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# APPROXIMATELY RIGHT?: GLOBAL V. LOCAL METHODS FOR OPEN-ECONOMY MODELS WITH INCOMPLETE MARKETS

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Approximately Right?: Global v. Local Methods for Open-Economy Models with Incomplete Markets
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## ABSTRACT

Global and local methods widely used to study open-economy incomplete-markets models yield very different cyclical moments, impulse responses, spectral densities and precautionary savings. Endowment and RBC model solutions obtained with first-order, higher-order, and risky-steady-state local methods are compared with fixed-point-iteration global solutions. Analytic and numerical results show that inaccuracies in the autocorrelation of Net Foreign Assets resulting from assumptions used to induce stationarity cause the local solutions' flaws. DynareOBC solutions of a Sudden Stops model with an occasionally binding collateral constraint yield similar flaws and sharply underestimate the effects of the constraint on financial premia and macroeconomic variables. Local methods yield much larger Euler equation errors and are of comparable speed for the Sudden Stops model.

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A online appendix is available at http://www.nber.org/data-appendix/w26426

## 1. Introduction

Major strands of the International Macroeconomics literature study topics in which incomplete asset markets play a key role (e.g., business cycles, sovereign default, Sudden Stops, global imbalances, nominal rigidities, macroprudential regulation, currency carry trade, etc.). Since the dynamics of external wealth or net foreign assets (NFA) generally lack analytic solutions, researchers rely on numerical methods. Choosing the appropriate method is difficult, however, because deterministic models yield stationary equilibria dependent on initial conditions and in stochastic models the evolution of wealth is state-contingent and driven by precautionary savings. Certainty equivalence fails and, if the interest rate equals the rate of time preference, precautionary savings make the NFA position infinitely large.

The literature follows two approaches to address these issues. The first, based on the influential work by Schmitt-Grohé and Uribe (2003), modifies the models by introducing "stationarity inducing" assumptions that yield a well-defined deterministic steady state for NFA, independent of initial conditions, and implements log-linear or first-order approximations (1OA) around that steady state, recovering certainty equivalence. They proposed introducing one of three assumptions: a Debt-Elastic Interest Rate (DEIR) function by which the real interest rate rises when NFA falls, preferences with endogenous discounting (ED), or quadratic NFA-adjustment costs.[1] Important innovations to local methods have occurred since then, including higher-order perturbation methods (e.g., Devereux and Sutherland, 2010; Fernández-Villaverde et al., 2011), the risky steady state (RSS) method proposed by Coeurdacier et al. (2011), and algorithms for solving models with occasionally binding constraints (e.g., OccBin by Guerrieri and Iacoviello (2015), DynareOBC by Holden (2016, 2019)). The second approach uses global (GLB) methods to solve for the nonlinear decision rules and long-run distribution of external wealth of the models in their original form. These methods are similar to those used in closed-economy models of heterogeneous agents with incomplete markets and their use dates back to the Mendoza (1991) RBC model of a small open economy.

Global and local methods have been widely used in research and policy applications. This is

[1] They showed that the business cycle moments of an RBC small-open-economy model solved using any of these assumptions are similar, and impulse response functions to a TFP shock are virtually identical.

documented in Appendix A using information from the 50 papers most cited in *Google Scholar* that cite Schmitt-Grohé and Uribe (2003), all quantitative papers in the references of this paper not in that top-50 list, several well-known papers going back to the early 1990s (when the first numerical solutions of open-economy models with incomplete markets were produced), and the models from eight policy institutions. When local methods are used, 1OA is the most common in research and is used in all eight policy models, and from the assumptions to induce stationarity, DEIR is the most common followed by NFA adjustment costs and ED preferences. From all DEIR solutions, the majority set the value of the debt elasticity parameter $\psi$ to an arbitrary small number, with the aim of preventing the DEIR function from playing a role other than inducing stationarity, since ideally this function should be endogenous.[2] The values range from 0.00001 to 0.01, and the most common is 0.001, as Schmitt-Grohé and Uribe proposed.[3] In other cases, $\psi$ is calibrated to a data target (six cases) or estimated (four cases) and it ranges from 0.00014 to 2.8.

With GLB methods, the existence of a well-defined stochastic steady state follows from the same condition as in the Bewley-Aiyagari models of heterogeneous agents (see Ch. 18 of Ljungqvist and Sargent (2018)): the rate of time preference must be lower than the interest rate. This is a general equilibrium result in multicountry models, because if the rate of interest equals the rate of time preference, all countries desire infinitely large NFA for self-insurance, which is inconsistent with market clearing (see Mendoza et al., 2009). This implies that the issues studied here are relevant also for quantitative multi-country and closed-economy models with incomplete markets. Moreover, in small-open-economy models, assuming an interest rate lower than the rate of time preference is an *implication* of the assumption that the interest rate is a world-determined price. With local methods, the DEIR function is constructed so that at a chosen deterministic steady state the rate of interest equals the rate of time preference.

While GLB methods solve the models in their original form and capture NFA dynamics

[2]Garcia-Cicco et al. (2010) explain that, following Schmitt-Grohé and Uribe (2003), the standard practice is to set $\psi$ to a small value because the DEIR function aims to obtain independence of the deterministic steady state from initial conditions without affecting cyclical dynamics. They also studied a model in which $\psi$ represents financial frictions, and in this case they estimated $\psi$ using Bayesian methods.

[3]DEIR functional forms are not always the same, so $\psi$ values are not directly comparable. We control for this by making comparisons in terms of the elasticity of the interest rate with respect to steady-state deviations of NFA.

accurately, they suffer from the traditional "curse of dimensionality" (i.e., they become exponentially inefficient with the number of endogenous state variables). In contrast, local methods can solve large-scale models but require the stationarity-inducing assumptions that are not part of the original models. These tradeoffs pose four key questions: Are local solutions accurate? If not, why not? Are the inaccuracies economically meaningful? Can they be reduced?

This paper answers these questions by conducting a theoretic and quantitative analysis comparing global with local solutions. For local methods, we consider 1OA, second-order approximation (2OA), RSS, and DynareOBC.[4,5] For GLB solutions, we use the fixed-point iteration (*FiPIt*) algorithm proposed by Mendoza and Villalvazo (2020).[6]

We compared solutions for three popular small open economy models: An endowment model, a real business cycle (RBC) model, and a model of Sudden Stops (SS) with an occasionally-binding collateral constraint. We solve "*baseline calibrations*" in which the local methods use DEIR with $\psi = 0.001$ and the center of approximation of 1OA, 2OA and DynareOBC is the deterministic steady state, and RSS is centered at its risky steady state.[7] Then we study "*targeted calibrations*" with $\psi$ calibrated to match the first-order autocorrelation of NFA in the global solutions.[8] For RSS and DynareOBC, we also solve variants without DEIR in which the rate of interest is lower than the rate of time preference, so that credit constraints bind at the deterministic steady state. We compare statistical moments and impulse response functions (IRFs), and in the online Appendix we compare spectral densities. For the SS model, we also compare credit constraint multipliers, financial premia and macro responses when the constraint binds.

The results show that global and local solutions differ significantly, and that this is due to differences in the decision rule of NFA, the main endogenous state variable in open economy

[4]Appendix B.3.7 shows that moments from pruned third-order-approximation (3OA) solutions and 2OA solutions are the same up to the second decimal for the targeted and baseline calibrations (except the variability ratios for the latter), but this result could change in models with stochastic volatility (see de Groot, 2016).

[5]We use first-order DynareOBC because it yields the same results as OccBin when the equilibrium is unique. DynareOBC has the advantages that it converges in finite time and tackles equilibium multiplicity.

[6]Solving the endowment model with value function iteration yields very similar results (see Appendix B.3.9).

[7]We also compared results for the endowment model using ED preferences (see Section 2 and appendices C.3 and E.3). ED and DEIR have equivalent 1OA solutions but 2OA solutions differ sharply. In addition, 2OA ED solutions still approximate poorly the magnitude of precautionary savings of the ED GLB solutions.

[8]We also studied an alternative in which the center of approximation is the average NFA of the global solutions, but targeting the autocorrelation produces a closer match to the global solutions (see Appendix B.3.6).

models. GLB and local methods coincide in that NFA is a near-unit-root process. In all baseline calibrations for the three models, NFA autocorrelations exceed 0.97. In the local solutions, however, we show that this coefficient is determined by $\psi$ and the center of approximation, whereas in the GLB solutions it is determined by the endogenous ergodic distribution of NFA. Because they are near-unit-roots, slight differences in NFA autocorrelations between GLB and local solutions cause large differences in long-run moments, IRFs, and spectral densities.

The effect on two key moments of open-economy models is particularly striking. First, the local solutions' inaccuracies in NFA autocorrelations yield large differences in precautionary savings or the long-run average of NFA. Local methods also sharply under- or over-predict the precautionary-savings effect of changes in parameters that alter incentives to self-insure. For instance, the GLB solution predicts large increases in mean NFA with higher variability of shocks, lower rate of time preference or higher coefficient of relative risk aversion (CRRA). In contrast, 1OA maintains certainty equivalence, keeping mean NFA equal to the DEIR's predetermined steady state, while 2OA and RSS produce mean NFA much larger or smaller, depending on the model, the parameter change considered, and whether we use baseline or targeted calibrations. Similarly, DynareOBC calibrated to a steady state in which the credit constraint binds (does not bind) yields mean NFA well below (above) the GLB solution.

Second, small differences in NFA autocorrelations yield large differences in the autocorrelations of net exports ($nx$), because $nx$ is a quasi first-difference of the near-unit-root NFA process. In the endowment model with the baseline calibration, GLB predicts that raising the persistence of income from near 0 to 0.8 increases the autocorrelation of NFA from 0.83 to 0.99 and that of $nx$ from -0.09 to 0.77. In contrast, 2OA and RSS predict that the autocorrelation of NFA always exceeds 0.99 while that of $nx$ varies from 0.24 to 0.95. For a given autocorrelation of income in the 0-0.8 range, the local solutions always overestimate the autocorrelations of NFA and $nx$. As a result, they also overestimate the variability of consumption and $nx$ and underestimate their income correlations.

GLB and local solutions also show large differences in IRFs and spectral densities in the three models. In contrast, GLB and local methods yield similar results for supply-side variables (i.e., output, investment and inputs) in the RBC and SS models. This is an implication of using preferences without a wealth effect on labor supply and of the small equity premium typical of RBC models. The latter keeps the capital decision rule close to the one featuring Fisherian separation of investment from saving due to arbitrage of asset returns.

Comparing across local methods, 1OA, 2OA and RSS solutions yield similar second- and higher-order moments and IRFs for all endogenous variables in the endowment and RBC models. To explain these results, we derive analytic solutions of local NFA decision rules for the endowment model. We show that i) the coefficient on lagged NFA is nearly the same in the RSS and 2OA solutions when $\psi$ is small (less than 0.1), unless the deterministic and risky steady states of NFA differ by a large margin (at least 40 percentage points of GDP); ii) the coefficients in the square and interaction terms of 2OA decision rules are very small.

Local methods with targeted calibrations (i.e., $\psi$ set to match the GLB autocorrelation of NFA) do better at matching the GLB solutions. However, this approach has two drawbacks. One, it requires obtaining first the global solution so as to find the first-order autocorrelation of NFA to calibrate $\psi$, and doing this again for any parametric change that alters the NFA autocorrelation. Two, targeted calibrations require increasing $\psi$ from the common calibration setting of 0.001 to values of 0.0469 (0.0109) and 0.0469 (0.008) for the 2OA and RSS methods applied to the endowment (RBC) model, respectively. This increases the elasticity of the DEIR function by factors of 8 to 47 and makes NFA "sticky," as deviations of NFA from steady state become too costly. As a result, the first moments of 2OA and RSS become similar and similar also to the 1OA solution (i.e., certainty equivalence approximately holds). Hence, 1OA becomes the preferable *local* method, but this also means that precautionary savings are disregarded.

We use DynareOBC to solve the SS model because the occasionally binding constraint rules out standard local methods. DynareOBC uses local approximations but introduces news shocks that hit every time the constraint is violated to push the relevant variables back to the constraint. For consistency with rational expectations, these news shocks are constructed as if they were expected by agents along a perfect-foresight path and so are akin to "endogenous news shocks." This method, when solved in first-order and without integrating over future uncertainty, ignores precautionary savings, the possibility of alternative future paths in which the constraint may or may not bind, and the equity risk premium.

Findings from the endowment and RBC solution comparisons extend to the SS model. GLB and DynareOBC yield large differences in the amount of precautionary savings induced by the credit constraint, business cycle moments, the probability of hitting the constraint, impulse responses, and spectral densities. Moreover, the near-unit-root nature of NFA increases DynareOBC execution time considerably, because it requires multiple, long perfect-foresight paths to form the news shock realizations that implement the constraint, and long time-series simulations to attain convergence of long-run moments. DynareOBC also underestimates significantly the tightness of the credit constraint and its effects on financial premia and macro responses. Lower equity returns imply higher equity prices and investment when the constraint binds, and hence higher borrowing capacity. As a result, DynareOBC with the constraint binding at steady state yields weaker Sudden Stop macro responses, and with the constraint not binding at steady state it cannot produce Sudden Stop effects.

In terms of computational performance, the global *FiPIt* algorithm is slower than local methods for solving the endowment and RBC models but of comparable speed to DynareOBC for solving the SS model. For all three models, however, the local methods yield much less accurate results in terms of larger Euler equation errors and large differences in decision rules.

This paper is related to recent studies comparing global v. local solutions of models with financial frictions. Holden (2016) shows that DynareOBC yields similar results as the GLB solution for a small open economy model with endowment income, quadratic utility (which rules out precautionary savings) and NFA adjustment costs. There are four occasionally binding constraints: minimum income, non-negative consumption and autonomous spending, and expected future income larger than debt service with probability 1. In contrast, we found that the GLB and DynareOBC solutions of our endowment economy model with an ad-hoc debt limit differ sharply. Our analysis differs from Holden's in that it uses CRRA utility (which allows for precautionary savings) and solves for the stochastic steady state without a cost of holding assets imposing a deterministic steady state. We also used DynareOBC to solve the SS model, which has two endogenous states (capital and NFA) and a collateral constraint that depends on both states and endogenous asset prices, and found that the results again differ markedly from the GLB solution. Dou et al. (2019) compared GLB v. 1OA, 2OA and OccBin methods for closed-economy models and found that local solutions poorly approximate nonlinear dynamics and yield biased IRFs. Rabitsch et al. (2015) compared the local method proposed by Devereux and Sutherland (2010) (henceforth, DS) for solving portfolio allocations in a two-country, incomplete-markets model v. a GLB method. In the DS method, NFA non-stationarity remains an issue, but given NFA it yields an accurate portfolio structure. They found that DS is accurate only with particular calibrations and with symmetric countries with long-run NFA set to 0. With asymmetric countries, and using ED preferences for stationarity, DS performs poorly unless the center of approximation matches the GLB solution, and more so if NFA decision rules are nonlinear. Our work differs from these studies in three key respects. We study 1OA, 2OA, RSS and DynareOBC methods using the dominant DEIR approach to induce stationarity; compare results in the time and frequency domains; and consider endowment, RBC and SS models, and for the latter we compare global v. DynareOBC solutions.

Local and global solutions with occasionally binding constraints have also been compared in the literature on closed-economy New-Keynesian models with a zero-lower-bound (ZLB) on interest rates. These models formulate a Taylor rule with the ZLB constraint (rather than studying constraints on the agents' optimization problems) and typically assume complete markets, private bonds in zero net supply and a rate of time preference equal to the steady-state interest rate. Hence, the effects of precautionary savings on the dynamics of bond positions and the center of approximation of local solutions, which are essential to our findings, are not at issue in this literature. Fernández-Villaverde et al. (2015) proposed an innovative use of projection methods based on the Smolyak collocation approach to obtain an efficient global solution of a ZLB model with one endogenous state (price dispersion) and four exogenous shocks.[9] They found that the ZLB yields important nonlinearities that local methods miss. Gust et al. (2017) also solved a ZLB model with projection methods and compared the results with the OccBin results. They found that the latter approximates poorly the GLB solution and that the differences have major implications for propagation of shocks and estimation results.[10] Atkinson

[9]In their model, the ergodic average and the deterministic steady state of the endogenous state are nearly identical, whereas a key finding of our analysis is that precautionary savings causes large differences in the ergodic average v. the deterministic steady state of NFA positions.

[10]Solving the SS model using these methods is difficult because the global basis functions are not defined in

et al. (2019) also examined model estimation but concluded that there are more gains in terms of accuracy from estimating a richer, less misspecified version of the model using OccBin than estimating a stylized version of the model using GLB methods.

The rest of the paper is organized as follows. Section 2 compares the endowment model solutions, providing analytic and numerical results. Section 3 compares global v. DynareOBC solutions of the SS model. Section 4 provides conclusions and an extensive online Appendix provides further details on the solution methods and the endowment, RBC and SS applications.

