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ABSTRACT

We build a quantitative model of trade with multistage manufacturing value chains, which features iceberg trade costs and technology differences across both goods and production stages. We estimate technology and trade costs via the simulated method of moments, matching bilateral shipments of final goods and inputs. Applying the model, we investigate how comparative advantage and trade costs shape the structure of global value chains and trade flows. As the level of trade costs falls, we show that the elasticity of bilateral trade to trade costs increases, due to the endogenous reorganization of value chains (increased export platform production). Surprisingly, however, the elasticity of world trade to trade costs is not magnified by multistage production.

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Global value chains (GVCs) are widely recognized as important conduits of international trade. Nonetheless, the exact definition of "value chain" is often elusive. Appealing to plain language, a chain is a sequence of connected elements. A value chain then is a sequence of connected production stages, in which value is added stage-by-stage to produce final goods. In a *global* value chain, these sequential production stages are sliced up and allocated across countries to minimize production costs, giving rise to international trade in inputs and final goods.

While this sequential, multistage description of global value chains is very general, the default approach to incorporating GVCs into quantitative trade models is instead quite particular. Specifically, the default is to assume that there is a roundabout input loop in the production process.[1] The roundabout assumption requires that inputs and final goods are interchangeable – they are produced with identical technologies, and output may be flexibly used either as an intermediate input or consumed as a final good. Though tractable and useful for many purposes, models that feature roundabout production fail to capture two basic forces that govern the structure of trade via global value chains.

First, countries differ in the cost at which they can perform individual production stages [Dixit and Grossman (1982); Sanyal (1983); Yi (2003)].[2] Some countries have comparative advantage in downstream stages (e.g., manufacturing assembly in China), while others have comparative advantage in upstream stages (e.g., electronic components in Japan). Therefore, within-sector comparative advantage across stages influences production and trade patterns, in addition to traditional comparative advantage across goods and sectors.

Second, trade costs may be particularly burdensome when production takes the sequential, multistage form. As inputs are shipped from country to country through the chain, trade costs are paid multiple times. Further, ad valorem trade costs (proportional to the gross value of goods shipped) are higher in absolute terms for the output of downstream stages, since the value of output accumulates along the value chain. These trade costs have a big impact on decisions about where to locate downstream production stages, because they are large relative to the cost savings from locating downstream stages in low wage or high productivity locations. As emphasized by Yi (2003, 2010), this aspect of multistage production may magnify the elasticity of trade flows to frictions.

In this paper, we build a quantitative model of trade with multistage production to study the role of comparative advantage and trade costs in shaping global value chains, trade patterns, and the elasticity of trade to frictions. The model features two sectors (manufacturing and non-

[1]See de Gortari (2019) for a comprehensive discussion of the relationship between roundabout and sequential models, which emphasizes the role of specialized input suppliers in multistage-type models.

[2]Dixit and Grossman (1982) emphasize comparative advantage based on differences in factor endowments within a multistage production process. As in Sanyal (1983) and Yi (2003), we focus on technological differences (Ricardian comparative advantage).

manufacturing), with many goods in each sector. Production of each manufactured good requires a discrete number of sequential stages, while non-manufactured goods are produced via a conventional Ricardian production process. In both sectors, manufactured and non-manufactured goods from home and abroad are also used as inputs in production, via a roundabout production loop. Thus, the model features input linkages across both countries and sectors, with both sequential and non-sequential value chains side by side.

To study the quantitative properties of the model, we estimate technology parameters and bilateral trade costs using a simulated method of moments procedure, matching bilateral trade flows of final goods and inputs in the model and data.[3] We then assess the response of value chains and trade flows to changes in technology and trade costs via counterfactual experiments.

One barrier to this estimation and counterfactual analysis is that multistage models are computationally burdensome in high dimensional environments (e.g., with many countries). The basic problem is that equilibrium outcomes are discontinuous in parameters, because the number of number of goods and/or production stages is discrete. This is a familiar problem in Ricardian models, which is compounded in our multistage context.[4]

To overcome this challenge, we apply a smoothing technique from the discrete choice literature, where similar issues arise in simulating choice probabilities [McFadden (1989)]. Specifically, we solve for an approximate equilibrium in the model, in which discrete (binary) sourcing choices are approximated by continuous (logit-type) functions of prices. With this technique, we are able to solve the model using standard, gradient-based optimization procedures in a multi-country environment, which in turn facilitates simulated method of moments estimation of the parameters. This new procedure allows a tighter mapping between theory and data than previous approaches to quantifying multistage models (e.g., Yi (2010)), so it paves the way for use of the multistage models in future applications.[5]

Applying this procedure, we estimate technology and trade cost parameters for 37 industrial and emerging market countries, plus a composite rest-of-the-world region, using data from Johnson and Noguera (2017). Our estimates show that there are substantial cross-country differences in relative productivity (comparative advantage) across stages. For example, we find that Vietnam has a strong comparative advantage in downstream production, whereas Chile has a comparative

[3]Remaining model parameters are calibrated to match various data targets, including final expenditure by sector and country, value-added to output ratios by sector, total GDP by country, and so on.

[4]In a two-country Ricardian model, Dornbusch et al. (1977) solve this problem by assuming that there is a continuum of goods. Because the continuum assumption alone is not enough in multi-country models, Eaton and Kortum (2002) develop a probabilistic approach to analyzing Ricardian models. Antràs and de Gortari (2016) adopt information assumptions to apply the Eaton-Kortum idea in the multistage context, as we discuss further below.

[5]Yi (2010) calibrates a related multistage model for the US and two Canadian regions using a mixture of data (on production, labor allocations, income, etc.) and parameter restrictions (e.g., equal productivity levels across stages). Yi (2010) also measures trade costs from auxiliary data, while we estimate trade costs to match trade shares.

advantage in upstream production. These differences in comparative advantage induce specialization, such that the export composition of countries with downstream comparative advantage is tilted toward final goods relative to inputs. We also find that estimated international trade costs are large in our multistage model (on the order of 350%), comparable in magnitude to estimated trade costs in standard gravity models.

Turning to trade elasticities, we start by characterizing bilateral trade elasticities for manufacturing in the model. Though our multistage model does not admit an exact gravity representation of trade flows, we interpret simulated data from our model through the lens of the gravity equation in order to compute elasticities that are comparable to estimates in the literature. Unlike CES-gravity models, our model generates endogenous, heterogeneous elasticities of bilateral trade to bilateral trade costs, which vary across country pairs, for final goods versus inputs, and as the level of trade costs change.[6]

As the level of trade costs falls (holding relative bilateral trade costs constant), we show that the average bilateral trade elasticity rises. This reflects a mixture of two forces. First, the elasticity of both final goods and input trade to bilateral trade costs rises. Because bilateral trade costs are a function of distance, this implies that trade flows become more geographically concentrated as the level of trade costs falls. Second, the decline in trade costs induces changes in the composition of trade, leading input trade to expand relative to final goods trade. Because input trade is more sensitive to trade costs than trade in final goods (this result is an endogenous outcome in the model, not a primitive assumption), the rise of input trade also raises the implied elasticity of trade to trade costs overall.

Both these bilateral elasticity results are fundamentally driven by the endogenous reorganization of value chains in response to falling trade frictions. Specifically, as trade costs fall, export platform production – in which inputs sold by country i to country j are re-exported (embedded in finished goods) to third destinations – increases. Moreover, the choice of where to set up export platforms is highly sensitive to trade costs: export platforms for value chains originating in country i are disproportionately located in countries j for which iceberg trade costs are low. Thus, as the importance of export platform production rises, the sensitivity of input trade to trade costs rises.

To evaluate the response of global trade to trade costs, we examine the elasticity of world trade to world GDP in the manufacturing sector. This analysis echos Yi (2003), which argued that multistage production explained the large, nonlinear increase in trade relative to GDP in response to declines in trade costs. Revisiting this idea, we compare the elasticity of trade to GDP in our multistage model to two benchmarks. The first benchmark is a counterfactual in our own model,

[6]While most of our analysis focuses on how the average bilateral elasticity depends on the level of trade costs, we also discuss implications of heterogeneity in bilateral elasticities for estimation of gravity regressions. Specifically, in Section 3.1.2, we quantify the heterogeneous coefficients bias embedded in gravity estimates of trade elasticities.

in which we lower trade costs but do not allow agents to reoptimize the location of production stages in response. The second benchmark is obtained from a multi-sector Eaton-Kortum model with roundabout input-output linkages across sectors, similar to Caliendo and Parro (2015).[7] We show that the ratio of manufacturing world trade to GDP rises non-linearly in all three cases, and that this nonlinearity is actually strongest in the Caliendo-Parro benchmark. This result prima facie contradicts the argument in Yi (2003). We argue this apparent contradiction is driven by the choice of benchmark model: we compare the multistage model to a roundabout model with input trade, while Yi compares a multistage model to models without any input trade. Our takeaway is that inputs magnify the the rise of trade to GDP as trade costs fall, but the particular form (multistage or roundabout) that input trade takes has relatively little impact on the aggregate trade response.

Our paper is obviously related to an active literature on input sourcing in general, and to work on multistage production in particular. Dixit and Grossman (1982) and Sanyal (1983) pioneered trade models with a continuum of production stages, which have been recently extended by Costinot et al. (2013) and Fally and Hillberry (2018). We instead adopt a model with a discrete number of stages, as in Yi (2003, 2010), Markusen and Venables (2007), and Baldwin and Venables (2013).[8]

A recent (contemporaneous) contribution by Antràs and de Gortari (2016) is also closely related to our work. Antràs and de Gortari study two variants of multistage trade models. They first analyze optimal stage locations for single value chains with many countries and stages, using computational techniques for solving high dimensional combinatorial problems. They then pivot to a second quantitative model with a small number of discrete stages, similar to our paper. In this model, they restrict the information that decision makers possess – assuming they only know particular features of the distribution of costs of production, not realized values – as they locate production stages. This probabilistic treatment is an analog to the Eaton and Kortum (2002) approach to Ricardian models.[9] This solution technique is one key difference between their paper and ours; Rather than relying on ex ante information and functional form assumptions to enable closed form aggregation ("smooth out" discontinuities in the model), we instead apply an ex post computational technique to smooth the equilibrium conditions directly. Importantly, because our

[7]We calibrate this benchmark model to match the equilibrium in our multistage model exactly, so we compute elasticities using different models that fit the exact same set of (simulated) data.

[8]Arkolakis and Ramanarayanan (2009) and Bridgman (2008, 2012) also study models with discrete numbers of stages, but they lack the the sequential stage allocation problem that is central to our analysis.

[9]Antràs and de Gortari have two alternative characterizations of uncertainty about the costs of locating production stages, which yield the same aggregate results. The first is that the unit cost for the entire value chain is uncertain, drawn from a Fréchet distribution with location parameter that depends on wages and technology levels in all countries in that value chain. The second is that firms face uncertainty about upstream productivity when they make decisions about where to locate their production stage – i.e., firms know their own productivity and the productivity of suppliers one stage upstream, but have imperfect information about suppliers further than one stage upstream, where uncertainty is again characterized by draws from the Fréchet distribution.

approach does not require particular information or functional form assumptions, it is applicable to models where Eaton-Kortum style aggregation fails.[10] A second major difference between our papers concerns their quantitative focus. Antràs and de Gortari analyze the relationship between geography (centrality) and downstream location in value chains, and gains from trade due to multistage production. In contrast, we place primary emphasis on trade elasticities, as in Yi (2003, 2010).

The paper proceeds as follows. We describe the model in Section 1. In Section 2, we discuss how we calibrate and estimate model parameters, and we describe the technology and trade cost estimates. We analyze trade elasticities in Section 3, and Section 4 concludes.

1 Framework

We start this section by laying out the basic elements of our multistage model, describing the economic environment first and defining the model equilibrium. We then discuss how we solve the model numerically, since the model does not admit an analytic solution.

1.1 Economic Environment

Consider a world economy with many countries and two sectors. Countries are indexed by $i, j, k \in \{1, \ldots, C\}$ and sectors are denoted by m and n, standing for manufacturing and non-manufacturing (including agriculture, natural resources, and services) respectively. Within each sector, there is a unit continuum of goods indexed by z. By way of notation, we put country labels in the superscript and good and sector labels in parentheses. We also assume that all goods and factor markets are perfectly competitive.

Manufacturing The manufacturing sector features a discrete multistage production process, as in Yi (2003, 2010). Each good requires $s \in \{1, \ldots, S\}$ production stages to be completed sequentially, and subscripts on each variable index the production stage.

Production in stage 1 uses labor and a composite input, and we assume the production function for good z in sector m is:

$$q_1^i(z,m) = T_1^i(z,m)\Theta_1(m)X^i(z,m)^{\theta_1(m)} l_1^i(z,m)^{1-\theta_1(m)}, \tag{1}$$

[10] Beyond the multistage application that we pursue, the discrete choice technique is applicable to many potential computational trade models. For example, one could use the same approach to solve high dimensional Ricardian models without assuming productivity is distributed according to the Fréchet distribution, or relaxing assumptions about functional forms and the structure of competition that enable Fréchet aggregation.

where $T_1^i(z,m)$ is the good-specific productivity of country i in manufacturing stage 1, $l_1^i(z,m)$ and $X^i(z,m)$ are the quantities of labor and the composite input used in production, $\theta_1(m)$ is the share of the composite input in production in stage 1, and $\Theta_1(m) = (1-\theta_1(m))^{1-\theta_1(m)}\theta_1(m)^{\theta_1}$ is a normalization.

