (36)-(38) implicitly give equilibrium solutions for c, c*,

α, and L. Equation (35) determines the nominal interest rate, while equation (34) with the definition (31) determines the price level.

We have only to complete the description of the equilibrium by giving an example of an assignment function that determines which individuals produce which sets of goods. Obviously, there are infinitely many assignment functions that will work. One is that agents of type j choose:

if $j+\alpha \leq 1$ then

$$e^j(i) = \begin{cases} 1 \text{ if } i \text{ is in } [j,\ j+\alpha) \\ 0 \text{ otherwise} \end{cases}$$

if $j+\alpha > 1$ then

$$e^j(i) = \begin{cases} 1 \text{ if } i \text{ is in } [j,1] \text{ or } [0,\alpha+j-1) \\ 0 \text{ otherwise.} \end{cases} \tag{39}$$

An increase in the mean growth rate of money would, according to equation (35), translate into a higher nominal interest rate. In order to determine the effects of greater money growth on equilibrium allocations, we consider some special cases of the model, and then report on results of simulations of the general model.

First, suppose that L and α are exogenously determined. Then c and c* are determined by (32) and (36). We find that an increase in the nominal interest rate (brought about by an increase in mean money growth) raises an individual's consumption of home-produced goods and reduces consumption of goods he purchases on the market:

$$dc/dR = (1-\alpha)U'(c)/\Omega > 0 \quad \text{and} \tag{40}$$

$$dc*/dR = -\alpha U'(c)/\Omega < 0, \tag{41}$$

where

$$\Omega = -\{\alpha U''(c*) + (1-\alpha)(1+R)U''(c)\} > 0. \tag{42}$$

These results reflect the higher relative cost of buying goods purchased on the market when the nominal interest is higher. This substitution from market goods to home goods resembles the results in Aschauer and Greenwood, where home goods are analogous to leisure (which can be thought of as productive time in the household) and the substitution out of "cash goods" into "credit goods" in Lucas and Stokey (1983, 1987), where "cash goods" are analogous to goods purchased on the market in our model and "credit goods" play the role of home-produced goods in our model. Indeed, if we allow labor services L to be endogenous then our model consists of equations (31), (36), and (38) which implicitly give c. c*, and L. The results are then

$$dc/dR = (1-\alpha)h''(\alpha L)U'(c)/\Omega' > 0, \tag{43}$$

$$dc*/dR = [(f(\alpha))^2U''(c) - \alpha h''(\alpha L)]/\Omega' < 0, \quad \text{and} \tag{44}$$

$$d(\alpha L)/dR \;=\; (1-\alpha)f(\alpha)u'(c)U''(c)/\Omega' \;<\; 0, \tag{45}$$

where

$$\Omega' = -\{(1-\alpha)h''(\alpha L)U''(c)(1+R) \;-\; U''(c*)[(f(\alpha))^2U''(c)-\alpha h''(\alpha L)]\} > 0. \tag{46}$$

The result for total labor supply, αL, is the same as in Aschauer and Greenwood; that for consumption of the two types of goods is the same as in Lucas and Stokey.

The comparative statics results just discussed require an exogenously fixed degree of specialization. In fact, we will show that results differ when the degree of specialization is chosen optimally by individuals. The simplest case to consider is the choice of α when total labor effort αL is exogenously fixed. Let

$$x \;\equiv\; c - c* -\alpha Lf'(\alpha) \;>\; 0 \tag{47}$$

where the sign follows from (29), concavity of U(), and f'<0. Then

$$dc/dR \;=\; \{xR + U''(c*)c*x \;-\; \alpha Lf''(\alpha)(1-\alpha)U'(c)\}/\Omega*, \tag{48}$$

$$dc*/dR = \{\alpha Lf''(\alpha)\alpha U'(c) \;+\; U''(c)c*(1+R)x\}/\Omega* \;<\; 0, \quad \text{and} \tag{49}$$

$$d\alpha/dR = -\{U''(c)(1+R)(1-\alpha)c* + U''(c*)\alpha c* + \alpha RU'(c)\}/\Omega*, \tag{50}$$

where

$$\Omega* = \alpha f''(\alpha)U''(c*)\alpha L - (1+R)U''(c)[xR - (1-\alpha)f''(\alpha)\alpha L] > 0. \tag{51}$$