## 2. Endowment economy

### *2.1. Model structure and equilibrium*

Consider first a small-open-economy model with stochastic endowment income. We use this setup to derive analytic results and characterize NFA dynamics under incomplete markets. The economy is inhabited by a representative agent with preferences given by:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}, \quad u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor, $c_t$ is consumption and $\sigma$ is the CRRA coefficient. The economy's resource constraint is given by

$$c_t = e^{z_t}\bar{y} - A + b_t - qb_{t+1}, \tag{2}$$

where $e^{z_t}\bar{y}$ is stochastic income that fluctuates around a mean $\bar{y}$ with shocks $z_t$ of exponential term $e^{z_t}$, $b_t$ denotes the NFA position in one-period, non-state-contingent discount bonds traded in a global credit market at constant price $q = \frac{1}{1+r}$, where $r$ is the world real interest rate, and $A$ is a constant that represents investment and government expenditures for model calibration.[11] Income shocks follow an AR(1) process: $z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_t^z$ where $\varepsilon_t^z$ is i.i.d.

The agent chooses the optimal sequences of bonds and consumption so as to maximize (1) subject to (2). This optimization problem is analogous to the one solved by a single individ-

points of the state space where it is not feasible to satisfy the collateral constraint with positive consumption. The boundary varies as capital, NFA and the capital pricing function vary. This problem can be avoided using uneven grids but this is also difficult because the debt limit imposed by the collateral constraint is not a pre-determined value. These hurdles do not arise in ZLB models and models with constant, unidimensional debt limits.

[11] We study later in this Section the implications of allowing $r$ to be stochastic.

ual in heterogeneous-agents models (e.g., Aiyagari, 1994). The Inada condition of CRRA utility implies that $u_c(c_t) \to \infty$ as $c_t$ goes to zero from above. This implies that the economy faces Aiyagari's Natural Debt Limit (NDL), by which the NFA position never exceeds the annuity value of the worst realization of net income $b_{t+1} \geq -\min(e^{z_t}\bar{y} - A)/r$, otherwise agents are exposed to the possibility of nonpositive consumption with positive probability. Following Aiyagari (1994), we can also impose a tighter ad-hoc debt limit $\varphi$, such that $b_{t+1} \geq \varphi \geq -\min(e^{z_t}\bar{y} - A)/r$, which is useful for model calibration.

Using the resource constraint, we can express the Euler equation for bonds as

$$u_c(e^{z_t}\bar{y} - A + b_t - qb_{t+1}) = (1+r)\beta E_t\left[u_c(e^{z_{t+1}}\bar{y} - A + b_{t+1} - qb_{t+2})\right] + \mu_t, \tag{3}$$

where $\mu_t$ is the Lagrange multiplier of the debt limit.

Under complete markets of contingent claims, and assuming income shocks are idiosyncratic to the small open economy, the economy diversifies away all the risk of its endowment fluctuations. The equilibrium features a constant consumption stream and the economy's wealth position is time- and state-invariant. The solution is akin to that of a perfect-foresight model with $\beta(1+r) = 1$ and wealth (the present value of income plus initial NFA) scaled to represent the same wealth as in the complete markets economy.

Under incomplete markets, the equilibrium differs sharply, because wealth becomes state contingent and consumption is not perfectly smoothed. Equation (3) implies that $M_t \equiv (1+r)^t\beta^t u_c(t)$ forms a supermartingale, which converges almost surely to a non-negative random variable because of the Supermartingale Convergence Theorem (see Chapter 18 of Ljungqvist and Sargent (2018)). If $\beta(1+r) \geq 1$, this convergence implies that consumption and NFA diverge to infinity because marginal utility converges to zero almost surely, which causes the non-stationarity problem that led to the use of the DEIR function in local methods. The economy builds an infinitely large stock of precautionary savings so that self-insurance can sustain a consumption process for which $M_t$ converges and $u_c(t) \geq \beta(1+r)E_t\left[u_c(t+1)\right]$ holds. In contrast, if $\beta(1+r) < 1$, the economy attains a well-defined stochastic steady state with finite long-run averages of assets and consumption, and the rest of the moments of the model's endogenous variables are also well-defined. Intuitively, the opposing forces of the pro-saving incentive for self-insurance and the pro-borrowing incentive due to $\beta(1+r) < 1$ keep NFA moving within an ergodic set. If NFA falls (rises) too much the first (second) force prevails.

### 2.2. *Global methods*

GLB methods solve the model in recursive form over a discrete state space of $(b, z)$ pairs. The AR(1) process of income is approximated as a discrete Markov chain with transition probability matrix $\pi(z', z)$. The goal is to solve for the NFA decision rule $b'(b, z)$, which together with the Markov process of the shocks produces the joint ergodic distribution of NFA and income $\lambda(b, z)$ (i.e., the stochastic steady state).

We solve for $b'(b, z)$ using the *FiPIt* method (see Mendoza and Villalvazo, 2020, for details).[12] For this model, *FiPIt* iterates on the following recursive representation of the Euler equation:

$$c_{j+1}(b, z) = \left\{ \beta\,(1+r) \sum_{z'} \pi(z', z) \left[ \left( c_j(\hat{b}'_j(b, z), z') \right)^{-\sigma} \right] \right\}^{-\frac{1}{\sigma}}. \tag{4}$$

Given a conjectured decision rule $\hat{b}'_j(b, z)$ in iteration $j$, the associated consumption function is $c_j(b, z) = e^z \bar{y} - A + b - q\hat{b}'_j(b, z)$. This consumption function is interpolated over its first argument in order to determine $c_j(\hat{b}'_j(b, z), z')$, so that eq. (4) solves directly for a new consumption function $c_{j+1}(b, z)$. Using the resource constraint, the new consumption function yields a new decision rule for bonds $b'_{j+1}(b, z)$, which is re-set to $b'_{j+1}(b, z) = \varphi$ if $b'_{j+1}(b, z) \leq \varphi$. Then the decision rule conjecture is updated to $\hat{b}'_{j+1}(b, z)$ as a convex combination of $\hat{b}'_j(b, z)$ and $b'_{j+1}(b, z)$, and the process is repeated until $b'_{j+1}(b, z) = \hat{b}'_j(b, z)$ up to a convergence criterion.

The global method solves the model without imposing assumptions to force stationarity. If $\beta(1+r) = 1$, the solution is that NFA diverges to infinity, which is unpleasant but *is* the equilibrium outcome. However, $\beta(1+r) < 1$ is the relevant case because, as noted above, it is implied by world general equilibrium. Note also that with $\beta(1+r) < 1$ the *deterministic* stationary state converges to the debt limit $\varphi$, with consumption falling at a gross rate of $(\beta\,(1+r))^{1/\sigma}$. Hence, without quantitative analysis, theory predicts that the long-run average of NFA in the stochas-

[12]This method is in the large class of global methods that iterate on Euler equations dating back to Coleman (1990) and Baxter (1991), including endogenous grids, time iteration and projection methods (see Rendahl, 2015, for a general discussion). Mendoza and Villalvazo show that *FiPIt* performs better than time iteration and endogenous grids, particularly for models with two endogenous state variables and occasionally binding constraints, because time iteration requires solving Euler equations nonlinearly and endogenous grids require interpolation techniques for irregular grids, while *FiPIt* solves Euler equations directly using standard linear interpolation.

tic, incomplete-markets model can differ significantly from the deterministic steady state and that the difference is due to precautionary savings.

*2.3. Local methods*

The local methods solve a local approximation of the optimality conditions (equations (2) and (3)) around the deterministic steady state ($b^{dss}$) for 1OA and 2OA or the risky steady state ($b^{rss}$) for RSS. Since assuming $\beta(1+r)=1$ implies that $b^{dss}$ depends on initial conditions and under uncertainty NFA diverges to infinity, 1OA and 2OA require a stationarity-inducing assumption. As noted earlier, the most common assumption is to introduce the DEIR function:

$$r_t = r + \psi \left[ e^{b^* - B_{t+1}} - 1 \right], \tag{5}$$

where $b^*$ and $\psi$ are parameters, with $\psi$ determining the elasticity of $r_t$ with respect to NFA, and $B_{t+1}$ is the "aggregate" NFA position (i.e., treated as exogenous by agents). At equilibrium, $b_{t+1} = B_{t+1}$. Since DEIR applications assume $\beta(1+r) = 1$, eq. (3) implies $b^* = b^{dss}$. The elasticity of $r_t$ with respect to (small) percent deviations of $b_{t+1}$ from $b^{dss}$ is $\eta^r \equiv -\psi b^{dss}$.

We implement the 1OA and 2OA methods using Dynare 4.5.6 and the RSS method following Coeurdacier et al. (2011). 1OA and 2OA solve for local approximations around $b^{dss}$ by solving a first- or second-order approximation to the decision rules jointly with approximations of the same order to the model's optimality conditions. In contrast, RSS uses $b^{rss}$ as center of approximation and assumes $\beta(1+r) < 1$ (see Appendix B.2.2 for details).

RSS aims to take into account future risk, so that the center of approximation may do better at capturing precautionary savings. The value of $b^{rss}$ is obtained from a second-order approximation to the conditional expectation of the steady-state Euler equation, solved jointly with the coefficients of a first-order approximation to the decision rules using a conditional second-order approximation of the full equilibrium conditions' Jacobian, which requires the third derivatives of those conditions. As explained by de Groot (2014), this second part of the solution is crucial to obtain stationary NFA dynamics in the RSS solution. We also consider a variant of RSS in which $b^{rss}$ is computed in the same way, but is then used together with the DEIR function and standard first-order approximations to the decision rules and equilibrium conditions to obtain stationary dynamics. We denote the original RSS method as "full RSS" and the alternative with the DEIR function as "partial RSS."

*2.4. Calibration & comparison of quantitative results*

***a) Calibration***

We use the same baseline calibration as in Durdu et al. (2009), which targeted annual data for Mexico (see their article for a full description of the calibration). Table 1 lists the parameter values separating those that are common to global and local solutions from those particular to each, including for the local methods *baseline* and *targeted* calibrations of $\psi$. For the income process, the local methods use the AR(1) process estimated by Durdu et al. with $\sigma_z = 0.0327$ and $\rho_z = 0.597$. In the GLB solution, we approximate it as a five-point Markov chain using the improvement of the Tauchen-Hussey quadrature method developed by Flodén (2008).[13]

The values of $\varphi$ and $\beta$ in the GLB calibration were set so that the model matches the $-0.44$ average of the NFA-GDP ratio from Mexican data together with Mexico's cyclical variability of private consumption of 3.28 percent over the 1965–2005 period. This implies $\varphi = -0.51$ and $\beta = 0.94$. Two parameters are required to identify the calibration, because while the average NFA-GDP ratio can be matched by simply adjusting $\varphi$, this can result in stochastic steady states in which the distribution of bond holdings is clustered near the debt limit and consumption fluctuates too much, or has a high variance and consumption fluctuates too little.

In the baseline calibration for the local methods (except for full RSS) we follow the standard practice of setting $\beta = 1/(1+r)$ so that $b^{dss} = b^*$. We set $b^*$ equal to $\varphi$ in the GLB calibration, hence $b^{dss} = -0.51$. This is done so that in both solutions $b^{dss} = \varphi$ (this is the case for the GLB solution because there $\beta(1+r) < 1$). The discount factors of the global and local calibrations differ only slightly (0.944 v. 0.940).[14] The *baseline* value of $\psi$ is the commonly-used value of 0.001. In the *targeted* calibrations, we set $\psi$ to values so that the solution for a given local method

[13]The Markov process is discrete with bounded support whereas the AR(1) process has normally-distributed innovations with unbounded support. Floden showed, however, that for other than highly-persistent shocks, Markov processes produced by quadrature methods match closely the unconditional moments of AR(1) processes even with few nodes. Appendix B.3.9 shows that increasing the nodes from 5 to 11 yields nearly identical results.

[14]An alternative calibration strategy could retain the GLB value of $\beta$ and choose $b^*$ such that eq. (5) yields $b^{dss} = -0.51$. This requires $b^* > b^{dss}$ and implies $r^{dss} > r$. The problem with this approach is that $\psi$ no longer determines the elasticity of $r_t$ with respect to NFA. To see this, note that $\eta^r = -\psi b^{dss} \cdot e^{b^* - b^{dss}}$. Under our calibration strategy (with $b^* = b^{dss}$), $\eta^r$ is independent of $b^*$. In contrast, this alternative strategy (with $b^* > b^{dss}$) is isomorphic to the one we proposed but with a higher $\psi$ value. This is problematic since $\psi$ is a key parameter and adjusting the wedge between $b^*$ and $b^{dss}$ blurs its interpretation.

matches the autocorrelation of NFA obtained with the GLB solution. This yields $\psi = 0.0469$ for both the 2OA and RSS. We do this because, as we show below, the targeted calibrations give the local methods the best chance to match the GLB solution.

***b) NFA decision rule and Net Exports***

We compare first the results for two key moments of open-economy models, namely the first-order autocorrelations of NFA and $nx$. Assuming that $b_{t+1}$ follows an AR(1) process with autocorrelation coefficient $\rho_b$, and since $nx$ is a quasi first-difference of NFA ($nx_t = qb_{t+1} - b_t$), Appendix B.3.2 shows that the first-order autocorrelation of net exports ($\rho_{nx}$) is:[15]

$$\rho_{nx} = \frac{q^2\rho_b + \rho_b - q - q\rho_b^2}{1 + q^2 - 2q\rho_b}. \tag{6}$$

Hence, if $\rho_b$ is close to 1, as is typical in incomplete-markets models, small differences in $\rho_b$ induce large differences in $\rho_{nx}$. Variances and correlations of $b$, $nx$ and other variables that depend on $b$ would also differ sharply. Thus, small errors in the local solutions for $\rho_b$ can yield large errors in $\rho_{nx}$ and other key moments. We show below that this is indeed the case.

The 2OA decision rule for NFA can be expressed as:

$$\tilde{b}_{t+1} = h_b\tilde{b}_t + h_z z_t + \frac{1}{2}\left(h_{bb}\tilde{b}_t^2 + h_{zz}z_t^2\right) + h_{bz}\tilde{b}_t z_t + \frac{1}{2}h_{\sigma\sigma}, \tag{7}$$

where $\tilde{b}_t \equiv b_t - b^{dss}$. The 1OA and RSS decision rules have similar expressions, except that they only have the first two terms in the right-hand side. For RSS, $b^{dss}$ is replaced with $b^{rss}$.

The key coefficient to analyze is $h_b$, because it is the main determinant of $\rho_b$. This is the case even for the 2OA solutions because in all of our quantitative applications $h_{bb}$, $h_{zz}$ and $h_{bz}$ are negligibly small.[16] The term $h_{\sigma\sigma}$ is also important because it isolates the effect of income variability on mean NFA. It is an estimate of the amount of precautionary savings that the 2OA solution captures. Moreover, since $h_{\sigma\sigma}$ is the only *quantitatively* relevant term of those that distinguish 2OA from 1OA and $h_b$ is the same in both, these results also imply that the 2OA

[15] NFA *is* an AR(1) process in the 1OA and RSS solutions. In the 2OA solution it includes squared and interaction terms in $b_t$ and $z_t$, but these are negligible for second- and higher-order moments in all our experiments.

[16] Appendix B.3.3 shows that this a robust result. In particular, $h_{bb}$, $h_{by}$, and $h_{zz}$ are irrelevant for the autocorrelation and standard deviation of NFA for a wide range of values of $\psi$, $\sigma$ and $\rho_z$. Even for mean NFA, those terms make a difference only if $\rho_z$ is high and/or $\psi$ is very low.

and 1OA solutions should be very similar, except in their first moments.

For the RSS method, de Groot (2014) showed that income variability matters also for determining $b^{rss}$ because the coefficient of variation of consumption (relative to its risky steady state) is constant, at a level that depends on $\beta$, $r$ and $\sigma$.[17] Intuitively, this captures precautionary savings because, if income variability rises and the shares of income allocated to savings v. consumption remain unchanged, the volatility of consumption would rise. But by increasing NFA relative to endowment income, more disposable income comes from interest income, so that the coefficient of variation of consumption can remain constant. Since the RSS decision rule follows from a first-order approximation, however, the $\rho_b$ value will differ from the 1OA and 2OA solutions only to the extent that $b^{dss}$ and $b^{rss}$ differ, and as we document below, this requires larger differences than those implied by our calibrations. Hence, 1OA, 2OA and partial RSS solutions are likely to be very similar, except for their first moments.

We show next how $\psi$ and the center of approximation determine $h_b$. Assuming log utility and i.i.d income for tractabilty, Appendix B.3.2 derives this solution for $h_b$:[18]

$$h_b(\psi, b^*) = \frac{R + e^{b^*\psi}(1 - b^*\psi + \psi) - \sqrt{R^2 + 2e^{b^*\psi}(b^*\psi + \psi - 1)R + e^{2b^*\psi}\left(1 - b^*\psi + \psi\right)^2}}{2e^{b^*\psi}}, \quad (8)$$

where $R \equiv 1 + r$ and $b^* = b^{dss}$ for 1OA and 2OA or $b^{rss}$ for RSS. Since $h_{bb}$, $h_{zz}$ and $h_{bz}$ are quantitatively irrelevant, it follows that $\rho_b(\psi, b^*) \approx h_b(\psi, b^*)$ for 1OA, 2OA and RSS methods. Hence, eq. (8) describes how $\psi$ and $b^*$ determine the autocorrelation of the equilibrium process of NFA produced by local methods. Moreover, it also implies that the value of $h_b$ obtained with 1OA and 2OA differs from the RSS solution only to the extent that $b^{dss}$ and $b^{rss}$ differ.