Production at stages $s > 1$ requires labor and output from stage $s-1$ as in intermediate input, and the production function is given by:

$$q_s^i(z,m) = T_s^i(z,m)\Theta_s(m)x_{s-1}^i(z,m)^{\theta_s(m)}l_s^i(z,m)^{1-\theta_s(m)}, \tag{2}$$

where $T_s^i(z,m)$ is productivity in stage s, $x_{s-1}^i(z,m)$ is the quantity of the stage $s-1$ input used, $l_s^i(z,m)$ is labor used, $\theta_s(m)$ is the cost share attached to the stage $s-1$ input, and $\Theta_s(m)$ is again a parameter normalization.[11]

Output in each stage may be produced in any location, but every time output is shipped between countries it incurs a bilateral, sector-specific, ad valorem iceberg trade cost $\tau^{ij}(m)$.[12]

Non-manufacturing The non-manufacturing sector features Ricardian production and trade, as in Eaton and Kortum (2002). Production of good z in sector n requires labor and the composite intermediate input:

$$q^i(z,n) = T^i(z,n)\Theta(n)X^i(z,n)^{\theta(n)}l^i(z,n)^{1-\theta(n)}. \tag{3}$$

where $T^i(z,n)$ is productivity, $l^i(z,n)$ and $X^i(z,n)$ are the quantities of labor and the composite input used in production, $\theta(n)$ is the share of the composite input in production, and $\Theta(n)$ is a parameter normalization. Each non-manufacturing good can be produced in any location, and shipping from source to destination incurs bilateral, ad valorem iceberg trade cost $\tau^{ij}(n)$, which may differ from the trade cost for manufactured goods.

Aggregation Within each sector, goods are aggregated to form non-traded composites $Q^i(m)$ and $Q^i(n)$, which are sold to final consumers and used to form the composite input.[13] Each sector-level

[11]Following Yi (2010), we adopt Cobb-Douglas production functions for stage output. This functional form facilitates calibration, but alternative functional forms are feasible. Further, we do not explicitly include capital in the model. Including endogenous capital stocks, as inYi (2003), would be a straightforward extension of the model.

[12]Extensions in which trade costs differ for final versus intermediate goods are feasible. This exenstion would allow one to consider the effects of input tariff liberalization in the model. Further, extensions with per unit trade costs, as in Irarrazabal et al. (2015), are also interesting. In particular, our iceberg formulation assumes that the proportional burden of trade costs is constant for all stages, but that the absolute value of trade costs incurred is higher for latter stages because the value of output is larger. In constrast, per unit trade costs would imply that the proportional burden of trade costs is lower for later stages.

[13]One can think of this aggregation step as an additional production stage with zero value added, or one can embed the aggregation directly into preferences and production functions. We choose the former route, but this choice has no consequences.

composite is a Cobb-Douglas combination individual goods:

$$Q^i(m) = \exp\left(\int_0^1 \log(\tilde{q}^i(z,m))dz\right) \tag{4}$$

$$Q^i(n) = \exp\left(\int_0^1 \log(\tilde{q}^i(z,n))dz\right), \tag{5}$$

where $\tilde{q}^i(z,m)$ and $\tilde{q}^i(z,n)$ are the quantities of each good purchased (from low cost sources at home or abroad) by country i. For manufacturing, $\tilde{q}^i(z,m)$ represents purchases of stage S goods.

These sector-level composite goods are combined to form an aggregate final good and the composite input. The aggregate final good is given by $F^i = A^i F^i(m)^{\alpha_i} F^i(n)^{1-\alpha_i}$, where $F^i(m)$ and $F^i(n)$ denote the amount of the sector-level composite good that is sold to final consumers, α_i is a country-specific manufacturing expenditure share, and $A^i = (1-\alpha_i)^{1-\alpha_i}\alpha_i^{\alpha_i}$. The composite input is given by $X^i = BX^i(m)^{\beta}X^i(n)^{1-\beta}$, with $X^i = \int_0^1 X^i(z,m)dz + \int_0^1 X^i(z,n)dz$ and $B = (1-\beta)^{1-\beta}\beta^{\beta}$. Finally, adding up requires that $Q^i(m) = F^i(m) + X^i(m)$ and $Q^i(n) = F^i(n) + X^i(n)$.

Households Consumers supply labor inelastically to firms and consume the composite final good F_i. The consumer budget constraint is: $w^i L^i = P_F^i F^i + TB^i$, where w^i is the wage, L^i is the labor endowment, P_F^i is the price of the final composite, and TB^i is the nominal trade balance. The trade balance appears here in the budget constraint, since we treat it as an exogenous nominal transfer necessary to equate income and expenditure for each country.

1.2 Model Equilibrium

To solve for an equilibrium, we need to describe the optimal sourcing decisions. We then define price indexes, collect market clearing conditions, and define the equilibrium.

Input Sourcing For non-manufacturing, this amounts to determining who the low cost suppliers are for each good to each destination. If $p^j(z,n)$ is the potential factory gate price that country j could supply non-manufacturing good z, and $p^{jk}(z,n) = \tau^{jk}(n)p^j(z,n)$ is the delivered price in country k, then realized price of good (z,n) in destination k is:

$$\tilde{p}^k(z,n) = \min_j p^{jk}(z,n). \tag{6}$$

The potential price at which j can supply non-manufacturing good z is itself given by $p^j(z,n) = \frac{(w^j)^{1-\theta(n)}(P_X^j)^{\theta(n)}}{T^j(z,n)}$, where P_X^j is the price of the composite input (defined below).

For manufacturing, we need to solve for the optimal assignment of stages to countries for production of all goods purchased by each destination, where the assignment of stages to countries depends on the destination in which that good is consumed. If $p_s^j(z,m)$ is the potential factory gate price that country j could supply stage-s of manufactured good z, and $p_s^{jk}(z,m) = \tau^{jk}(m) p_s^j(z,m)$ is the delivered price in destination k inclusive of trade costs, then optimal sourcing implies that the realized price in destination k is:

$$\tilde{p}_s^k(z,m) = \min_j p_s^{jk}(z,m). \tag{7}$$

For stages $s > 1$, the potential factory gate price at which country i can supply stage s output is:

$$p_s^j(z,m) = \frac{(w^j)^{1-\theta_s(m)} (\tilde{p}_{s-1}^j(z,m))^{\theta_s(m)}}{T_s^j(z,m)} \quad \text{with} \quad \tilde{p}_{s-1}^j(z,m) = \min_i p_{s-1}^{ij}(z,m). \tag{8}$$

At stage $s = 1$, the potential factory gate price of output from country i is:

$$p_1^i(z,m) = \frac{(w^i)^{1-\theta_1(m)} (P_X^i)^{\theta_1(m)}}{T_1^i(z,m)}, \tag{9}$$

where P_X^i is again the composite input price.

This sourcing problem has a recursive structure. The price at which k actually purchases stage s output for good (z,m) is the minimum of the set of possible prices at which each country j could deliver that output, conditional on country j choosing the minimum cost source its own purchases of stage $s-1$ output. Potential supply prices for stage $s-1$ inputs in turn depend on optimal sourcing further upstream, at stage $s-2$. And so on. Finally, at stage 1, input supply prices depend on the composite input price in country in each country, which itself is a function of the realized prices (given optimal sourcing) of stage S output. This structure gives rise to a combinatorial stage location problem, inside a general equilibrium problem. For each manufactured good and purchasing country, there are C^S possible combinations of the value chain.

Price Indexes and Market Clearing Given Equation 4, the price of the manufacturing composite good is $P^i(m) = \exp\left(\int_0^1 \log(\tilde{p}^i(z,m))dz\right)$, and the price of the non-manufacturing composite good is $P^i(n) = \exp\left(\int_0^1 \log(\tilde{p}^i(z,n))dz\right)$. Then, the price indexes for F^i and X^i are $P_F^i = (P^i(m))^{\alpha_i}(P^i(n))^{1-\alpha_i}$ and $P_X^i = P^i(m)^{\beta} P^i(n)^{1-\beta}$.

The market clearing conditions for output are:

$$q_S^i(z,m) = \sum_j \tau^{ij}(m)\tilde{q}^j(z,m)\mathbf{1}(p_S^{ij}(z,m) \leq p_S^{kj}(z,m)\,\forall k \neq i), \tag{10}$$

$$q_s^i(z,m) = \sum_j \tau^{ij}(m)x_s^j(z,m)\mathbf{1}(p_s^{ij}(z,m) \leq p_s^{kj}(z,m)\,\forall k \neq i) \text{ for } 1 \leq s < S, \tag{11}$$

$$q^i(z,n) = \sum_j \tau^{ij}(n)\tilde{q}^j(z,n)\mathbf{1}(p^{ij}(z,n) \leq p^{kj}(z,n)\,\forall k \neq i). \tag{12}$$

Market clearing conditions for the composite input and the labor market are:

$$X^i = \int_0^1 X^i(z,n)dz + \int_0^1 X^i(z,m)dz, \tag{13}$$

$$L^i = \int_0^1 l^i(z,n)dz + \int_0^1 l_s^i(z,m)dz. \tag{14}$$

Equilibrium Given parameters $\{\alpha_i, \theta(s), \beta, T_1^i(z,m), T_2^i(z,m), T^i(z,n), \tau^{ij}(m), \tau^{ij}(n), L^i, TB^i\}$, an equilibrium is a collection of prices $\{w^i, \tilde{p}_1^i(z,m), \tilde{p}_2^i(z,m), \tilde{p}^i(z,n), P^i(m), P^i(n), P_X^i, P_F^i\}$, aggregate quantities $\{F^i, X^i, Q^i(m), Q^i(n), F^i(m), F^i(n), X^i(m), X^i(n)\}$, and production, sourcing, and input use decisions $\{X^i(z,n), X^i(z,m), l^i(z,n), l_s^i(z,m), q^i(z,n), \tilde{q}^i(z,n), q_s^i(z,m), x_s^i(z,m), \tilde{q}_2^i(z,m)\}$ such that producers maximize profits, consumers maximize real final expenditure subject to their budget constraint, and product and labor markets clear.

1.3 Discussion

Prior to discussing how we translate this model into a quantitative framework for analysis, we comment on two aspects of the model.

First, the model features both sequential multistage production and roundabout production. Roundabout production introduces a loop in the production process, which amplifies the ratio of gross output to value added. That is, gross output will exceed value added both because multistage production implies that inputs are produced and used up in the production process, but also because production in each sector uses its own output as inputs. Roundabout production also gives rise to input linkages across sectors in the model. We have assumed that sequential production is confined to the manufacturing sector, and that all cross-sector input flows are non-sequential in nature. Both these aspects of the model are important for how we calibrate the model to match the data, discussed further below.

Second, the multistage component of the model is essential to understanding the behavior of the elasticity of trade flows to trade costs. One useful piece of intuition is that stages are more often co-located in the same country when trade costs are high. This implies that the model behaves more like a standard single-stage (multi-sector) Ricardian model as trade costs rise. Since the single-stage model features a constant partial elasticity of trade to changes in trade costs, the multistage model will also feature a near constant partial trade elasticity at high levels of trade costs. Further, it will also generate changes in trade and welfare that are similar to the Ricardian benchmark in response to marginal changes in trade costs when trade costs are initially high.

As trade costs fall, it is increasingly attractive to exploit cost differences and break up production stages across countries. The ability to substitute over the location of individual stages, rather than simply over the location of production for entire goods, tends to amplify the sensitivity of trade to trade costs. The key mechanism is that trade costs are paid on the full value of stage output, while cost savings of shifting the the location of a single stage of the production process apply only to the value added at that stage. For downstream production stages, the value of gross output – and thus the trade cost paid for exporting downstream output – is large relative to the marginal value added at that stage. This deters the formation of value chains in which inputs from home are used abroad and the final good is re-exported, either back home or to third countries. Yi (2010) refers to this as the "effective rate of protection" force.

At intermediate levels of trade costs, the model economy features both standard Ricardian trade, where consumers substitute across entire goods, and trade through multistage value chains in which agents substitute over production locations for each stage. Therefore, the aggregate model elasticity of trade to trade costs depends on the mix of Ricardian versus multistage trade. As trade costs fall, the share of trade via multistage value chains rises, so we expect the elasticity of trade to trade costs to rise as well.

1.4 Solving the Model

We describe solution of the model for the non-manufacturing sector and manufacturing sector separately, since we treat them in somewhat different ways. While we continue to assume that there is a continuum of goods in non-manufacturing, we introduce a discrete approximation to the continuum of goods in manufacturing, following Yi (2003, 2010).

Non-Manufacturing As in Eaton and Kortum (2002), we assume that technology parameters $\{T^i(z,n)\}$ independent draws from Fréchet distributions, which have shape parameter κ (common to all countries) and country-specific location parameters $\{T^i(n)\}$. We can then solve for trade shares and price indexes in closed form:

$$\pi^{ji}(n) = \frac{T^j(n)\left(\tau^{ji}(n)(w^j)^{1-\theta(n)}(P_X^j)^{\theta(n)}\right)^{-\kappa}}{\sum_j T^j(n)\left(\tau^{ji}(n)(w^j)^{1-\theta(n)}(P_X^j)^{\theta(n)}\right)^{-\kappa}}, \quad (15)$$

$$P^i(n) = \exp(\gamma/\kappa)\left(\sum_j T^j(n)\left(\tau^{ji}(n)(w^j)^{1-\theta(n)}(P_X^j)^{\theta(n)}\right)^{-\kappa}\right)^{-1/\kappa}. \quad (16)$$

The ability to solve for the non-manufacturing equilibrium in closed form is useful, because it facilitates computation of the full model equilibrium.

Manufacturing We assume that there are a large, finite number of manufactured goods, and let $r = \{1,\ldots,R\}$ index individual goods. The equilibrium of the model is essentially the same as described above, with r rather than z indexing goods in manufacturing and summations over this set of goods replacing integrals where appropriate.

