NBER WORKING PAPER SERIES

CORPORATE TAXES AND INCENTIVES AND THE STRUCTURE OF PRODUCTION: A SELECTED SURVEY

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Working Paper No. 2579

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
May 1988

The research reported here is part of the NBER's research program in Taxation. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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ABSTRACT

In this paper we develop a general intertemporal model of production, emphasizing the role of present and expected future corporate income taxes, credits and allowances along with costly adjustment and variable utilization of the quasi-fixed factors. Three specific issues are considered: 1) the direct and indirect effects of taxes operating through factor prices on the long-run input substitution, thus altering the structure of the production process; 2) the effects of tax policy changes on the rate and direction of technological change; and 3) the effects of tax policy on the intertemporal pattern of substitutions and complementarities among the inputs that arise due to presence of quasi-fixity of some inputs. The rates of utilization of the quasi-fixed factors are determined in the short-run in conjunction with the demands for the variable factors of production. Hence, utilization rates depend on product and factor prices and therefore on tax policy. We specialize the general model in order to highlight each of the three themes and their interaction with tax policy. We also discuss the various ways in which empirical implementation of the theoretical models and a brief summary of the empirical results in the literature is also provided. Lastly, we discuss some policy implications which emerge from the analysis and empirical results.

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1. Introduction*

Over the past few decades, tax policy has been an important instrument of the government to influence both fluctuations and growth in the economy. Initially, economists emphasized the link between taxes and demand as the channel through which tax policy affected the cyclical variability and secular trend of output and thereby employment and capital accumulation. Now, however, we have come to recognize that there are also significant direct influences of tax policy on production. Tax rates, credits and allowances affect the costs incurred by firms in hiring labour, investing in equipment and structure, using energy, and undertaking research and development. The purpose of this paper is to analyze the models used to evaluate the effects of tax policy on the structure of production and to discuss the substantive results arising from the empirical application of these models.

The predominant focus in studies of production has been related to the degree to which factors of production are substitutable and to the biases associated with technological change. Substitution possibilities characterize the manner in which firms alter their input demands in the face of changing relative factor prices. As a consequence, since tax policy operates through factor prices, the knowledge of the substitutability or complementarity of inputs permits us to determine how factor proportions change as tax policy initiatives are introduced.

Technological change generates new structures of production. In general, technological change is biased toward some factors of production and away from others, in the sense that the new production processes tilt relative input demands. However, technological change does not occur in a vacuum, but rather it is influenced by the same determinants as factor demands, namely product and factor prices. Hence, tax policy can affect technological change. For example, assume a credit is introduced on investment which lowers the effective factor price of capital. If technological change is of the variety which is biased towards capital, then the cost of undertaking technological change falls with the investment credit, causing the rate of technological progress to advance.

The ability of tax policy to influence factor demands and the rate of technological change may be severely hampered in the short-run because subtantial costs of adjustment may have to be incurred in order to change factor demands. The quasi-fixity of certain inputs (for example, skilled labour, equipment and structures, research and development) limits substitution possibilities and technological biases in the short-run. This implies that adjustment in the quasi-fixed factors occurs gradually and is not immediate.

In the short run, the effects of changes in tax policy on factor demands may be quite different from the effects occurring in the long-run. For example, a decrease in the corporate income tax rate can lower the effective cost of production, therefore causing output to expand. If there are some factors which are quasi-fixed, then increases in short-run output production occur more intensely, using the variable factors of production. However, as the adjustment costs are incurred (the lower tax rate could also help in this regard) and investment takes place, the quasi-fixed factors are substituted for the variable factors in the long-run. The existence of adjustment costs changes the manner in which taxes affect production and the effectiveness of the policy.

Adjustment costs fix the level of the quasi-fixed factors in the short-run and cause the gradual adjustment to their long-run magnitudes. However, the rate at which the quasi-fixed factors are utilized in the short-run may also vary. If utilization of the quasi-fixed factors is not costless (for example, due to overtime and shift wage premiums or greater depreciation costs), then firms may find it desirable to leave idle portions of the quasi–fixed factors in order to meet future production requirements. The rates of utilization of the quasi–fixed factors are determined in the short–run in conjunction with the demands for the variable factors of production. Hence, utilization rates depend on product and factor prices and therefore on tax policy. For example, a decrease in the corporate income tax rate which causes output to expand can generate increases in the demand for variable factors and increases in the rates of quasi–fixed factor utilization. Since the latter is now costly, resources will be redirected from investment and the future expansion of the quasi–fixed factors towards the greater utilization of the current stocks.

Corporate tax rates, credits and allowances influence the long–run substitution of all factors of production, the short–run utilization and the dynamic adjustment of the quasi–fixed factors. This survey is based upon these themes. We develop a general intertemporal model of production, emphasizing the role of present and expected future corporate income taxes, credits and allowances along with costly adjustment and utilization of the quasi–fixed factors. We then proceed to specialize the general model in order to highlight each of the three themes and their interaction with tax policy. We also discuss the various ways in which empirical implementation of the theoretical models has been undertaken, along with the relevant results from the empirical investigations. The empirical studies are restricted to those which include the array of corporate tax, credit and allowance rates, have emphasized the role of tax policy on production structure, and are explicitly based on an optimization model of firm production decisions. The latter criterion enables us to establish a clear link between the theoretical and empirical models, and to see the problems in empirical implementation. The survey is organized along the following lines. In section 2 we develop the general theoretical model. Section 3 focuses on the issues of tax policy, long–run factor substitution and the rate of technological change. In the fourth section we specifically discuss the issues of taxes and quasi–fixed factors adjustment costs, while in section 5 the topic centers on the short–run utilization of the quasi–fixed factors. Lastly, we discuss some policy implications which emerge from the analysis and empirical results.

2. A Model of Production, Investment and Taxation

There are two objectives of this section. First a model is developed in which the effect of taxes on production and investment can be analyzed within the general themes of factor substitution, adjustment and utilization, output expansion and technological change. The second objective is to provide a framework in which to organize and evaluate the empirical research of this topic.

We begin by characterizing production and investment decisions. We assume that a firm produces ℓ outputs using n non-capital inputs and m capital inputs. The technology is represented by

$$(1) \qquad T(y_t, v_t, K_t^N, K_t^0, I_t, A_t) = 0$$

where T is the transformation function, y_t is an ℓ dimensional vector of output quantities, v_t is a n-dimensional vector of non-capital input quantities, K_t^N is an m dimensional vector of 'new' capital (or beginning of period capital) input quantities, K_t^0 is a m dimensional vector of 'old' capital (or end of period capital) input quantities, I_t is a m dimensional vector of investment quantities and A_t represents an indicator of autonomous technological change.[1] (The subscript t represents the time period.) The transformation function is twice continuously differentiable, increasing in y_t, K_t^0, I_t and decreasing in v_t, K_t^N. Generally the transformation function is decreasing in A_t (in other words, technological progress). The transformation function is also concave in y_t, K_t^N, K_t^0, v_t and I_t.

The specification of the technology is flexible enough to include the costs associated with installation and utilization of capital. The costs associated with capital utilization are introduced in manner similar to the general approach developed by Hicks (1946), Malinvaud (1953), Bliss (1975), and Diewert (1980). Each time, the firm combines the beginning of period capital inputs (K_t^N) with the non-capital inputs to produce output and the capital inputs to be used for future production (K_t^0). Thus, the firm produces two kinds of output: one type for current sale (y_t) and one type for future production (K_t^0). Utilization is captured through the selection of capital for future production. The choice of the end of period capital reflects decisions on the using and repairing of the capital inputs which are available at the beginning of the period. The specific process of capital utilization is embedded or internal to the production process and is captured by the tranformation function.

Capital adjustment or installation is costly. This is reflected by the vector of investment flows (I_t) in the transformation function. The installation costs are internal to the production process since the specific process of capital installations is captured by the transformation function. Resources devoted to installing capital must be directed away from producing current output and repairing existing capital. The cost of installing additional capital is the opportunity cost of foregone current output and foregone capital repairs. The existence of installation costs for certain types of inputs implies that there is an adjustment process associated with these inputs, and so they are referred to as quasi-fixed factors. The other inputs are called variable factors.

In the present context, there are essentially two kinds of investment undertaken by the firm: one type arises through capital purchases and one type results from maintaining the existing capital stocks. Thus, there are two ways in which capital becomes available for future production: internal investment (repair) and external investment (purchase). This implies that the vector of capital inputs used in production accumulates by

$$(2) \qquad K_{t+1}^N = I_t + K_t^0.$$

Equation (2) generalizes the standard formulation of exogenous depreciation by evaporation. We can see this by noting that depreciation is $(K_t^N - K_t^0) = \delta_t K_t^N$ where δ_t defined as an m dimensional diagonal matrix of depreciation rates. Thus, equation (2) can be re-written as $K_{t+1}^N = I_t + (I_m - \delta_t)K_t^N$, where I_m is the m dimensional identity matrix. Clearly, if δ_t is time invariant and exogenous, then equation (2) becomes the usual formula of depreciation by evaporation.

The distinction between stock and flow decisions can be noted from equations (1) and (2). At any time t, the beginning of period capital stocks are predetermined. Thus there exists a given bundle of capital services (or quasi-fixed factors) embedded in each stock of capital available to the firm. The firm selects the flow of services from each of the given capital stocks or the rates of utilization to combine with the non-capital (or variable) inputs to produce output or to install additional capital stocks. The choice on the rates of utilization are captured through the decisions on the end of period capital stocks. These end of period stocks along with the newly installed capital represent the capital stocks available to the firm at the beginning of period t+1.[2]

The firm generates revenue, hires variable inputs, utilizes its capital stocks, invests and finances its operations such that the flow of funds is

$$(3)\quad p_t^T y_t - w_t^T v_t - q_t^T I_t + \Delta B_t + p_{st}\Delta N_{st} - r_{bt}B_t - T_{ct} - D_t = 0.$$

The vector of output prices is p_t, w_t is the vector of variable input prices, q_t is the vector of capital purchase prices, ΔB_t is the nominal value of new bond issues (not of retirements), p_{st} is the price of new shares, ΔN_{st} is the number of new shares, r_{bt} is the interest rate on the corporate bond, T_{ct} and D_t are corporate income taxes and dividends.[3] (The superscript T stands for vector transposition.) We assume that the firm is a price taker in all markets.

