NBER WORKING PAPER SERIES

A MODEL OF ENDOGENOUS RISK INTOLERANCE AND LSAPS: ASSET PRICES AND AGGREGATE DEMAND IN A “COVID-19” SHOCK

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Working Paper 27044
http://www.nber.org/papers/w27044

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 2020, Revised March 2021

We are grateful to the editor, Ralph Koijen, and two anonymous referees for constructive comments. We also thank Daron Acemoglu, Katrin Assenmacher, Dan Cao, Gabriel Chodorow-Reich, Eduardo Davila, Sebastian Di Tella, Emmanuel Farhi, Luca Fornaro, Sebnem Kalemi-Ozcan, Rohan Kekre, Yueran Ma, Plamen Nenov, Carolin Pflueger, Ludwig Straub, Faith Witryol, and seminar participants at the University of Chicago, HKUST, Baruch College, the AEA meeting 2021, BIS, the Central Bank of Chile, the ECB, the Fed, the IMF, the Bank of Italy, VMACS, the BI Norwegian Business School, the Office of Financial Research (U.S. Treasury), the Armenian Economic Association meetings, and the Florida Macroeconomics seminar series for their comments. Chris Ackerman, Alex Carrasco, Tishara Garg, Elliot Parlin, and Jilun Xing provided excellent research assistance. Caballero and Simsek acknowledge support from the National Science Foundation (NSF) under Grant Numbers SES-1848857 and SES-1455319, respectively. Simsek thanks the University of Chicago Booth School of Business for their hospitality during the later stages of the project. First draft: April 11, 2020. Previous title: “A Model of Asset Price Spirals and Aggregate Demand Amplification of a 'Covid-19' Shock” The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

A Model of Endogenous Risk Intolerance and LSAPs: Asset Prices and Aggregate Demand in a “Covid-19” Shock
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April 2020, Revised March 2021
JEL No. E00,E12,E21,E22,E30,E40,G00,G01,G11

ABSTRACT

In this paper we: (i) provide a model of the endogenous risk intolerance and severe asset price and aggregate demand contractions following an adverse real (non-financial) shock; and (ii) demonstrate the effectiveness of Large Scale Asset Purchases (LSAPs) in addressing these contractions. The key mechanism stems from heterogeneous risk tolerance: as a recessionary shock hits the economy and brings down asset prices, risk-tolerant agents’ wealth share declines and their leverage rises endogenously. This reduces the market’s risk tolerance and generates downward pressure on asset prices and aggregate demand. When monetary policy is unconstrained, it can offset the decline in risk tolerance with an interest rate cut that boosts the market’s Sharpe ratio. However, if the interest rate policy is constrained, new contractionary feedbacks arise: recessionary shocks lead to further asset price and output drops, which feed the risk-o¤ episode and trigger a downward loop. In this context, LSAPs improve asset prices and aggregate demand by transferring risk to the government’s balance sheet, which reduces the market’s required Sharpe ratio and reverses the contractionary feedbacks. Quantitatively, we show that aggregate shocks and LSAPs have large impacts on asset prices when the model is calibrated to fit the inelastic demand for aggregate assets uncovered in recent literature. We also show that heterogeneity in risk tolerance explains part of the demand inelasticity in normal times, and further reduces the elasticity after a recessionary shock. The Covid-19 shock and the large response by all major central banks provide a vivid illustration of the environment we seek to capture.

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Dynamic link to most recent draft is available at
https://www.dropbox.com/s/dg5lv6kqnckb71h/covidDemandAmplification_

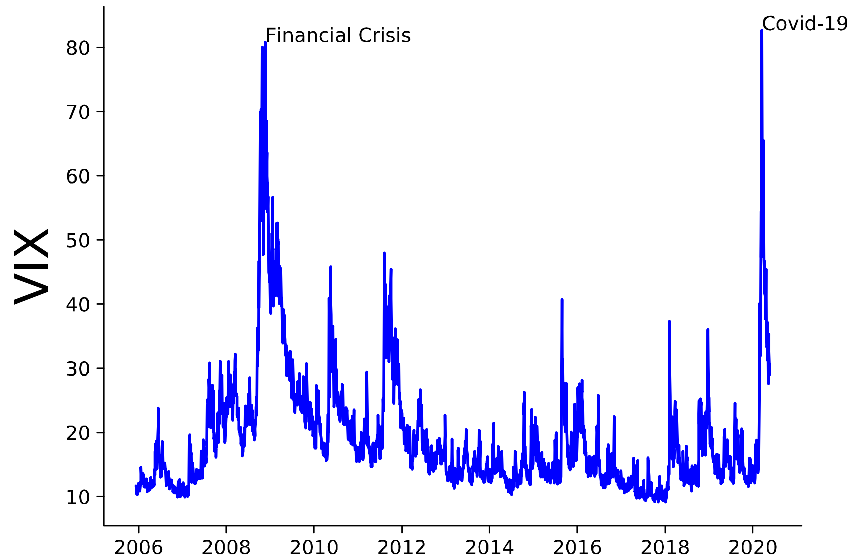

Figure 1: CBOE Volatility Index (VIX)

The Covid-19 shock is primarily a real (non-financial) shock with supply and demand elements. However, the shock also generated a large reaction in financial markets that had the potential to exacerbate the direct drop in economic activity caused by the real shock. Figure 1 illustrates that (implied) stock market volatility spiked to levels comparable to the global financial crisis of 2008–2009. Other indicators of financial distress exhibited similar patterns. For example, investment grade and high yield spreads tripled, and the S&P 500 dropped by 30% in a matter of weeks (a drop, per unit time, larger than the worst drop during the Great Depression). The Fed (with the backing of the Treasury) had to pledge close to 20% of US GDP in funding for a wide range of credit and market supporting facilities to stop the free fall.[1] Central banks in the Group of Seven countries purchased $1.4 trillion of financial assets in March alone. Beyond the Covid-19 episode, the structurally low safe interest rates suggest that asset market interventions are likely to become the norm for monetary policy response to large recessionary

[1] Here is a brief chronology of the Fed's main policy actions from early March until April 9th, 2020 (when the first version of this paper was released): On 03/03, implements a 50bps emergency rate cut; on 03/12, adds repos of up to $500b/week, purchases wider range of securities under current $60b/month program; on 03/15, cuts rates by 100bps to zero and initiates QE bond buying program of $700b, lowers swap lines with major central banks by 25bps; on 03/17, establishes a commercial paper funding facility to provide stability to short-term CP market; on 03/19, launches USD liquidity-swap lines with a broad range of countries, including major Emerging Markets; on 04/09, implements $2.3t emergency measures, among them a $500b Municipal Liquidity Facility for state and local governments, a $600b Main Street Lending program, and a Paycheck Protection Program Liquidity Facility for small businesses; expands the Primary and Secondary Market Corporate Credit Facilities and the term loan facility to buy ABS securities to $850b and includes asset purchases of HY bonds, HY ETFs, CLOs, and CMBS securities. All other major central banks around the world have also pursued unprecedented financial markets interventions.

shocks rather than the exception.

Motivated by these events and observations, in this paper we: (i) provide a model of the endogenous risk intolerance and severe asset price and aggregate demand contractions following a large real shock; and (ii) demonstrate the effectiveness of Large Scale Asset Purchases (LSAPs) in addressing these contractions.

Our model builds on the macroeconomic model in Caballero and Simsek (2020). That model is a variant of the New Keynesian model, but formulated in terms of a risk-centric decomposition. Specifically, in that model we decompose the demand block of the equilibrium into two relations: an *output-asset price relation* that captures the positive association between asset prices and aggregate demand; and a *risk balance condition* that describes asset prices given risks, risk attitudes, beliefs, and the interest rate. This decomposition facilitates studying the *macroeconomic* impact of a variety of forces that affect risky asset prices.[2] In the current model, we extend that analysis by splitting investors into risk-tolerant and risk-intolerant agents—we dub the risk tolerant agents "banks" (interpreted broadly to include the shadow financial system and other agents able/willing to hold substantial risk) and the risk intolerant agents "households" (also interpreted broadly).[3] The key implication of this assumption is that banks are levered in equilibrium, and therefore are highly exposed to aggregate shocks and the endogenous risk-off processes that these shocks may trigger.

To fix ideas, consider a large negative supply shock (e.g., the supply component of the Covid-19 shock).[4] This shock exerts downward pressure on risky asset prices (which include credit, equity, real estate, as well as other assets). As banks incur losses, their leverage rises. With higher leverage, banks require a higher Sharpe ratio (risk premium per unit of risk) to hold the same amount of risky assets. Risk-intolerant households also require a higher Sharpe ratio to hold the risky assets unloaded by banks wishing to reduce their leverage. Both of these channels reduce effective risk tolerance and increase the market's *required* Sharpe ratio.

As a benchmark, suppose that banks' initial leverage is not too high and the supply shock is small. In this case, effective risk tolerance does not decline by much following the shock. If the supply shock is also temporary, a small decline in asset prices may be all that is needed to increase the Sharpe ratio as much as the market demands. Asset prices and supply are temporarily low but they are expected to recover, which raises the expected return and the

[2]The decomposition is supported by a growing empirical literature that shows risky asset prices can substantially affect aggregate demand. See Mian and Sufi (2014) and Chodorow-Reich et al. (forthcoming) on the effect of house and stock prices, respectively, on consumption and (nontradable) employment; Pflueger et al. (2020) on the effect of financial market risk perceptions on economic activity and interest rates. See also Gilchrist and Zakrajšek (2012) on the effect of credit spreads on investment and consumption (while our main model is not about credit per se, credit spreads are closely correlated with overall risk perception).

[3]Note that the function of "banks" we seek to capture is risk absorption, not lending. Moreover, in practice agents' willingness to absorb risk can vary across asset classes. For instance, in the housing market homeowners absorb risk and therefore behave as the "banks" in our model.

[4]Adding an exogenous demand shock exacerbates our main results (see Remark 3 in Section 1.3). We chose to focus on the supply component of the Covid-19 shock because it allows us to isolate the endogenous component of the aggregate demand contraction.

Sharpe ratio. In fact, this expected recovery effect can even induce the central bank to *raise* the interest rate to keep asset prices and aggregate demand in line with the reduced supply.

In contrast, we focus on scenarios in which banks' initial leverage is high, or the supply shock is sufficiently large. In these cases, a supply shock substantially reduces effective risk tolerance and increases the required Sharpe ratio. This higher required Sharpe ratio can overwhelm the expected recovery effect (even if the shock is temporary) and induce a disproportionate decline in asset prices and demand that exceeds the decline in supply.

The first line of defense is conventional monetary policy that cuts the interest rate. Cutting interest rates provides the market with the greater Sharpe ratio that it requires and relieves the downward pressure on asset prices. Asset prices and aggregate demand decline in proportion to the reduction in supply but no more. However, if the interest rate is constrained there is a more acute decline in asset prices. Lower asset prices provide the market with a greater Sharpe ratio but they also generate a demand recession: output falls beyond the reduction in potential output. To make matters worse, the decline in asset prices further reduces banks' wealth share (and endogenously raises their leverage), which further increases the required Sharpe ratio and depresses asset prices, triggering a downward spiral. We show that, when banks' initial leverage is sufficiently high, the feedback between asset prices and risk intolerance becomes so strong that multiple equilibria are possible. In the worst of these equilibria, banks go bankrupt.

This description of events suggests that policies where the *consolidated* government (e.g., the Fed and the Treasury in the U.S.) absorbs some of the risk that banks are struggling to hold can be highly effective. We loosely refer to these policies as large-scale asset purchases (LSAPs). We show that, by transferring risk to the government's balance sheet, LSAPs reduce the market's *required* Sharpe ratio. This improves asset prices and aggregate demand and mitigates the recession. Moreover, LSAPs are powerful because they reverse the downward spiral. In particular, when the aggregate demand amplification of the supply shock is severe, the government might find it optimal to deploy LSAPs even if it is *less* risk tolerant than the market. The government also chooses larger LSAPs when it has greater future fiscal capacity. In the Covid-19 episode, the spike in VIX began to reverse immediately after the major central banks' policy actions (Figure 1 and Footnote 1), which suggests the interventions were effective in containing the initial downward spiral.

Although our model is highly stylized, we are able to gauge the quantitative potential of our mechanisms. For this exercise, we build on the recent literature documenting the low price elasticity of the demand for *aggregate* assets (see, e.g., Gabaix and Koijen (2020)). We find that the asset demand elasticity in our model is lower than its counterpart in a homogeneous-agent model, and this gap grows as banks become more levered. Therefore, heterogeneity in risk tolerance helps explain part of the demand inelasticity in normal times, and *endogenously reduces* demand elasticity after a recessionary shock. A low asset demand elasticity increases the quantitative strength of our mechanism because it implies that a given amount of asset outflows (triggered by a negative shock to levered banks) requires a larger drop in the equilibrium price to absorb the flows. For back-of-the-envelope calculations, we calibrate the (pre-shock) asset demand elasticity to match estimates from the recent literature, and calibrate banks' leverage to match the recent stress test scenarios used by the Fed. We find that negative aggregate supply shocks can cause large drops in the equilibrium asset price, and that relatively small LSAPs may be sufficient to overturn these effects. Moreover, for our calibrated parameters, the economy features regions of multiple equilibria and regions where the economy falls into a *unique* bad equilibrium with bankruptcy and low asset prices. LSAPs are powerful in the neighborhood of the high-price equilibrium (when there are multiple equilibria), and they significantly narrow the set of scenarios under which the economy falls into the unique bad equilibrium.

Finally, we extend the model to show how firms' debt overhang problems interact with our risk-centric mechanism. The corporate debt overhang problem creates a feedback mechanism between asset prices and aggregate supply. This feedback makes the market's effective risk tolerance (and hence the required Sharpe ratio) more sensitive to asset prices, which in turn strengthens our amplification mechanism and the effectiveness of LSAPs.

Section 1 describes the model and shows how supply shocks can trigger amplified drops in asset prices and aggregate demand. Section 2 shows how LSAPs operate in this environment. Section 3 derives the asset demand elasticities in our model and provides quantifiable formulas for the marginal price impact of supply shocks and LSAPs. This section also uses recent estimates of asset demand elasticities to assess the quantitative strength of the model's amplification mechanism and the power of LSAPs. Section 4 presents an extension with debt overhang. Section 5 concludes. The appendices contain the derivations and proofs omitted from the text as well as the details of our calibration exercise.

Literature review. Our paper contributes to a growing literature that emphasizes the role of leveraged financial intermediaries in asset pricing (see, e.g., Shleifer and Vishny (1997); Brunnermeier and Pedersen (2009); Geanakoplos (2010); Adrian and Shin (2010); Garleanu and Pedersen (2011); He and Krishnamurthy (2013, 2018); Brunnermeier and Sannikov (2014); Adrian et al. (2014); He et al. (2017); Haddad and Muir (forthcoming)).[5] Like most of this literature, we emphasize the financial health of "banks" as a key determinant of asset prices, but we also explore the interaction of this mechanism with nominal rigidities and monetary policy. In particular, our model generates macroeconomic effects driven by *aggregate demand fluctuations*, whereas the previous literature mostly emphasizes financial frictions on the supply side of the economy.[6] We uncover additional amplification mechanisms for asset prices when monetary policy is con-

[5] More broadly, our paper is part of a large finance literature that studies the effect of risk tolerance heterogeneity for asset prices (e.g., Dumas (1989); Wang (1996); Chan and Kogan (2002); Bhamra and Uppal (2009); Longstaff and Wang (2012); Gârleanu and Panageas (2015)). A related literature studies the asset pricing effects of limited market participation (e.g., Mankiw and Zeldes (1991); Heaton and Lucas (1996); Basak and Cuoco (1998); Vissing-Jørgensen (2002); Cao et al. (2005); Gomes and Michaelides (2008); Guvenen (2009); Iachan et al. (forthcoming)).

[6] Also related is Caballero and Krishnamurthy (2009), who show how the endogenous leverage of the US economy caused by the global demand for safe assets creates instability with respect to supply shocks. See also Kiyotaki and Moore (1997); Beaudry and Lahiri (2014); Di Tella (2017); Cao et al. (2019).

strained and highlight the role of LSAPs in reversing this amplification. Finally, we characterize the price elasticity of asset demand in this environment. We show that the heterogeneity in risk tolerance contributes to the inelastic demand for *aggregate* assets observed in normal times (see Gabaix and Koijen (2020); Koijen and Yogo (2020)), and it reduces the elasticity further after a large recessionary shock.

At a methodological level this paper adopts the risk-centric perspective in Caballero and Simsek (2020, 2021b). The novel ingredient is that the supply shock endogenously lowers risk tolerance. Kekre and Lenel (2020) features a similar mechanism, although they do not look at the effect of supply shocks or LSAPs. In particular, they calibrate a model in the spirit of Caballero and Simsek (2020) and show the power of conventional monetary policy in affecting the risk premium when agents have heterogeneous risk tolerance (or more broadly, heterogeneous marginal propensities to take risk). Similarly, Caballero and Farhi (2018) show that when a large share of wealth is allocated to extremely risk-intolerant agents (Knightians) in a New Keynesian framework with a zero lower bound on interest rates, the economy may fall into a "safety trap." Like our paper, they show that asset market policies where the government absorbs part of the risk of the economy (and replace it with safe assets) can be highly effective. However, their focus is on the macroeconomic implications of a chronic scarcity of safe assets rather than on the role of endogenous risk intolerance following a large real shock. More broadly, our paper is part of a New Keynesian literature that emphasizes the role of changes in risk premium in driving aggregate demand fluctuations (see, e.g., Ilut and Schneider (2014); Basu and Bundick (2017)).

Our paper is also related to a large macroeconomics literature that emphasizes the role of financial intermediaries and asset prices for aggregate demand and economic activity (see Gertler and Kiyotaki (2010) for a review). Our paper highlights that intermediaries and asset prices matter *even without financial frictions*, although adding conventional frictions would strengthen our mechanisms. Our approach shifts focus from the intermediaries' role in lending and credit allocation to their role in absorbing aggregate risk. We make predictions for prices of assets that are not intermediated by traditional banks (such as stock prices), and we show that aggregate demand *also* depends on the financial health of nontraditional "banks" (such as hedge funds, active mutual funds, investment banks, broker-dealers, and so on).[7]

Our analysis of LSAPs is related to a growing literature on the role of central bank asset purchases in stimulating aggregate demand when conventional monetary policy is constrained. Empirical evidence suggests these policies have a meaningful impact on asset prices (see Bernanke (2020)) but the underlying mechanisms are not fully understood. The literature emphasizes either financial frictions and credit (e.g., Gertler and Karadi (2011); Del Negro et al. (2017)), portfolio balance effects in segmented markets (e.g., Vayanos and Vila (2009); Ray (2019)), or

[7]Incidentally, the share of bank loans and mortgages in the U.S. nonfinancial corporations' total borrowing (the sum of their total loans and debt securities) has declined from nearly 50 percent before the mid-1970s to less than 20 percent today (source: Financial Accounts of the United States, series FL103168005, FL103165005, FL104123005, FL104122005).

signaling effects (see, e.g., Bhattarai et al. (2015)). The mechanism in our paper is different and relies on the government's ability to absorb aggregate risk using its future tax capacity in a non-Ricardian model (see also Silva (2016)).[8] We also provide a quantifiable formula for the asset price impact of these risky LSAPs. With realistic *aggregate* asset demand elasticities, our calibration suggests that central bank purchases of *risky* assets can affect asset prices substantially, which is consistent with the findings of a growing empirical literature (see, e.g., Caballero (1999); Charoenwong et al. (2019); Barbon and Gianinazzi (2019)).

In terms of whether demand factors can exacerbate the direct effect of a supply shock, the closest paper to ours is Guerrieri et al. (2020) (see also Baqaee and Farhi (2020); Bigio et al. (2020); Woodford (2020)). They provide a clean decomposition of the ingredients needed for an affirmative answer in a two-period, deterministic model. They conclude that, in such a model, aggregate demand cannot exacerbate the supply recession when the economy has a single sector, regardless of whether markets are complete or incomplete. In contrast, they show that in a multi-sector environment there are configurations of preference parameters where demand responds by more than supply, especially if markets are incomplete. Our risk-based mechanism is orthogonal to theirs. In fact, our model has a single sector.

Our paper is related to a growing empirical finance literature that analyzes the drivers of asset prices in the Covid-19 recession (e.g., Gormsen and Koijen (2020); Landier and Thesmar (2020); Ramelli and Wagner (2020); Croce et al. (2020); Davis et al. (2020)). This literature typically attributes the large decline and the subsequent recovery of risky asset prices in the Covid-19 recession to changes in the risk premium. From the lens of our model, the initial shock and its amplification increased the risk premium, whereas LSAPs helped reduce it. In line with this interpretation, the literature finds that the central bank asset purchases had a large positive impact on asset prices (Fed (2020); Cavallino and De Fiore (2020); Haddad et al. (2020)), even in emerging markets (Arslan et al. (2020)).

The Covid-19 shock also triggered a large response among macroeconomists. For example, Eichenbaum et al. (2020); Faria-e Castro (2020) embed pandemic shocks and their constraints on economic activity within DSGE models and study the role of fiscal policy and different containment strategies. Baker et al. (2020) document the dramatic spike in uncertainty and study its impact in a real business cycle model. Our analysis is complementary as we emphasize the excessive aggregate demand contraction that results from supply shocks—which is exacerbated by uncertainty—and we highlight the damage caused by *the pricing of uncertainty.* Fornaro and Wolf (2020a,b) provide a stylized New Keynesian model and capture the Covid-19 shock as a decline in (exogenous and endogenous) expected growth. Their mechanisms and policy analysis do not operate through endogenous spikes in risk intolerance and asset price spirals, which is

[8] In this broad sense, our policy mechanism builds upon the extensive literature spurred by Holmström and Tirole (1998) on the space created by the taxation power of the government to expand the supply of liquidity. Also, our model provides a distinct microfoundation for the portfolio balance theory of monetary policy pioneered by Tobin (1969). In our model, the central bank shifts the *aggregate* supply of risky assets that needs to be absorbed by the private sector.

our focus. Correia et al. (2020) use the 1918 flu pandemic to empirically analyze the economic costs of pandemics and find a role for both supply- and demand-side channels, consistent with our analysis.[9]

1. A Model of Endogenous Risk Intolerance

We present a model that illustrates how a supply shock can reduce risk tolerance in financial markets and induce an asset price and aggregate demand contraction that amplifies the supply shock. Later on, we discuss how adding exogenous demand shocks strengthens our results. Our mechanism operates through heterogeneous risk tolerance: the decline in asset prices due to the supply shock lowers risk-tolerant agents' wealth share and increases their leverage. As these agents attempt to lower their exposure to risk, effective risk tolerance declines. If the interest rate policy is constrained, risk intolerance leads to low asset prices and aggregate demand. The demand-induced decline in asset prices further lowers the wealth share of risk-tolerant agents, and so on.