While an increase in the nominal interest rate reduces consumption of each type of good purchased on the market, its effects on consumption of each type of good produced at home and on the degree of specialization are indeterminate in sign. Consider a special case in which

$$f(\alpha) = F - q\alpha, \tag{52}$$

where F and q are positive constants. Then c*, c, and α are given by

$$c* = q\alpha L/R, \tag{53}$$

$$c = U'^{-1}\{(1+R)U'(q\alpha L/R)\}, \text{ and} \tag{54}$$

$$\alpha = \{F\alpha L - q\alpha L/R\}/\{c+q\alpha L-q\alpha L/R\}. \tag{55}$$

If also the utility function is U(c) = ln(c), then c* is given by (53), and c and α are given by

$$c = (1+R)q\alpha L/R, \tag{56}$$

$$\alpha = (F - q/R)/2q. \tag{57}$$

In this case, a higher nominal interest rate causes less specialization (higher α), and, in contrast to the results for fixed α, reduces consumption of each type of home-produced good, c.

This example shows that the type of results obtained in a model like the one discussed by Lucas and Stokey, on the effects of changes in conditional expectations of money growth on demands for "cash goods" and "credit goods," is sensitive to the assumption of exogenous specialization (which translates in their model to exogeneity of the matching of types of goods with the type of payment required for them). Intuitively, a higher nominal interest rate raises the wedge between the costs of goods produced at home and goods purchased on the market. Given the degree of specialization, individuals respond to a higher wedge by buying less on the market and consuming more home-produced goods (selling less of them). But individuals can also respond by reducing the degree of specialization in order to reduce the set of goods purchased on the market. A reduction in the degree of specialization involves costs of forgoing the benefits of specialization, and this "wealth effect" leads to a fall in consumption of each type of good produced at home. If α is fixed exogenously, the individual consumes less of each market-purchased good and more of each home-produced good. In contrast, with α rising in our example, the individual consumes less of all goods that were previously produced at home, less of all market goods, and possibly more of goods previously purchased on the market and now produced at home. The loss from the higher nominal interest rate is "spread out" across a wider range of goods rather than concentrated on market goods alone. One might suspect that a similar argument would imply that results on the effects of a higher nominal interest rate on the labor/leisure choice can also be altered by allowing individuals to choose optimally the degree of specialization, and this suspicion is right.

Returning to the general case — but with fixed total labor effort αL — we can see that a higher nominal interest rate definitely raises α if it lowers c or leaves c unchanged. If $dc/dR \leq 0$, then

$$x[R+c*U''(c*)] \leq \alpha L(1-\alpha)f''(\alpha)U'(c). \tag{58}$$

But then

$$d\alpha/dR = -U''(c)(1-\alpha)(1+R)c* - U''(c*)\alpha c* - \alpha RU'(c) \tag{59}$$

$$\geq -U''(c)(1-\alpha)(1+R)c* - U''(c*)\alpha c* - \alpha Rx[R+c*U''(c*)]/\alpha L(1-\alpha)f''(\alpha)$$

$$= -U''(c)(1-\alpha)(1+R)c* - R^2x/Lf''(\alpha)(1-\alpha) - U''(c*)\alpha c*[1 - xR/\alpha L(1-\alpha)f''(\alpha)]$$

$$> 0.$$

So, although a higher nominal interest rate has an ambiguous effect on α, in order for it to reduce α we would require parameter values such that the higher nominal interest rate also raises c, the consumption of each type of home-produced good.