As in Ricardian models with a finite number of goods, the equilibrium of this discretized model is not continuous in the underlying parameters. This makes the model challenging to solve numerically, and hence also to estimate the model parameters via simulated method of moments. The standard approach to dealing with this complication would be to approximate the continuum with a "large" number of goods, where "large" means a high enough value for R so that the discretized model equilibrium conditions are sufficiently smooth to be accurately solved with standard (derivative-based) numerical methods.[14] In our multi-country setting, this standard approach is computationally burdensome, because we need to solve the combinatorial sourcing problem (described above) for every good and destination country. Therefore, we adopt a different procedure that allows us to use a "small" value for R.

Noting the similarity between sourcing decisions in the model and consumer optimization in discrete choice models, we borrow a smoothing technique developed to facilitate simulation of choice probabilities in the discrete choice literature. Specifically, we draw on the logit-smoothed accept-reject (AR) simulator, developed by McFadden (1989).[15] The key observation is that – like simulated choice probabilities – sourcing decisions and hence trade shares are discontinuous. These discontinuities are associated with the presence of indicator functions in the market clearing conditions of the model. The logit-smoothed AR simulator approximates the indicator function

[14] For example, Yi (2010) approximates the continuum using 1.5 million discrete goods.
[15] See Train (2009) for a lucid presentation of accept-reject simulators.

with a continuous logit function, as in:

$$\mathbf{1}(p_s^{ij}(r,m) \leq p_s^{kj}(r,m) \forall k \neq i) \approx \frac{e^{-p_s^{ij}(r,m)/\lambda}}{\sum_k e^{-p_s^{kj}(r,m)/\lambda}}, \tag{17}$$

where $\lambda > 0$ is a smoothing parameter that determines the accuracy of the approximation.[16] Intuitively, when country i is a relatively high cost supplier to j (near the max of the set $\{p_s^{kj}(r,m)\}$), the logit function takes on a value near zero. In contrast, as country i's price falls relative to its competitors, the logit function smoothly converges to one.

With this assumption, the market clearing conditions for manufacturing in the smoothed, discretized model become:

$$q_S^i(r,m) = \sum_j \tau^{ij}(m)\tilde{q}^j(r,m)\left(\frac{e^{-p_S^{ij}(r,m)/\lambda}}{\sum_k e^{-p_S^{kj}(r,m)/\lambda}}\right), \tag{18}$$

$$q_s^i(r,m) = \sum_j \tau^{ij}(m)x_s^j(r,m)\left(\frac{e^{-p_s^{ij}(r,m)/\lambda}}{\sum_k e^{-p_s^{kj}(r,m)/\lambda}}\right) \text{ for } 1 \leq s < S. \tag{19}$$

Thus far, we have not said anything about manufacturing technology parameters $\{T_s^i(z,m)\}$. To reduce the dimensionality of the parameter space, we impose distributional assumptions on them, so that we only need to estimate distribution parameters below. While our model can accommodate many possible distributions, we follow Yi (2010) and assume that $\{T_s^i(z,m)\}$ are independent draws from Fréchet distributions, which have shape parameter κ (common to all countries, and identical to the parameter in non-manufacturing) and country-stage-specific location parameters $\{T_s^i(m)\}$.[17]

Solution Procedure A brief overview of the procedure we use to solve the model is as follows. Given parameters $\{\alpha_i, \theta(s), \beta, T^i(n), T_s^i(m), \kappa, \tau^{ij}(m), \tau^{ij}(n)\}$, data $\{L^i, TB^i\}$, productivity draws $\{T_s^i(r,m)\}$, and an initial guess for the vector of wages, we can solve for the optimal assignment of stages to countries and equilibrium prices in manufacturing, along with equilibrium prices in non-manufacturing. Given this, we then construct manufacturing and non-manufacturing production, and thus labor demanded. An equilibrium vector of wages equates labor demand and labor supply, as in Equation (14). We describe in detail an algorithm to solve the model in Appendix A.

[16] As $\lambda \to 0$, the logit function converges to the indicator function. The choice of λ is guided by a trade-off between accuracy and computational speed, and there is little guidance on the appropriate level of λ in general. By trial and error, we find that $\lambda = 0.1$ yields a very good approximation to the exact equilibrium.

[17] One important difference relative to Yi (2010) is that we allow countries to have comparative advantage across sectors, while Yi instead restricts the location of the productivity to be the same in all stages, as in $T_s^i(m) = T^i(m)$.

2 Model to Data

In this section, we describe how we fit the model in Section 1 to the data. We first describe how we calibrate a subset of the parameters of the model and estimate the remainder via simulated method of moments. We then describe our data source. We conclude with a description of the estimated values for technology and trade cost parameters.

2.1 Fitting the Model

The free parameters in the model include technology parameters $\{T_s^i(m), T^i(n), \kappa\}$, trade costs $\{\tau^{ij}(m), \tau^{ij}(n)\}$, and share parameters in production functions and preferences $\{\theta_s(m), \theta_s(n), \beta, \alpha_i\}$. We mix calibration and estimation in pinning down these parameters.

Thus far, we have not specified the number of production stages (S) in the model, but we must take a stand on the number of stages to calibrate/estimate the remaining parameters. We assume there are two production stages in manufacturing ($S = 2$), following Yi (2003, 2010).[18] We also need to choose a value for the number of manufacturing goods (R), which yields an acceptable trade-off between simulation accuracy and computation time. In Monte Carlo simulations, we have found that our estimation procedure is able to recover the true parameters of the model when $R = 20,000$, so we use this value.

2.1.1 Calibrated Parameters

We calibrate $\{\theta_s(m), \theta(n), \beta, \alpha_i\}$ to match production and expenditure data. The parameter α_i is set to match the share of manufacturing in final expenditure in each country. The median value of α_i is 0.24, with values that range from 0.17 to 0.43 across countries. In the baseline calibration, we assume that $\theta(m) \equiv \theta_1(m) = \theta_2(m)$ and set $\theta(m) = 0.70$ and $\theta(n) = 0.41$ to match the ratio of value-added to output for the world as a whole in the manufacturing and non-manufacturing, respectively.[19] Given this, we calibrate β match input flows across manufacturing and non-manufacturing sectors for the world.[20] This yields $\beta = 0.16$, which means that the com-

[18] Antràs and de Gortari (2016) attempt to discriminate between models with 2 versus 3 stages based on model fit, and find that two stages are sufficient to fit bilateral trade data.

[19] For each country, value added in manufacturing is equal to the wage bill: $va^i(m) = \sum_{r=1}^{R} w^i(l_1^i(r,m) + l_2^i(r,m)) = (1-\theta(m))go^i(m)$, where $go^i(m) = \sum_{r=1}^{R}[p_1(r,m)q_1^i(r,m) + p_2(r,m)q_2^i(r,m)]$. Then $1-\theta(m) = \sum_i va^i(r,m)/\sum_i go^i(m)$, so we set $\theta(m)$ to match the ratio of value-added to output in manufacturing for the world as a whole. While we could allow $\theta(m)$ to vary across countries, we have chosen not to do so for two reasons. First, while differences in value-added to output ratios differ a lot between manufacturing and services, differences across countries within sectors are more muted. Second, imposing a common value for $\theta(m)$ across countries facilitates calibration of β.

[20] Let $y(m) = y_1(m) + y_2(m)$ be the value of total world output in manufacturing, where$y_1(m)$ and $y_2(m)$ denote stage 1 and stage 2 output. Whereas $y(m)$ is directly observable, $y_1(m)$ and $y_2(m)$ are not. In the model, $y_1(m) = \theta(m)y_2(m)$, so $y(m) = (1+\theta(m))y_2(m)$. Further, $y_2(m) = \sum_i \alpha_i P_F^i F^i + \beta \sum_i P_X^i X^i$, with $\sum_i P_X^i X^i = \theta(m)y_1(m) + \theta(n)y(n)$, where $y(n)$ is total world non-manufacturing output. Combining these observations, $(1+\theta(1))^{-1}y(m) =$

posite input is composed primarily of non-manufacturing output.

2.1.2 Estimation of Technology and Trade Cost Parameters

In Section 1.4, we assumed that $\{T_s^i(z,m), T^i(z,n)\}$ are independent draws from Fréchet distributions. We set the common shape parameter in these distributions to $\kappa = 4.12$, guided by Simonovska and Waugh (2014). We leave technology levels $\{T_s^i(m), T^i(n)\}$ as parameters to be estimated, and we set $T_1^1(s) = T_2^1(s) = T^1(n) = 1$ so that technology levels are measured relative to country 1.

We parameterize trade costs by assuming that bilateral trade costs are a power function of distance: $\tau^{ij}(m) = \tau^j \left(d^{ij}\right)^{\rho(m)}$ and $\tau^{ij}(n) = \tau \left(d^{ij}\right)^{\rho(n)}$, where d^{ij} is the distance between country i and country j, $\{\tau^j, \tau\}$ are a level parameters for trade costs, and $\rho(m)$ and $\rho(n)$ are sector-specific elasticities of trade costs to distance.[21] We set trade costs on domestic shipments to one in all countries ($\tau^{ii}(m) = \tau^{ii}(n) = 1$).

We estimate the parameters $\Theta = \left\{T_1^i(m), T_2^i(m), T^i(n), \tau^j, \tau, \rho(m), \rho(n)\right\}$ by minimizing the distance between trade flows in the model and data, given data on expenditure in each market $\{P_F^i F^i\}$, trade balances $\{TB^i\}$, and labor endowments $\{L^i\}$. Together, these data allow us to compute wages as $w^i = \left(P_F^i F^i + TB^i\right)/L^i$, and we set $w^1 = 1$ as our price normalization. Given wages and a candidate parameter vector $\tilde{\Theta}$, we draw $\{T_s^i(r,m)\}$, compute the model equilibrium, and form a vector of moments based on trade shares.

The first set of moments are based on bilateral trade shares for final manufactured goods. In the model, the share of final goods purchased by destination j from source i as a share of final expenditure in country j is equal to the probability that destination j sources stage 2 goods from country i, because expenditure per good is constant. Employing the same smoothing procedure described in Section 1.4, we compute these shares as:

$$\pi_F^{ij}(m) = \frac{1}{R}\sum_{r=1}^{R}\left(\frac{e^{-p_2^{ij}(r,m)/\lambda}}{\sum_k e^{-p_2^{kj}(r,m)/\lambda}}\right). \tag{20}$$

The second set of moments are based on trade shares for manufactured inputs. Input shipments from country i to j include both stage 1 goods and stage 2 goods destined for the composite input.

$\sum_i \alpha_i P_F^i F^i + \beta\theta(m)\left[\frac{\theta(m)}{1+\theta(m)}\right] y(m) + \beta\theta(n) y(n)$. Given parameters $\alpha_i, \theta(m), \theta(n)$ and data $y(m), y(n), P_F^i F^i$, this equation can be solved for β.

[21] We adopt a parsimonious specification for trade costs to limit the number of parameters that need to be estimated. While the specification we choose is sufficient to fit the data well, a more flexible specification would be feasible. The assumption that trade costs have a destination-specific component can be motivated in a number of ways. The most obvious is that it captures differences in multilateral import protection. In relation to prior work, Eaton and Kortum (2002) also assume that there is a destination-specific, but not source-specific, component of bilateral trade costs. We omit source-specific effects (in part) to reduce the dimensionality of the parameter space for estimation.

Input shipments from i to j of sector m goods are:

$$Inputs^{ij}(m) = \sum_{r=1}^{R}\left(\frac{e^{-p_1^{ij}(r,m)/\lambda}}{\sum_k e^{-p_1^{kj}(r,m)/\lambda}}\right)\left[\theta(m)p_2^j(r,m)q_2^j(r,m)\right] + \frac{1}{R}\sum_{r=1}^{R}\left(\frac{e^{-p_2^{ij}(r,m)/\lambda}}{\sum_k e^{-p_2^{kj}(r,m)/\lambda}}\right)\left[\beta P_M^j M^j\right]. \quad (21)$$

Then the share of inputs from source i in country j's total purchases of manufactured inputs is: $\pi_I^{ij}(m) = \frac{Inputs^{ij}(m)}{\sum_k Inputs^{ij}(m)}$. The third set of moments are based on trade shares in the non-manufacturing sector, which can be computed in closed form as in Equation (15).

Since the trade shares sum to one for each importer, we only use off diagonal trade shares ($i \neq j$) for estimation. This gives us $3\left(N^2 - N\right)$ moments to estimate $(3N-3)+N+3$ unknown parameters. Letting π^{ij} denote the vector of trade shares for pair ij, we stack log differences between actual and simulated trade shares $\pi^{ij}(\Theta) = \ln \pi^{ij} - \ln \hat{\pi}^{ij}(\Theta)$ in a column vector $\pi(\Theta)$. The moment condition is then $E\left[\pi\left(\Theta_0\right)\right] = 0$, where Θ_0 is the true value of Θ, so we estimate a $\hat{\Theta}$ that satisfies:

$$\hat{\Theta} = \arg\min\left\{\pi(\Theta)'\pi(\Theta)\right\} \quad (22)$$

One point worth noting here is that the algorithm we use to solve this minimization problem exploits the existence of closed forms in non-manufacturing to speed computation, by estimating non-manufacturing parameters via regression within the simulated MoM procedure for manufacturing parameters.