Equation (8) demonstrates that setting the value of $\psi$ imposes implicitly the equilibrium autocorrelation of NFA. In particular, given $b^*$ and $R$, choosing a very low $\psi$ implies a $\rho_b$ close to 1 (for the RSS method, we need to consider also that $b^* = b^{rss}$ and $b^{rss}$ is solved together with the coefficients of the decision rules for $\tilde{b}_{t+1}$ and $\tilde{c}_t$, which also depend on $\psi$). In fact, as the numerical results reported below show, $\rho_b$ falls (rises) with $\psi$ for relatively low (high) $\psi$.

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[17] Corollary 5 in de Groot (2014) shows that $\frac{var(c)}{(c^{rss})^2} = \frac{2}{\sigma(1+\sigma)} \frac{1-\beta R}{\beta R}$.

[18] This result applies for both full and partial RSS.

Equation (8) also illustrates the non-stationarity of the local solutions if a stationarity-inducing transformation is not used. If $\psi = 0$, the solution of $\rho_b(\psi, b^*)$ has two roots, $R$ or $1$, so NFA is non-stationary. In contrast (and assuming $b^* = 0$ for simplicity), if $\psi > 0$ the smaller of the two roots that solve $\rho_b(\psi, 0)$ is less than unitary, and thus yields a stable solution.[19]

We study numerically how variations in $\psi$ and $b^*$ alter $\rho_b(\psi, b^*)$. To this end, we use $R = 1.059$ from the baseline calibration and solve for $\rho_b(\psi, b^*)$ for a set of values of $\psi$ and $b^*$. Figure 1 plots $\rho_b(\psi, b^*)$ for $\psi$ in the interval $[0, 0.9]$ and three values of $b^*$: 0, -0.41 ($b^{rss}$ in the baseline calibration) and -0.51 ($b^{dss}$ in the baseline calibration).

Figure 1 yields a key finding: $\rho_b$ is nearly identical across 2OA and RSS for any $0 \leq \psi \leq 0.1$, which is an interval that includes the baseline and targeted calibration values and also all the values of $\eta^r$ implied by the $\psi$ values used in the literature reviewed in Appendix A.[20] Hence, for the values of $\psi$ used in the literature, the choice of approximating around $b^{dss}$ v. $b^{rss}$ and solving with 1OA, 2OA or partial RSS does not make a difference! The two steady-state estimates would have to differ much more than what the baseline calibration and small variations around it would predict. We start to notice a non-negligible difference only if $b^{dss}$ is more than forty percentage points of GDP below $b^{rss}$. Moreover, since in the baseline and targeted calibrations it is also the case that the quadratic and interaction terms of the 2OA decision rule of $b$ are nearly zero, it follows that we can expect the 2OA and RSS solutions to produce similar second and higher-order moments for all endogenous variables, as the results reported below confirm.[21]

The above findings indicate that the implications of $\rho_b$ for $\rho_{nx}$ conjectured in condition (6) by *assuming* that NFA follows an AR(1) process apply to the *equilibrium* processes produced by the local methods. The DEIR function with very small $\psi$ imposes values of $\rho_b$ near 1, and small differences between them and the GLB solutions result in large differences in $\rho_{nx}$, as we document next. In contrast, in the GLB solutions, $\rho_b$ and $\rho_{nx}$ are moments implied by the

[19]Schmitt-Grohé and Uribe (2003) obtained similar results by deriving the analytic solution of the NFA decision rule for an endowment economy assuming ED preferences with log utility.

[20]The highest $\psi$ in the literature is 2.8 in Garcia-Cicco et al. (2010), and with their value of $b^{dss} = -0.007$ yields $\eta^r = -0.0196$. For $0 \leq \psi \leq 0.1$ with our $b^{dss} = -0.51$, we obtain an interval of elasticities $0 \geq \eta^r \geq -0.051$. In our baseline (targeted) calibration, $\psi = 0.001$ $(0.0469)$ implies $\eta^r = -0.0051$ $(-0.0239)$. Figure 1 also shows that $\rho_b$ becomes increasing in $\psi$ for $\psi \geq 0.5$, but these $\psi$ values imply $\eta^r$ values much larger than in the literature.

[21]The analytic solution for $h_b(\psi, b^*)$ is strictly valid only for log utility and i.i.d. shocks, but these implications of the analysis still hold quantitatively in the solutions with AR(1) shocks.

endogenous limiting distribution of NFA ($\lambda(b,z)$), the NFA decision rule ($b'(b,z)$) and the definition of $nx$.

Table 2 compares values of $\rho_b$ and $\rho_{nx}$ produced by GLB and local solutions as $\rho_z$ varies from 0 to 0.8.[22] Panel i) shows GLB results for the baseline calibration. Panel ii) shows 2OA and partial RSS results for their baseline calibration with $\psi = 0.001$. Panel iii) shows local solutions for targeted calibrations with $\psi$ set to match $\rho_b = 0.977$ (the value in the GLB solution shown in Table 3) which implied $\psi = 0.0469$ for the 2OA and partial RSS solutions. Panel iv) shows a scenario in which, for each $\rho_b$ obtained with the GLB solution, we re-calibrate $\psi$ in the local solutions so as to match that value of $\rho_b$ (the $\psi$ values are also show in this panel).

The first result evident in Table 2 is that 2OA and partial RSS results are always very similar, because the gap between $b^{dss}$ and $b^{rss}$ and the quadratic and interaction terms in the 2OA decision rules are too small to yield larger differences in the values of $\rho_b$ and $\rho_{nx}$ that the two methods produce, for all combinations of $\rho_z$ and $\psi$ considered. Recall also that for the same reasons 1OA and 2OA solutions are nearly the same.

Panel i) shows that as $\rho_z$ rises from 0 to 0.8, the GLB solution indicates that $\rho_b$ rises from 0.82 to 0.99 and $\rho_{nx}$ rises from almost -0.1 to 0.77. Thus, as eq. (6) predicts, small variations in $\rho_b$ near 1 cause large changes in $\rho_{nx}$. In contrast, Panel ii) shows that with $\psi = 0.001$, the local solutions yield $\rho_b$ values always above 0.99, which in turn yield $\rho_{nx}$ values between 0.24 to 0.95. The differences relative to the GLB solutions are large. For $\rho_z = 0$, GLB yields $\rho_b$ and $\rho_{nx}$ of 0.83 and -0.1 respectively, while 2OA and RSS yield $\rho_b = 0.99$ and $\rho_{nx}$ of 0.27 and 0.24 respectively. For the calibrated value of $\rho_z = 0.597$, GLB yields $\rho_b = 0.977$ and $\rho_{nx} = 0.54$, while 2OA and RSS yield $\rho_b = 0.999$ and $\rho_{nx} = 0.82$ (see Table 3). Thus, these results confirm that local methods need very accurate approximations of $\rho_b$ to approximate $\rho_{nx}$ closely.

Panel iii) shows that local solutions perform better with the targeted calibrations ($\psi = 0.0469$), which match the $\rho_b$ of the GLB solution for the calibrated $\rho_z$ (0.597) by construction. For lower $\rho_z$, the local solutions overestimate slightly $\rho_b$ and $\rho_{nx}$ relative to the GLB solutions. Panel iv) shows that, if we re-calibrate $\psi$ as we change $\rho_z$ so that the local solutions match the $\rho_b$

[22]Since 1OA and 2OA solutions are nearly identical, we omit the former from the Table.

of the GLB solutions in each column of the Table, the local methods do a good job at matching the global solutions. This is true by construction for $\rho_b$, but the values of $\rho_{nx}$ are also close. In these results, however, $\psi$ has to rise as $\rho_z$ falls. The required values of $\psi$ range from 0.027 to 0.185, significantly larger than the ideal value of 0.001 that keeps the DEIR inessential, and effectively they make deviations of NFA from its steady state very costly. Moreover, knowing the value of $\rho_b$ needed as calibration target to set $\psi$ requires solving the model globally first.

***c) Long-run moments & impulse response functions***

Table 3 shows long-run moments.[23] Local solutions under the baseline calibration do poorly at matching the GLB moments. GLB yields $E(b/y)$ of -41 percent, nearly 10 percentage points above the -51 percent at the deterministic steady state (which is the 1OA average because of certainty equivalence). 2OA and partial RSS yield $E(b/y)$ of -28.2 and -45.1 percent, respectively. The former (latter) overestimates (underestimates) precautionary savings by about 13 (4) percentage points. Full RSS yields much lower $E(b/y)$ of nearly -1121 percent of GDP, because it has the same $\beta R < 1$ of the GLB method but lacks the debt limit $\varphi$ that allows the GLB solution to match $E(b/y)$ and the variability of consumption observed in the data.[24]

Since NFA is a near-unit-root process, the higher $\rho_b$ of the three baseline perturbation solutions implies that they also overestimate sharply the variability and persistence of $c$, $nx$ and $b$ and underestimate their GDP correlations.[25] Notably, given the literature's emphasis on explaining consumption variability in emerging markets, all the baseline local solutions overestimate significantly consumption variability relative to GDP.

The local methods again perform better at approximating the global results using targeted calibrations. The major exception is that they do *worse* at capturing precautionary savings, with RSS and 2OA solutions yielding $E(b/y)$ of nearly -0.51, very close to $b^{dss}$. This occurs because

[23] 1OA solutions are not shown because they are nearly the same as the 2OA solutions, except for the averages.

[24] Full RSS is closer to (but still below) the mean NFA of the GLB solution without ad-hoc debt limit (i.e., with $\varphi = NDL$), which yields $E(b/y) = -1080$ percent. These solutions, however, produce high variability and persistence and low GDP correlations in all the variables. We also considered re-calibrating $\beta$ in Full RSS to match $E(b/y)$ of the GLB solution. Since it requires a slightly higher $\beta$, $\rho_b$ moves even closer to a unit root and hence variability and persistence statistics are even higher and GDP correlations are near zero (see Appendix B.3.6).

[25] Since $\rho_b \approx h_b$ and higher-order terms other than the variance term are negligible, the variability of $b$ rises with $\rho_b$ because $\sigma(b) = h_z\sigma(z)/\sqrt{1-h_b^2}$ and the correlation with GDP falls because $\rho_{b,z} = [\rho_z/(1-h_b\rho_z)]\sqrt{1-h_b^2}$.

(as explained below) higher $\psi$ is akin to a higher cost of moving $b$ away from its steady state.[26] The local solutions with targeted calibrations also continue to overestimate the autocorrelation of consumption, but for the rest of the moments they approximate better the GLB solution than the baseline calibrations. As noted before, however, they require the GLB solution to determine the target values of $\psi$ and those values are much larger than $0.001$.

2OA and partial RSS yield similar results for nearly all moments under either baseline or targeted calibrations. The only exception is $E(b/y)$ under the baseline calibration, which is -0.28 with 2OA v. -0.45 with RSS, but under the targeted calibration even this moment is nearly the same. This is further evidence indicating that the different centers of approximation in these solutions and the extra terms in the 2OA decision rules have negligible quantitative effects.

Figure 2 provides further evidence of the inaccuracy of the local methods at accounting for precautionary savings by showing how $E(b/y)$ changes with $\sigma_z$. Recall that for 1OA solutions, certainty equivalence implies that $E(b/y) = b^{dss} = -0.51$ for all values of $\sigma_z$ and $\psi$, so there are no precautionary savings. The plots yield a key result: local methods cannot approximate accurately the values of $E(b/y)$ produced by the GLB solutions *in general*, and hence they yield incorrect measures of precautionary savings. The continuous, blue curves for the GLB solutions show that the model embodies a strong precautionary savings motive. Increasing $\sigma_z$ from 1 to 8 percent increases $E(b/y)$ from -0.5 to near zero. In contrast, the local solutions with the baseline calibrations (Panel (a)) show that 2OA overestimates the increase in precautionary savings significantly, with a gap that widens as $\sigma_z$ rises, while partial RSS mostly underestimates $E(b/y)$, although with a smaller error in absolute value than 2OA. Local methods with targeted calibrations do even worse than the baseline calibrations (see Panel (b)). $E(b/y)$ barely rises above $b^{dss}$ as $\sigma_z$ rises.

Since the quadratic and interaction terms of 2OA solutions are quantitatively irrelevant, the above result suggests that, except when $\psi$ is very low, the 1OA, 2OA and RSS solutions are nearly the same in all dimensions, even long-run averages. In addition, the 2OA and RSS

[26]We could target $\psi$ to match $E(b/y)$ in the GLB solution (-0.41) instead, but then the local methods do poorly at matching the GLB value of $\rho_{nx}$. Using this approach, $\rho_{nx} = 0.74$ and 0.88 for the 2OA and RSS solutions respectively, whereas $\rho_{nx} = 0.536$ in the global solution.

solutions also become nearly identical, since $b^{rss}$ becomes very similar to $b^{dss}$. Thus, while calibrating $\psi$ to match $\rho_b$ in the GLB solution improves the accuracy of second- and higher-order moments of the local solutions, it also removes precautionary savings almost entirely and renders 2OA and RSS solutions approximately consistent with certainty equivalence. Appendix B.3.3 demonstrates this using the analytic decision rules for log utility and i.i.d. shocks.

The intuition for why $E(b/y)$ stays close to $b^{dss}$ in the targeted calibrations follows from the argument by Schmitt-Grohé and Uribe (2003) showing that the DEIR setup is similar to a setup without DEIR but where agents incur a quadratic cost $(\tilde{\psi}/2)(b_{t+1} - b^{dss})^2$ for deviating from $b^{dss}$. The log-linearized Euler equations of the two formulations are equivalent if we set $\tilde{\psi} = \psi/(1 + r)$. Hence, a model with DEIR can be re-interpreted as a model in which agents are penalized for deviating from $b^{dss}$, and the cost increases with $\psi$.[27] Moreover, the cost has variable and fixed components, $(\tilde{\psi}/2)(b_{t+1} - 2b^{dss})b_{t+1}$ and $(\tilde{\psi}/2)b^{dss}$, respectively. If the fixed cost is larger than the benefit derived from precautionary savings, it would be suboptimal to let $E(b/y)$ deviate from $b^{dss}$. Thus, local solutions using targeted calibrations have the shortcoming that it takes only a modest increase in $\psi$ to make precautionary savings nearly vanish and render 1OA, 2OA and RSS solutions nearly identical.

The execution times of the different algorithms shown in Table 3 should be compared with caution.[28] Full RSS runs in 0.3 seconds because, given the simplicity of the endowment model, we could split the solution into a step that constructs a non-linear system of equations using Mathematica and a step that solves it using Matlab. Partial RSS takes longer (5.6 seconds) because it does both steps within Matlab using the toolkit developed by Schmitt-Grohé and Uribe (2004). The Dynare 2OA solutions run in 0.7 seconds and the *FiPIt* GLB solution in 2.5 seconds. Hence, the perturbation methods are significantly faster. The GLB solution, however, is much more accurate, as indicated by its much smaller maximum and mean Euler equation errors. Moreover, NFA local decision rules show average (maximum) differences relative to the GLB solution ranging from 7.5 to 22.5 (12.3 to 47.1) percent. Relaxing the *FiPIt* convergence criterion to yield Euler errors of similar magnitude as the local solutions lowers its execution

[27]With DEIR, for $b_{t+1} < b^{dss}$ ($b_{t+1} > b^{dss}$) agents pay more (get less) for borrowing (saving) more.

[28]The footnote to Table 3 provides details on hardware and software.

time to 1.7 seconds, but it yields moments that are not invariant to stricter convergence criteria.

Figure 3 compares IRFs for a negative, one-standard-deviation income shock. Consumption and output are shown in percent deviations from long-run means, while $b/y$ and $nx/y$ are in differences relative to long-run means (since these are GDP ratios already in percent). The IRFs for 1OA, 2OA and RSS are nearly identical, in line with the results that the $h_b$ coefficients of NFA decision rules are similar and the quadratic and interaction terms of 2OA solutions are negligible. On the other hand, local IRFs with the baseline calibration differ sharply from the GLB ones. GLB predicts a smaller decline in $b/y$ (i.e., less borrowing) and much faster mean reversion. Accordingly, consumption falls nearly twice as much on impact in the GLB solution, and continues to decline before recovering, displaying also faster mean reversion. These differences imply smaller trade deficits on impact and in the first periods of transition and a faster recovery into trade surpluses with the GLB solution. Local solutions with targeted calibrations yield IRFs that approximate better the GLB solutions, but still show discrepancies. In particular, they overestimate the fall in consumption on impact.

We also compared GLB and local solutions in the frequency domain using nonparametric periodograms of simulated data (see Appendix B.3.4). The results are very different. Local methods under the baseline calibration overestimate the contribution of low frequency movements to the variance of $b$, $c$ and $nx$, in line with their slower mean-reversion and higher $\rho_b$ relative to the GLB solution. Moreover, while the contribution of fluctuations at the business cycle frequency or higher for the variability of $b$ is slightly higher with the local solutions than in the GLB solution, for $nx$ the local methods overestimate it and for $c$ they underestimate it. For targeted calibrations, GLB and local periodograms of $b$ are nearly the same almost by construction, because the targeted calibrations have the same $\rho_b$ of the GLB solution. However, the local solutions still underestimate significantly the contribution of consumption fluctuations at business cycle and higher frequencies to overall consumption variance.