1.1. Environment and equilibrium

Consider an economy with infinitely many periods, $t \in \{0, 1, 2, ...\}$, a single consumption good, and a single factor, capital (see Appendix B for an extension that also features a non-capitalized factor such as labor). There is no investment or depreciation and capital is normalized to one unit. We let z_t denote the productivity of capital in period t. *Potential* output is equal to productivity, z_t, but actual output, y_t, can be below this level due to nominal rigidities and a shortage of aggregate demand.

Our focus is on period 0, which we view as the short run. The short run has three features. First, there is aggregate risk about future productivity (z_1), which leads to a risk premium and a non-trivial portfolio choice. Second, monetary policy might be constrained, which allows output to be influenced by aggregate demand. Finally, motivated by the Covid-19 recession, we focus on supply shocks that we model as a decline in initial productivity, z_0, relative to a normalized level, $z_0 = 1$. Periods $t \geq 1$ feature no aggregate risk about future productivity (z_{t+1}). This assumption also implies that monetary policy is unconstrained for periods $t \geq 1$. These features simplify the analysis by enabling us to focus on the equilibrium in period 0. Starting from period 1 onward, output is equal to its potential and agents consume a constant fraction of this output (determined by their wealth share in period 1).

Formally, agents observe the productivity, z_t, at the beginning of the corresponding period t (before they take decisions). Given the initial productivity, z_0, future productivity evolves

[9] There are also several papers that embed SIR type epidemiological models into macroeconomic models and study the optimal containment policy that balances health concerns and economic costs (e.g., Alvarez et al. (2020); Gourinchas (2020); Berger et al. (2020); Callum et al. (2020); Bethune and Korinek (2020)). We do not address this important trade-off and take as given the broad supply implications of the containment policies.

according to

$$\log z_1 \sim N\left(\log \overline{z}_1 - \frac{\sigma^2}{2}, \sigma^2\right) \text{ where } \begin{aligned} \log \overline{z}_1 &= \varphi \log z_0 + g \\ \text{and} \quad \log z_t &= \log z_{t-1} + g \text{ for } t > 1. \end{aligned} \tag{1}$$

At period 0, from the agent's perspective, productivity at period 1 is uncertain and log-normally distributed, where $\overline{z}_1$ is the expected productivity and σ is its volatility. The parameter $\varphi \in [0, 1]$ is the persistence of the initial productivity shock. When $\varphi = 0$, the initial shock is fully transitory and does not affect future productivity. When $\varphi = 1$, the shock is fully persistent and shifts future productivity one-to-one. For most of the analysis, we focus on persistent shocks, $\varphi = 1$, to streamline exposition. However, we set up the model with general φ to illustrate that our results hold for mildly persistent or even fully transitory shocks. In subsequent periods, $t > 1$, productivity is deterministic and grows at a constant rate.

The supply side of the economy features standard New Keynesian production firms described in Appendix A.1. These firms choose their capital utilization rate, $\eta_t \in [0, 1]$, where $y_t = \eta_t z_t$. They can increase capital utilization for free until $\eta_t = 1$ and cannot increase it beyond this level. For simplicity, firms have fully sticky nominal prices that they don't change (see Remark 1 for the case with partially sticky prices). With these assumptions, output is determined by the aggregate demand for goods (aggregate consumption) up to the capacity constraint,

$$y_t = \sum_i c_t^i \leq z_t, \tag{2}$$

where c_t^i denotes consumption by agent i at time t. Firms optimally meet the available demand at their preset price as long as the price is higher than their marginal cost (which is zero when $\eta_t < 1$ and becomes infinitely high when $\eta_t = 1$).

There are two types of financial assets. The "market portfolio" represents claims to all output (which accrues to production firms as earnings). We denote the (ex-dividend) price of the market portfolio at date t with $z_t P_t$, so that P_t corresponds to the price *per unit of productivity*. We let r_t denote the log return of the market portfolio:

$$r_t = \log\left(\frac{y_{t+1} + z_{t+1} P_{t+1}}{z_t P_t}\right). \tag{3}$$

There is also a risk-free asset in zero net supply whose (nominal and real) return is set by the central bank. We let r_t^f denote the log risk-free rate.

The demand side features two types of agents, $i \in \{b, h\}$. Type b agents ("banks") are more risk tolerant than type h agents ("households"). Formally, agents have Epstein-Zin utility with risk aversion parameters given by $1/\tau^i$ that satisfy $\tau^b > \tau^h$. We refer to τ^i as agent i's risk tolerance. Agents also have common elasticity of intertemporal substitution (EIS) equal to one, and common discount factor denoted by $e^{-\rho}$. We write agents' flow budget constraints as

$$\begin{aligned} c_t^i + a_t^i &= A_t^i, \\ A_{t+1}^i &= a_t^i \left(\omega_t^i \exp(r_t) + \left(1 - \omega_t^i\right) \exp\left(r_t^f\right) \right). \end{aligned}$$

Agents start each period with wealth A_t^i and decide how much to consume, c_t, and how much wealth to carry to the end of the period, a_t^i. They allocate a fraction of this end-of-period wealth to the "market portfolio," ω_t^i, and the residual fraction to the risk-free asset. This portfolio provides them with their next period wealth, A_{t+1}^i.

We formally state and solve the investors' problem in Appendix A.2. Because $EIS = 1$, agents spend a fixed fraction of their wealth in every period,

$$c_t^i = \left(1 - e^{-\rho}\right) A_t^i \quad \text{and} \quad a_t^i = e^{-\rho} A_t^i. \tag{4}$$

Consider agents' optimal portfolio choice. Starting from period 1 onward, there is no uncertainty and agents are indifferent between the two assets,

$$r_t = r_t^f \quad \text{for } t \geq 1. \tag{5}$$

In period 0, agents' weight on the market portfolio is approximately given by

$$\omega_0^i \sigma \simeq \tau^i \frac{\overline{r}_0 - r_0^f}{\sigma} \quad \text{where } \overline{r}_0 = E\left[r_0\right] + \frac{\sigma^2}{2}. \tag{6}$$

Here, $\overline{r}_0$ denotes the log of the expected gross return on the market portfolio. Eq. (6) is a standard mean-variance portfolio optimality condition that says the risk of agents' optimal portfolio (the left side) is proportional to the Sharpe ratio on the market portfolio (the right side). This equation holds exactly in continuous time but only approximately in discrete time. To simplify the analysis, we assume the equation is exact.[10]

The asset market clearing conditions are given by

$$\sum_i a_t^i = \sum_i \omega_t^i a_t^i = z_t P_t. \tag{7}$$

At the end of every period, aggregate wealth is equal to the (ex-dividend) price of the market portfolio, both before and after agents' portfolio decisions.

Finally, there is a central bank that manages demand by setting the nominal interest rate (which is the same as the real interest rate since prices are fully sticky). We assume the central bank sets the interest rate to replicate the supply-determined output level, subject to a lower

[10] Specifically, agents approximate their portfolio problem with problem ($A.14$) stated in the appendix, which leads to the optimality condition (6). This approximation works well for relatively short investment horizons and is widely used in the literature (see, e.g., Campbell and Viceira (2002)).

bound constraint, $r_t^f \geq 0$. Specifically, suppose the interest rate policy follows a standard Taylor rule, $r_t^f = \max(0, \psi(y_t - z_t))$. We focus on the limit $\psi \to \infty$, in which case this rule implies that either the interest rate is positive and output is at its potential, $r_t^f = r_t^{f*} > 0$ and $y_t = z_t$; or the interest rate is constrained and there is a demand recession, $r_t^f = 0$ and $y_t \leq z_t$. Here, r_t^{f*} ("rstar") denotes the natural interest rate consistent with making output equal to potential output, $y_t = z_t$ [see Eqs. (11) and (20)].

Given agents' initial positions, A_0^i, the equilibrium corresponds to a path of allocations and prices that satisfy Eqs. $(1-7)$ along with the central bank's policy rule. We next characterize the equilibrium in three steps. First, we describe a tight relationship between asset prices and output that applies in each period. We then describe the equilibrium in periods $t \geq 1$. Finally, we specify agents' initial positions, A_0^i, and characterize the equilibrium in period 0. In the rest of the section, we investigate how supply shocks in period 0 affect this equilibrium.

Output-asset price relation. Eq. (4) implies that consumption is a fraction of agents' end-of-period wealth

$$c_t^i = \frac{1-e^{-\rho}}{e^{-\rho}} a_t^i, \quad \text{which implies} \quad c_t \equiv \sum_i c_t^i = \frac{1-e^{-\rho}}{e^{-\rho}} z_t P_t. \tag{8}$$

Here, we have defined aggregate consumption, c_t, and used the asset market clearing condition (7). Combining this with Eq. (2), we further obtain,

$$y_t = c_t = \frac{1-e^{-\rho}}{e^{-\rho}} z_t P_t. \tag{9}$$

We refer to this equation as *the output-asset price relation*. This relation says that higher asset prices increase aggregate wealth and consumption, which leads to greater output (see Remark 2 for discussion and various extensions).

Setting $y_t = z_t$ in (9), we also solve for the efficient level of asset price per productivity as

$$P^* = \frac{e^{-\rho}}{1-e^{-\rho}}. \tag{10}$$

This is the asset price per unit of productivity that ensures output is at its potential. If there is a supply shock that reduces z_t, asset prices should fall proportionally to $z_t P^*$, but no further. Any further reduction in asset prices would trigger a demand recession as illustrated by (9).

Equilibrium in periods $t \geq 1$. We next solve the equilibrium backward. Suppose agents start period 1 with wealth levels, A_1^i. Appendix A.2.2 characterizes the continuation equilibrium as follows,

$$
\begin{aligned}
y_t &= z_t \quad \text{with} \quad z_t = z_1 e^{g(t-1)} \\
r_t &= r_t^{f*} = \rho + g > 0 \\
P_t &= P^* \\
c_t^i &= \left(1 - e^{-\rho}\right) A_t^i \quad \text{with} \quad A_t^i = A_1^i e^{g(t-1)}.
\end{aligned} \tag{11}
$$

Since there is no uncertainty, the economy immediately reaches a deterministic balanced growth path in which output is equal to its potential and grows at a constant rate [(1)]. Agents' wealth and their consumption grow at the same rate as productivity. The interest rate is positive and the asset price per productivity is at its efficient level. Intuitively, the interest rate constraint does not bind and the central bank ensures the economy operates at its potential.

Initial endowments. Our focus is on the equilibrium in period 0, which depends on (among other things) agents' initial endowments, A_0^i. In a model with endogenous history, risk tolerant banks would take a levered position on the market portfolio in the (unmodeled) period -1, with a leverage ratio that would depend on relative risk tolerances and anticipated shocks. Therefore, we endow banks with initial positions that feature leverage:

$$
A_0^b = \max\left(0, \tilde{A}_0^b\right) \quad \text{and} \quad A_0^h = \min\left(y_0 + z_0 P_0, \tilde{A}_0^h\right), \tag{12}
$$

where

$$
\begin{aligned}
\tilde{A}_0^b &= \kappa\left(y_0 + z_0 P_0\right) - \kappa l\left(1 + P^*\right) \\
\tilde{A}_0^h &= (1-\kappa)\left(y_0 + z_0 P_0\right) + \kappa l\left(1 + P^*\right),
\end{aligned} \tag{13}
$$

for some $\kappa, l \in (0,1)$.

Eq. (13) describes banks' endowments and wealth assuming they are not bankrupt. Banks initially hold a fraction of the market portfolio, κ. They also owe $\kappa l\left(1 + P^*\right)$ units of safe debt. We have normalized the debt level so that in *the benchmark*, defined as the case when there is no demand recession, $P_0 = P^*$, and the supply shock is normalized to one, $z_0 = 1$, banks' leverage ratio (defined as their debt-to-asset ratio) is l. Households hold the mirror image positions: they hold the residual fraction of the market portfolio, $1 - \kappa$, as well as banks' safe debt. Eq. (12) adjusts agents' wealth for the possibility of bankruptcy. When $\tilde{A}_0^b < 0$, the value of banks' assets is less than their outstanding debt. In this case, banks are bankrupt and their actual wealth is zero, $A_0^b = 0$. Households take over banks' assets and hold all of the market portfolio, so their wealth becomes $A_0^h = y_0 + z_0 P_0$.

Equilibrium in period 0. Next consider the characterization of the equilibrium asset price, $z_0 P_0$. To facilitate the analysis, we define banks' (end-of-first-period) wealth share as

$$\alpha \equiv \frac{a_0^b}{z_0 P_0}.$$

Households' wealth share is the residual, $1-\alpha \equiv \frac{a_0^h}{z_0 P_0}$. Using this notation, we can write the asset market clearing condition (7) as

$$\alpha \omega_0^b + (1-\alpha)\, \omega_0^h = 1. \tag{14}$$

The equilibrium asset price is determined by this condition together with agents' wealth shares, $\alpha, 1-\alpha$, and their optimal portfolio weights, ω_0^b, ω_0^h.

To calculate the wealth shares, we use the output-asset price relation in (9) together with agents' initial positions in (12) and their optimal saving rule in (4). For banks' wealth share, we obtain

$$\alpha = \alpha(z) \equiv \max\left(0, \left(1 - \frac{l}{z}\right)\kappa\right) \quad \text{where} \quad z = z_0 \frac{P_0}{P^*}. \tag{15}$$

To understand this expression, first consider the benchmark with $P_0 = P^*$ and the supply shock normalized to one, $z_0 = 1$. In this benchmark, $z = 1$ and banks' wealth share is given by $\alpha = (1-l)\,\kappa$: their initial share of assets net of their leverage. Now suppose asset valuations fall, $z = z_0 (P_0/P^*) < 1$, either because of a decline in productivity, z_0, or a decline in the asset price per productivity, P_0. This decline causes banks' wealth share to fall below the benchmark (and households' wealth share increases above the benchmark). Intuitively, since banks are levered, a decline in asset valuations reduces their wealth more than it reduces households' wealth. This mechanism will play an important role for our results. If asset valuations decline beyond banks' initial leverage, $z = z_0 (P_0/P^*) < l$, banks are bankrupt and their wealth share falls to zero.

To calculate the optimal portfolio weights, first note that Eqs. (3) and (9) imply the (log) return on the market portfolio is given by $r_0 = \rho + \log\left(\frac{z_1 P^*}{z_0 P_0}\right)$. Combining this with Eq. (1), we solve for the (log) expected return,

$$\overline{r}_0 = E[r_0] + \frac{\sigma^2}{2} = \rho + \log\frac{\overline{z}_1}{z_0} - \log\left(\frac{P_0}{P^*}\right). \tag{16}$$

Substituting this return into Eq. (6), we obtain

$$\omega_0^i \sigma = \tau^i \frac{\rho + \log\frac{\overline{z}_1}{z_0} - \log\left(\frac{P_0}{P^*}\right) - r_0^f}{\sigma}. \tag{17}$$

Combining Eqs. $(14-17)$, we arrive at the central equation of our analysis, *the risk balance condition*:

$$\frac{\sigma}{\tau(z_0 p_0)} = \frac{\rho + \log\frac{\overline{z}_1}{z_0} - \log(p_0) - r_0^f}{\sigma}, \tag{18}$$

where p_0 denotes the *normalized asset price* per productivity,

$$p_0 \equiv \frac{P_0}{P^*} \in [0, 1],$$

while $\tau(\cdot)$ corresponds to *the effective risk tolerance* function,

$$\begin{aligned} \tau(z) &\equiv \alpha(z)\tau^b + (1 - \alpha(z))\tau^h \\ &= \max\left(\tau^h, \tau^h + \left(1 - \frac{l}{z}\right)\kappa\left(\tau^b - \tau^h\right)\right). \end{aligned} \tag{19}$$

Eq. (18) says that the risk the economy generates normalized by the effective risk tolerance (the left side) should be compensated by a sufficiently high reward per unit of risk (the right side). Specifically, the right side is *the actual (expected) Sharpe ratio* on the market portfolio: the risk premium per unit of risk. In the rest of the paper, we refer to the expression on the left side as *the required Sharpe ratio*, and note that the equilibrium in risk markets obtains when the actual and required Sharpe ratios are equal.

Eq. (19) illustrates that effective risk tolerance depends on a wealth-weighted average of investors' risk tolerances. The second line, which uses Eq. (15), solves for the effective risk tolerance and shows that it is increasing in $z = z_0 p_0$. In particular, a decline in asset prices—either through reduced productivity, z_0, or reduced valuation per productivity, p_0—reduces the effective risk tolerance. Lower asset prices reduce banks' wealth share, which lowers the effective risk tolerance since $\tau^b > \tau^h$. If banks go bankrupt, the effective risk tolerance is the households' tolerance, $\tau(z) = \tau^h$.

The equilibrium normalized price, $p_0 = \frac{P_0}{P^*}$, and the risk-free rate, r_0^f, are then determined by the risk balance condition (18) (solved with the endogenous risk tolerance in (19)) and the monetary policy rule. Given the asset price, P_0, the equilibrium output is determined by the output-asset price relation (9).

Remark 1 (Partially sticky prices). *While we assume fully sticky prices, the analysis naturally extends to the case in which a fraction of firms adjust their prices in each period. In this case, aggregate output is still determined by aggregate demand [see* (2)*] and the central bank still manages demand by setting the nominal interest rate (which still influences the real interest rate). The main difference is that, when there is a demand recession in period* 0*, the equilibrium also features some disinflation—firms that get to adjust in period* 0 *cut their nominal prices (since recession lowers their marginal costs). Moreover, this downward price adjustment does not* necessarily *mitigate the demand recession. Adjusting firms increase their demand at the expense of other firms, but* aggregate demand *is ultimately determined by monetary policy. In fact, if the central bank follows a standard inflation targeting policy (once the economy recovers), partial flexibility can induce agents to* expect disinflation*, which can further* tighten *the lower bound on the real interest rate (see Caballero and Simsek (2020) for further discussion).*

Remark 2 (Output-asset price relation)**.** *The output-asset price relation can also be interpreted as a reduced form for various channels that link asset prices and aggregate demand. For example, suppose we split consumers (and income) between our agents (share γ) and a group of hand-to-mouth consumers (share $1-\gamma$). Then, our agents' consumption is still given by* (8) *and aggregate consumption is given by*

$$c_t = \gamma \frac{1-e^{-\rho}}{e^{-\rho}} z_t P_t + (1-\gamma)\, y_t.$$

Using $y_t = c_t$, we once again obtain Eq. (9)*. In Appendix B, we show that a version of this equation also holds in an extension in which there is a non-capitalized factor (such as labor), and hand-to-mouth households hold all of the income from that factor whereas our banks and households hold all of the market portfolio.*[11] *In Caballero and Simsek (2020) we show that adding investment also leaves the output-asset price relation qualitatively unchanged (due to a Q-theory mechanism); and in Section 4 we show that adding a corporate debt overhang problem strengthens the relation (output becomes even more sensitive to asset prices).*

1.2. Partly temporary supply shocks

We next consider the comparative statics of partly temporary supply shocks—a reduction in z_0 with partial anticipated recovery, $\log \overline{z}_1 = g + \varphi \log z_0$ [see (1)]. An example is the supply component of the Covid-19 shock. In this context, we illustrate how, when banks' effective leverage is sufficiently high, supply shocks *reduce* aggregate demand by *more* than the aggregate decline in potential output (i.e., $y_0 < z_0$, for an unchanged interest rate), and hence induce the central bank to cut the interest rate.

First suppose there is no lower bound on the interest rate. In this case, the central bank always ensures output is equal to its potential, $r_0^f = r_0^{f*}$ and $y_0 = z_0$. This outcome requires the asset price per productivity to be at its efficient level, $p_0 = \frac{P_0}{P^*} = 1$ [see (10)]. Combining this with Eqs. (18) and (1), we solve for the output-stabilizing interest rate ("rstar"),

$$r_0^{f*} = \rho + \log \frac{\overline{z}_1}{z_0} - \frac{\sigma^2}{\tau(z_0)} \text{ where } \log \frac{\overline{z}_1}{z_0} = g - (1-\varphi) \log z_0. \tag{20}$$

Consider a decline in z_0 in the case with $\varphi < 1$. Eq. (20) illustrates that this decline exerts two effects on "rstar." On the one hand, a decline in z_0 increases the expected capital gain, $\frac{\overline{z}_1}{z_0}$, which increases "rstar." Intuitively, while supply and asset prices are currently low, they are expected to partly recover. The expected recovery raises the actual Sharpe ratio and induces agents to invest in the market portfolio [see (17)], which exerts upward pressure on the asset

[11] Specifically, we obtain $y_t = c_t = \frac{1-e^{-\rho}}{e^{-\rho}} y_t^* P_t$ [see (B.1)], where $y_t^* = (n+1)\, z_t$ denotes the potential output that combines the fraction that accrues to the non-capitalized factor (nz_t) and the capitalized factor (z_t). This extension is useful for calibration purposes since in practice a large share of income accrues to labor or other non-capitalized factors. Accounting for this income via the parameter n provides additional flexibility in matching the aggregate ratio of capitalized wealth to consumption, $\frac{z_t P_t}{c_t}$ (see Appendix B).

price per productivity. That is, the central bank needs to *increase* the interest rate to keep asset prices at the efficient level. On the other hand, a decline in z_0 also reduces banks' wealth share [see (15)], which decreases "rstar." Since banks are levered, a decline in asset valuations reduces their wealth more than households' wealth. This decreases effective risk tolerance and puts downward pressure on the asset price per productivity. That is, the central bank needs to *cut* the interest rate to keep asset prices at the efficient level.