2.1.3 Data

We draw trade and input-output data from Johnson and Noguera (2017), covering forty-two major countries. We select 37 countries in 2005 to include our analysis, and we construct a composite rest-of-the-world region to close the world economy.[22] Thus, $C = 38$ overall. Further, we aggregate the data to the two-sector level, where manufacturing is category D in the ISIC (Revision 3.1) classification and non-manufacturing includes all other sectors of the economy. Consistent with the discussion above, we use data on aggregate final expenditure and the trade balance for each country from this data. We also use data on bilateral shipments of final goods and inputs to form trade shares, including each country's purchases from itself. In order to calibrate $\{\theta_s(m), \theta(n), \beta\}$,

[22] In the original Johnson and Noguera (2017) data, there is no information on input use in the rest of the world region. We extend the data here to close the world economy by imputing input linkages in the ROW. Details available on request. Further, while data in Johnson and Noguera is available through 2009, we select 2005 for analysis to avoid the impact of the 2008-2009 recession and trade collapse. In previous working papers, we have alternatively used data from the World Input-Output Database, which returns essentially similar results.

we aggregate across countries (including the rest of the world) to compute world-level data on sector-level value added, sector-level gross output, and cross-sector input shipments.

Finally, we measure aggregate labor endowments data from Penn World Tables 9.0. We define labor endowments $\{L^i\}$ in effective terms, equal to numbers of persons engaged (employment) times the level of human capital, which is constructed based on estimates of educational attainment and returns to schooling by the Penn World Tables. We also use bilateral distance data from CEPII to estimate trade cost parameters.

2.2 Estimates for Trade Costs and Technology

In this section, we present estimates for trade costs and technology parameters. We also briefly examine the role of comparative advantage in driving specialization across production stages, and thus export composition.

Trade Costs Our estimate of the elasticity of trade costs to distance for the manufacturing sector is $\rho(m) = 0.25$. Combining this distance elasticity with estimates of importer-specific levels of trade costs (τ^j), we compute implied iceberg frictions for manufacturing and plot the distribution of trade costs across bilateral pairs in Figure 1. Clearly, estimated iceberg frictions are both large on average (with median value of about 4.5) and very dispersed across partners. The high level of these trade cost estimates is broadly in line with findings in the previous literature.[23]

As one would expect, the level and heterogeneity in bilateral trade costs is tightly linked to variation in sourcing shares. For one, the level of trade costs is highly correlated with the home bias in each country's expenditure. Second, bilateral frictions are needed to match the wide variation in import shares across partners. To illustrate this variation in the data and model, we plot intermediate and final goods trade shares in Figure 2, with true trade shares on the x-axis and simulated trade shares on the y-axis (both log scales). The model clearly fits these targeted moments well.

Technology We present estimates for technology levels by stage for the 37 countries and the composite region in Table 1. The columns labeled Stage 1 and Stage 2 present the geometric means of $T_1^i(z,m)$ and $T_2^i(z,m)$ in each country, expressed relative to relative to Argentina.[24] Across

[23]The review by Anderson and van Wincoop (2004) argues that existing estimates implied that trade costs for a representative industrialized country are near 170% ($\tau = 2.7$). Our sample includes emerging markets as well, where trade costs are plausibly higher. Eaton and Kortum (2002) report estimated distance costs of roughly 300% for country pairs in the 3000 to 6000 mile distance range.

[24]Since $T_s^i(z,m)$ is drawn from a Fréchet, the unweighted geometric mean is given by $\exp(\gamma/\kappa)T_s^i(m)^{1/\kappa}$, where γ here is the Euler-Mascheroni constant. Since we report means for each country relative to Argentina, the numbers reported in the table are effectively $\left(T_s^i(m)/T_s^{ARG}(m)\right)^{1/\kappa}$.

countries, productivity levels are highly correlated with real income per capita. Productivity levels are also correlated across stages: countries with high absolute productivity in stage 1 tend to also have high absolute productivity in stage 2. Despite this correlation, there are sizable cross-country differences in relative productivity across stages.

The final column in Table 1 reports the ratio of mean technology in stage 2 relative to stage 1 in each country, where numbers greater than one indicate that a country has a technological comparative advantage in stage 2 (downstream) production relative to Argentina. Countries are ordered by comparative advantage in the table: Chile and South Africa at the top have comparative advantage in the upstream stage, while Greece and Vietnam at the bottom have comparative advantage in the downstream stage. Technological comparative advantage is naturally related to export composition in the data. In Figure 3, we plot the share of final goods in exports against our estimates of relative stage technologies. Though we do not directly target export composition in estimation, there is an evident positive correlation between export composition and stage comparative advantage, wherein countries with high productivity in stage 2 relative to stage 1 have higher shares of final goods in their exports.

Comparative Advantage and Export Composition While technological comparative advantage has an obvious influence on export composition, it is not the only determinant of it (e.g., trade costs matter too). To isolate the role of technological comparative advantage in driving export composition, we turn to a counterfactual simulation of the model in which we eliminate this comparative advantage, setting technology parameters to be equal across stages. Specifically, we set counterfactual technology in each country and stage ($\tilde{T}_s^i(m)$) equal to the geometric mean across stages: $\tilde{T}_s^i(m) = \left(T_1^i(m)\right)^{1/2}\left(T_2^i(m)\right)^{1/2}$ for $s \in 1,2$.

We compare export composition in the baseline estimated model and in this counterfactual simulation in Figure 4. In Panel (a), we plot the share of final goods exports for each country in the estimated model (with unequal stage technologies) versus the data as a benchmark; Unsurprisingly, these are highly positively correlated, as expected based on Figure 3 above. In Panel (b), we plot the share of final goods exports for each country from the counterfactual simulation, having removed comparative advantage across stages, versus the data. As is evident, the positive correlation between export composition in model and data largely disappears when we remove comparative advantage. To reinforce this point, we plot the difference between export composition in the estimated model and the counterfactual model versus estimated comparative advantage in Panel (c). For countries with upstream comparative advantage, removing comparative advantage leads to large increases in the share of final goods in their exports, while the opposite is true for countries who have downstream comparative advantage. Thus, we conclude that technological comparative advantage plays a significant role in explaining upstream vs. downstream specialization.

3 Trade Elasticities

In this section, we use counterfactual simulations to study trade elasticities in the model. We start with a discussion about how bilateral elasticities vary with the level of trade costs, and then we discuss global trade elasticities.

3.1 Bilateral Trade Elasticities

To compute bilateral trade elasticities in the multistage model that are comparable to existing empirical estimates, we interpret our simulated data through the lens of a general gravity equation. Following Head and Mayer (2014), let bilateral gross exports be given by $X^{ij} = GS^{i}M^{j}\phi^{ij}$, where G is a (gravitational) constant, S^{i} captures the supply capabilities of the exporter, and M^{j} captures characteristics of the import market. The parameter $\phi^{ij} = (\tau^{ij})^{-\zeta^{ij}}$ is the level of bilateral (iceberg) trade frictions raised to the power $-\zeta^{ij}$, where ζ^{ij} is the elasticity of trade flows to frictions.

In standard applications of the gravity equation, ζ^{ij} is a constant – as in $\zeta^{ij} = \zeta$ for all i and j – pinned down by the elasticity of substitution in the CES-Armington model or the Fréchet shape parameter in an Eaton-Kortum model. We relax this assumption by allowing the elasticity to vary by country pair, while maintaining the assumption that it is symmetric within each country pair ($\zeta^{ij} = \zeta^{ji}$) and that there are no domestic trade costs ($\tau_{ii} = 1$). Then, we solve for the bilateral trade elasticity as follows:

$$\zeta^{ij} = \zeta^{ji} = \frac{\ln\left(\frac{X^{ij}X^{ji}}{X^{ii}X^{jj}}\right)}{\ln\left(\tau^{ij}\tau^{ji}\right)}. \tag{23}$$

This approach to computing bilateral elasticities is the analog to the ratio-type estimation of trade costs, developed by Head and Ries (2001). Our application of it is similar to Caliendo and Parro (2015), who use measured barriers to trade (e.g., tariffs) and ratios of observed trade flows to estimate trade elasticities.

We compute elasticities using simulated trade data for a sequence of counterfactual equilibria in the multistage model, which differ only in terms of the level of trade costs. Starting from baseline trade cost estimates $\{\hat{\tau}^{ij}(m), \hat{\tau}^{ij}(n)\}$, we consider counterfactual equilibria with trade costs equal to $\{\delta\hat{\tau}^{ij}(m), \delta\hat{\tau}^{ij}(n)\}$, where $\delta \lessgtr 1$ is a scaling factor that raises or lowers the level of trade costs relative to the baseline estimates.[25] For each equilibrium, we use simulated trade flows and trade costs to compute bilateral elasticities, using Equation (23).

[25] Prior to changing trade costs, we close the exogenous aggregate trade imbalances and re-compute the baseline equilibrium with balanced trade (holding all other parameters at their baseline values). We then maintain balanced trade as we compute counterfactual equilibria and compare each of these counterfactuals to this initial balanced trade equilibrium.

Results In Figure 5, we plot mean values of ζ^{ij} for the manufacturing sector in the set of counterfactual equilibria, where the mean level of trade costs in each equilibrium is on the x-axis. In the baseline equilibrium, the mean trade elasticity is roughly 3.9, close to standard values in the literature. Bilateral elasticities also vary across trade partners in the cross-section: the 10th and 90th percentiles for bilateral gravity elasticities are about 3.6 and 4.2 in the baseline equilibrium.

As the level of trade costs falls (rises) from their estimated level, the mean elasticity rises (falls) in the model. This inverse correlation between the trade elasticity and the level of trade costs is the multistage magnification effect in action: bilateral trade appears to become more sensitive to trade frictions as the level of trade costs falls and the extent of value chain fragmentation rises. Quantitatively, a decline in trade costs from 550% ($\tau = 6.5$) to 200% ($\tau = 3.5$) raises the mean elasticity from about 3.84 to 3.98.[26] Thus, while the multistage magnification effect is present in the model, we think it reasonable to say that it is modest in size.

Underlying this average elasticity result, there is heterogeneity in elasticities across final goods versus inputs. We estimate elasticities using Equation (23) for final goods and inputs separately, and then plot the the mean values in Figure 6. The first point to note is that the elasticity for inputs is endogenously larger than for final goods in the model. This reflects the fact that most input trade dominated by stage 1 goods, which are more sensitive to trade costs than stage 2 goods.[27] Given that trade frictions increase with distance, this implies that input trade is concentrated among geographically proximate trade partners, relative to trade in final goods. The second point to note is that both the elasticities of input trade and final trade rise as trade costs fall. That said, the rise in the input elasticity is about 30% larger than the rise in the final trade elasticity. Though different in magnitude, these increases are intimately linked to one another – as GVC stages are reorganized in response to falling trade costs, both the pattern of input and final goods trade changes. In order to better understand bilateral trade elasticities, we now turn to unpacking these changes.

3.1.1 Unpacking GVC Adjustments

To analyze GVC adjustments in response to falling trade costs, we focus on exports of stage 1 goods. Conceptually, it is useful to distinguish between two types of stage 1 trade. On the one hand, some stage 1 goods are shipped to foreign countries and then embodied in stage 2 goods that are directly absorbed there. In this type of value chain, stages are fragmented across countries, but stage 1 imports are not used to produce exports. On the other hand, some stage 1 goods are shipped to foreign countries and then embodied in stage 2 goods that are exported, either back home or to

[26]While this may first appear like a radically large (≈45%) decline in trade frictions, a decline in iceberg frictions on the order of 30-40% is needed to rationalize the growth of world trade since 1970 [Jacks et al. (2011), Johnson and Noguera (2017)].

[27]To clarify, inputs here include both stage 1 goods, which are all used as inputs, and stage 2 goods dedicated to production of the composite input. Stage 1 goods constitute the bulk of input trade for manufacturing.

third countries.

Building on this distinction between different types of global value chains, we can decompose stage 1 exports from country i to j as follows:

$$EX_1^{ij}(m) = EX_1^{ij}(m,j) + EX_1^{ij}(m,k \neq j) \tag{24}$$

The first term $EX_1^{ij}(m,j)$ is the value of stage 1 exports from i that are used to produce stage 2 goods that are absorbed directly in j, and is given by:

$$EX_1^{ij}(m,j) = \sum_{r=1}^{R} \left(\frac{e^{-p_1^{ij}(r,m)/\lambda}}{\sum_l e^{-p_1^{lj}(r,m)/\lambda}} \right) \left[\theta(m) p_2^j(r,m) \tilde{q}^j(z,m) \left(\frac{e^{-p_2^{jj}(r,m)/\lambda}}{\sum_l e^{-p_S^{lj}(r,m)/\lambda}} \right) \right].$$

The second term $EX_1^{ij}(m,k \neq j)$ is the value of stage 1 exports from i that are used to produce stage 2 goods that are exported by j:

$$EX_1^{ij}(m,k \neq j) = \sum_{r=1}^{R} \sum_{k \neq j} \left(\frac{e^{-p_1^{ij}(r,m)/\lambda}}{\sum_l e^{-p_1^{lj}(r,m)/\lambda}} \right) \left[\theta(m) \tau^{jk}(m) p_2^j(r,m) \tilde{q}^k(z,m) \left(\frac{e^{-p_2^{jk}(r,m)/\lambda}}{\sum_l e^{-p_S^{lk}(r,m)/\lambda}} \right) \right].$$

In plain English, this term captures how much "round-trip trade" occurs between i and j – i selling stage 1 inputs to j that are embedded in stage 2 goods sold back to i – as well as the role of j as an "export platform" for value chains that originate from country i. Reflecting this breakdown, we can write $EX_1^{ij}(m,k \neq j) = EX_1^{ij}(m,i) + EX_1^{ij}(m,k \neq i,j)$, where first term is round-trip trade and the second is export platform trade.

As discussed in Section 1.3, trade costs deter organizing the value chain in a way that involves stage 2 being performed in a country where neither stage 1 is performed, nor where stage 2 output is consumed. The reason is that the cost savings of locating stage 2 abroad, rather than at home or where stage 2 output is consumed, will tend to be small relative to the gross trade costs incurred in exporting stage 2 output. Lower trade costs make it more profitable to exploit cost differences in locating stage 2 production. As a result, we should expect to see an increase in both round-trip and export platform trade in stage 1 goods as trade costs fall.