The flow of funds can be further decomposed by considering the nature of the corporate income taxes. These taxes are defined by a tax rate of $0 < u_{ct} < 1$, based on revenues net of variable input costs, interest payments, capital cost allowances, investment tax credits and allowances. Revenues net of variable input costs and interest payments are straightforward items. Next consider the capital cost allowances. In general, the firm is permitted depreciation deductions equal to $D_{i\tau}$ on one dollar of the original cost of the i^{th} capital of age τ. Since capital must be fully depreciated, it must be the case that $\Sigma_{\tau=0}^{\infty} D_{i\tau} = 1$, $i=1,\ldots,m$. The depreciation deductions at time t for a particular type of capital installed at different times is

$\Sigma_{\tau=0}^{\infty} q_{it-\tau} I_{it-\tau} D_{i\tau}$ for $i=1,\ldots,m$.

Governments generally offer incentives to undertake investment. These incentives are often in the form of tax credits such that at time t with a credit rate of $0 < u_{it} < 1$, $i=1,\ldots,m$, the investment tax credit is

$$(4) \qquad ITC_{it} = u_{it} q_{it} I_{it}, \quad i=1,\ldots,m.$$

Moreover, the investment tax credit can reduce the depreciation base for tax purposes of the capital stocks. This means that the depreciation deductions for tax purposes or the capital cost allowances are reduced by the investment tax credit. Hence the capital cost allowance at time t is

$$(5) \qquad CCA_{it} = \Sigma_{\tau=0}^{\infty} q_{it-\tau} I_{it-\tau} (1-\phi_{it} u_{it}) D_{i\tau}, \quad i=1,\ldots,m,$$

where ϕ_{it} is the proportion of the investment tax credit which reduces the depreciation base for tax purposes. In Canada, $\phi_{it} = 1$ and the U.S. $\phi_{it} = .5$.[4]

Besides the capital cost allowance and investment tax credit, a third type of investment incentive relates to additions to the rate of investment. For example, incentives of this nature have been introduced to stimulate R&D expenditures. In Canada, from 1978 to

1984, there was a tax allowance of 50 per cent on current R&D expenditures in excess of the average of the previous three years. In the U.S., a tax credit of 25 per cent exists since 1981, on current R&D expenditures in excess over the average expenditures undertaken during the previous three years. An allowance at time t based on incremental investment can be defined as

$$(6) \qquad IIA_{it} = \gamma_{it}\Sigma_{\tau=0}^{\infty}\mu_{it}q_{it-\tau}I_{it-\tau}, \quad i=1,...,m$$

where $\mu_{i0} = 1$, $\mu_{i\tau} = \mu_{is} < 0$, $\tau \neq s$, $\tau, s > 0$ and

$$\gamma_{it} = \begin{cases} a_{it} > 0 & \text{if } \Sigma_{\tau=0}^{\infty}\mu_{i\tau}q_{it-\tau}I_{it-\tau} > 0, \\ 0 & \text{otherwise.} \end{cases}$$

To see the magnitude of the incremental allowance, suppose that in order to obtain the allowance, current investment expenditure must exceed the average of the past three years. Thus, $\mu_{10} = 1$, $\mu_{1t} = -.33$, t=1,2,3. In addition, suppose current expenditures are \$1.00, while expenditures for the previous three years are \$.75, \$.50 and \$.25, respectively. Thus, the incremental expenditures upon which the allowance is based is \$1.00 – \$.50 = \$.50. If the allowance rate is .5, then the firm obtains an allowance of \$.25.

Combining equations (4), (5), and (6) yields corporate income taxes at time t to be

$$(7) \quad T_{ct} = u_{ct}[p_t^T y_t - w_t^T v_t - r_{bt}B_t - I_m(CCA_t + IIA_t)] - I_m ITC_t,$$

where I_m is the m dimensional identity matrix, CCA_t, IIA_t and ITC_t are m dimensional diagonal matrices of the capital cost allowances, incremental investment allowances, and the investment tax credits respectively.

Substituting equation (7) into the firm's flow of funds which is given by equation (3), we can write

(8) $F_t = [D_t/(p_{st}N_{st}) + \Delta p_{st}/p_{st}]p_{st}N_{st} + r_{bt}(1-u_{ct})B_t - \Delta(p_{st}N_{st}) - \Delta B_t,$

where $F_t = [p_t^T y_t - w_t^T v_t](1-u_{ct}) - q_t^T I_t + I_m[u_{ct}(CCA_t+IIA_t)+ITC_t],$

which is the flow of funds to the shareholders and bondholders.

Share market equilibrium requires that $r_{st} = D_t/p_{st}N_{st} + \Delta p_{st}/p_{st}$, where r_{st} is the rate of return on equity, and defining by $V_t = p_{st}N_{st} + B_t$ so that $\Delta V_t = \Delta(p_{st}N_{st}) + \Delta B_t$, then equation (8) can be rewritten as

(9) $F_t = [r_{st} + 1/(1+\theta_t) + r_{bt}(1-u_{ct})\theta_t/(1+\theta_t)]V_t - \Delta V_t,$

where $\theta_t = B_t/V_t$. The rate of return on financial capital can be defined as $p_t = r_{st}/(1+\theta_t) + r_{bt}(1-u_{ct})\theta_t/(1+\theta_t)$. Thus, equation (9) implies that the flow of funds to the shareholders and bondholders plus any capital gains equals the return on financial capital.

The objective of the firm is to operate in the interest of its shareholders by maximizing the expected present value of the flow of funds to the shareholders. In the present context, because the rates of return on bonds and shares are exogenous to the firm, and therefore cannot be influenced by shareholder behavior, the objective is equivalent to maximizing the expected present value of financial capital (or in other words, the expected present value of the flow of funds to shareholders and bondholders). The objective function which can be obtained from equation (9) by solving for the present value of financial capital and appplying expectations, can be written as

(10) $J_t = E_t\Sigma_{s=t}^{\infty}\alpha(t,s)[(p_s^T y_s - w_s^T v_s)(1-u_{cs}) - Q_s^T I_s + I_m M_s],$

where E_t is the expectation operator conditional on information known at time t, the discount rate is $\alpha(t,t) = 1$, $\alpha(t,t+1) = 1/(1+p_t)$, Q_s is an m dimensional vector of capital purchase prices net of taxes such that

$Q_{is} = q_{is}(1 - u_{is} - \Sigma_{\tau=0}^{\infty}\alpha(t,s+\tau)\alpha(t,s)^{-1}u_{cs+\tau}[(1-\phi_{is}u_{is})D_{i\tau} + \gamma_{is+\tau}\mu_\tau])$. M_s is an m dimensional diagonal matrix such that the diagonal in the i^{th} row is $u_{cs}\Sigma_{\tau=s}^{\infty}q_{is-\tau}I_{is-\tau}[(1-\phi_{is-\tau}v_{is-\tau})D_{i\tau} + \gamma_{is}\mu_\tau]$.[5] M_s represents the tax reduction due to the capital cost allowances and the incremental investment allowances arising from past investment expenditures. In deriving equation (10), we have made use of the fact that capital purchase prices are modified by the investment tax credit, the capital cost allowance (which may be reduced in part by the credit) and the incremental investment allowance. In addition, we have separated each type of investment expenditure into the portion at any time t which relates to the present and the portion which is a legacy of the past (given by the matrix M_s). Clearly, at any time t the latter does not figure into the firm's maximizing program because, from the vantage point of the present, it is predetermined.

The post–tax purchase prices contain the allowance on incremental investment. To see how the latter affects the post–tax purchase prices and reduces taxes, assume that the corporate income tax rate is fixed and equal to u_c, the allowance rate is fixed and equal to γ, and the discount rate is constant and equal to ρ. In addition, assume that the allowance is based on current investment expenditures in excess of the average of the past three years. Suppose there is one type of capital and a firm incurs an investment expenditure in year 1 of \$1 (q_1 = \$1). This expenditure will add \$1 to the incremental allowance in year 1. Thus, the tax reduction from the allowance is $u_c\gamma\$1$. In year 2, however, the \$1 expenditure will decrease taxes through the allowance by one–third of $u_c\gamma\$1$. Discounting the latter magnitude back to year 1 yields $u_c\gamma\$1(.33)/(1+\rho)$. In year 3 and 4, the discounted tax reductions from the allowance are $u_c\gamma\$1(.33)/(1+\rho)^2$ and $u_c\gamma\$1(.33)/(1+\rho)^3$ respectively. The \$1 expenditure increases the incremental allowance in the year the expenditure was increased and then reduces the allowance over the next three years. Thus the present value of the tax reduction due to the incremental allowance is

$\$1u_c\gamma(1-.33\Sigma_{t=1}^{3}1/(1+\rho)^t)$. If $u_c = .46$, $\gamma = .5$ and $\rho = .15$, then $\$1u_c\gamma.25 = .06$, which is the present value of the tax reduction from the investment allowance generated by the $1 expenditure.

The firm maximizes the right side of equation (10) by selecting the vectors of outputs, variable inputs, levels of investment and used (or end of period) capital stocks, subject to the technology (equation (1)) and the generation of new (or beginning of period) capital stocks (equation (2)). This program can be undertaken in two stages. First, conditional on the beginning of period capital stocks and the technology, the firm determines its output supplies, variable factor demands and end of period capital stocks. This is the set of short-run decisions. With this solution, the firm proceeds to the intertemporal problem in order to determine the beginning of period capital stocks.

The short-run problem is defined by

$$\text{(11)}\quad \max_{(y_t,v_y,K_t^0)} \quad (p_t^T y_t - w_t^T v_t)(1-u_{ct}) + Q_t^T K_t^0$$

$$\text{s.t.} \quad T(y_t,v_t,K_t^N,\ K_t^0,\ K_{t+1}^N-K_t^0,A_t) = 0.$$

The first order necessary conditions for any time period are (including the constraint in (11)):

$$\text{(12.1)}\quad p_t(1-u_{ct}) - \lambda vT_y = 0$$

$$\text{(12.2)}\quad -w_t(1-u_{ct}) - \lambda vT_v = 0$$

$$\text{(12.3)}\quad Q_t - \lambda v(T_0 - T_I) = 0,$$

where λ is the Lagrangian multiplier and vT_i represent the first order partial derivatives of outputs (i=y), variable inputs (i=v), end of period capital stocks (i=0) and investment levels (i=I). Equation sets (12.1) and (12.2) are standard. They imply that relative product prices equal the respective rates of product transformation and relative variable factor prices equal the respective rates of factor substitution. Equation (12.3) implies that relative net of tax capital stock purchase prices equal the respective relative marginal values of capital utilization (T_0) net of the marginal costs of capital installation (T_I).

