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LABOR MARKET POWER

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ABSTRACT

To measure labor market power in the US economy, we develop a tractable quantitative, general equilibrium, oligopsony model of the labor market. We estimate key model parameters by matching the firm-level relationship between labor market share and employment size and wage responses to state corporate tax changes. The model quantitatively replicates quasi-experimental evidence on (i) imperfect productivity-wage pass-through, (ii) strategic behavior of dominant employers, and (iii) the local labor market impact of mergers. We then measure welfare losses relative to the efficient allocation. Accounting for transition dynamics, we quantify welfare losses from labor market power relative to the efficient allocation as roughly 6 percent of lifetime consumption. An analytical decomposition attributes equal parts to dead-weight losses and misallocation. Lastly, we find that declining local concentration added 4 ppt to labor's share of income between 1977 and 2013.

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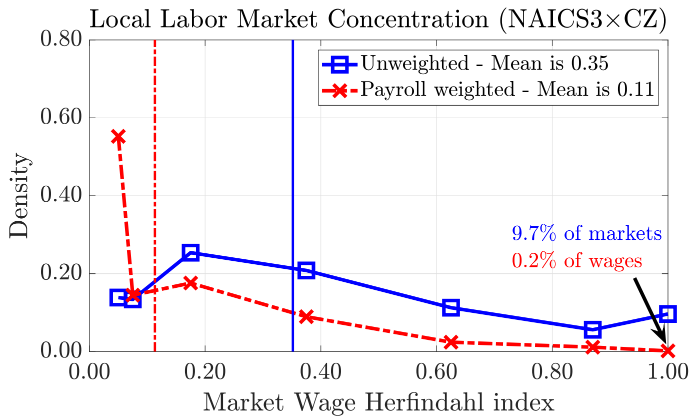

Figure 1: Cross-market distribution of concentration: Longitudinal Business Database, 2014.

Notes: Panel A plots the distribution of number of firms in markets. Panels B plots the across market distribution of the payroll Herindahl index (HHI_j^{wn}). Bins are determined by the following bounds: $\{0, 0.10, 0.25, 0.50, 0.75, 0.99, 1\}$. Horizontal axis gives the mean in each bin. Blue line (solid squares) gives the distribution of markets, red line (dashed crosses) gives the distribution of total wage payments. Data is Census LBD for the whole US economy in 2014. Market is defined as a commuting zone and NAICS 3-digit industry. See Appendix C for additional details. Table A2 provides additional data on employment HHI's.

In the average local labor market in the U.S., there are many firms but employment and wages are concentrated in only a few. Defining a labor market as a commuting zone and three-digit industry, the average number of firms is over 100, while the weighted average level of market concentration is 0.11, the same level of concentration one would observe with only 9 equally sized firms (Figure 1).[1] This has led to the growing concern that these firms may exert "labor market power" over their workers, generating large welfare losses.[2] In this paper, we measure the amount of oligopsony power in labor markets and quantify its consequences for welfare. We do so by developing a tractable, quantitative, general equilibrium model with differentially concentrated local labor markets in which firms behave strategically under an oligopsony equilibrium. These novel features allow the model to match empirical regularities in the labor literature such as incomplete wage pass-through and strategic competitor wage responses that a standard monopsony model misses. The model delivers a structurally consistent formulation of labor market power and a framework for understanding the mechanisms behind potential welfare losses.

Our benchmark oligopsony model features two sources of market power. The first is classical *monopsony*: atomistically small firms face upward sloping labor supply curves due to preference heterogeneity, which they internalize (Burdett and Mortensen, 1998; Manning, 2003; Card, Cardoso, Heining, and Kline, 2018; Lamadon, Mogstad, and Setzler, 2019). Optimal wages are a markdown relative to competitive wages, i.e. the marginal revenue product of labor. Second, motivated by Figure 1 and the focus of this paper, is *oligopsony*: firms are non-atomistic and compete strategically for workers, further internalizing how they expect other employers to respond to their hiring and wage policies. This strategic interaction

[1] Appendix Table F2 reports 113 firms per market across all industry codes. Appendix C provides additional market level summary statistics.

[2] For example: Azar, Marinescu, and Steinbaum (2020), Benmelech, Bergman, and Kim (2020), Card, Cardoso, Heining, and Kline (2018), and Lamadon, Mogstad, and Setzler (2019).

leads to large equilibrium markdowns at the most productive firms and provides a second source of welfare loss. Understanding the welfare consequences of labor market power requires understanding how these markdowns vary across firms. In our model, the markdown is an exact function of the *structural labor supply elasticity* that a firm faces in equilibrium which—via a closed-form—depends on the firm's observable labor market share and parameters that determine how easily labor is reallocated across- (θ) and within- (η) markets.

We estimate the model on U.S. Census data, and derive three main results. First, the framework is quantitatively consistent with documented empirical regularities suggestive of oligopsony: incomplete wage pass-through, strategic competitor wage responses, and size-dependent post-merger wage dynamics. A monopsony version of our model cannot qualitatively match these empirical regularities. Second, the model implies substantial welfare losses from labor market power, both across steady states and along the transition path to an efficient allocation. Welfare losses are large, ranging from 4 to 9 percent of lifetime consumption depending on wealth effects. A representative agent counterpart to our economy delivers equilibrium aggregate prices and quantities and decomposes welfare loss into two components: (1) a dead-weight loss due to average markdowns, (2) a misallocation effect due to wider markdowns at more productive firms. While the former exists under monopsony, the latter does not. We show that both channels account equally for welfare losses. Third, despite these large losses, we find that labor market power has not contributed to the declining labor share. Despite the backdrop of stable *national* concentration, we find that the model-consistent measure of *local* concentration, which we measure for the first time, has declined over the last 35 years, indicating that most local labor markets are more competitive than they were in the 1970s.[3]

In terms of the general equilibrium theory of the model, we prove two properties that are central to our main applications. First, we show that our model is block recursive, meaning that local labor market equilibrium is independent of aggregates. This allows us to estimate the model quickly and decompose welfare for arbitrary aggregate preferences. Second, we provide a closed-form relationship between labor's share of income and local payroll concentration. Our model-relevant measure of payroll concentration is new to the literature. We use our formula to measure the contribution of changes in local payroll concentration on labor's share of income.

In terms of estimation of the model, strategic interaction complicates the identification of the key parameters by violating exclusion restrictions that are otherwise applicable in monopsonistically competitive models. We address this issue by integrating into our structural estimation the first reduced-form estimates of the size dependence of employment and wage responses to state corporate taxes. We estimate our key parameters using U.S. Census Longitudinal Business Database (LBD) micro data (see Figure 2). Given a quasi-experiment that yields an identified shock to labor demand, a researcher can estimate *reduced-form labor supply elasticities* off of relative employment and wage responses. The literature so far has assumed a special case of our model: firms do not behave strategically, rationalized by

[3]In contemporaneous work Rinz (2018) also uses Census data and shows similar patterns for alternative measures of concentration. These measures are not exactly those that are welfare relevant for the model. Rossi-Hansberg, Sarte, and Trachter (2018) use NETS data and find similar patterns in sales and employment concentration.

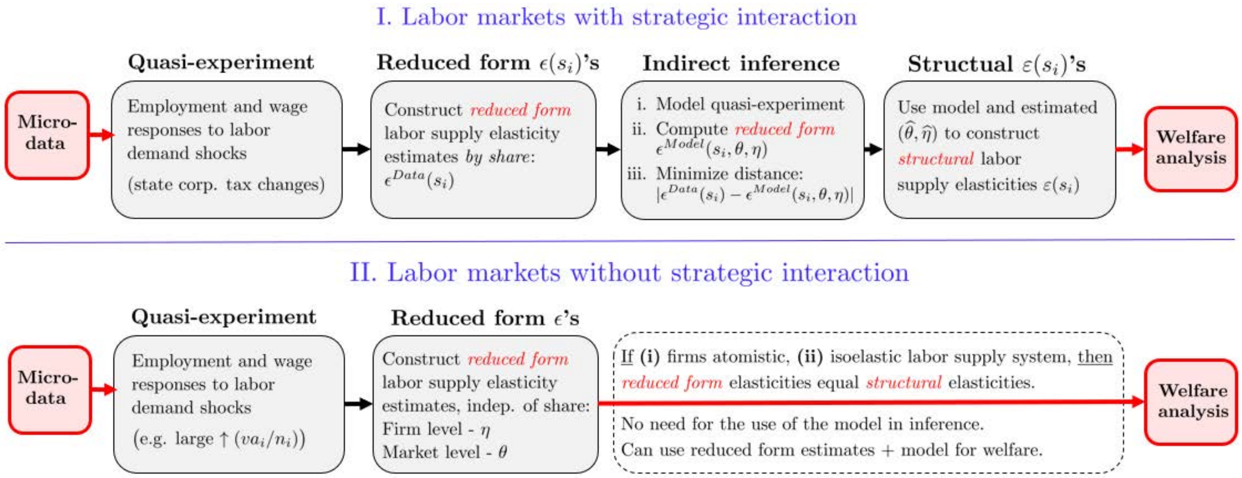

Figure 2: Estimation strategy

infinitely many firms in each labor market.[4] This assumption abstracts from competitor equilibrium best responses, and implies that empirically estimated *reduced-form elasticities* are equal to *structural elasticities*, so one can move directly from empirical analysis to welfare analysis. In the more general case of granular labor markets, there is no closed-form mapping between (observed) reduced-form elasticities and (unobserved) structural elasticities.[5] A model is needed to account for the equilibrium best responses that determine the mapping between underlying structural parameters and the reduced-form elasticities we observe.

Our approach is therefore indirect inference. Our quasi-experiment is an extension of Giroud and Rauh (2019). We exploit state corporate tax rate changes to estimate *reduced-form elasticities*. We extend their methodology to characterize how they relate to a firm's local labor market share. We then simulate tax changes in our model and determine the structural parameters that minimize the distance between the profile of reduced-form elasticities by market share in model and data. The estimated model is then used to compute structural elasticities, markdowns, and conduct welfare counterfactuals.[6]

This departure from the literature contributes three additional results. First, in the data, responses of firms to labor demand shocks vary systematically: firms with smaller market shares have statistically significantly larger reduced-form elasticities than firms with larger market shares. Second, in our particular experiment, *reduced-form elasticities* at small firms are around 2, but welfare-relevant *structural elasticities* are around 7. Filtering the data through the model is necessary to uncover the high labor supply elasticities faced by small firms. Third, we explore bias in more common empirical settings that

[4]Papers in the literature that study strategic behavior have been theoretical, which we discuss below.

[5]The finitely many firms case is indeed more general. That is, a 'competitive' monopsony model is indeed a special case of our model. Taking the number of firms in all markets in our model toward infinity smoothly yields the 'competitive' economy in which there is no strategic interaction. We let the data tell us where we are on this spectrum between one and infinitely many firms per market.

[6]This procedure has a direct counterpart in the estimation of linearized state-space systems in macroeconomics: $A\boldsymbol{X}_t = B\mathbb{E}[\boldsymbol{X}_{t+1}] + C\boldsymbol{X}_{t-1} + D\varepsilon_t$. The *structural model* implies a *reduced-form* VAR representation: $\boldsymbol{X}_{t+1} = H\boldsymbol{X}_t + F\boldsymbol{e}_{t+1}$. The researcher first estimates the reduced-form on the data to obtain *reduced-form* shocks $\{\widehat{\boldsymbol{e}}_t\}_{t=0}^T$. They then simulate *structural shocks* $\{\varepsilon_t\}_{t=0}^T$ in the model and jointly estimate *structural parameters* $\{A, B, C, D\}$ and *structural shocks* $\{\varepsilon_t\}_{t=0}^T$ such that the model implied reduced-form shocks match those obtained from the data.

exploit purely idiosyncratic variation. Here results are different; when we account for the new market equilibrium, *structural elasticities* are always less than empirically estimated *reduced-form elasticities*, often by a large amount. A researcher using reduced-form estimates for welfare analysis would infer flat labor supply curves and *understate* the degree of labor market power.

We validate the estimated model by replicating three reduced-form experiments that distinguish oligopsony from monopsonistic competition and find in all cases that our model estimates are within the 95% confidence interval of the published estimates. First, we replicate the 0.47 pass-through from log value added per worker to log wages in Kline, Petkova, Williams, and Zidar (2019), producing 0.61 in our model. Second, we replicate the 0.13 response elasticity of competing hospital's wages to VA hospital wage increases in Staiger, Spetz, and Phibbs (2010), producing 0.07 in our model. Third, we replicate the 0.8 percent decline in worker wages following a merger in Arnold (2020a), producing a 1.3 percent decline in our model, and matching a 3 times larger decline in more concentrated markets. Theoretically, we prove that a monopsonistically competitive economy features a pass-through of one, a competitor response elasticity of zero, and no effect of mergers on competitors. These tests provide evidence that oligopsony is necessary to fit key empirical regularities in the reduced-form literature.

With our model calibrated to aggregates and local labor markets, we define the *welfare loss* due to labor market power as the consumption subsidy required to make households indifferent between the oligopsonistic economy and the efficient allocation that a planner would choose. Comparing steady states at an aggregate Frisch elasticity of labor supply of 0.50, we measure a welfare loss of 7.0 percent. Competitive equilibrium wages, output and labor supply are significantly greater. Welfare losses are slightly lower (5.7 percent) when accounting for macroeconomic transition dynamics between these two labor market structures. We show that these results are robust to aggregate preferences being of Greenwood Hercowitz Huffman (1988, henceforth GHH) or balanced growth types.[7]

We explore the mechanisms underlying these large welfare losses using a novel representative agent counterpart to our economy. We decompose output losses into two components. The first component is an aggregate markdown which reflects pure dead-weight loss from oligopsony power. The second component is an aggregate efficiency loss that reflects misallocation. Productive firms have the most labor market power and widest markdowns. They therefore restrict employment the most. This results in an inefficient under allocation of employment at the most productive firms and the wider the productivity distribution, the larger the wage mark-downs and subsequent efficiency losses. Overall, we find that roughly 50 percent of welfare losses are driven by misallocation, 40 percent are due to pure markdowns, and the remainder is due to their interaction.[8] This would not be the case in a standard monopsonistically competitive environment. We show that the misallocation effect is zero under monopsonistic competition, so strategic interactions and markdown heterogeneity account for roughly half of the losses observed.

[7]With more significant wealth effects on labor supply, welfare losses are smaller, but still exceed 4 percent even with a coefficient of relative risk aversion of four. With a higher Frisch elasticity of labor supply, welfare losses are larger. Under an aggregate Frisch of 0.2 (0.8), welfare losses are 4.8 (9.2

[8]With more significant wealth effects on labor supply, welfare losses due to misallocation increases. With a higher Frisch elasticity of labor supply, welfare losses due to the aggregate markdown increases.

A symptom of the misallocation present in the benchmark economy is that the planner's solution has *greater* concentration, employment, and wages. In the oligopsonistic economy, large firms are inefficiently small, so any policy that decentralizes the efficient allocation would reallocate more employment to already large firms. Aggregate concentration roughly doubles, employment increases by 11 percent and the average wage increases by 48 percent.

We conclude by applying the model to study the relationship between local labor market concentration and the labor share. Despite large welfare losses from labor market power, we find that declining local labor market concentration between 1977 and 2013 *increased* labor's share of income. First, letting our model guide measurement, we show that the distribution of market-level payroll Herfindahls can be used to compute a sufficient statistic for labor's share of income, with a relationship that is independent of the aggregate labor supply elasticity and wealth effects.[9] Second, the model implies that these micro measures should be aggregated using market-level payroll weights, shown in red in Figure 1B. We construct this model relevant concentration measure directly from the Census LBD and find it has declined from 0.16 to 0.11 between 1977 and 2013.[10] Ignoring these weights would double the level of concentration and imply a stable trend.[11] We feed our measure into our formula for labor's share of income under the estimated preference parameters (θ, η). We find that declining local labor market concentration would have implied a counterfactual 4 percentage point increase in labor's share of income. Changing labor market concentration is not behind the declining labor share.[12]

We review the literature and then proceed as follows. Sections 1 lays out the model and characterizes the equilibrium. Section 2 provides empirical estimates of the relationship between reduced-form labor supply elasticities and market share, then combine this relationship and our new concentration statistics to parameterize the model. Section 3 validates the model via replication of three empirical studies. Section 4 presents our main welfare measurement exercises. Section 5 applies the model to measure welfare-relevant aggregate concentration and the labor share.

Literature. Our work is related to a growing literature that explores the implications of market power. In the product market, Gutiérrez and Philippon (2017); Autor, Dorn, Katz, Patterson, and Van Reenen (2020) all document an increase in national sales concentration and a fall in the labor share across many industries, while De Loecker, Eeckhout, and Unger (2020) document an increase in product market power more directly by measuring firm markups. Consistent with our findings, concurrent work by Rossi-Hansberg, Sarte, and Trachter (2018) documents declining regional employment concentration, despite rising national concentration. In the labor market, several concurrent studies have documented

[9] The market-level wage-bill Herfindahl is the sum of the squared payroll shares of all firms within the labor market

[10] These measures of concentration are equivalent to what would be obtained with 6.25 equally sized firms per market in 1977, and 9.43 equally sized firms per market in 2013.

[11] Our model replicates the distribution and means of both weighted and unweighted Herfindahls in the data. The large difference between weighted and unweighted Herfindahls is due to the fact that 11 percent of markets have one firm, and thus a Herfindahl of 1, yet these markets only comprise 0.18 percent of aggregate payroll. Moreover, the payroll share of concentrated markets is falling, presumably as individuals leave highly concentrated rural markets for less concentrated city markets.

[12] Interestingly, in their recent paper on the dynamics of the labor share, Kehrig and Vincent (2021) find evidence consistent with our results, as employment reallocation is roughly independent of output reallocation (see their Fig. III).

cross-sectional and time-series patterns of U.S. Herfindahls in employment (Benmelech, Bergman, and Kim, 2020; Rinz, 2018; Hershbein, Macaluso, and Yeh, 2020) and vacancies (Azar, Marinescu, Steinbaum, and Taska, 2020; Azar, Marinescu, and Steinbaum, 2020). Brooks, Kaboski, Li, and Qian (2019), Hershbein, Macaluso, and Yeh (2020), and Chan, Salgado, and Xu (2020) use tools from industrial organization to identify wage markdowns and heterogeneous pass-through rates consistent with the theory in this paper. Our contributions to this literature are (i) a new, model consistent, measure of U.S. labor market concentration, which we use to (ii) quantitatively measure the welfare losses associated with labor market power. In general, the exercises in our paper issue a warning against qualitatively mapping changes in concentration into a change in welfare.

Our work is also related to a large literature measuring reduced-form labor supply elasticities of individual firms (Staiger, Spetz, and Phibbs, 2010; Webber, 2015; Card, Cardoso, Heining, and Kline, 2018; Suárez Serrato and Zidar, 2016; Dube, Jacobs, Naidu, and Suri, 2020). We provide new estimates of measured labor supply elasticities by building on the approach of Giroud and Rauh (2019), who find significant effects of state corporate taxes on firm-state employment.[13] Our contributions to this empirical literature are (i) estimates of the share-dependency of measured elasticities that point to large firms having more market power (ii) to demonstrate that if markets have firms that interact strategically, there can be a large disconnect between measured labor supply elasticities and the structural elasticities that are relevant for welfare. This is a substantive point: the empirical literature cited above typically measures labor supply elasticities that are small. If structural elasticities were equal to these reduced-form elasticities, then labor market power would be extremely high.[14] We describe empirical designs under which (i) reduced-form estimates of labor supply elasticities may be biased downwards relative to structural elasticities, and even then, (ii) that structural elasticities vary systematically with the firm's labor market share, and show that this reconciles the range and level of empirical estimates.

Finally, our work is related to the large literature that models monopsony in labor markets. We depart from benchmark models of monopsony described in (Burdett and Mortensen, 1998; Manning, 2003; Card, Cardoso, Heining, and Kline, 2018; Lamadon, Mogstad, and Setzler, 2019; Kroft, Luo, Mogstad, and Setzler, 2020) by explicitly modeling a finite set of employers that compete strategically for workers. We demonstrate that this addition is crucial for identification: strategic interaction and finiteness of firms jointly imply that reduced-form empirical estimates of labor supply elasticities from *any* shock cannot be used to infer the (structural) labor supply elasticities firms face—and hence identify preference parameters—except in the limiting case of monopsonistic competition between infinitesimally sized firms. Additionally, our assumptions allow us to (i) interpret granular measures of concentration, such as Herfindahl indexes, and (ii) accommodate a planning problem that allows us to define an efficient benchmark.

[13]Conceptually, our approach is related to papers that estimate exchange rate pass-through (Amiti, Itskhoki, and Konings, 2014, 2019). The main difference is that this literature focuses exclusively on prices, whereas we look at both price and quantity responses.

[14]Consider Manning (2011) discussing the widely cited natural experiments of Staiger, Spetz, and Phibbs (2010) and others: *"Looking at these studies, one clearly comes away with the impression not that it is hard to find evidence of monopsony power but that the estimates are so enormous to be an embarrassment even for those who believe this is the right approach to labour markets."*

Our main quantitative contribution is to build a general equilibrium model of oligopsony and measure the welfare costs of current levels of U.S. labor market power.[15] Our framework extends the general tools developed in Atkeson and Burstein (2008) to the labor market, adding multiple non-trivial features: capital, corporate taxes, decreasing returns to scale, setting the model in general equilibrium, and studying transition dynamics between steady states. Recent related work by Jarosch, Nimcsik, and Sorkin (2019) considers non-atomistic firms, but adapts a random search model to construct a search-theoretic measure of labor market power. We view our papers as complementary.

Our model features firm-specific upward sloping labor supply curves. This is supported by numerous recent studies using (quasi-)experimental approaches.[16] Belot, Kircher, and Muller (2017) randomly assign higher wages to observationally equivalent vacancies on an actual job-board and find that higher wage vacancies attract more applicants. Dube, Jacobs, Naidu, and Suri (2020) and Banfi and Villena-Roldan (2018) also find job-specific upward sloping labor supply curves in well-identified contexts.[17]

Finally, our quantitative model features strategic complementarity between oligopsonists. Strategic complementarity in labor markets is not new to the theoretical literature. The earliest models used to motivate monopsony power were Robinson (1933) and the spatial economies of Hotelling (1990) and Salop (1979).[18] Our contribution relative to these stylized single-market models, is a quantitative general equilibrium framework. We incorporate firm heterogeneity, decreasing returns to scale, and general equilibrium across multiple markets, such that the model is rich enough to be estimated on U.S. Census data. Moreover, by modeling a finite set of employers, our model may be used in the future to understand the wage and welfare effects of mergers, firm exit, and other shocks to local labor market competition. Very recent work by Azkarate-Askasua and Zerecero (2020) and MacKenzie (2019) also estimate models with strategic interactions using French and Indian data, respectively. Our contribution is to develop a quantitative general equilibrium framework and develop a methodology to consistently estimate the underlying preference parameters governing oligopsony.

1 Model

1.1 Environment

Agents. The economy consists of a representative household and a continuum of firms. The household consists of a unit measure of atomistic, homogeneous workers each with one unit of labor supply. Firms are heterogeneous in two dimensions. First, firms inhabit a continuum of local labor markets $j \in [0,1]$, each with an exogenous and finite number of firms indexed $i \in \{1, 2, \ldots, m_j\}$. Second, firms' productivities $z_{ijt} \in (0, \infty)$ are drawn from a location invariant distribution $F(z)$. The *only ex-ante difference*

[15]Our work is therefore related to a literature measuring the welfare consequences of misallocation. There the focus has been on the product market (Baqaee and Farhi, 2020; Edmond, Midrigan, and Xu, 2018), and measures misallocation via heterogeneous markups. Our paper measures misallocation from heterogeneous mark-downs.

[16]See Ashenfelter, Farber, and Ransom (2010) for a summary of prior papers.

[17]We are unaware of experimental evidence regarding the market-share dependence of the elasticity of labor supply.

[18]Boal and Ransom (1997) and Bhaskar, Manning, and To (2002) provide excellent summaries of strategic complementarity in spatial models of the labor market.

between markets is the number of firms $m_j \in \{1, \ldots, \infty\}$. Time subscripts are necessary in that we study welfare counterfactuals on transition paths between steady-states, but productivity and number of firms are constant at the firm- and market-level, respectively.

Goods and technology. The continuum of firms produce tradeable goods that are perfect substitutes, and so trade in a perfectly competitive national market at a price P_t that we normalize to one. Firms operate a *value-added* production function that uses inputs of capital k_{ijt} and labor n_{ijt}.[19] A firm produces y_{ijt} units of net-output (value-added) according to the production function:

$$y_{ijt} = z_{ijt} \left(k_{ijt}^{1-\gamma} n_{ijt}^{\gamma} \right)^{\alpha} \quad , \quad \gamma \in (0,1) \quad , \quad \alpha > 0.$$

The degree of returns to scale α is unrestricted and later estimated. The household uses these goods for consumption and investment. Investment augments the capital stock K_t, which is rented to firms in a competitive market at price R_t and depreciates at rate δ. To the best of our knowledge this is the first paper to model imperfect competition, either in input or output markets, with finitely many firms and decreasing returns to scale in general equilibrium. To model imperfect competition we extend tools developed in the trade literature (Atkeson and Burstein, 2008).

1.2 Household

Preferences and problem. The household chooses the measure of workers to supply to each firm n_{ijt}, investment in next period capital K_{t+1}, and consumption of each good c_{ijt} to maximize their net present value of utility. Given an initial capital stock K_0, the household solves

$$\mathcal{U}_0 = \max_{\left\{n_{ijt}, c_{ijt}, K_{t+1}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U\Big(\boldsymbol{C}_t, \boldsymbol{N}_t\Big) \tag{1}$$

where the aggregate consumption and labor supply indexes are given by:

$$\boldsymbol{C}_t := \int_0^1 \Big[c_{1jt} + \cdots + c_{m_j jt}\Big]\, dj \ , \ \boldsymbol{N}_t := \left[\int_0^1 \boldsymbol{n}_{jt}^{\frac{\theta+1}{\theta}}\, dj\right]^{\frac{\theta}{\theta+1}} \ , \ \boldsymbol{n}_{jt} := \left[n_{1jt}^{\frac{\eta+1}{\eta}} + \cdots + n_{m_j jt}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}} \ , \ \eta > \theta > 0$$

and maximization is subject to the household's budget constraint in each period:

$$\boldsymbol{C}_t + \Big[K_{t+1} - (1-\delta)K_t\Big] = \int_0^1 \Big[w_{1jt}n_{1jt} + \cdots + w_{m_j jt}n_{m_j jt}\Big]\, dj + R_t K_t + \Pi_t. \tag{2}$$

Firm profits, Π_t, are rebated lump sum to the household. The function U is twice continuously differentiable with standard properties.[20] The consumption index captures perfect substitutability of consumption goods, such that our assumption of a single market price $P_t = 1$ is valid.[21]

[19] Since aggregating firm-level value-added yields aggregate output (GDP), we abuse terminology and refer to the output of this production function interchangeably in terms of goods and value-added. We carefully distinguish the two when comparing our results to empirical studies.

[20] Properties: $U_C > 0$, $U_{CC} < 0$, $U_N < 0$, $U_{NN} > 0$, $\lim_{C\to 0} U_C = -\lim_{N\to\infty} U_N = \infty$, $\lim_{C\to\infty} = -\lim_{N\to 0} U_N = 0$.