Building on these ideas, we plot the share of stage 1 exports that are absorbed in the destination, embedded in round-trip trade, or embedded in platform exports against the mean level of trade costs in each counterfactual equilibrium in Figure 7. When trade costs are high, stage 1 trade is dominated by input shipments that are absorbed in the destination. As trade costs fall, a larger share of stage 1 goods is dedicated to round-trip trade and platform exporting.[28] In particular, the

[28]Focusing on the levels, round-trip trade is relatively uncommon in the model. In our data, this type of trade only occurs in substantial amounts among the lowest trade cost pairs (e.g., France-Germany, US-Canada, China-Korea, etc.).

rise in stage 1 inputs dedicated to platform exporting comes at the expense of direct absorption of stage 1 inputs.

This rise in platform exporting is important for understanding how the elasticity of input trade behaves in the model. Across bilateral export destinations in the cross-section, stage 1 inputs dedicated to platform exports fall off quickly with respect to bilateral trade costs. That is, export platforms for value chains originating in country i tend to be located in countries j for which $\tau^{ij}(m)$ is low (e.g., countries that are close to country i itself). As the level of trade costs falls and platform exporting increases, the composition of stage 1 exports thus shifts toward the type of trade that is more sensitive to trade costs. The result is that the measured elasticity of trade increases.

To illustrate these mechanics, we turn to Figure 8. For each equilibrium in our model, we bin country pairs based on the quintiles of the distribution of bilateral trade costs. Because relative trade costs are fixed in the counterfactuals, the mapping from country pairs to quintile is stable across equilibria. For each quintile q, we compute log stage 1 trade broken down by destination use, given by $\ln(\sum_{ij\in q} EX_1^{ij}(m,j))$ and $\ln(\sum_{ij\in q} EX_1^{ij}(m,k\neq j))$, where this second summation pools round-trip and platform exports. We plot these values against the log of the mean trade cost in each quintile in Figure 8, for both the highest and lowest trade cost equilibria.

The slope of the line that connects the quintile values for each series corresponds (roughly speaking) to the magnitude of the bilateral elasticity for each type of trade. In both equilibria, stage 1 exports associated with round-trip trade and platform exporting fall off faster with respect to bilateral trade costs than do stage 1 exports that are absorbed in the destination. Further, comparing the right figure to the left, as trade costs fall, stage 1 exports dedicated to round-trip and export platform trade rise relative to those absorbed in the destination. The average response of stage 1 exports with respect to distance (again, roughly speaking) is the weighted mean slope of the two series. The mean slope is evidently higher in the low trade cost equilibrium, primarily because round-trip and platform trade becomes more important for low trade cost partners as trade costs fall. In the end, this endogenous supply chain re-organization drives the rise in the overall bilateral trade elasticity as trade costs fall.

3.1.2 Gravity Regressions

In this section, we briefly draw out additional implications of our results for interpretation of gravity regressions. In our model, the bilateral elasticity of trade to trade costs is heterogeneous. If one ignores this heterogeneity, and instead uses estimation methods that impose constant bilateral elasticities, the resulting elasticity estimates will suffer from heterogeneous coefficient bias (as in a random coefficients model).[29]

[29] See Appendix A in Card (1999) for a succinct discussion of OLS estimation of random coefficient models.

To fix ideas, consider the standard gravity regression:

$$\ln X^{ij} = \ln S^{i} + \ln M^{j} - \zeta \ln \tau^{ij} + \varepsilon^{ij}, \tag{25}$$

where $\ln S^i$ and $\ln M^j$ can be absorbed by exporter and importer specific fixed effects, and ε^{ij} is a residual. The researcher typically treats ordinary least squares (OLS) estimates of ζ from this equation as informative about the average elasticity of bilateral trade to trade costs. Importantly, this interpretation is not correct in our model.

Using the auxiliary gravity model with heterogeneous elasticities from above, we can write the residual as $\varepsilon^{ij} = -[\zeta^{ij} - \zeta]\ln \tau^{ij} + \nu^{ij}$, where ν^{ij} is a residual.[30] Because ζ^{ij} is heterogeneous, ε^{ij} will generally be correlated with $\ln \tau^{ij}$. In turn, the OLS estimate of ζ from Equation 25, defined here as $\hat{\zeta}_{OLS}$, will be a biased estimate of the true mean elasticity $\bar{\zeta}^{ij} = E(\zeta^{ij})$. If the bilateral elasticity ζ^{ij} is higher for pairs with higher trade costs, then $\hat{\zeta}_{OLS}$ will be biased away from zero.

In our baseline equilibrium, $\hat{\zeta}_{OLS} = 4.26$, which is larger than the mean elasticity $\bar{\zeta}^{ij} = 3.90$. Thus, the gravity elasticity is inflated by heterogeneity bias, by about 10%.[31] Further, we note that the extent of this bias is not constant as we vary the level of trade costs, because the underlying heterogeneous elasticities themselves change as the level of trade costs changes. We illustrate this in Figure 9. In Panel (a), we plot estimates of $\hat{\zeta}_{OLS}$ in counterfactual simulated data for different levels of trade costs, similar previous figures. As is evident, there is a U-shape in the estimates: $\hat{\zeta}_{OLS}$ rises as trade costs fall, and as they rise, relative to baseline. In Panel (b), we plot the value of the heterogeneous coefficients bias, defined as $\hat{\zeta}_{OLS} - \bar{\zeta}^{ij}$. The bias falls as trade costs decline, but the decline is non-linear – the bias is roughly constant as trade costs fall from baseline, but rises steeply as trade costs rise from baseline. To frame these results differently, note that $\bar{\zeta}^{ij}$ rises monotonically as trade costs decline. The gravity estimate of the trade cost elasticity ($\hat{\zeta}_{OLS}$) only reveals the rise in $\bar{\zeta}^{ij}$ as trade costs fall from baseline. As trade costs rise form baseline, the heterogeneous coefficients bias is sufficiently strong to lead $\hat{\zeta}_{OLS}$ to rise, even though $\bar{\zeta}^{ij}$ is falling over this interval.

Finally, we emphasize that these results have further interesting implications for interpreting the "distance puzzle" – the empirical fact that estimated distance elasticities have not declined over time, despite seemingly obvious declines in trade costs [Disdier and Head (2008)]. In our model,

[30] Pushing back one level, recall that we imposed $\zeta^{ij} = \zeta^{ji}$ to measure bilateral elasticities. Deviations from this assumption are implicitly included in ν^{ij}.

[31] This result is related to Yi (2010), who argues that multistage production explains the puzzlingly large impact of the national border – "the border effect" – on US-Canada trade. Nonetheless, the results are not directly comparable. The exercise in Yi (2010) computes the border effect using simulated data from models with and without multistage production, and it compares them to estimates from the true data. Our discussion of heterogeneous coefficient bias here uses only data from the simulated multistage model. Thus, we are quantifying pure econometric bias, rather than comparing multistage to non-multistage models.

log trade costs are: $\ln \tau^{ij} = \ln \tau^{j} + \rho(m) \ln d^{ij}$. Thus, the reduced form gravity distance elasticity equals $\zeta \rho(m)$, where we treat $\rho(m)$ as a fundamental constant from the model. In our model, regression estimates of the distance elasticity thus inherit the behavior of $\hat{\zeta}_{OLS}$. Further, the non-monotone behavior of $\hat{\zeta}_{OLS}$ means that one ought to interpret historical and prospective declines in trade costs somewhat differently. Our results suggest that the distance puzzle in historical data is indeed a puzzle: our model predicts that historical declines in trade costs (from previously higher levels to their current estimated level) would tend to drive the distance elasticity toward zero, which is not what one sees in data. On the flip side, as trade costs continue to decline from their current levels, our model suggests that the distance elasticity ought to start rising again, reflecting the fundamental role of fragmentation in leading to geographic concentration in trade. Thus, for prospective (rather than historical) declines in trade costs, the model generates a "distance puzzle" *because* the level of trade costs have fallen. Broadly, all these results suggestion that significant caution is warranted in interpreting gravity regression estimates as evidence for/against changes in the elasticity of trade costs to frictions.

3.2 World Trade Elasticity

We now turn to the elasticity of trade at the global level. To focus our discussion, we study how the ratio manufacturing trade to manufacturing GDP changes with the level of trade costs. This intentionally mimics the influential analysis of the role of multistage production in explaining the rise of world trade by Yi (2003). In a nutshell, Yi argued that multistage models magnify the response of aggregate trade to declines in trade costs, due to the fact that declines in trade costs trigger fragmentation of production stages across countries. Thus, he argues the multistage model plays an important role in explaining the large, non-linear rise in trade relative to GDP as trade costs gradually declined in the second half of the twentieth century. This point (in our view) has become "conventional wisdom" in the trade literature, so we revisit it here.

To assess the role of multistage production, we need to compare the response of trade in our model to benchmarks that shut down the multistage mechanism. We consider two reasonable benchmarks. The first benchmark is a restricted version of our own model, in which we shut down the ability of agents to reallocate stages across countries (from the estimated baseline equilibrium) as trade costs change. Put differently, it is our model with stage locations locked in place where they are initially located in the baseline equilibrium.

The second benchmark we use is an alternative model: the multi-sector Eaton-Kortum model with an input-output structure, as articulated by Caliendo and Parro (2015) (henceforth, we refer to this as the "EK-CP model"). We parameterize the initial equilibrium in this model so that it exactly matches simulated data on bilateral trade shares, income, sector-level production and expenditure, and sector-level input cost shares from our multistage model. We set the Fréchet technology shape parameters in the manufacturing to 4.26, based on the OLS gravity regression in simulated data from the multistage model.[32] Following Dekle et al. (2008), we compute exact changes in the model's endogenous variables for different levels of trade costs. Additional specification, calibration, and solution details are discussed in Appendix B.

In Figure 10, we plot the ratio of manufacturing exports to manufacturing GDP against the level of trade costs in counterfactual equilibria (as in previous sections) for the three models. Like Yi (2003), our multistage model yields a non-linear increase in the ratio in response to linear declines in trade cost – i.e., the curve is convex, such that marginal changes in trade costs have larger impacts on the trade to GDP ratio as the level of trade costs falls. However, the two alternative benchmark models also yield the same non-linearity in trade. Reorganization of GVC production staging does magnify the response of trade somewhat, as evidenced by the gap between the fixed stage location and full model simulations. This gap is relatively small, though. Furthermore, the EK-CP model actually yields larger increases in trade relative to GDP than does the multistage model as trade costs decline, though again the gap is not particularly large. Thus, we conclude that multistage production does not substantially magnify the response of trade to declines in trade costs.

This result prima facie contradicts the major quantitative takeaway from Yi (2003), and thus begs for explanation. Our explanation of this apparent contradiction boils down the observation that "the benchmark model matters." In particular, we compare the response of trade in our multistage model to two models that include input trade, with different microeconomic forces governing input use and trade. In contrast, Yi (2003) compares a multistage model to a single-stage model *without input trade.* This distinction is important. In a model without input trade, there is no scope for double counting in gross trade data, because goods (at most) cross borders once en route from source to destination. In contrast, models with input trade – either the multistage model or the EK-CP model – allow for goods to cross borders multiple times as they move through the production process, which inflates the value of gross trade relative to GDP in the model.

One way to read the results in Yi (2003) is that they point out that the double counting generated by input trade is important in explaining trade growth. That is, the multistage model in Yi (2003) generates larger increases in trade relative to GDP than the single-stage model (without input trade) precisely because it generates double counting in trade. While we agree on this point, it does not follow that multistage production itself – as opposed to other ways of modeling input trade (e.g., roundabout production) – is essential for explaining increases in trade relative to GDP. In our analysis, we compare our multistage model to two models that both allow for double counting, and thus they all generate inflated values for gross trade relative to GDP as trade costs fall. The

[32] The Fréchet shape parameter is 4.12 for the non-manufacturing sector, identical to our calibration in Section 2.

punchline then is that input trade, not multistage production per se, magnifies the growth of trade in response to declining trade costs.

4 Conclusion

Despite substantial academic and policy interest in the rise of global value chains, few quantitative models incorporate multistage value chains. In contrast, this paper puts the decision to collocate or fragment production stages at center stage. This allows us to quantify the role of technology and trade costs in driving fragmentation and the role of value chain structure in shaping trade elasticities.

We found that there are sizable differences in upstream versus downstream comparative advantage across countries, and that these are an important driver of the final versus input composition of exports. We also found that the response of value chain structure to frictions plays an important role in determining the endogenous bilateral elasticity of input trade. As trade costs decline, the input trade elasticity rises, leading the bilateral trade elasticity to increase as well. Nonetheless, the multistage model generates aggregate world trade elasticities that are quantitatively similar to more standard Ricardian models with input trade.

Digging beneath these aggregate results, global value chains play an interesting conceptual role in explaining how trade costs map into trade flows. Changes in trade frictions induce value chain re-organization, leading to the growth of more complex export platform sourcing strategies. This points to a broader message: multistage models generate a variety of predictions regarding the micro-structure of value chains that have yet to be explored fully. We expect that combining the type of model we have written down here with micro-data on value chain structure would be a fruitful path for future empirical work.