***d) Interest-rate shocks***

We examine next the effects of adding interest-rate shocks. This facilitates comparing the endowment model results with those of the RBC and SS models that also have interest-rate shocks. This is also important because, as Coeurdacier et al. (2011) and de Groot (2014) showed, RSS yields much higher precautionary savings with these shocks. The gross real interest rate is $R_t = e^{z_t^R}\bar{R}$, where $z_t^R$ is a shock with exponential support and $\bar{R}$ is the mean interest rate. The endowment and interest rate shocks follow a diagonal VAR representation:

$$\begin{bmatrix} z_t \\ z_t^R \end{bmatrix} = \begin{bmatrix} \rho_z & 0 \\ 0 & \rho_{z^R} \end{bmatrix} \cdot \begin{bmatrix} z_{t-1} \\ z_{t-1}^R \end{bmatrix} + \begin{bmatrix} \varepsilon_t^z \\ \varepsilon_t^R \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_{\varepsilon^z}^2 & \sigma_{\varepsilon^z,\varepsilon^R} \\ \sigma_{\varepsilon^z,\varepsilon^R} & \sigma_{\varepsilon^R}^2 \end{bmatrix}, \tag{9}$$

where $\Sigma$ is the variance-covariance matrix of the innovations.

The value of $\rho_z$ is 0.597, as in the original calibration. To minimize the size of the state space in the GLB solution, we use a bi-variate, two-point Markov chain defined by the Simple Persistence Rule, which imposes the same autocorrelation on both shocks (see Appendix B.3.5).[29] Hence, $\rho_{z^R} = 0.597$. The value of $\sigma_{\varepsilon^z}^2$ is set at 0.00069, so that the standard deviation of income is $\sigma_z = \sqrt{\sigma_{\varepsilon^z}^2/(1-\rho_z^2)} = 0.0327$, as in the original calibration. For the terms that involve the interest-rate process, we solve the model with values of $\sigma_{\varepsilon^R}^2$ and $\sigma_{\varepsilon^z,\varepsilon^R}$ such that $\sigma_{z^R}$ takes values ranging from 0 to 2.5 percent and the correlation between income and the interest rate is $\rho_{z,z^R} = -0.669$, which matches the correlation of the interest rate with TFP in Mendoza (2010), and is also the value used to calibrate the RBC and SS models. The values of $\sigma_{\varepsilon^z,\varepsilon^R}$ and $\sigma_{\varepsilon^R}^2$ change as we change $\sigma_{z^R}$, and they are given by: $\sigma_{\varepsilon^z,\varepsilon^R} = (1-\rho_z\rho_{z^R})\rho_{z,z^R}\sigma_z\sigma_{z^R}$ and $\sigma_{\varepsilon^R}^2 = \sigma_{z^R}^2/(1-\rho_{z^R}^2)$.

A well-defined limiting distribution of NFA now requires $\beta\bar{R} < 1$, otherwise $\beta^t\Pi_{j=1}^t R_j$ diverges to infinity (see Chamberlain and Wilson, 2000). In addition, there are long histories of realizations with $R_t$ lower (higher) than $\bar{R}$, which imply much weaker (stronger) precautionary savings incentives than with a constant interest rate. For example, histories with $\beta R_t > 1$ produce sequences where $b_{t+1}$ can grow very large, since there is no pro-borrowing effect due to $\beta R_t < 1$ offsetting the precautionary savings incentive.[30] At some point, each of these histories shifts to histories with sufficiently low $R_t$ to induce NFA mean-reversion. Note also that the NDL is now computed with the highest realization of $R_t - 1$, so it is tighter than when

[29]This is reasonable because in the data reported in Mendoza (2010) $\rho_z = 0.537$ and $\rho_{z^R} = 0.572$.

[30]Reducing $\bar{R}$ keeping $\sigma_{z^R}$ constant accentuates these effects, because histories with even larger gaps between $\beta$ and $R_t$ are possible and with higher probability.

computed with $\bar{R}-1$. These effects are at work only in the GLB solution, because they result from expectations of histories of future shocks that take the economy far from $E(b/y)$ and $b^{dss}$.

The DEIR function now takes this form:

$$1+r_t = e^{z_t^R}\bar{R} + \psi\left[e^{b^{dss}-b_{t+1}}-1\right]. \tag{10}$$

Table 4 shows key moments produced by the different solution methods for $\sigma_{z^R}$ in the [0,0.025] interval. The baseline and targeted calibrations are as in Table 1. For the GLB solution, we show results with both the calibrated ad-hoc debt limit ($\varphi=-0.51$) and the NDL, with the aim of comparing the roles of debt limits and interest-rate shocks in inducing higher mean NFA, and with the similar effect of interest-rate shocks in local solutions.

Considering the baseline calibration for partial RSS and 2OA, we find that the local solutions sharply overestimate the increase in $E(b/y)$ in response to higher $\sigma_{z^R}$ relative to the GLB solution. $E(b/y)$ increases by 140 (109) percentage points for the partial RSS (2OA) solution and turns from negative to positive, while in GLB it increases by about 3 percentage points. In addition, the ability of partial RSS v. 2OA solutions to generate precautionary savings changes as $\sigma_{z^R}$ rises. With low or no interest-rate variability, 2OA generates significantly more precautionary savings ($E(b/y)=-0.285$ v. $-0.451$), but for interest-rate variability of 2.5 percent the opposite is true ($E(b/y)=0.806$ v. $0.942$). Larger interest-rate shocks also alter the result that the baseline RSS and 2OA solutions have similar second- and higher-order moments.

The above findings suggest that interest-rate shocks in the partial RSS solutions with baseline $\psi$ could be helpful for matching mean NFA, playing the role of $\varphi$ in the GLB calibration. This strategy fails, however, because consumption fluctuates too much in all the scenarios for partial RSS and 2OA. All the local solutions shown in Table 4 overestimate the variability of consumption in the GLB solutions by ratios ranging from 1.04 (for partial RSS with targeted $\psi$ and $\sigma_{z^R}=0.5\%$) to 4.01 (for partial RSS with baseline $\psi$ and $\sigma_{z^R}=2.5\%$).

Comparing GLB with local solutions for targeted calibrations, the adjustment-cost-like effect of higher $\psi$ keeping NFA close to $b^{dss}$ still dominates. Local solutions yield small increases in $E(b/y)$ (with $\sigma_{z^R}<1.5\%$ there is almost no change) and second- and higher-order moments for RSS and 2OA are very similar. Hence, the result that higher $\psi$ removes precautionary savings and yields very similar 1OA, 2OA and RSS results is robust to adding interest-rate shocks.

Table 4 also shows that, with interest-rate shocks, full and partial (baseline) RSS do not yield similar second- and higher-order moments. Full RSS generates higher variability in consumption and NFA, higher autocorrelations in $nx$, and very low $E(b/y)$. In fact, full RSS is closer to the GLB solution that replaces the ad-hoc debt limit with the NDL than to the baseline or targeted partial RSS solutions. Full RSS and the GLB solution with the NDL have, however, the major shortcoming that they produce unreasonable NFA positions of -3 to -11 times GDP. Moreover, for $R$ higher than the calibrated 1.059 and such that $\beta R$ is almost 1, full RSS yields much lower $E(b/y)$ than GLB solutions with either ad-hoc or natural debt limits. Conversely, for low $R$, the RSS solution violates the NDL very often (e.g., for $\bar{R} = 1.01$, NDL is $-68.44$ while full RSS yields $E(b/y) = -69.62$). Hence, although at the calibrated $R$ full RSS gets closer to the mean NFA of the GLB solution with NDL, full RSS performs poorly in general at approximating $E(b/y)$, whether we use NDL or $\varphi$ as debt limit in the GLB solution.

***e) Endogenous discounting instead of DEIR***

As noted in the Introduction, most local solutions in the literature induce stationarity using DEIR, but some do use ED preferences. We report next results showing that, while 2OA decision rules using ED differ from those using DEIR, 2OA solutions still do a poor job at approximating $E(b/y)$ and precautionary savings of the GLB solutions. ED yields good approximations only when precautionary savings are negligible in the GLB solution.

Appendix B.3.8 compares analytical DEIR and ED local decision rules assuming log utility and i.i.d. shocks for an ED setup in which the discount factor depends on aggregate consumption (i.e., this dependency is disregarded by private agents). In line with the numerical findings in Schmitt-Grohé and Uribe (2003), 1OA solutions using DEIR or ED are equivalent: A nonlinear mapping determines the elasticity of the discount factor with respect to consumption ($\psi^{ED}$) for a given $\psi$ such that the decision rules are the same. 2OA solutions, however, are not equivalent. Varying $\psi$ while adjusting $\psi^{ED}$ so that the $h_b$ NFA decision-rule coefficients of DEIR and ED are equal, the $h_{bb}$ coefficient for DEIR is increasing and concave in $\psi$ while that for ED is slightly and nearly-linearly decreasing. The $h_{\sigma\sigma}$ coefficient of the ED case is decreasing and convex in $\psi$ but in the DEIR case it is nearly independent of $\psi$. Moreover, $h_{\sigma\sigma}$ is very sensitive to small changes in $r$ in the ED case but nearly invariant in the DEIR case. These differences are due to a critical difference between the two setups: When $c_t$ rises as the economy borrows (reduces $b_{t+1}$), $r_t$ rises in DEIR but $\beta_t$ falls in ED. As a result, the marginal benefit of savings $\beta_t(1 + r_t)u'(c_{t+1})$ rises in DEIR but falls in ED. The fall in the marginal benefit of savings with ED also weakens the precautionary savings incentive in the GLB solution relative to the GLB solution with standard preferences and $\beta R < 1$, as Durdu et al. (2009) showed.

Appendix B.3.8 also compares quantitative results for ED solutions using 2OA and GLB methods. We consider two GLB solutions, one with ED preferences and one that is the baseline case of Table 3 (which uses preferences with $\beta R < 1$). These are compared with 2OA solutions in which $\psi^{ED}$ is calibrated to match $b^{dss}$ in each of the GLB solutions. Case I for the GLB solution of Table 3 and Case II for the GLB solution with ED. The GLB solutions use the calibration proposed by Durdu et al. (2009) for the same two specifications of preferences. As explained in the Appendix, this calibration makes the two GLB solutions yield similar $E(b/y)$. Hence, when calibrating $\psi^{ED}$ for the comparable 2OA solutions we found that they require similar $\psi^{ED}$ values (0.109 in Case I and 0.11 in Case II). The results show that, for the calibrated $\sigma_z$, precautionary savings (i.e., $E(b/y) - b^{dss}$) are negligible in the GLB-ED solution (Case I), and thus the comparable 2OA ED solution approximates it well. As $\sigma_z$ rises and self-insurance incentives strengthen, however, the 2OA ED solution underestimates precautionary savings and by an increasing margin as $\sigma_z$ rises. Case II has stronger precautionary savings incentives in the GLB solution, because it uses standard preferences with $\beta R < 1$ instead of ED. Hence, the 2OA ED solution underestimates precautionary savings even more than in Case I and the gap again widens as $\sigma_z$ rises. Moreover, the higher $\psi^{ED}$ of Case II, albeit slightly above that of Case I, has effects similar to those of higher $\psi$ in DEIR solutions in that it makes deviations of NFA relative to $b^{dss}$ costlier and keeps NFA close to $b^{dss}$.

We also compared the 2OA ED solutions against the baseline and targeted 2OA DEIR solutions. As noted above, at the calibrated $\sigma_z$, the ED solutions in Cases I and II yield similar $E(b/y)$ as their comparable GLB solutions, and as we documented earlier the baseline DEIR overestimates $E(b/y)$ while the targeted DEIR underestimates it. The ED solutions appear to approximate better precautionary savings, but this is only because at the calibrated $\sigma_z$ precautionary savings are negligible in the GLB ED solution. For $\sigma_z$ high enough to make precautionary savings relevant, the 2OA ED solutions underestimate the $E(b/y)$ of the GLB solutions.

***f) Exact-solution model***

So far we have compared local and GLB solutions that approximate an unknown "exact solution." Under two special assumptions, however, the model can be solved in closed form so as to allow us to compare those solutions with the "exact" solution. The two assumptions are: i) income is a multiplicative return on a risky asset with a log-normal i.i.d process; and ii) consumption is chosen before the return is observed (see Appendix B.3.11 for details).[31] The analytic solutions are $c_t = \lambda(\sigma_\varepsilon)b_t$ and $b_{t+1} = (1-\lambda(\sigma_\varepsilon))R_t b_t$, where the savings rate is:

$$1-\lambda(\sigma_\varepsilon) = \beta^{1/\sigma}E(R)^{\frac{1-\sigma}{\sigma}}\exp\left(-(1-\sigma)\frac{\sigma_\varepsilon^2}{2}\right). \tag{11}$$

The precautionary savings effect is evident in that, if $\sigma > 1$, a mean-preserving spread of the $R_t$ process (i.e., higher $\sigma_\varepsilon^2$ keeping $E(R)$ constant) increases the savings rate. NFA (in logs) follow a random walk with drift ($ln(b_{t+1}) = ln(1-\lambda(\sigma_\varepsilon)) + ln(b_t) + ln(R_t)$), and hence consumption does as well, but consumption growth is a log-i.i.d. process: $c_{t+1}/c_t = (1-\lambda(\sigma_\varepsilon))R_t$.

Appendix B.3.11 implements the GLB solution and local solutions up to fourth order (4OA) plus RSS by expressing the model in ratios of $b_t$. We keep $\beta = 0.94$ (the same as in the baseline calibration), set $E(R) = 1.7$, in line with the assumption that $b$ is a risky asset that provides all of the economy's income, and vary $\sigma_\varepsilon$ from 0 to 0.45 keeping $E(R)$ unchanged. Higher $\sigma_\varepsilon$ yields unfeasible equilibria with $\lambda(\sigma_\varepsilon) < 0$. The GLB and exact solutions are virtually identical for all values of $\sigma_\varepsilon$ and 4OA is also very similar. RSS and 2OA are accurate only for $\sigma_\varepsilon < 0.3$ otherwise they understimate the savings rate by up to 15 and 5 percent, respectively. 1OA starts to do poorly with $\sigma_\varepsilon > 0.08$ and underestimates the savings rate by up to 40 percent. Moreover, 1OA, 2OA and RSS yield incorrect results indicating feasible saving rates when the true solution is unfeasible (for $\sigma_\varepsilon > 0.45$). Note that, since the theoretical model itself is non-stationary in this case, $\rho_b$ and the center of approximation of NFA do not contribute to the inaccuraccies of the local solutions. Their cause is only the approximation error of RSS, 2OA and 1OA when

[31] The resource constraint becomes $b_{t+1} = R_{t+1}(b_t - c_t)$, where $\log(R_t) = \mu + \sigma_\varepsilon\varepsilon_{t+1}$, $\varepsilon_{t+1} \sim N(0,1)$, and the Euler equation for assets is $c_t^{-\sigma} = \beta\mathbb{E}_t\left(c_{t+1}^{-\sigma}R_{t+1}\right)$. Note that $b$ is now a risky asset with return $R$.

expanding the Euler equation, which is negligible with 4OA. Hence, a higher-order approximation improves the accuracy of the local methods in this case, but for the other models we solved it does not because the flaws in pinning down $\rho_b$ and the center of approximation remain.

## 3. Sudden Stops Model

This section compares global and local solutions of the SS model with an occasionally binding collateral constraint proposed by Mendoza (2010). Without the constraint, the model reduces to a standard small-open-economy RBC model. The solutions for this RBC model are compared in Appendix C. Local solutions show similar flaws with regard to NFA, consumption and the external accounts as the endowment model, but they are accurate for supply-side variables because there is no wealth effect on labor supply and the equity premium is small, as is typical of RBC models. As Mendoza (1991) noted, these features render the capital decision rule similar to that implied by risk-neutral arbitrage of returns on capital and NFA, which implies that Fisherian separation of investment from consumption and savings decisions nearly holds. Hence, the coefficient of the capital decision rules on lagged NFA in the local solutions and the elasticities of the decision rule $k'(b, k, \varepsilon)$ with respect to $b$ in the GLB solution are negligible except when the debt limit binds (see Appendix C.3.1).

### *3.1. Model structure*

As in Mendoza (2010), the model's competitive equilibrium is represented as the solution to a representative firm-household problem. Gross output is produced with a Cobb-Douglas technology using capital, $k_t$, labor, $L_t$, and imported inputs, $\upsilon_t$:

$$e^{\varepsilon_t^A} F(k_t, L_t, \upsilon_t) = e^{\varepsilon_t^A} k_t^{\gamma} L_t^{\alpha} \upsilon_t^{\eta}, \quad 0 \leq \alpha, \gamma, \eta \leq 1, \quad \alpha + \gamma + \eta = 1. \tag{12}$$

Gross output is a tradable good sold at a world-determined price which is the numeraire and set equal to 1. The relative price of imported inputs is also determined in world markets and is given by $p_t = pe^{\varepsilon_t^P}$, where $p$ is the mean price and $\varepsilon_t^P$ is a terms-of-trade shock. There are also TFP shocks, $\varepsilon_t^A$, and interest-rate shocks $\varepsilon_t^R$. A standard working capital constraint requires a fraction $\phi$ of the cost of $L_t$ and $\upsilon_t$ to be paid in advance of sales. Working capital loans are obtained from foreign lenders at the beginning of each period and repaid at the end, so that the financing cost of inputs is the net interest rate $R_t - 1$. Physical capital is costly to adjust, with adjustment costs per unit of net investment $(k_{t+1} - k_t)$ given by $\Psi(\frac{k_{t+1}-k_t}{k_t}) = \frac{a}{2}\left(\frac{k_{t+1}-k_t}{k_t}\right)$, with $a \geq 0$. This functional form satisfies Hayashi's conditions so that the average and marginal Tobin Q's are equal at equilibrium.