[21] Observe that since we are solving the model with decreasing returns to scale in production, we are arbitrarily able to introduce monopolistic competition in the national market for goods. Let $\boldsymbol{C}_t = [\int \sum_{i\in j} c_{ijt}^{(\sigma-1)/\sigma} dj]^{\sigma/(\sigma-1)}$, then given household's

Notation. Aggregate variables are denoted in upper-case, and firm- and market-level in lower-case. Bold fonts are used for indexes, which are book-keeping devices, not directly observable in the raw data, but can be constructed from observables. For example, the disutility of labor supply $\boldsymbol{N}_t$ does not correspond to any aggregates reported by the Bureau of Labor Statistics. However, given parameters, $\boldsymbol{N}_t$ can be constructed from the universe of firm-level employment $\{n_{ijt}\}$. We denote aggregate labor computed by adding bodies as unbolded: $N_t = \sum_{ij} n_{ijt}$.

Optimality conditions. The first order necessary conditions of the household problem describe the supply of labor and capital:

$$-\frac{U_N\left(\boldsymbol{C}_t, \boldsymbol{N}_t\right)}{U_C\left(\boldsymbol{C}_t, \boldsymbol{N}_t\right)} \frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}} \frac{\partial \boldsymbol{n}_{jt}}{\partial n_{ijt}} = w_{ijt} \quad , \quad U_C\left(\boldsymbol{C}_t, \boldsymbol{N}_t\right) = \beta U_C\left(\boldsymbol{C}_{t+1}, \boldsymbol{N}_{t+1}\right)\left[R_t + (1-\delta)\right] \tag{3}$$

Labor supply. Under the assumed structure of preferences, we can express the set of labor supply conditions across all firms more economically as follows:

$$\underbrace{-\frac{U_N\left(\boldsymbol{C}_t, \boldsymbol{N}_t\right)}{U_C\left(\boldsymbol{C}_t, \boldsymbol{N}_t\right)} = \boldsymbol{W}_t}_{\text{Aggregate labor supply}} \quad \text{and} \quad \underbrace{n_{ijt} = \left(\frac{w_{ijt}}{\boldsymbol{w}_{jt}}\right)^{\eta} \left(\frac{\boldsymbol{w}_{jt}}{\boldsymbol{W}_t}\right)^{\theta} \boldsymbol{N}_t}_{\text{Firm labor supply for all } i = 1, \dots, m_j,\, j \in [0,1].} \quad \leftrightarrow \quad \underbrace{w_{ijt} = \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{1}{\eta}} \left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{1}{\theta}} \boldsymbol{W}_t.}_{\text{Inverse labor supply curve}} \tag{4}$$

Given aggregate labor supply, the firm labor supply curve includes two book-keeping terms: the *market wage index* $\boldsymbol{w}_{jt}$ and *aggregate wage index* $\boldsymbol{W}_t$. These are defined as the numbers that satisfy

$$\boldsymbol{w}_{jt}\boldsymbol{n}_{jt} := \sum_{i \in j} w_{ijt} n_{ijt} \quad , \quad \boldsymbol{W}_t \boldsymbol{N}_t := \int_0^1 \boldsymbol{w}_{jt}\boldsymbol{n}_{jt}\, dj.$$

Together with optimality conditions (4) the definitions imply

$$\boldsymbol{w}_{jt} = \left[\sum_{i \in j} w_{ijt}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \boldsymbol{W}_t = \left[\int_0^1 \boldsymbol{w}_{jt}^{1+\theta} dj\right]^{\frac{1}{1+\theta}}. \tag{5}$$

Since labor market competition is Cournot, firms choose quantities taking their inverse labor supply curve (4) into account. For full derivations see Appendix E.1.

Explicit Microfoundation. In Appendix B, we show that the supply system described by equations (4) and (5) can be obtained in an environment with heterogeneous workers making independent decisions, providing an exact map between η and θ and the distribution of relative net costs to individuals of moving between and across markets.[22] The micro-foundation makes clear that workers are not confined

optimal demand schedules, a firm would optimize a decreasing returns to scale *revenue function* as opposed to the decreasing returns to scale *production function* used here. Firms would charge identical time-invariant markups, and profits due to market power in the product market would be rebated to the household. To keep our analysis clean, we ignore this case.

[22] Recent (non-nested) logit formulations of individual decisions have also been used to model the supply of labor to a firm in competitive markets (Card, Cardoso, Heining, and Kline, 2018; Borovickova and Shimer, 2017). Our contribution is to adapt results in the discrete choice literature to demonstrate equivalence with our 'nested-CES' specification, and to set the problem in oligopsonistic markets. In particular, We adapt arguments from the product market case due to Verboven (1996). That paper the establishes the equivalence of nested-logit and nested-CES, extending the results of Anderson, De Palma, and Thisse (1987) which establishes an equivalence between single sector CES and single sector logit.

to particular markets. The limitation that markets impose is on the boundary of the strategic behavior of firms. Within markets firms are strategic, but with respect to firms in the continuum of other markets, firms are price takers.

Elasticities. The firm labor supply curve is upward sloping and features two elasticities of substitution $\eta > 0$ and $\theta > 0$. These jointly affect the labor market power of firms. Both across and within markets, the lower the degree of substitutability, the greater the market power of firms. Across-market substitutability θ stands in for mobility costs across markets, which are often estimated to be significant (Kennan and Walker, 2011). As such costs increase ($\theta \to 0$), the household minimizes labor disutility $\boldsymbol{N}_t$ by choosing an equal division of workers across markets: $\boldsymbol{n}_{jt} = \boldsymbol{n}_{j't}, \forall j, j' \in [0,1]$. This imparts the largest degree of local labor market power as market-by-market market-level employment becomes perfectly inelastic and unresponsive to across-market wage differences. As substitutability approaches infinity, the representative household optimally sends all workers to the market with the highest wage, eroding market power of firms in competing markets.

Within-market substitutability η stands in for within-market, across-firm mobility costs such as the job search process (Burdett and Mortensen, 1998), some degree of non-generality of accumulated human capital (Becker, 1962), or preference heterogeneity in the form of worker-firm specific amenities or commuting costs (Robinson, 1933). As these costs increase ($\eta \to 0$), the household minimizes within-market disutility $\boldsymbol{n}_{jt}$ by choosing an equal division of workers across firms: $n_{ijt} = n_{i'jt}, \forall i, i' \in \{1, 2, ...m_j\}$. This generates the largest degree of monopsony power to firms within a market. Regardless of its wage, firm-ij will employ the same number of workers, allowing it to pay less while maintaining its workforce. As substitutability increases, competition tightens as workers are reallocated toward firms with higher wages.

Regardless of θ, in the limit as $\eta \to \infty$, local labor markets tend to perfect competition. In this limit, marginal revenue products are equalized across firms at a single market wage $w_{ij} = \boldsymbol{w}_j$. This is possible with productivity heterogeneity due to decreasing returns as in Hopenhayn (1992). A model without decreasing returns would mistakenly infer labor market power from the fact that there is productivity heterogeneity and many firms operate in each market.

1.3 Firms

In order to maximize profits, firms choose how much capital to rent, k_{ijt}, and the number of workers to hire n_{ijt}. Infinitesimal with respect to the macroeconomy, firms take the aggregate wage $\boldsymbol{W}_t$ and labor supply $\boldsymbol{N}_t$ as given. Since the equilibrium concept is Cournot, they also take as given their competitors' employment decisions, which we denote n^*_{-ijt}.

The firm maximizes profits:

$$\pi_{ijt} \;=\; \max_{n_{ijt},k_{ijt}} \underbrace{z_{ijt}\left(k_{ijt}^{1-\gamma}n_{ijt}^{\gamma}\right)^{\alpha}}_{\text{Value added: } y_{ijt}} - R_t k_{ijt} - w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{N}_t, \boldsymbol{W}_t\right) n_{ijt}. \tag{6}$$

$$s.t. \qquad w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{N}_t, \boldsymbol{W}_t\right) \;=\; \left(\frac{n_{ijt}}{\boldsymbol{n}(n_{ijt}, n^*_{-ijt})}\right)^{\frac{1}{\eta}} \left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{1}{\theta}} \boldsymbol{W}_t \quad , \quad \boldsymbol{n}(n_{ijt}, n^*_{-ijt}) = \left[n_{ijt}^{\frac{\eta+1}{\eta}} + \sum_{k\neq i} n^{*\,\frac{\eta+1}{\eta}}_{kjt}\right]^{\frac{\eta}{\eta+1}}$$

The first order necessary conditions of the firm problem describe its demand for capital and labor:

$$R_t = \alpha(1-\gamma)\frac{y_{ijt}}{k_{ijt}} \quad , \quad \underbrace{w_{ijt} + \left.\frac{\partial w_{ijt}}{\partial n_{ijt}}\right|_{n^*_{-ijt}} n_{ijt}}_{\text{Marginal cost: } mc_{ij}} = \alpha\gamma\frac{y_{ijt}}{n_{ijt}}.$$

The firm has a standard competitive demand for capital, but since the firm has market power in the labor market, its marginal cost of labor accounts for both the wage and the additional cost associated with increasing wages. This requires an equilibrium marginal revenue product of labor that exceeds the wage alone. The standard re-arrangement of the labor demand condition yields a Lerner condition for the wage as a markdown $\mu_{ijt} \leq 1$ on the marginal product of labor:

$$w_{ijt} = \mu_{ijt}\alpha\gamma\frac{y_{ijt}}{n_{ijt}} \quad , \quad \mu_{ijt} = \frac{\varepsilon_{ijt}}{\varepsilon_{ijt}+1} \quad , \quad \varepsilon_{ijt} := \left[\left.\frac{\partial \log w_{ijt}}{\partial \log n_{ijt}}\right|_{n^*_{-ijt}}\right]^{-1}. \tag{7}$$

Under our specification of preferences, the elasticity and markdown have closed-form expressions that depend only on firms' payroll share s_{ijt} in the market, with larger firms having wider markdowns:

$$\varepsilon(s_{ijt}) = \left[\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)\left.\frac{\partial \log \boldsymbol{n}_{jt}}{\partial \log n_{ijt}}\right|_{n^*_{-ijt}}\right]^{-1} = \left[\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_{ijt}\right]^{-1} \quad , \quad s_{ijt} := \frac{w_{ijt}n_{ijt}}{\sum_{i=1}^{m_j} w_{ijt}n_{ijt}} = \frac{w_{ijt}n_{ijt}}{\boldsymbol{w}_{jt}\boldsymbol{n}_{jt}}.$$

We characterize the solution of the economy in three steps: partial equilibrium, market equilibrium, and general equilibrium.

1.4 Characterization - Partial equilibrium

It will be useful to substitute the firms' capital demand condition into its profits (6), which gives:

$$\pi_{ijt} = \max_{n_{ijt}} \; \widetilde{z}_{ijt} n_{ijt}^{\widetilde{\alpha}} - w_{ijt}n_{ijt} \quad , \quad \text{subject to the inverse labor supply curve (4),.}$$

where we introduce the auxiliary parameters $\{\widetilde{\alpha}, \widetilde{z}_{ijt}\}$:

$$\widetilde{\alpha} := \frac{\gamma\alpha}{1-(1-\gamma)\alpha} \quad , \quad \widetilde{z}_{ijt} := \left[1-(1-\gamma)\alpha\right]\left(\frac{(1-\gamma)\alpha}{R_t}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} z_{ijt}^{\frac{1}{1-(1-\gamma)\alpha}}.$$

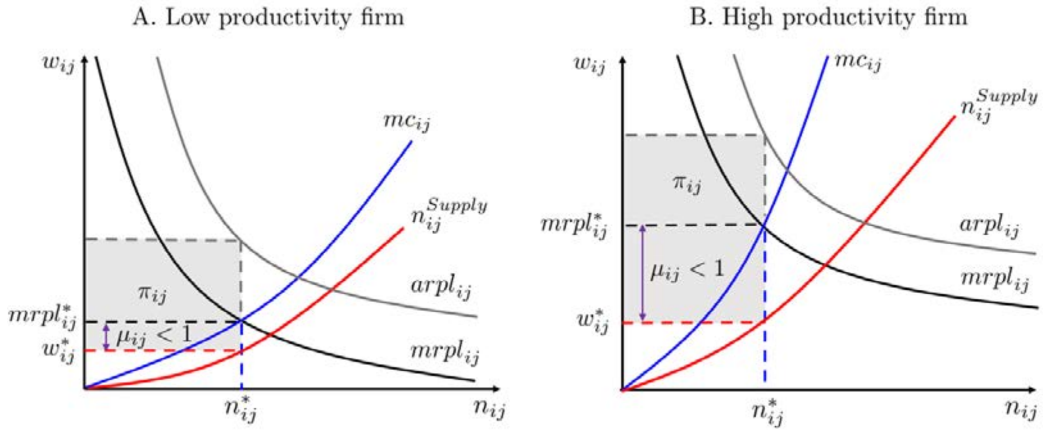

Figure 3: Firm level optimality

We can then express the *markdown* ($\mu_{ijt} \in (0,1)$), marginal and average product of labor as:[23]

$$w_{ijt} = \mu\left(s_{ijt}\right)\ mrpl_{ijt} \qquad , \qquad mrpl_{ijt} = \widetilde{\alpha}\widetilde{z}_{ijt}n_{ijt}^{\widetilde{\alpha}-1} \qquad , \qquad arpl_{ijt} = \frac{1}{\widetilde{\alpha}}mrpl_{ijt} \tag{8}$$

with the same formulas as above determining the markdown.

Figure 3 characterizes firm optimality. Decreasing returns implies a downward sloping marginal revenue product of labor strictly below the average revenue product. Firms internalize their upward sloping labor supply curve, so their marginal cost of labor is also upward sloping and lies strictly above labor supply which describes the average cost of labor. At the margin, a unit of labor costs more than just the higher wage paid to the marginal worker, since the firm must increase wages paid to all workers. As such, choosing n_{ijt} so that labor's marginal revenue product equals its marginal cost necessarily implies a *markdown* of the wage relative to marginal revenue product. The firm then earns profits of $arpl_{ij} - w_{ij} = (arpl_{ij} - mrpl_{ij}) + (mrpl_{ij} - w_{ij})$ from each worker, with a contribution due to the gap between average and marginal revenue products, and a gap due to the markdown.

These markdowns constitute our measure of *firm level labor market power*, and depend on firm characteristics. As we have established, in the Cournot Nash equilibrium, they are determined by the equilibrium (inverse) labor supply elasticity faced by the firm $(1/\varepsilon_{ijt})$ *at the equilibrium allocation*. This depends on a firm's own (observable) market share as well as the degree of within-market (η) and across-market (θ) labor substitutability. This can be seen by returning to Figure 3. Panel A describes the equilibrium outcomes for a low productivity firm. Relative to the high productivity firm in panel B, the low productivity firm has a lower $mrpl_{ij}$ for any n_{ij}. In equilibrium, it has both lower wages w_{ij}^{*}, and lower employment n_{ij}^{*}, so its share of wage payments s_{ij}^{*}, is smaller. With a smaller share of the labor market wage payments, its partial equilibrium elasticity of labor supply is higher, and its inverse labor supply curve is flatter. A flatter inverse supply curve yields a narrower markdown at its optimal labor demand, n_{ij}^{*}. The larger firm faces an endogenously steeper supply curve and hires more workers at higher wages but at a wider markdown. A key property of this equilibrium is that a larger share of employment is at

[23]Here we have abused description slightly since we are using a value-added production function and maximized out optimal capital, so this is really the marginal "revenue net of capital and intermediate input expense" product of labor.

wide markdown firms.

1.5 Characterization - Market equilibrium

Given firm optimality, we establish properties of the market equilibrium and provide an example which illustrates strategic interactions within the market.

Proposition 1.1. Block recursivity. *In each market $j \in [0,1]$, the equilibrium market shares $s_{1jt}, \ldots, s_{m_j jt}$ satisfy the following m_j equations:*

$$s_{ijt} = \frac{\left[\mu(s_{ijt})^{1-(1-\gamma)\alpha} z_{ijt}\right]^{\frac{\eta+1}{(1-\alpha)(\eta+1)+\alpha\gamma}}}{\sum_{k=1}^{m_j}\left[\mu(s_{kjt})^{1-(1-\gamma)\alpha} z_{kjt}\right]^{\frac{\eta+1}{(1-\alpha)(\eta+1)+\alpha\gamma}}}\ ,\ \mu(s_{ijt}) = \frac{\varepsilon(s_{ijt})}{\varepsilon(s_{ijt})+1}\ ,\ \varepsilon(s_{ijt}) = \left[s_{ijt}\theta^{-1} + (1-s_{ijt})\eta^{-1}\right]^{-1}\ , \forall i = 1,\ldots,m_j \tag{9}$$

This system is independent of aggregate variables, and hence the joint distribution $\{\mu_{ijt}, z_{ijt}\}_{\forall ij}$ is determined under market equilibrium. Moreover, the payroll share of labor at the market level and market payroll concentration are given by the following, and hence independent of aggregates:

$$ls_j = \frac{\sum_{i\in j} w_{ij}n_{ij}}{\sum_{i\in j} y_{ij}} = \Big[\sum_{i\in j} s_{ij} ls_{ij}^{-1}\Big]^{-1} = \alpha\gamma\Big[\sum_{i\in j} s_{ij}\mu_{ij}^{-1}\Big]^{-1} \quad , \quad hhi_j = \sum_{i\in j} s_{ij}^2.$$

Proposition (1.1) establishes that the equilibrium of the model is *block recursive* in that the market equilibrium can be solved without knowledge of aggregate variables. For the proof see Appendix E.3. This has three significant implications. First, solving the Nash equilibrium in a large J number of markets is computationally expensive. Proposition (1.1) says that this need only be done once. Second, the aggregate economy can be arbitrarily rich, and feature transition dynamics that do not require re-solving the J market equilibria. Third, if it can be shown that an aggregate moment of the economy only depends on the joint distribution of markdowns and productivity, then we know that such moments are robust to alternative assumptions on preferences and capital accumulation. Below we will use only these types of moments in our calibration, so that our calibration is robust to assumptions on preferences.

The logic underlying the proof of this proposition is that we can consider the equilibrium for the firm as a recursive set of equations that determine the marginal revenue product of labor:

$$mrpl_{ijt} = \widetilde{\alpha}\widetilde{z}_{ijt} n_{ijt}^{\widetilde{\alpha}-1} \quad , \quad n_{ijt} = \left(\frac{w_{ijt}}{\boldsymbol{w}_{jt}}\right)^{\eta} \boldsymbol{n}_{jt} \quad , \quad w_{ijt} = \mu(s_{ijt}) mrpl_{ijt}.$$

This system implies a multiplicative relationship between $mrpl_{ij}$ and the factors common to all firms in the market: $\boldsymbol{w}_{jt}$, $\boldsymbol{n}_{jt}$. Since payroll shares can be expressed in terms of relative wages $s_{ijt} = (w_{ijt}/\boldsymbol{w}_{jt})^{(1+\eta)}$, the homotheticity of $\boldsymbol{w}_{jt}$ implies that these common factors drop out. For a full proof see Appendix E.

Decreasing returns. The expression for equilibrium payroll shares in Proposition 1.1 is new, and extends such expressions in constant returns oligopoly models to the case of oligopsony, multiple inputs, and decreasing returns. It also provides a novel link between returns to scale and concentration. Consider starting with $\alpha < 1$ and $\gamma = 1$, such that labor is the sole input to production. Now consider the

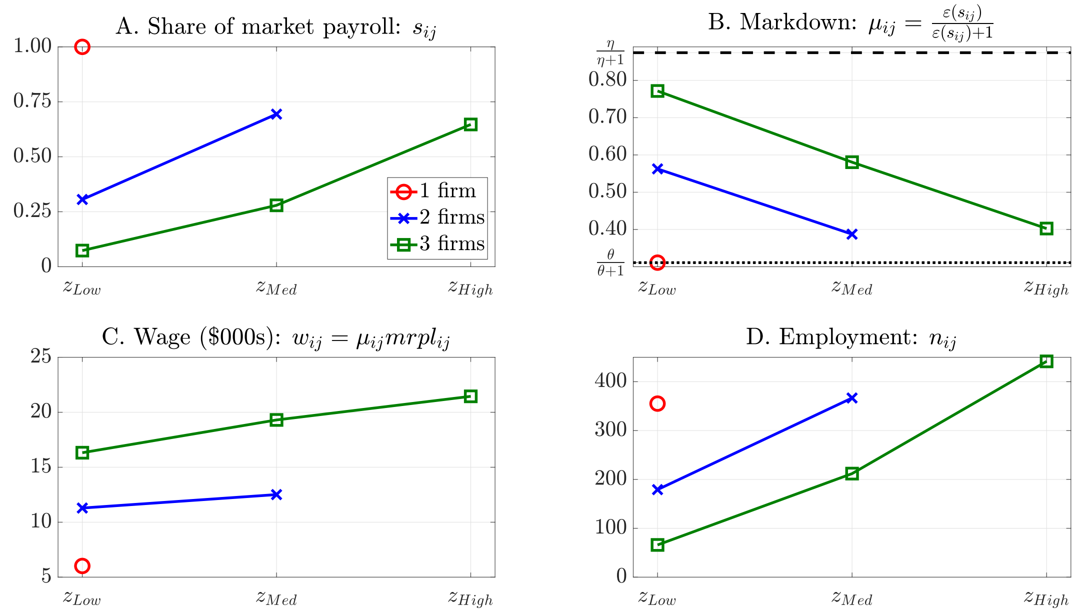

Figure 4: Oligopsonistic market equilibrium in three labor markets

Notes: Figure constructed from model under estimated parameters (Table 3). Low, medium and high productivities of the firms correspond to the 10^{th}, 50^{th} and 90^{th} percentiles of the productivity distribution.

comparative static of increasing α to $\alpha' \in (\alpha, 1]$. With less decreasing returns, more productive firms become larger, accrue a larger labor market share, and pay wider markdowns relative to marginal products. This increases the dispersion in market shares and markdowns in the market, reduces the labor share, and increases concentration.

Example. To show how strategic interaction shapes the market equilibrium, Figure 4 plots examples of the equilibrium shares, markdowns, wages, and employment in three markets. The first market has a single low productivity firm (red), the second adds a firm with median productivity (blue), the third an additional high productivity firm (green).[24]

Consider the market with a single firm (red). By construction, the wage bill share is one (Panel A). Panel B shows that the markdown on the marginal product of labor is 69 percent, which is equal to $\theta/(\theta+1)$ since the firm faces the lower bound on labor supply elasticities, $\varepsilon(1) = \theta$. Panel C shows that wages are low due to low productivity *and* a wide markdown. Despite this, the relatively inelastic labor supply across markets means the firm still employs many workers (panel D).

Consider the addition of a firm with higher productivity, a duopsony (blue). The low-productivity firm's labor market share drops to 31 percent, the more productive firm employs the majority of the market, and market employment is higher. As its share falls, the low-productivity firm's markdown narrows to 56 percent, as more competition increases their equilibrium labor supply elasticity toward η. Panel C shows that with no change to its productivity, but with a narrower markdown, the less productive firm's wage increases. Despite this wage increase, the higher wage at its new competitor bids

[24] Figure 4 is constructed from our benchmark calibration of the model (Section 2).

away labor, causing the low productivity firm's employment to fall. Adding another firm (green), the markdown at the low- and mid-productivity firms decline. The largest firm has the widest markdown (Panel B), but pays more (Panel C) and employs more workers (Panel D).

Figure A2 replicates this exercise with three firms but varying decreasing returns α. Consistent with our above description, higher α generates more concentration and wider markdowns at the leading firm.

Strategic interaction is not an assumption, it's an outcome of the environment, and leads to a negative covariance between markdowns and productivity—visible along the green line in Panel B. In equilibrium, strategic interaction occurs by definition of the Nash equilibrium concept when there is local labor market power ($\eta > \theta$) and finitely many firms. In a model of monopsonistic competition, the green line would be flat, as firms all pay identical markdowns. We now make precise how this negative covariance distorts the general equilibrium of the economy.

1.6 General equilibrium

Given equilibria in each market of the economy, which determines $\{\mu_{ijt}, z_{ijt}\}_{\forall ij}$, we state our main proposition characterizing the general equilibrium of the economy. For the proof see Appendix E.4.

Proposition 1.2. General equilibrium. *The general equilibrium of the model can be characterized in the following three steps:*

1. *Using the market equilibria $\{\mu_{ijt}, z_{ijt}\}_{i=1}^{m_j}$ from all $j \in [0,1]$ markets in the economy, define the following indexes:*

$$\textit{Productivity}: \quad \widetilde{\boldsymbol{Z}} = \left[\int_0^1 \widetilde{\boldsymbol{z}}_j^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}} \quad , \quad \widetilde{z}_j = \left[\sum_{i=1}^{m_j} \widetilde{z}_{ij}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}}$$

$$\textit{Markdown}: \quad \boldsymbol{\mu} = \left[\int_0^1 \left(\frac{\widetilde{\boldsymbol{z}}_j}{\widetilde{\boldsymbol{Z}}}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \boldsymbol{\mu}_j^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}} \quad , \quad \boldsymbol{\mu}_j = \left[\sum_{i=1}^{m_j} \left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}} \mu_{ij}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}}$$

$$\textit{Misallocation}: \quad \boldsymbol{\Omega} = \int_0^1 \left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \left(\frac{\boldsymbol{\mu}_j}{\mu}\right)^{\frac{\widetilde{\alpha}\theta}{1+\theta(1-\widetilde{\alpha})}} \boldsymbol{\omega}_j \, dj \quad , \quad \boldsymbol{\omega}_j = \sum_{i=1}^{m_j} \left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}} \left(\frac{\mu_{ij}}{\boldsymbol{\mu}_j}\right)^{\frac{\eta\widetilde{\alpha}}{1+\eta(1-\widetilde{\alpha})}}$$

2. *In steady-state the four aggregate quantities $\boldsymbol{Y}$, $\boldsymbol{N}$, $\boldsymbol{C}$, K and two prices $\boldsymbol{W}$, R are determined by six equations:*

$$\textit{Output and resource constraint:} \quad \boldsymbol{Y} = \boldsymbol{\Omega}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\alpha} = \boldsymbol{\Omega}^{1-(1-\gamma)\alpha}\boldsymbol{Z}\left(K^{1-\gamma}\boldsymbol{N}^{\gamma}\right)^{\alpha} \quad , \quad \boldsymbol{C} = \boldsymbol{Y} - \delta\boldsymbol{K}$$

$$\textit{Labor and capital demand:} \quad \boldsymbol{W} = \gamma\alpha\left(\frac{\boldsymbol{\mu}}{\boldsymbol{\Omega}}\right)\frac{\boldsymbol{Y}}{\boldsymbol{N}} \quad , \quad R = (1-\gamma)\alpha\frac{\boldsymbol{Y}}{K}$$

$$\textit{Labor and capital supply:} \quad \boldsymbol{W} = -\frac{U_N(\boldsymbol{C},\boldsymbol{N})}{U_C(\boldsymbol{C},\boldsymbol{N})} \quad , \quad 1 = \beta\left[R + (1-\delta)\right]$$

where aggregate productivity $\boldsymbol{Z}$ satisfies [25]

$$\boldsymbol{Z} = \left[\frac{R}{(1-\gamma)\,\alpha}\right]^{(1-\gamma)\alpha} \left[\frac{\widetilde{\boldsymbol{Z}}}{1-(1-\gamma)\,\alpha}\right]^{1-(1-\gamma)\alpha}$$

3. *Given aggregate quantities and prices, firm level variables can be obtained as follows. First, equating market labor demand and market labor supply determines $\boldsymbol{w}_j$ and $\boldsymbol{n}_j$. Then, equating firm labor demand and firm labor supply determines w_{ij} and n_{ij}:*

$$\boldsymbol{w}_j = \underbrace{\boldsymbol{\mu}_j \widetilde{\alpha} \widetilde{\boldsymbol{z}}_j \boldsymbol{n}_j^{\widetilde{\alpha}-1}}_{\text{Labor demand}} = \underbrace{\left(\frac{\boldsymbol{n}_j}{\boldsymbol{N}}\right)^{1/\theta} \boldsymbol{W}}_{\text{Labor supply}} \quad , \quad w_{ij} = \underbrace{\mu_{ij} \widetilde{\alpha} \widetilde{z}_{ij} n_{ij}^{\widetilde{\alpha}-1}}_{\text{Labor demand}} = \underbrace{\left(\frac{n_{ij}}{\boldsymbol{n}_j}\right)^{1/\eta} \boldsymbol{w}_j}_{\text{Labor supply}} .$$

An alternative, intuitive, representation of the aggregate equations can be obtained using the 'tilde' objects introduced previously, giving four equations determining consumption, output, labor and the wage:

$$\boldsymbol{W} = \underbrace{-\frac{U_N(\boldsymbol{C},\boldsymbol{N})}{U_C(\boldsymbol{C},\boldsymbol{N})}}_{\text{Labor supply}} = \underbrace{\boldsymbol{\mu} \widetilde{\alpha} \widetilde{\boldsymbol{Z}} \boldsymbol{N}^{\widetilde{\alpha}-1}}_{\text{Labor demand}} \quad , \quad \widetilde{\boldsymbol{Y}} = \boldsymbol{\Omega} \widetilde{\boldsymbol{Z}} \boldsymbol{N}^{\widetilde{\alpha}} \quad , \quad \boldsymbol{C} = \left[1 - \frac{\delta}{R}(1-\gamma)\,\alpha\right] \frac{\widetilde{\boldsymbol{Y}}}{1-\alpha\,(1-\gamma)} .$$

With respect to an aggregate production function with productivity $\widetilde{\boldsymbol{Z}}$, the markdown $\boldsymbol{\mu}$ is a wedge that pushes the wage below the marginal product of labor, meanwhile for a given productivity $\widetilde{\boldsymbol{Z}}$ and employment $\boldsymbol{N}$, misallocation $\boldsymbol{\Omega}$ represents a direct reduction in output.[26] Note that the two terms appear *independently*.