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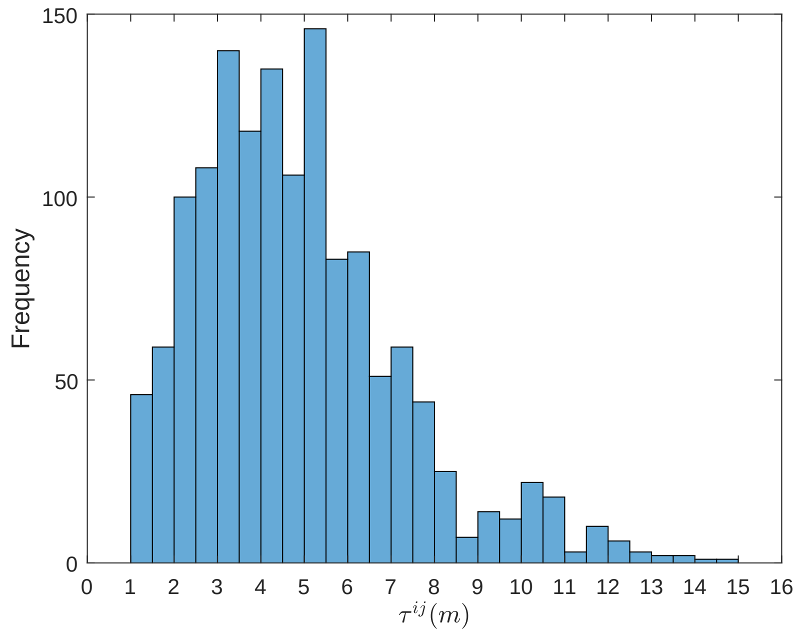
Table 1: Estimated Manufacturing Technology

	Abbreviation	Stage 1	Stage 2	$\frac{Stage2}{Stage1}$
Rest of World	RoW	2.61	1.94	0.74
Chile	CHL	1.28	1.13	0.88
South Africa	ZAF	1.55	1.49	0.96
Argentina	ARG	1.00	1.00	1.00
United States	USA	3.49	3.66	1.05
Australia	AUS	2.22	2.40	1.08
Belgium	BEL	2.24	2.50	1.12
Canada	CAN	2.01	2.28	1.14
Japan	JPN	2.77	3.46	1.25
Romania	ROM	0.65	0.93	1.42
France	FRA	2.25	3.23	1.43
China	CHN	1.20	1.74	1.45
Germany	DEU	2.19	3.34	1.53
Brazil	BRA	1.24	1.92	1.55
Netherlands	NLD	1.55	2.43	1.57
Switzerland	CHE	1.78	2.81	1.58
Spain	ESP	1.82	2.91	1.60
Israel	ISR	1.19	1.94	1.62
Sweden	SWE	1.75	3.02	1.72
South Korea	KOR	1.69	2.93	1.73
Finland	FIN	1.57	2.76	1.76
Norway	NOR	1.48	2.66	1.80
Italy	ITA	2.00	3.62	1.81
Denmark	DNK	1.41	2.70	1.91
Hungary	HUN	0.94	1.80	1.91
Poland	POL	0.95	1.85	1.95
Austria	AUT	1.44	2.89	2.01
United Kingdom	GBR	1.33	2.69	2.03
Mexico	MEX	1.07	2.30	2.14
New Zealand	NZL	1.18	2.53	2.14
Indonesia	IDN	0.63	1.35	2.14
Ireland	IRL	1.46	3.19	2.18
Portugal	PRT	1.06	2.35	2.23
Thailand	THA	0.73	1.66	2.26
Turkey	TUR	1.01	2.63	2.61
India	IND	0.53	1.50	2.81
Greece	GRC	0.72	2.28	3.14
Vietnam	VNM	0.23	1.06	4.56

Note: Columns labeled Stage 1 and Stage 2 report the geometric mean of the Fréchet distribution for each manufacturing stage, relative to the geometric mean of the Argentina's distribution. Relative productivity (Stage 2/Stage 1) measures comparative advantage across stages relative to Argentina's comparative advantage. Countries are ordered by relative stage 2 productivity (last column).

Figure 1: Distribution of Estimated Bilateral Iceberg Trade Costs for Manufacturing

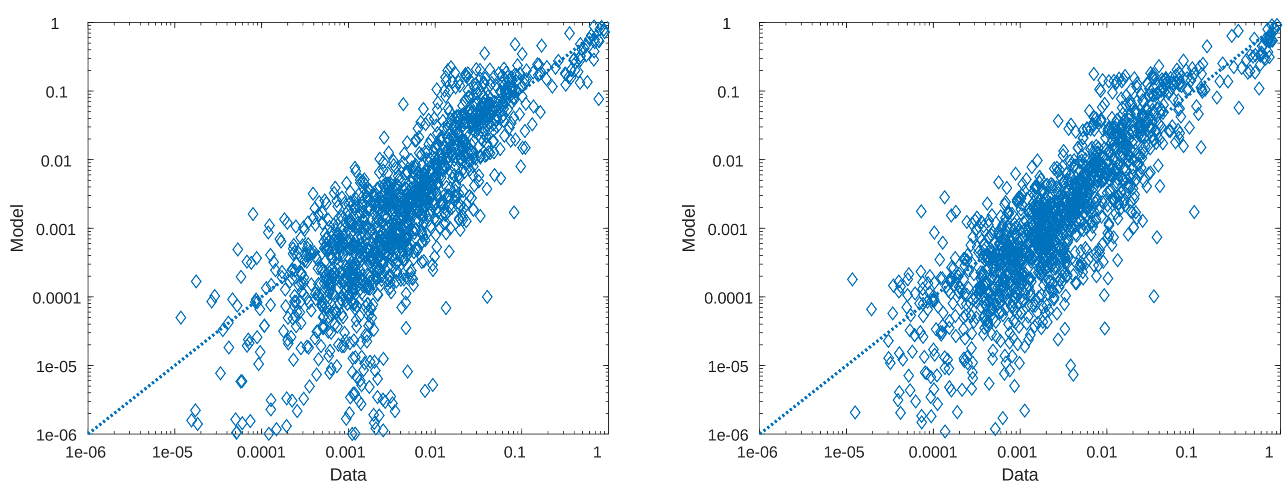


Note: Histogram displays the distribution of estimated values for bilateral manufacturing trade costs: $\tau^{ij}(m) = \tau^{j}\left(d^{ij}\right)^{\rho(m)}$, where d^{ij} is bilateral distance and τ^{j} is an importer-specific level. The point estimate for $\rho(m)$ is 0.25. The median value of $\tau^{ij}(m)$ is 4.49 and the mean is 4.85.

Figure 2: Manufacturing Trade Shares in Model and Data

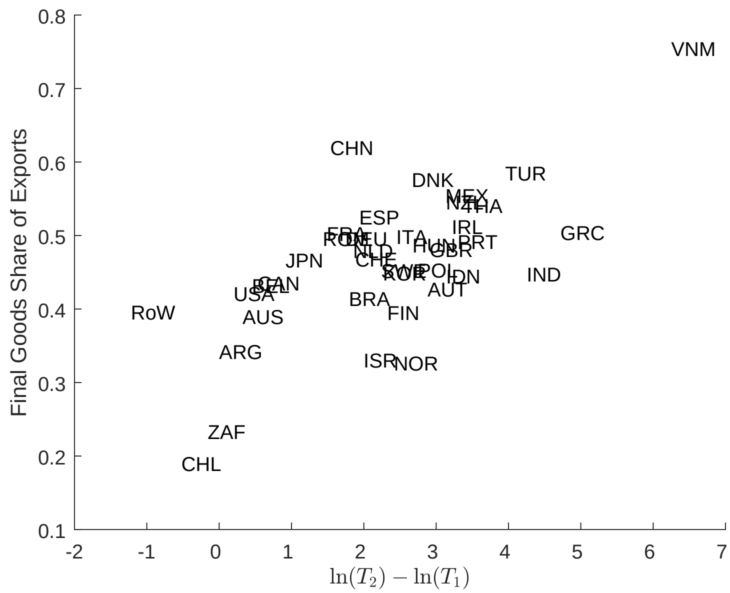
(a) Final Goods

(b) Inputs



Note: Fitted trade shares from the estimated model are on the y-axis, and target shares from data are on the x-axis. Axes are log scale, because the objective function minimizes log differences in trade shares in model versus data. Dashed line represents 45-degree line,

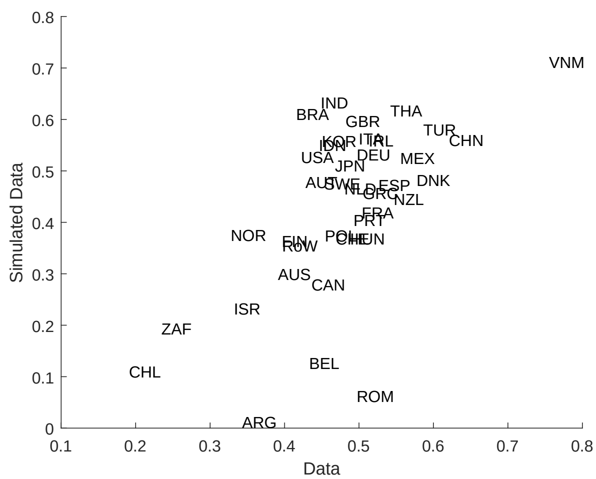
Figure 3: Comparative Advantage across Stages and Export Composition



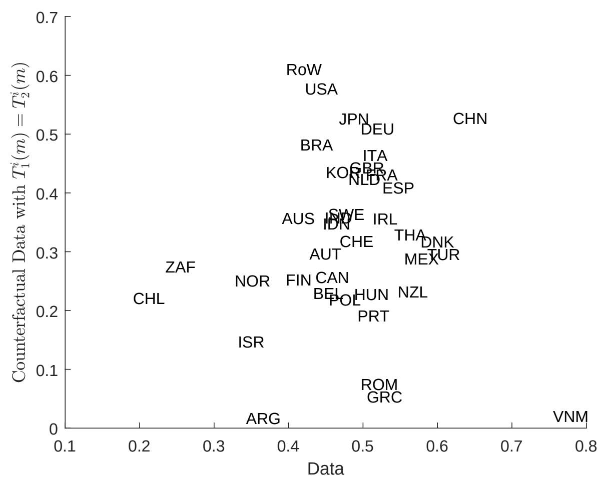
Note: The y-axis reports the share of final goods in manufacturing exports in the data. The x-axis reports the log difference between estimated stage 2 and stage 1 Fréchet location parameters for manufacturing in the multistage model: $\ln T_2^i(m) - \ln T_1^i(m)$.

Figure 4: Quantifying the Role of Stage Comparative Advantage in Export Composition

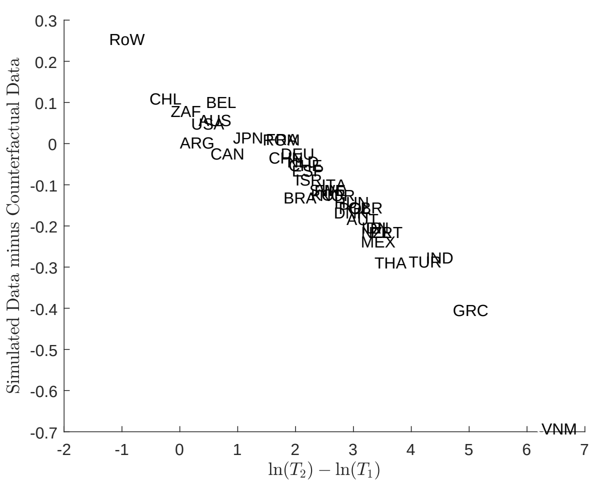
(a) Export Composition in Data and Model



(b) Counterfactual Export Composition without Stage Comparative Advantage

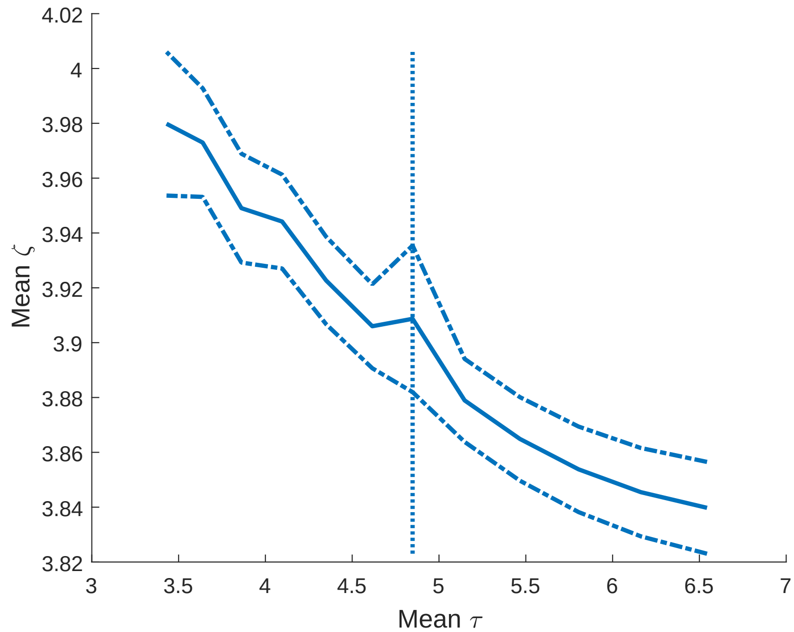


(c) Change in Export Composition due to Elimination of Stage Comparative Advantage



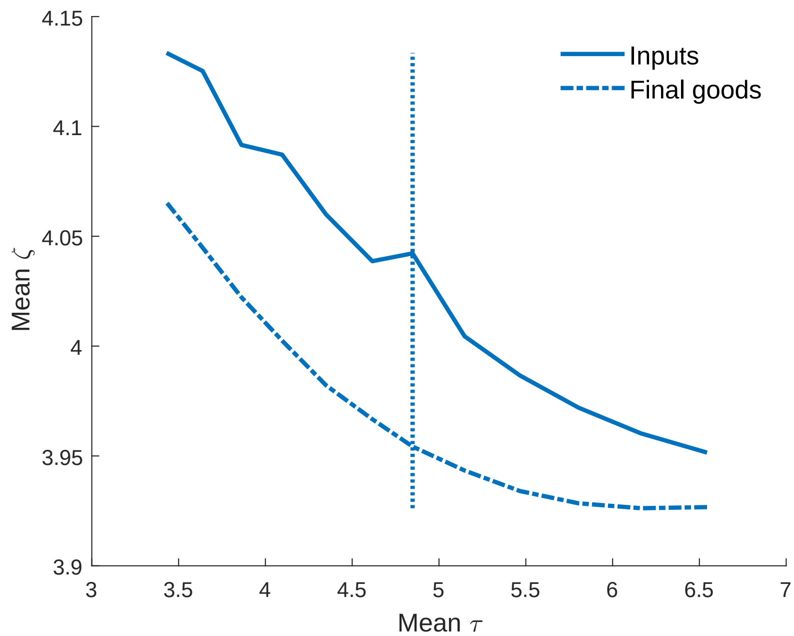
Note: In Panel (b), we plot simulated data from the model with counterfactual productivity in manufacturing equalized across stages, set to $\tilde{T}_s^i(m) = \left(T_1^i(m)\right)^{1/2} \left(T_2^i(m)\right)^{1/2}$ for $s \in 1,2$. Panel (c) plots differences between the share of final goods in exports for this counterfactual simulation and the baseline simulated data against technological comparative advantage in stage 2 production ($\ln T_2^i(m) - \ln T_1^i(m)$).