The second channel dominates (locally), $\frac{dr_0^{f*}}{dz_0} > 0$, as long as the parameters satisfy $z_0 > l$ (no bankruptcy) and

$$\frac{l/z_0}{\frac{\tau^h/\kappa}{\tau^b-\tau^h} + 1 - l/z_0} > (1-\varphi)\frac{1}{\overline{r}_0 - r_0^{f*}}. \tag{21}$$

Here, we have substituted the risk premium, $\overline{r}_0 - r_0^{f*} = \frac{\sigma^2}{\tau(z_0)}$ [see (16) and (20)]. As we will see, the inverse of the risk premium, $\left(\overline{r}_0 - r_0^{f*}\right)^{-1}$, controls the asset demand elasticity in our model (this is in fact its *functional role* in the model—see Section 3.1). Therefore, Eq. (21) says that supply shocks are more likely to reduce aggregate demand by more than the decline in potential output when: (i) the asset demand elasticity is relatively low, (ii) the shock is more persistent (higher φ), (iii) agents' risk tolerance is more heterogeneous (greater $\frac{\tau^b}{\tau^h}$), (iv) banks have greater initial leverage (greater l), and (v) the shock is more severe (lower z_0).

For the rest of the paper, we isolate our leverage mechanism and simplify the analysis by focusing on permanent supply shocks, $\varphi = 1$, so that $\log \overline{z}_1 = \log z_0 + g$. The (unconstrained) risk-free interest rate is then given by [cf. (20)]

$$r_0^{f*} = \rho + g - \frac{\sigma^2}{\tau(z_0)}. \tag{22}$$

In this case a decrease in z_0 always (weakly) reduces aggregate demand and the interest rate. This simplifies the equations but is not necessary for our results: we could have instead worked with parameters that satisfy (21). In Appendix B, we calibrate our model and show that condition (21) is plausible. The condition reduces to a joint restriction between the severity of the shock and its persistence. When the shock is mildly persistent ($\varphi \simeq 20\%$), the condition holds for *arbitrarily small shocks* (for each $z_0 \leq 1$) in our baseline calibration.

1.3. Supply shocks and asset price spirals

We next consider the case where there is a lower bound on the interest rate. In this case, the supply shock *can* cause a demand recession. We assume the parameters satisfy [see (19)]

$$\tau(1) \geq \frac{\sigma^2}{\rho + g} > \tau^h. \tag{23}$$

The first inequality ensures that when the supply is equal to its benchmark level, $z_0 = 1$, there is an equilibrium with an unconstrained (positive) interest rate. The second inequality ensures that, if households control all the wealth in the economy, the interest rate is constrained (zero).

To characterize the equilibrium more generally, we define two cutoffs for productivity that we denote with z^h and z^*. Below the first cutoff, z^h, there is an equilibrium where banks go bankrupt and households control all wealth. To calculate this cutoff, suppose there is bankruptcy. Using the risk balance condition (18) with $\tau = \tau^h$ (and $r_0^f = 0$), we obtain

$$p^h \equiv \exp\left(\rho + g - \frac{\sigma^2}{\tau^h}\right) < 1. \tag{24}$$

Note that p^h is the minimum normalized asset price. Suppose the normalized price falls to this level, $p_0 = p^h$. Then, Eq. (15) implies banks will indeed go bankrupt as long as productivity is sufficiently low:

$$z_0 < z^h \equiv \frac{l}{p^h}. \tag{25}$$

When $z_0 < z^h$, there is always a bankruptcy equilibrium. Note that the cutoff z^h is increasing in l: bankruptcy is more likely when banks have greater initial leverage.

The second cutoff, z^*, is the productivity level above which there is a supply determined equilibrium with the efficient price $p_0 = 1$. To calculate this cutoff, we use the risk balance condition (18) with $p_0 = 1$ and $r_0^f = 0$ to obtain the value of $z^* < 1$ that solves

$$\tau(z^*) = \frac{\sigma^2}{\rho + g}. \tag{26}$$

When $z_0 > z^*$, there is always an equilibrium with the efficient asset price. Our main result in this section characterizes the equilibrium for arbitrary z_0.

Proposition 1. *Consider the equilibrium with condition* (23)*. Let z^h and z^* denote the cutoffs defined by Eqs.* (25) *and* (26)*.*

(i) If $z_0 > z^h$, then the equilibrium is unique and does not feature bankruptcy. If $z_0 \in (z^h, z^*)$ (assuming the interval is nonempty), then the equilibrium features an interior asset price, $p_0 \in (p^h, 1)$, that solves

$$\frac{\sigma}{\tau^h + \left(1 - \frac{l}{z_0 p_0}\right)\kappa\left(\tau^b - \tau^h\right)} = \frac{\rho + g - \log p_0}{\sigma}. \tag{27}$$

Reducing productivity reduces the equilibrium price per productivity, $\frac{dp_0}{dz_0} > 0$. If $z_0 \geq z^*$ (as well as $z_0 > z^h$), the equilibrium features the efficient asset price, $p_0 = 1$.

(ii) If $z_0 \leq z^h$, then there is a bankruptcy equilibrium with the low asset price, $p_0 = p^h < 1$. There might also be other equilibria. When $z_0 \in [z^*, z^h]$ (assuming the interval is nonempty), there is also an equilibrium with the efficient asset price, $p_0 = 1$.

Proof. See Appendix A.2.4. □

The first part of Proposition 1 shows that the equilibrium is unique as long as the shock

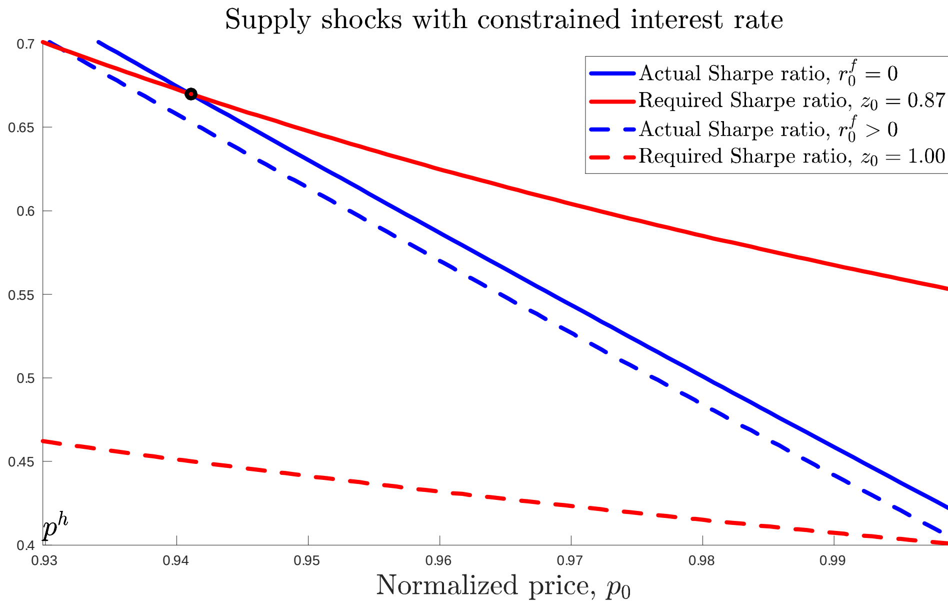

Figure 2: Effect of supply shocks when the interest rate is constrained—the case with a unique equilibrium.

is not severe enough to trigger bankruptcy ($z_0 > z^h$). In this region, when the supply shock is below a cutoff ($z_0 < z^*$), the equilibrium features a demand recession. More severe supply shocks lead to lower asset prices and more severe demand recessions. As we will see below, these supply shocks also generate downward spirals and have an amplified effect on asset prices and aggregate demand. The second part of Proposition 1 shows that, when the shock is severe enough to trigger bankruptcy ($z_0 < z^h$), these amplification mechanisms can lead to multiple equilibria.

To illustrate the first part, consider parameters that lead to a unique and interior equilibrium price, $z^h < z_0 < z^*$. Substituting $r_0^f = 0$ in the risk balance condition (18), we find that the normalized price solves Eq. (27). This equation has a natural interpretation. The right side is *the actual Sharpe ratio* (with constrained interest rate $r_0^f = 0$). It is decreasing in p_0: lower asset prices increase the risk premium and the Sharpe ratio. The left side is *the required Sharpe ratio* (assuming there is no bankruptcy). It is also decreasing in p_0: lower asset prices transfer (relative) wealth from banks to households, which reduces effective risk tolerance and requires a greater Sharpe ratio for agents to absorb the risk.

Figure 2 plots these curves and the resulting equilibrium for a particular parameterization that satisfies $z^h < z^*$. The dashed lines correspond to the benchmark productivity level, $z_0 = 1$, which satisfies $z_0 > z^*$ (by assumption). In this benchmark case, there is a corner equilibrium in which the asset price is efficient, $p_0 = 1$, and the interest rate is positive, $r_0^f > 0$. The solid lines consider a lower productivity level, $z_0 \in \left(z^h, z^*\right)$, and illustrate how a supply shock can generate severe downward spirals in asset prices. Starting from the benchmark, a decline in productivity reduces asset prices and effective risk tolerance. This shifts the curve for the

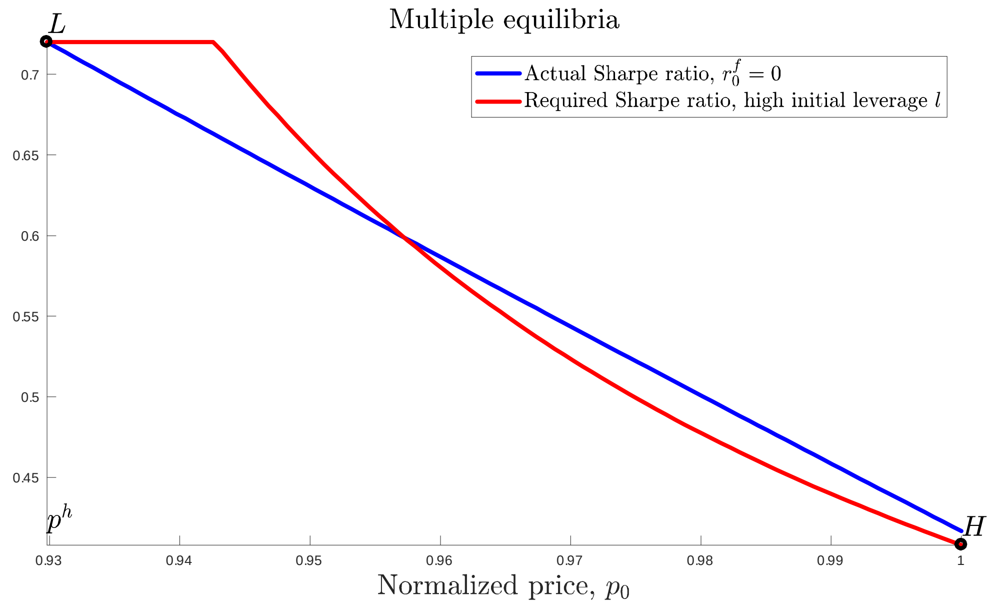

Figure 3: Effect of supply shocks when the interest rate is constrained—the case with multiple equilibria.

required Sharpe ratio upward. The central bank reacts by cutting interest rates, which increases the actual Sharpe ratio, but the central bank encounters a lower bound constraint, $r_0^f = 0$. When the risk free rate cannot fall any farther, asset prices and aggregate demand decline more than the decline in productivity—in order to increase the actual Sharpe ratio. The reduction in asset prices further damages banks' balance sheets and increases the required Sharpe ratio, which further reduces asset prices, and so on.

The figure also illustrates that, due to this amplification mechanism, the Sharpe ratio rises more than the initial impact of the shock (captured by the vertical shift from the dashed red line to the solid red line). Consequently, the asset price falls considerably more than the direct effect of the negative supply shock. Moreover, there is greater amplification when risk tolerance and the required Sharpe ratio are more sensitive to asset prices (when the solid red line is steeper), which happens with greater l or lower z_0 [see Eq. (27)]. Hence, supply shocks induce a larger contraction in asset prices and aggregate demand when banks have greater leverage or when the shock is more severe.

To illustrate the second part of Proposition 1, consider parameters that allow for bankruptcy and multiple equilibria, $z^* < z_0 < z^h$. Since $z_0 < z^h$, there is a bankruptcy equilibrium with the lowest asset price, $p_0 = p^h$. However, since $z_0 > z^*$, there is also an equilibrium with the efficient asset price, $p_0 = 1$.

Figure 3 illustrates these equilibria by plotting the required and the actual Sharpe ratio curves. The high- and the low-price equilibria are marked with H and L, respectively.[12] Starting

[12] There is also an interior equilibrium that corresponds to the intersection of the two curves. However, this equilibrium is unstable: small price deviations would bring the equilibrium to either H or L.

from the high-price equilibrium H, a decline in asset prices weakens banks' balance sheets substantially, which rapidly raises the required Sharpe ratio. This in turn reinforces the large fall in asset prices and culminates in the low-price equilibrium L that features bankruptcy. As this discussion suggests, multiplicity is more likely when banks have greater leverage. In fact, the parameters used in Figure 3 are the same as those used in Figure 2, with the difference that we raise banks' initial leverage l (and also adjust banks' risk tolerance τ^b to keep the benchmark risk tolerance $\tau(1)$ unchanged).

Remark 3 (Adding demand shocks)**.** *In our analysis we focus on the endogenous response of asset prices and aggregate demand to a large supply shock. However, most recessions (including the Covid-19 recession) are driven by a complex combination of supply and demand shocks. There are at least three ways to introduce demand shocks into our framework. First, as in Caballero and Simsek (2020), agents' risk perception, σ, may rise. Second, consumers may become more conservative and lower their discount rate, ρ (increase saving). Third, consumers may become more pessimistic about growth, g, as in Lorenzoni (2009); Caballero and Simsek (2021b). Eq.* (22) *illustrates that all these channels put direct downward pressure on r_0^{f*}, which translates into a larger aggregate demand recession once r_0^f reaches the lower bound.*

2. Large-scale Asset Purchases

The downward spiral caused by the endogenous decline in risk tolerance suggests that policy interventions that absorb some risky assets during severe risk-off events can be powerful. We now introduce unconventional monetary policy in the form of large-scale asset purchases (LSAPs) and demonstrate their effectiveness in reversing the spiral. We also briefly discuss the determinants of optimal LSAPs.

Modeling LSAPs requires introducing a fiscal authority: even if the asset purchases are made by the central bank, the gains and losses from these positions ultimately accrue to the treasury. We merge the fiscal and monetary authorities into a third agent which we refer to as *the government* and denote by superscript g.

Formally, the government is endowed with no resources in period 0. In each future period $t \geq 1$, the government has resources given by $y_t \eta^g$. We think of these resources as the government's future tax capacity. They can be "microfounded" by introducing a group of agents other than banks and households (e.g., the future generation) from which the government will be able to extract some taxes. We assume future tax capacity is proportional to future output, which simplifies the analysis but is not necessary for our results (in fact, making the government's tax capacity safer would strengthen our results).

With the proportionality assumption, the government can equivalently be thought of as being endowed with η^g units of the market portfolio at the end of period 0. In particular, the government starts with wealth

$$a_0^g = z_0 P_0 \eta^g. \tag{28}$$

In period 0, the government chooses the fraction of its wealth to allocate to the market portfolio, ω_0^g. This determines the government's wealth in period 1,

$$A_1^g = a_0^g \left(\omega_0^g \exp(r_t) + (1 - \omega_0^g) \exp r_t^f \right).$$

We restrict attention to portfolio allocations that satisfy $\omega_0^g \geq 1$. Choosing $\omega_0^g = 1$ replicates the government's initial endowment. The government is already fully exposed to the market portfolio through its future tax revenues, and it can further increase its exposure by borrowing and investing in risky assets.

The presence of the government changes the asset market clearing condition [cf. (7)]:

$$\sum_{i \in \{b,h,g\}} \omega_0^i a_0^i = z_0 P_0 (1 + \eta^g). \tag{29}$$

The right side illustrates that the government's tax capacity implicitly expands the supply of the market portfolio. The left side illustrates that the government also expands demand.

In subsequent periods $t \geq 1$, the government chooses its spending and asset holdings subject to flow budget constraints similar to the agents [see (A.23) in the appendix]. Since our focus is in period 0, we keep the equilibrium in future periods simple. Specifically, the government maintains a constant-growth spending path (like the agents). With these assumptions the balanced growth path equilibrium in (11) generalizes to this case (see Appendix A.3.2).

Given a government portfolio choice in the initial period, $\omega_0^g \geq 1$, our definition of equilibrium generalizes in straightforward fashion. In the rest of the section, we characterize the equilibrium and the asset price impact of LSAPs. At the end of the section, we discuss the determinants of optimal LSAPs.

2.1. Equilibrium with large-scale asset purchases

Investors' optimality conditions are the same. Therefore much of the earlier analysis applies in this setting. Specifically, Eqs. (9) , (15), and (17) still hold. Using Eq. (29), we obtain an analogue of the market clearing condition (14):

$$\alpha \omega_0^b + (1 - \alpha) \omega_0^h + \eta^g \omega_0^g = 1 + \eta^g. \tag{30}$$

Combining these observations, we obtain an analogue of the risk balance condition (18):

$$\frac{\sigma (1 - \lambda)}{\tau (z_0 p_0)} = \frac{\rho + g - \log (p_0) - r_0^f}{\sigma}, \tag{31}$$

where $\lambda \equiv \eta^g (\omega_0^g - 1)$ and $\tau(z)$ is given by the same expression as before [see (19)].

Eq. (31) illustrates that LSAPs effectively take some risk out of the market. Specifically, the risk balance condition is equivalent to an economy in which the risk is reduced by a fraction λ.

How much risk LSAPs remove depends on the government's tax capacity, η^g, and the riskiness of its portfolio, $\omega_0^g \geq 1$. When $\eta^g = 0$ or $\omega_0^g = 1$ the policy does not reduce risk, and the risk balance condition (and the equilibrium) is the same as before. In subsequent analysis, we refer to λ as the size of the LSAP program.

In this context, first consider the equilibrium when the interest rate constraint does not bind. Substituting $p_0 = 1$ into (31), we solve for the unconstrained interest rate [cf. Eq. (22)]:

$$r_0^{f*} = \rho + g - \frac{\sigma^2}{\tau(z_0)}(1-\lambda).$$

When the interest rate is not constrained, LSAPs do not affect asset prices, $z_0 P^*$, or output, $y_0 = z_0$, but they translate into higher interest rates. As LSAPs take risk out of the market, they exert upward pressure on asset valuations and aggregate demand. Conventional monetary policy responds by raising the interest rate to keep asset prices and aggregate demand consistent with potential output.

We next consider the case in which the interest rate can be constrained and generalize Proposition 1. We assume the following analogue of (23):

$$\tau(1) \geq \frac{\sigma^2}{\rho+g} > \frac{\tau^h}{1-\lambda}. \tag{32}$$

As before, we also define two cutoff productivity levels, $z^h(\lambda), z^*(\lambda)$. Let

$$z^h(\lambda) = \frac{l}{p^h(\lambda)} \quad \text{where} \quad p^h(\lambda) = \exp\left(\rho + g - \frac{\sigma^2(1-\lambda)}{\tau^h}\right) < 1. \tag{33}$$

Here, $z^h(\lambda)$ is the cutoff productivity below which there is a bankruptcy equilibrium, and $p^h(\lambda)$ is the normalized price in a bankruptcy equilibrium [cf. $(24-25)$]. Increasing λ increases the normalized price, $p^h(\lambda)$, and decreases the cutoff, $z^h(\lambda)$: LSAPs increase the worst-case asset price level and shrink the set of productivity realizations that allow for bankruptcy.

Let $z^*(\lambda) \in (0,1)$ denote the unique solution to

$$\frac{\tau(z^*)}{1-\lambda} = \frac{\sigma^2}{\rho+g}. \tag{34}$$

As before, $z^*(\lambda)$ is the cutoff productivity above which there is a supply determined equilibrium with the efficient price [cf. (26)]. LSAPs expand the set of productivity realizations that allow for an efficient price equilibrium. The next result characterizes the equilibrium when it is unique and interior. The case with multiple equilibria is similar to Proposition 1.

Proposition 2. *Consider the equilibrium with LSAPs,* $\lambda = \eta^g(\omega_0^g - 1) \geq 0$*, and conditions (32). Suppose* $z_0 \in \left[z^h(\lambda), z^*(\lambda)\right]$ *given the cutoffs in Eqs.* $(33-34)$*. There exists a unique*

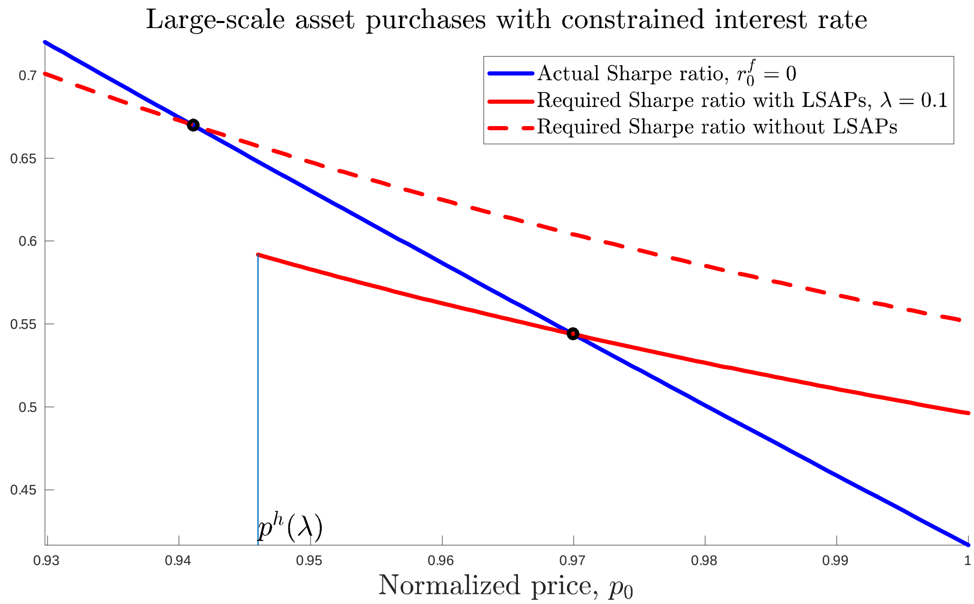

Figure 4: Effect of LSAPs when the interest rate is constrained—the case with a unique equilibrium.

equilibrium with an interior normalized price, $p_0 \in \left(p^h(\lambda), 1\right)$*, that solves*

$$\frac{\sigma(1-\lambda)}{\tau^h + \left(1 - \frac{l}{z_0 p_0}\right)\kappa\left(\tau^b - \tau^h\right)} = \frac{\rho + g - \log(p_0)}{\sigma}. \tag{35}$$

The normalized price is increasing in the size of the LSAP program, $\frac{dp_0}{d\lambda} > 0$*.*

Proof. See Appendix A.3.3. □

Consider Figure 4, which illustrates Eq. (35). LSAPs shift the required Sharpe ratio curve downward without affecting the actual Sharpe ratio curve. In equilibrium, this shift leads to a lower Sharpe ratio and a higher asset price. In fact, LSAPs have an *amplified* effect on the Sharpe ratio: the change in the equilibrium Sharpe ratio is much greater than the initial downward shift of the curve. As LSAPs increase asset prices, they improve banks' balance sheets, which further reduces the required Sharpe ratio and raises asset prices. Essentially, LSAPs help undo the downward spirals created by supply shocks illustrated in Figure 2.