Benchmark cases. Since welfare is determined by $\boldsymbol{C}$ and $\boldsymbol{N}$, and Proposition 1.2 allows us to restrict our attention to understanding markdowns $\boldsymbol{\mu}$ and misallocation $\boldsymbol{\Omega}$. Three benchmarks are useful:

- **Case I - Efficient allocation.** The efficient allocation coincides with an economy in which firm-by-firm wages and marginal revenue products of labor are aligned, that is $\mu_{ij} = 1$ for all firms. In this case $\boldsymbol{\mu} = 1$, and $\boldsymbol{\Omega} = 1$.

- **Case II - Monopsony limits.** A monopsonistically competitive economy attains under *either* of two limits: (1) $m_j \to \infty$ or (2) $\theta \to \eta$. Henceforth we simply refer to these conditions as the "*monopsony limits*". Under either limit, firms are infinitesimal in the markets in which they set wages. In the first limit, they face a highly competitive local market. In the second limit, they face a national

[25] Note that we could directly compute productivity $\boldsymbol{Z}$ using only primitives: $\boldsymbol{z}_j := \left[\sum_{i\in j} z_{ij}^{\frac{1+\eta}{1-(1-\gamma)\alpha+\eta(1-\alpha)}}\right]^{\frac{1-(1-\gamma)\alpha+\eta(1-\alpha)}{1+\eta}}$ and $\boldsymbol{Z} := \left[\int z_j^{\frac{1+\theta}{1-(1-\gamma)\alpha+\theta(1-\alpha)}} dj\right]^{\frac{1-(1-\gamma)\alpha+\theta(1-\alpha)}{1+\theta}}$. Using these as primitives leads to long exponents on the $\boldsymbol{\mu}_j$, $\boldsymbol{\mu}$, $\boldsymbol{\omega}_j$, and $\boldsymbol{\Omega}$ terms, hence we state the proposition in terms of effective productivities after the firms' optimal capital choice.

[26] Another way to see this is to define the following production function for competitive intermediate goods producers: $\widetilde{\boldsymbol{Y}} = \widetilde{\boldsymbol{Z}} \boldsymbol{N}^{\widetilde{\alpha}}$. The labor demanded by these producers is given by $\boldsymbol{W} = \boldsymbol{\mu} \widetilde{\alpha} \widetilde{\boldsymbol{Z}} \boldsymbol{N}^{\widetilde{\alpha}-1}$. A final goods producer with productivity $\boldsymbol{\Omega} < 1$ then converts intermediates into final goods.

market. Markdowns μ_{ij} are identical across firms and equal to $\eta/(\eta+1)$, as market shares $s_{ij} \to 0$. In this case $\boldsymbol{\mu} = \mathbb{E}\left[\mu_{ij}\right]$, and $\boldsymbol{\Omega} = 1$.

- **Case III - Full model.** In our full model, the negative correlation of productivity and markdowns within markets (recall Figure 4), leads to (i) $\boldsymbol{\Omega} < 1$, which reduces output, and (ii) a higher productivity weight on wide markdown firms, lowering $\boldsymbol{\mu} < \mathbb{E}\left[\mu_{ij}\right]$.

These special cases reveal that the oligopsonistic economy we have contributed distorts welfare relative to a monopsonistically competitive economy precisely through misallocation $\boldsymbol{\Omega}$. In a monoposonistically competitive economy, the labor supply elasticity to the firm η could be calibrated to generate the same $\boldsymbol{\mu}$, yet it would still feature $\boldsymbol{\Omega} = 1$. That $\boldsymbol{\Omega} < 1$ is an outcome of the counterpart of both limits (i) labor markets are concentrated, and (ii) market power via $\theta < \eta$.

This characterization of the model situates the remainder of our paper. First, we provide new empirical facts that allow us—along with the structure of the model—to credibly estimate θ and η. Second, we show that $\theta < \eta$ is necessary for the model to qualitatively and quantitatively match the sign and magnitude of non-targeted empirical evidence on pass-through and strategic wage-setting of firms. Third, we show that the implied misallocation $\boldsymbol{\Omega}$ due to $\theta < \eta$ accounts for around half of the welfare losses due to labor market power, and that this is robust to specifications of aggregate preferences.

1.7 Measurement

The general equilibrium of the model can be used to show that the following two measures of the labor market are independent of the specification of the macroeconomy. We use these results in our calibration exercise in the next section.

Proposition 1.3. Labor share and concentration.

- *The* aggregate labor share *depends only on the distribution of markdowns and productivity*

$$LS_t := \frac{\int_0^1 \sum_{i=1}^{m_j} w_{ij} n_{ij}\, dj}{\int_0^1 \sum_{i=1}^{m_j} y_{ij}\, dj} = \frac{\boldsymbol{WN}}{\boldsymbol{Y}} = \gamma\alpha\left(\frac{\boldsymbol{\mu}}{\boldsymbol{\Omega}}\right)$$

- *The* across market payroll-weighted average of payroll concentration, *which we simply refer to as the* wage-bill Herfindahl, *is defined*

$$HHI_t^{wn} = \int_0^1 s_{jt}\, hhi_{jt}^{wn} dj \quad , \quad hhi_{jt}^{wn} = \sum_{i=1}^{m_j} s_{ijt}^2 \quad , \quad s_{jt} = \frac{\sum_{i \in j} w_{ijt} n_{ijt}}{\int_0^1 \sum_{i \in j} w_{ijt} n_{ijt}\, dj} \quad ,$$

- *The two are linked by the following equation, and hence HHI_t^{wn} depends only on the distribution of markdowns and productivity*

$$LS_t = \int_0^1 s_{jt} ls_{jt}\, dj = \underbrace{\alpha\gamma}_{\text{Competitive LS}} \quad \underbrace{\left[HHI_t^{wn}\left(\frac{\theta}{\theta+1}\right)^{-1} + \left(1 - HHI_t^{wn}\right)\left(\frac{\eta}{\eta+1}\right)^{-1}\right]^{-1}}_{\text{Labor market power adjustment}} \tag{10}$$

For a full derivation of these results see Appendix E.5. Consider again the three benchmark cases. In an efficient economy, labor share is equal to the output elasticity $\gamma\alpha$ and concentration plays no role. Under monopsony due to $m_j \to \infty$, the Herfindahl in each market is zero, all firms have a markdown $\mu_{ij} = \eta/(\eta+1)$, but with $\mathbf{\Omega} = 1$ the labor share is $\gamma\alpha\mu$. Under monopsony due to $\theta \to \eta$, the Herfindahl in each market is positive but does not appear in the labor share. There is a meaningful relationship between concentration and the labor share only under oligopsony with $\theta < \eta$.

In such an economy, higher concentration reduces the labor share. Intuitively, this expression arises in two steps. At the market level, the HHI measures the payroll share of high payroll share firms, which in our model have wide markdowns and so low labor shares. Aggregating across firms within each market delivers (10) in the cross-section of markets. At the aggregate level, the aggregate labor share is the payroll weighted average of market labor shares, leading to (10).

Note that HHI_{jt}^{wn} and LS_t are not sufficient statistics for welfare, even when combined with all other parameters of the model. Combined they reveal the ratio $(\boldsymbol{\mu}/\mathbf{\Omega})$, but cannot be used to disentangle the two. Proposition 1.2 established that both are required independently in order to compute aggregate quantities and hence welfare. Intuitively, the labor share and Herfindahl capture the wedge in the labor demand condition, but still do not capture the output wedge $\mathbf{\Omega}$.

Nonetheless, this model-implied measure of labor market concentration differs from all existing studies. For example, recent work by Benmelech, Bergman, and Kim (2020) and Rinz (2018) use employment Herfindahls and various weighting schemes. Independent of our model framework, employment Herfindahls understate concentration since they ignore the positive relationship between wages and employment, which is a robust feature of the data (Brown and Medoff, 1989; Lallemand, Plasman, and Rycx, 2007; Bloom, Guvenen, Smith, Song, and von Wachter, 2018).[27] We return to study the model's implication for the labor share through the lens of changing HHI_t^{wn} and equation (10) in Section 5.

2 Estimation

Our key parameters to estimate are the degree of across- (θ) and within- (η) market labor substitutability. In this section, we describe our novel approach which integrates (i) new empirical estimates from a quasi-natural experiment and (ii) new moments from the cross-section of markets given in (for additional moments see Table D1), into (iii) a simulated method of moments routine in which all unknown parameters are estimated jointly.

2.1 Approach - Structural vs. reduced-form labor supply elasticities

Structural elasticities. We motivate our approach from the following observation. If a researcher could estimate the *structural elasticities of labor supply* that firms perceive at the Nash equilibrium level of employment, then they could combine data on payroll shares and one of the key model equations to esti-

[27] For a complete proof of this claim see Appendix E. The unconditional firm-level correlation of log employment and log wages is 0.30 in our 2014 tradeable industries LBD sample.

mate (θ, η):

$$\varepsilon\left(s_{ij}^{wn}, \theta, \eta\right) := \left[\frac{\partial \log w_{ijt}}{\partial \log n_{ij}}\left(s_{ij}^{wn}\right)\bigg|_{n_{-ij}^*}\right]^{-1} = \left[\frac{1}{\eta}\left(1 - s_{ij}^{wn}\right) + \frac{1}{\theta}s_{ij}^{wn}\right]^{-1}. \tag{11}$$

In particular, a decreasing relationship between ε_{ij} and s_{ij}^{wn} would identify $\eta > \theta$.

Reduced form elasticities. When firms behave strategically the structural elasticity cannot be measured using wage and employment responses to well identified firm-level shocks. As is clear from the notation above, the structural elasticity is a strictly partial equilibrium concept and answers the counterfactual: *How much will firm ij have to increase w_{ij} in order to expand n_{ij} by one percent, holding its competitors' employment fixed?* Given a shock to any firm in the market, an employment change at firm i will lead competitors to best-respond, which will cause i to best respond and so on. What an empiricist would measure in the data following a shock is therefore a *reduced-form elasticity* $\epsilon(s_{ijt}, \theta, \eta, \ldots)$, which include all other firms' employment and wage changes across market equilibria.[28]

Our insight is that, despite this, the reduced-form elasticities that we may aspire to measure, once filtered through our structural model, are still informative of (θ, η). To a first order approximation, the reduced-form elasticity of labor supply a researcher would measure for firm ij following a shock to it or a competitor is (for derivation see Appendix E.7):

$$\epsilon\left(s_{ijt}^{wn}, \theta, \eta, \ldots\right) := \frac{\mathrm{d}\log n_{ijt}}{\mathrm{d}\log w_{ijt}} = \left\langle \frac{1}{1 + \varepsilon\left(s_{ijt}^{wn}, \theta, \eta\right)\left(\frac{\eta-\theta}{\theta\eta}\right)\left\{\sum_{k\neq i} s_{kjt}^{wn}\frac{d\log n_{kjt}}{d\log n_{ijt}}\right\}} \right\rangle \times \varepsilon\left(s_{ijt}^{wn}, \theta, \eta\right). \tag{12}$$

A distinct property of (12) is that reduced form and structural elasticities coincide exactly under the monopsony limits. As $\theta \to \eta$, the term $\langle\cdot\rangle$ goes to one. As $s_{ijt} \to 0$, then the perturbed firm is infinitesimal so competitors do not respond and the equilibrium interaction term $\{\cdot\}$ goes to zero. Outside the monopsony limits, strategic interaction implies that reduced-form estimates of labor supply elasticities cannot be used to directly infer welfare-relevant labor supply elasticities.

Bias. The relationship between structural and reduced-form elasticities varies predictably depending on whether the underlying shock is idiosyncratic or common across multiple – but not all – firms in a market. A common shock to *all* firms drops out from the market equilibrium condition in Proposition 1.1 and could only be used to estimate the market level labor supply elasticity.

First, consider a positive idiosyncratic productivity shock to firm i in market j such that the firm expands employment. As the firm expands employment, its competitors respond. Since competition is Cournot, employment levels across firms are strategic substitutes so competitors reduce employment ($d\log n_{kjt} < 0$), implying that the equilibrium interaction term is negative, $\{\cdot\} < 0$, and the reduced-form elasticity exceeds the structural elasticity: $\epsilon(s_{ijt}^{wn}, \theta, \eta) > \varepsilon(s_{ijt}^{wn}, \theta, \eta)$. Figure 5A illustrates this case. The contraction in employment at competitors expands labor supply to the firm. An observer drawing conclusions about labor market power from the high reduced-form labor supply elasticity would con-

[28]We borrow the notation of ϵ for reduced-form elasticities and ε for structural elasticities from the estimation of structural macroeconomic models. In this literature *reduced-form shocks* which are empirical objects estimated out of VARs are often denoted ϵ, and *structural shocks* that are backed out of an estimated structural model are denoted ε.

clude labor markets are more competitive than they are. Later in this section, we show that this bias is quantitatively significant: inferred structural and reduced-form elasticities differ by up to 50 percent, even for perfectly idiosyncratic shocks.

For non-idiosyncratic shocks that are common across a subset of firms, we reach the opposite conclusion. Consider a tax cut that affects firm i in market j as well as the other large firms in market j. Call these affected firms *C-Corps*. Suppose the tax cut induces firm i and all affected C-Corps to expand employment, i.e. $d\log n_{ijt} > 0$ and $d\log n_{kjt} > 0$ for all firms-kj that are C-Corps. If non-C-Corp firms have small shares ($s_{kjt} \approx 0$), their strategic response is irrelevant. The equilibrium interaction term will be positive $\{\cdot\} > 0$, and the reduced-form elasticity understates the structural elasticity: $\epsilon\left(s_{ijt}^{wn}, \theta, \eta\right) < \varepsilon\left(s_{ijt}^{wn}, \theta, \eta\right)$. Figure 5B illustrates this case. The expansion in employment at competing C-Corps contracts labor supply to the firm. An observer drawing conclusions about labor market power from the low reduced-form labor supply elasticity would conclude that labor markets are less competitive than they are.

Indirect inference. The above demonstrates that reduced-form elasticities are informative of structural elasticities which are in turn informative about welfare relevant parameters, and that the equilibrium structure of the model is necessary to complete this mapping. Our approach recognizes this. We first use a quasi-natural policy experiment to estimate the relationship between payroll shares and average *reduced-form* labor supply elasticities in the data: $\widehat{\epsilon}^{Data}(s)$. We then replicate the same policy experiment in our model which yields

$$\widehat{\epsilon}^{Model}\left(s, \theta, \eta\right) := \mathbb{E}\left[\epsilon^{Model}\left(s, \theta, \eta, \ldots\right)\right],$$

where the expectation is being taken with respect to the distribution of all relevant labor market variables and shocks. We then choose (θ, η)—along with other parameters—to replicate the empirical relationship between average reduced-form elasticities and payroll shares.

2.2 Estimating reduced-form labor supply elasticities in the data: $\widehat{\epsilon}^{Data}(s)$

We estimate size-dependent reduced-form labor supply elasticities using state corporate tax changes in conjunction with the Census Longitudinal Business Database (LBD). The LBD provides high quality measures of employment, location, and industry with nearly universal coverage of the non-farm business sector. Data are carefully linked over time at the establishment and firm level. In order to proceed, we first define markets and firms. We then describe our regression approach.

Market. In our model, a *labor market* has two features: (i) a worker drawn at random from the economy will have a greater attachment to one labor market than others on the basis of idiosyncratic preferences, but will nonetheless be able to move across markets, and (ii) firms within a market compete strategically.

With these features in mind and given what we can observe in the LBD, we define a *local labor market* as a 3-digit NAICS (NAICS3) industry within a Commuting Zone (CZ).[29] Examples of adjacent 3-digit

[29] Using BLS Occupational Employment Statistics microdata, Handwerker and Dey (2018) show that when it comes to concentration there is little practical difference in defining a market at the occupation-city level rather than the industry-city level as these two measures are highly correlated. In particular, the across-city correlation of Herfindahl-Hirschman Indices at the CBSA-occupation and CBSA-industry level is 0.97.

NAICS codes are subsectors 323-325: *'Printing and Related Support Activities'*, *'Petroleum and Coal Products Manufacturing'* and *'Chemical Manufacturing'* which we regard as suitably different. Examples of adjacent commuting zones include the collection of counties surrounding downtown Minneapolis and those surrounding Duluth.[30]

Firm. We define a firm in a local labor market as the collection of establishments operated by that firm. We aggregate employment and annual payroll of all establishments owned by the same firm within the same NAICS3-CZ market.[31] For each resulting firm-market-year observation, we observe employment, payroll, and herein define the *wage* as payroll per worker.

Regression framework. To estimate the relationship $\widehat{\epsilon}^{Data}(s)$ in the data we use within-firm-market, across-time changes in wages and employment following state corporate tax changes.

Let i denote firm, j denote market, and t year. Let $s(j)$ denote the geographical state of market j. Let y_{ijt} denote a variable of interest at the firm-market-year level, such as employment or the wage. We are interested in coefficients on state corporate taxes $\tau_{s(j)t}$ and their interaction with lagged payroll shares. We use lagged payroll shares to avoid mechanical correlations between contemporaneous wages, employment and wage-bill shares, and control for lagged payroll shares s_{ijt-1} by themselves.[32] To isolate within-firm-market variation, we introduce firm-market fixed effects α_{ij}.[33] Lastly, we follow Giroud and Rauh (2019) and include controls X_{it} for the state unemployment rate and budget balance, along with a set of indicators for years in which state corporate income tax applied to gross receipts. Our regression specification is as follows:

$$\log y_{ijt} = \alpha_{ij} + \mu_t + \psi\, s_{ijt-1} + \beta^y\, \tau_{s(j)t} + \gamma^y \left(\tau_{s(j)t} \times s_{ijt-1}\right) + \Gamma X_{it} + \nu_{ijt}. \qquad (13)$$

The coefficients β^y and γ^y capture the average effect of state corporate tax rate changes and their differential effect by market share. We estimate (13) separately for log employment and log wages (total payroll per worker). We then show how coefficient estimates from (13) can be used to construct $\widehat{\epsilon}^{Data}(s)$.

Clustering. We provide two sets of estimates which cluster at the state-year and market-year levels. Our estimated labor supply elasticity is a combination of both (i) the direct effect of taxes, and (ii) the interaction between payroll share and taxes. The former varies at the state-year level suggesting that clustering at the state-year level is appropriate; the latter varies at the firm-market-year level and since payroll shares contain market level variation, clustering at the market-year level is appropriate.

Sample. To abstract from changes in product market power we restrict our sample to tradeable industries identified by Delgado, Bryden, and Zyontz (2014) and listed in Appendix C.[34] Plants owned by the same firm are aggregated within a market, such that an observation is a firm-market-year. Since we rely on state-level corporate tax variation to generate changes in labor demand, we restrict our sample to

[30]Many more examples are provided in Tables C2 and C3 in Appendix C.

[31]Firms are identified by the LBD variable *firmid*.

[32]State-level corporate taxes are proportional flat-taxes on firms' accounting profits. Our data for state-level corporate taxes comes from the data made publicly available by Giroud and Rauh (2019): (https://web.stanford.edu/ rauh/).

[33]In this exercise only, we exclude commuting zones that straddle multiple states.

[34]See additional discussion in Section C

C-Corporation firms (C-Corps) in the LBD from 1977 to 2011. Table C1 includes summary statistics of our 2.8 million observations at the firm-market-year level.

Estimates. Table 1 presents empirical estimates of (13). We start with (log) employment in year t as a dependent variable. Column (1) presents the full set of interaction terms between payroll shares and corporate taxes. Since $\tau_{s(j)t}$ is in units of percents, the coefficient on $\tau_{s(j)t}$ is an elasticity: a one percent increase in corporate taxes results in a 0.309 percent reduction in employment at firms that are atomistic within the market ($s_{ijt-1} = 0$). The interaction term is positive and significant. When combined with the negative direct effect, the interaction indicates a dampened response at larger firms. Compare the mean effect of a 1 ppt increase in $\tau_{s(j)t}$ on a firm with a mean payroll share (0.03) to a firm with a one standard deviation higher share (0.10). Employment declines by -0.26 percent at the small firm and -0.15 percent at the large firm. Consistent with Giroud and Rauh (2019), increases in corporate tax rates reduce employment. Our empirical finding is that this reduction is around 40 percent weaker at larger firms.

Column (2) illustrates estimates of (13) when the dependent variable is the wage. Qualitatively the signs echo the employment response: on average wages fall, and this decline is smaller at larger firms. Columns (3) and (4) provide estimates of (13) using year $t+1$ employment and wages as dependent variables. These specifications are designed to accommodate adjustment frictions in prices and quantities. We again find a negative effect of corporate taxes on employment and wages, with diminished effects at larger firms.

Share-dependent reduced-form labor supply elasticities. Table 2 combines the wage and employment responses to compute the relationship between the average reduced-form labor supply elasticity and payroll shares, which informs both θ and η. Differentiating (13) with respect to $\tau_{s(j)t}$ delivers share-dependent reduced-form wage and employment elasticities:

$$\frac{\mathrm{d}\log n_{ijt}}{\mathrm{d}\tau_{s(j)t}} = \beta^n + \gamma^n s_{ijt-1} \ , \ \frac{\mathrm{d}\log w_{ijt}}{\mathrm{d}\tau_{s(j)t}} = \beta^w + \gamma^w s_{ijt-1} \ , \ \widehat{\epsilon}^{Data}(s_{ijt-1}) = \frac{\widehat{\mathrm{d}\log n_{ijt}}}{\widehat{\mathrm{d}\log w_{ijt}}} = \frac{\widehat{\beta}^n + \widehat{\gamma}^n s_{ijt-1}}{\widehat{\beta}^w + \widehat{\gamma}^w s_{ijt-1}} \quad (14)$$

When we turn to indirect inference, we run the same regressions on model simulated data to compute $\epsilon^{Model}(s)$ in the same way.

Column (1) of Table 2A reports reduced-form labor supply elasticity estimates $\widehat{\epsilon}^{Data}(s_{ijt-1})$ based on the Table 1 Year t estimates for $s_{ijt-1} \in \{0.01, 0.05, 0.10\}$. At a wage bill share of 1 percent, the year t reduced-form labor supply elasticity is 1.20, and declines to 0.72 at a wage bill share of 10 percent. Columns (2) and (3) show that the elasticity is statistically significant at the 5 percent level under either assumption for clustering.

In Columns (4) and (5) of Table 2A, we test whether the estimated date t labor supply elasticities of larger firms are statistically different from atomistic firms. Formally, we test $H_0 : \widehat{\epsilon}^{Data}(s_{ijt-1}) = \widehat{\epsilon}^{Data}(0)$ for $s_{ijt-1} \in \{0.01, 0.05, 0.10\}$. In each case the year t reduced-form labor supply elasticities in column (1) is significantly different from that of an atomistic firm at the 1 percent level.

Table 2B repeats the same exercise for year $t+1$ employment and wage responses from columns (3)

		Year t		Year $t+1$	
		$\log n_{ijt}$	$\log w_{ijt}$	$\log n_{ijt+1}$	$\log w_{ijt+1}$
		(1)	(2)	(3)	(4)
State corporate tax	$\tau_{s(j)t}$	-0.00309***	-0.00249***	-0.00269***	-0.00109*
		(0.000641)	(0.000692)	(0.000738)	(0.000619)
		[0.000349]	[0.000303]	[0.000390]	[0.000316]
Payroll share	s_{ijt-1}	0.886***	0.130***	0.719***	0.116***
		(0.0207)	(0.0105)	(0.0204)	(0.0113)
		[0.0157]	[0.00893]	[0.0163]	[0.01000]
Interaction	$\tau_{s(j)t} \times s_{ijt-1}$	0.0158***	0.00396***	0.0146***	0.00376**
		(0.00271)	(0.00140)	(0.00267)	(0.00146)
		[0.00196]	[0.00112]	[0.00204]	[0.00125]
Fixed effects		Y	Y	Y	Y
R-squared		0.897	0.790	0.875	0.735
Firm-market-year observations		2.84m	2.84m	2.84m	2.84m

Table 1: Estimation results for equation (13)

Notes: All specifications include fixed effects for: (i) year, (ii) firmid×market. According to Census requirements, the number of observations is rounded to the nearest 1,000. Standard errors in round parentheses $(\cdot)$ are clustered at State $\times$ Year level. Standard errors in square parentheses $[\cdot]$ are clustered at Market $\times$ Year level. Sample includes tradeable *C*-Corps from 1977 to 2011.