It is clear from equation set (12.1) and (12.2) that tax policy influences output supplies and variable factor demands through its effect on the quasi-fixed factors. There are two reasons for this result. First, the corporate income rate does not effect output supplies and variable factor demands directly because it is based on revenues net of variable input costs or variable profits. The corporate income tax is a variable profits tax and as such it is based on a residual of the firm's income stream, given capital utilization, installation and accumulation. The second reason is that all allowances and credits are actually based on the quasi-fixed factors. As a consequence, output supplies and variable factor demands are affected by tax policy through their link with the intertemporal decisions governing the quasi-fixed factors.[5]

In this model there are three ways in which quasi-fixed factor decisions interact with output supplies and variable factor demands. First, there is the traditional route through factor substitution and output expansion. This is the link between y_t and v_t on the one hand and K_t^N on the other. Second, there is the interrelationship through capital installation which is the link between decisions on y_t and v_t and decisions on I_t. Third, there is the interaction between capital utilization, K_t^0 and output supply, y_t, and variable factor demand, v_t, decisions. To see the role of each of these interrelationships, let us assume for the moment that the costs of capital utilization and installation are separable from the production technology. In other words, $vT_{y0} = vT_{yI} = vT_{v0} = vT_{vI} = 0$. This means that changes in the corporate income tax, credit and allowance rates only affect output supplies and variable factor demands through changes in the beginnings of period quasi-fixed factors. The channel is as follows. A change in tax policy in period t elicits a change in capital utilization and installation in period t. This causes the quasi-fixed factors at the beginning of period t+1 to change, which in turn generates changes in period t+1 output supplies and variable factor demands. This channel may be termed the production channel. There is no direct link between capital utilization or installation and variable input demands and output supplies.

The other two channels arise from capital utilization and installation. If utilization and installation decisions are not separable from production decisions then from equation set (12) a change in tax policy generates contemporaneous effects on output supplies and variable factor demands. In addition, the effects on utilization and installation alter the quasi-fixed factors available for production in the succeeding period which in turn affects output supplies and factor demands in this later period.

The solution to the short-run program given by equation set (12) can be substituted into (11) to define the post tax variable profit function (see Diewert (1973)):

$$\pi_t^v = \Pi_t(P_t, W_t, Q_t, K_t^N, K_{t+1}^N, A_t) \quad (13.1)$$

where π_t is a twice continuously differentiable function which is increasing in $P_t = p_t(1-u_{ct})$, and Q_t, increasing in K_t^N and decreasing in $W_t = w_t(1-u_{ct})$ and K_{t+1}^N, convex and homogeneous of degree 1 in the prices P_t, Q_t and W_t, concave in K_t^N and K_{t+1}^N. The post-tax variable profit function is defined such that differentiating it with respect to the post tax prices (P_t,W_t, and Q_t) yields,

$$\nabla\Pi_P = y_t \quad (13.2)$$

$$\nabla\Pi_W = v_t \quad (13.3)$$

$$-\nabla\Pi_Q = K_t^Q. \quad (13.4)$$

This result, known as Hotelling's Lemma, implies that the short-run equilibrium can be better characterized by equation set (12) and the transformation function (defined by the constraint in (11)) or by equation set (13). The attractive feature of the latter approach is that reduced form output supply, variable factor demand, and quasi–fixed factor utilization functions are readily obtainable from the variable profit function.

The second stage of the firm's program involves the intertemporal determination of the beginning of period quasi–fixed factor demands. This can be obtained by substituting the post tax variable profit function into the expected present value of the firm's financial capital (which is the right side of equation (10)). Thus the firm desires to

$$(14)\quad \max_{(K^N_{s+1})} \quad E_t\Sigma^{\infty}_{s=t}\alpha(t,s)[\Pi_s(P_s,W_s,Q_s,K^N_s,K^N_{s+1},A_s) - Q^T_sK^N_{s+1}].$$

The first order necessary conditions for any time period are

$$(15)\quad E_t[\nabla\frac{\partial\Pi_t}{\partial K^N_{t+1}} - Q_t + \alpha(t,t+1)\nabla\frac{\partial\Pi_{t+1}}{\partial K^N_{t+1}}] = 0.$$

Equation set (15) implies that the present value of marginal variable profit of a quasi–fixed factor available for production must be balanced against the marginal cost of obtaining this input.[7] The marginal cost contains the post tax purchase price of additional capital and the decline in variable profits due to installing and maintaining the quasi–fixed factor for future production. This is the classic trade–off between higher future post–tax profits due to larger capital stocks versus lower present post tax–profits in order to obtain the larger capital stocks.

There are some interesting features contained in equation set (15). First, not only contemporaneous but all future tax, credit and allowance rates enter each equation through the post–tax purchase price of additional capital stocks. Second, embedded in the post–tax variable profit function is the manner in which the quasi–fixed factors interact with each other and with the variable input demands in determining output supplies. Third, utilization of the quasi–fixed factors is endogenous and governed by the post tax variable profit function. In other words, the specification of the post tax variable profit function implies a specification of quasi–fixed factor utilization.

The complete model consists of equation sets (13) and (15). We can see that empirical models which do not consider the potentially important influences of changes in present and future tax credit and allowance rates on output expansion, factor substitution, and quasi–fixed factor utilization and installation may be assuming away significant effects of tax policy on the structure of production.

3. Taxes, Factor Substitution and Productivity Growth

The theoretical model previously developed is complex in that it involves the analysis of corporate taxes and the structure of production in a dynamic context. The empirical literature on taxation and the structure of production has, in recent times, moved towards the implementation of a general model of production in order to address the issues related to the influence of taxes on factor substitution, adjustment, and utilization as well as output expansion and technological change. The purpose of this and the following sections of this paper is to analyse, within the context of the general theoretical model, the empirical work on the interaction between taxes and production decisions. We undertake this task by discussing the substantive empirical findings along with the nature of the models used to obtain these results.

The first issue we discuss pertains to the effects of taxes on factor substitution. We can address this issue by assuming that utilization and installation are costless and current prices and tax policy are always expected to persist. Thus the determination of production decisions can be simplified to the following two stage procedure. First, the problem defined by (11) is simplified to

$$(16) \quad \max_{(y_t, v_t)} \; (p^T y_t - w^T v_t)(1 - u_c)$$

$$\text{s.t.} \quad T(y_t, v_t, K_t^N, A_t) = 0.$$

This leads to equations similar to (12.1) and (12.2). In addition, a post-tax variable profit function can be defined in a similar fashion for equation (13.1) with the derived conditions similar to equations (13.2) and (13.3). In this simpler context, the variable profit and derived conditions (with respect to the post-tax prices) are

$$(17.1) \qquad \pi_t^v = \Pi(P,\ W,\ K_t^N,\ A_t)$$

(17.2) $\nabla\Pi_P = y_t$

(17.3) $-\nabla\Pi_W = v_t.$

Although the properties of the post–tax variable profit function are similar to those for the function given by the right side of (13.1), there are some differences. First, in the case defined by equation (17.1), post–tax variable profits are defined as revenue minus variable input costs. The value of the unutilized quasi–fixed factors does not have to be added to revenue, as in the general model, because utilization is costless and thereby exogenous. Moreover, this implies that the post tax purchase prices of the quasi–fixed factors do not enter the variable profit function. Second, because there are no installation costs, future quasi–fixed factors are not part of the domain of the variable profit function.

The second stage of the production problem is to

$$(18)\quad \max_{(K^N_{s+1})} \quad \Sigma^{\infty}_{s=t}(1+\rho)^{-s+t}(\Pi(P,W,K^N_s,A_s)-Q^T(K^N_{s+1}-(I_m-\delta)K^N_s).$$

The fact that quasi–fixed factor utilization is costless means that these factors are fully utilized and any depreciation can simply be defined to be exogenous and constant over time. The m dimensional diagonal matrix of constant depreciation rates is δ and capital accumulates by

$K^N_{s+1} = I_s + (I_m - \delta)K^N_s$. The first order necessary conditions for this program are

$$(19)\qquad \nabla\partial\pi/\partial K^N_{t+1} - W_k = 0$$

where W_k is vector of post–tax rental rates such that

$$W_{ki} = Q_i(\rho + \delta_i) = q_i(\rho + \delta_i)(1 - v_i - d_{ic} - d_{ii}) \quad i=1,...,m.$$

We have defined the present value of capital cost allowances as d_{ic} and the present value of incremental investment allowances as d_{ii}. Clearly equation (19) is just a special case of equation (15). The equilibrium of the firm consists of equation set (17) and equation (19).

In this model, the emphasis is on how output supplies and factor demands are influenced by tax policy. This can be described geometrically by assuming there is a simple output ($\ell = 1$) and two inputs (n=m=1 – there is no distinction here between variable and quasi–fixed factor). The analysis of an increase in the investment tax credit or capital cost allowance is straightforward. An increase in either of these policy instruments lowers the relative factor price of capital. The firm chooses a new cost minimizing mix of inputs for the given output. This mix is relatively more capital intensive. In addition, at the given output level, the marginal cost of production declines and therefore output supply expands.