LSAPs can have even more powerful effects when there are multiple equilibria. Figure 5 illustrates this by plotting the effect of LSAPs for parameters that lead to multiplicity. The dashed red line illustrates the risk premium curve without LSAPs, which leads to multiple equilibria (denoted by L and H in the figure). The solid red line illustrates the effect of LSAPs of the same size as in the previous case (with a unique equilibrium). In this case, LSAPs eliminate the low-price equilibrium. By removing risk from the market, the policy reduces the required Sharpe ratio and increases asset prices, which triggers a virtuous spiral that culminates

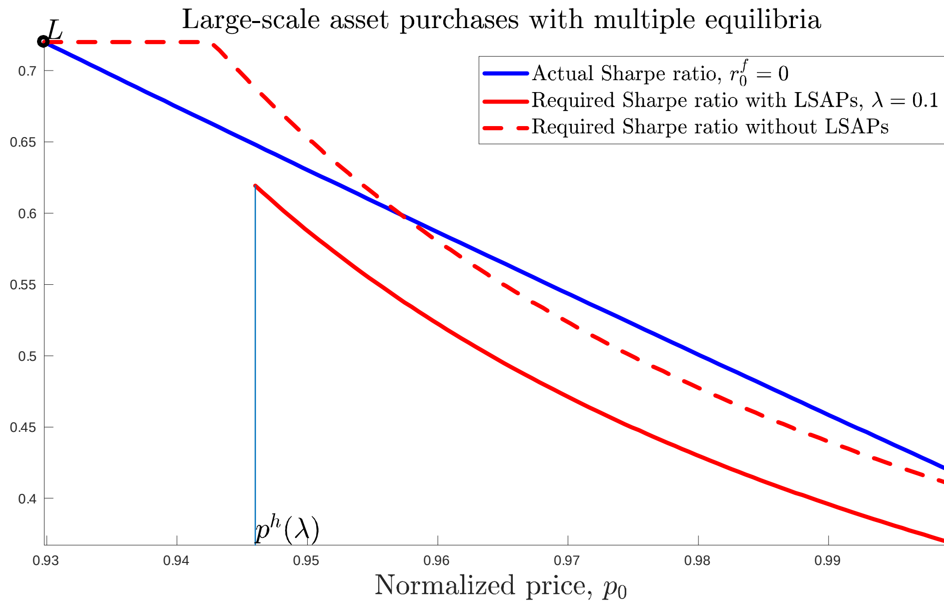

Figure 5: Effect of LSAPs when the interest rate is constrained—the case with multiple equilibria.

in the high-price equilibrium (denoted by H' in the figure).

2.2. Optimal LSAPs

When should central banks engage in greater LSAPs? We address this question in Appendix A.4. Here, we summarize the results. We endow the government with an Epstein-Zin utility function over its future spending, $\{c_t^g\}_{t=1}^{\infty}$, with its own risk tolerance parameter, τ^g. We then set up a constrained Pareto planning problem in a simpler version of the model in which we collapse the banks and households into a single representative agent (the market), taking the market's risk tolerance parameter as $\tau(1)$—the benchmark effective risk tolerance. This results in the following policy problem:

$$\max_{\lambda \geq 0} \left(1 - e^{-\rho}\right) \log\left(p_0(\lambda)\right) - \frac{1}{2} e^{-\rho} \sigma^2 \left(\eta^g \frac{1}{\tau^g} \left(1 + \frac{\lambda}{\eta^g}\right)^2 + \frac{1}{\tau(1)} (1 - \lambda)^2\right). \tag{36}$$

The government balances three terms. The first term, $(1 - e^{-\rho}) \log(p_0(\lambda))$, captures the government's desire to close the output gap in period 0. In our model, this is equivalent to closing asset price gaps [see (9)]. The second term captures disutility from the risk in the government's portfolio. This disutility depends on the government's risk tolerance, τ^g, and its leverage, $\omega_0^g = 1 + \frac{\lambda}{\eta^g}$. The last term captures the disutility from risk in the representative agent's (the market's) portfolio. Hence, the government trades off macroeconomic stabilization objectives with the optimal allocation of risk.

We characterize the optimal LSAPs assuming the government is weakly less risk tolerant than the market, $\tau^g \leq \tau(1)$. This ensures that, absent macroeconomic stabilization benefits (when $p_0'(\lambda) = 0$), the government would *not* use LSAPs, $\lambda = 0$. Nonetheless, in a demand recession the government might use LSAPs since they raise asset prices and output, $p_0'(\lambda) > 0$. Problem (36) implies that optimal LSAPs are greater when the government has greater risk tolerance (τ^g) and greater tax capacity (η^g). Greater capacity helps because it enables the government to achieve the same impact on financial markets with less leverage on its own portfolio ($\omega_0^g = 1 + \frac{\lambda}{\eta^g}$).

Importantly, problem (36) illustrates that optimal LSAPs depend on the marginal price impact of LSAPs, $\frac{d\log(p_0)}{d\lambda}$. This price impact is greater when the required Sharpe ratio is more sensitive to asset prices (when the solid red line in our figures is steeper), which happens with greater l or lower z_0 (see Proposition 3 in the next section for an analytical characterization). In Appendix A.4, we numerically verify that the government optimally engages in greater LSAPs when initial leverage is greater or the supply shock is more severe—as long as banks are not bankrupt under the optimal LSAPs.[13]

3. A Quantification Based on Asset Market (In)elasticity

A recent literature argues that in practice demand elasticities for *aggregate* assets are much lower than implied by standard models, and that these low elasticities are behind the large observed asset price fluctuations (see, e.g., Gabaix and Koijen (2020)). In this section, we calculate the asset demand elasticity in our model and use this elasticity to provide *quantifiable* formulas for the marginal price impact of supply shocks and LSAPs. We find that the asset demand elasticity in our model is lower than in a baseline model with homogeneous risk tolerance, and more so when banks are more levered. Therefore, heterogeneity in risk tolerance helps explain part of the demand inelasticity in normal times, and *endogenously reduces* the demand elasticity after a recessionary shock (that increases banks' effective leverage). These features provide a complementary intuition for our mechanisms. A large shock not only induces levered banks to sell a disproportionate amount of assets (at a given price), but it makes asset demand more inelastic and requires a larger price drop for the market to reach equilibrium. By the same token, LSAPs are especially powerful after a large shock because their marginal impact on asset prices depends inversely on the asset demand elasticity.

Aggregate asset market elasticity estimates also provide a direct target for quantifying our mechanisms. Later in this section, we calibrate the pre-shock elasticity to match estimates from the recent literature and banks' leverage to match the recent stress test scenarios used by the Fed. Our calibration and price impact formulas imply that supply shocks and LSAPs both induce *quantitatively* meaningful effects on the equilibrium asset price. These results are

[13] With bankruptcy, the optimal policy might feature a discontinuity and the local comparative statics do not necessarily apply. In particular, there are parameters where *improving* productivity z_0 *increases* optimal LSAPs. This happens when the government finds it too costly to save the banks via an LSAP program. As z_0 improves, the government at some point finds it optimal to save the banks, which induces a discrete upward jump in optimal LSAPs. Likewise, when banks are bankrupt, *decreasing* their initial leverage might *increase* optimal LSAPs.

consistent with an emerging empirical literature that finds central banks' *risky* asset purchases in recent decades have had sizeable effects on asset prices.[14]

3.1. Price impacts and asset demand elasticities

Our main result in this section characterizes the marginal price impact of supply shocks and LSAPs along with the asset demand elasticity. Consider the setup with LSAPs, $\lambda \geq 0$. Let q_0^i denote the demand for the market portfolio from type i agents, *after adjusting for the effect of the price on their wealth*:

$$q_0^b = \omega_0^b (p_0) \alpha (z_0 p_0) \text{ and } q_0^h = \omega_0^h (p_0) (1 - \alpha (z_0 p_0)).$$

The functions, $\alpha(\cdot)$ and $\omega_0^i(\cdot)$ do not explicitly depend on λ, since the LSAP policy affects the agents' wealth and portfolio choice only through its impact on prices and returns. We let $q_0 = q_0^b + q_0^h$ denote the *aggregate* asset demand and observe that the equilibrium obtains when $q_0 = 1 - \lambda$ [see (30)].

Proposition 3. *Consider the neighborhood of a stable and interior equilibrium for a fixed LSAP policy, $\lambda \geq 0$. The price impact of supply shocks and LSAPs are given by, respectively,*

$$\frac{d \log p_0}{d \log z_0} = \frac{\partial \log q_0}{\partial \log z_0} \left(\frac{\partial \log q_0}{-\partial \log p_0} \right)^{-1} \tag{37}$$

$$\frac{d \log p_0}{d\lambda} = \frac{1}{1-\lambda} \left(\frac{\partial \log q_0}{-\partial \log p_0} \right)^{-1}, \tag{38}$$

where $\frac{\partial \log q_0}{\partial \log z_0}$ denotes the aggregate asset demand impact of a supply shock given by

$$\frac{\partial \log q_0}{\partial \log z_0} = \frac{\frac{l}{z_0 p_0}}{\frac{\tau^h/\kappa}{\tau^b - \tau^h} + 1 - \frac{l}{z_0 p_0}}. \tag{39}$$

Likewise, $\frac{\partial \log q_0}{-\partial \log p_0}$ denotes the price elasticity of aggregate asset demand given by

$$\frac{\partial \log q_0}{-\partial \log p_0} = \frac{1}{\overline{r}_0 - r_0^f} - \frac{\frac{l}{z_0 p_0}}{\frac{\tau^h/\kappa}{\tau^b - \tau^h} + 1 - \frac{l}{z_0 p_0}}, \tag{40}$$

where $\overline{r}_0 - r_0^f = \rho + g - \log(p_0) - r_0^f$ denotes the risk premium in equilibrium.

Explanation. Proposition 3 considers comparative statics in the neighborhood of an interior and stable equilibrium. In particular, the interest rate is constrained, $r_0^f = 0$, and does not

[14] See, e.g., Charoenwong et al. (2019); Barbon and Gianinazzi (2019) for studies of the asset price impact of the Bank of Japan's stock purchases over the last decade, and Caballero (1999) for a description of the Hong Kong stock market intervention during the Asian crisis of the late 1990s.

react to the change. Eq. (37) says that the price impact of a supply shock is increasing in its asset demand impact, $\frac{\partial \log q_0}{\partial \log z_0}$, and decreasing in the price elasticity of asset demand, $\frac{\partial \log q_0}{-\partial \log p_0}$. When this price elasticity is lower, a given demand impact (the exogenous flow) triggers a bigger price drop in order to generate the counteracting demand change (the endogenous flow) necessary to equilibrate the market. For the same reason, Eq. (38) says that the price impact of LSAPs is decreasing in the aggregate demand elasticity. In fact, for relatively small interventions ($1-\lambda \simeq 1$), the LSAPs' price impact is the inverse of the aggregate demand elasticity.

Eqs. (39) and (40) characterize the asset demand impact and the elasticity of the asset demand. In our model, these terms follow a similar structure. In a baseline model with homogeneous risk tolerance, $\tau^b = \tau^h$, supply shocks do not have a demand impact, $\frac{\partial \log q_0}{\partial \log z_0} = 0$, and the demand elasticity is the inverse of the risk premium, $\frac{\partial \log q_0}{-\partial \log p_0} = \frac{1}{\bar{r}_0 - r_0^f}$. With heterogeneous risk tolerance, $\tau^b > \tau^h$, supply shocks have a positive demand impact, $\frac{\partial \log q_0}{\partial \log z_0} > 0$, and the demand elasticity is lower than in the baseline, $\frac{\partial \log q_0}{-\partial \log p_0} < \frac{1}{\bar{r}_0 - r_0^f}$. This reduced elasticity finding is relevant for a quantitative analysis. With standard calibrations of the equity risk premium, the baseline model typically features a demand elasticity that is too high relative to recent empirical estimates. Therefore, risk tolerance heterogeneity helps close the gap between the theory and the data (see Section 3.2 for further discussion).

Eqs. (39) and (40) show that the supply shocks have a greater demand impact, and demand is more inelastic, when banks have a greater *effective initial leverage*, $\frac{l}{z_0 p_0}$: the leverage at the moment when asset prices decline, $z_0 p_0 < 1$, and banks have *not* yet adjusted their initial positions. This term can be sizeable, especially after a large (and unanticipated) drop in z_0. Moreover, once asset prices decline endogenously, the effective leverage increases *endogenously*, further strengthening the effects.

Taken together, Eqs. $(37-40)$ provide a complementary intuition for our main results. When risk tolerance is heterogeneous and effective leverage is high, a negative productivity shock causes a large asset price decline through two distinct channels. First, the shock induces sizeable outflows at the initial price. Second, since the shock raises banks' effective leverage, it further reduces the market's demand elasticity and therefore its ability to absorb these flows. Both effects contribute to the large price drop illustrated in Figure 2. The decline in the market's demand elasticity is also behind the large impact of LSAPs illustrated in Figure 4.

Sketch of proof. To provide further intuition, we present a sketch of the proof of Proposition 3 (completed in Appendix A.5). Eqs. (37) and (38) follow from differentiating the equilibrium condition, $\log q_0 = \log(1-\lambda)$, with respect to $\log z_0$ and λ, respectively.

We next derive the asset demand impact of productivity shocks, $\frac{\partial \log q_0}{\partial \log z_0}$. First consider the impact on *banks'* asset demand, q_0^b. Using Eqs. (15) and (17), we have

$$\log q_0^b = \log\left(\omega_0^b \alpha\right) = \log\left(\frac{\tau^b}{\sigma^2}\left(\rho + g - \log(p_0) - r_0^f\right)\right) + \log\left(\left(1 - \frac{l}{z_0 p_0}\right)\kappa\right). \tag{41}$$

Differentiating with respect to log productivity, we obtain

$$\frac{\partial \log q_0^b}{\partial \log z_0} = \frac{\frac{l}{z_0 p_0}}{1 - \frac{l}{z_0 p_0}}. \tag{42}$$

Keeping the relative price p_0 constant, an increase in productivity, z_0, increases asset valuations without changing asset returns (and therefore agents' leverage ratios, ω_0^i). High valuations increase banks' wealth share, $\alpha(z_0 p_0)$, which increases their purchasing power and asset demand. The strength of this channel depends on banks' effective initial leverage, $\frac{l}{z_0 p_0}$.

In Appendix A.5, we follow similar steps to calculate the impact on *households'* asset demand, q_0^h, as well as *aggregate* asset demand, $q_0 = q_0^b + q_0^h$, and prove Eq. (39). A positive productivity shock *reduces* households' wealth share and asset demand. Nonetheless, a productivity shock still increases aggregate demand because its positive impact on banks' demand dominates its negative impact on households' demand. In fact, Eqs. (39) and (42) imply that the aggregate impact closely resembles the banks' demand impact. In the limit with highly heterogeneous risk tolerance ($\frac{\tau^b}{\tau^h} \to \infty$), the two expressions become identical. The aggregate asset demand impact is a *quantity-weighted* average of banks' and households' demand impacts. Since banks are more risk tolerant, they choose a leveraged position in the market portfolio and thus are more central for the aggregate asset demand impact. Recall that banks start with a levered position (due to their past investment decisions). Therefore, a positive productivity shock transfers relative wealth to banks and increases aggregate asset demand.[15]

Next consider the price elasticity of asset demand, $\frac{\partial \log q_0}{-\partial \log p_0}$. As before, it is useful to start by characterizing the elasticity of *banks'* demand, q_0^b. Differentiating Eq. (41) with respect to the log price, and evaluating at the equilibrium price, we obtain

$$\frac{\partial \log q_0^b}{-\partial \log p_0} = \frac{1}{\overline{r}_0 - r_0^f} - \frac{\frac{l}{z_0 p_0}}{1 - \frac{l}{z_0 p_0}}. \tag{43}$$

Here, we substituted the equilibrium expected return, $\overline{r}_0 = \rho + g - \log p_0$ [see (16)]. A decrease in the asset price affects banks' asset demand through *two* channels. First, low prices induce banks to take on greater risk, $\omega_0^b(p_0)$, which raises their desired asset purchases. This is the standard channel in a variety of finance models and it implies an elasticity given by the inverse of the risk premium, $\frac{1}{\overline{r}_0 - r_0^f}$. Our model features a second channel that reduces the demand elasticity. Low asset prices decrease banks' wealth share, $\alpha(z_0 p_0)$, which decreases their purchasing power and demand. In fact, the decline in the asset price $z_0 p_0$ has the same impact on banks' wealth share regardless of whether that decline comes from the the productivity, z_0, or the price per

[15] This result and its intuition are related to the analysis in Kekre and Lenel (2020). They show that a valuation shock (driven by a change in the policy interest rate) decreases the risk premium only if the shock redistributes wealth toward agents with a greater marginal propensity to take risk (MPR). They also find that, when agents' initial leverage reflects the positions they would like to take given their MPRs, a positive valuation shock (driven by a policy interest rate cut) redistributes wealth to high MPR agents and thus reduces the risk premium. In our setting banks have a greater MPR than households.

productivity, p_0. Therefore, the adjustment term in (43) is *exactly the same* as the demand impact that we characterized earlier, $\frac{\partial \log q_0^b}{\partial \log z_0}$ [cf. (42)].

In Appendix A.5, we further characterize the elasticity of *aggregate* demand, $q_0 = q_0^b + q_0^h$, and prove Eq. (40). As before, the aggregate demand elasticity is very similar to banks' elasticity and it becomes the same as banks' elasticity in the limit with highly heterogeneous risk tolerance ($\frac{\tau^b}{\tau^h} \to \infty$).

3.2. A quantitative exploration

Although our model is highly stylized, Proposition 3 is useful for gauging the *quantitative* importance of our mechanisms. We calibrate the model in Appendix B (after slightly extending the model to introduce a non-capitalized factor). Here, we summarize the calibration exercise and conduct back-of-the-envelope calculations using Eqs. (37 − 40).

We set $\tau^b = 1/3.7 = 0.27$ based on the empirical analysis in He et al. (2017). The parameters, τ^h and κ, do not make much of a difference as long as the term $\frac{\tau^h/\kappa}{\tau^b - \tau^h}$ is relatively small—which we assume since our focus is on heterogeneous risk tolerance. We set $\tau^h = \tau^b/10$ and $\kappa = 0.75$. The leverage parameter, l, plays a more important role. We calibrate this parameter to match the relationship between the rate of banks' losses and the decline in banks' equity capital in the adverse scenario of the Fed's June 2020 stress tests. This leads us to set $l = 0.71$, which implies an initial leverage ratio, $\frac{1}{1-l} \simeq 3.4$. With these assumptions, we calculate the demand impact of productivity shocks in Eq. (39) as

$$\left.\frac{\partial \log q_0}{\partial \log z_0}\right|_{z_0 p_0 = 1} = \frac{l}{\frac{\tau^h/\kappa}{\tau^b - \tau^h} + 1 - l} = 1.63. \tag{44}$$

At the pre-shock benchmark (when $z_0 p_0 = 1$), a 1% decline in asset valuations (driven by a negative productivity shock) induces a 1.63% reduction in aggregate asset demand.

It remains to calibrate the aggregate demand elasticity in (40). Recent empirical analyses suggest this elasticity could be up to two orders of magnitude smaller in the data than what is implied by standard models. For instance, Gabaix and Koijen (2020) estimate a demand elasticity of 0.2 for the U.S. stock market whereas the standard model without heterogeneity would predict something around 20 (when the risk premium is equal to 5%). Other studies suggest similarly low elasticities for *aggregate* assets in other contexts, e.g., around 0.45 for the Chilean stock market (Da et al. (2018)); between 0.15 and 0.38 for the Chinese stock market (Li et al. (2020)); and around 0.19 for the U.S. style portfolios (Ben-David et al. (2020)). To be conservative, we target an elasticity equal to 1 in the pre-shock benchmark (when $z_0 p_0 = 1$). Using Eqs. (40) and (44), we solve

$$\left(\overline{r}_0 - r_0^f\right)^{-1} - 1.63 = 1. \tag{45}$$

This requires setting $\overline{r}_0 - r_0^f = 38\%$. Clearly, this is an unrealistically high risk premium. However, matching the appropriate aggregate demand elasticity is an issue for *all* models that rely on the standard portfolio choice setup (as we do). Therefore, we view the high calibrated risk premium as capturing unmodeled frictions such as investment mandates that reduce the demand elasticity in practice (see Gabaix and Koijen (2020) for an extensive discussion of these frictions). Having said this, note that the heterogeneity in risk tolerance significantly reduces the gap between the models and the data: absent heterogeneity (or with $l = 0$), we would have to calibrate $\overline{r}_0 - r_0^f = 100\%$ to generate the same demand elasticity.