The representative firm-household chooses $[c_t, L_t, i_t, \upsilon_t, b_{t+1}, k_{t+1}]_{t=0}^{\infty}$ so as to maximize:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t - \frac{L_t^{\omega}}{\omega}\right)^{1-\sigma}}{1-\sigma} \right\}, \tag{13}$$

subject to

$$c_t(1+\tau) + i_t = e^{\varepsilon_t^A} F(k_t, L_t, \upsilon_t) - p_t \upsilon_t - \phi(R_t - 1)(w_t L_t + p_t \upsilon_t) - q_t^b b_{t+1} + b_t, \tag{14}$$

$$q_t^b b_{t+1} - \phi R_t (w_t L_t + p_t \upsilon_t) \geq -\kappa q_t k_{t+1}. \tag{15}$$

The utility function is of the Greenwood-Hercowitz-Huffman (GHH) form, which removes the wealth effect on labor supply. The market prices of labor and capital, which are taken as given by the agent, are denoted $w_t$ and $q_t$. As in the endowment model, $b_t$ is a non-state-contingent discount bond traded in world markets at price $q_t^b$. The left-hand-side of the resource constraint (14) is the sum of consumption, inclusive of an ad-valorem tax $\tau$ used to calibrate the ratio of government expenditures to GDP, plus gross investment, $i_t$, where $i_t = \delta k_t + (k_{t+1} - k_t)\left[1 + \Psi\left(\frac{k_{t+1}-k_t}{k_t}\right)\right]$ and $\delta$ denotes the depreciation rate. The right-hand-side equals total supply, which consists of GDP ($y_t \equiv e^{\varepsilon_t^A} F(k_t, L-t, \upsilon_t) - p_t \upsilon_t$) net of foreign interest payments on working capital loans ($\phi(R_t - 1)(w_t L_t + p_t \upsilon_t)$) minus (plus) net resources lent (borrowed) abroad ($q_t b_{t+1} - b_t$). The net exports is $nx_t = q_t b_{t+1} - b_t + \phi(R_t - 1)(w_t L_t + p_t \upsilon_t) = y_t - c_t(1+\tau) - i_t$. Condition (15) is a Fisherian collateral constraint by which debt and working capital credit cannot exceed a fraction $\kappa$ of the market value of capital.

The competitive equilibrium is defined by sequences of allocations $[c_t, L_t, k_{t+1}, b_{t+1}, \upsilon_t, i_t]_0^{\infty}$ and prices $[w_t, q_t]_0^{\infty}$ such that (a) the representative firm-household solves its optimization problem given $[w_t, q_t]_0^{\infty}$ and initial conditions $(k_0, b_0)$, and (b) $[w_t, q_t]_0^{\infty}$ satisfy their corresponding market equilibrium conditions.

### 3.2. *Solution Methods*

Local and GLB solutions of this model require considering the additional endogenous state variable ($k_t$) and handling the occasionally binding constraint. For the global solution, we use *FiPIt* with grids of $k$ and $b$ with 30 and 80 nodes, respectively. Mendoza and Villalvazo (2020) provide full details, including Matlab codes and a Users Guide.

The DynareOBC solution of the SS model is similar to the one described in Appendix B.3.10. This method treats the occasionally biding constraint as a source of endogenous news about the future along perfect-foresight paths. If the constraint is (is not) binding at the deterministic steady state, it uses news shocks to solve for unconstrained (constrained) periods along those paths by solving a mixed-integer linear programming problem. For instance, if the constraint does not bind at steady state, when agents anticipate that the constraint will bind at some date $t + j$ conditional on the date-$t$ state variables and the deterministic evolution of the exogenous shocks, this provides "news" that $b_{t+1}$ will follow a path higher than otherwise. This approach is akin to assuming that there is no constraint, but whenever agents are on a path that would lead them to borrow above what the constraint allows, a series of news shocks hit that makes them borrow only what is allowed and moderate their borrowing before that happens.[32]

The main output of DynareOBC is a time-series simulation constructed by stitching together the date-$t$ values of perfect-foresight paths conditional on $(k_t, b_t)$ and the date-$t$ realizations of the exogenous shocks. Each path is obtained using an extended path algorithm that traces equilibrium dynamics up to $T$ periods ahead of $t$, with the shocks following their deterministic VAR dynamics. The extended path can be obtained using first- or higher-order approximations, but we report here results based on the former.[33] The path computed for a given starting date $t$ determines the values of $(k_{t+1}, b_{t+1})$ and these together with the realizations of the shocks at $t$ and the optimality conditions determine the date-$t$ values of all the endogenous variables. The

[32]The model with the constraint is similar to the same model without the constraint but with sequences of news shocks chosen to yield the same equilibrium as the model with the constraint. This equivalence holds exactly if the model is linear and shock variances are zero, such that any shocks that occur are truly "unexpected."

[33]Holden (2016) showed that a second-order approximation integrating over future uncertainty can approximate precautionary savings in models with simple constraints, but this method is significantly slower and for the SS model produced results that deviate sharply from the GLB and first-order DynareOBC solutions. In particular, investment and net exports had negative serial autocorrelation and NFA had near-zero autocorrelation.

rest of the path is discarded and the process is repeated at $t+1$ to generate the values of the time-series simulation for that period. The efficiency of this method hinges on three factors: (a) $T$ needs to be large enough so that for $t > T$ no further news shocks are needed (if the constraint does not bind at the deterministic steady state $T$ needs to be large enough so that the constraint never binds again, and if it binds at steady state $T$ needs to be large enough so that it always binds); (b) for each path requiring news shocks, the algorithm needs to find the sequence of news shocks that supports the correct equilibrium path; and (c) the time-series simulation needs to be long enough for long-run moments of the endogenous variables to converge. The algorithm is less efficient in models with persistent dynamics, which require large $T$ and a long simulation length, and models in which the news shocks are needed frequently.

Figure 4 illustrates the DynareOBC method using the endowment model, for which $b_{t+1} \geq \varphi$ is an occasionally binding constraint, including the DEIR function so that $\varphi$ does not bind at the deterministic steady state (see Appendix B.3.10 for details). Panels (a) and (b) show the solutions for $c_t$ and $b_{t+1}$ for t=90 to 250 (black, solid curves) and eleven of the perfect-foresight paths (red, dashed curves) that generated them, with the corresponding date-t solution marked with a red circle. In panels (b) and (d), the shaded area corresponds to $b_{t+1} < \varphi$. The constraint never binds in seven of the perfect-foresight paths shown in Panel (b) and in four it does.

Panels (c) and (d) isolate periods t=140 to 180 and show the extended path that generates the results for t=141 (red, dashed curve). DynareOBC computes a sequence of news shocks that sustains this path as an equilibrium. The comparable path of $b_{t+1}$ in the solution without credit constraint is also provided in Panel (d) (black, dotted curve). The constraint first becomes binding along the perfect-foresight path at t=144. Relative to the model without constraint, agents choose higher $b_{t+1}$ (less debt) earlier, in anticipation of the constraint becoming binding with perfect foresight (i.e., the red, dashed curve is above the black, dashed curve at t=142,143). Since income rises gradually back to its deterministic steady state, the constraint continues to bind for several periods, until income is high enough for $b_{t+1}$ to also start rising back towards its steady state (after t=170).

It is critical to note that first-order DynareOBC ignores the *risk* of hitting the constraint and moving across states where it binds or not. At each date $t$, it does not consider the histories of future shocks and associated allocations and prices that can occur, it only considers the perfect-foresight path conditional on date $t$ and the date-$t$ shock. If the constraint binds (does not bind) at the deterministic steady state, agents anticipate deterministically hitting (escaping) the constraint at some date $t+j$ if unconstrained (constrained) at $t$ and adjust their decisions before $t+j$ accordingly. Hence, wealth and precautionary-saving effects of the constraint are ignored, and forward-looking objects like asset prices and excess returns also abstract from them. This is central to SS models, because a financial crisis with a deep recession and collapsing prices occurs when the constraint binds, and the risk of these events strengthens precautionary savings and alters asset prices even in "good times" (see Mendoza, 2010; Durdu et al., 2009).

*3.3. Calibration*

Table 5 shows the calibration parameters, most of which were taken from Mendoza (2010) (see Appendix C.2 for details). The main difference is that $\varphi$ and $\beta$ in the GLB solution are set following a strategy similar to that used in the endowment model, by targeting them so that the RBC version of the model approximates the mean NFA-GDP ratio and the variability of consumption in Mexican data.

The model's three shocks follow the same diagonal VAR from Mendoza (2010):

$$\begin{bmatrix} \varepsilon_t^A \\ \varepsilon_t^R \\ \varepsilon_t^p \end{bmatrix} = \begin{bmatrix} \rho^A & 0 & 0 \\ 0 & \rho^R & 0 \\ 0 & 0 & \rho^p \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{t-1}^A \\ \varepsilon_{t-1}^R \\ \varepsilon_{t-1}^p \end{bmatrix} + \begin{bmatrix} u_t^A \\ u_t^R \\ u_t^p \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_{u^A}^2 & \sigma_{u^A,u^R} & 0 \\ \sigma_{u^A,u^R} & \sigma_{u^R}^2 & 0 \\ 0 & 0 & \sigma_{u^p}^2 \end{bmatrix}. \tag{16}$$

In this VAR, the co-movement between TFP and interest-rate shocks is driven only by the covariance of their innovations and price shocks are independent of the other two shocks, following Mendoza's empirical evidence. The elements of the autocorrelation and variance-covariance matrices also take the same values as in Mendoza (2010). The discrete approximation to the VAR in the GLB solution is constructed using the Simple Persistence Rule, which requires $\rho^A = \rho^R$ (see Appendix C.2 for details). In the local solutions, we impose the same autocorrelation and innovation matrices on the VAR specification of the shocks.

It is important to note that when DynareOBC is used assuming that the constraint binds at the deterministic steady state, the steady-state equilibrium is well-defined without the DEIR function. The bonds Euler equation becomes $1 = \beta R + \mu_{ss}/u'(c_{ss})$, where $\mu$ is the multiplier on the borrowing constraint. Since $\beta R < 1 \iff \mu_{ss} > 0$, having the constraint bind at steady state requires $\beta R < 1$ and viceversa. The Euler equation is solved jointly with the other steady-state equilibrium conditions to fully solve the stationary equilibrium.

We study DynareOBC solutions with $\mu_{ss} > 0$ (labeled "DynareOBC-$\beta R < 1$") and $\mu_{ss} = 0$ (labeled "DynareOBC-DEIR," because the DEIR function is used to induce stationarity). For the former, $\beta$ is the same as in the GLB solution, and hence DynareOBC and GLB calibrations are identical. For the DynareOBC-DEIR case, $b^{dss}/y^{dss}$ in the DEIR function is set so that it matches $E(b/y)$ from the GLB solution, with $\beta = 1/R$ and $\psi = 0.001$. The rationale for looking at this case is that in the GLB solution the constraint binds rarely and $E(b/y)$ is much higher than $b^{dss}/y^{dss}$. Hence, a local approximation around an unconstrained steady state is more in line with the unconstrained long-run equilibrium of the GLB solution.

*3.4. Comparison of quantitative results*

***a) Long-run moments, impulse responses & performace metrics***

Table 6 shows that several moments of the DynareOBC solutions differ from their GLB counterparts, with smaller differences for supply-side variables. The latter occurs because, around the stochastic steady state, the model is still close to Fisherian separation of savings and investment. Many of the moments that are underestimated with DynareOBC-$\beta R < 1$ relative to the GLB solution tend to be overestimated with DynareOBC-DEIR.

The credit constraint causes a large increase in precautionary savings. In the GLB solution, $E(b/y)$ rises from -0.37 in the RBC model to 0.015 in the SS model. In contrast, DynareOBC-$\beta R < 1$ (DynareOBC-DEIR) underestimates (overestimates) mean NFA significantly, yielding $E(b/y) =$ -0.1 (0.206). This has important implications for research and policy. For example, quantifying optimal macroprudential regulation or foreign reserves to manage Sudden Stop risk (e.g., Durdu et al., 2009; Bianchi and Mendoza, 2018) requires determining first how NFA responds to this risk without policy intervention. DynareOBC's results are sharply above or below the GLB solution and would call for policies that are too weak or too strong, respectively.

Certainty equivalence fails in the DynareOBC solutions even though the perfect-foresight paths are first-order approximations. In the DynareOBC-$\beta R < 1$ (DynareOBC-DEIR) solution,

$b^{dss}/y^{dss} = -0.192$ (0.015) while $E(b/y) = -0.1$ (0.206). This is not due to precautionary savings, since DynareOBC ignores them, but to asymmetric responses to shocks induced by the constraint in DynareOBC even without risk. This asymmetry can be illustrated using Figure 4 (see also Appendix B.3.10). A negative shock that causes the constraint to bind along the perfect-foresight path determining the date-t value of the solution reduces $b_{t+1}$ by less than the increase in $b_{t+1}$ in response to a positive shock of the same size. As a result, upward movements in $b_{t+1}$ when positive shocks hit are larger than downward movements when negative shocks hit if $b_t$ is near or at a point where the constraint binds. Moreover, $b_{t+1}$ cannot move below the constraint but it can wander off to high values after sequences of positive shocks. Hence, the DynareOBC time-series is "biased" above $b^{dss}$, which implies a mean above $b^{dss}/y^{dss}$.[34]

The GLB solution has a similar asymmetry but it also takes into account precautionary savings effects due to the risk of future shocks and the constraint becoming binding. It does not follow, however, that DynareOBC always yields mean bond positions *lower* than the GLB solution (DynareOBC-$\beta R < 1$ yields lower mean NFA but DynareOBC-DEIR higher). Both DynareOBC results ignore precautionary savings, but in the DynareOBC-DEIR solution $b^{dss}/y^{dss}$ is set equal to the value of $E(b/y)$ in the GLB solution (0.015) and the constraint does not bind at steady state. As a result, the solution is "biased" above 0.015 and must yield $E(b/y) > 0.015$.

Choosing between Dynare-$\beta R < 1$ and Dynare-DEIR, the former is preferable. Both yield moments that differ from the GLB solution, but as we show later in this Section, Dynare-$\beta R < 1$ does better at approximating the effects of the collateral constraint. It also uses the same calibration as the GLB solution and does not require extra assumptions to impose stationarity.

We compare next performance metrics.[35] The speed advantage of the local methods shrinks considerably, particularly for DynareOBC-$\beta R < 1$, which has a speed ratio of 0.90 relative to the GLB solution. This is due to the three determinants of DynareOBC efficiency noted earlier and the fact that NFA follows a near-unit-root process. Each extended path required at least 60 periods and the full simulations needed 100,000 periods to converge to invariant moments.[36]

[34] Recall that the constraint in this example is a fixed debt limit while in the SS model it depends on $q_t k_{t+1}$.

[35] See footnote to Table 3 for details on hardware and software.

[36] Intuitively, consider that the estimators of the mean and autocorrelation of an AR(1) process are consistent but biased in finite samples. The bias is higher the higher the true autocorrelation but it falls as the sample size

Speed comparisons of DynareOBC and *FiPIt* need to be pondered carefully. *FiPIt* suffers from the curse of dimensionality and it is slower in models that require a root-finder when the constraint binds.[37] But once the decision rules are solved, generating time-series simulations is fast. In contrast, the number of state variables is less of an issue for DynareOBC, but execution time rises with the length of perfect-foresight paths, the iterations needed to compute news-shocks sequences that implement the constraint, and the length of the time-series simulation needed for convergence of long-run moments. As Appendix C.4.2 shows, DynareOBC-$\beta R < 1$ is much slower than *FiPIt* with a simulation length of 150,000 periods (350 seconds v. 268 seconds), TFP shocks only (230 v. 42 seconds), or $\kappa = 0.3$ (228 v. 137 seconds).[38] Solving in second-order and/or integrating over future uncertainty slows down DynareOBC further.

In terms of accuracy, *FiPIt* produces accurate GLB results with small maximum errors in the bonds and capital Euler equations. Since DynareOBC solutions only produce a time-series simulation, we follow Holden (2016) to evaluate their accuracy by constructing consumption simulations of the GLB solution for the same initial conditions and sequence of shocks as in the DynareOBC solutions, and computing the maximum absolute values of the differences across them. The maximum differences in log base 10 are about 1.3 for both DynareOBC solutions, much larger than Holden's estimates for an endowment model.

Figure 5 shows IRFs for a one-standard deviation, negative TFP shock. The IRFs for the GLB solution are conditional on starting at the long-run averages of $k$ and $b$, and those for Dynare-OBC solutions on starting at the deterministic steady state (which for DynareOBC-DEIR are the same as the GLB averages). DynareOBC IRFs differ sharply from their GLB counterparts. With DynareOBC-$\beta R < 1$, $b/y$ hardly moves and $nx/y$ moves into a surplus on impact, reflecting reduced demand for imported inputs. This occurs because the constraint binds at date 0 and the TFP shock tightens the constraint more. For DynareOBC-DEIR, $b/y$ declines, offsetting

rises. For a near-unit-root process, the sample needs to be quite large to make the estimation bias negligible.

