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TERMS OF TRADE AND THE TRANSMISSION OF OUTPUT SHOCKS IN A RATIONAL EXPECTATIONS MODEL

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ABSTRACT

This paper analyses the effects of productivity shocks on the current and future terms of trade and on output in a two-country framework. An overlapping-generations model is used in which individuals allocate their savings between domestic and foreign capital assets according to their preferences for risk and return. Since production in both countries is specialized, changes in the terms of trade affect investment returns in both countries; rational expectations regarding such changes are assumed and a new approach to analyzing the comparative statics of rational expectations equilibria is developed. It is concluded that a temporary, positive productivity shock to the home country will cause the domestic terms of trade to depreciate initially and then to appreciate slowly back towards its trend level. The depreciation causes foreign output to fall below trend, and causes a symmetric rise in domestic output, via its effects on capital stocks. The impact of a permanent productivity shock differs, however. In this case investors will reallocate their portfolios and increase their holdings of domestic assets, which are expected to earn higher returns. If the portfolio shifts are strong enough, they cause the terms of trade to appreciate initially. Foreign output falls and domestic output rises in this case as well, this time because of the portfolio shifts towards domestic capital.

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The real exchange rate of the United Kingdom has been appreciating over the last few years. Most observers associate this with strong output growth associated, in turn, with the legislative and regulatory overhaul engineered by Prime Minister Thatcher. This paper proposes one explanation for the recent real appreciation of the pound that does connect a positive supply shock with an appreciating terms of trade; more generally, the paper discusses the interdependence between the level of output and the terms of trade.

The correspondence of rising output and an appreciating terms of trade strikes observers outside of academia as entirely consistent: intuitively they agree that a stronger economy should be associated with a stronger currency. Economists, on the other hand, have differing opinions regarding whether and when the association is to be expected. There is general agreement that an appreciation of the terms of trade should be expected when output rises as a result of a fiscal stimulus, other things equal. But the U.K. has been holding government spending in check during the last few years, rather than the opposite. Looking at the rise in output as a "supply shock" does not typically lead to the conclusion that such a conjunction is to be expected. Laursen and Metzler (1950) consider the effects of a positive supply shock (in the U.K., no less) on the terms of trade, and conclude that the rise in output would increase imports and thus drive down the terms of trade, rather than appreciate it. While their analysis applied to an economy without capital flows, a supply-side augmented Mundell-Flemming model of a large-country would also imply a depreciation of the terms of trade in response to a positive supply shock (Flemming 1962; Mundell 1968). A simple monetary model of a floating exchange rate (Dornbusch 1976) would imply an appreciation of the nominal exchange rate upon an expansion of long run supply, but no change in the terms of trade because these models impose purchasing power parity. More recent to analyses of the effects of supply shocks, including Flood and Marion (1982) and Aizenman and Frenkel (1985), also imply a nominal appreciation but not a real one.

We find here that the effects of a current productivity shock on the terms of trade depends on whether and how strongly the shock is expected to persist. More specifically, we find that if the shock is not expected to persist, then the relative price of domestic output must initially fall -- the terms of trade must depreciate -- to induce consumers to purchase the excess supply. On the other hand, if the shock is expected to persist, portfolios will shift towards domestic equities in anticipation of higher domestic equity returns; if the rise in future expected productivity, and portfolio demand, is strong enough, then this shift in asset demand will cause the terms of trade to appreciate.

In addition to considering the effect of output shocks on the terms of trade, we consider a reverse association: the effect of changes in the terms of trade on the level of output, via their effects on capital stocks. To illustrate this connection concretely, if simplistically, note that with the dollar at its recent level of 125 ¥/$, 5 trillion ¥ buys twice as many assembly plants as it did just a few years ago. This has numerous implications, the most important of which is that a temporary increase in domestic output, by depreciating the terms of trade, will decrease the foreign capital stock and reduce expected foreign output, with the opposite effect on expected domestic output, for a period extending beyond the duration of the shock itself. Thus, via the terms of trade and international equities markets, a positive output disturbance at home can be transformed into a future decline in foreign output and a rise in domestic output.

Laursen and Metzler also analyze the effects of an output shock in one country on the output of another country, given flexible exchange rates. While their paper and this one both conclude that a positive output shock in one country will tend to cause a contraction in output in the other country, the driving factors behind the results in these two works could hardly be more dissimilar. For Laursen and Metzler the crucial factor was their assertion that a depreciation of a country's terms of trade would, by raising the real income of foreigners, cause them to save more out of a given money income. Through the multiplier, this decline in foreign consumption would be reflected in a decline in their output. In the present paper a change in the terms of trade has no immediate effect on foreign savings or output, but instead affects future capital stocks and output in the manner described above.

While the focus of this paper is on the interaction between output and the level of the terms of trade, we undertake this analysis in a framework in which the affects of current and prospective changes in the terms of trade on portfolios, consumption, and the balance of payments is incorporated scrupulously. We modify Diamond's (1965) OLG model to include two countries who produce specialized goods under conditions of stochastic technology. The shocks to productivity, which are imperfectly correlated across countries, cause domestic and foreign capital returns to be randomly distributed and imperfectly correlated, as well. The claims on these returns, referred to as "equities," are freely traded across countries. Consumers allocate their current expenditures across domestic and foreign goods and also allocate their savings among domestic and foreign equities, as well. Anticipated changes in the terms of trade, which impinge on expected total returns to portfolio investments, are important determinants of investor choices. To operationalize our assumption that these expectations are formed rationally we develop a new approach to approximating the comparative statics of rational expectations equilibria. This approach is tested via simulations of the model and is found to work, in the sense that for small disturbances the equilibria, including the implied time path of the terms of trade, are consistent with consumers' first-order conditions.

Our model is one in which individuals live for two "periods." While the interpretation of a "period" is left intentionally vague, it is certainly long enough for money neutrality to be a reasonable assumption. In consequence, the model has no monetary elements, and instead our analysis highlights the importance of equities markets, which we show have effects distinct from those of bonds and money markets.

The paper is divided into four parts. The next section, Part II, describes the model. This is followed by an analysis of the effects of output disturbances; we consider first a temporary rise in domestic output, and then rise in output which is expected to persist indefinitely, thirdly an anticipated change in the distribution of output, and finally an unanticipated shock to output that changes the anticipated distribution of output in the future. In Part IV we conclude.

PART II: THE MODEL

Production

There are two countries, a "home" country and a "foreign" country, which produce distinct goods and freely trade both output and equities. Taking the domestic good as numeraire, the relative price of the foreign good in terms of the domestic good, which we will refer to as the "terms of trade" or the "real exchange rate"[1], will be denoted p_0. (Unless otherwise specified, the word "domestic" will refer to the home country, and foreign variables will be denoted by a superscript ^.)

Output of each domestic firm, where firms are indexed by i, is generated from inputs of labor and capital according to the stochastic production function

$$Q_t^i = F(K_t^i, L_t^i) + \alpha_t K_t^i \quad ,$$

where α_t is an i.i.d. random variable distributed over the interval $[-d,d]$, $0 < d < 1$, with mean zero and variance σ^2.[2,3] Under the assumption that $F(K,L)$ is linear-homogeneous we can re-express production in terms of per-worker output, $q = Q/L$, and the capital-labor ratio, $k = K/L$:

$$q_t^i = f(k_t^i) + \alpha_t k_t^i \quad .$$

We will also assume that $\lim_{k \to \infty} -kf''(k) = 0.$

The exogenously determined labor force is fully employed in each period, as is the endogenously determined stock of capital. Factor markets are assumed to be perfectly competitive,[4] in consequence of which we can express aggregate output in terms of total capital and labor:

$$Q = F(K,L) + \alpha K = Lq = L[f(k) + \alpha k].$$

Another implication of perfect competition in the factor markets is that labor and capital will always be paid their marginal product: wages will be $w = f(k) - kf'(k)$, and the actual return to capital will be $\underline{r} = f'(k) + \alpha$, with expectation $r = f'(k)$.

Foreign production technology will also be determined according to a stochastic technology:

$$\hat{Q} = F(\hat{K}, \hat{L}) + \hat{\alpha}\hat{K}.$$

The random productivity shocks, α at home and $\widetilde{\alpha}$ abroad, have mean zero, variances σ^2 and $\widetilde{\sigma}^2$, respectively, and correlation coefficient ρ, which we assume is unequal to unity. We will assume throughout that the two countries are symmetric, which implies $\sigma^2 = \widetilde{\sigma}^2$

Consumption

During any period t a new generation of individuals is born in each country which is n percent larger than the previous generation. Each member of the new generation of the home country, who will live for two periods, works during period t earning the prevailing wage, w_t, consumes some (proper) fraction of his income, and invests the rest. Savings nust be allocated between home country investments and investments abroad. It is convenient to imagine that savings is used to purchase equities from firm managers, who in turn employ the income derived from equity sales as capital. Domestic equities are denominated in terms of the domestic good while foreign equities are denominated in terms of the foreign good; their stochastic return is identical to the own-good return per unit of capital employed. The members of the older generation do not work and consume their savings, which will have accrued interest as well as capital gains -- due to changes in the terms of trade -- on the foreign investments .

All domestic individuals have the same utility function: $U(c_1) + \beta U(c_2)$, where $0 < \beta \leq 1$. We assume that $U(\cdot)$ is twice continuously differentiable, with $U' > 0$, $U'' < 0$. To simplify the mathematics of uncertainty, we specify the following functional form for $U(\cdot)$:

$$U(c) = \frac{c^{1-\gamma}}{1-\gamma} ,$$

where

$$c = c_h^{\mu} c_f^{1-\mu} .$$

This form is convenient in part because expenditure on the domestic good will always be the fraction μ of total current expenditure, with share 1 - μ for the foreign good. This allows imperfect substitutability between the two outputs but is also analytically tractable under uncertainty.