The analysis is somewhat different when the incremental investment allowance increases. The reason is that the firm can only take advantage of the incremental allowance if current investment expenditure exceeds an average of past expenditures. In the following Figure, the firm produces output defined by the isoquant y^1. The minimum cost equilibrium in the absence of any taxes or tax incentives is denoted by E^1, with relative factor prices reflected by the isocost line AB. Suppose an incremental allowance on capital is introduced. This has the effect of lowering the rental rate such that the new isocost line CD reflects the relative factor prices inclusive of the allowance. Thus the isocost line CD is steeper than AB. In addition, CD has been drawn so that it is tangent to the isoquant y^1 at E^2. The point E^2 represents the minimum cost equilibrium to produce y^1 inclusive of the incremental allowance. Next, let us assume that the capital stock upon which the incremental allowance is based is K_b^1, where by construction the isocost lines AB and CD intersect. In this situation, with capital stock levels greater than K_b^1, the relevant isocost line is AB. Thus the effective isocost curve is CB^1B. Moreover, this isocost curve represents the same production costs as those given by the isocost line AB (measuring cost in labour units). Hence, the firm is indifferent between the equilibria given by E^1 and E^2 as each represents the identical minimum cost to produce y^1. The

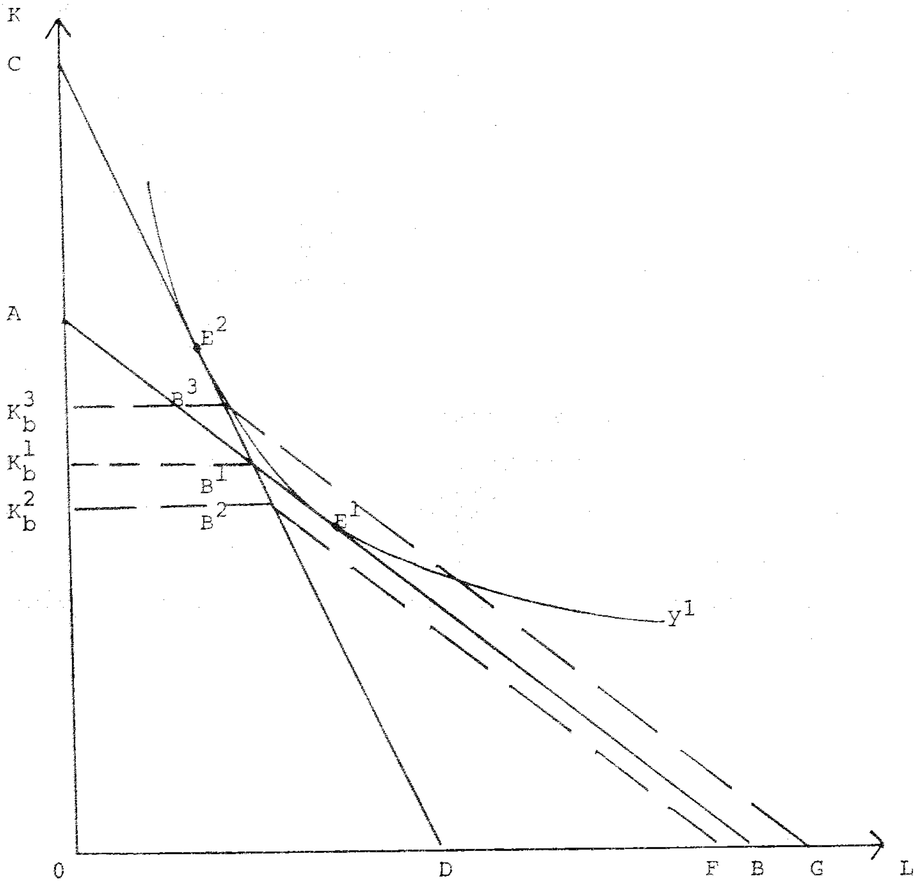

Figure: Incremental Investment Allowance and Factor Demands firm produces y^1 with the same cost with or without the incremental allowance. Suppose now that the base for the allowance declines to K_b^2. In this case, the effective isocost curve is CB^2F, which represents lower production costs relative to CB^1B. Thus, the firm produces y^1 at a minimum cost by using the incremental allowance. The equilibrium point is E^2. With a base of K_b^3, the effective isocost curve is CB^3G. The firm produces y^1 at a minimum cost given by the isocost line AB and so the equilibrium point is E^1. The firm does not use the incremental allowance. Notice that if the base quantities of capital are always less than the undistorted cost minimizing level, then the firm will always utilize the incremental allowance.

The empirical implementation of the model defined by equations (17) and (19) necessitates a functional form for the post–tax variable profit function. Moreover, because the post–tax rental rates of capital are time invariant, we can combine the two stages of production decisions and correspondingly define a post–tax profit function. In addition, empirical implementation is often mainly concerned with factor substitution, and so it is generally assumed that output levels are predetermined. In this instance, only the cost function needs to be specified. Many different functional forms for the cost function have been introduced over the years (see Berndt and Khaled [1979]). Probably the one most often used in this context is the translog (see Christensen, Jorgenson and Lau [1973]), Fuss and McFadden [1978] and Diewert [1980]),

$$(20)\quad \ell n\, c = \alpha_0 + \Sigma_{i=1}^{\ell}\alpha_i \ell n y_i + \Sigma_{j=1}^{n}\alpha_j \ell n W_j + \alpha_t t + .5\Sigma_{i=1}^{\ell}\Sigma_{r=1}^{\ell}\alpha_{ir}\ell n y_i \ell n y_r$$

$$+ .5\Sigma_{j=1}^{n}\Sigma_{s=1}^{n}\alpha_{js}\ell n W_j \ell n W_s + .5\Sigma_{i=1}^{\ell}\Sigma_{j=1}^{n}\alpha_{ij}\ell n y_i \ell n W_j + .5\alpha_{tt}t^2$$

$$+ \Sigma_{i=1}^{\ell}\alpha_{it}\ell n y_i t + \Sigma_{j=1}^{\ell}\alpha_{jt}\ell n W_j t + u_c,$$

where $c = \Sigma_{j=1}^{n}W_j v_j$ is the after–tax cost and the parameters satisfy

$\alpha_{ir} = \alpha_{ri}$, i,r=1,...$\ell$, $\alpha_{js}=\alpha_{sj}$, s,j=1,...,n, $\alpha_{ij}=\alpha_{ji}$, i=1,...,ℓ, j=1,...,n by symmetry and $\Sigma_j\alpha_j = 1$, $\Sigma_j\alpha_{js} = 0$, s=1,...,n, $\Sigma_j\alpha_{ij} = 0$, i=1,...ℓ, $\Sigma_j\alpha_{jt} = 0$ by homogeneity of degree 1 in the factor prices.[8] In addition, the cost function is concave and nondecreasing in the factor prices and nondecreasing in output. Applying the equivalent of Hotelling's Lemma, known as Shepherd's Lemma, to the cost function, the conditional factor demands (conditional since outputs are exogenous) are derived by differentiating the cost function with respect to the factor prices. Thus,

$$(21)\ s_j = \alpha_j + \Sigma_{s=1}^{n}\alpha_{js}\ell nW_s + \Sigma_{i=1}^{\ell}\alpha_{ji}\ell ny_i + \alpha_{jt}t + u_j,\ j=1,...,n$$

where $s_j = W_jv_j/c$ is the j^{th} input cost share. Stochastic disturbances have been appended to equations (20) and (21). These disturbances reflect errors of optimization and errors in the data. The disturbance in the cost function can also reflect stochastic shocks (for example, productivity shocks) to the technology.[9]

The model consists of equations (20) and (21). However, in estimating the unknown parameters, only n of the n+1 equations are used because one of the errors can always be written as a linear combination of the others and therefore one of the equations adds no new information. The easiest way to see this is to use equation set (21). The cost function is C(y,W) and
$\partial \ln C(y,W)/\partial \ln W_j$ are the terms on the right side of (21), not including the stochastic error. Thus, from (21), $\Sigma_js_j = \Sigma_jW_j\partial C(y,W)/\partial W_jC(y,W) + \Sigma_ju_j$, j=1,...,n. Since the cost shares sum to unity $\Sigma_js_j = 1$ and since the cost function is homogeneous of degree 1 in the factor prices, $\Sigma_jW_j\partial C(y,W)/\partial W_j = C(y,W)$, then it must be true that $\Sigma_ju_j = 0$.

There has been a great deal of empirical work over the years estimating the cost structure for firms and industries. To various degrees, tax rates, credits, and allowances have been included in the factor prices. However, few studies have explicitly investigated the effect of changes in tax policy on variable factor demands. An exception is the paper by Kesselman, Williamson and Berndt (1977). In this study, a single output, three–factor translog cost function is estimated in the absence of technological change and for a technology which exhibits constant returns to scale. Thus, in terms of the cost function given by the right side of equation (20), $\ell = 1$ (single output and so let $\alpha_i = \alpha_y$), $\alpha_y = 1$, $\alpha_{yy} = 0 = \alpha_{yt} = \alpha_{yj}$, j=1,...,n–1 by constant returns to scale, $\alpha_t = \alpha_{tt} = \alpha_{yt} = \alpha_{jt} = 0$, j=1,...,n–1 by the absence of technological change, and n = 3 (three inputs). The inputs are blue–collar workers, white collar workers and capital. The effects of an investment tax credit along with two types of employment tax credits on factor demand were simulated. One employment tax incentive was an employment tax credit and the other was a marginal (or incremental) employment tax credit.

The results from the elimination of the investment tax credit for the period 1962 to 1971 for U.S. manufacturing were that total labour demand would have been around .7% higher over the period. Employment of blue–collar workers would have been about 1.1% higher, while employment of white–collar workers would have fallen about .3 percent. These results reflect the findings that white and blue–collar workers are mildly substitutable, capital and blue–collar workers are substitutes and capital and white–collar workers are complements. Also, average costs and thereby product price costs would have been about .8 percent higher.

Next, Kesselman, Williamson and Berndt considered the effects of the imposition of an employment tax credit. First, the imposition was on a per man–hour basis and second on the wage bill. In each simulation, the cost of the employment tax credit was set equal to the revenue gain from eliminating the investment tax credit. In both cases, the effects were quite small and the tax credit on a per man–hour basis was relatively more favourable to blue–collar workers compared to white–collar workers. The converse is true for the credit based on the wage bill. The greatest influence of the employment tax incentives arose from the incremental tax credit. A base of .5 of the previous year's wage bill doubles the impact on factor demands relative to the effects of an employment tax credit based on a percentage of the wage bill. This result occurs because the incremental employment–tax credit channels subsidies to the firm for additional employment beyond a base magnitude. Hence, the same policy cost can generate a larger percentage change in the price of subsidized units of labour through an incremental credit. Provided, of course, that the firm utilizes the incremental tax credit.

The previous empirical analysis focused on the effects of tax incentives on factor demands. However, in the long–run equilibrium framework (defined by equations (20) and (21)), which admit multiple outputs, non–constant returns to scale and non–neutral technological change, it is also possible to investigate the effects of tax policy on scale economies, scope economies, and the rate of productivity growth. There has not been an empirical analysis of the effects of tax policy on scale and scope economies, but Fraumeni and Jorgenson (1980) and Jorgenson (1981) have studied the dependency of productivity growth on tax rates and incentives.

To see how productivity growth can be affected by tax policy, refer to equation (20). Since the rate of productivity growth is defined as the proportional decline in production costs over time, this rate can be obtained by differentiating equation (20) with respect to t,

$$(22) \quad -\partial \ell nc/\partial t = -[\alpha_t + \alpha_{tt}t + \Sigma_{i=1}^{\ell}\alpha_{it}\ell ny_i + \Sigma_{j=1}^{\ell}\alpha_{jt}\ell nw_j]$$

We can observe then that the rate of productivity growth is a function of the government's tax policy. Tax policy operates through the factor prices which, in turn, influence the rate of productivity growth.