[37] As Mendoza and Villalvazo (2020) explain, this is not needed for several specifications (e.g., the same Mendoza SS model but without working capital in the constraint, which reduces the *FiPIt* run time by 57 percent).

[38] DynareOBC also poses logistical hurdles. Updates to Dynare can make older versions of DynareOBC inoperable, and some versions of Dynare operate only in certain operating systems and software environments. For instance, the DynareOBC toolbox we used operates with Dynare 4.4.3 and only with Matlab2016a. Dynare 4.4.3 operates with the Ubuntu 14.04 Linux operating system but not with Ubuntu 18.04.

the fall in imported inputs to yield an almost unchanged $nx/y$. In contrast, in the GLB solution $nx/y$ jumps on impact nearly twice as much as under DynareOBC-$\beta R < 1$ and $b/y$ rises gradually to peak roughly 150 basis points above its mean, and after that it falls slowly to a trough 400 basis points below its mean before gradually reverting to its mean. For $k$, both DynareOBC solutions yield a decline on impact, while in the GLB solution it is nearly unchanged. Then $k$ declines slightly and starts recovering in the two local solutions, while in the GLB solution it falls by nearly three times as much reaching nearly 1.5 percent below average before starting to recover. Qualitatively, the responses of $c$, $i$, $L$, $v$ and $y$ are similar in all solutions, but the declines on impact are significantly larger in the GLB solution.[39]

Appendix C.4.1 compares periodograms for the DynareOBC and GLB solutions and shows that they differ sharply. DynareOBC assigns significantly less consumption variability to business cycle and lower frequencies than the GLB solution. Net exports show higher persistence in the DynareOBC-DEIR solution while DynareOBC-$\beta R < 1$ and GLB have similar persistence. The GLB solution has less variability at all frequencies. Investment has higher persistence in the GLB than in the local solutions, and it has uniformly higher variability at all frequencies.

***b) Credit constraint multipliers, Sudden Stops and risk effects***

Local and global solutions also differ sharply in that the constraint binds much more often in the former (51 to 71 percent of the time in the two local solutions v. only 2.6 percent in the GLB solution). This is in part because DynareOBC disregards precautionary savings, but it is also the case that it yields multipliers that are too small when the constraint binds and this alters results significantly, as we document next. We compare results for credit constraint multipliers and their effects on financial premia and sudden-stop responses of macro variables. Financial premia include the shadow interest rate premium ($SIP$), the equity premium ($EP$), its components due to unpledgeable capital ($(1-\kappa)SIP$) and risk premium ($RP$), and the Sharpe ratio ($S$). For sudden-stop responses, we compare deviations from long-run averages

[39]For the GLB solution, the IRFs of the RBC and SS models are very similar, because the constraint binds only in the left tail of the ergodic distribution (see Appendix C.3.3 for IRFs of the RBC model). Hence, IRFs, which are triggered by shocks of standard magnitudes and start from long-run means, are nearly unaffected by the credit friction. In contrast, the DynareOBC IRFs for the SS model are very different from the IRFs that all the local methods produce for the RBC model.

in $c$, $nx/y$, $i$, $y$, $L$ and $\upsilon$.

$SIP$ is the amount by which the intertemporal marginal rate of substitution $u'(t)/[\beta E_t(u'(t+1))]$ exceeds $R_t$. The bonds' Euler equation yields:

$$SIP_t = \frac{R_t \mu_t (1+\tau)}{u'(t) - \mu_t(1+\tau)}. \tag{17}$$

$SIP_t$ is relevant only when $\mu_t > 0$ and it rises as the constraint becomes more binding, because $\mu_t$ rises and $E_t(u'(t+1))$ falls, since the constraint forces agents to defer consumption.

The equity premium is $EP_t \equiv E_t[R^q_{t+1}] - R_t$, where $R^q_{t+1} \equiv (d_{t+1} + q_{t+1})/q_t$ is the return on equity and $d_{t+1}$ is the dividend payment, where $d_t \equiv \exp(\epsilon^A_t) F_k(t) - \delta + \frac{a}{2}\frac{(k_{t+1}-k_t)^2}{k_t^2}$. Using the Euler equations for bonds and capital it follows that:

$$EP_t = (1-\kappa) SIP_t + RP_t, \qquad RP_t \equiv -\frac{COV_t[u'(t+1), R^q_{t+1}]}{E_t[u'(t+1)]}. \tag{18}$$

$EP_t$ has two components: the standard risk premium ($RP_t$) driven by $COV_t[u'(t+1), R^q_{t+1}]$ and the fraction of $SIP_t$ pertaining to the share of $k_{t+1}$ that cannot be pledged as collateral $(1-\kappa)$. $EP_t$ rises when $\mu_t > 0$ for two reasons: First, $SIP_t$ rises, as explained above. Second, $RP_t$ rises, because $COV_t[u'(t+1), R^q_{t+1}]$ becomes more negative as consumption is harder to smooth and $E_t[u'(t+1)]$ falls as the credit constraint forces consumption into the future. Thus, $EP$ reflects both the tightness of the constraint via $SIP_t$ and the larger *risk* premium that the constraint induces. The Sharpe ratio measures the compensation for risk-taking, defined as $S_t = E[EP]/\sigma(R^q)$. Following standard practice, we compute $S_t$ using uconditional moments.

For the GLB solution, the financial premia are computed for each triple $(b, k, \varepsilon)$ in the state space (see Appendix C.4.3). Averages are then computed using the conditional and unconditional distributions of $(b, k, \varepsilon)$. For the DynareOBC solutions, $SIP_t$ is computed using the time-series simulation they produce. The equity premium is then generated as $EP_t = (1-\kappa)SIP_t$ because $RP_t = 0$ by construction, since each date-$t$ solution is determined by a perfect-foresight path (the simulated datasets also produce very small values for $COV[u'(\cdot), R^q]$).

Table 7 reports quintile distributions of $\mu$ conditional on $\mu > 0$, the associated within-quintile averages of financial and macro variables, their overall means and medians, and the

Sharpe ratios.[40] $\mu$ is very small in all five quintiles of all solutions, but this is because $\mu$ is in units of marginal utility with $CRRA$ preferences and $\sigma = 2$. For instance, at the unconditional means of $c$ and $L$, marginal utility is about 2.05E-05 (-4.688 in log base 10). Hence, small $\mu$ values do not imply that the constraint is irrelevant, as shown below.

The multipliers, financial premia and sudden-stop responses when the collateral constraint binds are significantly smaller in the local solutions than the GLB solution. The differences grow larger for higher $\mu$ (i.e., in the fourth and fifth quintiles), and they are larger relative to the local solution with unconstrained deterministic steady state (DynareOBC-DEIR) than the one with constrained steady state (DynareOBC-$\beta R < 1$).

For financial premia, GLB yields overall means of 2.6, 2.2, 2.1 and 0.1 percent for $SIP$, $EP$, $(1-\kappa)SIP$ and $RP$, respectively, while DynareOBC-$\beta R < 1$ (DynareOBC-DEIR) yields 0.8, 0.6, 0.6 and 0 (0.13, 0.10, 0.10 and 0) percent, respectively. In the GLB solution, $RP$ is about 0.1 percent on average in each of the five quintiles of $\mu$, but $EP$ still increases sharply with $\mu$ because $(1-\kappa)SIP$ rises sharply. In the fifth quintile, GLB yields averages of 6.6, 5.4, and 5.3 percent for $SIP$, $EP$, and $(1-\kappa)SIP$, respectively, while DynareOBC-$\beta R < 1$ (DynareOBC-DEIR) yields 3.3, 2.7 and 2.7 (0.64, 0.51 and 0.51) percent, respectively. The local solutions sharply underestimate $SIP$ and $EP$. They also miss the risk premium, but this accounts for a small fraction of the gap in $EP$. GLB yields a Sharpe ratio of 1.16, nearly 5 and 30 times the DynareOBC-$\beta R < 1$ and DynareOBC-DEIR results, respectively. Since $RP$ is small in the GLB solution and zero in the local solutions, the differences in $S$ are due to the large gap in $SIP$.

Large differences in $SIP$ and $EP$ result in very different sudden-stop responses. To explain why, we follow Mendoza and Smith (2006) in expressing the price of capital as:

$$q_t = E_t \left( \sum_{i=1}^{\infty} \left[ \prod_{j=0}^{i} \frac{1}{E_t(R^q_{t+1+j})} \right] d_{t+1+i} \right). \tag{19}$$

Since eq. (18) implies that $E_t[R^q_{t+1}] = (1-\kappa)SIP_t + RP_t + R_t$, lower financial premia with DynareOBC imply higher $q_t$ when $\mu_t > 0$, which in turn imply weaker Fisherian deflation effects of the binding credit constraint. Moreover, since $q_t$ and investment are monotonic func-

[40] Variables are assigned into quintiles according to the quintile distribution of $\mu$. If a given $\mu_i$ belongs to a particular quintile of $\mu$, then the corresponding values of the other variables are assigned to that same quintile.

tions of each other due to the Tobin Q nature of the investment setup, $k_{t+1}$ is higher and so is borrowing capacity ($\kappa q_t k_{t+1}$), which is key for determining allocations when $\mu_t > 0$. This also affects future dividends, causing further feedback effects into $q_t$ and borrowing capacity.

The differences in sudden-stop responses reported in Table 7 reflect the above arguments. In the GLB solution, the responses are in line with standard features of Sudden Stops (i.e., large recessions and sharp reversals in the external accounts). The mean percent declines (relative to long-run averages) are -3.6 in $c$, -4.1 in $i$, -1.0 in $y$, -0.7 in $L$, and -1.8 in $v$ while $nx/y$ rises 2.6 percentage points. The responses are generally larger when the constraint binds more, reaching means of -4.9 for $c$ and -13.5 for $i$ with a trade balance reversal of 5.1 percentage points for the top quintile of $\mu$. The DynareOBC-$\beta R < 1$ solution underestimates the mean responses of consumption and net exports (-1.9 v. -3.6 for $c$ and 1.2 v. 2.6 for $nx/y$) and overestimates those for $L$, $v$ and $y$. It also fails to match the property that the responses should be larger when the constraint binds more, as it yields the largest responses in the third quintile of $\mu$. DynareOBC-DEIR performs worse, producing *positive* mean responses for $c$ and $i$ and a *negative* mean response for $nx/y$. Moreover, these counterfactual responses grow larger when the constraint binds more, in the 4th and 5th quintiles of $\mu$. This failure to produce Sudden Stops when the constraint binds is a major shortcoming of DynareOBC-DEIR.

## 4. Conclusions

We found major differences between global and local solutions of open-economy models with incomplete markets (an endowment economy, an RBC model and a Sudden Stops model with an occasionally binding credit constraint). Local solutions were produced using 1OA, 2OA, 3OA, RSS and DynareOBC methods and the global solutions were generated using the *FiPIt* method. Most local methods need a stationarity-inducing assumption, for which we chose the widely-used DEIR function that makes the real interest rate a decreasing function of the NFA position. We considered the standard "inessential" approach to set a very low DEIR elasticity so that the interest rate remains close to the world interest rate and a variation in which the elasticity is targeted to match the autocorrelation of NFA in the global solution.

The key limitation of local methods is that they approximate poorly the effects of precautionary savings on NFA, net exports and consumption, even using higher-order methods such as 2OA and RSS. For the Sudden Stops model, first-order DynareOBC has two additional disadvantages: it underestimates the tightness of the credit constraint and its effects on financial premia and macro variables, and it does not capture *risk* effects of the credit constraint and their implications for precautionary savings and forward-looking variables like asset prices. Local methods are faster for the endowment and RBC models, but for the Sudden Stops model *FiPIt* and DynareOBC are of comparable speed. *FiPIt* yields significantly smaller Euler equation errors but the curse of dimensionality remains a limitation.

NFA is a near-unit-root process in the three models. Analytical and quantitative results for local solutions show that small errors in calculating the NFA autocorrelation cause sizable errors in the long-run averages of NFA, consumption and net exports, and various features of business cycle moments, impulse responses and spectral densities. Local solutions with targeted calibrations perform better but imply DEIR elasticities akin to imposing large costs in moving NFA from its steady state, which remove precautionary savings completely, and require knowing the global solution. Interestingly, 1OA, 2OA, and RSS produce very similar second- and higher-order moments, impulse responses and periodograms, because they yield decision rules that differ mainly in their intercepts but have similar first-order terms and negligible higher-order terms. Hence, if first moments are not central to the question under study, the 1OA method is the preferable *local* method.

These results are robust to several modifications, including setting the DEIR elasticity to its inessential low value v. targeting it to the global solution, replacing DEIR with a rate of interest lower than the rate of time preference or with an endogenous discount factor, introducing different shocks and changing their variability, and examining a model with an exact solution.

Our findings argue strongly in favor of using global methods unless the curse of dimensionality makes them unfeasible. The exception are models or variables within models for which wealth and precautionary-savings effects of incomplete markets are irrelevant (e.g., we found that, without wealth effects in labor supply and trivial risk premia, moments for factor allocations, output and investment are well-approximated by local methods even if they fail to approximate those for NFA, consumption and net exports). Our findings also suggest caution in assessing existing results obtained with local solutions. For example, this matters for predictions regarding average NFA positions to assess global imbalances and optimal foreign reserves, assessments of a model's ability to explain key cyclical moments such as GDP-correlations and autocorrelations of net exports and consumption-to-GDP variability ratios, IRF analysis to study the effects of shocks and policy changes, and evaluation of macroprudential policies for reducing the frequency and magnitude of Sudden Stops.

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Table 1: Calibration of the Endowment Economy Model

| Notation | Parameter/Variable | Value |
|---|---|---|
| **1. Common parameters** | | |
| $\sigma$ | Coefficient of relative risk aversion | 2.0 |
| $y$ | Mean endowment income | 1.00 |
| $A$ | Absorption constant | 0.28 |
| $R$ | Gross world interest rate | 1.059 |
| $\sigma_z$ | Standard deviation of income (percent) | 3.27 |
| $\rho_z$ | Autocorrelation of income | 0.597 |
| **2. Global solution parameters** | | |
| $\beta$ | Discount factor | 0.940 |
| $\varphi$ | Ad-hoc debt limit | $-0.51$ |
| **3. Local solution parameters** | | |
| *Common parameters* | | |
| $\beta$ | Discount factor | 0.944 |
| $\bar{b}$ | Deterministic steady state value of NFA | $-0.51$ |
| *Baseline calibration* | | |
| $\psi$ | Inessential DEIR coefficient | 0.001 |
| *Targeted calibration* | | |
| $\psi$ | DEIR coefficeint for 2OA | 0.0469 |
| $\psi$ | DEIR coefficient for RSS | 0.0469 |

Note: 2OA and RSS denote the second-order and risky-steady state solutions, respectively.

Table 2: Autocorrelations of Net Exports, NFA, and Income in the Endowment Economy

| | $\rho^{\epsilon}$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 |
| GLB | | | | | | | | | |
| $\rho_b$ | 0.827 | 0.866 | 0.899 | 0.926 | 0.947 | 0.964 | 0.977 | 0.987 | 0.993 |
| $\rho_{nx}$ | -0.088 | 0.010 | 0.110 | 0.213 | 0.321 | 0.432 | 0.547 | 0.661 | 0.768 |
| Baseline | | | | | | | | | |
| 2OA | | | | | | | | | |
| $\rho_b$ | 0.995 | 0.996 | 0.997 | 0.998 | 0.998 | 0.998 | 0.999 | 0.999 | 0.999 |
| $\rho_{nx}$ | 0.265 | 0.375 | 0.479 | 0.578 | 0.670 | 0.754 | 0.830 | 0.896 | 0.949 |
| RSS | | | | | | | | | |
| $\rho_b$ | 0.995 | 0.996 | 0.997 | 0.997 | 0.998 | 0.998 | 0.999 | 0.999 | 0.999 |
| $\rho_{nx}$ | 0.239 | 0.35 | 0.457 | 0.559 | 0.655 | 0.745 | 0.826 | 0.896 | 0.952 |
| Targeted | | | | | | | | | |
| 2OA | | | | | | | | | |
| $\rho_b$ | 0.914 | 0.929 | 0.942 | 0.953 | 0.962 | 0.971 | 0.978 | 0.984 | 0.990 |
| $\rho_{nx}$ | -0.013 | 0.086 | 0.186 | 0.286 | 0.386 | 0.486 | 0.586 | 0.687 | 0.789 |
| RSS | | | | | | | | | |
| $\rho_b$ | 0.912 | 0.928 | 0.941 | 0.952 | 0.961 | 0.97 | 0.977 | 0.984 | 0.990 |
| $\rho_{nx}$ | -0.010 | 0.089 | 0.188 | 0.287 | 0.386 | 0.485 | 0.585 | 0.684 | 0.784 |
| Targeted for all $\rho^{\epsilon}$ | | | | | | | | | |
| 2OA | | | | | | | | | |
| $\psi$ | 0.185 | 0.158 | 0.13 | 0.106 | 0.083 | 0.064 | 0.046 | 0.034 | 0.027 |
| $\rho_b$ | 0.827 | 0.866 | 0.899 | 0.926 | 0.947 | 0.964 | 0.977 | 0.987 | 0.993 |
| $\rho_{nx}$ | -0.029 | 0.068 | 0.166 | 0.267 | 0.37 | 0.476 | 0.586 | 0.698 | 0.807 |
| RSS | | | | | | | | | |
| $\psi$ | 0.185 | 0.158 | 0.13 | 0.106 | 0.083 | 0.064 | 0.046 | 0.034 | 0.027 |
| $\rho_b$ | 0.827 | 0.866 | 0.899 | 0.926 | 0.947 | 0.964 | 0.977 | 0.987 | 0.993 |
| $\rho_{nx}$ | -0.030 | 0.067 | 0.166 | 0.266 | 0.369 | 0.475 | 0.585 | 0.696 | 0.804 |

Note: GLB, 2OA and RSS denote the global, second-order and risky-steady state solutions, respectively.