Foreign utility will be: $\widehat{U}(\widehat{c}_1) + \beta\widehat{U}(\widehat{c}_2)$, where

$$\widehat{U} = \frac{\widehat{c}^{1-\gamma}}{1-\gamma} , \quad \text{and} \quad \widehat{c} = \widehat{c}_h^{\mu} \widehat{c}_f^{1-\mu} .$$

It is convenient to express retirement utility as an indirect function of a consumers savings and its return. To evaluate this, we must first discuss the appropriate units of account. With the price of the domestic good at unity, the perfect price index for domestic consumers is $p_0^{1-\mu}$: that is, a correct gauge of the consumption-value of any amount of income received in domestic goods, such as w, is $w/p_0^{1-\mu}$ For foreigners, the equivalent expression for income received in terms of the foreign good is $\hat{w}p_0^{\mu}$. In essence we have provided a measure of consumption in a composite currency. Thus the "real value", as we will refer to it, of savings in the first period of a domestic resident's life is

$$(w - c_h - p_0 c_f)/p_0^{1-\mu} = s/p_0^{1-\mu} \tag{3}$$

Real retirement consumption will be:

$$W^R = \left(\frac{s}{p_1^{1-\mu}}\right)\left[\pi(1+r+\alpha) + (1-\pi)(1+\hat{r}+\hat{\alpha})\left(\frac{p_1}{p_0}\right)\right] . \tag{4}$$

The gross return to savings is then (4) divided by (3), or

$$R = \pi(1+r+\alpha)\left(\frac{p_0}{p_1}\right)^{1-\mu} + (1-\pi)(1+\hat{r}+\hat{\alpha})\left(\frac{p_1}{p_0}\right)^{\mu} .$$

Using this expression we can construct the indirect utility function:

$$U(c_2) = U\left\{\left[\mu\frac{sR}{p_0^{1-\mu}}p_1^{1-\mu}\right]^{\mu}\left[(1-\mu)\frac{sR}{p_0^{1-\mu}}\left(\frac{1}{p_1^{\mu}}\right)\right]^{1-\mu}\right\}$$

$$= V\left(\frac{sR}{p_0^{1-\mu}}\right)$$

It is possible to simplify the expression for real returns further. Denote the rate of change of the terms of trade as $x_0 = (p_1 - p_0)/p_0$, with expected value $\bar{x}_0$ and variance σ_x^2. Assuming x_0 is sufficiently small, we can re-express the return to domesic savings as folows:

$$R = 1 + \pi(r+\alpha) + (1-\pi)(\hat{r}+\hat{\alpha}) + (\mu-\pi)x_0 ,$$

with expected value

$$\bar{R} = 1 + \pi(1+r) + (1-\pi)(1+\hat{r}) + (\mu-\pi)\bar{x}_0 . \tag{5}$$

and variance

$$Var(R) = \pi^2\sigma^2 + (1-\pi)^2\hat{\sigma}^2 + (\mu-\pi)^2\sigma_x^2$$
$$+ 2[\pi(1-\pi)\sigma\hat{\sigma}\rho + \pi(\mu-\pi)\eta + (1-\pi)(\mu-\pi)\hat{\eta}] . \tag{6}$$

The derivation of this approximation for R is spelled out in Appendix A. It comprises a weighted average real return to domestic and foreign equities, measured in their own goods, and a term which measures the changes in portfolio purchasing power associated with changes in the terms of trade. This latter term, $(\mu - \pi)x_0$, can be understood by rewriting it as $[(1 - \pi) - (1 - \mu)]x_0$. $(1 - \mu)$ will be the share of second-period income spent on the foreign good, while $(1 - \pi)$ will be, roughly, the proportion of income derived from equities denominated in that good. If the relative price of the foreign good increases between this period and next, and the share of equity income from abroad, $(1-\pi)$, equals the share of expenditure that will be devoted to foreign goods, $(1-\mu)$, the consumer will experience no loss of purchasing power. If $(1 - \pi) > (1 - \mu)$, there is an increase in the amount of the domestic good which can be purchased, if purchases of the foreign good are kept constant. This increase in real purchasing power can be approximated as $[(1 - \pi) - (1 - \mu)]x_0$ or $(\mu - \pi)x_0$.[5]

Domestic residents will be paid in domestic output; they choose consumption of the home good, c_h, savings measured in terms of the home good, s, and the share of domestic equities in their portfolio, π, to

$$\text{Maximize} \quad U(c_h, c_f) + \beta E\{V(s^R R)\}$$

subject to the following budget constraint:

$$c_f = (w - s - c_h)/p_0 \ .$$

Foreigners are paid in terms of foreign output and choose c_f, s and π subject to the constraint that

$$\hat{c}_h = (\hat{w} - \hat{s} - \hat{c}_f)p_0 \ .$$

Notice that we have attached a time subscript to the terms of trade, but not to other variables. Subscripts for the other variables have been eliminated whenever possible to reduce notational clutter. The subscripts for p and for x are maintained because both today's terms of trade and tomorrow's are important to current decisions. "p_0" will denote the current terms of trade, the one that determines current consumption of domestic and foreign goods; "x_0" refers to the change in terms of trade relevant to today's portfolio decisions, $x_0 = (p_1 - p_0)/p_0$.

Domestic consumers' three first-order conditions are:

$$U_h = \frac{U_f}{p_0} \tag{7a}$$

$$\frac{U_f}{p_0} = E\{V'R\} \tag{7b}$$

$$E\{V'\}(r - \hat{r} - \bar{x}_0) = - E\{V'[\alpha - \hat{\alpha} - (x_0 - \bar{x}_0)]\} \ . \tag{7c}$$

The first of these is the standard expenditure allocation condition equating the marginal rate of substitution between domestic and foreign goods with the price ratio. Equation (7b), which describes consumers' optimal savings level, requires that the gain in first period utility from a decline in savings equals the loss in expected retirement utility. Equation (7c) describes equilibrium portfolio shares. A change in π_t will affect expected retirement utility through the expected return to savings, R_{t+1} and through the rest of the distribution of returns. Equilibrium condition (7c) states that at the margin, the change in expected utility from these factors should be equal and opposite.

Portfolio shares will be independent of individual wages and savings, another convenient property of the utility function we have chosen. They depend on the distributions of expected returns, including the distribution of the terms of trade, on risk preference, and on consumption preference. The importance of consumption preference is best illustrated with an example: if all expected returns are the same, as is the variability of returns, then a consumer with a preference for domestic goods ($\mu > 1/2$) will hold a portfolio with

$$\mu \ > \ \pi \ > \ 1/2 \ .$$

The benefits of diversification would always give individuals a preference for $\pi = 1/2$. In this case, these benefits compete with the fact that by tailoring asset portfolios exclusively to goods preferences, individuals can minimize their exposure to terms of trade risk, which would lead individuals to prefer $\pi = \mu$. Though this is an important issue, it is not one with which this paper is concerned, so we will assume throughout that $\mu = \hat{\mu} = 1/2$. Since we are also assuming identical risk preference across countries, we can conclude that domestic and foreign portfolios will always be identically distributed among the two types of assets, or $\pi = \hat{\pi}$.

Equilibrium: An Overview

An important property of each equilibrium is that claims to the ownership of capital will be disbursed around the world, except under unusual circumstances. For example, the domestic capital stock will comprise investments by both domestic and foreign consumers, as will its foreign counterpart:

$$k_1 = \frac{\pi_0 s_0 + \hat{\pi}_0 \hat{s}_0 p_0}{1 + n} \ , \qquad \hat{k}_1 = \frac{(1-\pi_0)s_0/p_0 + (1-\hat{\pi}_0)\hat{s}_0}{1 + n} \ .$$

Note that the composition of these capital stocks is significantly affected by the terms of trade. For example, foreign purchases of domestic equities, $\widehat{\pi s}$, are chosen in terms of the foreign good, but when the domestic assets are purchased, the number of units of domestic capital represented by $\widehat{\pi s}$ is $\widehat{\pi s}p/(1+n)$. This implies that a depreciation (rise) today's terms of trade will raise tomorrow's domestic capital stock, other things equal; likewise it will reduce tomorrow's foreign capital stock. Other things won't be equal, since the terms of trade is endogenous and it won't change unless other things are also changing, but this connection between capital stocks and the terms of trade will have numerous implications for economic equilibria.

Balance-of-payments equilibrium requires:

$$
\begin{aligned}
&(w_0 - s_0 - c_{h0}) + (1-\mu)\left(\frac{s_{-1}}{1+n}\right)\left[\pi_{-1}(1+r_0+\alpha_0) + (1-\pi_{-1})(1+\hat{r}_0+\hat{\alpha}_0)\right] \\
&\qquad\qquad + (1-\pi_0)s_0 - \left(\frac{s_{-1}}{1+n}\right)(1-\pi_{-1})(1+\hat{r}_0+\hat{\alpha}_0)\left(\frac{p_0}{p_1}\right) \\
&= p_0\left\{(\hat{w}_0-\hat{s}_0-\hat{c}_{f0}) + \hat{\mu}\left(\frac{\hat{s}_{-1}}{1+n}\right)\left[\hat{\pi}_{-1}(1+r_0+\alpha_0)\left(\frac{p_{-1}}{p_0}\right) + (1-\hat{\pi}_{-1})(1+\hat{r}_0+\hat{\alpha}_0)\right]\right\} \\
&\qquad\qquad + p_0\hat{s}_0\hat{\pi}_0 - \left(\frac{\hat{s}_{-1}\hat{\pi}_{-1}}{1+n}\right)(1+r_0+\alpha_0)p_{-1} \ .
\end{aligned}
$$

This condition says that net domestic demand for foreign output (the left-hand-side), including consumption demand as well as net domestic demand for foreign assets, must equal net foreign demand for domestic goods (the right-hand-side), when both sides are measured in a common unit of account (in this case, domestic output). More explicitly, the first line on each side of the equals sign refers to goods demand while the second line refers to capital purchases, capital repatriation, and serivce income from capital. Together with the two budget constraints this balance of payments equilibrium condition suffices to ensure that demand equals supply in each goods market.