Quantifying the price impact of supply shocks. With the calibration in hand, we next consider Eq. (37) that describes the marginal price impact of a supply shock. We first evaluate this equation at the *pre-shock* asset price, $z_0 p_0 = 1$, to obtain

$$\left.\frac{d \log p_0}{d \log z_0}\right|_{z_0 p_0 = 1} = \frac{1.63}{2.63 - 1.63} = 1.63. \tag{46}$$

This shows that our mechanism can be *quantitatively* strong: For small shocks, a 1% decline in productivity results in a 1.63% reduction in the asset price per productivity (when the interest rate does not or cannot react). Since banks are levered, a negative shock to asset valuations induces a sizeable reduction in asset demand [see (44)]. With realistic demand elasticities [see (45)], this translates into a sizeable additional drop in asset prices.

In Appendix B, we present an exact solution for a calibration that also matches $r_0^f = 0.01$ when $z_0 p_0 = 1$: in the pre-shock benchmark, there is room to cut interest rates by 1 percent point (similar to the degree of monetary policy room at the onset of the Covid-19 shock). Without LSAPs (when $\lambda = 0$), the calibrated model features multiple equilibria, since $p^h = 0.6, z^h = l/p^h = 1.18$ and $z^* \simeq 0.985$ (see Proposition 1). Figure 6 plots the asset price per productivity in the best (highest-price) equilibrium as a function of productivity. A 1.5 percent negative productivity shock is sufficient to push the interest rates to zero (since $z^* \simeq 0.985$). Additional declines in productivity leave the interest rate unchanged and reduce the asset price per productivity. In this range ($z_0 \leq z^*$), the price impact formulas we developed earlier—under the assumption that the interest rate does not react—apply. The dashed line illustrates the predicted price decline based on the pre-shock approximation in (46). This approximation works well for small shocks around the cutoff productivity z^* (even though we evaluate the formula for a slightly higher price than at the cutoff, $z_0 p_0 = 1 > z^*$).

For larger shocks, the actual price decline exceeds the predicted decline from the pre-shock approximation. For instance, consider $z_0 = 0.97$, which corresponds to about a 1.5% decline in productivity from z^*. The exact equilibrium features $p_0 \simeq 0.95$, which corresponds to a 5% decline in price per productivity—a much bigger decline than the approximation predicts. To understand this discrepancy, suppose $z_0 = 0.97$ and consider Eq. (37) *evaluated at the exact*

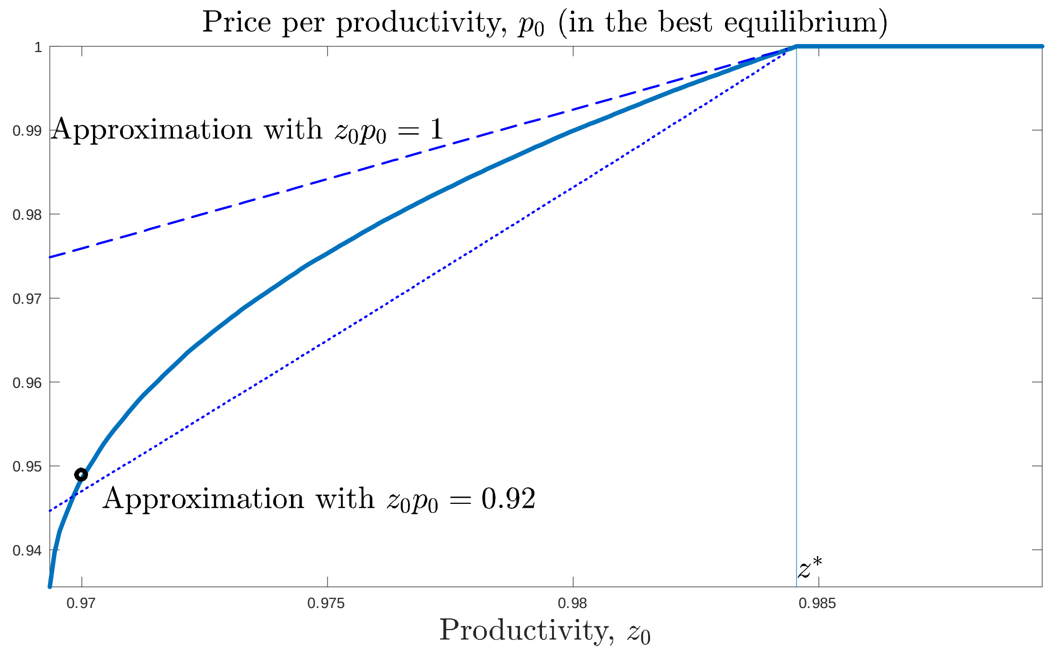

Figure 6: **Equilibrium with calibrated parameters and no LSAPs.** The solid line plots the asset price per productivity in the highest-price equilibrium. The dashed line (resp. the solid line) illustrates the predicted price based on the pre-shock (resp. the post-shock) log-linear approximation around the cutoff productivity, z^* [see Eqs. (46) and (47)].

equilibrium price $z_0 p_0 \simeq 0.92$.[16] This gives a *post-shock* approximation:

$$\left.\frac{d \log p_0}{d \log z_0}\right|_{z_0 p_0 = 0.92} = \frac{2.07}{2.63 - 2.07} \simeq 3.66. \tag{47}$$

This approximation suggests that a 1.5% decline in productivity induces approximately a 5.5% decline in the price per productivity, which is not far from the exact effect (see the dotted line in Figure 6). The reason is that the endogenous drop in asset prices increases banks' effective leverage ($\frac{l}{z_0 p_0} = 0.77 > l = 0.71$). This endogenous increase in leverage not only increases the direct impact of the shock on asset demand, as illustrated by the numerator of (47), but it also reduces aggregate demand elasticity, as illustrated by the denominator of (47). Put differently, *large shocks trigger disproportionately large asset sales while simultaneously reducing the market's ability to absorb those sales.* In fact, for the calibrated parameters, these nonlinear dynamics are quite powerful: when z_0 falls beyond the range plotted in Figure 6, the high-price equilibrium disappears and the unique equilibrium features bankruptcy with a very low price per productivity ($p_0 = p^h = 0.6$).

The upshot of this analysis is that our amplification mechanism can be quantitatively large,

[16] We still use the pre-shock level of the baseline elasticity, $\left(\overline{r}_0 - r_0^f\right)^{-1} = 2.63$. That is, we ignore the changes in this term due to changes in the risk premium (driven by the asset price decline). This is because we view the term $\left(\overline{r}_0 - r_0^f\right)^{-1}$ as capturing unmodeled factors that drive the baseline elasticity.

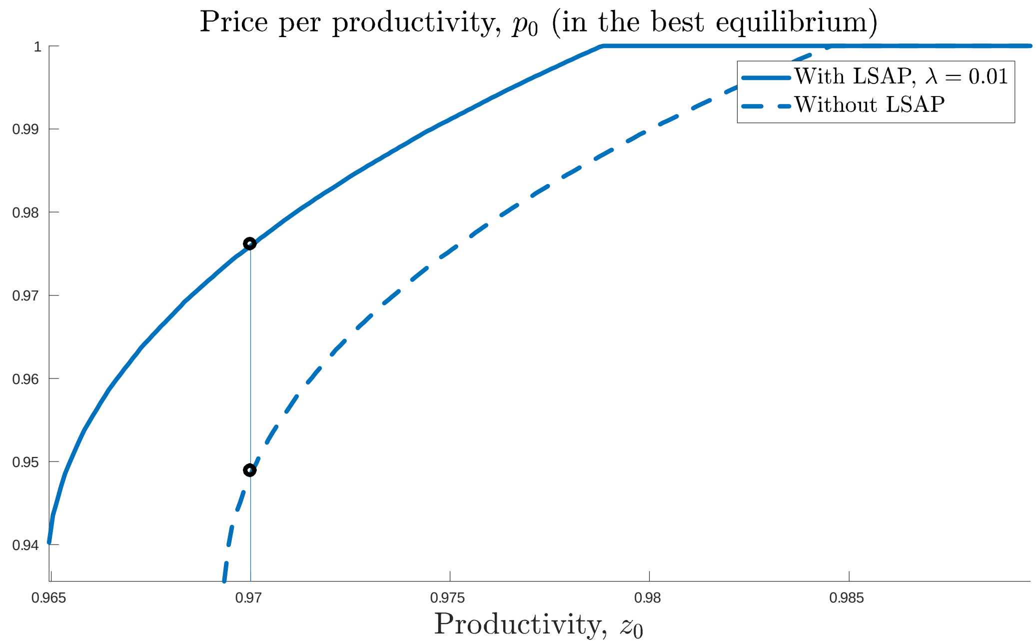

Figure 7: **The price impact of LSAPs with calibrated parameters.** The solid (resp. the dashed line) line plots the asset price per productivity in the highest-price equilibrium with LSAPs (resp. without LSAPs).

even with a relatively low level of initial leverage, *as long as the aggregate asset demand elasticity is calibrated to a level consistent with recent estimates.* Moreover, risk-tolerance heterogeneity helps explain the low asset demand elasticity observed even during normal times.

Quantifying the price impact of LSAPs. Next consider Eq. (38), which describes the marginal price impact of LSAPs. We evaluate this equation at the equilibrium with $z_0 = 0.97$. The asset demand elasticity evaluated at the equilibrium asset price is approximately given by $\left.\frac{\partial \log q_0}{-\partial \log p_0}\right|_{z_0 p_0 = 0.92} = 2.63 - 2.07 = 0.56$ [see (47)]. Therefore, Eq. (38) implies that the price impact of LSAPs at this equilibrium is approximately $\frac{d \log p_0}{d\lambda} = \frac{1}{1-\lambda}\frac{1}{0.56}$. In particular, if the government purchases 1% of the asset supply, the price per productivity increases by approximately 1.8%.

Figure 7 illustrates the impact of a 1% government asset purchase in the exact solution for the range of productivity shocks plotted in Figure 6. For $z_0 = 0.97$, the exact impact is close to the level predicted by the approximation (the discrepancy is driven by the endogeneity of the baseline elasticity, $\left(\overline{r}_0 - r_0^f\right)^{-1}$, as we describe in Footnote 16). LSAPs have a sizeable impact on the equilibrium price as long as the interest rate is constrained. LSAPs also expand the range of productivities that allow for a high-price equilibrium. In fact, the marginal impact of LSAPs is extremely large for the range, $z_0 \in [0.964, 969]$. In these cases, the best equilibrium with a 1% government asset purchase features a relatively high price (plotted) whereas the unique equilibrium without LSAPs features bankruptcy with a very low price, $p_0 = p^h = 0.6$ (outside the plot). We conclude that LSAPs can have a *quantitatively* meaningful effect on asset prices and therefore on *aggregate demand and output*.

4. Debt Overhang and Firm Insolvency

Since our main goal in this paper is to isolate the feedback between investors' endogenous risk tolerance and a large supply shock, we removed all other financial mechanisms. One financial mechanism that is particularly concerning in the context of the Covid-19 shock is firms' debt overhang. In this section we add debt overhang and show how it interacts with our risk-centric mechanism. Effectively, the corporate debt overhang problem creates a feedback between asset prices and productivity. This feedback makes the market's effective risk tolerance (and hence the required Sharpe ratio) more sensitive to asset prices, which strengthens our amplification mechanism and makes LSAPs more effective.

Recall that our baseline model features (New Keynesian) production firms that manage capital, produce (according to demand), and distribute their earnings to their owners. The market portfolio (which the agents trade among themselves) is a financial claim on all production firms. In this section, we assume production firms not only manage capital but also have debt liabilities (or debt claims) on each other. The market portfolio consists of the outstanding equity shares of all production firms. The value of an individual firm's equity is the value of its capital net of its debt liability (or plus its debt claim). Firms' debt liabilities and claims sum to zero, so the value of the market portfolio is still equal to the value of aggregate capital. However, the value of an indebted firm's equity share is less than the value of its capital. If the outstanding debt is too large, then the firm becomes insolvent.

Formally, there is a continuum of mass one of firms denoted by $\nu \in [0,1]$. Each firm manages one unit of capital and starts with an outstanding debt position, $b_0(\nu)$, that must be settled in period 0. If $b_0(\nu) > 0$, the firm has a debt liability to other firms. If $b_0(\nu) < 0$, the firm has debt claims on other firms. These outstanding positions are distributed according to a cumulative distribution function $dF(\cdot)$ that satisfies $\int_\nu b_0(\nu)\, dF(\nu) = 0$.

The firm can pay its debt using its earnings $y_0(\nu)$, or by issuing new claims backed by the (end-of-period) value of its assets (capital) $z_0 P_0$. To make the analysis stark, we assume the firm faces *no borrowing constraints*. For concreteness, consider a firm whose debt exceeds its earnings, $b_0(\nu) > y_0(\nu)$. First suppose the firm's debt is not too large,

$$b_0(\nu) \le y_0(\nu) + z_0 P_0. \tag{48}$$

We assume this firm issues new equity shares without frictions so that (at the end of the period) the firm becomes entirely equity financed and previous debtholders own a fraction of the firm $\zeta \in [0,1]$ that satisfies $b_0(\nu) - y_0(\nu) = \zeta P_0 z_0$.[17] Next consider a firm with more debt that

[17] While we describe a specific financing arrangement, other arrangements would also work and would lead to

violates condition (48). These firms cannot fully pay back their debt: they become insolvent and go through a bankruptcy process that restructures their debt.

We assume insolvency is costly: specifically, insolvent firms' productivity shrinks to a fraction of solvent firms' productivity, $\gamma \in [0,1]$ (for all periods $t \geq 0$). The parameter γ captures the efficiency of bankruptcy (or reallocation, when bankruptcy is not available). If $\gamma = 1$, an insolvent firm continues to operate at the same productivity as before. If $\gamma < 1$, which is empirically more likely, insolvency lowers the firm's productivity permanently.[18]

To close the model, we assume aggregate demand in period 0 is distributed among the solvent and insolvent firms according to their relative productivity levels. Specifically, let y_0 denote the output of a solvent firm. We assume the output of an insolvent firm is given by γy_0. Letting $S \in (0,1)$ denote the fraction of solvent firms, aggregate output is given by

$$\overline{y}_0 = \overline{S} y_0 \quad \text{where} \quad \overline{S} = S + (1-S)\,\gamma = \gamma + (1-\gamma)\,S.$$

Likewise, we denote the value of a solvent firm's assets by $P_0 z_0$. Then, the aggregate value of assets (or the market portfolio) is given by $\overline{S} z_0 P_0$. Note that the asset price per (effective) productivity is still given by P_0. The rest of the model is unchanged.

Most of the analysis is similar to Section 1. In periods $t \geq 1$, Eqs. (11) apply with the difference that $y_t = \overline{S} z_t$. Insolvencies that take place in period 0 permanently reduce productivity and output in subsequent periods. Consider period 0 where the interest rate policy might be constrained and output might also be influenced by aggregate demand. We have the following analogue of Eq. (9):

$$\overline{y}_0 = \sum_i c_0^i = \frac{1-e^{-\rho}}{e^{-\rho}} \overline{S} z_0 P_0.$$

Aggregate demand is still determined by aggregate wealth, but the latter has shrunk by a factor of $\overline{S}$. Recall that aggregate supply has also shrunk by the same factor. Using $\overline{y}_0 = \overline{S} y_0$, we obtain $y_0 = \frac{1-e^{-\rho}}{e^{-\rho}} z_0 P_0$: the output-asset price relation in (9) applies for a solvent firm. Therefore, the efficient asset price per productivity is still given by $P^* = \frac{e^{-\rho}}{1-e^{-\rho}}$. As before, we define $p_0 = P_0/P^* \in [0,1]$ as the normalized price per productivity.

Combining the output of a solvent firm, $y_0 = \frac{1-e^{-\rho}}{e^{-\rho}} z_0 P_0$, with the solvency constraint (48), we solve for the fraction of solvent firms:

$$S = \Pr\left\{ b\left(\nu\right) \leq \frac{z_0 P_0}{e^{-\rho}} \right\} = F\left(\frac{z_0 P_0}{e^{-\rho}}\right) = F\left(\frac{z_0 p_0}{1-e^{-\rho}}\right).$$

identical allocations for firms that meet condition (48). Under no arbitrage (which holds in our model) and no borrowing constraints (which we assume), the firm's value is independent of whether it issues debt or equity (or other claims) and from whom it borrows.

[18] The parameter γ is likely to have been especially low in the Covid-19 recession because the virus and lockdown measures restricted bankruptcy courts' capacity.

This in turn implies the following *aggregate* output-asset price relation [cf. (9)]:

$$\overline{y}_0 = \overline{S}\left(z_0 p_0\right) z_0 p_0 \quad \text{where} \quad \overline{S}\left(z_0 p_0\right) \equiv \gamma + (1-\gamma) F\left(\frac{z_0 p_0}{1-e^{-\rho}}\right). \tag{49}$$

Intuitively, debt overhang strengthens the output-asset price relation. Higher asset prices not only increase aggregate demand, as in our earlier analysis, but they also increase *aggregate supply* by enabling a greater fraction of indebted firms to remain solvent.

Next consider the characterization of the normalized asset price per productivity, $p_0 \in [0,1]$. Most of the analysis from Section 1 applies in this case. The main difference concerns banks' wealth share, which is now given by

$$\alpha = \alpha\left(z\right) \text{ where } z = \overline{S}\left(z_0 p_0\right) z_0 p_0.$$

Here, $\alpha\left(z\right)$ is the same function as before [see (15)]. Consequently, the risk balance condition is now given by [cf. (18)]

$$\frac{\sigma}{\tau\left(\overline{S}\left(z_0 p_0\right) z_0 p_0\right)} = \frac{\rho + g - \log\left(p_0\right) - r_0^f}{\sigma}. \tag{50}$$

Intuitively, debt overhang strengthens the impact of asset prices on risk tolerance. An increase in firm insolvencies (a decrease in $\overline{S}$) reduces the aggregate value of assets, which in turn reduces banks' wealth share. This reduces the market's effective risk tolerance and increases the required Sharpe ratio.[19]

The equilibrium is characterized by Eqs. (49) and (50) and the interest rate policy. Figure 8 illustrates the equilibrium for the earlier example that features a constrained interest rate $r_0^f = 0$. We assume the outstanding claims are uniformly distributed over $[-b, b]$ for some $b \geq 0$. As before, the left panel shows the equilibrium as the intersection of the required and actual Sharpe ratios. The dashed red line plots the required Sharpe ratio for the baseline case in which firms do not have outstanding debt ($b = 0$). The solid red line shows the required Sharpe ratio when firms are indebted ($b > 0$) and insolvency is costly ($\gamma < 1$). Debt overhang shifts the curve for the required Sharpe ratio upward. This lowers the normalized asset price and exacerbates the demand recession (solid blue and red lines).

The right panel sheds further light on the mechanism by plotting the fraction of solvent firms, S. With debt overhang, the supply shock in this example would induce some insolvencies even if there were no demand recession, $p_0 = 1$. However, the equilibrium features more insolvencies. Intuitively, low demand and asset prices ($p_0 < 1$) push a greater fraction of firms into distress by reducing their earnings and asset prices.

[19] The actual Sharpe ratio remains unchanged. To see this, note that the market portfolio return is given by $r_0 = \rho + \log\left(\frac{\overline{S} z_1 P^*}{\overline{S} z_0 P_0}\right)$. This expression is the same as before since the current price and the future productivity (future payoffs) both scale with $\overline{S}$ [see (16)].

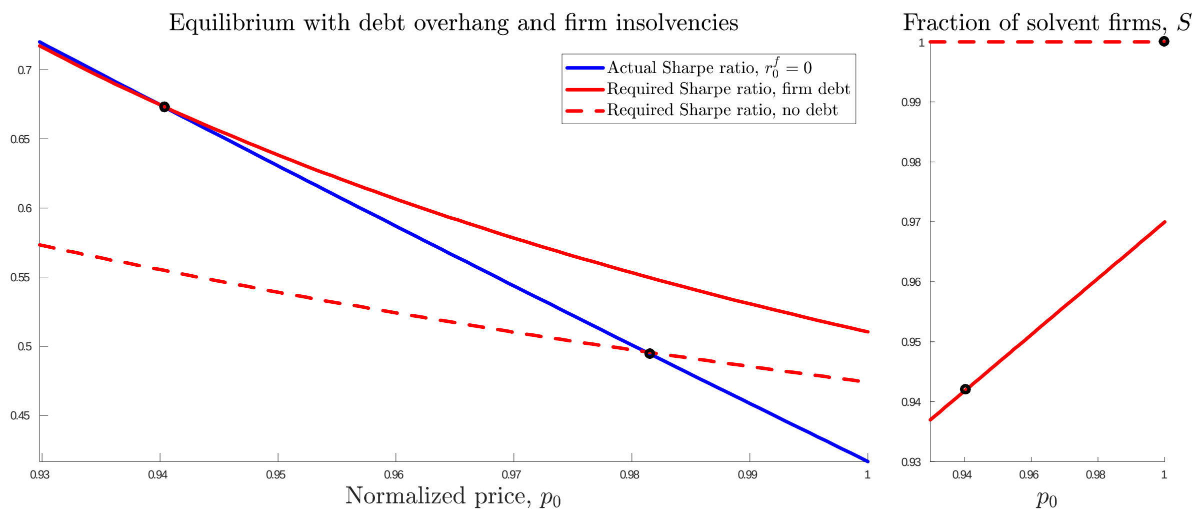

Figure 8: The effect of supply shocks when the interest rate is constrained and firms have a debt overhang problem and face costly insolvencies.

Importantly, Figure 8 illustrates that debt overhang also worsens asset price spirals (captured by the steepening of the required Sharpe ratio curve), as it makes the effective risk tolerance more sensitive to asset prices [see (50)]. This feature, together with our analysis in the previous sections, suggests that debt overhang also increases the marginal impact of LSAPs (e.g., Figure 4), which we verify in numerical simulations.