Table 3: Long-run Moments: Endowment Economy Model

| | GLB | Baseline Calibration | | | Targeted Calibration | |
|---|---|---|---|---|---|---|
| | | 2OA | RSS | | 2OA | RSS |
| | | DEIR | $\beta R < 1$ | DEIR | DEIR | DEIR |
| $\psi =$ | n.a. | 0.001 | n.a. | 0.001 | 0.0469 | 0.0469 |
| *Averages* | | | | | | |
| $E(c)$ | 0.694 | 0.702 | 0.093 | 0.692 | 0.689 | 0.689 |
| $E(nx/y)$ | 0.022 | 0.015 | 0.625 | 0.025 | 0.028 | 0.028 |
| $E(b/y)$ | -0.410 | -0.286 | -11.210 | -0.451 | -0.502 | -0.506 |
| *Variability relative to variability of income* | | | | | | |
| $\sigma(c)/\sigma(y)$ | 0.995 | 1.577 | 1.161 | 1.617 | 1.000 | 0.997 |
| $\sigma(nx)/\sigma(y)$ | 0.663 | 1.335 | 1.202 | 1.346 | 0.730 | 0.730 |
| $\sigma(nx/y)/\sigma(y)$ | 0.647 | 1.319 | 1.161 | 1.331 | 0.710 | 0.709 |
| $\sigma(b)/\sigma(y)$ | 7.497 | 63.033 | 1.706 | 40.078 | 6.648 | 6.576 |
| $\sigma(b/y)/\sigma(y)$ | 7.777 | 62.711 | 1.892 | 40.213 | 7.178 | 7.118 |
| *Income correlations* | | | | | | |
| $\rho(y, c)$ | 0.751 | 0.200 | 0.188 | 0.197 | 0.684 | 0.684 |
| $\rho(y, nx)$ | 0.726 | 0.584 | 0.312 | 0.567 | 0.725 | 0.708 |
| $\rho(y, nx/y)$ | 0.704 | 0.568 | 0.006 | 0.567 | 0.705 | 0.708 |
| $\rho(y, b)$ | 0.266 | 0.126 | 0.070 | 0.124 | 0.489 | 0.488 |
| $\rho(y, b/y)$ | 0.064 | 0.154 | 0.445 | 0.149 | 0.592 | 0.592 |
| *First-order autocorrelations* | | | | | | |
| $\rho_c$ | 0.838 | 0.995 | 0.996 | 0.995 | 0.929 | 0.929 |
| $\rho_{nx}$ | 0.536 | 0.821 | 0.934 | 0.823 | 0.582 | 0.582 |
| $\rho_{nx/y}$ | 0.544 | 0.828 | 0.995 | 0.830 | 0.591 | 0.590 |
| $\rho_b$ | 0.977 | 0.999 | 0.999 | 0.999 | 0.977 | 0.977 |
| $\rho_{b/y}$ | 0.980 | 0.997 | 0.953 | 0.998 | 0.958 | 0.959 |
| *Performance metrics* | | | | | | |
| Execution time (secs.) | 2.5 | 0.7 | 0.3 | 5.6 | 0.7 | 5.7 |
| ratio rel. to GLB | 1.0 | 0.280 | 0.120 | 2.920 | 0.280 | 2.880 |
| Max. Abs. Euler eq. errors | 1.22E-04 | 2.27E-04 | 3.42E-03 | 2.41E-04 | 5.27E-04 | 7.17E-04 |
| Mean Abs. Euler eq. errors | 6.44E-12 | 1.17E-08 | 3.35E-03 | 1.13E-04 | 2.79E-07 | 5.95E-04 |
| Decision rule diff $b$ | | 0.120 (0.248) | 0.225 (0.471) | 0.099 (0.378) | 0.086 (0.127) | 0.075 (0.123) |
| Decision rule diff $c$ | | 0.025 (0.049) | 0.053 (0.157) | 0.028 (0.055) | 0.019 (0.037) | 0.020 (0.039) |

Note: GLB, 20A and RSS refer to the global, second-order and risky steady state solutions, respectively. $\sigma(\cdot)$ denotes the coefficient of variation for variables in levels and the standard deviation for variables in ratios ($nx/y$, $b/y$ and the leverage ratio $lev/rat.$). The results were obtained using Matlab 2020a in a Linux cluster with 128gb of RAM, two 10-core Intel(R) Xeon(R) CPU E5-2690 v2 @ 3.00GHz processors, and a Samsung SSD 840 512GB hard drive. The number of CPUs called by the parallel computing toolbox was set to minimize execution time. Execution times include elapsed time up to the solution of decision rules. Euler equation errors and decision rule differences are computed for all $(b, z)$ pairs in the state space of the GLB solution. Decision rule differences in the last two rows are differences between the local and GLB solutions in percent of the latter. We report mean and maximum (maximum in brackets) differences conditional on bond values that have positive probability in the ergodic distribution of the GLB solution.

Table 4: Endowment Economy Model with Income and Interest-Rate Shocks

| | Std Dev of Int Rate (percent) | | | | | |
|---|---|---|---|---|---|---|
| | 0.0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 |
| *Global calibrated* | | | | | | |
| $E(b/y)$ | -0.411 | -0.410 | -0.408 | -0.403 | -0.396 | -0.384 |
| $\sigma(c)/\sigma(y)$ | 0.995 | 0.977 | 1.009 | 1.058 | 1.126 | 1.214 |
| $\sigma(b)/\sigma(y)$ | 7.497 | 7.169 | 7.465 | 8.009 | 8.874 | 10.311 |
| $\rho(y,nx)$ | 0.726 | 0.681 | 0.617 | 0.527 | 0.415 | 0.298 |
| $\rho(nx)$ | 0.535 | 0.540 | 0.542 | 0.546 | 0.551 | 0.559 |
| $\rho(b)$ | 0.977 | 0.973 | 0.975 | 0.976 | 0.978 | 0.981 |
| *Global with NDL* | | | | | | |
| $E(b/y)$ | -10.778 | -9.249 | -7.445 | -5.991 | -4.875 | -3.956 |
| $\sigma(c)/\sigma(y)$ | 9.747 | 7.375 | 6.962 | 6.189 | 5.563 | 4.906 |
| $\sigma(b)/\sigma(y)$ | 1.682 | 2.418 | 4.232 | 5.771 | 7.194 | 8.374 |
| $\rho(y,nx)$ | 0.684 | 0.457 | 0.343 | 0.308 | 0.297 | 0.301 |
| $\rho(nx)$ | 0.858 | 0.880 | 0.924 | 0.931 | 0.927 | 0.914 |
| $\rho(b)$ | 0.999 | 0.998 | 0.998 | 0.998 | 0.998 | 0.997 |
| *Full RSS w.* $\beta\bar{R} < 1$ | | | | | | |
| $E(b/y)$ | -11.21 | -9.098 | -7.182 | -5.577 | -4.226 | -3.075 |
| $\sigma(c)/\sigma(y)$ | 12.484 | 11.171 | 9.672 | 8.209 | 6.745 | 5.322 |
| $\sigma(b)/\sigma(y)$ | 19.067 | 38.394 | 49.967 | 53.952 | 52.038 | 45.600 |
| $\rho(y,nx)$ | 0.315 | 0.077 | 0.011 | -0.021 | -0.044 | -0.066 |
| $\rho(nx)$ | 0.933 | 0.987 | 0.993 | 0.994 | 0.992 | 0.986 |
| $\rho(b)$ | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |
| *Partial RSS w. baseline* $\psi$ | | | | | | |
| $E(b/y)$ | -0.451 | -0.426 | -0.279 | -0.018 | 0.381 | 0.942 |
| $\sigma(c)/\sigma(y)$ | 1.617 | 1.645 | 1.773 | 2.085 | 2.894 | 4.969 |
| $\sigma(b)/\sigma(y)$ | 40.078 | 43.072 | 71.486 | 1327.807 | 94.562 | 71.228 |
| $\rho(y,nx)$ | 0.567 | 0.560 | 0.531 | 0.469 | 0.357 | 0.217 |
| $\rho(nx)$ | 0.823 | 0.823 | 0.830 | 0.856 | 0.910 | 0.965 |
| $\rho(b)$ | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |
| *2OA w. baseline* $\psi$ | | | | | | |
| $E(b/y)$ | -0.286 | -0.319 | -0.179 | 0.056 | 0.384 | 0.806 |
| $\sigma(c)/\sigma(y)$ | 1.577 | 1.612 | 1.664 | 1.747 | 1.857 | 1.990 |
| $\sigma(b)/\sigma(y)$ | 63.033 | 55.583 | 100.480 | 313.282 | 47.421 | 23.101 |
| $\rho(y,nx)$ | 0.584 | 0.568 | 0.555 | 0.536 | 0.512 | 0.485 |
| $\rho(nx)$ | 0.821 | 0.816 | 0.809 | 0.798 | 0.785 | 0.771 |
| $\rho(b)$ | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |
| *Partial RSS w. targeted* $\psi$ | | | | | | |
| $E(b/y)$ | -0.506 | -0.507 | -0.505 | -0.501 | -0.495 | -0.487 |
| $\sigma(c)/\sigma(y)$ | 0.997 | 1.016 | 1.068 | 1.150 | 1.254 | 1.375 |
| $\sigma(b)/\sigma(y)$ | 6.576 | 6.571 | 6.657 | 6.805 | 7.022 | 7.315 |
| $\rho(y,nx)$ | 0.708 | 0.695 | 0.663 | 0.619 | 0.571 | 0.523 |
| $\rho(nx)$ | 0.582 | 0.580 | 0.576 | 0.570 | 0.564 | 0.559 |
| $\rho(b)$ | 0.977 | 0.977 | 0.977 | 0.977 | 0.977 | 0.977 |
| *2OA w. targeted* $\psi$ | | | | | | |
| $E(b/y)$ | -0.502 | -0.505 | -0.502 | -0.498 | -0.492 | -0.484 |
| $\sigma(c)/\sigma(y)$ | 1.000 | 1.020 | 1.073 | 1.157 | 1.264 | 1.391 |
| $\sigma(b)/\sigma(y)$ | 6.648 | 6.612 | 6.694 | 6.833 | 7.030 | 7.287 |
| $\rho(y,nx)$ | 0.725 | 0.693 | 0.660 | 0.615 | 0.564 | 0.514 |
| $\rho(nx)$ | 0.582 | 0.581 | 0.577 | 0.572 | 0.566 | 0.561 |
| $\rho(b)$ | 0.977 | 0.977 | 0.977 | 0.977 | 0.977 | 0.977 |

Note: The variability and persistence of endowment shocks are kept as in Table 1. The correlation between endowment and interest-rate shocks is set to $-0.669$, for all columns with the exception of the first column for which the correlation is set to 0. GLB, 20A and RSS refer to the global, second-order and risky-steady state solutions, respectively.

Table 5: Calibration of the Sudden Stops Model

| Notation | Parameter/Variable | Value |
|---|---|---|
| **1. Common parameters** | | |
| $\sigma$ | Coefficient of relative risk aversion | 2.0 |
| $R$ | Gross world interest rate | 1.0857 |
| $\alpha$ | Labor share in gross output | 0.592 |
| $\gamma$ | Capital share in gross output | 0.306 |
| $\eta$ | Imported inputs share in gross output | 0.102 |
| $\delta$ | Depreciation rate of capital | 0.088 |
| $\omega$ | Labor exponent in the utility function | 1.846 |
| $\phi$ | Working capital constraint coefficient | 0.2579 |
| $a$ | Investment adjustment cost parameter | 2.75 |
| $\tau$ | Consumption tax | 0.168 |
| $\kappa$ | Collateral constraint coefficient | 0.20 |
| $\rho^A$ | TFP autocorrelation | 0.555 |
| $\rho^R$ | Interest rate autocorrelation | 0.555 |
| $\rho^p$ | Input price autocorrelation | 0.737 |
| $\sigma^2_{u^A}$ | Variance of TFP innovations | $1.0273e-04$ |
| $\sigma^2_{u^R}$ | Variance of interest rate innovations | $2.4387e-04$ |
| $\sigma^2_{u^p}$ | Variance of input price innovations | $5.1097e-04$ |
| $\sigma_{u^A,u^R}$ | Covariance of TFP and interest rate innovations | $-0.0047$ |
| $y^{dss}$ | GDP at the deterministic steady state | 396 |
| **2. Global solution parameters** | | |
| $\beta$ | Discount factor | 0.920 |
| $\varphi$ | Ad-hoc debt limit as a share of $y^{dss}$ | $-0.505$ |
| $b^{dss}/y^{dss}$ | NFA/GDP at the deterministic steady state | $-0.192$ |
| **3. Local solutions parameters** | | |
| *DynareOBC with* $\beta R < 1$ | | |
| $\beta$ | Discount factor | 0.920 |
| $b^{dss}/y^{dss}$ | NFA/GDP at the deterministic steady state | $-0.192$ |
| *DynareOBC with DEIR* | | |
| $\beta$ | Discount factor | 0.9211 |
| $\psi$ | Inessential DEIR coefficient | 0.001 |
| $b^{dss}/y^{dss}$ | NFA/GDP at the deterministic steady state | 0.015 |

Note: For the Sudden Stops model, the GLB solution has two credit constraints, namely $\varphi$ and the collateral constraint. Credit is constrained at the deterministic steady state, since $\beta R < 1$, but $\varphi$ is set low enough so that the collateral constraint binds first.

Table 6: Long-run Moments: Sudden Stops model

| | GLB | DynareOBC-$\beta R < 1$ | DynareOBC-DEIR |
|---|---|---|---|
| *Averages* | | | |
| $E(y)$ | 393.619 | 391.390 | 395.230 |
| $E(c)$ | 274.123 | 269.610 | 279.970 |
| $E(i)$ | 67.481 | 66.714 | 67.897 |
| $E(nx/y)$ | 0.015 | 0.025 | 0.000 |
| $E(b/y)$ | 0.015 | -0.100 | 0.206 |
| $E(lev.rat.)$ | -0.102 | -0.157 | -0.011 |
| $E(\upsilon)$ | 42.617 | 42.263 | 42.712 |
| $E(L)$ | 18.426 | 18.364 | 18.469 |
| *Variability relative to variability of GDP* | | | |
| $\sigma(y)$ | 0.039 | 0.032 | 0.032 |
| $\sigma(c)/\sigma(y)$ | 1.023 | 0.937 | 1.207 |
| $\sigma(i)/\sigma(y)$ | 3.383 | 3.492 | 3.777 |
| $\sigma(nx/y)/\sigma(y)$ | 0.746 | 0.927 | 1.262 |
| $\sigma(b/y)/\sigma(y)$ | 4.980 | 3.703 | 9.595 |
| $\sigma(lev.rat.)/\sigma(y)$ | 2.340 | 1.705 | 4.498 |
| $\sigma(\upsilon)/\sigma(y)$ | 1.495 | 1.632 | 1.612 |
| $\sigma(L)/\sigma(y)$ | 0.599 | 0.571 | 0.569 |
| *Correlations with GDP* | | | |
| $\rho(y, c)$ | 0.842 | 0.823 | 0.557 |
| $\rho(y, i)$ | 0.641 | 0.309 | 0.224 |
| $\rho(y, nx/y)$ | -0.117 | 0.176 | 0.223 |
| $\rho(y, b/y)$ | -0.120 | 0.027 | -0.054 |
| $\rho(y, lev.rat.)$ | -0.111 | 0.008 | -0.056 |
| $\rho(y, \upsilon)$ | 0.832 | 0.777 | 0.775 |
| $\rho(y, L)$ | 0.994 | 0.987 | 0.986 |
| *First-order autocorrelations* | | | |
| $\rho(y)$ | 0.825 | 0.752 | 0.754 |
| $\rho(b)$ | 0.990 | 0.980 | 0.995 |
| $\rho(c)$ | 0.829 | 0.826 | 0.910 |
| $\rho(i)$ | 0.500 | 0.470 | 0.502 |
| $\rho(nx/y)$ | 0.601 | 0.456 | 0.651 |
| $\rho(lev.rat.)$ | 0.992 | 0.988 | 0.996 |
| $\rho(\upsilon)$ | 0.777 | 0.753 | 0.756 |
| $\rho(L)$ | 0.801 | 0.761 | 0.774 |
| Prob.($\mu$>0) | 2.58 | 51.80 | 71.06 |
| *Performance metrics* | | | |
| Time in sec. | 268.0 | 243.5 | 187.4 |
| Max. Abs. $b$ Euler eq. error | 2.62E-04 | na | na |
| Max. Abs. $k$ Euler eq. error | 4.25E-16 | na | na |

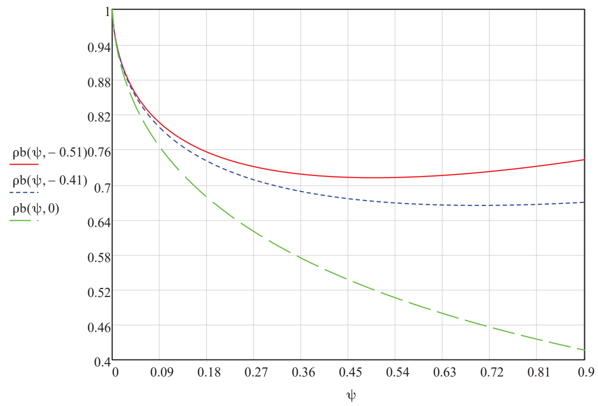
Note: See note to Table 3.