	A. Year t elasticities				
	$\hat{\epsilon}^{Data}(s_{ijt-1})$	p-value: $\epsilon^{Data}(s_{ijt-1}) = 0$		p-value: $\epsilon^{Data}(s_{ijt-1}) = \epsilon^{Data}(0)$	
		state-year	market-year	state-year	market-year
	(1)	(2)	(3)	(4)	(5)
1% payroll share, $s_{ijt-1} = 0.01$	1.20	0.00	0.00	0.01	0.00
5% payroll share, $s_{ijt-1} = 0.05$	1.00	0.00	0.00	0.01	0.00
10% payroll share, $s_{ijt-1} = 0.10$	0.72	0.02	0.00	0.01	0.00
	B. Year $t+1$ elasticities				
	$\hat{\epsilon}^{Data}(s_{ijt-1})$	p-value: $\epsilon^{Data}(s_{ijt-1}) = 0$		p-value: $\epsilon^{Data}(s_{ijt-1}) = \epsilon^{Data}(0)$	
		state-year	market-year	state-year	market-year
	(1)	(2)	(3)	(4)	(5)
1% payroll share, $s_{ijt-1} = 0.01$	2.42	0.05	0.00	0.32	0.17
5% payroll share, $s_{ijt-1} = 0.05$	2.17	0.10	0.01	0.28	0.15
10% payroll share, $s_{ijt-1} = 0.10$	1.72	0.23	0.05	0.23	0.10

Table 2: Elasticities and hypothesis testing

Notes: Panel A Column (1) constructs elasticities based on the Date t estimates in Columns (1) and (2) in Table 1 using equation (14). Column (2) reports p-value of the hypothesis test $H0 : \epsilon^{Data}(s) = 0$ using standard error clustered at the state-year level. Column (3) clusters at the market-year level. Column (4) reports p-value of the hypothesis test $H0 : \epsilon^{Data}(s) = \epsilon^{Data}(0)$ using standard error clustered at the state-year level. Column (5) clusters at the market-year level. Panel B repeats the exercise based on the Date $t+1$ estimates in Columns (3) and (4) in Table 1.

and (4) of Table 1. At year $t+1$ the implied reduced-form labor supply elasticities are larger, potentially due to slow employment adjustment. However, the estimates are noisier. Based on year $t+1$ estimates we cannot statistically distinguish $\widehat{\epsilon}^{Data}(s_{ijt-1})$ from $\widehat{\epsilon}^{Data}(0)$ for $s_{ijt-1} \in \{0.01, 0.05, 0.10\}$. At a wage-bill share of 10 percent, we come closest to distinguishing the year $t+1$ reduced-form labor supply elasticity from that of an atomistic firm at the 10 percent level.

In summary, our more precise year t estimates of the size-dependent wage and employment response indicate (i) less responsiveness of larger firms, and (ii) significantly lower reduced-form labor supply elasticities of larger firms. Our year $t+1$ estimates imply greater labor supply elasticities across all firm sizes, consistent with frictional adjustment. In both cases we find that larger firms have lower labor supply elasticities; however, we lack the power to statistically distinguish the labor supply elasticity of large firms from small firms in the year $t+1$ case.

Additional results. Appendix G.3 provides additional results. First, model estimation simply requires *consistent* auxiliary moments that can be simulated. The threat to *consistency* when we estimate equation (13) is that other forces move employment and wages at the state-year level (e.g. taxes are cut when unemployed is low). Table G3 shows that our main interaction between corporate taxes and the wage-bill share is robust to the inclusion of state-year fixed effects, thus removing all common state-year variation. Second, we directly compute the ratio of wage changes to employment changes at the firm-level and study their relationship with firms' wage-bill share. Following corporate tax cuts, we estimate statistically significantly different labor supply elasticities at large relative to atomistic firms. Third, using the 2012 Census of Manufacturers, we show that variation in non-wage compensation is unable to explain the large movements in markdowns implied by our baseline labor supply elasticity estimates. Finally, we show that systematic variation in capital intensity by market share cannot explain our results: within markets, capital intensity and payroll shares are only weakly correlated.

2.3 Simulating reduced-form labor supply elasticities in the model: $\widehat{\epsilon}^{Model}(s, \theta, \eta)$

To construct $\widehat{\epsilon}^{Model}(s, \theta, \eta)$, we add corporate taxes to the environment and show how they shift marginal revenue products of labor. We make several modifications to our theory. Corporate taxes are a tax on profits, net of interest payments on debt. Firms finance $\lambda_K \in [0,1]$ of their capital using debt and maximize post-tax profits:

$$\pi_{ijt} = \Big(1-\tau_C\Big) z_{ijt} \Big(k_{ijt}^{1-\gamma} n_{ijt}^{\gamma}\Big)^{\alpha} - \Big(1-\tau_C\lambda_K\Big) R k_{ijt} - \Big(1-\tau_C\Big) w(n_{ijt}) n_{ijt},$$

A random fraction $\omega_C \in [0,1]$ of firms in each market are C-corps and subject to τ_C; all other firms face $\tau_C = 0$. In the data, C-corps are larger on average. To capture this we assume a productivity premium $\Delta_C > 1$:

$$\log(z_{ijt}) \sim \begin{cases} N\Big(1, \sigma_z^2\Big) & \text{if } i \text{ is not a } C\text{-corp (i.e. with probability } 1-\omega_C) \\ N\Big(\Delta_C, \sigma_z^2\Big) & \text{if } i \text{ is a } C\text{-corp (i.e. with probability } \omega_C) \end{cases}$$

For C-corps, the corporate tax distorts their capital decision, which reduces the marginal product of labor. Under the firm's optimal capital demand, effective productivity $\widetilde{z}_{ijt}$ is decreasing in τ_C if $\lambda_K > 0$:

$$\frac{\pi_{ijt}}{1-\tau_C} = \max_{n_{ijt}} \widetilde{z}_{ijt} n_{ijt}^{\widetilde{\alpha}} - w(n_{ijt}) n_{ijt} \quad , \quad \widetilde{z}_{ijt} = \left[\frac{1-\tau_C}{1-\tau_C \lambda_K}\right]^{\frac{\alpha(1-\gamma)}{1-\alpha(1-\gamma)}} \times \left\langle \left[1-\alpha\left(1-\gamma\right)\right] \left(\frac{\alpha\left(1-\gamma\right)}{R_t}\right)^{\frac{\alpha(1-\gamma)}{1-\alpha(1-\gamma)}} z_{ijt}^{\frac{1}{1-\alpha(1-\gamma)}} \right\rangle,$$

see Appendix E.9. With these modifications to the theory, we can simulate an increase in corporate taxes and estimate reduced-form elasticities consistent with our approach to the data to obtain $\widehat{\epsilon}^{Model}(s, \theta, \eta)$.

2.4 Indirect inference

We estimate the model using 2014 Census data and proceed in two steps. First, to match the reduced-form elasticities measured in a sample of tradeable firms, we estimate a tradeable-only version of our economy. This includes the corporate tax experiment and yields estimates of the key preference parameters η and θ. Second, holding η and θ fixed, we drop the corporate tax experiment and estimate the remaining parameters to match economy-wide moments. The tradeable sector is more concentrated than the economy on the whole, so the second step is necessary for measuring labor market power in the US economy. We add to the model a parameter that shifts firm productivity $\overline{Z}$, and a preference parameter that shifts labor supply $\overline{\varphi}$.

Common external parameters. On an annual basis, the discount rate is 4 percent ($\beta = 0.9615$), and the depreciation rate is 10 percent ($\delta = 0.10$). Throughout we simulate $5,000$ markets and verify that our results are not sensitive to this choice. The moments used in our estimation are robust to alternative specifications of aggregate preferences $U(\boldsymbol{C}, \boldsymbol{N})$, so we defer specifying U until we evaluate welfare.

Tradeable only - External parameters. To capture the distribution of tradeable firms across markets, $m_j \sim G(m_j)$ we combine a Pareto distribution with a discrete mass at $m_j = 1$ to capture single firm markets. The mass of tradeable markets with a single firm is 16 percent (Table F1). We fit the remaining Pareto parameters to match the first three moments of the distribution of firms across markets. Appendix Table F1 provides moments and parameter estimates.

The fraction of capital financed by debt is chosen to match the debt to capital ratio among tradeable firms. For this we use tradeable firms in Compustat and obtain $\lambda_K = 0.213$. A fraction $\omega_C = 0.42$ of firms are C-corps based on the 2014 County Business Patterns data for tradeable sectors (CBP).

Given parameters we simulate a three-period panel from the model. The first two periods are given by the model's steady state with τ_C set to the mean state corporate tax rate of 6.9 percent (the average over the sample period 1977 to 2011). In the third period, we increase taxes by $\Delta_\tau =$ one percentage point, which is approximately one standard deviation of the distribution of state corporate tax changes observed in our sample period.[35] Treating model output as panel data, we estimate (13) with firm fixed effects and lagged payroll shares (hence the requirement for three periods). We replicate our treatment of the data, and transform point estimates into average reduced-form elasticities by payroll share using equation (14). Appendix F includes additional details on simulating the tax experiment.

[35] We use data made publicly available by Giroud and Rauh (2019)

Parameter		Value	Moment	Model	Data
I. TRADEABLE INDUSTRIES ONLY					
$G(m_j)$	Pareto and point mass at $m_j = 1$		Mean, Variance, Skewness of distribution 15 percent of markets have 1 firm		
ω_C	Share of firms that are C-corps	0.42	Share of estabs. that are C-corps (CBP, 2014)		
τ_C	State corporate tax rate	0.069	Mean of state corp. tax rate $\tau_{C,st}$		
Δ_τ	State corporate tax rate increase	0.010	Std. dev. of annual $\tau_{C,st}$		
λ_K	Fraction of capital debt financed	0.213	Tradeable industries (Compustat, 2014)		
Estimated					
θ	Across market substitutability	0.45	Average $\hat{\varepsilon}^{Data}(s)$ for $s \in [0.05, 0.10]$	1.48	1.38
η	Within market substitutability	6.96	Average $\hat{\varepsilon}^{Data}(s)$ for $s \in [0, 0.05]$	1.55	1.70
Δ_C	Relative productivity of C-corps	1.34	Emp. share of C-corps	0.66	0.66
σ_z	Productivity dispersion	0.248	Payroll weighted $\mathbb{E}[hhi_j^{wn}]$	0.17	0.17
α	Decreasing returns to scale	1.000	Labor share	0.53	0.54
γ	Exponent on labor	0.799	Capital share	0.19	0.19
$\overline{Z}$	Productivity shifter	1.53×10^4	Mean firm size	34.6	34.6
$\overline{\varphi}$	Labor disutility shifter	2.261	Mean worker earnings (\$000)	58.3	58.3
II. ALL INDUSTRIES					
$G(m_j)$	Pareto and point mass at $m_j = 1$		Mean, Variance, Skewness of distribution 9 percent of markets have 1 firm		
θ	Across market substitutability	0.45	Held fixed at estimated tradeable value		
η	Within market substitutability	6.96	Held fixed at estimated tradeable value		
Estimated					
σ_z	Productivity dispersion	0.327	Payroll weighted $\mathbb{E}[hhi_j^{wn}]$	0.11	0.11
α	Decreasing returns to scale	0.957	Labor share	0.57	0.57
γ	Exponent on labor	0.812	Capital share	0.18	0.18
$\overline{Z}$	Productivity shifter	1.59×10^4	Mean firm size	22.8	22.8
$\overline{\varphi}$	Labor disutility shifter	3.081	Mean worker earnings (\$000)	43.8	43.8

Table 3: Summary of Parameters

Tradeable only - Estimated parameters. We now estimate $\boldsymbol{\psi} = \{\theta, \eta, \gamma, \alpha, \sigma_z, \Delta_C, \overline{Z}, \overline{\varphi}\}$. Our strategy is to use moments that are independent of aggregate preferences.

To estimate θ and η, we target the reduced-form labor supply elasticities in Table 2. Year t and $t+1$ elasticities have different merits. Year t elasticities are less likely to include confounding factors, whereas date $t+1$ elasticities alleviate concerns regarding adjustment frictions. As a compromise we target the average of year t and year $t+1$ estimates. Rather than targeting the entire function (14), we compute the average reduced-form labor supply elasticity of firms with payroll shares between 0 and 5 percent, and 5 and 10 percent. This captures the bulk of variation in our data. The value for small firms is most informative of η. The value for large firms is most informative of θ. We estimate $\theta = 0.45$ and $\eta = 6.96$.

We estimate productivity dispersion σ_z to match the payroll weighted wage-bill Herfindahl of 0.17 (Table D1). Increasing σ_z increases the market power of large firms, increasing concentration. We pin down α and γ using the capital and labor share of income.[36] As can be seen from equation (10), conditional on HHI_t^{wn}, α shifts the labor share. Our estimate of α implies constant returns to scale in the

[36]We use BEA data to compute the tradeable labor share of 53.9 percents. The remaining non-labor income is apportioned according to the share of capital and profits in the aggregate economy. The aggregate capital share is 18 percent based on (Barkai, 2020). Apportioning yields a tradeable capital share of 19.3 percent.

tradeable sector. Parameter γ matches the aggregate capital share. Table 3 summarizes all parameters and the model's fit to the target moments. In Appendix E.6 we provide a closed form solution of the model and prove that in any equilibrium $(\overline{Z}, \overline{\varphi})$ normalize units of wages and labor, so are chosen to match average firm size (34.6) and payroll per worker (\$58,300) (Table D1). Finally, Δ_C matches the 66 percent employment share of C-corps (CBP).

Economy-wide calibration. Holding our estimates of preference parameters η and θ fixed, we recalibrate our model to match economy-wide moments. We update the distribution of firms across markets $G(m_j)$, which almost halves the number of markets with one firm to 9 percent. We remove C-corps, setting $\omega_C = 0$ and estimate $\boldsymbol{\psi} = \{\gamma, \alpha, \sigma_z, \overline{Z}, \overline{\varphi}\}$ to match, the (i) labor share, (ii) capital share, (iii) payroll-weighted wage-bill Herfindahl, (iv) average firm size, and (v) average payroll per worker. Notably, in the overall economy, concentration is lower, the labor share is higher and wages and average firm size are smaller. With less concentration market power is lower, reducing profits, hence a lower value of α is required to increase profits. The model matches the data exactly and yields decreasing returns to scale, $\alpha = 0.957$.

2.5 Discussion of estimated θ and η

Figure 5C plots $\widehat{\varepsilon}^{Data}(s_{ijt-1})$ over $s_{ijt-1} \in [0, 0.10]$. The model generates a downward sloping reduced-form labor supply elasticity similar to the data. Notably, the reduced-form estimates for atomistic firms are roughly four times smaller than the structural estimates. Thus, a naive estimation would conclude that the labor market is less competitive than it actually is, and infer markdowns at atomistic firms of 0.65. Our structural estimates of the labor supply elasticity at atomistic firms imply a markdown of only 0.87 percent, roughly 3 times narrower. The upward bias in market power implied from naive use of reduced-form estimates is less pronounced among larger firms. These predictions are in line with our theory of non-idiosyncratic shocks in Panel B.

Entry and Exit. One concern may be that following a tax increase some firms may exit, and this may affect our estimates of θ and η. To address this we conduct two exercises. First, in Appendix G we estimate linear probability models of firm-market exit in year $t+1$ as a function of corporate taxes in year t. We find economically insignificant results. Giroud and Rauh (2019) find that firms adjust their *total number of plants* in the state. Our results imply that they do not appear to be exiting commuting-zone markets *entirely* in response to corporate tax changes. Second, despite these insignificant empirical results, we estimate the model under the extreme counterfactual assumption that the smallest 5 percent of C-corps in each market exit after the tax increase. Our estimates of θ and η are unchanged. Details of this exercise are in Appendix G.2.

Idiosyncratic shocks and bias. So far, our analysis has focused on bias between reduced-form and structural elasticities when the observed shock is non-idiosyncratic, illustrated in Figure 5B. In this section, we quantify the bias under idiosyncratic shocks, illustrated in Figure 5A. Many existing papers rely on estimation strategies that assume atomistic firms and infer monopsony from firm-level responses to

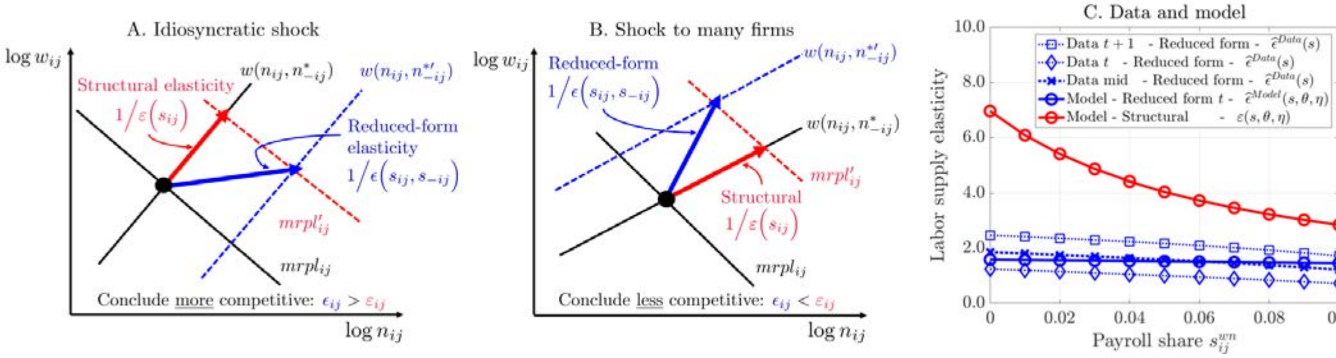

Figure 5: Reduced form and structural labor supply elasticties

Notes: **Panel A** demonstrates the relationship between reduced-form and structural labor supply elasticities for an idiosyncratic shock. **Panel B** demonstrates the relationship between reduced-form and structural labor supply elasticities for a non-idiosyncratic shock that affects a proper subset of firms. **Panel C** compares reduced-form and structural labor supply elasticities by firm payroll share in response to a corporate tax shock of 1 percentage point. 'Data $t+1$ - Reduced form' plots the date $t+1$ empirical labor supply elasticity estimates in Table 2. 'Data t - Reduced form' plots the date t empirical labor supply elasticity estimates in Table 2. 'Data mid - Reduced form' is an equally weighted average of the date $t+1$ and date t empirical labor supply elasticity estimates in Table 2. 'Model - Reduced form t' plots reduced-form labor supply elasticity estimates, estimated on simulated model data as described in Appendix F.2. 'Model - Structural' plots $\varepsilon(\cdot)$ from equation (11).

idiosyncratic shocks (e.g., see the articles surveyed in Card, Cardoso, Heining, and Kline (2018)). We show that using data on employment and wage changes in response to identified firm-level shocks to infer key structural parameters may generate sizeable bias.

Consider a truly idiosyncratic shock to a single randomly selected firm in our economy. Drawing the treated firm at random, compute the reduced form elasticity $\widehat{\epsilon}_{ij}$ and compare it to the structural elasticity ε_{ij}. We repeat this 5,000 times for small (one percent), medium (10 percent) and large (50 percent) productivity shocks. We plot the results in Figure 6. This Monte Carlo exercise reveals a significant difference in reduced-form and structural labor supply elasticities for firms with market shares not equal to 0 or 1, even when the identifying variation is *perfectly idiosyncratic*. The bias between reduced-form and structural elasticities is 18 percent for firms with market shares between 0 and 10% that receive small shocks (1% productivity increase). This bias increases to over 23% for medium and large shocks. Accounting for the Nash equilibrium of the local labor market is quantitatively important for recovering welfare relevant parameters.

This exercise implies that even if a researcher aims to use perfect idiosyncratic variation in productivity to infer structural elasticities and do welfare analysis, they would have to deflate their *reduced-form elasticity* estimates substantially in order to recover the true *structural elasticities*. Inferring structural elasticities that are too large one would infer narrower markdowns which would bias *downward* the welfare losses due to labor market power. The details of our Monte Carlo exercise are included in Appendix F.3.

Figure 6 shows that two important caveats apply, both summarized in equation (12). If the firm has a share of one, then reduced-form and structural elasticity estimates coincide and reveal θ. If the firm has an infinitesimal share, then reduced-form and structural elasticity estimates coincide and reveal η. Finally, a *market level* shock will directly reveal θ, so long as the market itself is not large. If the market is very large then a market level shock will also effect the macroeconomic equilibrium of the labor market,

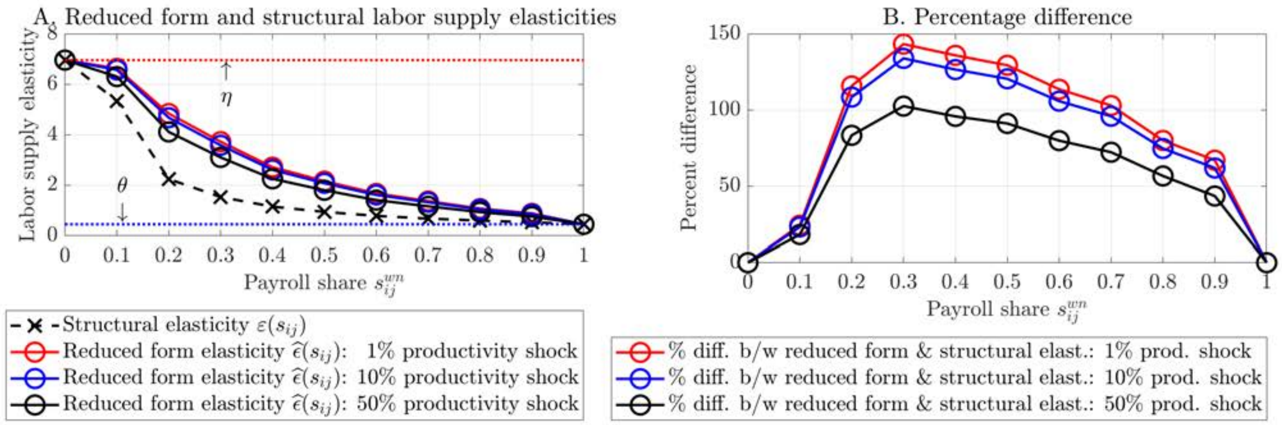

Figure 6: Reduced form and structural elasticities in response to idiosyncratic productivity shocks.

Notes: Panel A plots Monte Carlo results which compare reduced-form to structural labor supply elasticities in response to a perfectly idiosyncratic shock to a single firm. The lines labeled 'Reduced form elasticity' plot the average estimated reduced-form labor supply elasticity $\widehat{\varepsilon}(s)$ as detailed in Appendix F.3. The dashed line labeled 'Structural elasticity' plots $\varepsilon(s)$ from equation (11). Panel B reports the error of the average reduced-form elasticity relative to the structural elasticity: $100 \times (\widehat{\varepsilon}(s) - \varepsilon(s))/\varepsilon(s)$.

and reduced-form elasticities will be contaminated by φ.

3 Validation

In this section we show that our oligopsony model with $\theta < \eta$ is qualitatively and quantitatively consistent with independent evidence by comparing the model's implications for (i) pass-through of value added to wages, (ii) strategic responses of firms to competitors' wage changes, and (iii) the effects of mergers on employment and wages. In each case we show how the standard monopsony model ($\theta = \eta$) qualitatively and quantitatively fails, while our estimated model matches the data. A summary is as follows:

1. **Pass-through** - Under $\theta = \eta$, pass-through from value added per worker to wages is equal to one. Kline, Petkova, Williams, and Zidar (2019) produce an estimate of 0.47. In a replication of their exercise the model produces an estimate of 0.61
2. **Competitor responses** - Under $\theta = \eta$, a firm's response to the wage increase of a competitor will be close to zero. Staiger, Spetz, and Phibbs (2010) produce an estimate of 0.128. In a replication of their exercise the model produces an estimate of 0.065.
3. **Mergers** - Under $\theta = \eta$, the merging of two firms in a labor market has no effect on market wages or employment. Arnold (2020a) finds that worker earnings fall by 0.8 percent, and 3.1 percent in highly concentrated markets. In a replication of their exercise the model produces estimates of 1.3 percent and 4.1 percent, respectively.

Figure A1 shows that the model replicates the distribution of markets by concentration, both unweighted and payroll weighted.

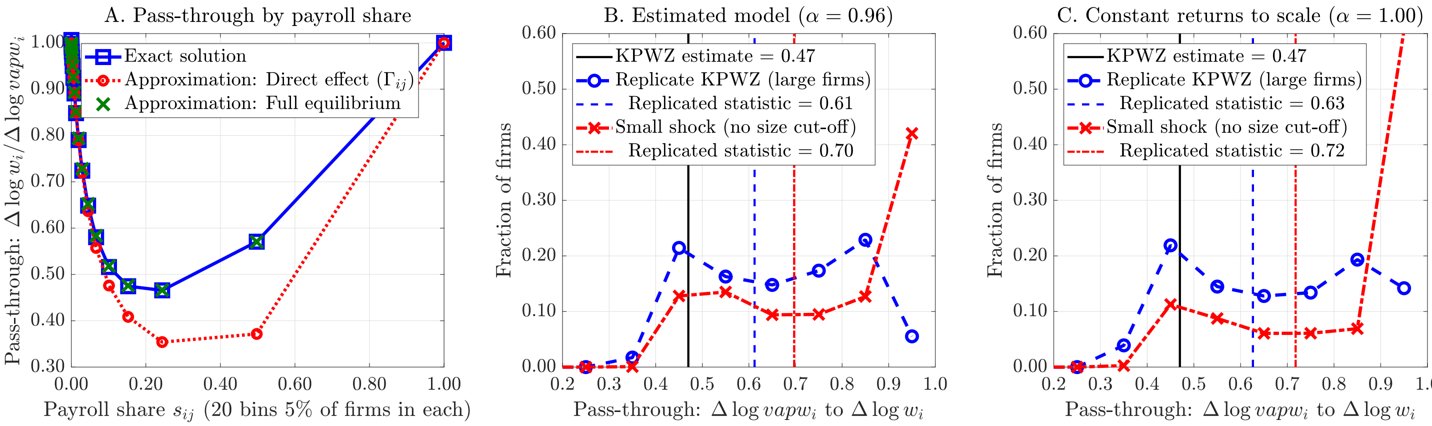

Figure 7: Pass-through and replication of KPWZ

Notes: Panel A computes average pass-through in bins by 20 ventiles of the payroll share distribution. We draw one firm from each market at random and increase its productivity by 1 percent. We resolve the market equilibrium, keeping general equilibrium aggregates fixed. Within each bin we compute the mean of $\Delta \log w_i / \Delta \log vapw_i$ of these firms (blue solid line, squares) We use equation (16) and compute Γ_{ij} for each firm based on initial market shares, and again take averages within each bin (red dotted line, circles). We then compute $\Delta \log w_{kjt}$ for all other firms in the market, and use these to compute the full approximation from (16) and again take averages within each bin (green squares). The histograms in Panel B and C plot the fraction of firms with firm-level pass-through $\Delta \log w_i / \Delta \log vapw_i$ in bins of width 0.10, both with (blue, circles) and without (red, crosses) the size restrictions imposed to match the sample statistics of KPWZ.

3.1 Pass-through - Kline, Petkova, Williams, and Zidar (2019)

Theory. A body of recent empirical evidence documents that the elasticity of worker wages with respect to value added per worker following shocks to firm productivity is less than one (Kline, Petkova, Williams, and Zidar, 2019; Card, Cardoso, Heining, and Kline, 2018). Under our theory, equation (7) implies:

$$\underbrace{w_{ijt} = \alpha\gamma \times \mu_{ijt} \times vapw_{ijt}}_{\text{A. Levels}} \quad , \quad \underbrace{\Delta \log w_{ijt} = \Delta \log \mu_{ijt} + \Delta \log vapw_{ijt}.}_{\text{B. Log changes}} \tag{15}$$

The literature discusses pass-through in two ways, in levels as in (15)A. or in log changes as in (15)B. Imperfect pass-through in the first case says nothing about labor market power, since pass-through may be less than one due to markdowns, $\mu < 1$, or returns to scale, $\alpha\gamma < 1$. Imperfect pass-through in the second case, however, is a test of the oligopsony model. In order for log wages to respond less than 1 for 1 with changes in log value added per worker, e.g. $\Delta \log w_{ijt} < \Delta \log vapw_{ijt}$, markdowns must increase. Our oligopsony model naturally generates variations in time variation in markdowns: following an increase in firm productivity firms hire more workers, pay higher wages, but with an expanding market share the firm's markdown widens, which dampens the wage increase. In either of our monopsony limits, markdowns are constant and $\Delta \log w_{ijt} = \Delta \log vapw_{ijt}$, as in monopsony models of Manning (2003) and Card, Cardoso, Heining, and Kline (2018).