The equations for the capital account and the trade balance in period 0 are:

$$
\kappa_0 = \left(\hat{\pi}_0\hat{s}_0 p_0 - (1-\pi_0)s_0\right) - \left(\frac{1}{1+n}\right)\left(\widehat{\pi_{-1}}\widehat{s_{-1}}p_{-1} - \frac{p_0}{p_{-1}}(1-\pi_{-1})s_{-1}\right),
$$

$$
tb_0 = \left(\frac{1}{1+n}\right)\left((r_0+\alpha_0)\widehat{\pi_{-1}}\widehat{s_{-1}}p_{-1} - (\hat{r}_0+\hat{\alpha}_0)\left(\frac{p_0}{p_{-1}}\right)(1-\pi_{-1})s_{-1}\right) - \kappa_0 .
$$

The world enters each period with predetermined capital stocks. Via competitive equilibrium, these will determine the wages of the young and, in conjunction with realizations of α and $\hat{\alpha}$, they also determine the actual return to equities. Individual consumption, savings, and portfolio choices, as well as the terms of trade, will be simultaneously determined each period. These will jointly determine the capital stocks for the succeeding period.

There will be no steady-state equilibrium in the true sense, since output, the returns to capital, and the terms of trade are all random. We can characterize the long-run by looking at the values of k and $\hat{k}$ to which these random variables would tend were the output shocks, α and $\hat{\alpha}$, to happen to be realized at their expected values of zero for a long period of time. This is a measure of central tendency, though not the expected value.

$$k(w,\hat{w}) = \frac{\pi^*(.)s^*(.) + \hat{\pi}^*(.)\hat{s}^*(.)p^*}{1 + n}$$

$$\hat{k}(w,\hat{w}) = \frac{[1-\pi^*(\cdot)]s^*(\cdot)/p^* + [1-\hat{\pi}^*(\cdot)]\hat{s}^*(\cdot)}{1 + n}$$

where

$$w^* = f[k(w^*,\hat{w}^*)] - k(w^*,\hat{w}^*)f'[k(w^*,\hat{w}^*)]$$

and

$$\hat{w}^* = f[\hat{k}(w^*,\hat{w}^*)] - \hat{k}(w^*,\hat{w}^*)f'[\hat{k}(w^*,\hat{w}^*)]$$

and the arguments in $s^*(.)$ and $\pi^*(.)$ are w^*, $\hat{w}^*$, r^*, $\hat{r}^*$, σ^2, $\hat{\sigma}^2$, ρ, η, and n. Since k and $\hat{k}$ are bounded,[6] this system has a nonempty set of fixed points (LaSalle 1976). For the case of symmetric countries, we will assume that the solution $k^* = \hat{k}^*$ is one of these fixed points, and in fact is the unique fixed point.

Equilibrium: An In-Depth View

Having sketched out briefly the nature of equilibria in this world, we will now analyse more closely the determination of equilibrium in each period. Specifically, we will consider the equilibrium conditions which characterize each period and we analyse how a change in the equilibrium today affects the equilibrium tomorrow.

The capital stock definitions and the balance of payments conditions represent three equilibrium conditions for the temporary equilibrium in our international system. The consumers' first-order conditions corresponding to c_{h0}, s_0 π_0, $\hat{c}_{f0}$, $\hat{s}_0$, and $\hat{\pi}_0$, represent six more. The factor-market clearing conditions, corresponding to w_0, z_0, $\hat{w}_0$, and $\hat{z}_0$, are another four:

$$w_0 = f(k_0) - k_0 f'(k_0) \qquad \hat{w}_0 = f(\hat{k}_0) - \hat{k}_0 f'(\hat{k}_0)$$

$$r_0 = f'(k_0) + \alpha_0 \qquad \hat{r}_0 = f'(\hat{k}_0) + \hat{\alpha}_0$$

It is also important that expected returns to capital in the next period be consistent with the actual supplies of that capital:

$$r_1 = f'(k_1) \qquad \hat{r}_1 = f'(\hat{k}_1)$$

This brings to fifteen the number of equilibrium conditions. Unfortunately, there are sixrteen endogenous variables: c_{h0}, s_0, π_0, $\hat{c}_{f0}$, $\hat{s}_0$, $\hat{\pi}_0$, k_0, $\hat{k}_0$, r_0, $\hat{r}_0$, r_1, $\hat{r}_1$, w_0, $\hat{w}_0$, p_0, and $\bar{x}_0$. So far we have no equation corresponding to $\bar{x}_0$. In fact, potentially there is an infinite sequence of relations needed to characterize this equilibrium: we must know $\bar{x}_0$ in order to characterize p_0; but to characterize $\bar{x}_0$, we need to know p_1, and to characterize p_1 we need to know $\bar{x}_1$ and therefore p_1 . . .

The fifteen relations do provide quite a bit of information, despite their limitations. Specifically, they tell us the effects on the equilibrium of a change in a state variable, or in a system parameter, if expectations of x_0 were exogenous.[7] We will refer to these "with $\bar{x}_0$ unchanged" effects as "direct effects", and they will be of central importance as our analysis progresses. Completely ignoring the induced changes in $\bar{x}_0$ cannot be justified, however. For example, suppose the direct effect of some shock to the system were to depreciate the current terms of trade, increasing p_0.. This in itself will tend to reduce $\bar{x}_0$; a lower $\bar{x}_0$ will change current portfolio allocations which in turn will put additional pressures on p_0. Further, changes in todays terms of trade and in portfolio shares will certainly have effects on tomorrows equilibrium, including tomorrow's terms of trade, which also affect x_0.

Since we must deal explicitly with the formation of expectations about x_0, and we have already assumed implicitly that expectations about r_1 and $\hat{r}_1$ are formed rationally, it is natural to assume that expectations about x_0 are formed rationally as well. However, none of the available approaches to solving for rational expectations equilibria seem suitable. Lag operators were inappropriate since the system isn't linear, and it has too many dimensions, even in its most compact form, for phase diagrams. The model cannot be solved using dynamic stochastic programming since some of the functional specifications have been left fairly general; even if they were further specified the system would be too complex to be solved explicitly. In consequence, a new approach to analyzing rational expectations equilibria was tried here, which focuses on characterizing their comparative statics. This approach is outlined in the following subsection, with some of the details reserved for Appendix B. That the approach is correct for small changes was demonstrated by simulations which are discussed in Part III.

Approximating the Comparative Statics of a Rational Expectations Equlibrium

We need to find the matrix representing the effects of changes in today's state variables, $\{s_{-1}, \hat{s}_{-1}, \pi_{-1}, \hat{\pi}_{-1}, p_{-1}\}$, on tomorrow's state variables, $\{s_0, \hat{s}_0, \pi_0, \hat{\pi}_0, p_0\}$. If expectations were not a problem we could simply take the total differential of our equilibrium conditions and solve for these elements via Cramer's Rule. Under rational expectations, however, such results represent only the "direct" effects of a change in a state variable, in the sense defined above. We will consider the direct and indirect effects to be additive and approximate the total effects linearly. Thus each entry of the transition matrix will comprise two terms, for example:

$$\frac{ds_0}{ds_{-1}} = \frac{\partial s_0}{\partial s_{-1}} + \frac{\partial s_0}{\partial \bar{x}_0}\frac{d\bar{x}_0}{ds_{-1}} . \tag{8}$$

The first term on the right-hand-side, $\partial s_0/\partial s_{-1}$, represents the direct effect of a change in s_{-1} on s_0. The second term, $(\partial s_0/\partial \bar{x}_0)*(d\bar{x}_0/ds_{-1})$, gives us the indirect effects of s_{-1} on s_0: s_{-1} affects p_0 and $E\{p_1\}$, changing $\bar{x}_0$ and, in turn, changing s_0. We will use the notation "∂" to refer to direct effects and "d" to refer to the composite effects throughout the rest of the paper.

To carry out the Cramer's Rule part of the exercise we must totally differentiate the first-order conditions evaluated at some initial equilibrium. Since there is no true steady-state equilibrium to this system, we will use as our starting point the state of the system if the random variables' realizations for an extended period of time had been their expected values. Together with the assumption of symmetry across countries, we can infer that portfolios in the initial equilibrium will be evenly divided between domestic and foreign equities, and that the terms of trade will be unity.[8]

The signs of the direct effects are given in the table below. In the rest of the paper, we denote two of these as follows:

$$\theta = \frac{\partial \pi_0}{\partial \bar{x}_0} < 0, \ \omega = \frac{\partial p_0}{\partial \bar{x}_0} > 0$$

Table 1

<u>An increase in:</u>		s_{-1}	$\hat{s}_{-1}$	π_{-1}	$\hat{\pi}_{-1}$	p_{-1}	$\bar{x}_0$
implies:	Δs_0	+	+	+	+	+	0
	$\Delta \hat{s}_0$	+	+	-	-	-	0
	$\Delta \pi_0 = \Delta \hat{\pi}_0$	0	0	-	-	-	-
	Δp_0	0	0	+	+	+	-

In considering the effects of a rise in $\bar{x}_0$ we parallel closely Persson and Svensson's (1985) analysis of the effects of an anticipated terms of trade deterioration on a small country. In our model such a change in $E\{p_1\}$ would be interpreted as an increase in $\bar{x}_0$. In both models, this anticipated change causes investors to shift towards foreign assets ($d\pi_0 < 0$). Domestic savings rises in the small country, because the expected capital gains on foreign assets, combined with their fixed own-good returns, require an increase in domestic returns, as well. In the two-country setting, own-good returns to foreign capital are endogenous, and portfolio shifts towards foreign assets cause them to decline while own-good returns to domestic assets rise. Since portfolios are initially divided evenly across domestic and foreign equities, these changes and those associated with capital gains offset each other and overall portfolio expected returns remain unchanged.