We now turn to the general model, and report results from simulations. For purposes of the simulations, we assumed the following functional forms:

$$U(c) := \left[\frac{1}{1-\Gamma}\right] c^{1-\Gamma}, \tag{60}$$

$$F(\alpha) := 1.1 - \left[\frac{.1}{1.0901-\alpha}\right], \text{ and} \tag{61}$$

$$H(L) := L^{j}. \tag{62}$$

We then solved the model for c, c*, α, $l=\alpha L$ (= total labor effort), and m for various parameter values, Γ and j, and for various nominal interest rates R. The nominal interest rate can be treated as the exogenous variable because, given β and equations (33) and (35), it is simply a transformation of the rate of money growth. In each case, we varied the nominal interest rate from .10 to 5.00.

For $\Gamma=1$, that is, the case of U(c)=log(c), and j=2, we found that an increase in the rate of money growth (i) reduces total labor effort, l, (ii) reduces consumption of each type of good that continues to be purchased on the market, c*, (iii) reduces consumption of each type of good produced by the household, c, and (iv) increases the number of different types of goods (strictly speaking, the measure of the set of goods) produced by the household, α. (Recall the α is inversely related to the degree of specialization.)

The real demand for money falls with increases in the nominal interest rate. The implied interest-elasticity of the demand for money in the general equilibrium with these parameters is reasonable in magnitude and is a function of the level of the nominal interest rate. At R–.10, the elasticity is –.60, and rises in absolute value along with the level of R; at R=5.00 the elasticity is –.90. Our result on the behavior of the elasticity as R rises differs from that of Svensson (1985), in which the response of real money demand to the nominal money growth rate is negative for sufficiently small money–growth rates, but rises to zero (and the transactions velocity rises to unity) as the growth rate increases. In contrast, our model implies that the interest–elasticity of money demand may rise as the money growth rate rises.

For $\Gamma=.50$ and $j=2$, we found that an increase in the money growth rate (i) reduces total labor effort and (ii) reduces consumption of each type of good purchases on the market. But we also found some nonmonotonic behavior of c and α: (iii) for money–growth rates such that the nominal interest rate is smaller than.25, an increase in the growth rate lowers consumption of each type of good produced by the household and raises the number of goods produced by the household. For $.25 < R < 1.50$, an increase in the money growth rate raises consumption of each type of good produced by the household, and continues to raise the number of different goods each household produces. Finally, for $R > 1.50$, an increase in the money growth rate raises consumption of each type of good produced by the household but reduces the number of different types of goods that the household produces, leaving more goods to be purchased on the market. Despite the fact that the household then purchases more types of goods on the market, the decline in consumption of each type is sufficiently large that increases in the money growth rate always lowers the real demand for money. The implied interest–elasticity of money demand, in equilibrium with these parameters, is

−.65 at an interest rate of R=.10, and rises in absolute value as R rises, reaching −1.03 at R = 1.00 and −1.13 at R = 3.00.

For smaller values of Γ the response of c, the level of consumption of each type of good produced at home, also changes. For Γ=.10 and j=2, we found that an increase in the money growth rate (i) lowers total labor effort, (ii) lowers consumption of each type of good purchased on the market, but (iii) raises consumption of each type of good produced by the household. Finally, (iv) an increase in the money-growth rate raises the number of types of goods produced by the household if R < .20, but reduces this number of types if R > .20. The implied interest-elasticity of money demand varies monotonically from −.82 at R=.10 to −1.46 at $=1.00.

For g=2 and j=2, we found that an increase in the rate of money growth raises total labor effort. This differs from the results in Wilson (1979) and Aschauer and Greenwood (1983), where higher inflation leads households to substitute away from market goods into leisure, which — like the "credit goods" of Lucas and Stokey (1983, 1987), is not purchased with money. In our model, in contrast, there are two opposing effects on labor effort: the substitution effect that reduces it and also a wealth effect associated with the reduction in output when households optimally vary the degree of specialization. With these (and other) parameter values, greater rates of inflation can be associated with more, rather than less, labor effort. In addition, greater rates of money growth reduce consumption of each type of good (whether produced by the household or purchased on the market) and raises the number of types of goods the household produces. Implied interest-elasticities of money demand range from −.59 at $−.10 to −.90 at 5.00.