Totally differentiating the market equilibrium system yields the following first order approximation for pass-through following any perturbation (for derivation see Appendix E.8):

$$\underbrace{\frac{\Delta \log w_{ij}}{\Delta \log vapw_{ij}}}_{\text{Pass-through}} = \underbrace{\Gamma^*_{ij}}_{\text{Direct}} + \underbrace{(1-\Gamma^*_{ij}) \sum_{k \neq i} \frac{s^*_{kj}}{1-s^*_{ij}} \frac{\Delta \log w_{kjt}}{\Delta \log vapw_{ijt}}}_{\text{Indirect}} \quad , \quad \Gamma^*_{ij} = \frac{s^*_{ij}(\eta-\theta)+\theta(\eta+1)}{[1+(1+\eta)(1-s^*_{ij})]s^*_{ij}(\eta-\theta)+\theta(\eta+1)}, \tag{16}$$

where Δ's are taken with respect to the initial equilibrium, which is denoted by asterisks. Clearly from this expression under either monopsony limit ($\theta \to \eta$ or $s_{ij} \to 0$) $\Gamma^*_{ij} = 1$ and so pass-through is one.

Figure 7 plots the average measure of the exact value of pass-through by payroll share bins, and shows that the approximation (16) is highly accurate. As firms become larger in a market two offsetting forces shape pass-through. First, the direct effect declines, as increases in productivity into increasing market power and widening markdowns, reducing pass-through. Second, when the firm is large its competitors respond more aggressively, increase their wages, which indirectly leads to further wage increases at the firm, increasing pass-through. On net, the direct effect dominates and pass-through is less than one.

Replication. Estimates for wage pass-through from a paper with sufficient details for us to replicate come from Kline, Petkova, Williams, and Zidar (2019).[37] KPWZ exploit patent issuance as an instrument, comparing consequent changes in value added per worker and wages. To replicate their quasi-experiment we solve the model in general equilibrium, then draw one firm from each market and increase their productivity by ψ_1^{KPWZ} percent. We solve the new market equilibria, keeping aggregates constant. We limit our sample of firms that we shock to firms with more than ψ_2^{KPWZ} workers. We calibrate the replication parameters $\{\psi_1^{KPWZ}, \psi_2^{KPWZ}\}$ to match two moments of their study: a median firm size in sample of 25.36, which is larger than in our baseline calibration, and a mean increase in value added per worker relative to mean value added per worker of 13 percent.[38] Table A1 compares summary statistics of our regression sample to theirs.

Measurement. To measure pass-through, we adopt the procedure in KPWZ.[39] We treat the pre- and post- observations from the model as a panel with two observations per firm.[40] We then regress w_{it} on $vapw_{it}$ in levels with firm-specific fixed effect. The regression coefficient is a semi-elasticity which is converted into an elasticity using the initial period mean wage and initial period mean value added per worker (see their Section 7). With this procedure their point estimate implies pass-through of 0.47,

[37] Recent work by Card, Cardoso, Heining, and Kline (2018) uses lagged log sales per worker as an instrument for log value added per worker. From Table 2 (panel A, row IV, column 1) their estimate of pass-through is 32.7 percent, however the paper contains insufficient information in order for us to replicate it, for example the size of changes in value added per worker. A structural approach is taken by Friedrich, Laun, Meghir, and Pistaferri (2019), who estimate pass-through of 31 percent from permanent shocks in a model of worker and firm dynamics estimated on Swedish employer-employee data. See their Table 12, column 1.

[38] See KPWZ. We take the *Median firm size* of 25.36 from their Table II, panel A, column 7. The percentage increase in VAPW is 0.13=15.74/120.16, where 15.74 is the mean increase in value added per worker (Table V, column 4), and 120.16 is the mean value added per worker (Table II, panel A, column 5). This is exactly equal to our value-added production function which represents sales minus costs of intermediate inputs.

[39] They describe this procedure in Section VII, and footnotes to Table VIII.

[40] *Value-added* in KPWZ is defined as sales minus 'costs of goods sold net of labor costs'. This is consistent with our measure.

with a standard error of 0.23. We verify that under monopsony, i.e. $\theta = \eta$, this approach delivers a pass-through of one.

Results. Figure 7B provides the results of this exercise. Replicating the KPWZ statistic, our estimate of pass-through is 0.61. To put this in context, our estimate lies at the 70^{th} percentile of the distribution of the point estimate of KPWZ, and hence within even the 25 percent confidence interval. We view this as a success of the model. We also plot the distribution of pass-through across firms, showing rich cross-sectional heterogeneity. Relatively smaller firms have higher pass-through, and the support of firm pass-through extends below the KPWZ estimate. Ignoring the fact that the KPWZ sample is biased toward large firms would increase our estimate of pass-through by around 10 percentage points (red crossed lines) as smaller firms have higher pass-through, revealing that the bias introduced by the sample in KPWZ leads to lower pass-through estimates. Figure 7C verifies that decreasing returns is not behind these results. Conducting the exercise under $\alpha = 1$ increases pass-through only slightly.

3.2 Strategic responses - Staiger, Spetz, and Phibbs (2010)

Theory. An important paper by Staiger, Spetz, and Phibbs (2010, henceforth SSP) provides direct empirical evidence regarding the response of firms in one labor market to increases in wages of other firms in the same labor market. Consider either monopsony limit where a firm exogenously narrows its markdown to $\mu' \in [\mu, 1]$, where $\mu = \eta/(\eta+1)$. In either limit, the fact that the firm is infinitesimal would imply zero effect on competitor's wages within the same geographic area. Contrary to this, SSP find that when Department of Veterans Affairs (VA) hospitals increased their wages due to a change in policy, competitors increased their wages in response. In an environment with $\eta > \theta$, the above pass-through formula (16) shows how our model is consistent with this fact as an increase in wage at firm $k \neq i$ causes firm i to increase its wage. The mechanism is as follows: a VA hospital increases its wage, which increases its employment and increases its market share $s_{VA,jt}$, this tightens competition leading non-VA hospitals to narrow their markdowns, which increases their wages.

Replication. Key properties of the sample and quasi-experiment in SSP are as follows: (i) markets—defined as a 15-mile radius of the focal VA hospital—had on average 10.9 hospitals, (ii) the VA hospital was on average paying nurses 1.9 percent below the average wage for nurses at non-VA hospitals, (iii) the policy increased nurse wages of VA hospitals paying below the local average up to the average wage of nurses at non-VA hospitals. To replicate this experiment we take our baseline economy which we call period zero. We then isolate markets j with between 9 and 13 firms, draw one firm i at random in each of these markets from the set of firms with a wage between 1 and 3 percent less than the average market wage, and then increase this firm's productivity by ψ_{ij}^{SSP} percent. Holding aggregates fixed, we then solve the new market equilibria. We choose ψ_{ij}^{SSP} firm-by-firm such that in the new equilibrium the wage w_{ij1} at firm i equals the initial period average wage at competitors.[41] On average ψ_{ij}^{SSP} is 3.12 percent, and ranges from 1.42 percent to 5.10 percent.

[41] An alternative would have been to have narrowed the VA hospital's markdown. From the perspective of the competing firms, both are equivalent, since they only take into account competitor's wages.

	Model	Data
A. Replication statistics		
Average log difference (gap) between VA hospital wage and average competitor wage	0.020	0.019
Average number of firms in a market	10.9	10.9
Average productivity increase to set gap to zero		1.0
B. Result		
Elasticity of competitor wages to VA hospital wage	0.065	0.128
(Standard error)		(0.033)

Table 4: Strategic interaction and replication of Staiger, Spetz, and Phibbs (2010)

Notes: Model simulation selects firms (the 'VA hospital') whose wages are between 1% and 3% lower than the average market wage and are in a market with 9 to 13 firms. The exercise is to raise the VA hospital wage in period one up to the average market wage in period zero, and then to compute the response to competitor wages. Pooling across markets, we report a cross-sectional elasticity obtained by regressing log changes of average competitor wages on log changes of VA hospital wages. We compare our estimates to Table 1 (summary statistics) and Table 2 (point estimates) in Staiger, Spetz, and Phibbs (2010).

Measurement. To measure employer wage responses, we adopt the procedure in SSP. We treat the data from the model as a panel with two periods. From this we compute $\Delta \log w_{VA,j}$ at the 'VA hospital' in each market, and the change in log wages at non-*VA* hospitals $\Delta \log w_{ij}$. We then pool across markets and estimate regression equation (6) of SSP which produces a coefficient α_1 comparable to their Table 2, column 1:

$$\Delta \log w_{ij} = \alpha_0 + \alpha_1 \Delta \log w_{VA,j} + e_{ij}$$

Results. Table 4 compares our results to SSP. Quantitatively, the model generates a response of competitors' wages of 6.46 percent, which is within the 95 percent confidence interval around the SSP estimate: $[6.33, 19.27]$. We conclude that the structure of labor markets and our estimates of θ and η generate strategic complementarities in wage setting that are consistent with this important empirical evidence.

3.3 Mergers - Arnold (2020a)

Theory. A recent paper by Arnold (2020a) documents the employment and wage effects of mergers on employment and wages, and how these vary by market concentration. Our theory of a merger between two firms in a labor market is simple: following a merger between firm $i = 1$ and $i' = 2$ in market j, employment in both firms is chosen 'centrally', in order to maximize $\pi_{1j} + \pi_{2j}$. This results in changes in employment and wages consistent with the data.

Proposition 3.1. Mergers. *(i) Following a merger, the markdowns at the merged firms are equalized and depend on the total market share, $\mu_{1j} = \mu_{2j} = \mu(s_{1j} + s_{2j})$. (ii) The wages of each merging firm decreases, and their total employment decreases. (iii) The total market share of the merging firms decreases and the market share of every non-merging firm increases, (iv) Market wage $\mathbf{w}_j$ and employment $\mathbf{n}_j$ decline, so total market pay $\sum_{i\in j} w_{ij}n_{ij}$ declines. (v) Under either monopsony limit a merger has no effect on any labor market variables.*

Proof is in Appendix E.10. A merger has two negative welfare effects on a labor market. Directly, the merging firms' increase in market power leads them to set markdowns according to their (new) combined market share, reducing their wages which lowers $\boldsymbol{\mu}_j$. Indirectly, this reallocates employment

Moment	A. Arnold (2020) Reference	Value	B. Replicate Value
Part I. Outcomes at merging firms			
Target: Median employment pre-merger	Table 1	116.0	116.3
Change in log employment (weighted)	Table 3(1)	-0.144	-0.070
Change in log payroll (weighted)	Table 3(4)	-0.121	-0.083
Change in log worker earnings	Table 5(2)	-0.008	-0.013
... high impact market	Table 6(1)	-0.031	-0.041
... medium impact market	Table 6(2)	-0.008	-0.012
$\Delta HHI_j = \alpha + \beta\Delta\widehat{HHI}_j$	Table 8(1)	0.834	0.904
Part II. Market outcomes in markets with large predicted changes in hhi_j			
Target: Average change in log HHI_j	Figure 8A	0.170	0.171
Elasticity of market wage to HHI	Table 10(3)	-0.219	-0.475
... above median HHI	Table 10(6)	-0.259	-0.505

Table 5: Mergers and replication of Arnold (2020a)

Notes: Bracketed terms indicate column numbers in the relevant table of Arnold (2020a). Change in employment is computed as change in combined firm employment pre- and post-merger. We follow a similar procedure for payroll and worker earnings. *High and medium impact* markets are within the top quartile of predicted Δhhi_j, and high (medium) impact markets are above (below) the median concentration within this group. See text for further discussion.

toward competing firms, increasing their market power which reduces their wages, further reducing $\boldsymbol{\mu}_j$. Reallocation effects are ambiguous. If the two merging firms are highly productive, reallocation lowers $\boldsymbol{\omega}_j$, further reducing welfare. If the two firms are low productivity then reallocation away from the merging firms improves $\boldsymbol{\omega}_j$. In on-going work, we determine how large merger productivity gains must be in order to offset these negative effects.

Under either monopsony limit there are no market wage or employment effects from mergers. Markdowns are always constant and employment at both firms is chosen to set the marginal product of labor equal to its marginal cost. Hence we can again use mergers to assess the qualitative and quantitative relevance of our model.

Replication. We randomly draw two firms in markets with more than two firms and 'merge' these firms. We then solve the new market equilibria keeping aggregates fixed. In the data, merging firms skew heavily toward large firms. To sample large firms we drop all markets in which average employment at the merging firms is below ψ_1^A, and choose ψ_1^A to match median pre-merger employment of 116 (Table 1).[42] Second, the second part of Arnold (2020a)'s analysis focuses on mergers that have a large impact on concentration, using only mergers that have a 95^{th} percentile and above effect on concentration. In this sample, the average change in concentration is 0.17 (Figure 8A). We choose a cut-off percentile ψ_2^A that delivers the same average increase in concentration. We carefully follow the measurement of market and firm variables in Arnold (2020a) .

Results. Table 5 gives our results. The first part of Arnold (2020a) 's analysis considers changes in wages and employment across firms at all mergers. In employment weighted regressions he finds aver-

[42] We also consider dropping markets where the smaller of the merging firms is below a cut-off, both approaches give the same cut-off.

age firm log employment declines by 14 percent and log payroll by 12 percent (Table 3). Our results are of similar magnitude: employment declines by 7 percent and payroll by 8 percent. Focusing on workers rather than establishments, Arnold (2020a) then shows that the wages of workers that remained at merging establishments fell by 0.8 percent (Table 5), our effects are slightly larger: -1.3 percent.

A key result in Arnold (2020a) is that worker wage losses are larger in highly concentrated markets (Table 6). He categorizes markets on two variables. First, whether a market is below or above median concentration pre-merger. Arnold (2020a) first splits mergers based on whether the predicted changes in concentration from the merger are inside or outside the top quartile.[43] *High and medium impact* markets are within the top quartile of predicted Δhhi_j, and high (medium) impact markets are above (below) the median concentration within this group. Repeating the analysis for these groups, he finds that wage losses are 3.9 times larger (-3.1 vs -0.8 percent) in high relative to medium impact markets (Table 6). The model produces 3.4 times larger wage losses in high impact markets (-4.1 vs -1.2 percent). Finally, Arnold (2020a) finds that increases in concentration are less than predicted, with a coefficient of 0.83, which our model also produces. The simple 'predicted change' in concentration measure does not account for the equilibrium response of market shares. After a merger, competitors' market shares increase and the merging firms' market shares decrease, implying that concentration increases by less than the simple prediction. This unique prediction is consistent with oligopsony, $\eta > \theta$, and would not obtain in the absence of strategic interactions.

Finally, we follow the analysis in the second part of Arnold (2020a) and focus on the relationship between concentration, wages and employment induced by mergers.[44] Around merger events Arnold (2020a) finds a significant, negative elasticity of average market wages with respect to the induced change in market concentration. On average the elasticity is -0.219, which increases to -0.259 in markets above median concentration. Our effects are larger, with an elasticity of -0.475 but with a similar proportional increase to -0.505 in markets above median concentration.

4 Measuring and decomposing welfare

Now that we have validated our model quantitatively using three empirical studies, we use it to measure labor market power in the U.S. at the micro- and macro-level and explore it's implications for welfare.

4.1 Microeconomic measurement

A firm's markdown is an economically meaningful measure of labor market power. The markdown at the firm measures the wedge between the firm's marginal revenue product of labor and its wage. At the efficient allocation, workers are paid their entire marginal revenue product and thus markdowns are equated to one.

[43] The predicted change in concentration is computed by comparing the pre-merger hhi_j to one that takes the pre-merger market shares of firms, and then simply combines those of the merging firms and re-computes concentration.

[44] Arnold (2020a) considers a sub-sample of markets with 95^{th} percentile and above changes in concentration. We generate a similar sub-sample by keeping markets in which concentration increases by more than 0.025, which matches the average change in concentration in his sub-sample ($+0.17$) (Figure 8A).

Figures 8A and 8B plot the distribution of firms and wage payments across structural labor supply elasticities ε_{ij} and markdowns μ_{ij}. In an economy that matches the distribution of firms and concentration across markets, as well as salient pass-through and wage setting facts, we find that most firms in the economy are highly competitive, with narrow markdowns attributable to low market shares and high labor supply elasticities. Taking an unweighted average across firms, the mean labor supply elasticity is more than six, while the markdown is narrower than 20 percent.

Despite this, the distribution of wage payments in the economy is highly skewed toward firms with more labor market power. Weighted by payroll, the average labor supply elasticity drops to less than four, and the average markdown is around 0.76. As we have shown in Section 1.6, however, what matters for welfare is the aggregate markdown $\boldsymbol{\mu}$. This is a particular productivity weighted average of firm markdowns, and skews even further, with a value of 0.71. We can contextualize this by computing what we call the *representative labor supply elasticity*, which is the elasticity of labor supply to a firm that would lead a firm to set a markdown of $\boldsymbol{\mu}$. This value $\mathcal{E}$ is around 2.50, which is less than half the cross-sectional average of ε_{ij}. In short, the distribution of wage payments in the economy is crucial for determining the mapping from labor supply elasticities to welfare.

Reduced form elasticities. Next we investigate our model's ability to rationalize the dispersion observed in existing studies of the labor supply elasticity. As we have already shown, our empirical exercise generates reduced-form labor supply elasticities that are lower than the high structural labor supply elasticities that matter for welfare. Figure 8C compares the distribution of reduced-form labor supply elasticities from our estimation exercise with a survey of reduced-form elasticity point estimates in 23 recent papers on labor market power (we list the studies and point estimates in Appendix A). The distribution of reduced-form elasticities we estimate rationalizes the distribution of point estimates found in empirical work. Our model suggests that much of the empirical work estimates reduced-form elasticities that are polluted with strategic interactions via the market equilibrium.[45]

4.2 Macroeconomic measurement

Efficient allocation. To measure and decompose the welfare losses due to labor market power we first must define an efficient benchmark. The planner's problem is to choose employment at all firms $\{n_{ijt}\}$ and capital K_t to maximize the present discounted value of utility subject to the definitions of preferences and technology and the aggregate resource constraint:

$$\mathbf{C}_t + \left[K_{t+1} - (1-\delta) K_t\right] = \int_0^1 \sum_{i=1}^{m_j} \overline{Z} z_{ijt} \left(k_{ijt}^{\gamma} n_{ijt}^{1-\gamma}\right)^{\alpha} dj \tag{17}$$

The efficient allocation is characterized by the following first order condition for n_{ijt}:

$$-U_N\left(\mathbf{C}_t, \mathbf{N}_t\right) \left(\frac{n_{ijt}}{\mathbf{N}_{jt}}\right)^{\frac{1}{\eta}} \left(\frac{\mathbf{N}_{jt}}{\mathbf{N}_t}\right)^{\frac{1}{\theta}} = U_C\left(\mathbf{C}_t, \mathbf{N}_t\right) mpl_{ijt} \quad , \quad mpl_{ijt} = \alpha\gamma \frac{y_{ijt}}{n_{ijt}} \quad , \quad \text{for all} \quad ij \tag{18}$$

[45]Unfortunately, none of the studies surveyed includes estimates of firms' relative size in the market (either employment share, wage-bill share, etc.), thus we can only compare our model to a histogram of existing data estimates.

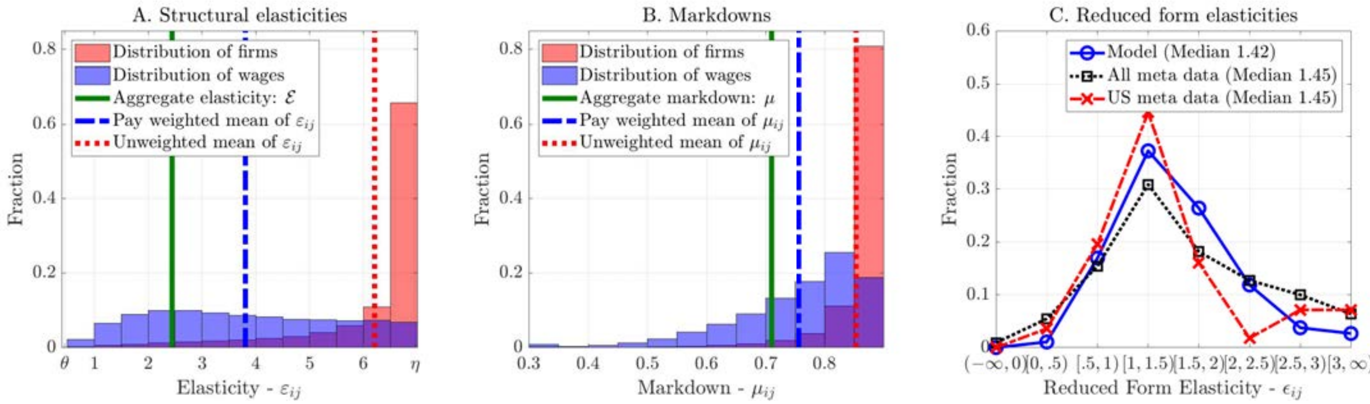

Figure 8: Distribution of labor market power at the firm level

Notes: Panel A plots the distribution of equilibrium structural labor supply elasticities $\varepsilon(\cdot)$ from equation (11), unweighted ('Firms') and weighted by payroll ('Wages'). $E[\varepsilon_{ij}]$ is the unweighted mean structural elasticity, and $\mathcal{E}$ is the aggregate structural labor supply elasticity consistent with an aggregate markdown $\boldsymbol{\mu}$, i.e. $\boldsymbol{\mu} = \mathcal{E}/(\mathcal{E}+1)$. Panel B conducts the same exercise for markdowns. $E[\mu_{ij}]$ is the unweighted mean markdown, and $\boldsymbol{\mu}$ is the aggregate markdown.

On the right is the marginal product of labor at firm ij, converted into utils, while on the left is the disutility of supplying that labor transformed into utils. The marginal product of capital is equated across firms.[46] In this economy the aggregate markdown is $\boldsymbol{\mu}^* = 1$ and misallocation $\boldsymbol{\Omega}^* = 1$

Competitive equilibrium. Note that the efficient allocation can be decentralized under a *competitive equilibrium* concept that yields $\mu_{ijt} = 1$. This is obtained if firms take their wage w_{ijt} as given, in which case (18) corresponds to the firm's first order condition for n_{ijt}, combined with the household's aggregate labor supply curve. The wages that would be obtained in this case obviously correspond to the shadow wages of the planner, as such we use them to compute objects like the HHI_t^{wn} implied by the efficient allocation. This also justifies our description of the efficient allocation having *more competition* than the benchmark economy, since in the corresponding decentralization firms are competitive, taking their wages as given. Figure A3 compares a firm behaving monopsonistically (Panel A) and competitively (Panel B), as in the decentralization just described.

Measurement. We measure the *welfare loss / gain*, which we denote λ, as the percentage increase in consumption in the benchmark economy, that would be required to make the household indifferent with respect to a counterfactual allocation.[47] First, we compute the welfare gain from competition across steady-states: λ_{SS}. Let $\{\boldsymbol{C},\boldsymbol{N}\}$ denote consumption and disutility of labor in the benchmark economy. Let $\{\boldsymbol{C}^*,\boldsymbol{N}^*\}$ denote consumption and disutility associated with the efficient allocation. Then

[46] First order condition for capital k_{ijt} equates the marginal product of capital at all firms to the shadow value of capital R_t^* which satisfies $U_C(\mathbf{C}_t,\mathbf{N}_t) = \beta U_C(\mathbf{C}_{t+1},\mathbf{N}_{t+1})\,[R_t^* + (1-\delta)]$.

[47] Note that aggregate consumption incorporates the effect of competition on wages, employment and firm profits. Recall that $\boldsymbol{W}$ is defined by $\boldsymbol{WN} = \int \sum_{i\in j} w_{ij} n_{ij}\, dj$, and $\boldsymbol{C}$ is defined by $\boldsymbol{C} = \int \sum_{i\in j} c_{ij}\, dj$. Therefore, aggregating firms' profit conditions $\left(\pi_{ij} = y_{ij} - w_{ij}n_{ij} - Rk_{ij}\right)$ under goods market clearing and these definitions returns the household budget constraint $(\Pi = \boldsymbol{C} - \boldsymbol{WN} - RK)$, so $\boldsymbol{C} = \Pi + \boldsymbol{WN} + RK$.

λ_{SS} equates

$$U\big((1+\lambda_{SS})\,\boldsymbol{C},\boldsymbol{N}\big) = U\big(\boldsymbol{C}^{*},\boldsymbol{N}^{*}\big) \tag{19}$$

Second, we compute the welfare gain from competition along the transition path between steady states: λ_{Trans}. We assume that market structure changes, unexpectedly, at date 0, and compare permanently being in the benchmark economy to the transition to the efficient allocation:

$$\sum_{t=0}^{\infty}\beta^{t}U\big((1+\lambda_{Trans})\,\mathbf{C},\mathbf{N}\big) = \sum_{t=0}^{\infty}\beta^{t}U\big(\mathbf{C}_{t}^{*},\mathbf{N}_{t}^{*}\big). \tag{20}$$

Preferences. Proposition 1.2 established that the inputs into solving for aggregate $\boldsymbol{\mu}$ and $\boldsymbol{\Omega}$ were independent of other aggregates and so independent of the specification of $U(\cdot)$. For our baseline results we consider GHH preferences, and then introduce wealth effects (WE):

$$U_{GHH}(\mathbf{C}_t,\boldsymbol{N}_t) = \log\left(\boldsymbol{C}_t - \overline{\varphi}^{-1/\varphi}\frac{\boldsymbol{N}_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right) \quad , \quad U_{WE}(\mathbf{C}_t,\boldsymbol{N}_t) = \frac{\boldsymbol{C}_t^{1-\sigma}}{1-\sigma} - \overline{\varphi}^{-1/\varphi}\frac{\boldsymbol{N}_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}.$$

We show in Appendix E that $\boldsymbol{\mu}$ and $\boldsymbol{\Omega}$ are also independent of the scale parameters $\overline{Z}$ and $\overline{\varphi}$. When giving comparative statics with respect to preference parameters we recalibrate these scale parameters to match the same average worker wage and average firm payroll in the benchmark oligopsony economy.

Results. Table 6 presents our baseline results. We focus on an aggregate Frisch elasticity of 0.50, which we vary within the range considered by the Congressional Budget Office in assessing policy: $\varphi \in [0.20, 0.80]$. First, the steady state welfare gains are large, around 7 percent of consumption. Second, these gains are moderated slightly due to transition dynamics. Reaching higher steady-state capital is costly and gradual due to decreasing marginal utility. Transition dynamics are straight-forward to compute, since Proposition 1.2 tells us that $\boldsymbol{\mu}_t$ and $\boldsymbol{\Omega}_t$ jump at date zero to their efficient levels. Figure A4 and its footnote describe the dynamics of the economy in detail. Third, under a higher labor supply elasticity, it less costly for the representative household to supply more labor in the competitive allocation, and so welfare and output gains are larger.

We now leverage Proposition 1.2 to decompose the welfare effects of labor market power into the aggregate markdown $\boldsymbol{\mu}$ and misallocation $\boldsymbol{\Omega}$. We do so by changing one at a time, and recomputing the aggregate economy using the general equilibrium conditions. At the benchmark $\varphi = 0.50$ welfare gains are roughly half due to misallocation $\boldsymbol{\Omega}$, 42 percent due to markdowns $\boldsymbol{\mu}$, with the remaining 8 percent due to their interaction.