In both models a rise in p_1 causes an immediate current account deficit, but the proximate cause of the deficit are completely different. In the small country this cause is the rise in savings, while when two large countries interact it is the associated increase the current terms of trade, a response which Persson and Svensson rule out by assumption. This increase in p_0 is due to the portfolio shifts, and is less than one-for-one. Its affect on the current account can best be understood by considering instead how it impinges on the capital account, which goes into surplus: the domestic-good value of foreign investment, $p_0\hat{\pi}_0\hat{s}_0$, rises, and does so by more than the increase in capital outflows, $(1-\pi_0)s_0$.

Having dealt with the direct effects of changes in the state variables on the temporary equilibrium we can move on to consider the "indirect effects." Since savings levels are independent of expected changes in the terms of trade, there are no "indirect effects" of changes in the state variables on current savings in both countries. In consequence, we can fill in the 10 transition matrix elements corresponding to the responses of current savings levels to the state variables:

$$\frac{ds_0}{dz_{-1}} = \frac{\partial s_0}{\partial z_{-1}}, \quad \text{and} \quad \frac{d\hat{s}_0}{dz_{-1}} = \frac{\partial \hat{s}_0}{\partial z_{-1}} \tag{9}$$

for z_{-1} representing any one of the five state variables. This leaves us with 15 of the original 25 elements still to pin down.

Current portfolio shares and the current terms of trade will both respond to a rise in $\tilde{x}_0$, so we must proceed with endogenizing this variable. To do this we consider the effects of each state variable on $E\{p_1\}$, which requires resolving the problems of infinite recursion discussed above.

We begin by approximating these effects linearly. Consider the effects of a change in p_{-1} on $E\{p_1\}$. a change in p_{-1} will affect current savings, current portfolio shares, and the current terms of trade, all of which will, in turn, affect p_1.

$$\frac{dE\{p_1\}}{dp_{-1}} = \frac{dE\{p_1\}}{ds_0}\frac{ds_0}{dp_{-1}} + \frac{dE\{p_1\}}{d\tilde{s}_0}\frac{d\tilde{s}_0}{dp_{-1}} + \frac{dE\{p_1\}}{d\pi_0}\frac{d\pi_0}{dp_{-1}} + \frac{dE\{p_1\}}{d\hat{\pi}_0}\frac{d\hat{\pi}_0}{dp_{-1}} + \frac{dE\{p_1\}}{dp_0}\frac{dp_0}{dp_{-1}} \qquad (10)$$

Our linear approach implies:

$$\frac{dE\{p_1\}}{ds_0} = \frac{dp_1}{ds_0}, \quad \frac{dE\{p_1\}}{d\pi_0} = \frac{dp_1}{d\pi_0}, \quad \text{etc.}$$

which permits us to re-express equation (10) as follows:

$$\frac{dp_1}{dp_{-1}} = \frac{dp_1}{ds_0}\frac{ds_0}{dp_{-1}} + \frac{dp_1}{d\tilde{s}_0}\frac{d\tilde{s}_0}{dp_{-1}} + \frac{dp_1}{d\pi_0}\frac{d\pi_0}{dp_{-1}} + \frac{dp_1}{d\hat{\pi}_0}\frac{d\hat{\pi}_0}{dp_{-1}} + \frac{dp_1}{dp_0}\frac{dp_0}{dp_{-1}} \ .$$

There is one important observation that allows us to collapse a potentially infinite series of unknowns into a finite one: in the neighborhood of the initial equilibrium, with all state variables unchanging and all random variables at their expected values, the composite effects of changes in one of today's state variables, say p_{-1}, on tomorrow's state variables -- $\{s_0, \tilde{s}_0, \pi_0, \hat{\pi}_0, p_0\}$ -- must be the same as the effects of changes in the latter on the state variables for the two periods hence, $\{s_1, \tilde{s}_1, \pi_1, \hat{\pi}_1, p_1\}$. Thus,

$$\frac{dp_1}{dp_0} = \frac{dp_0}{dp_{-1}}, \quad \frac{dp_1}{d\pi_0} = \frac{dp_0}{d\pi_{-1}}, \quad \text{etc.} \qquad (11)$$

As will become apparent as we procede, it is this observation that powers the approximation algorithm.

Using equations (11) we re-express (10) once again:

$$\frac{dp_1}{dp_{-1}} = \frac{dp_0}{ds_{-1}}\frac{ds_0}{dp_{-1}} + \frac{dp_0}{d\tilde{s}_{-1}}\frac{d\tilde{s}_0}{dp_{-1}} + \frac{dp_0}{d\pi_{-1}}\frac{d\pi_0}{dp_{-1}} + \frac{dp_0}{d\hat{\pi}_{-1}}\frac{d\hat{\pi}_0}{dp_{-1}} + \frac{dp_0}{dp_{-1}}\frac{dp_0}{dp_{-1}} \ .$$

Combining this expression with the definition of $\bar{x}_0$, and using the fact that the initial value of p_0 is unity, generates the following relationship between $\bar{x}_0$ and the state variable p_{-1}:

$$\frac{d\bar{x}_0}{dp_{-1}} = \frac{dp_0}{ds_{-1}}\frac{ds_0}{dp_{-1}} + \frac{dp_0}{d\hat{s}_{-1}}\frac{d\hat{s}_0}{dp_{-1}} + \frac{dp_0}{d\pi_{-1}}\frac{d\pi_0}{dp_{-1}} + \frac{dp_0}{d\hat{\pi}_{-1}}\frac{d\hat{\pi}_0}{dp_{-1}} + \frac{dp_0}{dp_{-1}}\frac{dp_0}{dp_{-1}} - \frac{dp_0}{dp_{-1}} \quad . \tag{12}$$

There are five versions of equation (12): the one above and those corresponding to $d\bar{x}_0/ds_{-1}$, $d\bar{x}_0/d\hat{s}_{-1}$, $d\bar{x}_0/d\pi_{-1}$, and $d\bar{x}_0/d\hat{\pi}_{-1}$. Substituting these into the versions of equations (8) corresponding to $d\pi_0/dz_{-1}$, $d\hat{\pi}_0/dz_{-1}$, and dp_0/dz_{-1}, would give us fifteen equations in these fifteen unknowns (once again, "z_{-1}" refers to any and all of the state variables.)

The symmetry of the system allows us to conclude that $d\bar{x}_0/d\pi_{-1} = d\bar{x}_0/d\hat{\pi}_{-1}$. Furthermore, as is shown in Appendix B, $d\bar{x}_0/ds_{-1} = d\bar{x}_0/d\hat{s}_{-1} = 0$. This tells us that there are no indirect effects of changes in s_{-1} and $\hat{s}_{-1}$ on π_0, $\hat{\pi}_0$, and p_0. Since there are also no "direct" effects of such changes on portfolio shares and the terms of trade (see Table 1), we conclude that:

$$\frac{d\pi_0}{ds_{-1}} = \frac{d\hat{\pi}_0}{ds_{-1}} = \frac{d\pi_0}{d\hat{s}_{-1}} = \frac{d\hat{\pi}_0}{d\hat{s}_{-1}} = \frac{dp_0}{ds_{-1}} = \frac{dp_0}{d\hat{s}_{-1}} = 0 \quad . \tag{13}$$

Intuitively, changes in s_{-1} and $\hat{s}_{-1}$ affect both country's current wages and also the incomes of the elderly of each country exactly symmetrically since at our initial equilibrium portfolios are divided evenly among the two types of equities. This leaves no cause for the terms of trade or portfolio shares to change in the present or the future.

We are left with 9 elements of the transition matrix yet to pin down: the effects of π_{-1}, $\hat{\pi}_{-1}$, and p_{-1} on π_0, $\hat{\pi}_0$, and p_0. The symmetry of our system allows us to infer that the effects of π_{-1} on π_0, $\hat{\pi}_0$, and p_0 will be identical to the effects of $\hat{\pi}_{-1}$, leaving us with four unknowns: $d\pi_0/d\pi_{-1}$, $dp_0/d\pi_{-1}$, $d\pi_0/dp_{-1}$, and dp_0/dp_{-1}.

Our earlier steps have given us the following relations:

$$\frac{d\pi_0}{dp_{-1}} = \frac{\partial\pi_0}{\partial p_{-1}} + \frac{\partial\pi_0}{\partial\bar{x}_0}\frac{d\bar{x}_0}{dp_{-1}}, \qquad \frac{d\pi_0}{d\pi_{-1}} = \frac{\partial\pi_0}{\partial\pi_{-1}} + \frac{\partial\pi_0}{\partial\bar{x}_0}\frac{d\bar{x}_0}{d\pi_{-1}},$$

$$\frac{dp_0}{dp_{-1}} = \frac{\partial p_0}{\partial p_{-1}} + \frac{\partial p_0}{\partial\bar{x}_0}\frac{d\bar{x}_0}{dp_{-1}}, \qquad \frac{dp_0}{d\pi_{-1}} = \frac{\partial p_0}{\partial\pi_{-1}} + \frac{\partial p_0}{\partial\bar{x}_0}\frac{d\bar{x}_0}{d\pi_{-1}}. \tag{14}$$

These allow us to solve for $dp_0/d\pi_{-1}$ in terms of $d\pi_0/d\pi_{-1}$, and for dp_0/dp_{-1} in terms of $d\pi_0/dp_{-1}$. Modifying (12) to take account of relations (9), (13), and (14), we get two equations in the two unknowns $d\pi_0/d\pi_{-1}$ and $d\pi_0/dp_{-1}$. The exact equations are presented in Appendix B. Each of them is quadratic in one of the variables; more specifically, each equation represents a hyperbola. The hyperbolae which correspond to one set of system parameters ($\beta = .35$, $\sigma^2 = .8$, $\rho = 0$) are presented below:

There are only three possible solutions, all of which are real. Of the solutions, only "A" is stable, in the sense that the eigenvalues of the transition matrix associated with "A" are all below 1 in absolute value. (The stability condition for the system is presented in Appendix B.) Assuming that the economy will always find a stable path allows us to rule out solutions B and C.