We also examined the effects of variations in j, the disutility-of-labor parameter. An increase in j corresponds to more curvature of the implied utility-of-leisure function at each level of labor effort. With $\Gamma=2$ and $j=20$, for example, an increase in the rate of money growth reduces consumption of each type of good, reduces total labor effort, and raises the number of types of goods produced by the household if $R < 1.50$ or lowers this number if $R > 1.50$.

These results show that a variety of responses of real variables to the money growth rate are possible in this model, despite the simplicity, and that these responses are not always even monotonic.

4. Alternative Transactions Methods (ATMs)

We now alter the model to permit an alternative transactions method, ATM. Individuals may barter goods. Barter involves transporting goods to a central market (in the middle of the circle where individuals live), participating in centralized exchange there at Walrasian prices, and returning home. Transporting goods requires e units of labor per good carried to the central market.[4] Individuals are also permitted to use money in the central market, but they would not choose to do so because it involves the extra cost of the nominal interest rate and has no benefits because the costs of transacting in the central market are zero.

Assume, as before, that there is a stationary rational expectations equilibrium. Let z(i) denote the number of goods of type i that the individual acquires through barter. The maximization problem is the same as before (12), except the instantaneous utility function is now

$$\int U[c(i)+c*(i)+z(i)]\ di \quad - \quad h[\alpha L + \sigma\int z(i)di] \tag{63}$$

and the budget constraint (15) becomes

$$[B_t - (1+R_{t-1})B_{t-1} + M_t - (M_{t-1}+\tau_t)]/p_t \tag{64}$$

$$\leq \ f(\alpha_t)\alpha_t L_t - \int c_t(i)+c_t*(i)+z(i)\ di,$$

and we have a nonnegativity constraint on z(i),

$$z(i) \geq 0 \qquad \text{for all i.} \tag{65}$$

The necessary conditions for each individual's maximization problem are now the same as in Section 2, but with the following changes:

(1) $U(c*)$ and its derivatives replaced by $U(c*+z)$, where $z = z(i)$, for all i not in $[0,\alpha)$, is the consumption of goods acquired through barter.

(2) $h(\alpha L)$ and its derivatives are replaced by $h[\alpha L+(1-\alpha)\sigma z]$ and its derivatives.

(3) Equation (24) becomes

$$\Theta_t(\alpha)[c_t*(\alpha)+z(\alpha)] + \Gamma_t f'(\alpha_t)\alpha_t L_t = 0. \tag{66}$$

(4) The new condition associated with the optimal choice of z(i) is

$$U'(c(i)+c*(i)+z(i)) - \Gamma - \sigma h' - w(i) = 0 \tag{67}$$

where w(i) is the multiplier on the new constraint (65).

The real labor cost associated with the barter system drives a wedge between the costs of consuming home-produced goods and market-purchased goods, as before. Obviously, barter and monetary exchange can coexist in equilibrium only if $\delta = \sigma h'()$.

The form of (66) reflects the choice that an individual has if he chooses a smaller α. A smaller α implies more specialization, which means that more types of goods are purchased on the market. Unlike the model in Sections 2 and 3, the model with an ATM permits the individual to choose the best method of buying goods on the market. The form of (66) reflects this option. If the economy is at an interior equilibrium where both money and barter are used to purchase market goods, then in the (stochastic) steady state equilibrium we have $R = \delta/\Gamma$ and (66) implies

$$(\delta/\Gamma)(c*+z) = R(c*+z) = -f'(\alpha)\alpha L. \tag{68}$$

An equilibrium is a set of functions that now includes z() and w(). There are three types of possible equilibria in the model, associated with which of the two systems of transactions are used. Two of these types involve corner solutions, either without the ATM (as in Sections 2 and 3) or without money. Assuming barter and monetary exchange are both used in equilibrium, then we have

$$P_{t+1}/P_t = \mu_t, \tag{69}$$

$$m = (1-\alpha)c*, \tag{70}$$

$$1+R = 1/\beta E(\mu) > 0, \tag{71}$$

$$(1+R)U'(c) = U'(c*+z), \tag{72}$$

$$U'(c*+z) - \sigma h'[\alpha L+(1-\alpha)\sigma z] = U'(c), \tag{73}$$

$$h'[\alpha L+(1-\alpha)\sigma z] / f(\alpha) = U'(c), \tag{74}$$

$$f'(\alpha)\alpha L = -R(c*+z), \text{ and} \tag{75}$$

$$\alpha c + (1-\alpha)(c*+z) = f(\alpha)\alpha L \tag{76}$$

which determine c, c*, z, α, and αL. Given total labor effort, αL, we have the system excluding equation (79).