Figure 5: Mean Gravity Trade Elasticity for Multistage Manufacturing



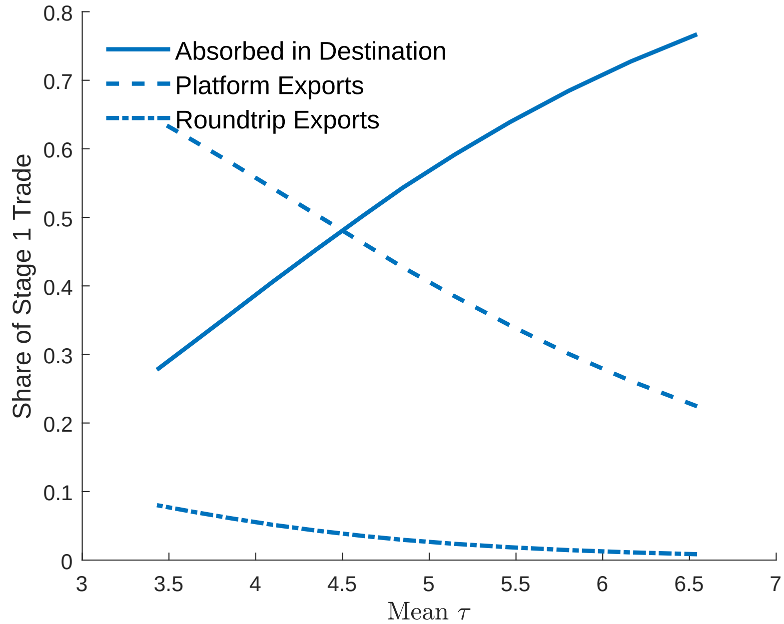
Note: Figure reports the simple mean (with 95% confidence interval) of the bilateral gravity elasticities ζ^{ij}, defined in Equation (23), for equilibria of the multistage model with different levels of trade costs. The mean level of trade costs in each equilibrium is on the x-axis. Vertical dashed line indicates baseline equilibrium with estimated trade costs.

Figure 6: Mean Gravity Trade Elasticity for Final Goods vs. Inputs



Note: Figure reports the simple mean of the bilateral gravity elasticities ζ^{ij} computed for final goods and inputs separately. The mean level of trade costs in each equilibrium is on the x-axis. Vertical dashed line indicates baseline equilibrium with estimated trade costs.

Figure 7: Decomposition of Stage 1 Exports by Destination Use

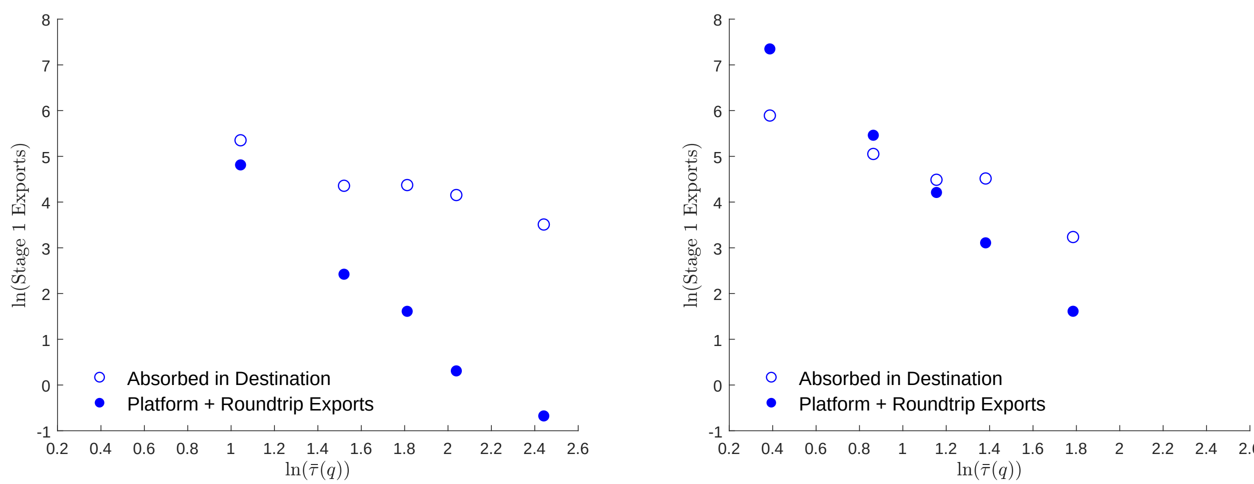


Note: "Absorbed in destination" is the share of bilateral stage 1 exports that are embodied in stage 2 goods absorbed in the destination. "Platform exports" is the share of bilateral stage 1 exports that are embodied in stage 2 goods that are exported to third destinations. "Roundtrip exports" is the share of bilateral stage 1 exports that are embodied in stage 2 goods that are exported back to the stage 1 source. The mean level of trade costs in each equilibrium is on the x-axis.

Figure 8: Decomposition of Stage 1 Exports by Destination Use and Bilateral Trade Costs

(a) High Trade Costs

(b) Low Trade Costs

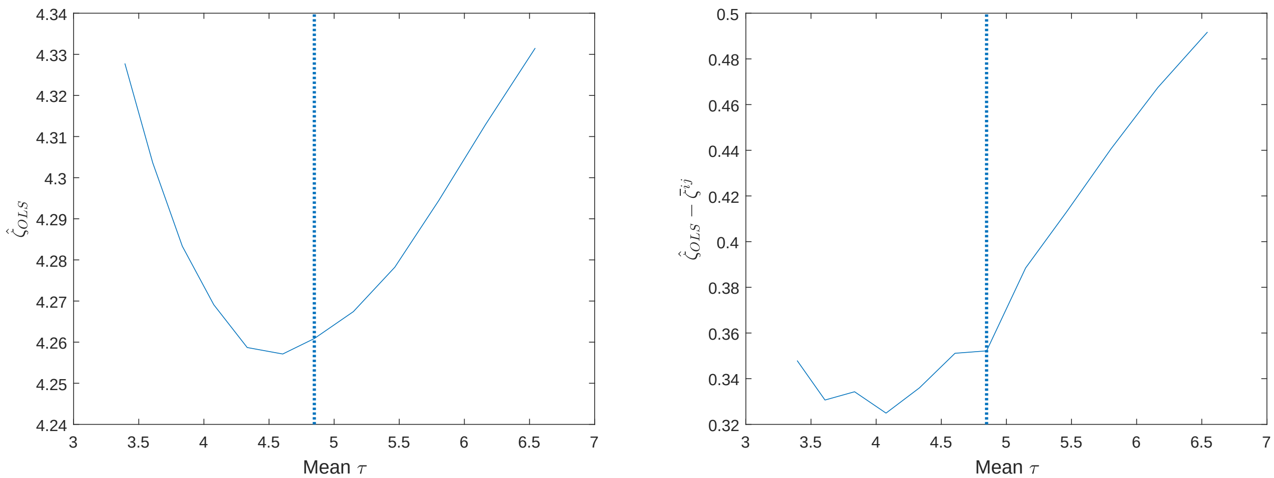


Note: Each point in the figure represents the mean trade cost and log value of stage 1 exports broken down by destination use for country pairs in a given quintile of the distribution of bilateral trade costs. Formally, for quintile q the data on the y-axis are: (i) Absorbed in Destination $= \ln(\sum_{ij\in q} EX_1^{ij}(m,j))$, and (ii) Platform + Roundtrip Exports $= \ln(\sum_{ij\in q} EX_1^{ij}(m,k\neq j))$. On the x-axis, we record the log of mean trade costs within each quintile, given by $\bar{\tau}(q) = \frac{1}{Q}\sum_{ij\in q}\tau^{ij}$, where Q here represents the number of observations in quintile q (1/5 the sample size). The left figure is for data from an equilibrium with "high trade costs" (mean τ near 6.5) and the right reports results from an equilibrium with "low trade costs" (mean tau near 3.5).

Figure 9: Bias in Gravity Regression Estimate of Mean Trade Cost Elasticity

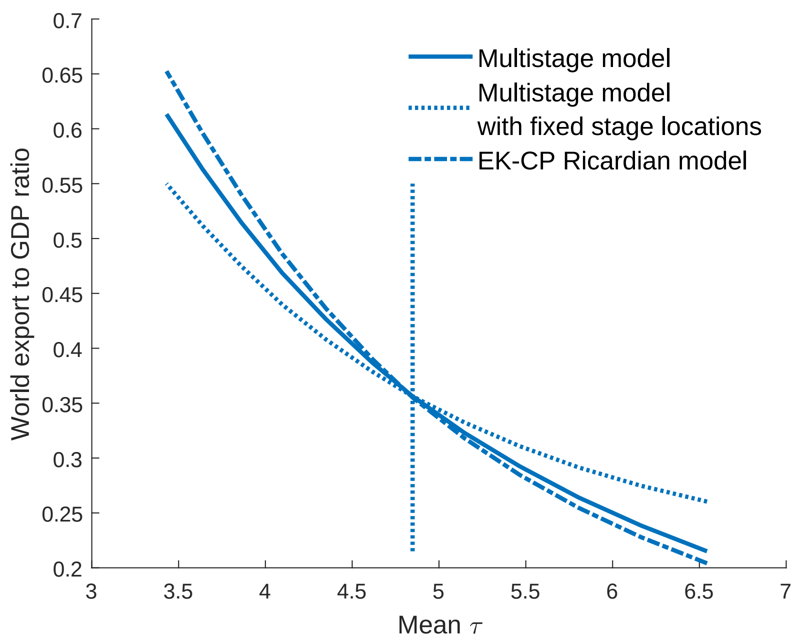
(a) Gravity Regression Estimate of Trade Cost Elasticity

(b) Difference between Gravity Regression Estimate of Trade Cost Elasticity and Mean Bilateral Elasticity



Note: Panel (a) plots estimates of $\hat{\zeta}_{OLS}$ from Equation 25 in simulated data. Panel (b) plots the difference between $\hat{\zeta}_{OLS}$ and the mean bilateral elasticity ($\bar{\zeta}^{ij}$) in simulated data. The mean level of trade costs in each equilibrium is on the x-axis. Vertical dashed line indicates baseline equilibrium with estimated trade costs.

Figure 10: Ratio of Manufacturing Trade to GDP for the World



Note: World export to GDP ratio is the ratio of world manufacturing trade to world manufacturing GDP. The ratio is reported for the multistage model, the multistage model with fixed stage locations (no re-optimization of stage locations from baseline equilibrium), and a Ricardian model with input-output linkages (EK-CP model). The mean level of trade costs in each equilibrium is on the x-axis. Vertical dashed line indicates baseline equilibrium with estimated trade costs.

A Solving the Model

In this appendix, we provide details concerning the algorithms we use to solve the model.

Picking up on the discussion in Section 1.4, we solve a discrete approximation to the model with a continuum of manufacturing goods, using a smoothing technique borrowed from the discrete choice literature in the process. We proceed here assuming that we know the model parameters $\{\alpha_i, \theta(s), \beta, T^i(n), T^i_s(m), \kappa, \tau^{ij}(m), \tau^{ij}(n)\}$, have realized productivity draws $\{T^i_s(r,m)\}$ for $r = 1,\ldots,R$ in hand, and data on exogenous factor endowments and trade balances $\{L^i, TB^i\}$.

For a given an initial value for the vector of wages $\{w^i\}$, we solve for the optimal assignment of stages to countries and hence prices of manufactured goods $\{\tilde{p}^k_2(r,m)\}$. One complication in doing so is that the composite input price P^i_X is a function of these prices, and simultaneously the cost of producing manufactured goods depends on P^i_X itself. In addition, P^i_X is also a function $P^i(n)$, which depends on production costs in the non-manufacturing sector and P^i_X due to the input loop. In the end, this problem has a fixed point structure.

Starting with a guess for the vector of composite input prices $\{\dot{P}^i_X\}$, we calculate the the optimized stage 1 input price that would prevail in each country j if it were to produce stage 2 output:

$$\tilde{p}^j_1(r,m) = \min_i \tau^{ij}(m) p^i_1(r,m), \quad \text{with} \quad p^i_1(r,m) = \frac{\left(w^i\right)^{1-\theta(m)} \left(\dot{P}^i_X\right)^{\theta(m)}}{T^i_1(r,m)}.$$

Then we compute the optimized price at which country k purchases stage 2 goods, given decisions optimized decisions at stage 1:

$$\tilde{p}^k_2(r,m) = \min_j \tau^{jk}(m) p^j_2(r,m), \quad \text{with} \quad p^j_2(r,m) = \frac{\left(w^j\right)^{1-\theta(m)} \left(\tilde{p}^j_1(r,m)\right)^{\theta(m)}}{T^j_2(r,m)}.$$

This yields $\{\tilde{p}^k_2(r,m)\}$, assuming that $\dot{P}^i_X$ is the composite input price, and tracing backwards the optimal location of stage 2 and stage 1 production for serving each destination k.