The coefficients, in equation (22), relating to the factor prices characterize how the rate of productivity growth responds to changes in the tax, credit and allowances rates. For example, suppose a credit is offered to the jth input which causes its factor price to decline by 1 percent. The effect on the rate of productivity growth is found by differentiating (22) with respect to ℓnw_j. Thus, in the case α_{jt} characterizes the manner in which the rate of technological change is influenced by an increase in the tax credit on the jth factor of production. If $\alpha_{jt}>0$, then the rate of productivity growth increases as the tax credit increases, while if $\alpha_{jt}<0$, the converse arises.

The α_{jt} coefficients show the biases of technological change. They indicate the effect of changes in technology on the input cost shares. For example, technological change for the jth input gives the change in the cost share of the jth input in response to changes in technology represented by time. This can be seen from equation set (21). If we differentiate the jth share by time, the effect is determined by α_{jt}. Hence the factor biases of technological change characterize how the rate of productivity growth is influenced by tax policy.

Generally, we define technological change as factor–using if the bias of technological change for the factor is positive (that is for the jth input $\alpha_{jt}>0$). In other words, if changes in technology result in an increase in the cost share of the jth input, then technological change is jth factor–using. Conversely, if changes in the technology result in a decrease in the cost share of the jth input, then technological change is jth factor–reducing (or saving).

The biases of technological change express the dependence of factor cost shares on the technology and also characterize the dependence of the rate of productivity growth on the input prices and thereby on tax policy. For example, technological change, which is the jth factor–using, means that an increase in the factor price of the jth input decreases the rate of productivity growth. Similarly, technological change which is the jth factor–reducing means that an increase in the factor price of the jth input increases the rate of productivity growth. The lesson to be learned from this analysis is that it is not sufficient for the government to provide tax incentives in order to improve productivity performance.

The factor biases associated with technological change must be determined in order to characterize how the rate of productivity is influenced by the factor prices.

Fraumeni and Jorgenson [1980] have estimated the biases of technological change for 35 industries in the U.S. for the period 1952–1979. They assume that the technology exhibits constant returns to scale, and so $\alpha_{it} = 0$, $i=1,...,\ell$ in equation (22); also, there is a single output and four inputs, which are capital, labour, energy and materials. The pattern of technology change that occurred most frequently is capital–using, labour–using, energy–using and material–reducing. This pattern arose for 19 of 35 industries. This implies that increases in the factor price of capital, labour and energy decrease the rate of productivity growth.

The overall conclusion of Jorgenson and Fraumeni is that effective tax rates on corporate income are inversely correlated with the rates of productivity growth. This result arises from the fact that tax policy has reduced the rental rate on capital which has increased the rate of productivity growth because the latter is capital–using. They found that effective tax rate declined sharply between 1960 and 1965 while the rate of productivity growth attained the postwar peak of 2.11 percent during this period. From 1965–1969, effective tax rates rose substantially while the rate of productivity growth declined to 0.05 percent. Effective tax rates declined from 1969 to 1972 and have remained relatively constant since that time but productivity growth increased slightly from 1969 to 1972 and fell dramatically from 1973. They attribute the latter decline to the energy price increases. In light of this concluson, which has been the subject of much debate (see Nadiri and Schankerman [1981], Baily [1981] and Clark [1982]), they recommend that tax policy should be introduced to decrease the factor prices of capital and labour.

In Canada, little work has been done on investigating the effects of tax, credit and allowance rates on factor substitution and productivity growth.[10] In general, much more empirical work needs to be done, even in the context of long–run equilibrium. First, little is known about the effects of tax policy on scale and scope economies. In order to capture these effects, it is necessary to estimate cost (or profit) functions which do not incorporate the maintained hypotheses of constant return to scale (or for that matter homotheticity) and of a single output. Second, the treatment of technological change is quite simplistic. Technological development does not usually occur autonomously; it is also part of production and investment decisions. Indeed, the demand for research and development capital, which is an important element of technological change, is itself a function of the array of factor prices and the quantities of outputs.[11] Thus, as is the case of the other factors of production, the demand for R&D capital depends on the various taxes, credits and allowances.

4. Taxes and Factor Adjustment

In the previous section we considered the effects of taxes on factor substitution and productivity growth in the context of our general model by assuming that factors of production could be costlessly adjusted and utilized. Suppose now it is assumed that a subset of factors of production can be costlessly adjusted while for the remaining inputs, installation cost must be incurred and so the latter are quasi-fixed factors.

Generally, two types of models have been developed which relate to factor adjustment. The first type emphasizes the trade-off between future increases in the quasi-fixed factors (and thereby future increases in output levels) and higher present costs associated with increasing adjustments speeds. The higher costs appear either as higher purchase prices of the quasi-fixed factors or as higher costs of financing the accumulation of these factors. The former costs have been considered by Lucas [1967], Gould [1968] and Mussa [1977], while the latter costs have been considered by Steigum [1983]. These models are able to capture the positive correlation between capital costs and investment and the magnitude of adjustment speeds associated with the quasi-fixed factors.

The short-run determination of investment in the quasi-fixed factors is the mechanism by which the adjustment process of these inputs are determined. There is, however, no relationship between the variable factor demands and investment in the quasi-fixed factors. Thus the costs of faster adjustment are not reflected in the lower current production levels.

The second type of model of factor adjustment recognizes that changes in the quasi-fixed factor investments alters variable factor demands and thereby current output supplies. In this context, the costs of adjustment are reflected in lower current output levels. Thus, in adjusting quasi-fixed factors, the benefits of increased future output supplies are balanced by the costs of decreased present output supplies. This type of model emphasizes internal costs of adjustment through the technology and is represented by foregone current output. The other model type emphasizes external adjustment costs, represented either by rising quasi–fixed factor purchase prices or by rising financing costs. The models incorporating internal adjustment costs have been developed by Treadway [1971], [1974], Mortensen [1973] and Epstein [1981].

The model developed in this paper incorporates internal adjustment costs. With costly quasi–fixed factor adjustment but costless utilization, the first stage of the production decisions is given by (16) except the transformation function is now defined as $T(y_t, v_t, K_t^N, K_{t+1}^N - (I_m - \delta)K_t^N, A_t) = 0$ and prices are not time invariant. Quasi–fixed factor utlization is costless and consequently depreciation is exogenous and constant over time, so that investment is $I_t = K_{t+1}^N - (I_m - \delta)K_t^N$. The first order necessary conditions are similar to equation set (13) such that the variable profit function and derived conditions (with respect to the post–tax prices) are

(23.1) $\pi_t^v = \Pi_t(P_t, W_t, K_t^N, K_{t+1}^N, A_t)$

(23.2) $\nabla\Pi_P = y_t$

(23.3) $-\nabla\Pi_W = v_t.$

In this case, after tax variable profits are defined as revenue minus variable inputs costs and future capital services enter the domain of the post–tax variable profit function because quasi–fixed factor adjustments are costly to undertake.

The second stage of the production problem is to

(24) $$\max_{(K_{s+1}^N)} E_t\Sigma_{s=t}^{\infty}\alpha(t,s)(\Pi_s(P_s,W_s,K_s^N,K_{s+1}^N,A_s) - Q_s^T(K_{s+1}^N-(I_m-\delta)K_s^N).$$

The first order necessary conditions for any time period are

$$(25)\quad E_t[\nabla\frac{\partial\Pi_t}{\partial K^N_{t+1}} - Q_t + \alpha(t,t+1)\ (\nabla\frac{\partial\Pi_{t+1}}{\partial K^N_{t+1}} + (I_m - \delta)Q_{t+1})] = 0$$

The equilibrium of the firm consists of equation sets (23) and (25). The empirical implementation of the model is generally quite complex and a number of procedures have been introduced in the literature. The complexity of the model relates to equation set (25) and the first procedure confronts this difficulty by placing enough structure on the technology and expectations of the firms in order for equation set (25) to have a closed form solution. We shall deem this procedure the direct approach. The direct approach restricts the technology represented by the variable profit function (or variable cost function if output is exogenous) to a quadratic specification and adjustment costs depend only on the first order changes in the quasi–fixed levels.[12] In addition, the expectations process must be specified in the model. Berndt, Fuss and Waverman [1979], Denny, Fuss and Waverman [1981] and Berndt and Morrison [1981] impose static expectations. Sargent [1978], Meese [1980] and Hansen and Sargent [1980] have imposed rational expectations. Static expectations are to be understood in the context of continuously revising plans and always expecting that current prices, tax credit and allowance rates are to persist. Current period plans are the only ones that are actually carried out. Rational expectations are to be understood in the context of generating forecasts of prices, taxes, credit and allowance rates which are the ones that best fit the actual time series. In this case, restrictions are imposed on the model (in other words, cross–equation restrictions on parameters) which reflect the maintained expectations processes.[13]

The direct approach can be presented in the following context. Assume that there is a single output (ℓ=1) and so the technology can be represented by a production function which is assumed to be

$$(26)\quad Y_t = \alpha^T V_t + 0.5 V_t^T A V_t + 0.5(V_{t+1} - V_t)^T B (V_{t+1} - V_t) + H(t)$$

where V_t is the n+m vector of inputs which may be variable or quasi–fixed. α is an n+m vector, A and B are symmetric and negative definite matrices, and H captures autonomous technological change as a function of time. The matrix B is diagonal and represents the costs of adjustment in terms of foregone output.[14] If a factor is variable then the relevant diagonal in B is zero, while if the factor is quasi–fixed then the relevant diagonal is positive. In this manner, variable and quasi–fixed factors are distinguished.

Prices evolve according to the following process:

$$(27)\quad S_t = \Psi + \Sigma_{i=1}^{h}\theta_i S_{t-i} + G(t) + \xi_t$$

where S_t is the n+m vector of the post–tax factor prices normalized by the post–tax price of output $(p_t(1-u_{ct}))$. Indeed, S_t is a vector of both variable and quasi–fixed post–tax prices (in other words, it contains both W_t and Q_t). Also Ψ is a m+n vector, θ_i is a m+n dimensional matrix and ξ_t is a m+n vector of white noise processes, and G reflects the trend.