Combining the two non-linear equations in $d\pi_0/d\pi_{-1}$ and $d\pi_0/dp_{-1}$ gives a cubic equation. It was not feasible to show analytically whether the nice properties of solution "A" would hold for all plausible combinations of parameters and endogenous variables. Instead, a broad spectrum of such combinations were examined in computer simulations of the model. A constant elasticity production function was assumed, $f = dk^a$, with elasticity $a = 0.3$ and scale parameter $d = 10$. The rate of population growth was set at 0.7, and simulations were run at all combinations of the following parameters values:

$$\beta = .25, .35, .5, 1$$

$$\sigma^2 = .1, .8, 2;$$

$$\rho = -.8, .8.$$

For each of these 24 cases the solution sets conformed to those depicted in the figure: all solutions were real, only one of them stable. In addition, the absolute values of own effects were in all cases between zero and unity.

The signs of the elements of the transition matrix are in each case the same as the signs of the direct effects displayed in Table 1. The size of these elements differ from the direct effects, however, according to the sign of associated changes in $\bar{x}_0$. In particular, a rise in π_{-1} will tend to cause the terms of trade to appreciate between the current period and the next ($d\bar{x}_0/d\pi_{-1} < 0$), which in turn reduces the amount by which current portfolio shares fall below 1/2 and also reduces the amount by which the current terms of trade depreciates. A rise in the previous terms of trade, p_{-1}, will have the opposite effects.

PART III: THE EFFECTS OF UNANTICIPATED OUTPUT DISTURBANCES

III.A: A Temporary, Unanticipated Shock to Output

We will analyse the effects of an unanticipated, transitory positive shock to domestic output using the framework derived above, considering first the "direct" effects and then composite effects, which incorporate endogenous changes in expected change in the terms of trade.

Direct Effects

The increased output associated with a higher realization of α_0 tends to depreciate the terms of trade because the rise in the quantity of domestic goods relative to the available quantity of foreign goods drives down the relative price of the former. According to the production function, the entire increase in output represents a rise in the realized own-good returns to domestic capital. Since half of that capital is owned by domestic investors and half by foreign investors, the increase in domestic output accrues equally to domestic and foreign residents. All of these recipients will consume half of their increased income as domestic goods and try to turn the other half into foreign goods, driving down the price of domestic goods. Thus, the depreciation can be attributed to an increase in domestic imports, as in Laursen and Metzler (1950), only if we ignore an equally large decline in foreign imports.

Investors shift their portfolios towards foreign equities because their expected own-good returns rise relative to those for domestic capital. As noted earlier, the current terms of trade depreciation will increase next-period domestic capital and cuase a symmetric decline in foreign capital. This in turn reduces expected own-good equity returns in the home country relative to those abroad, eliciting the portfolio shift.

Composite Effects

Will this same pattern emerge when the expected terms of trade depreciation, $\bar{x}_0$, is endogenized? Using our completed transition matrix, we can answer this question.

To begin our analysis of $d\bar{x}_0/d\alpha_0$, we define agents' expected value for the terms of trade in period 1. Since the shock to output will not be repeated, it can only affect the terms of trade in period 1 via its effects on current endogenous variables. If the output shock causes today's terms of trade to depreciate -- $dp_0/d\alpha_0 > 0$ -- then this will tend to increase tomorrow's terms of trade by the amount $(dp_1/dp_0)(dp_0/d\alpha_0) = (dp_0/dp_{-1})(dp_0/d\alpha_0)$. Any portfolio shifts caaused by the output shock, $d\pi_0/d\alpha_0 = d\hat{\pi}_0/d\alpha_0$, will change tomorrow's terms of trade by the amount $2(dp_1/d\pi_0)(d\pi_0/d\alpha_0) = 2(dp_0/d\pi_{-1})(d\pi_0/d\alpha_0)$. Any changes in savings have no effects on tomorrow's terms of trade, since $dp_0/ds_{-1} = dp_0/d\hat{s}_{-1} = 0$. In sum,

$$\frac{dE\{p_1\}}{d\alpha_0} = 2\frac{dp_0}{d\pi_{-1}}\frac{d\pi_0}{d\alpha_0} + \frac{dp_0}{dp_{-1}}\frac{dp_0}{d\alpha_0} \tag{15a}$$

This implies

$$\frac{d\bar{x}_0}{d\alpha_0} = 2\frac{dp_0}{d\pi_{-1}}\frac{d\pi_0}{d\alpha_0} + \frac{dp_0}{dp_{-1}}\frac{dp_0}{d\alpha_0} - \frac{dp_0}{d\alpha_0}\,. \tag{15b}$$

According to our linearization, current portfolio shares and the terms of trade will be affected by changes in $\bar{x}_0$ as follows:

$$\begin{aligned}\frac{d\pi_0}{d\alpha_0} &= \frac{\partial\pi_0}{\partial\alpha_0} + \theta\frac{d\bar{x}_0}{d\alpha_0}\\ \frac{dp_0}{d\alpha_0} &= \frac{\partial p_0}{\partial\alpha_0} + \omega\frac{d\bar{x}_0}{d\alpha_0}\end{aligned} \tag{16}$$

Substituting these into (14b) gives an equation in $d\bar{x}_0/d\alpha_0$ which can be solved to yield:

$$\frac{d\bar{x}_0}{d\alpha_0} = \frac{2\dfrac{dp_0}{d\pi_{-1}}\dfrac{\partial\pi_0}{\partial\alpha_0} - \left(1-\dfrac{dp_0}{dp_{-1}}\right)\dfrac{\partial p_0}{\partial\alpha_0}}{1-2\theta\dfrac{dp_0}{d\pi_{-1}} + \omega\left(1-\dfrac{dp_0}{dp_{-1}}\right)} < 0\,. \tag{17}$$

Finally, subsituting this expression back into equations (16) allows us to analyse the short-run effects of this temporary output shock. From this we conclude that the terms of trade will definitely depreciate in response to a one-shot rise in domestic output. It will then appreciate between the current period and the next ($dx_0/d\alpha_0 < 0$), though the next period's terms of trade will also have depreciated relative to its initial value of unity. It is not clear whether portfolios allocations will shift towards foreign equities, as indicated by the "direct effects" discussed above, or whether domestic equities will be favored. The motivation for a shift towards foreign equities has already been discussed, but it's worth repeating: the current depreciation of the terms of trade will raise the domestic capital stock relative to the foreign capital stock, which in turn reduces expected domestic equity returns relative to those abroad. A shift towards domestic equities would be associated with the anticipated appreciation in the domestic terms of trade. Despite the ambiguity surrounding the direction of change of portfolio allocations, it can be ascertained that the current account moves into deficit.

Laursen and Metzler (1950) also conclude that acurrent account deficit will arise in response to a domestic output shock; their motivation for the change and ours are entirely different. The concluded that an appreciation in the terms of trade faced by foreigners will increase their real income, leading them to save more, and thereby cause the domestic current account deficit. Our result relies on the fact that a depreciation in the terms of trade increases the value of foreign investment in the domestic economy and also raises the value of capital repatriated from abroad, causing a surplus on the capital account.

Though current foreign output is independent of current shocks to domestic output, such a shock will definitely affect foreign output in the next period. The depreciation of the terms of trade will tend to raise the domestic capital stock for the next period, and lower the foreign capital stock; since portfolio shifts are ambiguous they could affect capital stocks in either direction. It can be ascertained analytically that the domestic capital stock will rise, the foreign capital stock will fall, and there will be parallel effects on expected domestic and foreign output levels. Though future foreign output is likely to fall as a result of the current domestic output shock, foreign welfare may actually rise since their terms of trade will be stronger relative to its initial value of unity. It's not possible to determine the direction in which welfare changes.

This negative relation between foreign output and domestic output shocks is exactly the reverse of the relationship discussed by Stockman and Svensson (1987). In their analysis a small country imports all its investment goods from the rest of the world. When foreign output rises, the cost of that output, and therefore domestic capital imports, is reduced by the associated terms of trade depreciation. This tends to increase domestic investment and domestic output. The difference between the results of these two models is not related to the fact that theirs is a small-country analysis, but is due instead to different assumptions about the origins of capital. While all physical capital must be imported in the model of Stockman and Svensson, physical capital cannot be imported in our model. In reality, physical capital comprises both domestic output and imported output. For example, the U.S. is a major capital goods producer, but Japanese firms have been known to import specialized capital when building manufacturing plants here. The relevant question, then, is not whether capital is imported at all, but whether the amount of imported foreign direct investment is sufficiently large to outweigh the terms of trade effect on direct investment that is not imported and on purchases of equities themselves. In this respect, it is interesting to note that Ray (1988) finds the terms of trade to be one of the major determinants of the volume of foreign investment, and that such investment is increased when the dollar is "cheap" (depreciated) (p. 24): this is consistent with the approach adopted here.

What are the subsequent effects of a temporary output shock? If properties of the transition matrix could be pinned down beyond simply their sign, we could do an analytical impulse-response analysis and trace these effects through time.[9] It is possible to infer in this way that in the next period domestic savings definitely rises, savings abroad declines, and the terms of trade remain above unity but decline (appreciate) relative to their value in the initial period. Beyond this, all the comparative statics are ambiguous. To ascertain the likely direction of these changes, the model was simulated using the same specification that was used to analyze the transition matrix, with results that are consistent for all parameterizations.[10]

In the first period, "direct effects dominate" for portfolio shares as well as for the terms of trade, and portfolio shares shift towards foreign assets: the increase in the expected own-good return to domestic assets relative to the own-good return of foreign assets, caused by the terms of trade depreciation, dominates the expected appreciation in the terms of trade.