That misallocation $\boldsymbol{\Omega}$ plays a significant role is a key result. Recall that a monopsony economy would deliver $\boldsymbol{\mu} = \eta/(\eta+1)$. Hence η could be parameterized to match the $\boldsymbol{\mu}$ from the oligopsony economy. This monopsony economy would have the same welfare gains due to markdowns shifting to their efficient level. However the monopsony economy would feature $\boldsymbol{\Omega} = 1$ and miss the welfare gains from resolving misallocation. These welfare effects account for around one half of the total welfare effects of labor market power.

In terms of measurable features of the labor market, the average worker wage increases by about 50 percent, with employment changes ranging from 1 to 22 percent, depending on the Frisch elasticity, φ.

Frisch elasticity	A. Welfare				B. Labor market		C. Concentration
	Steady state	Transition	Due to $\boldsymbol{\omega}$	Due to $\boldsymbol{\mu}$	Ave. wage	Agg. emp.	Weighted
φ	$\lambda_{SS} \times 100$	$\lambda_{Trans} \times 100$	λ^{ω}/λ	λ^{μ}/λ	% change	% change	ΔHHI^{wn}
0.20	4.8	4.2	0.71	0.24	49.1	0.8	0.19
0.50	7.0	5.7	0.49	0.42	48.4	11.3	0.19
0.80	9.2	7.2	0.37	0.52	47.6	22.5	0.19

Table 6: Welfare gains from competition with GHH preferences

Notes: Preferences are GHH. Welfare gain λ_{SS} is given by (19), λ_{Trans} is given by (20). Both correspond to moving from the benchmark oligopsony to competitive equilibrium. Average wage and aggregate employment are expressed in percentage increases from oligopsony to competitive steady-state. Average wage is total wage payments divided by total employment, and aggregate employment is in 'bodies' not disutility.

Welfare gains are significantly larger for higher values of the Frisch elasticity, as working more in the efficient allocation is less costly. Wage increases are slightly smaller under higher φ. Our aggregation results showed that $\boldsymbol{W} = \boldsymbol{\mu}\widetilde{\alpha}\boldsymbol{N}^{\widetilde{\alpha}-1}$. Regardless of φ, the aggregate markdown $\boldsymbol{\mu}$ increases to one, which increases wages. With a higher Frisch elasticity φ, however, the larger increase in $\boldsymbol{N}$ reduces the marginal product of labor which dampens the increase in wages.

Finally, concentration and welfare increase in unison as we move to the efficient allocation. The benchmark economy has a payroll weighted wage-bill Herfindahl of 0.11 (Table 3). Under the shadow wages implied by the efficient allocation, the Herfindahl more than doubles, increasing by 0.19. This is consistent with the large role of misallocation $\boldsymbol{\Omega}$ in welfare. In the efficient allocation, large firms no longer use their market power to cut back on quantities and lower wages. Wages and employment therefore increase most at the largest firms who had the widest markdowns in our benchmark economy. This reduces misallocation (i.e. $\boldsymbol{\Omega}$ rises) but also increases concentration. Consistent with Proposition 1.3, this increase in concentration is independent of the specification of aggregate preferences. Similarly the labor share increases to $\widetilde{\alpha}$ in all economies, since $LS = \widetilde{\alpha}(\boldsymbol{\mu}/\boldsymbol{\Omega})$.

Reallocation. Figure 9(I) illustrates the reallocation of employment that underlies the increase in $\boldsymbol{\Omega}$ under the efficient allocation. We plot the change in employment across the productivity distribution of firms (panel I). Panel I shows that the reallocation consists of a significant shift in employment away from low productivity firms and toward the highest decile of firms. To visualize how reallocation occurs at a market level, Panel II returns to our two and three firm example Figure 4 and adds the efficient allocation. With markdowns equated at one, wages increase at all firms, more than doubling at the most productive firm. Since it had the widest markdown to begin with, the wage increase is largest at the most productive firm, which reallocates employment toward it and away from the medium and low productivity firms.

Wealth effects. We can learn more about the role of misallocation and the markdown by studying the economy with wealth effects. With wealth effects on labor supply, the increase in consumption under the efficient allocation leads to a reduction in household labor supply, dampening welfare gains. Despite this, our results are robust. Compared to our baseline welfare gain of 7 percent under $\varphi = 0.50$, Figure

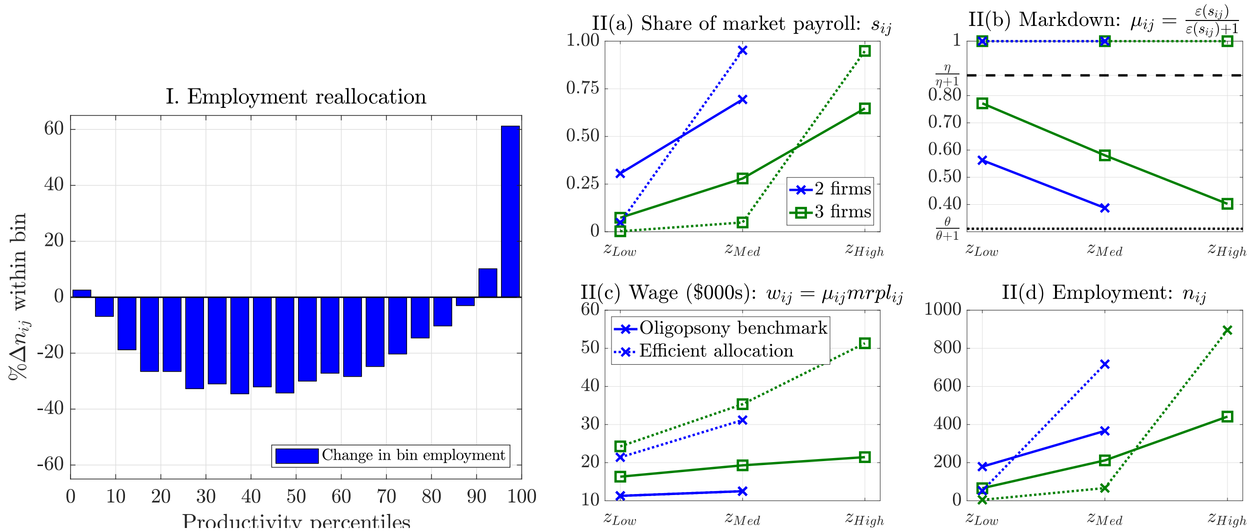

Figure 9: Employment reallocation from the benchmark economy to the efficient economy

Notes: Panel I: Firms are grouped by 5 percent bins of the productivity distribution in the benchmark economy. Panel II: Solid line replicates benchmark oligopsony economy Figure 4, dashed lines plot outcomes under the efficient allocation. Low, medium and high productivities of the firms correspond to the 10^{th}, 50^{th} and 90^{th} percentiles of the productivity distribution.

10A shows that shifting to $U_{WE}(\cdot)$ under log preferences ($\sigma = 1$), the welfare gain narrows slightly to 5.38 percent. Further increases in σ as far as 4 still lead to significant welfare gains of more than 4 percent.

Interestingly, wealth effects have a significant impact on the decomposition of welfare gains into misallocation and markdowns. Figure 10B shows that the fraction of welfare gains attributable to the resolution of misallocation $\mathbf{\Omega}$ jumps to nearly 60 percent under log preferences, and increases further as wealth effects become more pronounced. Recall that in general equilibrium $\mathbf{Y} = \mathbf{\Omega}\widetilde{\mathbf{Z}}\mathbf{N}^{\widetilde{\alpha}}$. An increase in $\mathbf{\Omega}$ delivers a direct increase in output. If σ is larger, the increase in utility due to the associated increase in consumption is larger. In the limit as $\varphi \to 0$ labor supply becomes perfectly inelastic and $\mathbf{\Omega}$ accounts for the entirety of the welfare gains from competition.

Summary. In summary, we measure that the welfare costs of labor market power in the U.S. economy are between 4 and 9 percent of aggregate consumption, with at least half of those gains coming from reallocation. Our model can measure these reallocation effects due to the structure of labor markets and strategic interaction among firms under our estimates of $\eta > \theta$. A model of monopsony would miss these welfare gains since in such a model misallocation $\mathbf{\Omega} = 1$ under the decentralized equilibrium and efficient allocation.

5 Application- Labor market concentration and labor's share, 1977 - 2013

As an application of our framework, we use the model implied relationship between concentration and the labor share to show how alternative measurements of concentration can lead to different counterfac-

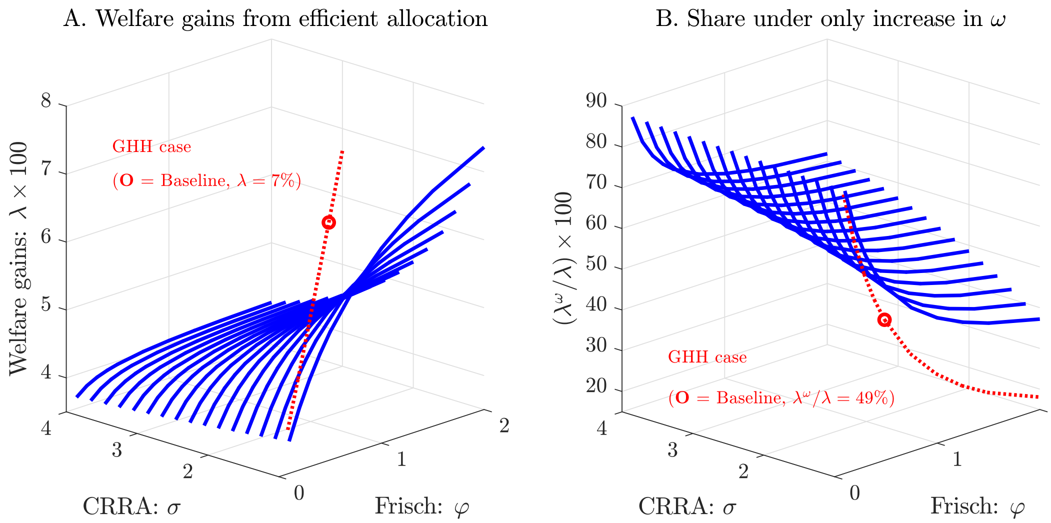

Figure 10: Welfare costs of labor market power with wealth effects

Notes: Panel A plots the percent welfare gains $\lambda \times 100$ associated with the efficient allocation as the Frisch elasticity of labor supply φ and the coefficient of relative risk aversion σ are varied. Each economy has the exact same level of concentration, average firm size, average worker wage, and all other moments used in our calibration 3. Panel B plots the percent welfare gains under only an increase in $\boldsymbol{\omega}$ to its efficient level $\boldsymbol{\omega}^* = 1$, divided by the overall welfare gains associated with the efficient allocation. The red dashed line corresponds to the case of GHH preferences.

tual predictions. We leverage the model's mapping from concentration to the labor share:

$$LS_t = \int_0^1 s_{jt} ls_{jt}\, dj = \alpha\gamma \left[HHI_t^{wn} \left(\frac{\theta}{\theta+1}\right)^{-1} + \left(1 - HHI_t^{wn}\right) \left(\frac{\eta}{\eta+1}\right)^{-1} \right]^{-1}, \; HHI_t^{wn} = \int_0^1 s_{jt} hhi_{jt}^{wn}\, dj. \quad (21)$$

The model clearly implies a welfare relevant measure of labor market concentration: payroll weighted wage-bill Herfindahl. Figure 11A shows how this has evolved from 1977 to 2013, using our definition of a *local* labor market: a 3-digit industry and commuting zone.[48] Tradeables, non-tradeables, and the combined economy all decline over time. In tradeables the decline is roughly 20 percent from 0.217 to 0.175. Concentration in non-tradeables is lower, and declines with a slight increase at the end of our sample, but by 2013 is half its level in 1984.

Figure 11B demonstrates the importance of weighting and compares payroll and employment concentration, considering only the tradeable sector.[49] First, not weighting across markets inflates the measure of concentration by a factor of around 2.5 for both payroll and employment concentration. Many markets have few employers but they account for a very small fraction of wage payments. Second, the weighted payroll and employment Herfindahls display similar trends, with a time-series correlation of 0.75 between 1977 and 2013. Despite this, the positive size wage premium leads employment concentration to be 20 percent less than payroll concentration.

Figure 11C repeats this exercise disregarding the local nature of labor markets. We first compute concentration at the national industry level, and then weight across industries. According to this mea-

[48] To meet Census disclosure requirements, we show detailed summary statistics in 1976 and 2014 in Appendix D. Our time series graphs cover the complementary years from 1977 to 2013.

[49] We have been unable to disclose the corresponding statistics for non-tradeable sectors.

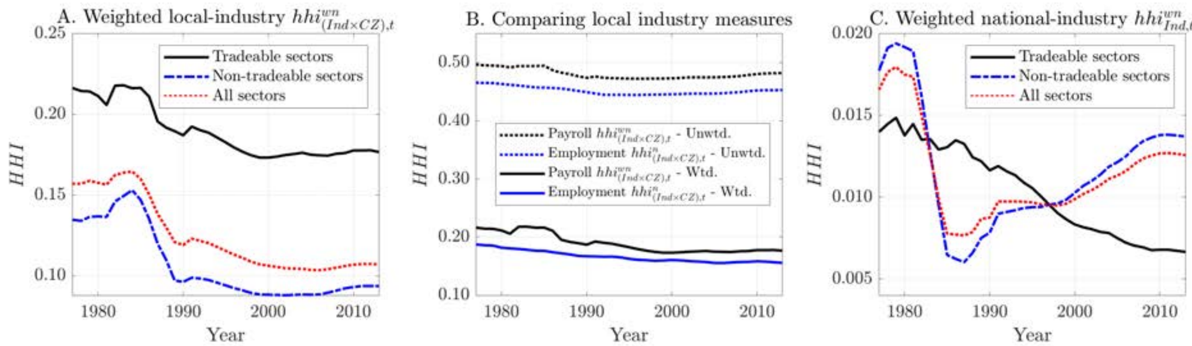

Figure 11: Measures of labor market concentration, 1977 to 2013

Notes: Data is plotted using a centered 5-year moving average in all panels. Panel A plots the payroll weighted average of the wage-bill Herfindahl computed at the commuting zone × NAICS3, $HHI_t^{wn} = \int_0^1 s_{jt} hhi_{jt}^{wn}\, dj$. Panel A includes three lines for tradeables (NAICS2 codes of 11,21,31,32,33,55), non-tradeables (all other NAICS2 codes), and the whole economy. Panel B compares the tradeable payroll weighted and unweighted CZ×NAICS3 wage-bill Herfindahl: hhi_{jt}^{wn}. Panel B also compares the employment weighted and unweighted CZ×NAICS3 employment Herfindahl: hhi_{jt}^{n}. Panel C plots the national payroll weighted wage-bill Herfindahl. National Herfindahls are computed at the NAICS3 level, ignoring geography, then weighted by industry payroll.

sure, which is irrelevant for welfare, labor market concentration increased over this period, following a sharp drop in the early 1980s. While our payroll Herfindahl measure is distinct, other contemporaneous work has documented a disconnect between national and local employment Herfindahls using different definitions of markets and aggregation (e.g. Rossi-Hansberg, Sarte, and Trachter (2018), Rinz (2018)).[50]

5.1 Counterfactual labor share, 1977 - 2013

We can now combine three of the novel contributions of this paper to link the dynamics of labor's share of income to labor market power: (i) the closed-form expression for labor's share of income given by equation (21), (ii) our estimates of θ and η, and (iii) our new time-series of aggregate concentration (Figure 11).

Our counterfactual holds $\{\gamma, \alpha, \eta, \theta\}$ fixed and varies the payroll weighted wage-bill Herfindahl HHI_t^{wn} from 1977 to 2013, using this to compute the implied labor share from equation (21). At our estimated parameters, the declining wage-bill Herfindahl between 1977 and 2013 contributed to increase the labor share by around 4 percentage points. Figure 12 plots the implied changes in labor share holding all else fixed except for the payroll weighted wage-bill Herfindahl. The predicted upward pressure of declining local Herfindahls on labor's share of income is similar for tradeables, non-tradeables, and the overall economy. We conclude that changes in labor market concentration are unlikely to have contributed to the declining labor share in the United States (e.g. Karabarbounis and Neiman (2013)).

[50] First, Rinz (2018) describes employment concentration in a number of non-tradeable sectors using a NAICS4×Commuting zone definition of a labor market. Second, Rinz (2018) does not aggregate establishments within firms when computing employment shares at the local level. When averaged at the 2-digit level, he finds similar trends in tradeable and non-tradeable sectors.

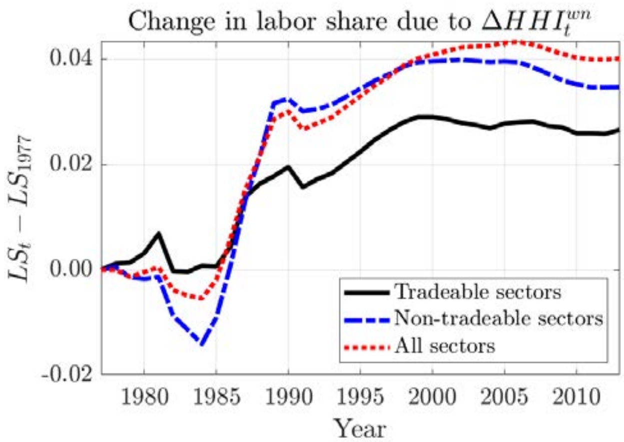

Figure 12: Change in labor share attributable to change in payroll Herfindahl, 1977 to 2013

Notes: Figure constructed by using estimates of payroll weighted wage-bill Herfindahl (Figure 11) and expression for labor's share of income (21). $\{\gamma, \alpha, \eta, \theta\}$ held fixed at values in Table 3.

6 Conclusion

We measure oligopsony in administrative U.S. Census data through the lens of a structural model. By doing so, we make several contributions. We develop a general equilibrium model of labor market oligopsony. We prove that the model is block recursive and provide a closed-firm link between labor market concentration and labor's share of income. We show how to estimate the underlying preference parameters that govern labor market power in the presence of strategic interactions. We provide novel measures of firm size-dependent labor supply elasticities. We rationalize empirical evidence suggestive of oligopsony by quantitatively replicating three empirical papers whose results cannot be obtained from theoretic models that abstract from oligopsony. Under a variety of aggregate preferences, we compute sizeable welfare gains worth 4 to 9 percent of lifetime consumption from moving to the efficient allocation. We show that roughly half of the gains are attributable to misallocation by using a novel representative agent counterpart of our economy. Lastly, we show that the model relevant measure of concentration is the payroll weighted wage-bill Herfindahl, which we measure, and use to show that changes in labor market concentration are unlikely to have contributed to a falling labor share in the U.S.

We believe our framework and empirical findings provide many avenues for future research. By establishing the empirical relevance of our framework through a battery of validation tests, we provide the literature with a useful point of departure. In ongoing work, we demonstrate the framework can be modified to contribute to debates on minimum wage policy and merger policy. The model can also incorporate firm entry/exit and worker heterogeneity, accommodating use of occupation or worker-level data such as the Longitudinal Employer Household Dynamics database to estimate oligopsony.

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Appendix For Online Publication

This Appendix is organized as follows. Section A provides additional tables and figures references in the text. Section B provides our micro-foundation for nested-CES preferences used in the main text and references in Section 1. Section C contains details about the data and sample selection criteria. Section D contains summary statistics used and references in the paper and additional concentration measures. Section E contains derivations of all mathematical expressions in the text, including proofs of Propositions. Section F provides additional details regarding the calibration. Section G provides additional discussion of our empirical results and robustness on a number of dimensions.

A Additional tables and figures

Description	**Model**	**Data (KPZW)**
Log change in VAPW (VAPW=$\widetilde{Z}\tilde{z}_{ij}n_{ij}^{\tilde{\alpha}-1}$)	0.13	0.13
Median firm size	34.59	25.26
Mean firm size	61.85	61.49
Median VAPW (dollars)	57152	86870
Mean VAPW (dollars)	57716	120160
Model Simulation Parameters		
Size cutoff (Employees)	11.00	
Fraction of Firms Shocked	0.01	
Shock size ($dlog(\tilde{z}_{ij})$)	0.17	

Table A1: Wage pass-through experiment details

Notes: Summary statistics for replication of Kline, Petkova, Williams, and Zidar (2019) regressions. We randomly sample one percent of firms in our benchmark economy. We draw firms with employment greater than $\underline{n}$. We increase the productivity of treated firms by a factor $\Delta log \widetilde{z}_i j$. The values of $\underline{n}$ and Δ are calibrated to match the KPWZ (1) median firm size of 25 employees, (2) increase in post-tax value added per worker of 13 percent. We keep aggregates fixed and solve the new market equilibrium. We treat the untreated and treated observations for each firm as a panel with two observations per firm of wages $\left\{w_{ij0}, w_{ij1}\right\}$ and value added per worker, $\left\{\frac{y_{ij0}}{n_{ij0}}, \frac{y_{ij1}}{n_{ij1}}\right\}$. We then regress the wages in levels on VAPW in levels and a firm-specific fixed effect. The regression coefficient is converted into an elasticity using untreated mean wages and mean value added per worker.

	Model	Data
Wage bill Herfindahl – Payroll weighted average	0.17	0.17
Wage bill Herfindahl – Unweighted average	0.33	0.48
Wage bill Herfindahl correlation with market employment	-0.86	-0.25
Employment Herfindahl – Payroll weighted average	0.15	0.15
Employment Herfindahl – Unweighted average	0.32	0.45
Employment Herfindahl correlation with Wage-bill Herfindahl	1.00	0.98

Table A2: Concentration and competition, model versus data

Notes: Data is from 2014 LBD, tradeable sectors. Model is for tradeable calibration. The market level wage-bill Herfindahl is given by: $HHI_j^{wn} := \sum_{i\in j}\left(s_{ij}^{wn}\right)^2$, $s_{ij}^{wn} = \frac{w_{ij}n_{ij}}{\sum_{i\in j} w_{ij}n_{ij}}$. When aggregating, we weight by the market's payroll share $s_j = \frac{\sum_{i\in j} w_{ij}n_{ij}}{\int \sum_{i\in j} w_{ij}n_{ij}dj}$ so that $HHI^{wn} = \int s_j HHI_j^{wn} dj$. The market level employment Herfindahl is given by: $HHI_j^n := \sum_{i\in j}\left(s_{ij}^n\right)^2$, $s_{ij}^n = \frac{n_{ij}}{\sum_{i\in j} n_{ij}}$. We weight the market level employment Herfindahls similarly.

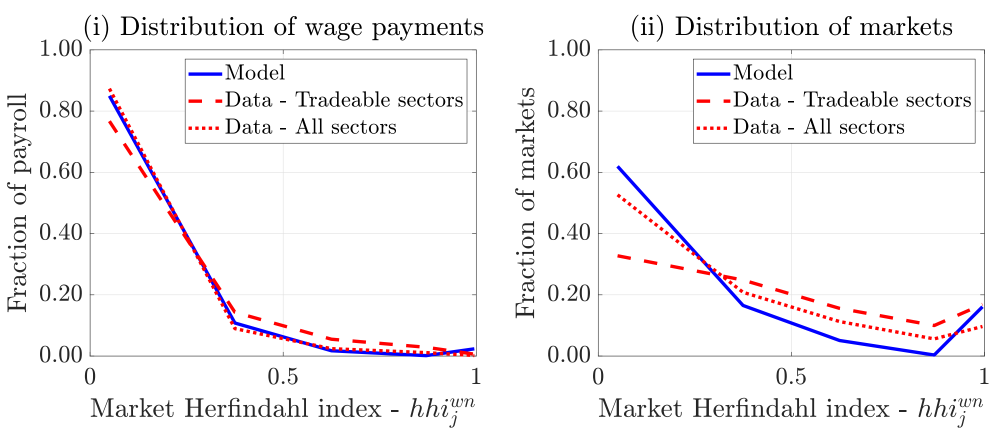

Figure A1: Cross market distribution of concentration model v. data

Notes: This figure plots the market-level distribution of the payroll Herindahl index (HHI_j^{wn}). Model corresponds to the all sectors model. Bins are determined by the following bounds: $\{0, 0.25, 0.50, 0.75, 0.99, 1\}$. The horizontal axis gives the center of each bin. Panel (i) plots the fraction of total payroll in each bin. Panel (ii) plots the fraction of markets in each bin. Data is Census LBD. See Appendix C for additional details.