5. Final Remarks

In this paper we show that real (non-financial) shocks can endogenously reduce the market's risk tolerance and induce large contractions in asset prices and aggregate demand, and we demonstrate the effectiveness of LSAPs in mitigating these contractions. The key ingredient is heterogeneity in investors' risk tolerance. As aggregate conditions worsen, asset prices and the wealth share of risk tolerant agents decline. Thus, the "representative agent" becomes less risk tolerant and demands a higher Sharpe ratio to hold risky assets. With unconstrained monetary policy, a cut in interest rates is the most effective mechanism to increase the market's Sharpe ratio. If the central bank cannot cut interest rates, asset prices drop further and drag down aggregate demand and the wealth share of risk tolerant agents, triggering a downward spiral. LSAPs improve asset prices and aggregate demand by transferring risk to the government's balance sheet, and can be powerful since they reverse the downward spiral. Optimal LSAPs are larger when the government has greater future fiscal capacity and when the economy is more unstable at the outset, which happens when the risk tolerant agents' initial leverage is greater and when the shock is more severe. Corporate debt overhang problems strengthen our mechanisms by making the wealth share of risk tolerant agents (and thus the market's risk tolerance) more sensitive to asset prices.

Although our model is highly stylized, we can use it to gauge the quantitative importance of our mechanisms by focusing on the price elasticity of asset demand. We find that heterogeneity in risk tolerance helps explain the inelastic demand for *aggregate* assets observed in normal times, and it further reduces demand elasticity after a recessionary shock. When we calibrate the model to match recent estimates of aggregate asset demand elasticity and banks' leverage implied by the Fed's recent stress test scenarios, we find that aggregate shocks can induce a severe decline in asset prices absent policy intervention. In this context, we also find that LSAPs have a quantitatively meaningful impact on asset prices and output. The LSAPs have a sizeable impact as long as the economy is in a demand recession, and their marginal impact increases with the severity of the shock. In addition, the calibrated economy has multiple equilibria and LSAPs help shrink the range of shocks that plunge the economy into a unique bad equilibrium with bankruptcy and low asset prices.

Our analysis lends support to the unprecedented (in terms of size and speed) asset market interventions by the Fed and other major central banks around the world in response to the financial distress caused by the Covid-19 shock, and it highlights the importance of targeting assets held by levered investors. Importantly, the rationale for this policy in our framework is not to protect "the financial pipeline," however important this may be, but *to boost aggregate demand when conventional monetary policy is constrained.* While we focus on LSAPs, our analysis also supports other policy mechanisms that help reestablish equilibrium in risk markets. For example, loosening capital requirements is likely to increase effective risk tolerance and hence reduce the required Sharpe ratio. Likewise, any public guarantee or put policy that reduces perceived volatility is likely to reduce the gap between the required and actual Sharpe ratios at any given asset price level. We will explore some of these policies in future work.

An important practical concern with policies that support asset markets is the perception that they are distributionally unfair. Two observations diminish these concerns. First, in our framework the goal of these policies is *not* to transfer resources to risk-tolerant agents ("banks") but to boost aggregate demand. As such, these policies increase *everyone*'s income (see Remark 2 for an example where hand-to-mouth consumers can be a main beneficiary). Second, the wealth share of "banks" in our model declines more than in a benchmark frictionless model in which outcomes are supply determined. Appropriately designed LSAPs (as well as conventional monetary policy) do not make "banks" wealthier—they only mitigate the additional decline in their wealth share that results from a demand recession.

A similar argument mitigates the concern that LSAPs can exacerbate moral hazard (see Bornstein and Lorenzoni (2018) for a formal analysis in the context of conventional monetary policy). The goal of the policy is not to insure "banks" against aggregate shocks, but only against the excessive decline in asset prices due to the constraint on monetary policy. Naturally, for shocks that are more predictable than the Covid-19 shock, LSAPs could be complemented with ex-ante macroprudential policies in order to reduce the magnitude of the required ex-post intervention. However, macroprudential policies are beneficial because of the aggregate demand externalities present in these models (e.g., Korinek and Simsek (2016); Farhi and Werning (2016); Caballero and Simsek (2020)), that might be compounded by fire-sale externalities (e.g., Lorenzoni (2008); Dávila and Korinek (2018)), rather than because of moral hazard concerns.

In isolating our key mechanisms, we ignored realistic dynamic aspects of the transmission of monetary policy. We complement this analysis in Caballero and Simsek (2021a), where we abstract from endogenous risk intolerance and focus on a version of the model where asset prices affect aggregate demand with a lag, as is well documented. In that context, we show that the optimal monetary policy response to the emergence of a negative output gap is front-loaded. In response to the policy, asset prices rise rapidly while aggregate demand recovers with a lag, which creates a large temporary gap between asset prices and real activity. This gap is consistent with the wide Wall Street/Main Street disconnect observed after the massive Fed intervention following the Covid-19 shock.

Finally, we do not argue that asset market policies should substitute for all other aggregate demand policies. In fact, the global expansion in fiscal policy in response to the Covid-19 shock has been as fast and remarkable as the response by central banks, and this seems appropriate to us. A pragmatic response to any severe recessionary shock mixes monetary and fiscal policy responses. Our paper highlights that LSAPs share many features with conventional monetary policy, and therefore provide an appropriate response when conventional monetary policy is constrained.

A. Appendix: Omitted derivations and proofs

This appendix presents the analytical derivations and proofs omitted from the main text. We first provide a New Keynesian microfoundation for nominal rigidities that ensure output is determined by demand [see Eq. (2)]. We then present the details of the baseline model without LSAPs. Next, we consider the model with LSAPs. We then characterize the optimal LSAPs and derive the results we discuss in Section 2.2. Finally, we calculate the asset demand elasticities and the marginal price impact formulas that we use in Section 3.1.

A.1. New Keynesian microfoundation for nominal rigidities

The supply side features monopolistically competitive New Keynesian firms. Specifically, a continuum of measure one of production firms denoted by ν own the capital stock (in equal proportion). They produce differentiated goods, $y_t(\nu)$, subject to the technology,

$$y_t(\nu) = \eta_t(\nu) z_t. \tag{A.1}$$

Here, $\eta_t(\nu) \in [0,1]$ denotes the firm's choice of capital utilization. We assume utilization is free up to $\eta_t(\nu) = 1$ and infinitely costly afterwards. The production firms sell their output to a competitive final goods firm with the CES technology,

$$y_t = \left(\int_0^1 y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\varepsilon/(\varepsilon-1)} \text{ for some } \varepsilon > 1. \tag{A.2}$$

This implies the demand for a production firm satisfies,

$$\begin{aligned} y_t(\nu) &\leq q_t(\nu)^{-\varepsilon} y_t \qquad\qquad (A.3) \\ \text{where } q_t(\nu) &= Q_t(\nu)/Q_t \text{ and } Q_t = \left(\int Q_t(\nu)^{1-\varepsilon} d\nu \right)^{1/(1-\varepsilon)}. \end{aligned}$$

Here, $q_t(\nu)$ denotes the firm's relative price, which depends on its nominal price, $Q_t(\nu)$, as well as the ideal nominal price index, Q_t.

When capital is underutilized, $\eta_t(\nu) < 1$, the marginal cost of production is zero. Therefore, without nominal rigidities, the equilibrium always features full factor utilization, $\eta_t(\nu) = 1$ and $y_t = y_t(\nu) = z_t$ (see Caballero and Simsek (2020) for a derivation).

In contrast, we assume production firms have a preset nominal price that is common across firms, $Q_t(\nu) = Q$. Thus, the relative price of a firm is fixed and equal to one, $q_t(\nu) = 1$. The firm chooses the remaining variables, $\eta_t(\nu) \in [0,1], y_t(\nu)$, to maximize its earnings, $y_t(\nu)$, subject to Eqs. $(A.1)$ and $(A.3)$. The firm's problem becomes,

$$\max_{\eta_t(\nu)} \eta_t(\nu) z_t \text{ s.t. } 0 \leq \eta_t(\nu) \leq 1 \text{ and } \eta_t(\nu) z_t \leq y_t.$$

The solution is given by, $\eta_t(\nu) = \min\left(1, \frac{y_t}{z_t}\right)$. The firm optimally increases its production until the supply or the demand constraint binds. Using Eq. $(A.2)$ to aggregate across all firms, we obtain Eq. (2) in the main text: that is, output is determined by aggregate demand subject to the capacity constraint.

A.2. Baseline model without policy

In this section, we complete the characterization of the equilibrium for the baseline model analyzed in Section 1. First, we formally state agents' preferences and problem. We then derive the optimality conditions and characterize the equilibrium for periods $t \geq 1$ that feature no uncertainty. Then, we derive the (approximate) optimality conditions for period 0, characterize the equilibrium, and prove Proposition 1.

A.2.1. Agents' preferences and problem

Type i agents have recursive utility defined as,

$$\begin{aligned} \log U_t^i &= \left(1 - e^{-\rho}\right) \log\left(c_t^i\right) + e^{-\rho} \log V_{t+1}^i \qquad (A.4) \\ \text{where } V_{t+1}^i &= \left(E\left[\left(U_{t+1}^i\right)^{1-1/\tau^i}\right]\right)^{1/\left(1-1/\tau^i\right)}. \end{aligned}$$

Here, V_{t+1}^i captures a certainty-equivalent measure of the next period's continuation utility. Note that the EIS is equal to one and the RRA is equal to $1/\tau^i > 0$. The case with $\tau^i = 1$ is equivalent to time-separable log utility. Absent an approximation, agents choose the path of consumption and portfolio allocations, $\left\{c_t^i, \omega_t^i\right\}_{t=0}^{\infty}$, that maximize $(A.4)$ subject to the following budget constraints

$$\begin{aligned} c_t^i + a_t^i &= A_t^i \qquad (A.5) \\ A_{t+1}^i &= a_t^i \left(\omega_t^i \exp\left(r_t\right) + \left(1 - \omega_t^i\right) \exp\left(r_t^f\right)\right). \end{aligned}$$

We also impose a nonnegative wealth requirement, $A_{t+1}^i \geq 0$, which is sufficient to rule out Ponzi schemes.

A.2.2. Optimality conditions and equilibrium in periods $t \geq 1$

Starting period 1 onward, there is no residual uncertainty. Therefore, the utility function in $(A.4)$ reduces to time-separable log utility. To see this, note that $V_{t+n}^i = U_{t+n}^i$ for each $t + n \geq t > 1$. Substituting this into $(A.4)$, and iterating forward, we obtain:

$$\log U_t^i = \left(1 - e^{-\rho}\right) \sum_{n=0}^{\infty} \left(e^{-\rho}\right)^n \log c_{t+n}^i \text{ for } t \geq 1. \qquad (A.6)$$

It is then easy to check that the optimal consumption is given by,

$$c_t^i = \left(1 - e^{-\rho}\right) A_t^i.$$

This verifies Eq. (4) for periods $t \geq 1$. Since there is no uncertainty, agents allocate their portfolio to the asset that has the higher return. In equilibrium, the asset returns are equated for asset markets to clear, $r_t = r_t^f$ for $t \geq 1$. This verifies Eq. (5).

We next characterize the equilibrium for periods $t \geq 1$ and establish (11). We conjecture an equilibrium in which the returns are strictly positive, $r_t = r_t^f > 0$, and the output is at its efficient level, $y_t = z_t$. In view of the output-asset price relation (9), this implies $P_t = P^* = \frac{e^{-\rho}}{1-e^{-\rho}}$ [see (10)]. Substituting this into the asset return (3), and using (1), we obtain,

$$r_t = \log\left(\frac{z_{t+1}}{z_t}\frac{1+P^*}{P^*}\right) = \rho + g \text{ for } t \geq 1.$$

This also implies $r_t^f = r_t = \rho + g$, verifying that the interest rate is strictly positive. Substituting this into agents' flow budget constraint in ($A.5$), and using Eq. (4), we obtain,

$$A_{t+1}^i = a_t^i \exp(\rho + g) = A_t^i \exp(g) = A_1^i \exp(gt).$$

This establishes that the continuation equilibrium is given by (11).

We also calculate agents' continuation utility in period 1 along the equilibrium path. Substituting $c_t^i = (1 - e^{-\rho}) A_t^i$ and $A_t^i = A_1^i \exp(g(t-1))$, Eq. ($A.6$) implies

$$\log U_1^i\left(A_1^i\right) = \log U_1(1) + \log A_1^i. \tag{A.7}$$

Here, $U_1(1)$ captures agents' (common) unit-wealth continuation utility. It depends on the parameters (ρ, g) but *it is independent of the realization of the productivity shock in period* 1, z_1. This enables us to capture the agent's preferences in period 0 with the following shifted utility function,

$$\begin{aligned} u_0^i\left(A_0^i\right) &\equiv \log U_0^i - e^{-\rho}\log U_1(1) \\ &= \left(1 - e^{-\rho}\right)\log\left(c_0^i\right) + e^{-\rho}\log\left(E\left[\left(A_1^i\right)^{\left(\tau^i-1\right)/\tau^i}\right]\right)^{\tau^i/\left(\tau^i-1\right)}. \end{aligned} \tag{A.8}$$

A.2.3. Optimality conditions in period 0

Next consider the agent's problem in period 0. Using Eqs. ($A.8$) and ($A.5$), we can write the agent's problem as

$$u_0^i\left(A_0^i\right) = \max_{c_0, a_0, \omega}\left(1 - e^{-\rho}\right)\log c_0 + e^{-\rho}\log\left(E\left[A_1^{\left(\tau^i-1\right)/\tau^i}\right]\right)^{\tau^i/\left(\tau^i-1\right)} \tag{A.9}$$

$$\text{s.t.} \quad c_0 + a_0 = A_0^i$$

$$\text{and} \quad A_1 = a_0\left(\omega\exp(r_0) + (1-\omega)\exp\left(r_0^f\right)\right). \tag{A.10}$$

Here, A_1, denotes the wealth in period 1 as a function of the initial and the realized productivity. Recall that agents' initial endowments, A_0^i, are given by (12).

In view of the Epstein-Zin functional form, agents can be thought of as solving the intertemporal problem,

$$u_0^i\left(A_0^i\right) = \max_{a_0}\left(1 - e^{-\rho}\right)\log\left(A_0^i - a_0\right) + e^{-\rho}\log\left(R_0^{CE,i}a_0\right). \tag{A.11}$$

Here, $R_0^{CE,i}$ denotes investors' certainty-equivalent portfolio return per dollar. Absent an approximation, it would correspond to the solution to the following portfolio optimization problem:

$$\begin{aligned} R_0^{CE,i} &= \max_{\omega}\left(E\left[\left(R_0^p\right)^{\left(\tau^i-1\right)/\tau^i}\right]\right)^{\tau^i/\left(\tau^i-1\right)} \\ \text{and } R_0^p &= \omega\exp(r_0) + (1-\omega)\exp\left(r_0^f\right). \end{aligned} \tag{A.12}$$

The variable, R_0^p, denotes the realized portfolio return per dollar.

The first order condition for problem $(A.11)$ implies Eq. (4) in the main text. That is, regardless of her certainty-equivalent portfolio return, the investor consumes and saves a constant fraction of her lifetime wealth.

The portfolio problem $(A.12)$ is more complicated. To simplify the problem, we note that Eqs. (3), (9) and (1) imply,

$$r_0 = \rho + \log\left(\frac{z_1 P^*}{z_0 P_0}\right) \sim N\left(\rho + \log\frac{\overline{z}_1}{z_0} - \log\left(\frac{P_0}{P^*}\right) - \frac{\sigma^2}{2}, \sigma^2\right). \quad \text{(A.13)}$$

Thus, the market portfolio return follows a log-Normal distribution. In general, the portfolio return in $(A.12)$ does not follow a log-Normal distribution. However, for relatively short horizons, the portfolio return remains close to a log-Normal distribution. This motivates an approximation that is widely used in the literature (see Campbell and Viceira (2002)). This approximation becomes exact if the portfolio return follows a log-Normal distribution ($\omega = 1$) as well as in the continuous time limit.

Specifically, we assume agents choose portfolios (and evaluate the resulting certainty-equivalent return, $R_0^{CE,i}$) by solving the following approximate portfolio problem:

$$\begin{aligned} \log R_0^{CE,i} - r_0^f &\simeq \max_{\omega} \omega\pi - \frac{1}{2}\frac{1}{\tau^i}\omega^2\sigma^2 \\ \text{where } \pi &= E[r_0] + \frac{\sigma^2}{2} - r_0^f. \end{aligned} \quad \text{(A.14)}$$

Here, π denotes the risk premium on the market portfolio and σ is its standard deviation (measured in log returns). The problem says that the agent trades off its portfolio mean (in excess of the risk-free rate), $\omega\pi$, with its portfolio variance, $\omega^2\sigma^2$. The first order condition for this problem implies Eq. (6) in the main text.

A.2.4. Equilibrium in period 0

Recall that the equilibrium is characterized by Eqs. $(18-19)$ along with the monetary policy rule. In the main text, we derive these equations. We also characterize the equilibrium with partly temporary supply shocks (see Section 1.2). We next characterize the equilibrium with permanent supply shock, assuming $\frac{\overline{z}_1}{z_0} = g$, and prove Proposition 1.

We first describe the equilibrium in terms of an auxiliary function. Consider the function:

$$\begin{aligned} F(p_0; z_0) &= \frac{\sigma^2}{\overline{\tau} - \frac{\kappa l(\tau^b - \tau^h)}{z_0 p_0}} + \log(p_0) - (\rho + g) \\ \text{where } \overline{\tau} &= \tau^h + \kappa\left(\tau^b - \tau^h\right). \end{aligned} \quad \text{(A.15)}$$

This function is defined over the domain $p_0 \in \left(\underline{p}_0, \infty\right)$, where $\underline{p}_0 = \frac{\kappa l\left(\tau^b - \tau^h\right)}{z_0 \overline{\tau}}$. Eq. (18) implies that every interior equilibrium, $p_0 \in \left(p^h, 1\right)$, corresponds to a zero of this function. Conversely, every zero of the function that falls in the range, $p_0 \in \left(p^h, 1\right)$, corresponds to an interior equilibrium. The zeros that fall outside this range do not correspond to an equilibrium. Finally, there is a corner equilibrium with $p_0 = 1$ (and $r_0^f \geq 0$) iff $F(1; z_0) \leq 0$; and there is a corner equilibrium with $p_0 = p^h$ (and bankruptcy) iff $F\left(p^h; z_0\right) \geq 0$.

We next establish some properties of the auxiliary function that facilities the proof. Consider the monotone change of variables:

$$\tau = \overline{\tau} - \frac{\kappa l\left(\tau^b - \tau^h\right)}{z_0 p_0} \;\Leftrightarrow\; p_0 = \frac{\kappa l\left(\tau^b - \tau^h\right)/z_0}{\overline{\tau} - \tau}. \tag{A.16}$$

In terms of the new variable, the auxiliary function corresponds to the transformed function:

$$f(\tau) \equiv \begin{pmatrix} \frac{\sigma^2}{\tau} - \log(\overline{\tau} - \tau) + \log l - \log z_0 \\ -(\rho + g) + \log\left(\kappa\left(\tau^b - \tau^h\right)\right) \end{pmatrix}. \tag{A.17}$$

This function has the domain $\tau \in (0, \overline{\tau})$, and it is strictly convex, that is:

$$f''(\tau) = \frac{2\sigma^2}{\tau} + \frac{1}{(\overline{\tau} - \tau)^2} > 0$$

The function also satisfies $\lim_{\tau \to 0} f(\tau) = \lim_{\tau \to \overline{\tau}} f(\tau) = \infty$. These observations imply that the zeros of the transformed function $f(\cdot)$ have the same characteristics as an upward-pointing parabola. The original function $F(\cdot; z_0)$ adopts the same characteristics. In particular, the function either does not have any (interior) zero:

$$F(p_0; z_0) \geq 0 \text{ for } p_0 \in \left(\underline{p}_0, \infty\right), \tag{A.18}$$

or it has exactly two interior zeros:

$$\begin{aligned} F\left(p_0^1; z_0\right) &= F\left(p_0^2; z_0\right) = 0 \text{ for } \underline{p}_0 < p_0^1 < p_0^2 \\ \text{with } F(p_0; z_0) &< 0 \text{ for } p_0 \in \left(p_0^1, p_0^2\right) \text{ and } F(p_0; z_0) > 0 \text{ otherwise.} \end{aligned} \tag{A.19}$$

Proof of Proposition 1. Consider the first part that concerns the case, $z_0 > z^h = \frac{l}{p^h}$ [see (25)]. This condition implies the auxiliary function in $(A.15)$ satisfies:

$$\begin{aligned} F\left(p^h; z_0\right) &= \frac{\sigma^2}{\tau\left(z_0 p^h\right)} + \log\left(p^h\right) - (\rho + g) \\ &< \frac{\sigma^2}{\tau^h} + \log\left(p^h\right) - (\rho + g) = 0. \end{aligned} \tag{A.20}$$

Here, the inequality follows since $z_0 > z^h = \frac{l}{p^h}$ implies $\tau\left(z_0 p^h\right) > \tau(l) = \tau^h$. The equality follows from the definition of p^h. This rules out the corner equilibrium with $p_0 = p^h$. Combining this observation with Eq. $(A.19)$ also implies that we must have the case $(A.19)$ with p^h falling between the two zeros. This in turn implies there is a unique equilibrium that depends on the sign of $F(1; z_0)$. When $F(1; z_0) > 0$, there is an interior equilibrium with $p_0 \in \left(p^h, 1\right)$. When $F(1; z_0) \leq 0$, there is a corner equilibrium with $p_0 = 1$. Note also that $F(1; z_0) = \frac{\sigma^2}{\tau(z_0)} - (\rho + g)$ implies that the condition, $F(1; z_0) > 0$, is equivalent to $z_0 < z^*$ from the definition of z^* [see (26)]. This proves that there is a unique interior equilibrium when $z_0 < z^*$ (and $z_0 > z^h$) and there is a unique corner equilibrium when $z_0 \geq z^*$ (and $z_0 > z^h$).

Next consider the comparative statics of the interior equilibrium with respect to z_0. Note that $F(p_0; z_0)$ is decreasing in z_0. Therefore, greater z_0 shifts $F(p_0; z_0)$ downward, which increases the (greater) zero of the function that corresponds to the equilibrium. This establishes $\frac{dp_0}{dz_0} > 0$ and completes the proof of the first part.