In the second period, we know from our analytical results that the rise in domestic output relative to foreign output once again depreciates the terms of trade, though by less than the first period. The simulations show that the depreciated terms of trade causes domestic output to rise relative to foreign output in the third period, which causes the terms of trade to depreciate once again . . . In this way, changes in the terms of trade acts to perpetuate an initial productivity shock into the future, causing domestic output to rise relative to foreign output throughout the period of return to the initial equilibrium. The difference between domestic and foreign output declines period by period, the terms of trade depreciates by less each period, and the system returns incrementally to its initial equilibrium.

The current account moves into surplus after the initial deficit, and remains there until the effects of the shock have worn off completely. This shift can best be understood by thinking of the current account in terms of net capital flows. For convenience, the expression for the capital account is reproduced below:

$$\kappa_0 = \left(\widehat{\pi_0 s_0} p_0 - (1-\pi_0)s_0\right) - \left(\frac{1}{1+n}\right)\left(\widehat{\pi_{-1} s_{-1}} p_{-1} - \frac{p_0}{p_{-1}}(1-\pi_{-1})s_{-1}\right),$$

The current account surpluse in period two is due in part to repatriation of the positive net foreign investment of the first period. The increase in domestic savings relative to foreign savings, due in turn to the change in relative wages, also helps create the deficit on capital account, and contributes to its persistence during the period of adjustment. This pattern of current account adjustment, represented schematically below, does not correspond to any of the patterns associated with terms of trade shocks in Persson and Svensson's small country analysis.

These simulations represented an opportunity to test the solution algorithm described in Part II.B. Two approaches were possible: one was to take a very small change in α and test whether consumers' first-order conditions are satisfied at the resulting equilibria; the other was to take a good-sized change in α and test to see whether the subsequent equilibria are consistent with the linearized first-order conditions that constitute the Hessian matrix of the system, and from which the analytical comparative statics are actually derived. The latter approach was chosen and the algorithm proved accurate in every case. The (linearized) first-order conditions were consistently fulfilled within rounding errors, and the system was always stable. This is true for all the simulations described in the rest of the paper.

III.B. A Persistent Productivity Shock

In this section we analyze the effects of an unanticipated positive shock to domestic output which is expected to persist. Suppose for concreteness that the shocks follow a random walk:

$$\alpha_t = \alpha_{t-1} + v_t \ ,$$

with v_t distributed with zero mean and variance σ^2.

To understand the implications of this assumption, let us return to our derivation of the short-run effects of a temporary shock to domestic productivity, and modify it slightly to incorporate the persistence of the shock at a constant level for the indefinite future. First we must note that the "direct effects" of an unanticipated but permanent productivity shock include the direct effects of today's shock and the direct effect of tomorrow's repeat of the shock. We modify Equations 16 as follows:

$$\frac{d\pi_0}{d\alpha_{0p}} = \frac{\partial\pi_0}{\partial\alpha_0} + \frac{\partial\pi_0}{\partial\alpha_1} + \theta\frac{d\bar{x}_0}{d\alpha_{0p}}$$

$$\frac{dp_0}{d\alpha_{0p}} = \frac{\partial p_0}{\partial\alpha_0} + \frac{\partial p_0}{\partial\alpha_1} + \omega\frac{d\bar{x}_0}{d\alpha_{0p}} \tag{16'}$$

where the subscript "p" refers to the permanence of the shock. The "direct effect" now comprises two parts which have opposite signs, since

$$\frac{\partial\pi_0}{\partial\alpha_1} > 0,\ \frac{\partial p_0}{\partial\alpha_1} < 0 .$$

These signs derive from higher expected returns to domestic capital which are associated with higher expected domestic output . The portfolio shift towards domestic assets will in turn appreciate the domestic terms of trade. Here we begin to see why the terms of trade could initially appreciate in response to a positive output shock. If portfolios were to shift towards domestic assets sufficiently strongly, then the terms of trade might appreciate despite the increase in domestic supply. How strong is "sufficiently strongly"? Unfortunately, the relative sizes of the two parts of the direct effects cannot be compared analytically.

To consider this possibility further we must incorporate the endogeneity of x_0 into our analysis. Equation 15a can be modified to incorporate the effect of the repeat shock:

$$\frac{dE\{p_1\}}{d\alpha_{0p}} = \frac{dp_0}{d\alpha_{0p}} + 2\frac{dp_0}{d\pi_{-1}}\frac{d\pi_0}{d\alpha_{0p}} + \frac{dp_0}{dp_{-1}}\frac{dp_0}{d\alpha_{0p}}$$

Here we are using the fact that in our linear approach the effect of the repeat shock on the terms of trade one period hence will be the same as the effect of the current shock on current terms of trade, or $dp_1/d\alpha_1 = dp_0/d\alpha_0$. This expression implies that

$$\frac{d\bar{x}_0}{d\alpha_{0p}} \approx \frac{dp_0}{d\alpha_{0p}} + 2\frac{dp_0}{d\pi_{-1}}\frac{d\pi_0}{d\alpha_{0p}} + \frac{dp_0}{dp_{-1}}\frac{dp_0}{d\alpha_{0p}} - \frac{dp_0}{d\alpha_{0p}} .$$

$$= 2\frac{dp_0}{d\pi_{-1}}\frac{d\pi_0}{d\alpha_{0p}} + \frac{dp_0}{dp_{-1}}\frac{dp_0}{d\alpha_{0p}} .$$

Substituting expressions (16') into this expression for $d\bar{x}_0/d\alpha_0$ we find:

$$\frac{d\bar{x}_0}{d\alpha_{0p}} = \frac{2\frac{dp_0}{d\pi_{-1}}\left(\frac{\partial\pi_0}{\partial\alpha_{0p}} + \frac{\partial\pi_0}{\partial\alpha_{1p}}\right) + \frac{dp_0}{dp_{-1}}\left(\frac{\partial p_0}{\partial\alpha_{0p}} + \frac{\partial p_0}{\partial\alpha_{1p}}\right)}{1 - 2\theta\frac{dp_0}{d\pi_{-1}} - \omega\frac{dp_0}{dp_{-1}}}$$

Though this expression cannot be signed.analytically because of the ambiguities mentioned above, simulated versions of this model can help us understand how these ambiguities are likely to be resolved. We find that the terms of trade will always be expected to depreciate between the current subsequent periods, $dx_0/d\alpha_{0p} > 0$, but initially the terms of trade may appreciate or depreciate, $dp_0/d\alpha_{0p} \gtrless 0$. In all cases portfolios shift towards domestic assets despite the expected depreciation. The likelihood that the increase in portfolio demand for domestic goods exceeds their increased supply depends on two factors: (i) the value of diversification, and (ii) the aggregate size of portfolios/savings relative to consumption. Diversification provides greater benefits to investors when equity riskiness is relatively high (say, $\sigma^2 = 0.8$ instead of 0.1), and also when equity returns are less well correlated (when $\rho = -.8$ rather than .8). As the value of diversification rises, portfolios shift less strongly towards domestic assets for a given increase in expected domestic equity returns relative to foreign returns. Portfolios are larger relative to consumption when consumer discount rates are higher (say, $\beta = 1$ rather than .35). For a given increase in the share of assets devoted to domestic equities, the increase in demand for domestic output will be higher as total assets rise.

Suppose instead that a current shock were expected to be followed by a persistent shock of greater magnitude. What then would be the association between productivity shocks and the terms of trade? To answer this question with our model we must take it in pieces, beginning with an increase in the future expected values of α which is unassociated with any current change. It is to this topic that we turn next.

III.C Anticipated Rise in the Distribution of Domestic Output

Since shocks to productivity in this model are unanticipated by their very nature, we cannot consider an anticipated productivity shock, temporary or permenant, in the way that Persson and Svensson (1985) consider an anticipated shock to the terms of trade. What can be anticipated in our model is a change in the distribution of productivity shocks.

We approach this analysis just as we did the previous cases, considering first the direct effects of a permanent rise in the distribution of α, and then endogenizing the expected change in the terms of trade, $\bar{x}_0$.

The "direct effects" of an anticipated increase in the expected value of α are those of the higher expected domestic output in period 1, which have already been discussed. To endogenize the indirect effects, we modify Equations (15) once again. The change in the expected terms of trade one period hence, relative to its original value of unity, can be expressed as the sum of three components:

$$\frac{dE\{p_1\}}{dE\{\alpha_a\}} = \frac{dp_1}{d\alpha_{1p}} + 2\frac{dp_0}{d\pi_{-1}}\frac{d\pi_0}{dE\{\alpha_a\}} + \frac{dp_0}{dp_{-1}}\frac{dp_0}{dE\{\alpha_a\}}, \tag{15'a}$$

where the subscript "a" refers to the anticiapted nature of the change in the distribution of α. The first component is due to the expected high realization of α_1 itself, the next is associated with the changes in current portfolio shares, and the last due to the change in the current terms of trade. The complete expressions for changes in portfolio shares and the current terms of trade are:

$$\frac{d\pi_0}{dE\{\alpha_a\}} = \frac{\partial \pi_0}{\partial \alpha_1} + \theta\frac{d\bar{x}_0}{dE\{\alpha_a\}}$$

$$\frac{dp_0}{dE\{\alpha_a\}} = \frac{\partial p_0}{\partial \alpha_1} + \omega\frac{d\bar{x}_0}{dE\{\alpha_a\}}$$

The expected depreciation in the terms of trade between the current period and the next is:

$$\frac{d\bar{x}_0}{dE\{\alpha_a\}} \approx \frac{dp_1}{d\alpha_{1p}} + 2\frac{dp_0}{d\pi_{-1}}\frac{d\pi_0}{dE\{\alpha_a\}} + \frac{dp_0}{dp_{-1}}\frac{dp_0}{dE\{\alpha_a\}} - \frac{dp_0}{dE\{\alpha_a\}} \tag{15'b}$$

or, solving:

$$\frac{d\bar{x}_0}{dE\{\alpha_a\}} = \frac{2\frac{dp_0}{d\pi_{-1}}\frac{\partial \pi_0}{\partial \alpha_1} + \frac{dp_0}{d\alpha_{1p}} - \left(1 - \frac{dp_0}{dp_{-1}}\right)\frac{\partial p_0}{\partial \alpha_1}}{1 - 2\theta\frac{dp_0}{d\pi_{-1}} + \omega\left(1 - \frac{dp_0}{dr_{-1}}\right)}$$

This is ambiguous because $dp_1/d\alpha_{1p}$ is ambiguous. Simulations results, on the other hand, display one consistent pattern. Initially, portfolios shift towards domestic assets and the increased demand for domestic output appreciates the domestic terms of trade. This is not surprising, since there is now no direct reason for the terms of trade to depreciate -- domestic output has not yet risen relative to foreign output. Once the shift in the distribution of α occurs, and domestic output does rise, the terms of trade depreciates well beyond its original value. Thus there is in this case a short run appreciation of the currency which is not expected to continue, a pattern which could be characterized as the inverse of "overshooting": in the short run the currency moves in the direction opposite to the one it will follow in the long run.