With these prices, we construct $P^i(m) = \exp\left(\frac{1}{R}\sum_r \log(\tilde{p}^k_2(r,m))\right)$, and use Equation (16) to compute $P^i(n)$. With these, we construct an updated value for the composite input price: $\ddot{P}^i_X = P^i(m)^\beta P^i(n)^{1-\beta}$. We iterate on these steps until $\ddot{P}^i_X = \dot{P}^i_X$.

Having converged on a value for P^i_X, we can easily compute the solution for all equilibrium prices, as well as the allocation of stages to countries for manufactured goods. The next step is to compute equilibrium quantities. Total demand for the sector-level composite goods is given by:

$$P^k(m)Q^k(m) = \alpha_k P_F^k F^k + \beta P_X^k X^k$$
$$P^k(n)Q^k(n) = (1-\alpha_k)P_F^k F^k + (1-\beta)P_X^k X^k.$$

Since we have taken the wage as given and treat the trade balance as an exogenous parameter, we can compute final demand as $P_F^k F^k = w^k L^k - TB^i$. We cannot immediately compute expenditure on the composite input, because we do not yet know X^k. However, we can solve for it as follows. Given a guess for $\dot{X}^k$, we can compute $P^k(m)Q^k(m)$ and$P^k(n)Q^k(n)$. Demand for stage 2 goods in manufacturing in destination k is then:

$$\tilde{q}^k(r,m) = \frac{\frac{1}{R}P^k(m)Q^k(m)}{\tilde{p}_2^k(r,m)}.$$

Tracing these demands back to the countries that supply those goods, the quantity of stage 2 goods produced in each source j is:

$$q_2^j(r,m) = \sum_k \tau^{jk}(m)\tilde{q}^k(r,m)\left(\frac{e^{-p_2^{jk}(r,m)/\lambda}}{\sum_k e^{-p_2^{jk}(r,m)/\lambda}}\right).$$

Given this stage 2 production in country j, demand for stage 1 inputs in country j is:

$$x_1^j(r,m) = \frac{\theta(s)p_2^j(r,m)q_2^j(r,m)}{\tilde{p}_1^j(r,m)}.$$

These input demands allow us to then solve for the quantity of each stage 1 good supplied by country i as:

$$q_1^i(r,m) = \sum_j \tau^{ij}(m)x_1^j(r,m)\left(\frac{e^{-p_1^{ij}(r,m)/\lambda}}{\sum_k e^{-p_1^{kj}(r,m)/\lambda}}\right).$$

Finally, we can compute an updated value for demand for the composite input:

$$\ddot{X}^i = \frac{1}{P_X^i}\left[\sum_r \theta(m)p_1^i(r,m)q_1^i(r,m) + \theta(n)\sum_j \pi^{ij}(n)P^j(n)Q^j(n)\right],$$

where $\pi^{ij}(n)$ is computed as in Equation (15). We iterate on this fixed point problem until $\ddot{X}^i = \dot{X}^i$. Upon convergence, we have the entire equilibrium for a given wage vector.

Lastly, we need to check whether the wage vector clears the labor market. We can calculate labor demand as:

$$
\begin{aligned}
L_D^i(\mathbf{w}) &= \int_0^1 l^i(z,n)dz + \sum_r \left[l_1^i(r,m) + l_2^i(r,m)\right] \\
&= \frac{1}{w^i}(1-\theta(n))\sum_j \pi^{ij}(n)P^j(n)Q^j(n) \\
&\quad + \frac{1}{w^i}(1-\theta(m))\sum_r \left(p_1^i(r,m)\,q_1^i(r,m) + p_2^i(r,m)\,q_2^i(r,m)\right)
\end{aligned}
$$

The equilibrium wage vector then sets labor demand equal to labor supply: $L_D^i(\mathbf{w}) = L^i$ for $i = 2,..,N$ (where market 1 is dropped appealing to Walras' law).

B Ricardian Trade Model with Input-Output Linkages

This appendix describes the benchmark two-sector Ricardian model against which we evaluate the multistage model. We present the key equilibrium conditions here and refer the reader to Caliendo and Parro (2015) for details regarding this class of models.

We define $E^i(m)$ and $E^i(n)$ to be total spending on final goods plus intermediates goods from the manufacturing and non-manufacturing sectors. Otherwise, the notation used here matches that use in the main text, with slight (obvious) modifications in the meaning of variables as necessary. For example, $c^i(s)$ denotes unit costs and $P^i(s)$ denotes an aggregate price level of an aggregate of sector s goods, but the functional forms are different here than in the main text reflecting differences between this model and the multistage model. The equilibrium of the model can be written as:

$$c^i(m) = (w^i)^{1-\theta(m)} \left[P^i(m)^{\gamma^i(m)} P^i(n)^{1-\gamma^i(m)}\right]^{\theta(m)} \tag{26}$$

$$c^i(n) = (w^i)^{1-\theta(n)} \left[P^i(m)^{\gamma(n)} P^i(n)^{1-\gamma(n)}\right]^{\theta(n)} \tag{27}$$

$$P^i(m) = \left[\sum_j T^j(m) \left(c^j(m)\tau^{ji}(m)\right)^{-\bar{\kappa}(m)}\right]^{-1/\bar{\kappa}(m)} \tag{28}$$

$$P^i(n) = \left[\sum_j T^j(n) \left(c^j(n)\tau^{ji}(n)\right)^{-\bar{\kappa}(n)}\right]^{-1/\bar{\kappa}(n)} \tag{29}$$

$$\pi^{ij}(m) = T^i(m) \left[\frac{c^i(m)\tau^{ij}(m)}{P^j(m)}\right]^{-\bar{\kappa}(m)} \tag{30}$$

$$\pi^{ij}(n) = T^i(n) \left[\frac{c^i(n)\tau^{ij}(n)}{P^j(n)}\right]^{-\bar{\kappa}(n)} \tag{31}$$

$$E^i(m) = \sum_s \gamma^i(s)\theta(s) \left[E^i(s) + TB^i(s)\right] + \alpha_i P_F^i F^i \tag{32}$$

$$E^i(n) = \sum_s (1-\gamma^i(s))\theta(s) \left[E^i(s) + TB^i(s)\right] + (1-\alpha_i) P_F^i F^i \tag{33}$$

$$E^i(m) = \sum_j \pi^{ij}(m) E^j(m) \tag{34}$$

$$E^i(n) = \sum_j \pi^{ij}(n) E^j(n) \tag{35}$$

$$P_F^i F^i = w^i L^i \tag{36}$$

There are several new parameters here. The parameters $\{\gamma^i(m), \gamma(n)\}$ are Cobb-Douglas input shares, equal to the share of input expenditure that each sector dedicates to inputs from sector m. The parameter $\bar{\kappa}(s)$ is a sector-specific trade elasticity. The parameters $\theta(s)$ and α_i are defined as in the main text. Lastly, note that the equilibrium above features balanced trade, to be consistent with the balanced trade assumption imposed in simulations of the multistage model.

Following Dekle et al. (2008), the equilibrium system of equations can be re-written in terms of changes relative to an initial equilibrium. Defining $\hat{x} \equiv \frac{x'}{x}$, where x' is the value of a variable in the new equilibrium and x is the value in the initial equilibrium, the equilibrium in changes is:

$$\hat{c}^i(m) = (\hat{w}^i)^{1-\theta(m)} \left[\hat{P}^i(m)^{\gamma^i(m)}\hat{P}^i(n)^{1-\gamma^i(m)}\right]^{\theta(m)} \quad (37)$$

$$\hat{c}^i(n) = (\hat{w}^i)^{1-\theta(n)} \left[\hat{P}^i(m)^{\gamma(n)}\hat{P}^i(n)^{1-\gamma(n)}\right]^{\theta(n)} \quad (38)$$

$$\hat{P}^i(m) = \left[\sum_j \pi^{ji}(m)\hat{T}^j(m)\left(\hat{c}^j(m)\hat{\tau}^{ji}(m)\right)^{-\bar{\kappa}(m)}\right]^{-1/\bar{\kappa}(m)} \quad (39)$$

$$\hat{P}^i(n) = \left[\sum_j \pi^{ji}(n)\hat{T}^j(n)\left(\hat{c}^j(n)\hat{\tau}^{ji}(n)\right)^{-\bar{\kappa}(n)}\right]^{-1/\bar{\kappa}(n)} \quad (40)$$

$$\hat{\pi}^{ij}(m) = \hat{T}^i(m)\left[\frac{\hat{c}^i(m)\hat{\tau}^{ij}(m)}{\hat{P}^j(m)}\right]^{-\bar{\kappa}(m)} \quad (41)$$

$$\hat{\pi}^{ij}(n) = \hat{T}^i(n)\left[\frac{\hat{c}^i(n)\hat{\tau}^{ij}(n)}{\hat{P}^j(n)}\right]^{-\bar{\kappa}(n)} \quad (42)$$

$$E^i(m)\hat{E}^i(m) = \sum_s \gamma^i(s)\theta(s)\left[E^i(s)\hat{E}^i(s) + TB^i(s)\hat{TB}^i(s)\right] + \alpha_i P_F^i F^i \widehat{P_F^i F^i} \quad (43)$$

$$E^i(n)\hat{E}^i(n) = \sum_s (1-\gamma^i(s))\theta(s)\left[E^i(s)\hat{E}^i(s) + TB^i(s)\hat{TB}^i(s)\right] + (1-\alpha_i) P_F^i F^i \widehat{P_F^i F^i} \quad (44)$$

$$E^i(m)\hat{E}^i(m) = \sum_j \pi^{ij}(m)E^j(m)\hat{\pi}^{ij}(m)\hat{E}^j(m) \quad (45)$$

$$E^i(n)\hat{E}^i(n) = \sum_j \pi^{ij}(n)E^j(n)\hat{\pi}^{ij}(n)\hat{E}^j(n) \quad (46)$$

$$P_F^i F^i \widehat{P_F^i F^i} = w^i L^i \hat{w}^i \hat{L}^i. \quad (47)$$

In all simulations, we assume that labor input is fixed in all countries $\hat{L}^i = 1$, and changes in trade costs ($\hat{\tau}^{ij}(s)$) and technology ($\hat{T}^j(s)$) are exogenous forcing variables. This leaves $10+2N^2$ endogenous variables $\{\hat{w}^i, \widehat{P_F^i F^i}, \hat{c}^i(s), \hat{P}^i(s), \hat{E}^i(s), \pi^{ij}(s)\}$ and $10+2N^2$ equations, before choosing a normalization.[33]

To solve for these endogenous variables, we need parameters $\{\alpha, \bar{\kappa}(s), \gamma^i(m), \gamma(n), \theta(s)\}$ and values for $\{w^i L^i, P_F^i F^i, TB^i, E(s), \pi^{ij}(s)\}$ in an initial equilibrium. We set these parameters based on simulated data generated by the multistage model – i.e., we treat equilibrium values from our estimated model as data, which implies that we start simulations from an "observationally equivalent" equilibrium in the multistage model and Ricardian models. We need $\{P_F^i F^i, E^i(s), \pi^{ij}(s)\}$ in the balanced trade equilibrium of the multistage model, plus values for the structural parameters $\{\alpha, \bar{\kappa}(s), \gamma^i(m), \gamma(n), \theta(s)\}$, to compute changes in equilibrium variables. We now describe the

[33]Note that we treat nominal final expenditure $P_F^i F^i$ as one variable, hence the wide-hat notation on $\widehat{P_F^i F^i}$. We do not need to separate the final price level and real final expenditure to compute the counterfactuals that interest us.

details regarding how we obtain values for these parameters.

Reading values for $\{P_F^i F^i, E^i(s), \pi^{ij}(s)\}$ from our simulated data is completely straightforward. Further, $\{\theta(s), \alpha_i\}$ are set to the same values as in the multistage model. The parameter $\gamma^i(m)$ for sector m (the manufacturing sector) is equal to the value of inputs from sector m used by sector m as a share of total input use by sector m. Denoting the value of stage s output produced by country i in sector m as $y_s^i(m)$, then total input use by sector m is equal to use of stage 1 inputs by stage 2, which are equal to $\theta(m)y_2^i(m)$, plus use of the composite input, which is equal to $\theta(m)y_1^i(m)$. Then all stage 1 inputs used by stage 2 in sector m originate from sector m, but only a fraction (β) of the composite input originates from sector m. This implies that:

$$\gamma^i(m) = \frac{\theta(m)y_2^i(m) + \beta\theta(m)y_1^i(m)}{\theta(m)y_2^i(m) + \theta(m)y_1^i(m)}$$

The value of this parameter varies across countries to the extent that the mix of stage 1 versus stage 2 output varies across countries. Turning to $\gamma(n)$, sector n uses sector m inputs only embodied in the composite input, and the composite input itself is the only input in production. Therefore, $\gamma(n) = \beta$ from the multistage model.

Finally, turning to trade elasticities, we obtain $\bar{\kappa}(m) = 4.26$ for the manufacturing sector by regressing simulated sector-level bilateral trade from the multistage model on log bilateral trade costs, controlling for importer and exporter fixed effects, as in Section 3.1.2. We then set $\bar{\kappa}(n) = 4.12$ for non-manufacturing, which is identical to the multistage model.