The objective of the firm is to

$$(28)\quad \max_{(\Gamma V_{s+1},\Phi V_s)} \; E_t\Sigma_{s=t}^{\infty}(1+\rho)^{-s+t}[y_s - S_s^T(\Gamma(V_{s+1} - (I_{m+n}-\delta)V_s) + \Phi V_s)]$$

with Γ an m+n dimensional diagonal matrix with a 1 in the diagonal if the factor is quasi–fixed and a 0 if the factor is variable, Φ is a diagonal matrix defined conversely to Γ, δ is the diagonal matrix of constant depreciation rates, and the diagonal is zero for a variable factor. If the ith factor is quasi–fixed then V_{is} is given. In addition, it must be assumed that the discount rate is known with certainty. This assumption is unavoidable if closed form solutions are to be obtained for multiple quasi–fixed factor production programs. The firm maximizes (28) by selecting the relevant factor demands subject to the technology (26) and price expectations (27).[15]

The solution to this problem (see Kushner [1971] or Astrom [1970]) is the set of flexible accelerator factor demand equations,

$$(29) \quad V_{t+1} - V_t = M(V_t - V^e_{t+1})$$

where $V^e_{t+1} = A^{-1}(\omega_t - \alpha)$, $\omega_t = C\Sigma^{\infty}_{s=t}(I_{m+n} + C)^{-s+t}[E_tS_s - \Gamma(I_{m+n} - \delta)E_tS_{s+1}]$,

$C = AB^{-1}(1+\rho) + R + M^T$, R is the m+n diagonal matrix with ρ in the diagonal and M is the stable adjustment matrix which solves the quadratic

$M^2 - (1+\rho)B^{-1}AM - \rho M - B^{-1}A(1+\rho) = 0.$

The model which can be estimated consists of equations (26), (27) and (29) with stochastic error terms appended to equations (26) and (29). The disturbance terms in these latter two equations can reflect optimization or measurement errors. In addition, the disturbance in the production function, (26), can also reflect shocks to the technology.[16] Berndt, Fuss and Waverman (1977) developed a special case of the above model which in–corporated the corporate income tax credit, the physical investment tax credit and the physical capital cost allowance. They assumed that there was a single quasi–fixed factor and static price expectations. Under the assumption of exogenous output, the first of the two stages relating to the production decisions can be determined by the specification of a variable cost function (as opposed to a variable profit function when output supplies are endogenous). Assuming a quadratic variable cost function which is normalized by the first variable factor

$$(30) \quad c^v/W_1 = \alpha_0 + \alpha_y y + \Sigma^n_{j=2}\alpha_j W_j + \alpha_k K^N + \alpha_t t + .5\alpha_{yy}y^2 +$$

$$.5\Sigma^n_{j=2}\Sigma^n_{s=2}\alpha_{js}W_jW_s + .5\alpha_{kk}(K^N)^2 + .5\alpha_{tt}t^2 + \Sigma^n_{j=2}\alpha_{yj}yW_j + \alpha_{yk}yK^N + \alpha_{yt}yt +$$

$$\Sigma^n_{j=2}\alpha_{jk}W_jK^N + \Sigma^n_{j=2}\alpha_{jt}W_jt + \alpha_{kt}K^Nt + \alpha_{II}(\Delta K^N)^2 + u_c,$$

where $c^v/W_1 = v_1 + \Sigma_{j=2}^n W_j v_j$, W_j is the normalized after–tax variable factor price, c^v/W_1 is the normalized after–tax variable cost, $\Delta K^N = K_{t+1}^N - K_t^N$, and the parameters satisfy $\alpha_{js} = \alpha_{sj}$, j,s=2,...,n by symmetry. Normalizing the variable cost function has the effect of imposing homogeneity of the first degree in the factor prices. The normalized variable cost function must also be nondecreasing and concave in the factor prices, non–increasing and convex in the quasi–fixed factor, nondecreasing in output and nondecreasing and convex in net investment. Applying Shepherd's Lemma to the normalized variable cost function yields the conditional variable factor demand functions

$$(31) \qquad v_j = \alpha_j + \Sigma_{s=2}^n \alpha_{js} W_s + \alpha_{yj} y + \alpha_{jk} K + \alpha_{jt} t + u_{jn}, \qquad j=2,...,n.$$

The stochastic disturbances u_c and u_j, j=2,...,n have been added to the variable cost and conditional variable factor demand functions. The error terms reflect the same kind of phenomena as described for the errors of equations (20) and (21). Equations (30) and (31) represent the first stage of the production decisions or the short–run equilibrium.

The determination of investment is governed by a flexible accelerator because this model is a special case of (28). The investment equation is

$$(32) \qquad K_{t+1}^N - K_t^N = M(K_t^N - K_{t+1}^{Ne}) + u_k,$$

where M is the stable adjustment coefficient which solves the quadratic $M^2 + (\alpha_{kk}/\alpha_{II} + \rho)M - \alpha_{kk}/\alpha_{II} = 0$, $K_{t-1}^{Ne} = (-1/\alpha_{kk})[\alpha_k + \alpha_{yk} y + \Sigma_{j=2}^n \alpha_{jk} W_j + \alpha_{kt} t + W_k]$ is the long–run equilibrium demand for the quasi–fixed factor, and $W_k = Q_k(\rho+\delta)$ is the after–tax rental rate on this factor, and a stochastic disturbance has been added to the investment equation.[17]

The model consists of equations (30), (31) and (32). Moreover, because the variable cost function is normalized, the errors in equations (30), (31) and (32) are linearly inde–

pendent. The first variable factor conditional demand function has already been eliminated. Thus equations (30), (31) and (32) can be used to estimate the unknown parameters.

Berndt, Fuss and Waverman estimated this model for U.S. manufacturing for the period 1947–1974 and incorporated the corporate income tax rate, investment tax credit and capital cost allowance into the post–tax rental rate on capital. From our point of view, the most significant result of this paper is that the long–run price elasticities on the conditional factor demands (both variable and quasi–fixed) are considerably smaller than their counterparts obtained in models with no adjustment costs. This means that, for U.S. manufacturing, the influence of tax policy on long–run factor demands is significantly smaller than previous empirical evidence showed. The misspecification caused by assuming all factors can be costlessly adjusted caused an upward bias in the influences of factor prices, and thereby tax policy, on input demands.

The second approach to the empirical implementation of the intertemporal production model is the dual approach developed by Rockafeller (1970), Benveniste and Schienkman (1979), McLaren and Cooper (1980) and Epstein (1981). The focus of this approach is not the variable profit function defined by (23.1) (or the variable cost function) but rather the value function defined by equation (10). Unlike the direct approach, dynamic duality can handle much more general specifications of the technology, including the quasi–fixed factor adjustment mechanisms. However, the treatment of expectations formation processes is much more limited using the dual approach.

The dual approach can be presented in the following context. Assume that there is a single output (ℓ=1) and the technology is represented by the general production function

$$(33) \quad y_t = F(v_t, K^N_t, K^N_{t+1} - (I_m - \delta)K^N_t, A_t).$$

In addition, assume that there are static expectations on the prices, tax, credit and al– lowance rates and the firm's discount rate is constant.

The objective of the firm is to

$$(34) \quad \max_{(v_s, K^N_{s+1})} \quad \Sigma^{\infty}_{s=t}(1+\rho)^{-t+s}[F(v_s, K^N_s, K^N_{s+1}-(I_m-\delta)K^N_s, A_s) - W^T v_s - Q^T(K^N_{s+1}-(I_m-\delta)K^N_s)],$$

with K^N_t given, and the post–tax prices of the variable factors (W) and of the quasi–fixed factors (Q) are normalized by the post–tax price of output. This problem is a special case (combined into a single stage) of the one defined by (23) and (24). Rather than proceeding directly, we can use the Hamilton–Jacobi equation (see Arrow and Kurz (1970) and Dreyfus (1965)). Define the maximized value of (34) as $J(K^N_t, W, Q)$ and thus

$$(35) \quad (1+\rho)J(K^N_t, W, Q) = F(v_t, K^N_t, K^N_{t+1}-(I_m-\delta)K^N_t, A_t) - W^T v_t$$
$$- Q^T(K^N_{t+1} - (I_m-\delta)K^N_t) + J_k(K^N_{t+1} - K^N_t),$$

where the factor demands are evaluated at the solution to the problem defined by (34). The solution to the problem (in other words, the factor demands) are found by differentiating both sides of (35) by the post–tax factor prices. Thus

$$(36.1) \quad K^N_{t+1} = J^{-1}_{KQ}[(1 + \rho)J^T_Q + K^N_t] - K^N_t$$

$$(36.2) \quad v_t = -(1 + \rho)J^T_W + J_{KW}(K^N_{t+1} - K^N_t)$$

$$(36.3) \quad y_t = (1 + \rho)[J(K^N_t, W, Q) - J_W W - J_Q Q] - [J_K - W^T J_{KW} - Q^T J_{KQ}](K^N_{t+1} - K^N_t).$$

Equation (36.3), which is the output supply function, is derived by substituting equations (36.1) and (36.2) into (35).

By appending error terms to the equations set (36) and postulating a functional form for the value function, $J(K^N_t, W, Q)$, the model can be implemented empirically. Epstein and Denny (1983) have investigated investment behaviour for U.S. manufacturing, Bernstein and

Nadiri (1985) have estimated the spillovers that are associated with R&D investment for U.S. firms, and Bernstein (1986) has estimated the effects of physical and R&D investment tax incentives for Canadian firms using dynamic duality. In all cases, an intertemporal cost minimizing approach was used, because the stream of output was assumed to be exogenous.

In his model of tax incentives and the structure of production, Bernstein (1986) assumes that labour is the sole variable factor, while physical and R&D capital are the quasi-fixed factors. The firm's discount rate is treated as a constant and there are static expectations on the prices.[18] The value function was assumed to be of the form

$$(37)\quad J(K_t^N, W, Q, y) = .5[Q^T \quad W^T] \begin{bmatrix} B_{QQ} & B_{QW} \\ B_{WQ} & B_{WW} \end{bmatrix} \begin{bmatrix} Q \\ W \end{bmatrix} y + [Q^T A^{-1} + a^T] K^N + [Q^T A^{-1} h + h_0](1+\rho)^{-1},$$

where the matrices B_{QQ}, B_{QW}, B_{WQ}, B_{WW} and A, the vectors a and w and the scalar h_0 represent the unknown parameters. The matrices B_{QQ} and B_{WW} are symmetric and negative definite, B_{QQ} is an m dimensional matrix (since there are m quasi-fixed factors) and B_{WW} is an n-dimensional matrix (since the are n variable factors). The stable adjustment matrix is given by $[(1+\rho)I_m - A]$, where A is an m dimensional matrix and I_m is the m dimensional identity matrix. This functional form for the value function is linear in output and the quasi-fixed factors and quadratic in the post-tax factor prices.[19]

The results from the empirical work based on a sample of about 30 firms over the period 1975-1980 are that physical and R&D capital are complements both in the short and long-runs, while each type of capital is a substitue for labour. Both types of capital respond to changes in their own post-tax purchase prices. However, the demands for capital are quite price inelastic. Even in the long-run, the own price elasticities of the cpaital inputs are less than .4. Labour demand is relatively more price responsive in both the short and long-runs. The adjustment process for physical capital is shorter than for

R&D capital. The latter takes about six years to adjust while the former takes about four years. Moreover, the capital stocks are complementary to each other along the adjustment path. In other words, increases in the stock of physical capital shorten the adjustment period of R&D capital.