Labor Supply Elasticity	Industry	Gender	Country (Region)	Paper	Table/Page
1.31	Agriculture	US	Weber(2018)	Table4	
1.6	Mining/oil/natural gas	US	Weber(2018)	Table4	
1.4	Utility	US	Weber(2018)	Table4	
1.59	Construction	US	Weber(2018)	Table4	
1.72	Manufacturing	US	Weber(2018)	Table4	
1.52	Wholesale trade	US	Weber(2018)	Table4	
1.07	Resale trade	US	Weber(2018)	Table4	
1.45	Transportation	US	Weber(2018)	Table4	
1.22	Information	US	Weber(2018)	Table4	
1.38	Finance and insurance	US	Weber(2018)	Table4	
1.13	Real estate and rental	US	Weber(2018)	Table4	
1.3	Profession/Scientific/technical services	US	Weber(2018)	Table4	
1	Management of companies	US	Weber(2018)	Table4	
0.97	Administrative support	US	Weber(2018)	Table4	
0.96	Educaitonal Services	US	Weber(2018)	Table4	
0.97	Health Care and Social Assistance	US	Weber(2018)	Table4	
0.93	Art and entertainment	US	Weber(2018)	Table4	
0.96	Accommodation and food services	US	Weber(2018)	Table5	
1.19	Other services	US	Weber(2018)	Table5	
1.11	Public administration	US	Weber(2018)	Table5	
3.7	Public school teachers	US	Ransom and Sims(2010)	Table5	
4.6	Retail stores	US	Dube et all(2019)	Table5	
1.43	Agriculture	US	Weber(2015)	Table5	
1.52	Mining/oil/natural gas	US	Weber(2015)	Table5	
1.18	Utility	US	Weber(2015)	Table5	
1.42	Construction	US	Weber(2015)	Table5	
1.82	Manufacturing	US	Weber(2015)	Table5	
1.48	Wholesale trade	US	Weber(2015)	Table5	
1.03	Resale trade	US	Weber(2015)	Table5	
1.47	Transportation	US	Weber(2015)	Table5	
1.17	Information	US	Weber(2015)	Table5	
1.27	Finance and insurance	US	Weber(2015)	Table5	
1.01	Real estate and rental	US	Weber(2015)	Table5	
1.17	Profession/Scientific/technical services	US	Weber(2015)	Table5	
1.17	Management of companies	US	Weber(2015)	Table5	
0.72	Administrative support	US	Weber(2015)	Table5	
0.91	Educaitonal Services	US	Weber(2015)	p1	
0.78	Health Care and Social Assistance	US	Weber(2015)	p26	
0.94	Art and entertainment	US	Weber(2015)	p1	
0.85	Accomodation and food services	US	Weber(2015)	p30	
1.04	Other services	US	Weber(2015)	p1	
1.19	Public administration	US	Weber(2015)	Table9	
1.09	All	US	Weber(2016)	Table9	
0.94	All	US	Weber(2016)	Table9	
1.86	Manufacture Skilled (Market FE)	US	Tortarolo and Zarate (2020)	p1	
4	Manufacture Unskilled (Market FE)	US	Tortarolo and Zarate (2020)	p1	
2.74	Manufacture All	US	Tortarolo and Zarate (2020)	p1	
0.1	Nurse at VA	US	Staiger et all(2010)	Table7	
0.1	Amazon Online Task Market	US	Dube et all(2020)	Table5	
2.7	Retail stores	US	Ransom and Oaxaca(2010)	Table5	
1.5	Retail stores	US	Ransom and Oaxaca(2010)	Table5	
3.94	Ford	US	Depew and Sorensen(2013)	p6	
2.52	Byers	US	Depew and Sorensen(2013)	p6	
2.37	Documented	US	Hotchkiss and Quispe-Agnoli(2012)	Table2	
1.85	Undocumented	US	Hotchkiss and Quispe-Agnoli(2013)	Table3	
2.574	All	US	Dobbelaere and Mairesse(2013)	Table5	

Table A3: Meta-analysis: US Data

Labor Supply Elasticity	Industry	Gender	Country (Region)	Paper	Table/Page
0.71	All	Australia	Booth and Katic(2011)	p1	
2.175	All(rais)	Brazil	Vick(2017)	Table7	
1.502	All(rais)	Brazil	Vick(2017)	Table7	
1.292	Agriculture	East Germany	Bachmann and Fringes (2017)	Table7	
2.28	Mining and utilities	East Germany	Bachmann and Fringes (2017)	Table7	
2.482	Manufacturing of food products	East Germany	Bachmann and Fringes (2017)	Table7	
3.369	Manufacturing of consumer products	East Germany	Bachmann and Fringes (2017)	Table7	
1.343	Manufactuying of industial products	East Germany	Bachmann and Fringes (2017)	Table7	
2.507	Manufacturing of capital goods	East Germany	Bachmann and Fringes (2017)	Table7	
2.762	Construction	East Germany	Bachmann and Fringes (2017)	Table7	
0.717	Wholesale	East Germany	Bachmann and Fringes (2017)	Table7	
0.344	Retailing	East Germany	Bachmann and Fringes (2017)	Table7	
1.427	Transportation	East Germany	Bachmann and Fringes (2017)	Table7	
-1.095	Hotel and restaurants	East Germany	Bachmann and Fringes (2017)	Table7	
1.794	Financial services	East Germany	Bachmann and Fringes (2017)	Table7	
1.896	Liberal professions	East Germany	Bachmann and Fringes (2017)	Table7	
2.918	Education	East Germany	Bachmann and Fringes (2017)	Table7	
2.109	Health	East Germany	Bachmann and Fringes (2017)	Table7	
2.786	Other services	East Germany	Bachmann and Fringes (2017)	Table7	
0.932	Nonindustrial organization	East Germany	Bachmann and Fringes (2017)	Table7	
1.563	Public administration	East Germany	Bachmann and Fringes (2017)	p1	
1.788	All	East Germany	Bachmann and Fringes (2017)	p1	
3.072	All	Germany	Hirsch, Schank and Schnabel (2010a)	Table4	
2.225	All	Germany	Hirsch, Schank and Schnabel (2010a)	Table4	
1.875	Foreigner	Germany	Hirsch, Jahn(2012)	p34	
1.645	Germans	Germany	Hirsch, Jahn(2012)	p34	
2.044	All	Germany	Hirsch, Schank and Schnabel (2013)	Table3	
0.52	All	Indonesia	Brummund(2011)	Table5	
0.4	All	Italy	Sulis et all (2011)	Table9	
0.307	All	Italy	Sulis et all (2011)	Table9	
2.15	Public Sector	Mexico	Bo, Finan and Rossi(2013)	p3	
1.4	Teacher	Norway	Falch(2010)	Table5	
1.25	Teacher	Norway	Falch(2011)	p6	
1.4	Teacher	Norway	Falch(2017)	p1	
1.41	All(Foreign Workers)	United Arab Emirates	Naidu, Nyarko and Wang(2014)	Table5	
0.883	Agriculture	West Germany	Bachmann and Fringes (2017)	p1	
2.577	Mining and utilities	West Germany	Bachmann and Fringes (2017)	Table3	
2.227	Manufacturing of food products	West Germany	Bachmann and Fringes (2017)	Table3	
1.757	Manufacturing of consumer products	West Germany	Bachmann and Fringes (2017)	p6	
2.249	Manufactuying of industial products	West Germany	Bachmann and Fringes (2017)	p6	
2.841	Manufacturing of capital goods	West Germany	Bachmann and Fringes (2017)	Table5	
2.167	Construction	West Germany	Bachmann and Fringes (2017)	Table5	
1.728	Wholesale	West Germany	Bachmann and Fringes (2017)	Table5	
0.737	Retailing	West Germany	Bachmann and Fringes (2017)	Table5	
2.178	Transportation	West Germany	Bachmann and Fringes (2017)	Table5	
0.274	Hotel and restaurants	West Germany	Bachmann and Fringes (2017)	Table5	
2.526	Financial services	West Germany	Bachmann and Fringes (2017)	Table5	
2.473	Liberal professions	West Germany	Bachmann and Fringes (2017)	Table5	
3.498	Education	West Germany	Bachmann and Fringes (2017)	Table5	
1.407	Health	West Germany	Bachmann and Fringes (2017)	Table5	
1.445	Other services	West Germany	Bachmann and Fringes (2017)	Table5	
1.607	Nonindustrial organization	West Germany	Bachmann and Fringes (2017)	Table5	
1.597	Public administration	West Germany	Bachmann and Fringes (2017)	Table5	
2.021	All	West Germany	Bachmann and Fringes (2017)	Table5	

Table A4: Meta-analysis: Non-US data

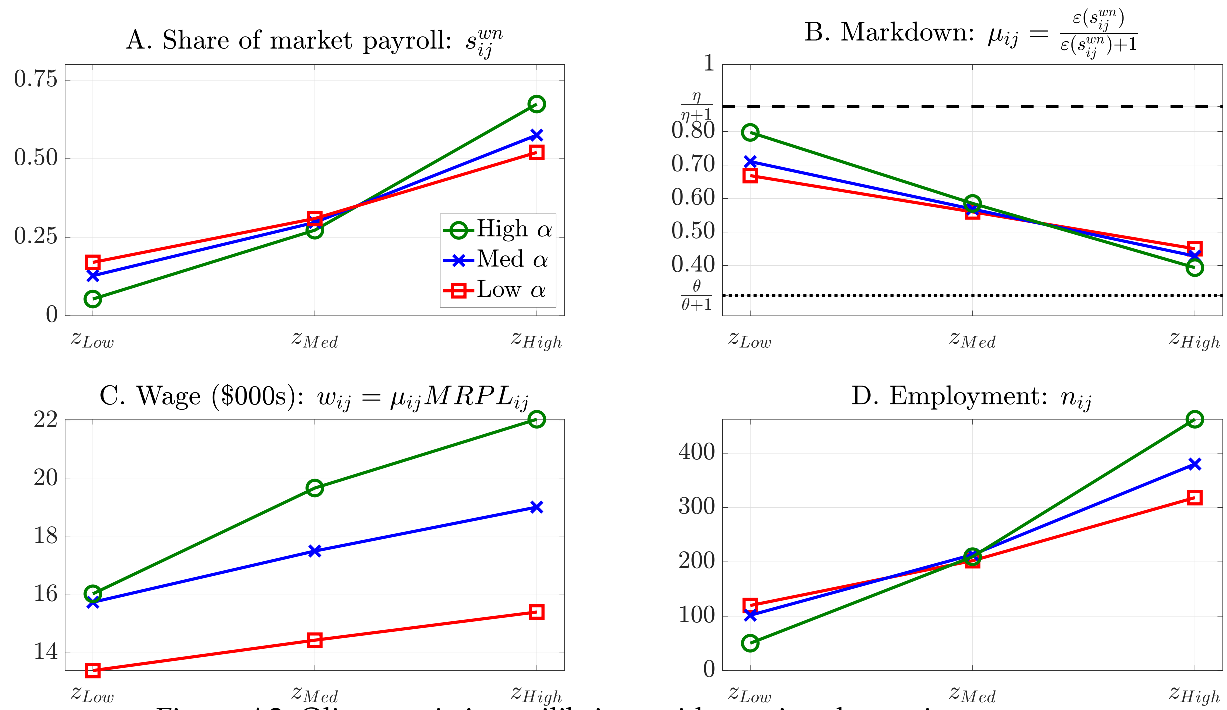

Figure A2: Oligopsonistic equilibrium with varying decreasing returns α

Notes: Figure constructed from model under estimated parameters (Table 3). Low, medium and high productivities of the firms correspond to the 10^{th}, 50^{th} and 90^{th} percentiles of the productivity distribution.

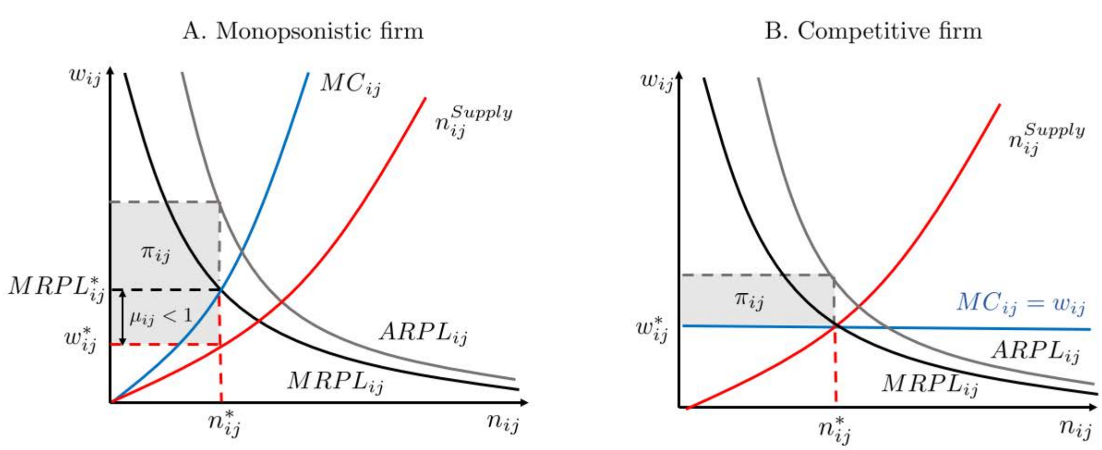

Figure A3: Oligopsonistic vs. Competitive equilibrium

Notes: In a *oligopsonistic equilibrium* (Panel A) the firm understands that its marginal cost MC_{ij} is increasing in its employment. In a *competitive equilibrium* (Panel B) the firm perceives that its marginal cost MC_{ij} is simply equal to its wage, which it takes as given. The true labor supply curve to the firm, however, is still upward sloping, reflecting household preferences.

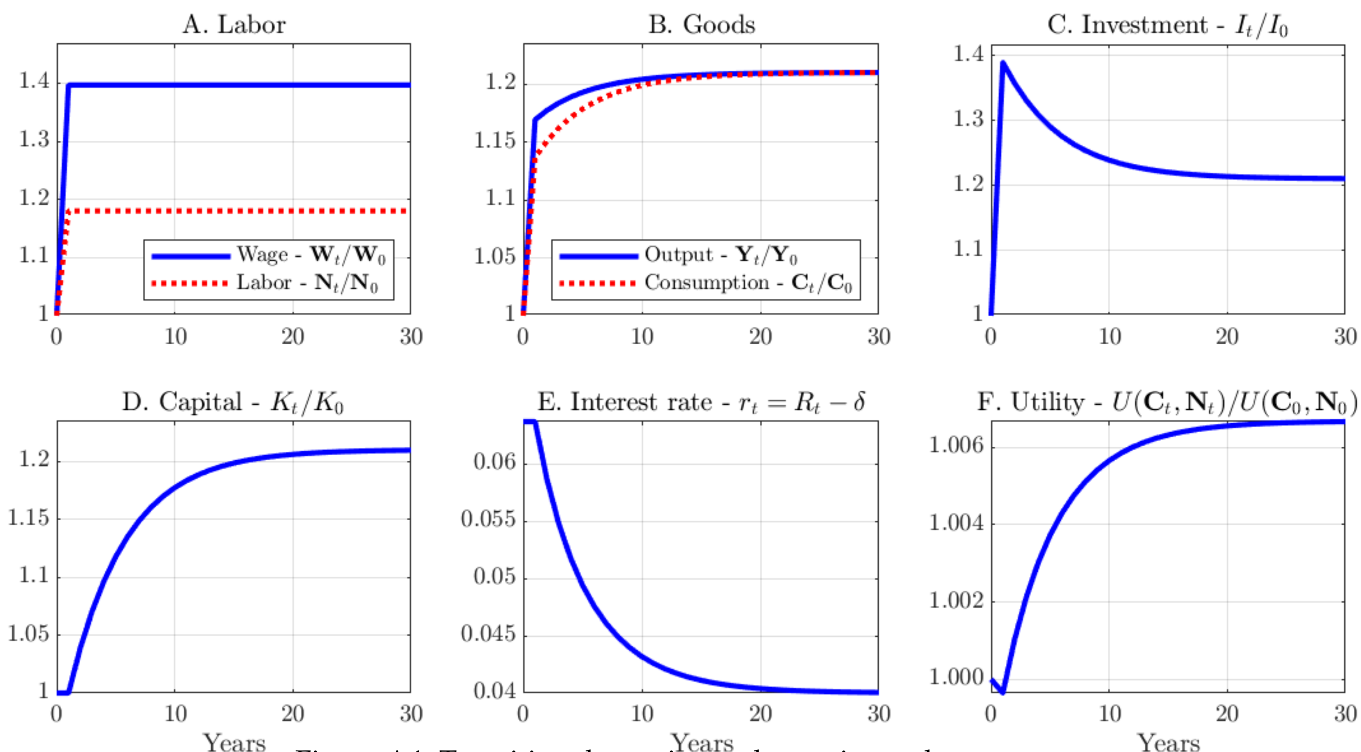

Figure A4: Transition dynamics to change in market structure.

Notes: This figure provides transition dynamics of aggregates to an unexpected change to the competitive market structure in period $t = 1$. Transition dynamics are computed under $\varphi = 0.50$. Preferences are GHH. Both $\boldsymbol{\Omega}$ and $\boldsymbol{\mu}$ immediately jump to 1 at $t = 1$. Since the model is block recursive, the market equilibrium allocations jump. Given shares, we can compute $\boldsymbol{W}_t$. So $\boldsymbol{W}_t$ jumps, as does $\boldsymbol{N}_t$ given the labor supply curve $\boldsymbol{N}_t = \overline{\varphi}\boldsymbol{W}_t^{\varphi}$. The path for capital and consumption is then determined by the resource constraint $\boldsymbol{C}_t = \boldsymbol{Y}_t + (1-\delta)K_t - K_{t+1}$, household's Euler equation $u_C(\boldsymbol{C}_t, \boldsymbol{N}_t) = \beta u_C(\boldsymbol{C}_{t+1}, \boldsymbol{N}_{t+1})\left[R_{t+1} + 1 - \delta\right]$, and equilibrium price of capital $R_t K_t = (1-\gamma)\alpha \boldsymbol{Y}_t$. Since capital is undistorted, it is paid the competitive factor share, which is equal to its output elasticity.

B Microfounding the nested CES labor supply system

In this section we provide a micro-foundation for the nested CES preferences used in the main text. The arguments used here adapt those in Verboven (1996). We begin with the case of monopsonistic competition to develop ideas and then move to the case of oligopsonistic labor markets studied in the text. We then show that the same supply system occurs in a setting where workers solve a dynamic discrete choice problem and firms compete in a dynamic oligopoly.

B.1 Static discrete choice framework

Agents. There is a unit measure of ex-ante identical individuals indexed by $l \in [0,1]$. There is a large but finite set of J sectors in the economy, with finitely many firms $i \in \{1,\ldots,M_j\}$ in each sector.

Preferences. Each individual has random preferences for working at each firm ij. Their disutility of labor supply is *convex* in hours worked h_l. Worker l's disutility of working h_{lij} hours at firm ij are:

$$\nu_{lij} = e^{-\mu\xi_{lij}} h_{lij} \qquad , \qquad \log \nu_{lij} = \log h_{ij} - \mu \xi_{ij},$$

where the random utility term ξ_{lij} is distributed *iid* across individuals according from a multi-variate Gumbel distribution:

$$F(\xi_{i1},...,\xi_{NJ}) = \exp\left[-\sum_{ij} e^{-(1+\eta)\xi_{ij}}\right].$$

The term ξ_{lij} is a worker-firm specific term which reduces labor disutility and hence could capture (i) an inverse measure of commuting costs, or (ii) a positive amenity.

Decisions. Each individual must earn $y_l \sim F(y)$, where earnings $y_l = w_{ij} h_{lij}$. After drawing their vector $\left\{\xi_{lij}\right\}$, each worker solves

$$\min_{ij} \left\{\log h_{ij} - \xi_{lij}\right\} \equiv \max_{ij} \left\{\log w_{ij} - \log y_l + \xi_{lij}\right\}.$$

This problem delivers the following probability that worker l chooses to work at firm ij, which is independent of y_l:

$$Prob_l\left(w_{ij}, w_{-ij}\right) = \frac{w_{ij}^{1+\eta}}{\sum_{ij} w_{ij}^{1+\eta}}. \tag{B1}$$

Aggregation. Total labor supply to firm ij, is then found by integrating these probabilities, multiplied by the hours supplied by each worker l:

$$n_{ij} = \int_0^1 Prob_l\left(w_{ij}, w_{-ij}\right) h_{lij} dF\left(y_l\right) \qquad , \qquad h_{lij} = y_l / w_{ij}$$
$$n_{ij} = \frac{w_{ij}^{\eta}}{\sum_{i\in j} w_{ij}^{1+\eta}} \underbrace{\int_0^1 y_l dF\left(y_l\right)}_{:=Y} \tag{B2}$$

Aggregating this expression we obtain the obvious result that $\sum_{i\in j} w_{ij} n_{ij} = Y$. Now define the following indexes:

$$\boldsymbol{W} := \left[\sum_{i\in j} w_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \boldsymbol{N} := \left[\sum_{i\in j} n_{ij}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}.$$

Along with (B2), these indexes imply that $\boldsymbol{WN} = Y$. Using these definitions along with $\boldsymbol{WN} = Y$ in (B2) yields the CES supply curve:

$$n_{ij} = \left(\frac{w_{ij}}{\boldsymbol{W}}\right)^{\eta} \boldsymbol{N}.$$

We therefore have the result that the supply curves that face firms in this model of individual discrete choice are equivalent to those that face the firms when a representative household solves the following income maximization problem:

$$\max_{\{n_{ij}\}} \sum_{i \in j} w_{ij} n_{ij} \quad s.t. \quad \left[\sum_{i \in j} n_{ij}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}} = \boldsymbol{N}.$$

Since at the solution, the objective function is equal to $\boldsymbol{WN}$, then the envelope condition delivers a natural interpretation of $\boldsymbol{W}$ as the equilibrium payment to total labor input in the economy for one additional unit of aggregate labor disutility. That is, the following equalities hold:

$$\frac{\partial}{\partial \boldsymbol{N}} \sum_{i \in j} w_{ij} n^*_{ij}(w_{ij}, w_{-ij}) = \Lambda = \boldsymbol{W} = \frac{\partial}{\partial \boldsymbol{N}} \boldsymbol{WN}.$$

Nested logit and nested CES. Consider changing the distribution of preference shocks as follows:

$$F(\xi_{i1}, ..., \xi_{NJ}) = \exp\left[-\sum_{j=1}^{J}\left(\sum_{i=1}^{M_j} e^{-(1+\eta)\xi_{ij}}\right)^{\frac{1+\theta}{1+\eta}}\right].$$

We recover the distribution (B1) above if $\eta = \theta$. Otherwise, if $\eta > \theta$ the problem is convex and the conditional covariance of within sector preference draws differ from the economy wide variance of preference draws. We discuss this more below.

In this setting, choice probabilities can be expressed as the product of the conditional choice probability of supplying labor to firm i conditional on supplying labor to market j, and the probability of supplying labor to market j:

$$Prob_l\left(w_{ij}, w_{-ij}\right) = \underbrace{\frac{w_{ij}^{1+\eta}}{\sum_{i=1}^{M_j} w_{ij}^{1+\eta}}}_{Prob_l(\text{Choose firm } i \,|\, \text{Choose market } j)} \times \underbrace{\frac{\left[\sum_{i=1}^{M_j} w_{ij}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{l=1}^{J}\left[\sum_{k=1}^{M_l} w_{kl}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}}_{Prob_l(\text{Choose market } j)}.$$

Following the same steps as above, we can aggregate these choice probabilities and hours decisions to obtain firm level labor supply:

$$n_{ij} = \frac{w_{ij}^{\eta}}{\sum_{i=1}^{M_j} w_{ij}^{1+\eta}} \frac{\left[\sum_{i=1}^{M_j} w_{ij}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{l=1}^{J}\left[\sum_{k=1}^{M_l} w_{kl}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} Y. \tag{B3}$$

We can now define the following indexes:

$$\boldsymbol{W}_j = \left[\sum_{i=1}^{M_j} w_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \boldsymbol{N}_j = \left[\sum_{i=1}^{M_j} n_{ij}^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}},$$

$$\boldsymbol{W} = \left[\sum_{j=1}^{J} \boldsymbol{W}_j^{1+\theta}\right]^{\frac{1}{1+\theta}} \quad , \quad \boldsymbol{N} = \left[\sum_{j=1}^{J} \boldsymbol{N}_j^{\frac{1+\theta}{\theta}}\right]^{\frac{\theta}{1+\theta}}.$$

Using these definitions and similar results to the above we can show that $\boldsymbol{W}_j\boldsymbol{N}_j = \sum_{i=1}^{M_j} w_{ij}n_{ij}$, and $Y = \boldsymbol{WN} = \sum_{j=1}^{J} \boldsymbol{W}_j\boldsymbol{N}_j$.

Consider the thought experiment of adding more markets J (which is necessary to identically map these formulas to our model). While the min of an infinite number of draws from a Gumbel distribution is not defined (it asymptotes to $-\infty$), the distribution of choices across markets is defined at each point in the limit as we add more markets J (Malmberg, 2013). As a result, the distribution of choices will have a well defined limit, and with the correct scaling as we add more markets (we can scale the disutilities at each step and not affect the market choice), as described in (Malmberg, 2013), the limiting wage indexes will be defined as above. We can then express (B3) as:

$$n_{ij} = \left(\frac{w_{ij}}{\boldsymbol{W}_j}\right)^{\eta}\left(\frac{\boldsymbol{W}_j}{\boldsymbol{W}}\right)^{\theta}\boldsymbol{N},$$

which completes the CES supply system defined in the text.

Comment. The above has established that it is straightforward to derive the supply system in the model through a discrete choice framework. This is particularly appealing given recent modeling of labor supply using familiar discrete choice frameworks first in models of economic geography and more recently in labor (Borovickova and Shimer (2017), Card, Cardoso, Heining, and Kline (2018), Lamadon, Mogstad, and Setzler (2019)). Since firms take this supply system as given, we can then work with the nested CES supply functions as if they were derived from the preferences and decisions of a representative household. This vastly simplifies welfare computations and allows for the integration of the model into more familiar macroeconomic environments.

The second advantage of this micro-foundation is that it provides a natural interpretation of the somewhat nebulous elasticities of substitution in the CES specification: η and θ. Returning to the Gumbel distribution we observe the following

$$F(\xi_{i1},...,\xi_{NJ}) = \exp\left[-\sum_{j=1}^{J}\left(\sum_{i=1}^{M_j} e^{-(1+\eta)\xi_{ij}}\right)^{\frac{1+\theta}{1+\eta}}\right]$$

A higher value of η *increases* the correlation of draws within a market (McFadden, 1978). Within a market if η is high, then an individual's preference draws are likely to be clustered. With little difference in non-pecuniary idiosyncratic preferences for working at different firms, wages dominate in an individual's labor supply decision and wage posting in the market is closer to the competitive outcome. A higher value of θ *decreases* the overall variance of draws across all firms (i.e. it *increases* the correlation across any two randomly chosen sub-vectors of an individual's draws). An individual is therefore more likely to find that their lowest levels of idiosyncratic disutility are in two different markets, increasing across market wage competition.

In the case that $\eta = \theta$, the model collapses to the standard logit model. In this case the following obtains. Take an individual's ξ_{lij} for some firm. The conditional probability distribution of some other draw $\xi_{li'j'}$ is the same whether firm i' is in the same market ($j' = j$) or some other market ($j' \neq j$). Individuals are as likely to find somewhere local that incurs the same level of labor disability as finding somewhere in another market. In this setting economy-wide monopsonistic competition obtains. When an individual is more likely to find their other low disutility draws in the *same* market, then firms within that market have local market power. This is precisely the case that obtains when $\eta > \theta$.

B.2 Dynamic discrete choice framework

We show that the above discrete choice framework can be adapted to an environment where some individuals draw new vectors $\boldsymbol{\xi}_l$ each period and reoptimize their labor supply. Firms therefore compete in a dynamic oligopoly. Restricting attention to the stationary solution of the model where firms keep employment and wages constant—as in the tradition of Burdett and Mortensen (1998)—we show that the allocation of employment and wages once again coincide with the solution to the problem in the main text. To simplify notation we consider the problem for a market with M firms $i \in \{1,\dots,M\}$ which may be generalized to the model in the text.

Environment. Every period a random fraction λ of workers each draw a new vector $\boldsymbol{\xi}_l$. Let n_i be employment at firm i. Let $\overline{w}_i$ be the *average wage* of workers at firm i, such that the total wage bill in the firm is $\overline{w}_i n_i$. Let the equilibrium labor supply function $h(w_i, w_{-i})$ determine the amount of hires a firm makes if it posts a wage w_i when its competitors' wages in the market are given by the vector $\boldsymbol{w}_{-i}$.

Value function. Let $V(n_i, \overline{w}_i)$ be the firm's present discounted value of profits, where the firm has discount rate $\beta = 1$. Then $V(n_i, \overline{w}_i)$ satisfies:

$$
\begin{aligned}
V\left(n_i, \overline{w}_i\right) &= \left(Pz_i - \overline{w}_i\right)(1-\lambda) n_i + \max_{w_i'} \left\{ \left(Pz_i - w_i'\right) h\left(w_i', w_{-i}'\right) + V\left(n_i', \overline{w}_i'\right) \right\} \quad , && \text{(B4)} \\
n'\left(n_i, w_i', \boldsymbol{w}_{-i}'\right) &= (1-\lambda) n_i + h\left(w_i', \boldsymbol{w}_{-i}'\right) \quad , && \text{(B5)} \\
\overline{w}'\left(n_i, \overline{w}_i, w_i', w_{-i}'\right) &= \frac{(1-\lambda)\overline{w}_i n_i + h\left(w_i', \boldsymbol{w}_{-i}'\right) w_i'}{(1-\lambda) n_i + h\left(w_i', \boldsymbol{w}_{-i}'\right)}. && \text{(B6)}
\end{aligned}
$$

The firm operates a constant returns to scale production function. Of the firm's n_i workers, a fraction $(1-\lambda)$ do not draw new preferences. The total profit associated with these workers is then average revenue (Pz_i) minus average cost $(\overline{w}_i)$. The firm chooses a new wage w_i' to post in the market. In equilibrium, given its competitor's wages $\boldsymbol{w}_{-i}'$, it hires $h(w_i', \boldsymbol{w}_{-i}')$ workers. The total profit associated with these workers is again average revenue (Pz_i) minus average cost (w_i'). The second and third equations account for the evolution of the firm's state variables.