Since the terms of trade initially appreciate, does an anticipated increase in $E\{\alpha\}_{t=1}^{\infty}$ have a positive effect on foreign output, rather than the negative effect we have observed in previous cases? No. In this case the portfolio shifts are strong enough to dominate, and foreign output declines in the next period despite the terms of trade appreciation. In both cases the mechanism by which the positive domestic impulse is transmitted as a negative impulse to foreign output relies on equities markets. The terms of trade depreciation of the unanticipated productivity shock can only affect foreign output so long as the foreign capital stock is owned in part by domestic investors. The anticipated increase in the expected value of domestic productivity affects foreign output insofar as equity portfolios shift out of foreign capital.

The current account goes immediately into deficit; in the long run the deficit is reduced but not eliminated. The initial deficit is due to the domestic capital inflow, and its reduction is associated with the subsequent terms of trade depreciation.

At this point we can answer the question that originally prompted us to analyse an anticipated increase in $E\{\alpha\}_{t=1}^{\infty}$: What if a current shock were expected to be followed by a permanent increase in $E\{\alpha\}_{t=1}^{\infty}$ of greater magnitude? In our modelling format we can recreate the consequences of such a scenario by taking linear combinations of the effects of (i) a temporary productivity shock and (ii) an anticipated rise in $E\{\alpha\}_{t=1}^{\infty}$. A permanent shock such as we analysed in the previous section represents a combination in which these effects have equal weights. In our simulations, this is always sufficient to cause the terms of trade to appreciate initially when $\beta = 1$. If $\beta = .5$, this occured only for $\rho = -.8$, and for lower values of β the terms of trade always depreciated initially. When β is at its base value of .35 (individuals' rate of time preference is about .03 and one period corresponds to 35-40 years), the subsequent increase in $E\{\alpha\}$ must be roughly one third above its current value for the terms of trade to appreciate initially; for β as low as .25 the ratio $E\{\alpha\}_{t=1}^{\infty}:\alpha_0$ must be around 1.7.

Let's observe this situation through some other prism to see what our analysis has contributed. The terms of trade will depreciate in the next period in all of our scenarios, regardless of the relative sizes of α_0 and $E\{\alpha\}_{t=1}^{\infty}$. Suppose we had treated this future, expected depreciation of the terms of trade as exogenous. As we noted in Part II, the small country analysis of Persson and Svensson indicates that a rise in p_1 would lead portfolios to shift towards foreign assets and the domestic capital stock to decline, exactly the opposite of what we find here. Our own two-country analysis would have implied a current depreciation in the terms of trade, once again the opposite of what we actually find, had we treated expectations as exogenous. Since the future terms of trade change is in fact an endogenous response to a more fundamental disturbance, this is an example of a principle familiar to all economists; analysing the effects of a change in an endogenous variable in isolation from the fundamental disturbance itself can be misleading.

IV. CONCLUSION

In this paper we have considered productivity shocks in a two-country, intertemporal maximizing model, focusing on the mechanism by which these affect the terms of trade and future domestic and foreign output. We first considered a temporary, unanticipated shock to domestic output which, by increasing the relative supply of this output, will drive down its price -- depreciate the domestic terms of trade. In turn, this depreciation causes an increase in the domestic capital stock relative to the foreign capital stock, so long as international equities markets are non-trivially integrated. When domestic output is once again higher than foreign output in the next period, as a result of the relatively large capital stock at home, the terms of trade once again depreciates relative to its original value, raising domestic capital once again relative to foreign capital in the next period, and causing the terms of trade to depreciate relative to its original value once again in the next period . . . As is clear from our analysis, the propagation of the initial output shock through time and across countries would not occur without integrated national equities markets.

A productivity shock that causes investors to revise upward their expected value of domesic productivity in the future may cause the terms of trade to appreciate initially, rather than to depreciate. This appreciation will be the result of portfolio shifts towards domestic equities in anticipation of higher own-good returns to these assets. Whether the terms of trade appreciates or depreciates depends on three factors: (i) the size of the increase in the expected value of productivity at home relative to the initial shock; (ii) the size of the shift portfolio in portfolio shares, which in turn depends on the variability of equity returns (productivity) and on the correlation between domestic and foreign equity returns; and (iii) the size consumer savings relative to their consumption.

Even when a domestic output shock causes an initial appreciation of the terms of trade, foreign output in the future can be expected to decline relative to future domestic output. This is because if the terms of trade do appreciate, the portfolio shifts towards domestic assets that cause the appreciation will also be reducing foreign capital relative to domestic capital. Once again, the mechanism by which a productivity shock is propagated relies on the integration of international equities markets.

Though we follow Persson and Svensson (1985), and many other authors, in analysing temporary and permanent, anticipated and unanticipated shocks, our two-country format requries that we locate the exogenous determinants of economic activity at a more fundamental level than the terms of trade itself. In our analysis, we take productivity shocks to be the exogenous disturbance. The terms of trade responses to these shocks will not necessarily follow the pattern of the shocks themselves. For instance, when we consider a temporary shock to domestic output, we find that the terms of trade response takes many periods to die out though the shock lasts but one period. In fact, the terms of trade in our economies will never follow the pattern of a one-period change considered by Persson and Svensson. In this and other respects the conclusions of small country analyses with an exogenous terms of trade do not carry over well to the two-country setting.

We began this paper expressing an interest in the current real exchange rate appreciation in the United Kingdom, which seems to be associated with a simultaneous productivity increase there. We have shown that this appreciation could be due to an increase worldwide in investors' expectations of long-run productivity in that country. Such a suggestion is entirely plausible for the U.K.: current strong output growth is generally agreed to be the result of "supply side" reforms of the Thatcher goverment, and this government is expected to be in power for a number of years to come and to pursue these same goals vigorously during the rest of its tenure. Any reading of the business press will show how far investor perceptions of the British economy have shifted in the last decade. Further support for this hypothesis comes from the fact that this country experienced positive net foreign equity purchases during 1987 far out of proportion to its share of the world equity market (Merrill Lynch, presentation at the Tuck School, May 1988).

NOTES

[1] The terms of trade is typically defined as the price of the home good in terms of the foreign good, which is the inverse of our definition. According to the definition used here, an appreciation will imply a decline in p. The real exchange rate is more authentically defined as the relative price of traded and nontraded goods. In a model such as this one, without nontraded goods, the real exchange rate becomes synonymous with the terms of trade.

[2] The choice of functional form for the production function deserves some comment. It is more common to analyze a linear homogenous function multiplied by a stochastic term with unit mean: $Q = \alpha F(K,L)$. Examples include Batra 1975; Mayer 1976; Baron and Forsythe 1979; Helpman and Razin 1978a and b; Grossman and Razin 1984, 1985. In this case the variance of the return to capital is $(f')^2\sigma^2$, implying that any change in per-worker capital stock affects not only the return to capital but also its variance. Since this is a complication with which this thesis is not concerned, our alternative formulation was adopted. Another advantage of the additive form is that it mimics reality more closely than the model with multiplicative uncertainty, since in actuality the return to capital does absorb most of the variance in output. The risk-sharing arrangements responsible for this asymmetry between labor and capital, which are the focus of the implicit contracts literature (Bailey 1974), are not readily incorporated into this particular model.

[3]The requirement that $0 < d < 1$ ensures that members of the older generation will always consume a positive amount. Since we assume $r > 0$, their retirement resources will be positive whenever α, $\alpha \succ 1$.

[4]Strictly speaking, firm managers in this world try to maximize returns to their shareholders rather than economic profits. That is, when choosing their level of capital for the next period, a manager's objective function is:

$$\max_{L,K} E_t\{\frac{F(K,L) + \alpha K - wL}{K}\}$$

This has first-order conditions

$$F_L(K,L) = w,$$

$$F_K(K,L) = \frac{F(K,L) - wL}{K},$$

Since these conditions imply that both factors of production should be paid their marginal product, the change in objective function from the norm of excess profit maximization does not imply any change in firm behavior. (The implications of residual maximization do differ from those of economic profit maximization under certain conditions of asymmetric information.)

[5]It is interesting to distinguish this expression for the real return to domestic savings from a related expression used in both Persson and Svensson (1985) and Frenkel and Razin (1985). Their expression corresponds to the case where returns to capital are always equal across countries, once changes in the terms of trade are accounted for. Using an approximation approach analogous to the one employed for R in the text, their expression for the real returns to consumers' investments, call it "R'", is $R' = r + \mu x$. When $r = \hat{r} + x$ in our model, then $R = R'$. However, this is unlikely to be satisfied very often in this model.

[6]This is implied when we assume $\lim_{k \to \infty} -kf''(k) = 0$.

[7]To derive the comparative statics of this model, the consumers' first-order conditions were approximated using a second-order Taylor's expansion. This effectively rules out the possibility for third and higher moments of the distributions of α and $\hat{\alpha}$ to affect the equilibrium. Since $-1 \le \alpha \le 1$, these moments and their possible effects are likely to be quite small.