Changes in three types of tax incentives are considered in this study. First, a 1 percent increase in the physical investment tax credit generates increases in the demand for physical capital of .022 percent in the short-run and .055 percent in the long run. Similarly, the demand for R&D capital increases by .010 percent in the short-run and .029 percent in the long-run. Moreover, when the output effects of the physical investment tax credit increase are considered, the demands for all the inputs increase.

Second, an increase in the R&D investment tax credit also affects the structure of production. However, these effects are smaller relative to an equivalent increase in the physical investment tax credit. The third incentive is the R&D incremental investment allowance. An increase in this allowance affects the structure of production, but generates the smallest effects of all three incentives.

The fact that the empirical results are based on a dynamic model permits the investigation of short- and long-run effects on factor demands. In addition, the speed of the adjustment process is estimated. Bernstein determines the annual adjustment from the short to the long-run effect of any tax policy initiative. In the study, this type of analysis is conducted for R&D expenditures, because of its focus on policies influencing R&D investment. However, the analysis applies equally to the other factor demands.

Changes in tax credit and allowance rates decrease post-tax factor prices and thereby decrease production and adjustment costs. Using an intertemporal application of Shepherd's Lemma based on the value function permits the determination of the cost to the government, in terms of foregone tax revenues, of increases in the tax credit and allowance rates. However, this analysis does not necessarily capture changes in efficiency associated with changes in tax policy (see Diewert (1985(a))), and Jorgenson and stoker (1985). Bernstein investigates the relative effectiveness of alternative tax policies on the structure of production when the cost to the government across tax policy changes is equalized. In additon, a calculation is made of the actual cost to the government of alternative tax policy initiatives. The calculations show that changes in tax credit and allowance rates directed towards R&D investment generate about $.82 of R&D expenditure per dollar of lost tax revenue at the existing level of output. Moreover, an increase in the physical investment tax credit generates around $.06 of R&D expenditure per dollar of lost tax revenue. This figure increases to around $.15 when output effects are considered. Hence, there may be important cross effects arising from government tax policy changes directed towards a particular factor of production or type of investment. Excluding these cross effects biases the cost estimates and the influence of tax policy on production and investment.

The third approach to the implementation of the model given by equations (23) and (25) is to treat the first order conditions for the quasi–fixed factors as implicit equations and not obtain closed form solutions. This is the approach developed and implemented by Kennan [1979], Hansen and Singleton [1982], Pindyck and Rotemberg [1983], and Bernstein and Nadiri [1986]. This approach, which may be reffered to as the implicit approach, specifies a functional form for the variable profit (or variable cost function) which is jointly estimated with the reduced form variable factor demand equations and the implicit equations for the quasi–fixed factors. This approach permits a great deal of flexibility in the specifications of the technology and the expectations generating processes because the first order conditions for the quasi–fixed factors do not have to be solved.

There are two difficulties with this implicit approach. First, because closed form solutions are not obtained for the quasi–fixed factors, there are no conditions in the model guaranteeing the optimality (existence and uniqueness) of the factor demands for any set of price trajectories. In other words the terminal or transversality conditions are ignored, as only the first order conditions are used. In terms of the estimation of the model, since the estimator ignores the information contained in the transversality conditions it must not be asymptotically efficient. However, the direct and dual approaches require the choice of particular expectation generating processes (as well as the choice of a technology). This necessitates that these processes be incorporated into the restrictions imposed in the estimation. An incorrect choice leads to inconsistent, as well as asymptotically inefficient estimates (see Gourieroux, Laffont and Monfort [1979]).

The second difficulty with the implicit approach is that because the quasi–fixed factor demands are not determined, we cannot characterize the properties of these demand functions through time. We can only investigate the long–run properties of the quasi–fixed factor demands. Wickens [1982] has suggested a solution to this difficulty. Replace all expected values of future variables with their realizations to produce an observable but incomplete system of equations. The system is then completed by adding equations characterizing the determinants of future values of the variables in terms of any variables known in the current period. Estimation of the complete system will be consistent but not asymptotically efficient. Moreover, through this augmented system of equations we can determine the short as well as the long–run properties of the quasi–fixed factor demands. This method has not as yet been used to estimate models of production structure and to determine the effects of tax policy on this structure.

5. Taxes and Factor Utilization

It has long been recognized that although adjustment costs cause the quasi–fixity of factors of production, the rates at which these factors are utilized are variable in the short–run. Indeed, these rates are part of the firm's production plan which are dependent on the stocks of quasi–fixed factors and post–tax product and factor prices. Thus, in the short–run changes in tax policy do not affect the stocks of quasi–fixed factors, they do influence the rates at which these stocks are accumulated and utilized. This is precisely the model developed in section 2 of this paper and is represented by equation sets (13) and (15).

Generally, there have been two types of models relating to the factor utilization. The first type due to Lucas [1970], Winston and McCoy [1974], Abel [1981] and Bernstein [1983] emphasize the trade–off between increased output and higher labour costs that utilization generates. The increase in costs manifest themselves in terms of overtime and shift wage premiums. These models are able to capture the positive correlation between real wages and labour utilization and the positive correlation between capital utilization and capital stock (see Foss [1981]).

The short–run interrelationship between utilization and investment is the mechanism in these models by which utilization affects the accumulation of the quasi–fixed factor. There is no connection between capital utlization and depreciation rates or between labour utilization and quit rates. Thus the costs of higher utilization rates are not reflected in the lifetime of the quasi–fixed factors.

The second type of model of factor utilization recognizes that changes in the rate of utilization alter the lifetime of a quasi–fixed factor. In this case, the benefits of increased current output are balanced by the costs of decreased future output. The cost of factor utilization is foregone future output. The cost of factor utilization are analogous to the two types of models pertaining to factor adjustment. One emphasizes external costs represented by rising wage rates or capital purchase prices, the other emphasizes internal costs through the technology represented by foregone output. In the case of installation costs, it is current output which is foregone, while in the case of utilization costs it is future output. The models incorporating internal utilization costs have been developed by Smith (1970), Taubman and Wilkinson (1970), Diewert (1980), Epstein and Denny (1980), Everson (1982), Schworm (1983) and Bernstein and Nadiri (1984).

It is generally difficult to empirically implement models with variable factor utilization because measures of utilization rates (especially for capital) are usually not available. In practice, different approaches have been used to overcome the lack of capital utilization data.[20] The first approach is to develop a measure of the 'potential' capital (or capital–output ratio). This is a statistical construct based on a trend through cyclical variations in actual capital (or capital–output ratio). This type of measure has been used in many macroeconometric models, and by Klein and Preston (1967), Nadiri and Rosen (1969), Coen and Hickman (1970), and Brechling (1975). The difficulty with this is approach is that the trend itself is a function of relative prices and the magnitudes of the quasi–fixed factors. As these variables change, the trend varies and must be revised. However, the revisions to the trend occur extraneously to the model which is in fact supposed to explain utilization variation.

A second approach recognizes that inventories (or at least departures from some long–run level) are linked to the rate at which capital (and other quasi–fixed factors) is utilized. For example, an increase in inventories relative to the long–run level signifies a fall in product demand and decrease in factor utilization. This method has been explored recently by Helliwell and Chung (1985). The integration of the theory of optimal inventory holdings with the theory of factor utilization and investment offers the potential of an important avenue in which to investigate the role of tax policy on the structure of production. At the present time, the application is limited to postulating the existence of a partial adjustment process characterizing inventory accumulation, rather than the explicit integration (in an optimizing framework) of inventory and factor utilization decisions.

The third approach is due to Epstein and Denny (1980) and it can be discussed within the context of the general theoretical model already developed. Because the focus is on utilization decisions which are determined in the short–run, we shall concentrate on the first stage production decisions of the the firm. Thus we return to equation set (13) and recall that in the short–run, given the beginning of period quasi–fixed factors, output prices and variable factor prices, output supplies, variable factor demands and end of period quasi–fixed factors (or the implied utilization rates) are determined. Once a functional form is specified for the post–tax variable profit function, the short–run supply and demand functions can be determined. Suppose the variable profit function is given as a Generalized Leontieff so that

$$(38)\quad \pi_t^v = K_t^N[2\alpha_1 p_t^{.5} W_t^{.5} + 2\alpha_2 p_t^{.5} W_{2t}^{.5} + 2\alpha_3 W_{1t}^{.5} W_{2t}^{.5} + \beta_1 p_t + \beta_2 W_{1t} + \beta_3 W_{2t} + 2\alpha_4 p_t^{.5} Q_t^{.5} + 2\eta_1 W_t^{.5} Q_t^{.5} + 2\eta_2 W_{2t}^{.5} Q_t^{.5} + \beta_4 Q_t] + u_\pi,$$

where α_i, i=1,2,3,4, β_i, i=1,2,3,4, and η_i, i=1,2 are the unknown parameters. The post–tax variable profit denoted by equation (38), incorporates the assumption of constant returns to scale, with a single output, single quasi–fixed factor and two variable factors. There is also a stochastic disturbance, u_π, appended to the variable profit function. Using equation (38) and by Hotelling's Lemma, equation set (13) becomes

$$(39.1)\quad y_t = K_t^N[\alpha_1 p_t^{-.5} W_{1t}^{.5} + \alpha_2 p_t^{-.5} W_{2t}^{.5} + \beta_1 + \alpha_4 p_t^{-.5} Q_t^{.5}] + u_y.$$

$$(39.2)\quad v_{it} = K_t^N[\alpha_i p_t^{.5} W_{it}^{-.5} + \alpha_3 W_{it}^{-.5} W_{jt}^{.5} + \beta_i + \eta_i W_{it}^{-.5} Q_t^{.5}] + u_i, \quad i,j=1,2, \quad i \neq j$$

$$(39.3)\quad K_t^0 = K_t^N[\alpha_4 p_t^{.5} Q_t^{-.5} + \eta_1 W_{1t}^{.5} Q_t^{-.5} + \eta_2 W_{2t}^{.5} Q_t^{-.5} + \beta_4] + u_K,$$

where u_y, u_i and u_K are stochastic disturbances.[21]

Equation set (39) defines the short–run equilibrium. Let us focus on equation (39.3) which captures the determinants of end period capital services and thereby (implicitly) the utilization rate. Clearly capital utilization depends on the set of prices and the quasi–fixed factor. Moreover, embodied in (39.3) is a generalization of the traditional model with a constant exogenous geometric depreciation rate. If $\alpha_4 = \eta_1 = \eta_2 = 0$, then $K_t^0 = K_t^N\beta_4$. As discussed in section 2 of the paper, $1-\beta_4$ is the depreciation rate. Although depreciation is price independent and constant, it is still to be determined within the model. If, however, a value of β_4 is exogenously given, then equation (39.3) collapses to the traditional model.