If monetary growth is low, then (71) implies that the nominal interest rate will be low. In that case, the costs of monetary exchange are sufficiently low that no barter occurs. Then the model collapses to that of Section 2. At a higher rate of monetary growth, both barter and money are used to acquire market goods. Then, for a range of nominal interest rates, there are equilibria with both barter and monetary exchange. While the per-unit leisure cost of barter is constant, diminishing marginal utility of leisure (h'' positive) implies that the utility cost of barter rises in the volume of barter exchange. This prevents individuals from suddenly switching from an equilibrium in which all market exchange is monetary to an equilibrium in which all market exchange is through barter as the nominal interest rate rises. In this range, z, consumption of each type of good acquired by barter, rises with the nominal interest rate R, and c*, consumption of each type of good purchased with money, falls with R. As in the model of Section 2, consumption of each type of home-produced good, c, may rise or fall with R depending on the parameters. The ability to trade with the ATM also affects the responsiveness of consumption of each type of good, and the degree of specialization, to a change in the nominal interest rate. Obviously, a greater cost of barter σ reduces the amount of barter and the consumption of each type of barter good. Clearly, shifts in σ over time due to financial innovations would alter the demand for money, the degree of specialization in production, and the total volume of market exchange. We believe that extensions of this model may be useful in obtaining predictions about changes in other variables that would accompany shifts in the money demand function in response to financial innovation.

Finally, if the nominal interest rate is very high then monetary exchange vanishes and all market exchange is through the ATM. In this case the equilibrium is described by the set of equations above with $c^*=0$, without equation (63), and with equation (6) replaced by

$$U'(c)f'(\alpha)\alpha L = -\sigma h'[\alpha L+(1-\alpha)\sigma z]z. \tag{77}$$

4. Conclusions

We have examined a model of differentiated products with monetary exchange in which individuals may consume their own output, buy other individuals' output with money, or use an alternative transactions method, barter, to acquire other individuals' output. With a low nominal interest rate or a high cost of the ATM, technological considerations dictate that the use of money is the lowest-cost alternative for market transactions. With lower costs of the ATM or a higher nominal interest rate, some or all market transactions may occur with the ATM. In contrast to previous models, individuals are identical ex ante but choose to differ ex post in equilibrium to take advantage of gains from specialization, the specialization choice is endogenous, and monetary exchange occurs for market transactions that involve exchange of differentiated gods at a point in time (so that a nonzero balance of trade for an individual is not a prerequisite for monetary exchange). In contrast to models that exogenously tie certain types of consumption goods to certain methods of exchange, and in contrast to models with a fixed degree of specialization, we find that consumption of home-produced goods may actually fall rather than rise with a rise in the nominal interest rate. Finally, our model can be used to obtain implications regarding the effects of a decrease in the cost of using alternative transactions methods, i.e. to technical innovations of the kind that have accompanied recent changes in financial markets and information technology.

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Footnotes

1. The cash-in-advance constraint in the King-Plosser model requires that a commodity they call gold coin" be exchanged for goods; gold coin is money in their model because it is assumed that buyers can verify its value at a lower cost than they could verify the value of other goods that might be used as payment. In our model, fiat money is recognized by the vending machines as legitimate payment at a much lower cost than if the vending machines had to be built to recognize the value of various goods that might be offered as payment.

2. Some features of our model are shared by Schreft (1987), who allows trade credit as an alternative to monetary transactions in an overlapping-generations model with spatially separated agents.

3. These costs could also be interpreted as resulting from explicit taxes on market transactions, costs of shopping, etc.

4. We assume goods are consumed at the central market, as at each vending machine, so we do not include costs of carrying goods back home.