Next suppose $z_0 < z^h = \frac{l}{p^h}$. We have the opposite of $(A.20)$, which implies that there is a corner equilibrium with $p_0 = p^h$. In this case, there can also be other equilibria. To see this, consider $z_0 \in \left(z^*, z^h\right)$ (assuming the interval is nonempty). Then, we have:

$$F(1; z_0) = \frac{\sigma^2}{\tau(z_0)} - (\rho + g) < \frac{\sigma^2}{\tau(z^*)} - (\rho + g) = 0. \tag{A.21}$$

Here, the inequality follows since $z_0 > z^*$ and the equality follows from the definition of z^*. This implies that there is a corner equilibrium with $p_0 = 1$. In particular in this case $p_0 = p^h$ and $p_0 = 1$ are both corner equilibria. This completes the proof of the proposition. □

A.3. Model with large-scale asset purchases

In this section, we present the details of the extended model with LSAPs that we analyze in Section 2. We first describe the government's budget constraints. We then describe the government's decisions in periods $t \geq 1$ and characterize the equilibrium in these periods. Then, we complete the characterization of the equilibrium in period 0 and prove Proposition 2. Section A.4 at the end of this appendix characterizes the optimal LSAPs that we discuss in Section 2.2.

A.3.1. Government's budget constraints

The government is endowed with some income in future periods $t \geq 1$ given by $y_t \eta^g$. These endowments can be thought of as tax claims on future generations that are not active in financial markets in period 0. In particular, future government taxes (or spending) do not directly affect the agents that are active in financial markets in period 0, which implies that Ricardian equivalence does not apply in this period. We also assume future tax capacity is proportional to future output, which simplifies the analysis but is not necessary for our results (in fact, making the government's tax capacity safer would strengthen our results).

Specifically, that the government can be equivalently thought of as being endowed with η^g units of the market portfolio at the end of period 0 (excluding the dividend income in period 0). Thus, the government's wealth at the end of period 0 is given by Eq. (28),

$$a_0^g = z_0 P_0 \eta^g.$$

In period 0, the government chooses the fraction of its wealth to allocate to the market portfolio, ω_0^g. This determines the government's wealth in period 1, that is,

$$A_1^g = a_0^g \left(\omega_0^g \exp(r_0) + (1 - \omega_0^g) \exp r_0^f\right). \tag{A.22}$$

Note that choosing $\omega_0^g = 1$ replicates the government's initial endowment. Choosing a greater portfolio weight, $\omega_0^g > 1$,corresponds to investing additional units of the risky asset by issuing safe assets.

In periods $t \geq 1$, the government chooses its spending, c_t^g, assets a_t^g, and portfolio allocation, ω_t^g. Therefore, its budget constraints satisfy the following analogue of agents' budget constraints $(A.10)$,

$$\begin{aligned} c_t^g + a_t^g &= A_t^g \\ A_{t+1}^g &= a_t \left(\omega_t^g \exp(r_t) + (1 - \omega_t^g) \exp\left(r_t^f\right)\right) \text{ for } t \geq 1. \end{aligned} \tag{A.23}$$

A.3.2. Equilibrium in periods $t \geq 1$ with government

Note that government tax revenues effectively expand the supply of the market portfolio by η^g units. Therefore, agents start period 1 with initial wealth levels, A_1^i (determined by their past investment decisions), that satisfy the resource constraint

$$\sum_{i \in \{g,b,h\}} A_1^i = (y_1 + z_1 P_1)(1 + \eta^g). \tag{A.24}$$

Likewise, asset market clearing conditions for each $t \geq 1$ are given by [cf. (7)],

$$\sum_{i \in \{g,b,h\}} a_t^i = \sum_{i \in \{g,b,h\}} \omega_t^i a_t^i = z_t P_t (1 + \eta^g). \tag{A.25}$$

The equilibrium depends on the spending path the government chooses, $\{c_t^g\}_{t=1}^{\infty}$. Recall that absent government the economy is on a balanced growth path in which output is at its potential and agents' consumption and assets grow at the constant rate, g [see Eqs. (11)]. We assume the government also chooses to grow its spending at the constant rate, g (subject to its lifetime budget constraint). This assumption is natural, and it is also optimal given the government's objective function that we introduce later in the appendix.

With this assumption, we conjecture that the equilibrium in periods $t \geq 1$ satisfies the equations in (11) as before. In particular, all agents—including the government—spend a constant fraction of their wealth and maintain constant wealth growth. It is easy to check that these allocations are optimal for agents $i \in \{b, h\}$. The allocations also satisfy the government budget constraints in $(A.23)$. Finally, the allocations satisfy the market clearing conditions in $(A.25)$ given initial allocations that satisfy $(A.24)$. The portfolio allocations are indeterminate since agents are indifferent between the market portfolio and the risk-free asset. This verifies the equilibrium for periods $t \geq 1$.

A.3.3. Equilibrium in period 0 with government

We next characterize the equilibrium in period 0 with LSAPs and establish Proposition 2. Consider the analogue of the function $(A.15)$ that incorporates LSAPs:

$$\begin{aligned} F(p_0; z_0, \lambda) &= \frac{\sigma^2 (1-\lambda)}{\overline{\tau} - \frac{l\kappa(\tau^b - \tau^h)}{z_0 p_0}} + \log(p_0) - (\rho + g) \\ \text{where } \overline{\tau} &= \tau^h + \kappa\left(\tau^b - \tau^h\right). \end{aligned} \tag{A.26}$$

Every interior equilibrium, $p_0 \in \left(p^h(\lambda), 1\right)$, corresponds to a zero of this function. Conversely, any zero of the function that falls in the interior range, $p_0 \in \left(p^h(\lambda), 1\right)$, corresponds to an equilibrium. The zeros that fall outside this range do not correspond to an equilibrium. There is a corner equilibrium with $p_0 = 1$ iff $F(1; z_0, \lambda) \leq 0$; and there is a corner equilibrium with $p_0 = p^h$ iff $F\left(p^h; z_0, \lambda\right) \geq 0$. Finally, the function $F(p_0; z_0, \lambda)$ satisfies the same property that we established for the special case with $\lambda = 0$: one of cases $(A.18)$ and $(A.19)$ holds.

Proof of Proposition 2. Suppose $z^h(\lambda) < z^*(\lambda)$ and consider a shock $z_0 \in \left(z^h(\lambda), z^*(\lambda)\right)$. Following the same steps as in Proposition 1, there exists a unique equilibrium that corresponds to the (greater) zero of the function, $F(p_0; z_0, \lambda)$, that falls in the range, $p_0 \in \left(p^h(\lambda), 1\right)$. Consider the comparative statics with respect to the size of the LSAPs, λ. Eq. $(A.26)$ implies that increasing λ shifts the function, $F(p_0; z_0, \lambda)$, downward. This increases the (greater) zero and raises the equilibrium price, that is, $\frac{dp_0}{d\lambda} > 0$. $\square$

A.4. Optimal LSAPs

In the main text, we focus on the impact of the LSAPs on equilibrium. In this appendix, we analyze the optimal LSAPs and derive the results we discuss in Section 2.2. Recall that the government chooses the initial leverage of its portfolio, $\omega_0^g \geq 1$ (and maintains a constant-growth spending path in subsequent periods), but otherwise does not interfere with the equilibrium. To analyze the welfare impact of this policy, we first introduce the government's own utility function. We then set up a constrained Pareto planning problem in a simpler version of the model in which we collapse the banks and households into a single representative agent. Finally, we use the objective function from the simpler model to characterize the optimal LSAPs in the original model and we describe the comparative statics of these optimal LSAPs.

A.4.1. Government's utility function

Recall that the government chooses a path of spending, $\{c_t^g\}_{t=1}^{\infty}$. Suppose the government's utility function over this path is similar to the other agents' utility function [cf. $(A.4)$],

$$\begin{aligned} \log U_0^g &= e^{-\rho} \log V_1^g \\ \text{and } \log U_t^g &= \left(1 - e^{-\rho}\right) \log\left(c_t^g\right) + e^{-\rho} \log V_{t+1}^g \text{ for } t \geq 1, \\ \text{where } V_{t+1}^g &= \left(E\left[\left(U_{t+1}^g\right)^{1-1/\tau^g}\right]\right)^{1/(1-1/\tau^g)}. \end{aligned} \tag{A.27}$$

That is, the government has Epstein-Zin preferences with EIS equal to 1 and RRA equal to τ^g.

First consider the government's optimal choice in periods $t \geq 1$. In these periods, the government's decision does not affect output (which is already at its potential) so the government maximizes its own utility, U_t^g, subject to the flow budget constraints in $(A.23)$. Consequently, the analysis in Section A.2.2 applies also for the government. In particular, the constant-growth spending allocations described in Section A.3.2 are optimal for the government.

Next consider the government's own utility in period 0. As before, the government's continuation utility satisfies Eq. $(A.7)$. Thus, we can capture the government's preferences in period 0 with the shifted utility function [cf. $(A.8)$],

$$\begin{aligned} u_0^g\left(a_0^g\right) &= \log U_0^g - e^{-\rho} \log U_1(1) \\ &= e^{-\rho} \log\left(E\left[A_1^g\right]^{(\tau^g-1)/\tau^g}\right)^{\tau^g/(\tau^g-1)} \text{ where } A_1^g = R_0^p a_0^g \\ &= e^{-\rho} \log\left(R_0^{CE,g} a_0^g\right) \text{ where } R_0^{CE,g} = \left(E\left[\left(R_0^p\right)^{(\tau^g-1)/\tau^g}\right]\right)^{\tau^g/(\tau^g-1)} \end{aligned}$$

Here, the last line writes the utility function in terms of the certainty equivalent return. Applying the log-Normal approximation to this return, similar to the agents, we have [cf. $(A.14)$],

$$\begin{aligned} \log R_0^{CE,g} &\simeq r_0^f + \omega_0^g \left(E[r_0] + \frac{\sigma^2}{2} - r_0^f\right) - \frac{1}{2}\frac{1}{\tau^g}\left(\omega_0^g\right)^2 \sigma^2 \\ &= \left(1 - \omega_0^g\right) r_0^f + \omega_0^g \left(\rho + g - \log\frac{P_0}{P^*}\right) - \frac{1}{2}\frac{1}{\tau^g}\left(\omega_0^g\right)^2 \sigma^2. \end{aligned} \tag{A.28}$$

Here, the second line substitutes for the expected return from $(A.13)$. Combining these observations and using $a_0^g = z_0 P_0 \eta^g$ [see (28)], we calculate the government's approximate utility as,

$$\begin{aligned} u_0^g &= e^{-\rho}\left(\log\left(R_0^{CE,g}\right) + \log\left(z_0\eta^g\right) + \log P_0\right) \\ &= ... + e^{-\rho}\left(\left(1-\omega_0^g\right) r_0^f + \omega_0^g\left(\rho + g - \log\frac{P_0}{P^*}\right) - \frac{1}{2}\frac{1}{\tau^g}\left(\omega_0^g\right)^2\sigma^2 + \log P_0\right). \end{aligned} \tag{A.29}$$

Here, the last line ignores the terms that are exogenous to the equilibrium.

A.4.2. Government's constrained Pareto problem in a simpler model

Note that, in period 0, the government's choice *can* affect output. Thus, the government takes into account other agents' utilities as well as its own utility. To simplify the setup, we merge the other agents (banks and households) into a single agent, which we refer to as *the market*, with risk tolerance τ^m. This enables the government to evaluate agents' expected utility with an exogenous risk tolerance (as opposed to an endogenous risk tolerance that depends on, among other things, distributional considerations that are not our focus). We next calculate the market's equilibrium utility in period 0. We then combine it with the government's utility in period 0 and describe a constrained Pareto problem.

The simpler model adopts many features of the original model. In particular, the market's equilibrium utility is given by [cf. $(A.11)$],

$$\begin{aligned} u_0^m\left(A_0^m\right) &= \left(1-e^{-\rho}\right)\log\left(A_0^m - a_0^m\right) + e^{-\rho}\log\left(R_0^{CE,m} a_0^m\right) \\ &= \left(1-e^{-\rho}\right)\log\left(\left(1-e^{-\rho}\right)A_0^m\right) + e^{-\rho}\log\left(R_0^{CE,m} e^{-\rho} A_0^m\right) \\ &= ... + \left(1-e^{-\rho}\right)\log A_0^m + e^{-\rho}\left(\log R_0^{CE,m} + \log A_0^m\right). \end{aligned}$$

Here, the second line substitutes the optimal consumption from (4). The last line simplifies the expression and ignores the exogenous terms. In equilibrium, the market is endowed all of the initial wealth, $A_0^m = y_0 + z_0 P_0 = \frac{z_0 P_0}{e^{-\rho}}$ [see (9)]. In addition, the market's certainty equivalent return satisfies an analogue of Eq. $(A.28)$. Combining these observations, the market's (approximate) utility is given by,

$$\begin{aligned} u_0^m &= ... + \left(1-e^{-\rho}\right)\log\frac{z_0 P_0}{e^{-\rho}} + e^{-\rho}\left(\log R_0^{CE,m} + \log\frac{z_0 P_0}{e^{-\rho}}\right) \\ &= ... + \left(1-e^{-\rho}\right)\log P_0 + e^{-\rho}\left(\begin{array}{c} \left(1-\omega_0^m\right) r_0^f + \omega_0^m\left(\rho + g - \log\frac{P_0}{P^*}\right) - \frac{1}{2}\frac{1}{\tau^m}\left(\omega_0^m\right)^2\sigma^2 \\ + \log P_0 \end{array}\right) \end{aligned} \tag{A.30}$$

We next turn to the government's constrained Pareto problem. Let ξ denote the government's Pareto weight on its own utility relative to the market's utility. In general, the government solves,

$$\begin{aligned} &\max_{\omega_0^g} \xi u_0^g + u_0^m \\ &\text{s.t. } P_0\left(\omega_0^g\right), r_0^f\left(\omega_0^g\right) \text{ are determined in equilibrium} \\ &\text{and } \omega_0^m + \eta^g \omega_0^g = 1 + \eta^g. \end{aligned} \tag{A.31}$$

The second line explicitly states the dependence of the price and the interest rate on the government's portfolio choice. The last line is the asset market clearing condition [cf. (30)].

To simplify the problem further, we focus on a special case in which the government's relative Pareto weight (on own utility) coincides with the relative size of its endowment,

$$\xi = \eta^g. \tag{A.32}$$

For this case, using Eqs. $(A.29)$, $(A.30)$, and $(A.31)$, the government's objective function becomes,

$$\eta^g u_0^g + u_0^m = ... + \left(1 - e^{-\rho}\right) \log\left(P_0\right) - \frac{1}{2} e^{-\rho} \sigma^2 \left(\eta^g \frac{1}{\tau^g} \left(\omega_0^g\right)^2 + \frac{1}{\tau^m} \left(\omega_0^m\right)^2\right).$$

As before we have ignored constant terms (including $\rho + g + \log P^*$). Note that the Pareto weight in $(A.32)$ helps cancel most of the terms that feature the asset price or the interest rate. Intuitively, changes in the asset price or the interest rate result in pecuniary externalities that raise one agent's utility while reducing the other agent's utility. With an appropriate choice of the Pareto weight, these pecuniary externalities "net out." The asset price for the current period, $\log\left(P_0\right)$, *does not cancel* because it represents *aggregate demand externalities*. Specifically, this term captures the impact of asset price changes on output and the market's consumption in period 0 [see $(A.30)$].

In sum, when the market consists of a representative agent and the government uses the relative Pareto weight in $(A.32)$, the government solves the constrained Pareto problem,

$$\max_{\omega_0^g} \left(1 - e^{-\rho}\right) \log\left(p_0\left(\omega_0^g\right)\right) - \frac{1}{2} e^{-\rho} \sigma^2 \left(\eta^g \frac{1}{\tau^g} \left(\omega_0^g\right)^2 + \frac{1}{\tau^m} \left(1 - \eta^g \left(\omega_0^g - 1\right)\right)^2\right). \tag{A.33}$$

Here, we substituted the normalized price, $p_0 = \frac{P_0}{P^*}$ (and ignored the constant term $\log P^*$). We have also substituted $\omega_0^m = 1 - \eta^g \left(\omega_0^g - 1\right)$ from the market clearing condition $(A.31)$. Note that the government's objective function features three terms. The first term, $\left(1 - e^{-\rho}\right) \log\left(p_0\left(\omega_0^g\right)\right)$, captures the government's desire to close the output gap in period 0. In our model, this is equivalent to closing asset price gaps [see (9)]. The second term, $\frac{1}{\tau^g}\left(\omega_0^g\right)^2$, captures the disutility from the risk in the government's portfolio. The remaining term captures the disutility from the risk in the representative agent's (the market's) portfolio, $\frac{1}{\tau^m}\left(1 - \eta^g\left(\omega_0^g - 1\right)\right)^2$. Hence, the government trades off the macroeconomic stabilization objectives with the optimal allocation of risk.

A.4.3. Optimal LSAPs and comparative statics

We next use the constrained Pareto problem for the simpler model to set up a planning problem for the original model. Specifically, we assume the government maximizes the objective function in $(A.33)$ with $\tau^m = \tau(1)$—the benchmark effective risk tolerance, $\tau(1)$. Put differently, that the government effectively ignores the changes in the effective risk tolerance due to the supply shock.[20] Formally, the government solves Eq. (36) that we state in the main text,

$$\max_{\lambda \geq 0} \left(1 - e^{-\rho}\right) \log\left(p_0(\lambda)\right) - \frac{1}{2} e^{-\rho} \sigma^2 \left(\eta^g \frac{1}{\tau^g} \left(1 + \frac{\lambda}{\eta^g}\right)^2 + \frac{1}{\tau(1)} (1 - \lambda)^2\right).$$

Here, we have also written the problem in terms of the size of the LSAPs program, $\lambda = \eta^g \left(\omega_0^g - 1\right)$.

[20] This can be viewed as a conservative assumption since the shock reduces the effective risk tolerance, which would make the government even more willing to absorb risk via LSAPs.

We solve this problem for the case in which the government is weakly less risk tolerant than the market,

$$\tau^g \leq \tau^m = \tau(1). \tag{A.34}$$

This ensures that, if there was no demand recession, the government would *not* use LSAPs. That is, the reason for LSAPs in our model is *not* a financial friction. Instead, the government uses LSAPs to respond to the demand recession when it cannot cut interest rates. To see this, suppose there is a unique and interior equilibrium price denoted by $p_0(\lambda)$ [see Proposition 2]. The condition for an optimum with a positive LSAPs, $\lambda > 0$, is then given by

$$\left[\frac{1}{\tau^g}\left(1+\frac{\lambda}{\eta^g}\right) - \frac{1}{\tau(1)}(1-\lambda)\right]\sigma^2 = \frac{1-e^{-\rho}}{e^{-\rho}}\frac{d\log p_0(\lambda)}{d\lambda}. \tag{A.35}$$

Eq. (A.35) says that the government stops purchasing risky assets when its marginal cost of portfolio risk relative to the market (the left side) is proportional to the marginal price impact, $\frac{d\log p_0(\lambda)}{d\lambda}$. If this price impact was zero, then the corner solution $\lambda = 0$ would be optimal since the government is relatively less risk tolerant. When the economy is in a demand recession, the price impact is strictly positive, $\frac{d\log p_0(\lambda)}{d\lambda} > 0$ [see Proposition 2], so the government might find it optimal to use LSAPs.

Eq. (A.35) also suggests that the size of the optimal LSAPs satisfies intuitive comparative statics (which we verify in numerical simulations). The optimal LSAPs is increasing in the government's risk tolerance, $\tau^g > 0$, and its tax capacity, $\eta^g > 0$. Greater capacity helps because it enables the government to achieve the same impact on financial markets with a smaller impact on its own risk exposure.

More subtly, Eq. (A.35) suggests that factors that increase the asset price impact of LSAPs, $\frac{d\log p_0(\lambda)}{d\lambda}$, raise the optimal LSAPs. Proposition 3 in Section 3.1 suggests this price impact is greater when the supply shock is more severe (lower z_0) or the private sector initially has greater leverage (greater l). We verify that these comparative statics typically hold in numerical simulations (as long as banks are not bankrupt under the optimal LSAPs). Figure 9 illustrates these results for the parameters in our earlier analysis (see Figures 2 and 4). We set the government's risk tolerance to be the same as the market's risk tolerance, $\tau^g = \tau(1)$. The government optimally chooses to use LSAPs. The left panel shows that increasing the severity of the shock increases the size of the optimal LSAPs. The right panel shows that increasing banks' initial leverage has the same effect. In this panel, as we increase l we also adjust banks' risk tolerance to keep the effective benchmark risk tolerance $\tau(1)$ unchanged (which leads to a more meaningful comparison).