The model to be estimated in order to determine the unknown parameters consists of equation set (39). Equation (38) can be eliminated since the error in this equation is a linear combination of the errors in equation set (39). However, the empirical implementation is not straightforward because there is no data on K_t^0 and K_t^N. This problem is solved by assuming that equation (39.3) is non–stochastic, which is in line with the traditional assumption that depreciation is exogenous and non–stochastic, and that the initial capital stock at the start of the sample period is equal to the measured capital stock. Thus, using the estimated parameters based on the technology, along with equation (39.3), the accumulation equation (2) and the initial capital stock, Epstein and Denny are able to construct both beginning and end of period capital stocks and the implied depreciation rate $(K_t^N - K_t^0)/K_t^N$. Thus equations (39.1) and (39.2) are to be estimated.

In order to implement this approach it is necessary that there is only a single quasi–fixed factor, the technology exhibits constant returns to scale and there is either static expectations or perfect foresight with respect to the prices, tax, credit and allowance rates. The fact that beginning and end of period capital stocks are unobservable variables means that the system of equations is underidentified. The existence of only a single quasi–fixed factor along with constant returns to scale technology implies that the firm is really con–

cerned only with the ratio of end to beginning period capital or the depreciation rate. This rate only depends on observable variables and the parameters characterizing the technology, and hence if the equation is non–stochastic then the system becomes identified (using the capital accumulation equation and given the initial stock). If there was more than a single quasi–fixed factor with variable utilization then either the technology would have to be specified with sufficient parameter restrictions in order to identify the system of equations or estimation methods allowing for errors in variables would have to be adopted.

The importance of this model is that it explicitly captures the manner in which prices and thereby tax policy affect capital utilization. It is of interest to note that Epstein and Denny use the corporate income tax rate, the investment tax credit rate, and capital cost allowance in computing the post–tax purchase price of capital. They estimated their model for U.S. manufacturing for the period 1947–1971. The most significant result from our point of view of this survey is that price elasticities on factor demands and output supply with variable capital utilization are substantially smaller than those found using the standard model of exogenous depreciation. In addition, capital utilization is price sensitive, although the elasticities are highly inelastic. This means (at least for U.S. manufacturing) that the influence of tax policy on short–run factor demands in significantly smaller than previous evidence led us to believe. The potential misspecification arising from assuming capital utilization is exogenous causes factor price influences to be borne by the variable inputs. Indeed, capital utilization does respond to changes in tax policy in the short–run.

6. Tax Policy Implications

Corporate tax, credit and allowance rates can influence factor demands, output supplies and the rate of technological change. These policies affect production decisions because the prices firms pay for their factors and charge for their products are modified. In order to evaluate the effectiveness of tax policy in this context, the authorities should have knowledge of how these prices are altered by their tax policies.

Firms respond to price changes initiated through tax policy and these responses are defined by the various product and factor (own and cross) price elasticities. This implies that the behavioral response of firms must be known by the tax authorities so as to determine tax policy effectiveness. However, obtaining estimates of the relevant elasticities is by no means a simple task. Indeed, we have stressed the problems of interaction and adjustment in production decisions for the estimation reflecting input demands and output supplies.

The interaction of factor demands and product supplies is a crucial element in capturing the impact of tax policy. Tax policy initiatives levied upon a particular production activity in general cause cross effects on other activities. For example, an investment tax credit on equipment and structures generates effects not only on the demand for capital but also for the demands for labour and intermediate inputs. This result, in fact, has been obtained in empirical studies. The evaluation of any particular tax policy initiative must reflect the contemporaneous interaction between inputs and outputs.

Current changes in tax, credit and allowance rates also cause future production plans to be altered. We have discussed and seen how the long-run effects of tax changes are quite distinct from the short-run influences; both the magnitude and nature of the effects differ. In the short-run variable factor demands, the utilization and accumulation of the quasi-fixed factors interact in light of tax policy changes. However, only in the long-run are firms able to vary the stocks' relating to the quasi–fixed factors. In addition, the distinction between the short– and long–run implies that there exists adjustment processes. These processes capture the expansion (or contraction) in the quasi–fixed factors and are also influenced by tax policy. Empirical results have highlighted the distinction between the short– and long–run effects and the biases involved in ignoring adjustment processes. The evidence seems to be that tax policy evaluation must explicitly recognize the important features of interaction and adjustment governing production activities.

NOTES

* The authors would like to express their thanks to Erwin Diewert, Mel Fuss, John Helliwell, Robert Lawrence, Nick Rowe and Larry Summers for their comments and suggestions on this paper. They gratefully acknowledge the assistance of Kristofer Hogg of the C.V. Starr Center for Applied Economics in the preparation of this manuscript. Financial support was provided by NSF grant PRA 810635 and by the C.V. Starr Center's Focal Program for Capital Formation, Technological Change, Financial Structure and Tax Policy.

1. All variables in the transformation are measured as flows of services. The term 'capital inputs' is meant to suggest factors of production obtained from stocks which can be accumulated. These stocks can represent the traditional equipment, structures and land and also pertain to research and development and various types of skilled labour.

2. In this paper, vintages of capital stocks are not distinguished. The reasons are first that the empirical work in this area has focused on the putty–clay type of vintage model (see Bischoff (1971), King (1972), Sumner (1974), and Malcomson (1982)). In other words, once installed factor proportions are fixed. This implies that changes in tax rates and incentives cannot affect the rate of factor substitution of installed capital. Second, these studies assume that the service life of capital is constant, which means that tax policy does not affect the rate of capital utilization. The model in the text could be modified to allow for alternative vintages of capital. The transformation function in this case would depend on the vector of all past investment flows for all types of investment rather than on the vectors of beginning– and end–period capital. (See Diewert (1985)).

3. The focus is not on the financial decisions of the firm and so it is assumed that the firm issues one kind of bond and one kind of share.

4. Under the Long Amendment in the U.S., which was repealed in 1964, the depreciation base was reduced by the amount of the ITC_{it} so that $\phi_{it} = 1$. We can introduce without any difficulty allowances or credits for the variable as well as the quasi–fixed factors in this model. However, the complexity of the tax issues relates to intertemporal resource allocation decisions. In Canada, $\varnothing_{it} = 1$ and the U.S. $\varnothing_{it} = .5$.

5. In the finite horizon model, we would have to specify terminal values of the capital stocks. See Diewert (1985(b)).

6. If tax credits or allowances are defined on the variable factors of production then these instruments of tax policy would directly affect output supplies and variable factor demands.

7. We also assume that $\lim_{s \to 0} \alpha(t,s) Q_{is} K^{N}_{is} = 0$, $i=1,\dots,m$.

8. Since there are no adjustment costs, all inputs are variable. Also, time (t) designates the rate of autonomous technological change. The time subscript is deleted from each of the variables.

9. The disturbances in the share equations could also reflect technology shocks. However, in this case, the disturbance in the cost equation must be contemporaneously correlated with each of the factor prices in order for technology shocks to appear in each of the share

equations. This does not pose any theoretical difficulties but adds to the estimation problems.

10. Recently Rao and Preston (1983), using the same framework as Fraumeni and Jorgenson (1980) have investigated the effects of factor prices on factor demands and the rate of productivity growth for 9 Canadian manufacturing industries and 8 non–manufacturing Canadian industries for the period 1957–1979. They did not investigate the effects of tax policy on the structure of production. Surprisingly, their results were quite different than obtained by Fraumeni and Jorgenson. In particular, technological change generally appears to be capital–reducing.

11. An excellent survey on the role of R&D capital in production activities is by Griliches (1979).

12. Hansen and Sargent (1981) have developed a model where adjustment costs do not have to depend on first order differences in the quasi–fixed factors. Their procedure has not as yet been implemented.

13. In a recent paper, Epstein and Yatchew (1985) develop and estimate a model which assumes that the technology is quadratic with adjustment costs based on first order differences and expectations are based on autoregressive processes. Because they estimate the quadratic production function and autoregressive expectations equations along with the derived factor demand equations, they could test all of the cross equation restrictions implied by the firm's programming plan and expectations processes.

14. The specification of adjustment costs, which depend on net rather than gross changes in the quasi–fixed factors and separable from the production technology, is not a significant difference, provided that aggregation over firms need not be theoretically justifiable. If firm aggregation is to be rigorously treated, then parameter restrictions must be imposed on the technology. However, as Blackorby and Schworm (1983) have shown, these restrictions are inconsistent with flexible accelerator factor demands when both positive and negative changes in the Qs occur. This inconsistency can be avoided by the use of gross investment.

15. Here both stages of the production decisions are combined into a single stage. In addition, a production function is specified because there is only a single output. We could just as easily have tackled this special case of the general model in two stages.

16. The disturbances in the factor demand equations (29) can also reflect technology shocks. However, by a similar argument to that presented in footnote 9, estimation problems arise. The error in the production function, from which the factor demands are derived, must be contemporaneously correlated with each of the factors in order for technology shocks to appear in each of the factor demand functions.

17. W_K is also defined in the discussion after equation (19). It is an outcome of the static price and tax expectations assumption. The disturbance in the investment equation represents optimizing or measurement errors. If the disturbance reflects technology shocks, then the error in the normalized variable cost function is contemporaneously correlated with the quasi–fixed factor.

18. Epstein and Denny (1983) estimate models with both static expectations and expectations generated by first order autoregressive processes. However, in the latter case the processes were estimated independently of the production decisions.

19. In addition, this functional form is consistent with aggregation conditions guaranteeing the existence of a representative firm (see Diewert (1980), Epstein and Denny (1983) and Blackorby and Schworm (1983)).

20. Usually average hours worked is the measure of labour utilization.

21. The stochastic disturbances in equations (38) and (39) represent optimization and measurement errors. The disturbance in the variable profit function can also represent technology shocks. However, if the disturbances in equation (39) represent technology shocks, then the error in the variable profit function is contemporaneously correlated with the factor and product prices.

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