Table 7: Credit Constraint Multiplier, Macro & Financial Variables Conditional on $\mu > 0$

| | $\log(\mu)$ | | Financial Premia *means in each quintile of* $\mu$ | | | | Macro variables *means of deviations from long-run averages in each quintile of* $\mu$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | upper limit | mean | $SIP$ | $EP$ | $(1-\kappa)SIP$ | $RP$ | $c$ | $nx/y$ | $i$ | $y$ | $L$ | $v$ |
| **Panel a. GLB** | | | | | | | | | | | | |
| *Quintiles of* $\mu$ | | | | | | | | | | | | |
| First | -6.563 | -6.941 | 0.32 | 0.37 | 0.26 | 0.10 | -2.76 | 1.98 | -1.76 | -0.60 | -0.26 | 0.35 |
| Second | -6.320 | -6.428 | 1.07 | 0.96 | 0.85 | 0.11 | -2.17 | 1.37 | 2.70 | 0.08 | 0.15 | -1.25 |
| Third | -6.088 | -6.187 | 1.82 | 1.56 | 1.46 | 0.10 | -3.80 | 2.30 | -3.00 | -1.35 | -0.82 | -1.29 |
| Fourth | -5.843 | -5.968 | 2.98 | 2.48 | 2.38 | 0.09 | -4.72 | 2.58 | -5.46 | -2.26 | -1.42 | -3.35 |
| Fifth | -3.374 | -5.636 | 6.59 | 5.38 | 5.27 | 0.10 | -4.86 | 5.10 | -13.45 | -1.21 | -1.37 | -2.98 |
| *Overall mean* | | -6.038 | 2.59 | 2.17 | 2.07 | 0.10 | -3.64 | 2.64 | -4.05 | -1.04 | -0.73 | -1.78 |
| *Overall median* | | -6.198 | 1.79 | 1.52 | 1.43 | 0.11 | -3.22 | 1.60 | -1.64 | -1.02 | -0.57 | -2.15 |
| | | | *Ex-post Sharpe ratio = 1.16* | | | | | | | | | |
| **Panel b. DynareOBC-**$BetaR < 1$ | | | | | | | | | | | | |
| *Quintiles of* $\mu$ | | | | | | | | | | | | |
| First | -9.000 | -9.155 | 0.00 | 0.00 | 0.00 | 0.00 | -0.78 | 0.73 | -3.17 | -0.40 | -0.30 | -0.53 |
| Second | -8.523 | -8.638 | 0.01 | 0.01 | 0.01 | 0.00 | -1.74 | 1.38 | -6.36 | -1.10 | -0.77 | -1.58 |
| Third | -8.155 | -8.319 | 0.02 | 0.01 | 0.01 | 0.00 | -3.23 | 2.33 | -11.28 | -2.24 | -1.52 | -3.13 |
| Fourth | -6.295 | -6.707 | 0.66 | 0.53 | 0.53 | 0.00 | -2.02 | 1.24 | -4.04 | -1.13 | -0.68 | -1.54 |
| Fifth | -5.523 | -6.005 | 3.32 | 2.65 | 2.65 | 0.00 | -1.85 | 0.38 | -1.35 | -1.38 | -0.81 | -1.92 |
| *Overall mean* | | -6.623 | 0.800 | 0.64 | 0.64 | 0.00 | -1.92 | 1.21 | -5.24 | -1.25 | -0.82 | -1.74 |
| *Overall median* | | -8.337 | 0.015 | 0.01 | 0.01 | 0.00 | -1.87 | 0.93 | -4.71 | -1.23 | -0.81 | -1.74 |
| | | | *Ex-post Sharpe ratio = 0.25* | | | | | | | | | |
| **Panel c. DynareOBC DEIR** | | | | | | | | | | | | |
| *Quintiles of* $\mu$ | | | | | | | | | | | | |
| First | -15.380 | -15.683 | 0.00 | 0.00 | 0.00 | 0.00 | -1.09 | 1.05 | -0.27 | 0.12 | 0.06 | 0.20 |
| Second | -15.064 | -15.198 | 0.00 | 0.00 | 0.00 | 0.00 | -0.14 | -0.13 | 0.62 | -0.13 | -0.05 | -0.22 |
| Third | -14.856 | -14.952 | 0.00 | 0.00 | 0.00 | 0.00 | 1.10 | -1.44 | 1.20 | -0.30 | -0.13 | -0.36 |
| Fourth | -14.659 | -14.756 | 0.00 | 0.00 | 0.00 | 0.00 | 2.67 | -3.09 | 1.68 | -0.55 | -0.25 | -0.68 |
| Fifth | -5.603 | -6.723 | 0.64 | 0.51 | 0.51 | 0.00 | 2.22 | -3.20 | 1.83 | -0.99 | -0.47 | -1.34 |
| *Overall mean* | | -7.422 | 0.13 | 0.10 | 0.10 | 0.00 | 0.95 | -1.36 | 1.01 | -0.37 | -0.16 | -0.48 |
| *Overall median* | | -14.954 | 0.00 | 0.00 | 0.00 | 0.00 | 0.89 | -1.13 | 0.96 | -0.37 | -0.18 | -0.48 |
| | | | *Ex-post Sharpe ratio = 0.04* | | | | | | | | | |

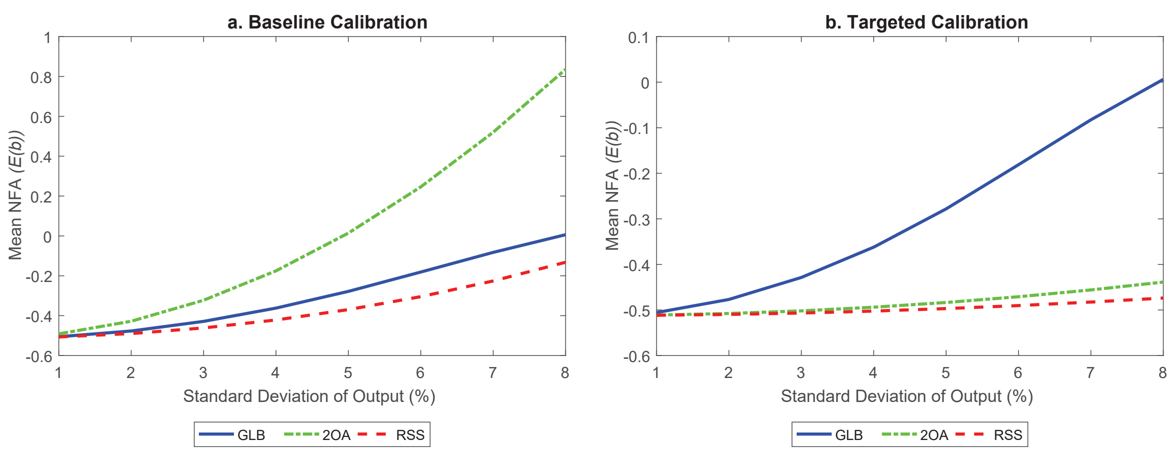
Note: $SIP$ is the Shadow interest premium, $EP$ is the equity premium, and $RP$ is the risk premium component of $EP$. The quintile distribution of $\mu$ is conditional on $\mu > 0$. Means for other variables are computed using the distribution of $\mu$ within each quantile and the overall distribution of $\mu$ conditional on $\mu > 0$. $\log(\mu)$ is the base-10 logarithm of the multiplier on the credit constraint. The moments for GLB are computed using the recursive equilibrium decision rules and ergodic distribution of the model. For $EP$, we compute the equity premium conditional on all date-t states $(b, k, s)$ and then average across them using the ergodic distribution. The Sharpe ratio is computed using the conditional ergodic distribution for $\mu > 0$. The moments for DynareOBC are ex-post moments, computed using time-series simulation output of DynareOBC. $RP$ is set to zero because the covariance between future equity returns and marginal utility is zero along the perfect-foresight paths that determine each date-t solution in DynareOBC.

Figure 1: The first-order coefficient of NFA decision rules as the elasticity of the DEIR function varies



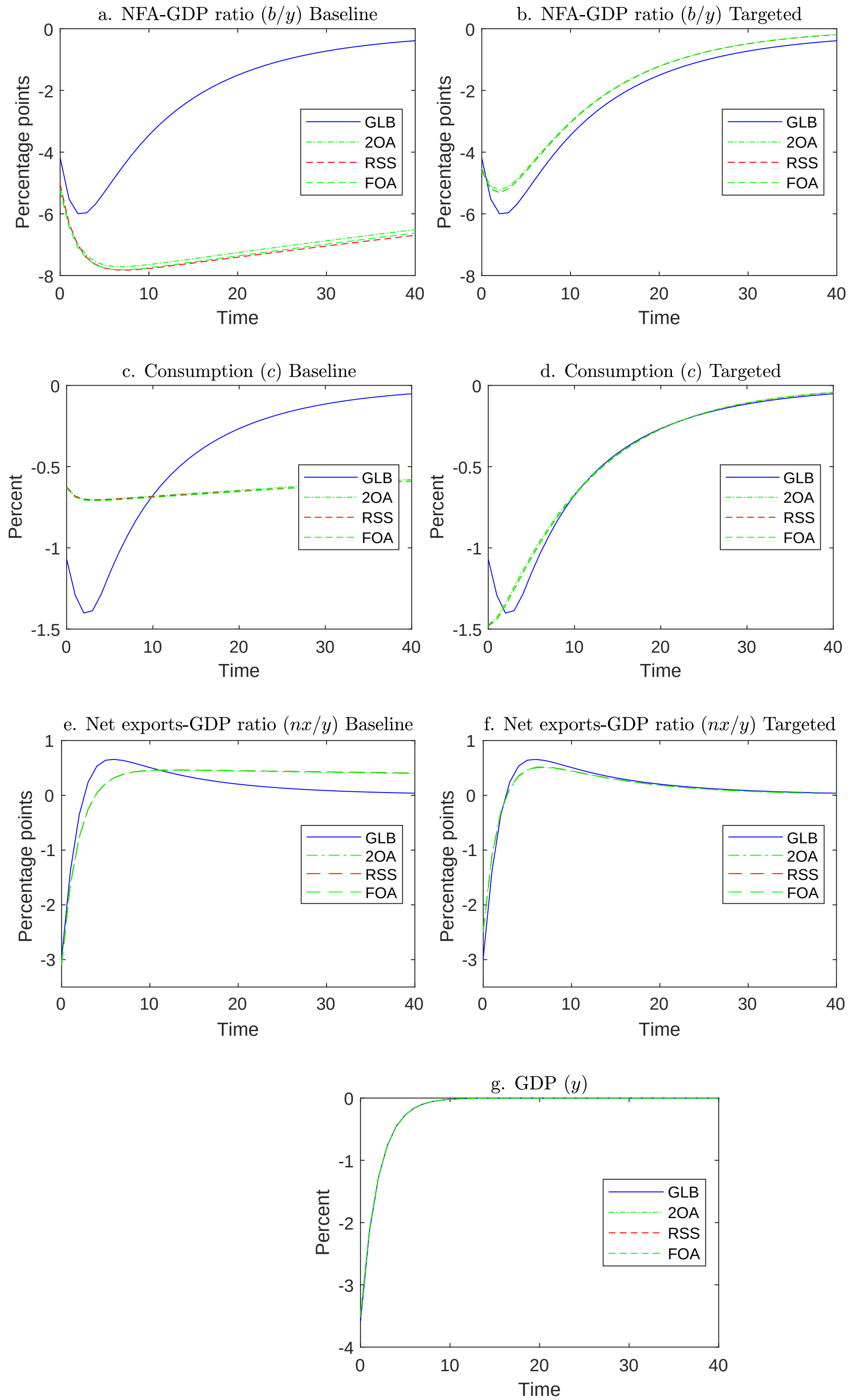

Note: This figure shows how the first-order coefficient of the NFA decision rules, $\rho_b(\psi, b^*)$, varies with $\psi$ for three values of $b^*$: -0.51 (deterministic steady state), -0.41 (risky steady state) and zero.

Figure 2: Average NFA in the endowment economy as the variability of output rises



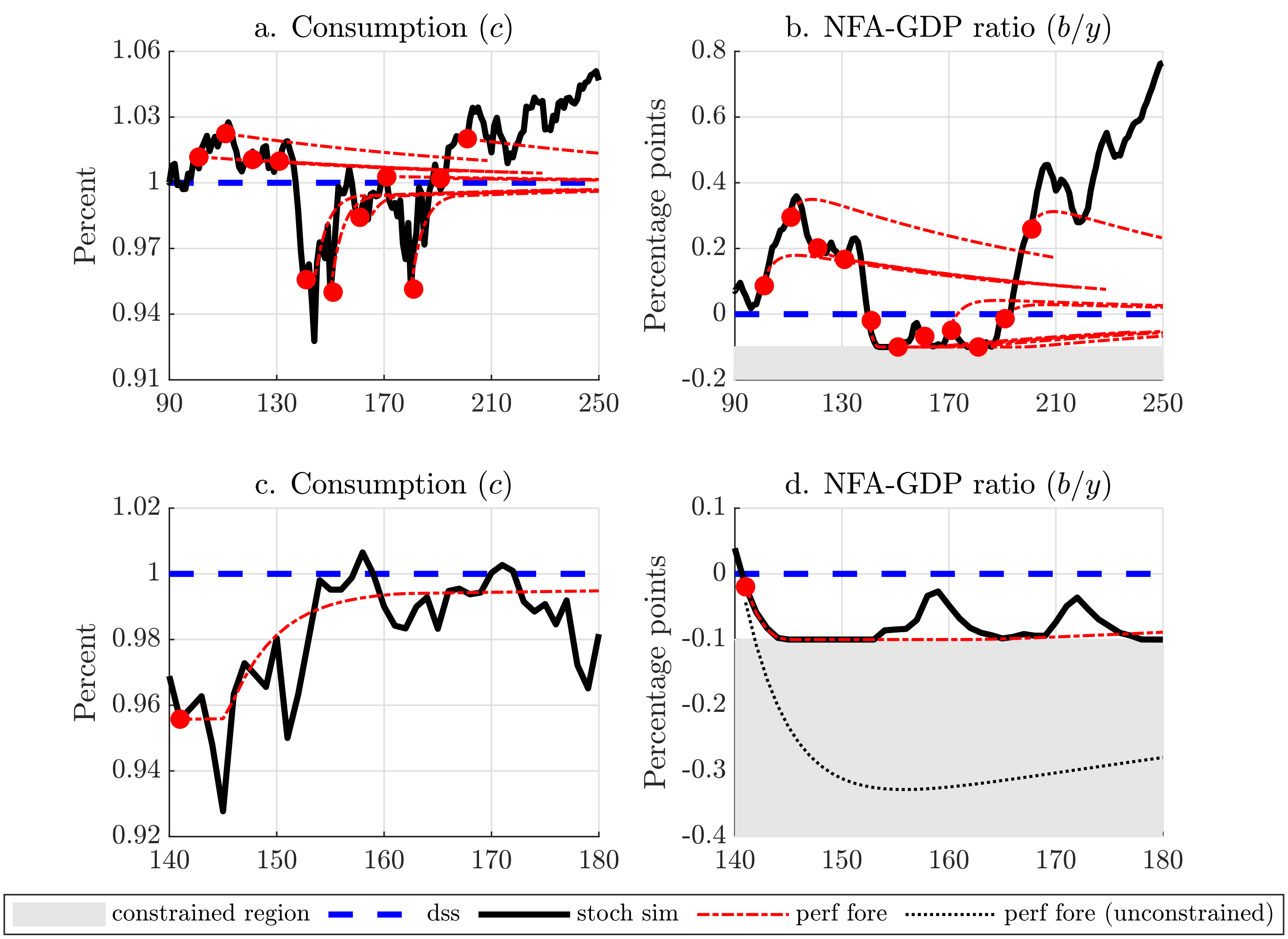

Note: GLB refers to global solution, 2OA refers to second-order solution, RSS refers to risky-steady state solution.

Figure 3: Endowment Model Impulse Response Functions to a Negative Income Shock




Note: GLB, 1OA, 2OA and RSS denote global, first-order, second-order and risky-steady state solutions, respectively. GLB impulse responses are forecast functions of the equilibrium Markov processes of the endogenous variables with initial conditions set to $E[b]$ and a value of $z$ equal to a one-standard-deviation shock.

Figure 4: DynareOBC Solution for Endowment Economy




Note: The top row plots one draw of the stochastic simulation ("stoch sim") from period 90 to 250 and plots corresponding perfect foresight ("perf fore") paths for select periods. The bottom row focussed on period 140 to 180 and plots both the constrained and unconstrained perfect foresight path from period 141.

Figure 5: Sudden Stops Model: Impulse Response Functions to a Negative TFP Shock