A.5. Price impacts and asset demand elasticities

In this appendix, we prove Proposition 3 that characterizes the price impact of supply shocks and LSAPs we well as the asset demand elasticities in our setting. Consider the neighborhood of a stable and interior equilibrium (e.g., the case with a unique equilibrium) given $\lambda \geq 0$ (that is, with or without LSAPs). We let q_0^i denote the demand for the market portfolio from type i agents, *after adjusting for the effect of the price on their wealth*, that is

$$q_0^b = \omega_0^b(p_0)\,\alpha(z_0 p_0) \text{ and } q_0^h = \omega_0^h(p_0)\,(1-\alpha(z_0 p_0)).$$

Recall that with LSAPs agents' portfolio weights, ω_0^i, are still given by Eq. (17). Likewise, agents' wealth shares, α and $1-\alpha$, are still given by Eq. (15). Therefore, the size of the LSAPs policy, $\lambda \geq 0$, does not

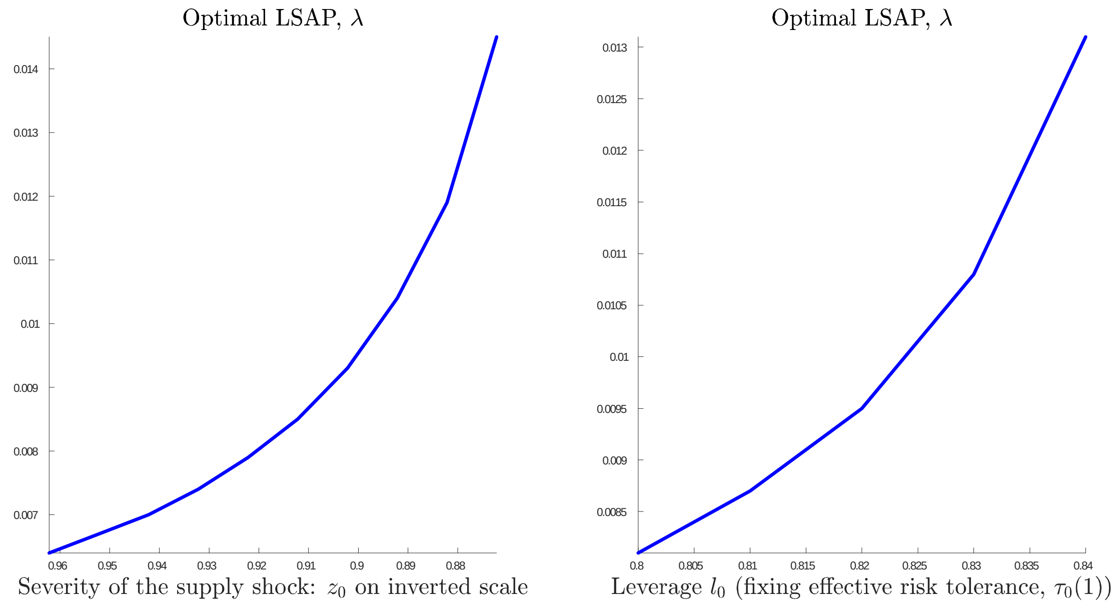

Figure 9: Optimal LSAP as a function of productivity, z_0 (left panel, inverted scale) and banks' initial leverage, l_0 (right panel).

directly affect agents' demand (it affects demand indirectly through the equilibrium price, p_0). Recall also that we use $q_0 = q_0^b + q_0^h$ to denote the *aggregate* asset demand. The equilibrium obtains when $q_0 = 1 - \lambda$ [see (30)].

Proof of Proposition 3. We first derive the price impact of supply shocks and LSAPs. Totally differentiating the equilibrium condition, $\log q_0 = \log(1-\lambda)$, with respect to $\log z_0$, we prove Eq. (37),

$$\frac{d\log p_0}{d\log z_0} = \frac{\partial \log q_0}{\partial \log z_0}\left(\frac{\partial \log q_0}{-\partial \log p_0}\right)^{-1}.$$

Totally differentiating the same equilibrium condition with respect to λ, we also prove Eq. (38),

$$\frac{d\log p_0}{d\lambda} = \frac{1}{1-\lambda}\left(\frac{\partial \log q_0}{-\partial \log p_0}\right)^{-1}.$$

Here, we have used that the partial derivative of agents' demand with respect to the size of the LSAPs program is zero, $\frac{\partial q_0^i}{\partial \lambda} = \frac{\partial q_0}{\partial \lambda} = 0$, because the LSAPs affect agents' demand only through their impact on the asset price.

We next derive the asset demand impact of a supply shock, $\frac{\partial \log q_0}{\partial \log z_0}$. In the main text, we characterize the impact on banks' demand, q_0^b. Consider the impact on households' demand, q_0^h. Using Eqs. (15) and (17), we calculate

$$\begin{aligned} \log q_0^h &= \log\left(\omega_0^h(p_0)\left(1-\alpha(z_0 p_0)\right)\right) \\ &= \log\left(\frac{\tau^h}{\sigma^2}\left(\rho + g - \log p_0 - r_0^f\right)\right) + \log\left(1 - \kappa + \frac{l}{z_0 p_0}\kappa\right). \end{aligned} \tag{A.36}$$

Differentiating with respect to log productivity, and evaluating at the equilibrium price, we obtain,

$$\frac{\partial \log q_0^h}{\partial \log z_0} = -\frac{\frac{l}{z_0 p_0}\kappa}{1-\kappa+\frac{l}{z_0 p_0}\kappa}. \tag{A.37}$$

Unlike with banks, an increase in asset valuations (driven by a positive productivity shock) decreases households' demand [cf. (42)]. High asset prices increase households' wealth less than banks' wealth. Therefore, high asset prices decrease households' wealth share, $1-\alpha(z_0 p_0)$, which in turn decreases their purchasing power and demand.

Next consider the impact on the aggregate demand, $q_0 = q_0^b + q_0^h$. We have,

$$\begin{aligned}
\frac{\partial \log q_0}{\partial \log z_0} &= \frac{q_0^b}{q_0}\frac{\partial \log q_0^b}{\partial \log z_0} + \frac{q_0^h}{q_0}\frac{\partial \log q_0^h}{\partial \log z_0} \\
&= \frac{q_0^b}{1-\lambda}\frac{\frac{l}{z_0 p_0}}{1-\frac{l}{z_0 p_0}} - \frac{q_0^h}{1-\lambda}\frac{\frac{l}{z_0 p_0}\kappa}{1-\kappa+\frac{l}{z_0 p_0}\kappa} \\
&= \left(\tau^b-\tau^h\right)\kappa\frac{\rho+g-\log(p_0)-r_0^f}{(1-\lambda)\sigma^2}\frac{l}{z_0 p_0} \\
&= \frac{\left(\tau^b-\tau^h\right)\frac{l}{z_0 p_0}\kappa}{\tau^h+\left(\tau^b-\tau^h\right)\left(1-\frac{l}{z_0 p_0}\right)\kappa}.
\end{aligned} \tag{A.38}$$

Here, the second line substitutes the demand impact formulas from Eqs. (42) and (A.37) as well as the market clearing condition, $q_0 = 1-\lambda$ [see (30)]. The third line substitutes for the equilibrium quantities using Eqs. (41) and (A.36). The last line substitutes the risk balance condition (35). This proves (39).

We finally derive the price elasticity of the aggregate asset demand, $\frac{\partial \log q_0}{-\partial \log p_0}$. In the main text, we characterize banks' demand elasticity. Differentiating Eq. (A.36) with respect to the log price, and evaluating at the equilibrium price, we obtain,

$$\frac{\partial \log q_0^h}{-\partial \log p_0} = \frac{1}{\overline{r}_0 - r_0^f} + \frac{\frac{l}{z_0 p_0}\kappa}{1-\kappa+\frac{l}{z_0 p_0}\kappa}. \tag{A.39}$$

The first term is the same as its counterpart with banks' elasticity, while the second term is different [cf. (43)]. Unlike with banks, the endogenous wealth channel *increases* households' demand elasticity. Intuitively, the decline in asset valuations, $z_0 p_0$, has the same impact on households' wealth share regardless of whether it is driven by a decline in productivity, z_0, or price per productivity, p_0. Therefore, through the wealth channel, a decline in p_0 has the same impact on households' demand as a decline in z_0 that we discussed earlier [cf. (A.37)].

Next consider the elasticity of the aggregate asset demand, $q_0 = q_0^b + q_0^h$. Following the same steps as in $(A.38)$, we obtain,

$$
\begin{aligned}
\frac{\partial \log q_0}{-\partial \log p_0} &= \frac{q_0^b}{q_0}\frac{\partial \log q_0^b}{-\partial \log p_0} + \frac{q_0^h}{q_0}\frac{\partial \log q_0^h}{-\partial \log p_0} \\
&= \frac{1}{\overline{r}_0 - r_0^f} - \left[\frac{q_0^b}{1-\lambda}\frac{\frac{l}{z_0 p_0}}{1-\frac{l}{z_0 p_0}} - \frac{q_0^h}{1-\lambda}\frac{\frac{l}{z_0 p_0}\kappa}{1-\kappa+\frac{l}{z_0 p_0}\kappa}\right] \\
&= \frac{1}{\overline{r}_0 - r_0^f} - \left(\tau^b - \tau^h\right)\kappa\frac{\rho + g - \log(p_0) - r_0^f}{(1-\lambda)\sigma^2}\frac{l}{z_0 p_0} \\
&= \frac{1}{\overline{r}_0 - r_0^f} - \frac{\left(\tau^b - \tau^h\right)\frac{l}{z_0 p_0}\kappa}{\tau^h + \left(\tau^b - \tau^h\right)\left(1 - \frac{l}{z_0 p_0}\right)\kappa}.
\end{aligned} \tag{A.40}
$$

This establishes Eq. (40) and completes the proof of the proposition. □

B. Appendix: Details of the Quantitative Exploration

In this appendix, we present the details of the calibration exercise we discuss in Remark 2 and Sections 1.2 and 3.2. Since the model is stylized, this calibration is suggestive: its main purpose is to show that the mechanisms we emphasize in the main text can be *quantitatively* large—at least in terms of the order of magnitudes.

To facilitate the calibration, we first slightly extend the model to introduce a *non-capitalized* factor such as labor and entrepreneurial capital. This extension allows for a more flexible target for the aggregate value of capitalized assets relative to the aggregate consumption. We then calibrate the model and discuss the plausibility of condition (21) (from Section 1.2) that determines whether partly temporary shocks ($\varphi \leq 1$) can induce a demand recession. We then consider the case with permanent shocks, $\varphi = 1$, and provide a graphical illustration of the calibrated equilibrium (the calibrated analogues of Figures 2 and 3). Finally, we provide a graphical illustration of the calibrated equilibrium with LSAPs (the calibrated analogue of Figures 4 and 5). These figures complement Figures 6 and 7 from Section 3.2 that plot the asset price in the calibrated equilibrium without and with LSAPs, respectively, for a range of productivity shocks.

Extension with a non-capitalized factor. Consider a version of the model in which the economy is endowed with n units of a non-capitalized factor in addition to one unit of capital (as before). For simplicity, the two factors are separable and equally affected by productivity. In particular, potential output is given by,

$$y_t^* = (n+1)\, z_t.$$

Absent nominal rigidities, factor incomes would be given by $y_t^n = \frac{n}{n+1} y_t^*$ and $y_t^k = \frac{1}{n+1} y_t^*$, respectively. We assume that a demand recession reduces the income that accrues to each factor proportionally, that is:

$$y_t^n = \frac{n}{n+1} y_t \quad \text{and} \quad y_t^k = \frac{1}{n+1} y_t.$$

As before, total output is determined by aggregate spending, $y_t = c_t$. To maintain the basic structure of the model, we also introduce hand-to-mouth agents that receive all of the non-capital income (see Remark 2). The remaining agents (banks and households) hold and trade the market portfolio—which now represents a claim on the capital income. The rest of the model is unchanged.

In this version of the model, aggregate spending is given by,

$$c_t = \frac{n}{n+1} y_t + \frac{1 - e^{-\rho}}{e^{-\rho}} z_t P_t.$$

After substituting $y_t = c_t$ and rearranging terms, we obtain,

$$y_t = c_t = \frac{1 - e^{-\rho}}{e^{-\rho}} y_t^* P_t, \text{ where } y_t^* = (n+1)\, z_t. \tag{B.1}$$

Hence, a version of the output-asset price relation (9) still applies. Setting $y_t = y_t^*$, the efficient asset price is the same as before, $P^* = \frac{e^{-\rho}}{1-e^{-\rho}}$ [see (10)]. However, the efficient capitalized wealth to consumption ratio is different, $\frac{P^*}{n+1}$, which allows for a more flexible calibration. The term, $n+1$, corresponds to a *Keynesian multiplier*: one dollar spending induced by capitalized wealth increases the equilibrium consumption and output by $n+1$ dollars—assuming the interest rate remains unchanged (see Remark 2 for an intuition).

The rest of the analysis is unchanged. In particular, in future periods monetary policy is unconstrained and capital income is equal to productivity, $y_t^k = z_t$ for each $t \geq 1$. Consequently, the asset price in period 0 is still determined by the risk balance condition (18). It follows our results from the main text (in particular, Propositions 1-3) apply also in this extension.

Calibration. We next calibrate the model to the US data. First consider the macroeconomic variables. Every period corresponds to a year. We identify output with aggregate consumption: specifically, we exclude government spending, investment, and net exports since these variables are driven by forces outside our model. We set ρ to target the (yearly) MPC out of wealth based on recent empirical estimates. Chodorow-Reich et al. (forthcoming) estimate an MPC out of *stock* wealth equal to 3 cents, and Mian et al. (2013) estimate an MPC out of *housing* wealth equal to 5-7 cents. We target the average of the two estimates and set $\frac{1-e^{-\rho}}{e^{-\rho}} \simeq \rho = 0.045$. We set n to target the aggregate household net worth to consumption ratio in the last quarter of 2019, $\frac{P^*}{n+1} \simeq 8$.[21] With $P^* = \frac{1}{0.045}$, this implies $n \simeq 1.7$. The implied share of non-capitalized income to total income is given by, $\frac{n}{n+1} = 0.63$, which is close to the labor share of income in the data. The implied Keynesian multiplier, $n + 1 = 2.7$, is relatively high but not too far from the typical empirical estimates. For instance, the meta analysis in Chodorow-Reich (2019) suggests that the aggregate zero lower bound multiplier is at least 1.7 (and it could be considerably greater than this level since the empirical estimates often identify a cross-sectional multiplier, and the aggregate zero lower bound multiplier exceeds the cross-sectional multiplier in standard models).

Next consider the asset pricing variables. We set banks' risk tolerance based on He et al. (2017). Specifically, they proxy "banks"' financial health with primary dealers' equity capital ratio, and they show that shocks to this measure explain the cross-section of returns for a broad set of asset classes. Their evidence and model suggests $\tau^b = \frac{1}{3.7} = 0.27$ (see their Table 5). We set households' risk tolerance to a relatively small fraction of banks' tolerance, $\tau^h = \tau^b/10$. We set $\kappa = 0.75$ to match the fraction of non-real-estate wealth relative to total household net worth at the end of 2019 (source: Financial Accounts of the US). This loosely captures the idea that most non-real-estate risk is intermediated, whereas most real-estate risk is directly held by households. Our analysis is robust to reasonable variations in τ^h and κ as long the ratio, $\frac{\tau^h/\kappa}{\tau^b - \tau^h}$, is relatively small.

The leverage parameter plays a more central role. Conceptually, it captures the decline in banks' wealth for a given decline in the value of their assets. We calibrate this parameter to match the relationship between the banks' losses and the banks' capital in the adverse scenario described in the Fed's June 2020 stress tests. The scenario projects an average loan loss rate of 6.3% for banks' portfolios and a decline in banks' (tier 1) capital ratio from 12% to 10%. In our model, banks' capital ratio is proportional to their wealth share, $\alpha(z) = 1 - \frac{l}{z}$. We match the projected loan loss in the scenario by assuming that z declines by 6.3%. We then match the decline in bank capital by solving, $\alpha(1 - 6.3\%) = \frac{10\%}{12\%}\alpha(1)$. This implies $l \simeq 0.7$ and $\frac{1}{1-l} \simeq 3.4$. Intuitively, a 6.3% portfolio loss generates a roughly 20% decline in bank capital when banks have a leverage ratio of slightly above 3.[22]

As we discuss in the main text, we also target a baseline (homogeneous-agent) demand elasticity,

[21] Specifically, in 2019 Q4, household net worth was \$118 trillion (source: Financial Accounts of the U.S., series FL152090005.Q), and the aggregate personal consumption expenditure was \$14.7 trillion (source: U.S. National Income and Product Accounts, table 1.1.5).

[22] For the details of the stress tests, see: https://www.federalreserve.gov/publications/files/2020-dfast-results-20200625.pdf

$\left(\overline{r}_0 - r_0^f\right)^{-1} = 2.63$, to achieve a pre-shock elasticity equal to 1 [see $(44-45)$]. This implies a large risk premium that we interpret as a stand-in for unmodeled frictions such as investment mandates that reduce the asset demand elasticity in practice (discussed in Gabaix and Koijen (2020)). For the risk-free interest rate, we target $r_0^f = 0.01$ to capture the distance between the policy interest rate and the zero lower bound at the onset of the Covid-19 shock. We hit these targets by choosing σ and g that jointly solve,

$$\begin{aligned}\overline{r}_0 - r_0^f &= \frac{\sigma^2}{\tau(1)} = 0.38 \\ \overline{r}_0 &= \rho + g = 0.39,\end{aligned}$$

where $\tau(1) = \tau^h + (1-l)\kappa\left(\tau^b - \tau^h\right) \simeq 0.08$ is already calibrated. The implied levels of the parameters, σ and g, are higher than their real-world counterparts. These parameters shouldn't be interpreted literally: they stand in for frictions missing from our stylized model (such as investment mandates that reduce the demand elasticity).

Can partly temporary supply shocks induce a demand recession? Consider the case in which shocks can be partly temporary, $\varphi \leq 1$. Recall from Section 1.2 that partly temporary shocks reduce demand more than the supply—and therefore induce the Fed to cut the interest rate—as long as condition (21) holds,

$$\frac{l/z_0}{\frac{\tau^h/\kappa}{\tau^b-\tau^h} + 1 - l/z_0} > (1-\varphi)\frac{1}{\overline{r}_0 - r_0^f}.$$

With our calibration, this condition becomes,

$$\frac{0.7/z_0}{0.16 + 1 - 0.7/z_0} > (1-\varphi)\, 2.63.$$

Hence, the condition implies a joint restriction about the severity of the supply shock, z_0, and its persistence, φ, that is:

$$z_0 < 0.62 + \frac{0.23}{1-\varphi}.$$

This condition is plausible. In the extreme case when the shock is fully transitory, $\varphi = 0$, the condition holds as long as $z_0 < 85\%$. When the shock is mildly persistent, $\varphi \simeq 0.19$, the condition holds for each productivity level below the benchmark, $z_0 < 1$. Hence, in the calibrated equilibrium, either sufficiently (though not unreasonably) large shocks or mildly persistent shocks is sufficient for supply shocks to reduce demand more than the supply.

Calibrated equilibrium without LSAPs. Next consider fully persistent shocks, $\varphi = 1$. Proposition 1 in the main text characterizes the equilibrium analytically. Figure 10 provides a graphical illustration of the equilibrium by plotting the actual and the required Sharpe ratios (cf. Figures 2 and 3). As before, the dashed lines correspond to the benchmark productivity level, $z_0 = 1$, and the solid lines correspond a lower productivity level, $z_0 = 0.97$. The equilibrium features multiplicity (both in the benchmark and after the shock). We focus on the comparative statics of the highest-price equilibrium. The figure illustrates that a relatively small productivity shock is sufficient to induce a demand recession with a low asset price per productivity, $p_0 \simeq 0.95$, even in the best equilibrium. Figure 6 in the main text plots the equilibrium price per productivity for a wider range of productivity shocks, z_0 (that include

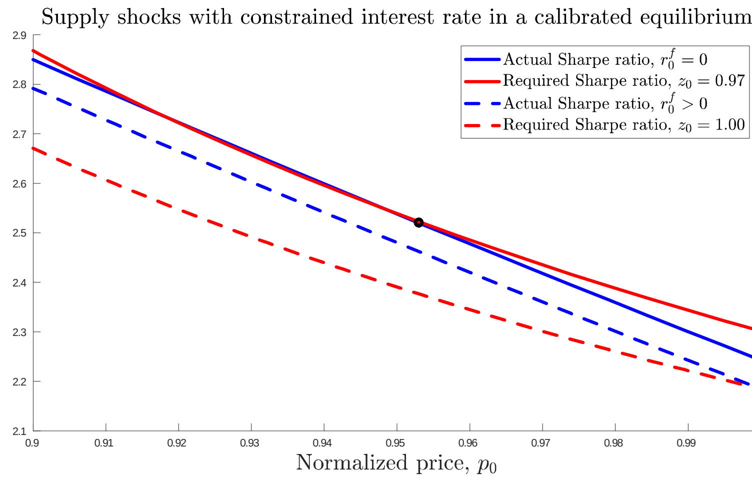

Figure 10: Effect of supply shocks in a calibrated equilibrium.

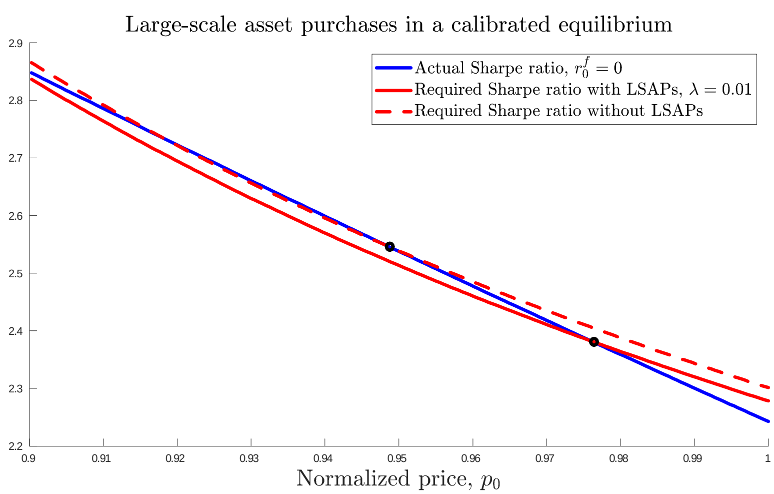

Figure 11: Asset price impact of LSAPs in a calibrated equilibrium.

$z_0 = 0.97$).

Calibrated equilibrium with LSAPs. We next consider the case with LSAPs, $\lambda = \eta^g (\omega_0^g - 1) \geq 0$. Proposition 2 in the main text characterizes the equilibrium analytically. We consider the productivity shock, $z_0 = 0.97$, and suppose the government purchases 1% of the asset supply, $\lambda = \eta^g (\omega_0^g - 1) = 1\%$. As before, we focus on the highest-price equilibrium when it exists. Figure 11 illustrates how the LSAPs affect this equilibrium. The LSAPs reduce the required Sharpe ratio and increase the asset price (in the highest-price equilibrium). The figure illustrates that a relatively small LSAPs policy has a sizeable impact on the equilibrium asset price (and therefore output). Figure 7 in the main text illustrates the asset price impact of this policy for a wider range of productivity shocks, z_0 (that include $z_0 = 0.97$).

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