[8]The use of the assumption of symmetric countries together with the assumption of a pooled portfolio equilibrium originated with Lucas (1980).

[9] For example, the effect of the output disturbance on the major endogenous variables for period one are:

$$\frac{ds_1}{d\alpha_0} = \frac{ds_0}{ds_{-1}}\left(\frac{ds_0}{d\alpha_0} + \frac{d\hat{s}_0}{d\alpha_0}\right) + 2\frac{ds_0}{d\pi_{-1}}\frac{d\pi_0}{d\alpha_0} + \frac{ds_0}{dp_{-1}}\frac{dp_0}{d\alpha_0}$$

$$\frac{d\hat{s}_1}{d\alpha_0} = \frac{ds_0}{ds_{-1}}\left(\frac{ds_0}{d\alpha_0} + \frac{d\hat{s}_0}{d\alpha_0}\right) - 2\frac{ds_0}{d\pi_{-1}}\frac{d\pi_0}{d\alpha_0} - \frac{ds_0}{dp_{-1}}\frac{dp_0}{d\alpha_0}$$

$$\frac{d\pi_1}{d\alpha_0} = 2\frac{d\pi_0}{d\pi_{-1}}\frac{d\pi_0}{d\alpha_0} + \frac{d\pi_0}{dp_{-1}}\frac{dp_0}{d\alpha_0}$$

$$\frac{dp_1}{d\alpha_0} = 2\frac{dp_0}{d\pi_{-1}}\frac{d\pi_0}{d\alpha_0} + \frac{dp_0}{dp_{-1}}\frac{dp_0}{d\alpha_0}.$$

[10] To accomplish this we began with the simulations described in Part II. Each set of parameters was associated with a long-run equilibrium solution and with a stable transition matrix. The solution values were used to calculate the "direct effects" of a rise in α_0, which were combined with the transition matrix elements in equations 15 and 16 to calculate the total initial effects of the shock. Thereafter the equations listed in the previous footnote were used to calculate the subsequent equilibria.

APPENDIX A

Using relations (3) and (4) we can re-express real wealth of retirees as follows:

$$W^R = \left(\frac{s}{p_0^{1-\mu}}\right)\left[\pi(1+r+\alpha)\left(\frac{p_0}{p_1}\right)^{1-\mu} + (1-\pi)(1+\hat{r}+\hat{\alpha})\left(\frac{p_1}{p_0}\right)^{\mu}\right] . \qquad \text{(A1)}$$

Define:

$$(1+z_h) = \left(\frac{p_0}{p_1}\right)^{1-\mu}, \text{ and } (1+z_f) = \left(\frac{p_1}{p_0}\right)^{\mu}$$

We can restate (A1) this way:

$$W^R = \left(\frac{s}{p_0^{1-\mu}}\right)\left[\pi(1+r+\alpha)(1+z_h) + (1-\pi)(1+\hat{r}+\hat{\alpha})(1+z_f)\right]$$

$$\approx \left(\frac{s}{p_0^{1-\mu}}\right)\left[\pi(1+r+\alpha+z_h) + (1-\pi)(1+\hat{r}+\hat{\alpha}+z_f)\right] .$$

Finally, note that:

$$z_h \approx z_f - x \ , \text{ and } z_f \approx \mu x$$

which implies

$$W^R \approx \left(\frac{s}{p_0^{1-\mu}}\right)\left[1 + \pi(r+\alpha) + (1-\pi)(\hat{r}+\hat{\alpha}) + (\mu-\pi)x\right] .$$

APPENDIX B

1. Proof that $\dfrac{d\bar{x}_0}{ds_{-1}} = \dfrac{d\bar{x}_0}{d\hat{s}_{-1}} = 0$

Let

$$\theta = \frac{\partial \pi_0}{\partial \bar{x}_0} \ , \ \omega = \frac{\partial P_0}{\partial \bar{x}_0} \ , \ \tilde{e} = \tilde{c} - \frac{\omega}{\theta}\tilde{a}$$

Begin by noting that $\frac{dp_0}{ds_{-1}} = \omega \frac{d\bar{x}_0}{ds_{-1}}$, or

$$\frac{dp_0}{ds_{-1}} = \omega\left[\frac{dp_0}{ds_{-1}}\frac{ds_0}{ds_{-1}} + \frac{dp_0}{d\hat{s}_{-1}}\frac{d\hat{s}_0}{ds_{-1}} + 2\frac{dp_0}{d\pi_{-1}}\frac{d\pi_0}{ds_{-1}} + \frac{dp_0}{dp_{-1}}\frac{dp_0}{ds_{-1}} - \frac{dp_0}{ds_{-1}}\right]$$

or

$$\frac{dp_0}{ds_{-1}} = \omega\left[-\frac{dp_0}{ds_{-1}}\left(1 - \frac{dp_0}{dp_{-1}} - \frac{ds_0}{ds_{-1}}\right) + 2\frac{dp_0}{d\pi_{-1}}\frac{d\pi_0}{ds_{-1}} + \frac{dp_0}{d\hat{s}_{-1}}\frac{d\hat{s}_0}{ds_{-1}}\right]$$

or

$$\frac{dp_0}{ds_{-1}}\left[1 + \omega\left(1 - \frac{dp_0}{dp_{-1}} - \frac{ds_0}{ds_{-1}}\right)\right] = \omega\left[\frac{dp_0}{d\hat{s}_{-1}}\frac{d\hat{s}_0}{ds_{-1}} + 2\frac{dp_0}{d\pi_{-1}}\frac{d\pi_0}{ds_{-1}}\right]$$

Further, $\frac{d\pi_0}{ds_{-1}} = \theta\frac{d\bar{x}_0}{ds_{-1}} = \frac{\theta}{\omega}\frac{dp_0}{ds_{-1}}$, so

$$\frac{dp_0}{ds_{-1}}\left[1 + \omega\left(1 - \frac{dp_0}{dp_{-1}} - \frac{ds_0}{ds_{-1}}\right)\right] = \omega\frac{dp_0}{d\hat{s}_{-1}}\frac{d\hat{s}_0}{ds_{-1}} + 2\theta\frac{dp_0}{ds_{-1}}\frac{d\pi_0}{ds_{-1}}$$

or

$$\frac{dp_0}{ds_{-1}}\left[1 - 2\theta\frac{d\pi_0}{ds_{-1}} + \omega\left(1 - \frac{dp_0}{dp_{-1}} - \frac{ds_0}{ds_{-1}}\right)\right] = \omega\frac{dp_0}{d\hat{s}_{-1}}\frac{d\hat{s}_0}{ds_{-1}}$$

Similarly, $\frac{dp_0}{d\hat{s}_{-1}} = \omega\frac{d\bar{x}_0}{d\hat{s}_{-1}}$ and $\frac{d\pi_0}{d\hat{s}_{-1}} = \theta\frac{d\bar{x}_0}{d\hat{s}_{-1}} = \frac{\theta}{\omega}\frac{dp_0}{d\hat{s}_{-1}}$, so

$$\frac{dp_0}{d\hat{s}_{-1}}\left[1 - 2\theta\frac{d\pi_0}{d\hat{s}_{-1}} + \omega\left(1 - \frac{dp_0}{dp_{-1}} - \frac{d\hat{s}_0}{d\hat{s}_{-1}}\right)\right] = \omega\frac{dp_0}{ds_{-1}}\frac{ds_0}{d\hat{s}_{-1}}$$

This gives us dp_0/ds_{-1} as a function of $dp_0/d\hat{s}_{-1}$ and vice versa. Solving, and using the fact that $ds_0/ds_{-1} = ds_0/d\hat{s}_{-1} = d\hat{s}_0/ds_{-1}$, we find that:

$$\frac{dp_0}{ds_{-1}} = \omega^2\left(\frac{ds_0}{ds_{-1}}\right)^2\left[1 - 2\theta\frac{d\pi_0}{ds_{-1}} + \omega\left(1 - \frac{dp_0}{dp_{-1}} - \frac{ds_0}{ds_{-1}}\right)\right]^{-2}\frac{dp_0}{ds_{-1}}$$

A similar expression holds for $dp_0/d\hat{s}_{-1}$. The only way these expressions can be satisfied for arbitrary parametrizations of the economies is for $dp_0/ds_{-1} = dp_0/d\hat{s}_{-1} = 0$. QED

2. The two hyperbolae relating $d\pi_0/d\pi_{-1}$ and $d\pi_0/dp_{-1}$.

Let

$$\tilde{a} = \frac{\partial\pi_0}{\partial\pi_{-1}},\ \tilde{b} = \frac{\partial\pi_0}{\partial p_{-1}},\ \tilde{c} = \frac{\partial p_0}{\partial\pi_{-1}},\ \tilde{d} = \frac{\partial p_0}{\partial p_{-1}}$$

$$a = \frac{d\pi_0}{d\pi_{-1}},\ b = \frac{d\pi_0}{dp_{-1}},\ c = \frac{dp_0}{d\pi_{-1}},\ d = \frac{\partial p_0}{\partial p_{-1}}$$

Equation 1:

$$0 = 2\omega a^2 + a[2\theta\tilde{e} - \omega(1 - \tilde{e}/2) - 1] + \theta\left(\frac{\omega}{\theta}\right)^2 ab + \omega\tilde{e}b + [\tilde{a} - \theta\tilde{e}(1\text{-}\tilde{e}/2)]$$

Equation 2:

$$0 = \theta\left(\frac{\omega}{\theta}\right)^2 b^2 + b[2\theta\tilde{e} - \omega(1 - \tilde{e}) - 1] + 2\omega ab + [\tilde{b} - \frac{\theta\tilde{e}(1\text{-}\tilde{e}/2)}{2}]$$

3. Stability Condition

Intertemporal stability of the system requires that the eigenvalues of the transition matrix lie between -1 and 1. There are 5 eigenvalues, three of which equal zero and the other two of which must satisfy:

$$\lambda^2 - \lambda(d + 2a) + 2(ad - bc) = 0.$$

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