Optimality. Given its competitor's prices, the first order condition with respect to w_i' is:

$$
\left(Pz_i - w_i'\right) h_1\left(w_i', w_{-i}'\right) - h\left(w_i', w_{-i}'\right) + V_n\left(n_i', \overline{w}_i'\right) n_w'\left(n_i, w_i', w_{-i}'\right) + V_{\overline{w}}\left(n_i', \overline{w}_i'\right) \overline{w}_w\left(n_i, \overline{w}_i, w_i', w_{-i}'\right) = 0
$$

The relevant envelope conditions are

$$
\begin{aligned}
V_n\left(n_i, \overline{w}_i\right) &= \left(Pz_i - \overline{w}_i\right)(1-\lambda) + V_n\left(n_i', \overline{w}_i'\right) n_n'\left(n_i, w_i', \boldsymbol{w}_{-i}'\right) + V_{\overline{w}}\left(n_i', \overline{w}_i'\right) \overline{w}_n'\left(n_i, \overline{w}_i, w_i', \boldsymbol{w}_{-i}'\right) \\
V_{\overline{w}}\left(n_i, \overline{w}_i\right) &= -(1-\lambda) n_i + V_{\overline{w}}\left(n_i', \overline{w}_i'\right) \overline{w}_{\overline{w}}'\left(n_i, \overline{w}_i, w_i', \boldsymbol{w}_{-i}'\right)
\end{aligned}
$$

In a stationary equilibrium $\overline{w}_i = w_i'$, and $n_i' = n_i$. One can compute the partial derivatives involved in these expressions, and evaluate the conditions under stationarity to obtain

$$
\left(Pz_i - w_i\right) h_1\left(w_i, w_{-i}\right) = h\left(w_i, w_{-i}\right).
$$

Rearranging this expression:

$$
w_i = \frac{\varepsilon_i(w_i, w_{-i})}{\varepsilon_i(w_i, w_{-i}) + 1} Pz_i \qquad , \qquad \varepsilon_i(w_i, w_{-i}) \quad := \quad \frac{h_1(w_i, w_{-i}) w_i}{h(w_i, w_{-i})}
$$

The solution to the dynamic oligopsony problem for a *given* supply system is identical to the solution of the static problem. In this setting, the supply system is obviously that which is obtained from the individual discrete choice problem in the previous section.

Comments. This setting establishes that the model considered in the main text can also be conceived as a setting where individuals periodically receive some preference shock that causes them to relocate, and firms engage in a dynamic oligopoly given these worker decisions. When $\eta > \theta$ the shock causes a worker to consider all firms in one market very carefully to the exclusion of other markets when they are making their relocation decision. When $\eta = \theta$ the individual considers all firms in all markets equally.

C Data

This section provides additional details regarding the data sources used in the paper, sample restrictions, and construction of a number of variables.

C.1 Census Longitudinal Business Database (LBD)

The LBD is built on the Business Register (BR), Economic Census and surveys. The BR began in 1972 and is a database of all U.S. business establishments. The business register is also called the Standard Statistical Establishment List (SSEL). The SSEL contains records for all industries except private households and illegal or underground activities. Most government owned entities are not in the SSEL. The SSEL includes single and multi unit establishments. The longitudinal links are constructed using the SSEL. The database is annual.

C.2 Sample restrictions

For both the summary statistics and corporate tax analysis, we isolate all plants (lbdnums) with non missing firmids, with strictly positive pay, positive employment, and non-missing county codes for the continental US (we exclude Alaska, Hawaii, and Puerto Rico). The units of payroll were manually changed from dollars to tens-of-thousands of dollars in the SSEL from 1976-1981 and 1983-1989. As a result we must remove data errors associated with this manual coding. We do so by removing firms that are in the upper two percentiles of the wage distribution while simultaneously being in the upper percentile of firm size. We then isolate all lbdnums with non-missing 2 digit NAICS codes equal to 11, 21, 31, 32, 33 or 55. These are the top tradeable 2-digit NAICS codes as defined by Delgado, Bryden, and Zyontz (2014). We use the consistent 2012 NAICS codes provided by Fort and Klimek (2018) throughout the paper. We winsorize the wage and employment at the 1% level to remove remaining outliers. Each plant has a unique firmid which corresponds to the owner of the plant.[51] Throughout the paper, we define a firm to be the sum of all establishments in a commuting zone with a common firmid and NAICS3 classification.

Summary Statistics Sample: Our summary statistics include all observations that satisfy the above criteria in 1976 and 2014 (Tables D1 and D2).

Corporate Tax Sample: The corporate tax analysis includes all observations that satisfy the above criteria between 1977 and 2011. We additionally require the firm to have at least 5 employees in order to compute direct elasticities (see Section G.3.2). The LBD begins in 1976, but we require information on lags of the wage bill share, thus 1977 is our first usable year. The tax series ends in 2012 but the 'Year t+1' estimates require information on forward lags, thus our final usable year is 2011. We further restrict the sample to firmid-market-year observations which are corporations. To build a consistent corporation definition over time, we use both the SSEL and LBD. We identify corporations as those with SSEL Form 1120 codes which indicate '*C*-Corporation' status and LBD legal form of organization codes that also indicate '*C*-Corporation' status. Table C1 provides summary statistics for this sample.

Sample NAICS Codes and Commuting Zones: Table C2 describes the NAICS 3 codes in our sample. Table C3 provides examples of commuting zones and the counties that are associated with those commuting zones.

[51] Each firm only has one firmid. The firmid is different from the EIN. The firmid aggregates EINS to build a consistent firm identifier in the cross-section and over time.

Variable		Mean	Std. Dev.
Corporate tax rate (percent)	$\tau_{s(j)t}$	7.45	2.96
Change in corporate tax rate	$\Delta\tau_{s(j)t}$	0.03	0.62
Total Pay At Firm (Thousands)	$w_{ijt}n_{ijt}$	1,750	12,920
Employment	n_{ijt}	54.20	281.80
Wage bill Herfindahl	HHI_{jt}^{wn}	0.12	0.17
Wage bill share	s_{ijt}^{wn}	0.04	0.12
Wage bill share, Lagged 1 yr	s_{ijt-1}	0.03	0.12
Number of firms per market	M_j	532	855
Log number of firms per market	$\log M_j$	5.13	1.71
Log employment	$\log n_{ijt}$	3.14	1.06
Log wage	$\log w_{ijt}$	3.70	0.53
Observations			2,844,000

Table C1: Regression sample summary statistics

Notes: Tradeable C-Corps from 1977 to 2011.

Table C2: NAICS 3 digit examples

NAICS3	Description	NAICS3	Description
111	Crop Production	322	Paper Manuf.
112	Animal Production and Aquaculture	323	Printing and Related Support Activities
113	Forestry and Logging	324	Petroleum and Coal Products Manuf.
114	Fishing, Hunting and Trapping	325	Chemical Manuf.
115	Support Activities for Agriculture and Forestry	326	Plastics and Rubber Products Manuf.
211	Oil and Gas Extraction	327	Nonmetallic Mineral Product Manuf.
212	Mining (except Oil and Gas)	331	Primary Metal Manuf.
213	Support Activities for Mining	332	Fabricated Metal Product Manuf.
311	Food Manuf.	333	Machinery Manuf.
312	Beverage and Tobacco Product Manuf.	334	Computer and Electronic Product Manuf.
313	Textile Mills	335	Electrical Equipment, Appliance, Component Manuf.
314	Textile Product Mills	336	Transportation Equipment Manuf.
315	Apparel Manufacturing	337	Furniture and Related Product Manuf.
316	Leather and Allied Product Manuf.	339	Miscellaneous Manuf.
321	Wood Product Manuf.	551	Management of Companies and Enterprises

Table C3: Commuting Zone (CZ) examples: Census commuting zones numbers 58 and 47

CZ ID, 2000	County Name	Metro. Area, 2003	County Pop. 2000	CZ Pop. 2000
58	Cook County	Chicago-Naperville-Joliet, IL Metro. Division	5,376,741	8,704,935
58	DeKalb County	Chicago-Naperville-Joliet, IL Metro. Division	88,969	8,704,935
58	DuPage County	Chicago-Naperville-Joliet, IL Metro. Division	904,161	8,704,935
58	Grundy County	Chicago-Naperville-Joliet, IL Metro. Division	37,535	8,704,935
58	Kane County	Chicago-Naperville-Joliet, IL Metro. Division	404,119	8,704,935
58	Kendall County	Chicago-Naperville-Joliet, IL Metro. Division	54,544	8,704,935
58	Lake County	Lake County-Kenosha County, IL-WI Metro. Division	644,356	8,704,935
58	McHenry County	Chicago-Naperville-Joliet, IL Metro. Division	260,077	8,704,935
58	Will County	Chicago-Naperville-Joliet, IL Metro. Division	502,266	8,704,935
58	Kenosha County	Lake County-Kenosha County, IL-WI Metro. Division	149,577	8,704,935
58	Racine County	Racine, WI MSA	188,831	8,704,935
58	Walworth County	Whitewater, WI Micropolitan SA	93,759	8,704,935
47	Anoka County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	298,084	2,904,389
47	Carver County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	70,205	2,904,389
47	Chisago County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	41,101	2,904,389
47	Dakota County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	355,904	2,904,389
47	Hennepin County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	1,116,200	2,904,389
47	Isanti County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	31,287	2,904,389
47	Ramsey County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	511,035	2,904,389
47	Scott County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	89,498	2,904,389
47	Washington County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	201,130	2,904,389
47	Wright County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	89,986	2,904,389
47	Pierce County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	36,804	2,904,389
47	St. Croix County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	63,155	2,904,389

A. Firm-market-level averages	1976	2014
Total firm pay ($1,000s)	673.30	2018.00
Total firm employment	54.94	34.63
Pay per employee	$ 12,255	$ 58,273
Firm-level observations	375,000	465,000

B. Market-level averages	1976	2014
Wage-bill HHI, Unweighted	0.50	0.48
Wage-bill HHI, Weighted by market's share of total payroll)	0.22	0.17
Firms per market	28.00	33.86
Percent of markets with 1 firm	16.5%	16.4%
National payroll share of markets with 1 firm	0.58%	0.43%
Market-level observations	13,000	14,000

C. Across market correlations with wage-bill HHI	1976	2014
Number of firms	-0.26	-0.36
Employment Herfindahl	0.98	0.98
Market Employment	-0.21	-0.25
Market-level observations	13,000	14,000

Table D1: Summary Statistics, U.S. Census Longitudinal Employer Database 1976 and 2014

Notes: Tradeable NAICS2 codes (11,21,31,32,33,55). Market defined to be NAICS3 within Commuting Zone. Observations rounded to nearest thousand and numbers rounded to 4 significant digits according to Census disclosure rules. Firm-market-level refers to a 'firmid by Commuting Zone by 3-digit NAICs by Year' observation. Market-level refers to a 'Commuting Zone by 3-digit NAICs by Year' aggregation of observations.

D Labor market concentration 1976 and 2014

Table D1A describes characteristics of the firm-market observations in 1976 and 2014. Average nominal payroll was $673,300 in 1976 and $2,018,000 in 2014. Average firm-market employment was 55 workers in 1976 and 35 workers in 2014. Average nominal wage was $12,255 in 1976 and $58,273 in 2014.

Table D1B shows that different weighting schemes of across-market averages imply different levels and trends. The unweighted average wage-bill Herfindahl for wages is between two and three times larger than its payroll weighted counterpart. Little employment or payroll is located in highly concentrated markets. In both periods, 16 percent of markets have only one employer and so HHIs equal to one. However, these single firm markets only account for roughly one half of one percent of national payroll. In terms of the time-series, unweighted average wage-bill Herfindahl declines marginally between 1976 and 2014. In contrast, payroll weighted wage-bill Herfindahl declines by 23% from 0.22 to 0.17.

Table D1C confirms that the number of firms and total market employment are negatively correlated with concentration. This is important for understanding why weighted and unweighted Herfindahls are so different and will be used as an over-identifying test of the estimated model. Moreover, employment and wage-bill Herfindahls are highly correlated.

Table D2 includes summary statistics of labor market concentration across all industries. Similar to tradeable industries, the market-level unweighted and weighted Herfindahls decline. The unweighted wage-bill Herfindahl declines from 0.36 to 0.34. The payroll weighted wage-bill Herfindahl declines from 0.16 to 0.11.

	(A) Firm-market-level averages	
	1976	**2014**
Total firm pay (000s)	202.10	1000.00
Total firm employment	19.35	22.83
Pay per employee	$ 10,444	$ 43,802
Firm-level observations	3,746,000	5,845,000

	(B) Market-level averages	
	1976	**2014**
Wage-bill Herfindahl (Unweighted)	0.36	0.34
Wage-bill Herfindahl (Weighted by market's share of total payroll)	0.16	0.11
Firms per market	75.71	113.10
Percent of markets with 1 firm	10.4%	9.4%
Market-level observations	49,000	52,000

	(C) Market-level correlations	
	1976	**2014**
Correlation of Wage-bill Herfindahl and number of firms	-0.20	-0.17
Correlation of Wage-bill Herfindahl and Employment Herfindahl	0.98	0.97
Correlation of Wage-bill Herfindahl and Market Employment	-0.15	-0.16
Market-level observations	49,000	52,000

Table D2: Summary Statistics, Longitudinal Employer Database 1976 and 2014

Notes: All NAICS. Market defined to be NAICS3 within Commuting Zone. Observations rounded to nearest thousand and numbers rounded to 4 significant digits according to Census disclosure rules. Firm-market-level refers to a 'firmid by Commuting Zone by 3-digit NAICs by Year' observation. Market-level refers to a 'Commuting Zone by 3-digit NAICs by Year' aggregation of observations.

E Mathematical derivations

This section provides detailed derivations of mathematical formulae that appear in the main text. It covers: (i) the household problem (Section E.1) (ii) the firm problem (Section E.2), (iii) market equilibrium (Proposition 1.1) (Section E.3), (iv) general equilibrium and aggregation (Proposition 1.2) (Section E.4), (v) relationship between the labor share and concentration (Proposition 1.3) (Section E.5), (vi) closed form general equilibrium solution and scaling properties used in calibration (Section E.6), (vii) reduced form and structural labor supply elasticities (Section E.7), (viii) pass-through expression (Section E.8), (ix) expressions used in the discussion of corporate taxes (Section E.9), (x) proofs of merger results (Proposition 3) (Section E.10).

E.1 Household problem - Section 1.2

- The household's problem is

$$\max_{\left\{n_{ijt},c_{ijt},K_{t+1}\right\}} \sum_{t=0}^{\infty} \beta^t U\left(\boldsymbol{C}_t, \boldsymbol{N}_t\right)$$

  where

$$\boldsymbol{N}_t = \left[\int \boldsymbol{n}_{jt}^{\frac{\theta+1}{\theta}} dj\right]^{\frac{\theta}{\theta+1}}$$

$$\boldsymbol{n}_{jt} = \left[\sum_{i\in j} n_{ijt}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$$

$$\boldsymbol{C}_t = \int \sum_{i\in j} c_{ijt} dj$$

  subject to the initial endowment $K_0 > 0$, and the following budget constraint in each period, in which it takes all prices as given, these include the wage w_{ijt} at all firms-ij, rental rate R_t and profits Π_t as given:

$$\boldsymbol{C}_t + K_{t+1} - (1-\delta) K_t = \int \sum_{i\in j} w_{ijt} n_{ijt} dj + R_t K_t + \Pi_t$$

E.1.1 First order conditions

- The first order conditions for consumption and capital give

$$U_C\left(\boldsymbol{C}_t, \boldsymbol{N}_t\right) = \beta U_C\left(\boldsymbol{C}_{t+1}, \boldsymbol{N}_{t+1}\right)\left[R_{t+1} + (1-\delta)\right]$$

- The first order conditions for consumption and labor supply to firm-ij gives

$$w_{ijt} = \frac{\partial \boldsymbol{n}_{jt}}{\partial n_{ijt}} \frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}} \left(-\frac{U_N\left(\boldsymbol{C}_t, \boldsymbol{N}_t\right)}{U_C\left(\boldsymbol{C}_t, \boldsymbol{N}_t\right)}\right)$$

E.1.2 Deriving supply system

- Define the following terms. The *market wage* $\boldsymbol{w}_{jt}$ is the number that satisfies $\boldsymbol{w}_{jt}\boldsymbol{n}_{jt} = \sum_{i\in j} w_{ijt} n_{ijt}$. The *aggregate wage* $\boldsymbol{W}_t$ is the number that satisfies $\boldsymbol{W}_t \boldsymbol{N}_t = \int \boldsymbol{w}_{jt}\boldsymbol{n}_{jt} dj$.
- We can write the first order condition as:

$$w_{ijt} n_{ijt} = \left(\frac{\partial \boldsymbol{n}_{jt}}{\partial n_{ijt}} \frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)\left(\frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}} \frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)\left(-\frac{U_N\left(\boldsymbol{C}_t, \boldsymbol{N}_t\right)}{U_C\left(\boldsymbol{C}_t, \boldsymbol{N}_t\right)}\right)\boldsymbol{N}_t$$

- Using the labor disutility indexes, note that

$$\frac{\partial \boldsymbol{n}_{jt}}{\partial n_{ijt}} \frac{n_{ijt}}{\boldsymbol{n}_{jt}} = \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{\eta+1}{\eta}} \quad \text{, therefore,} \quad \sum_{i \in j} \frac{\partial \boldsymbol{n}_{jt}}{\partial n_{ijt}} \frac{n_{ijt}}{\boldsymbol{n}_{jt}} = 1$$

$$\frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}} \frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t} = \left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{\theta+1}{\theta}} \quad \text{, therefore,} \quad \int \frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}} \frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t} dj = 1$$

- Using these results and aggregating the first order condition over $i \in j$, then over $j \in [0,1]$

$$\text{Aggregate over } i \in j: \quad \boldsymbol{w}_{jt}\boldsymbol{n}_{jt} = \left(\frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}} \frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)\left(-\frac{U_N(\boldsymbol{C}_t, \boldsymbol{N}_t)}{U_C(\boldsymbol{C}_t, \boldsymbol{N}_t)}\right)\boldsymbol{N}_t$$

$$\text{Aggregate over } j \in [0,1]: \quad \boldsymbol{W}_t\boldsymbol{N}_t = -\frac{U_N(\boldsymbol{C}_t, \boldsymbol{N}_t)}{U_C(\boldsymbol{C}_t, \boldsymbol{N}_t)}\boldsymbol{N}_t$$

$$\boldsymbol{W}_t = -\frac{U_N(\boldsymbol{C}_t, \boldsymbol{N}_t)}{U_C(\boldsymbol{C}_t, \boldsymbol{N}_t)}$$

- Aggregating the first order condition over markets $j \in [0,1]$, and then substituting the aggregate inverse labor supply curve back into the first order condition we can obtain the *market supply curve*:

$$\boldsymbol{w}_{jt}\boldsymbol{n}_{jt} = \left(\frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}} \frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)\left(-\frac{U_N(\boldsymbol{C}_t, \boldsymbol{N}_t)}{U_C(\boldsymbol{C}_t, \boldsymbol{N}_t)}\right)\boldsymbol{N}_t$$

$$\boldsymbol{w}_{jt}\boldsymbol{n}_{jt} = \left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{\theta+1}{\theta}} \boldsymbol{W}_t\boldsymbol{N}_t$$

$$\boldsymbol{n}_{jt} = \left(\frac{\boldsymbol{w}_{jt}}{\boldsymbol{W}_t}\right)^{\theta} \boldsymbol{N}_t$$

which also implies that

$$\frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}} \frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t} = \frac{\boldsymbol{w}_{jt}\boldsymbol{n}_{jt}}{\boldsymbol{W}_t\boldsymbol{N}_t}.$$

- Substituting this into the first order condition we can obtain the *firm supply curve*:

$$w_{ijt}n_{ijt} = \left(\frac{\partial \boldsymbol{n}_{jt}}{\partial n_{ijt}} \frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)\left(\frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}} \frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)\left(-\frac{U_N(\boldsymbol{C}_t, \boldsymbol{N}_t)}{U_C(\boldsymbol{C}_t, \boldsymbol{N}_t)}\right)\boldsymbol{N}_t$$

$$w_{ijt}n_{ijt} = \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{\eta+1}{\eta}} \left(\frac{\boldsymbol{w}_{jt}\boldsymbol{n}_{jt}}{\boldsymbol{W}_t\boldsymbol{N}_t}\right)\boldsymbol{W}_t\boldsymbol{N}_t$$

$$n_{ijt} = \left(\frac{w_{ijt}}{\boldsymbol{w}_{jt}}\right)^{\eta} \left(\frac{\boldsymbol{w}_{jt}}{\boldsymbol{W}_t}\right)^{\theta} \boldsymbol{N}_t$$

- We can now compute expressions for the wage indexes $\boldsymbol{w}_{jt}$ and $\boldsymbol{W}_t$.

- Take the labor supply curve to the firm and aggregate

$$\begin{aligned} n_{ijt} &= \left(\frac{w_{ijt}}{\boldsymbol{w}_{jt}}\right)^{\eta} \boldsymbol{n}_{jt} \\ w_{ijt} n_{ijt} &= w_{ijt}^{1+\eta} \boldsymbol{w}_{jt}^{-\eta} \boldsymbol{n}_{jt} \\ \sum_{i \in j} w_{ijt} n_{ijt} &= \left[\sum_{i \in j} w_{ijt}^{1+\eta}\right] \boldsymbol{w}_{jt}^{-\eta} \boldsymbol{n}_{jt} \\ \boldsymbol{w}_{jt} \boldsymbol{n}_{jt} &= \left[\sum_{i \in j} w_{ijt}^{1+\eta}\right] \boldsymbol{w}_{jt}^{-\eta} \boldsymbol{n}_{jt} \\ \boldsymbol{w}_{jt} &= \left[\sum_{i \in j} w_{ijt}^{1+\eta}\right]^{\frac{1}{1+\eta}} \end{aligned}$$

- Applying the same to the labor supply curve to the market we get

$$\boldsymbol{W}_t = \left[\int \boldsymbol{w}_{jt}^{1+\theta} dj\right]^{\frac{1}{1+\theta}}$$

- Therefore we have the set of results used in the body of the paper:

$$\boldsymbol{W}_t = -\frac{U_N(\boldsymbol{C}_t, \boldsymbol{N}_t)}{U_C(\boldsymbol{C}_t, \boldsymbol{N}_t)} \quad , \boldsymbol{n}_{jt} = \left(\frac{\boldsymbol{w}_{jt}}{\boldsymbol{W}_t}\right)^{\theta} \boldsymbol{N}_t \quad n_{ijt} = \left(\frac{w_{ijt}}{\boldsymbol{w}_{jt}}\right)^{\eta} \left(\frac{\boldsymbol{w}_{jt}}{\boldsymbol{W}_t}\right)^{\theta} \boldsymbol{N}_t$$

$$\boldsymbol{W}_t = \left[\int \boldsymbol{w}_{jt}^{1+\theta} dj\right]^{\frac{1}{1+\theta}} \quad , \boldsymbol{w}_{jt} = \left[\sum_{i \in j} w_{ijt}^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

- Using the above we can invert the labor supply curve to the firm in two steps. At the market level

$$\boldsymbol{w}_{jt} = \left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{1}{\theta}} \boldsymbol{W}_t$$

  and then at the firm level

$$w_{ijt} = \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{1}{\eta}} \boldsymbol{w}_{jt} \quad , \quad w_{ijt} = \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{1}{\eta}} \left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{1}{\theta}} \boldsymbol{W}_t$$

- This is delivers the set of partial equilibrium conditions specified in the text in Section 1.2

E.2 Proof of Nash equilibrium expressions - Section 1.3

- We can write the arguments of the firm's labor supply curve as the employment at competing firms in the same market which we denote by the vector n_{-ijt}, aggregate employment $\boldsymbol{N}_t$ and the aggregate wage $\boldsymbol{W}_t$
- **Definition** - The *Nash equilibrium labor demand* of each firm $\left\{n_{ijt}^*\right\}_{i \in j}$ must satisfy the following set of conditions:

$$n_{ijt}^* = \arg\max_{n_{ijt}} \widetilde{z}_{ijt} n_{ijt}^{\widetilde{\alpha}} - w\left(n_{ijt}, n_{-ijt}^*, \boldsymbol{W}_t, \boldsymbol{N}_t\right) n_{ijt} \quad \forall i \in j$$

  where the inverse labor supply curve is given by the household optimality condition:

$$w\left(n_{ijt}, n_{-ijt}^*, \boldsymbol{W}_t, \boldsymbol{N}_t\right) = \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{1}{\eta}} \left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{1}{\theta}} \boldsymbol{W}_t \quad , \quad \boldsymbol{n}_{jt} = \left[n_{ijt}^{\frac{\eta+1}{\eta}} + \sum_{k \neq i} n_{kjt}^{*\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$$

- The first order condition for each firm is as follows, where we write the marginal revenue product of labor $mrpl_{ijt} = \widetilde{\alpha}\widetilde{z}_{ijt}n_{ijt}^{\widetilde{\alpha}-1}$:

$$\begin{aligned} mrpl_{ijt} &= \frac{\partial w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right)}{\partial n_{ijt}} n_{ijt} + w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right) \\ mrpl_{ijt} &= w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right)\left[\frac{\partial w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right)}{\partial n_{ijt}} \frac{n_{ijt}}{w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right)} + 1\right] \end{aligned}$$

- The elasticity is

$$\begin{aligned} \frac{\partial \log w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right)}{\partial \log n_{ijt}} &= \frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) \frac{\partial \log \boldsymbol{n}_{jt}\left(n_{ijt}, n^*_{-ijt}\right)}{\partial \log n_{ijt}} \\ \frac{\partial \log w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right)}{\partial \log n_{ijt}} &= \frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)\left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{\eta+1}{\eta}} \end{aligned}$$

- We can write this in terms of the payroll share of the firm. Using our expression for the labor supply curve to the firm

$$s_{ijt} = \frac{w_{ijt}n_{ijt}}{\sum_{i\in j} w_{ijt}n_{ijt}} = \frac{\left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{1}{\eta}}\left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{1}{\theta}} n_{ijt}}{\sum_{i\in j}\left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{1}{\eta}}\left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{1}{\theta}} n_{ijt}} = \frac{n_{ijt}^{\frac{\eta+1}{\eta}}}{\sum_{i\in j} n_{ijt}^{\frac{\eta+1}{\eta}}} = \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{\eta+1}{\eta}}.$$

- This gives

$$\frac{\partial \log w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right)}{\partial \log n_{ijt}} = s_{ijt}\frac{1}{\theta} + \left(1 - s_{ijt}\right)\frac{1}{\eta}$$

- Define the *equilibrium inverse labor supply elasticity* ε^*_{ijt} as

$$\varepsilon^*_{ijt} = \left[s_{ijt}\frac{1}{\theta} + \left(1 - s_{ijt}\right)\frac{1}{\eta}\right]^{-1}$$

- Then we can write the wage as

$$\begin{aligned} w^*_{ijt} &= \mu^*_{ijt} mrpl_{ijt} \\ \mu^*_{ijt} &= \frac{1}{s_{ijt}\frac{1}{\theta} + \left(1 - s_{ijt}\right)\frac{1}{\eta} + 1} \\ \mu^*_{ijt} &= \frac{\left[s_{ijt}\frac{1}{\theta} + \left(1 - s_{ijt}\right)\frac{1}{\eta}s_{ijt}\right]^{-1}}{\left[s_{ijt}\frac{1}{\theta} + \left(1 - s_{ijt}\right)\frac{1}{\eta}\right]^{-1} + 1} \\ \mu^*_{ijt} &= \frac{\varepsilon^*_{ijt}}{\varepsilon^*_{ijt} + 1} \end{aligned}$$

- This is delivers the set of partial equilibrium conditions specified in the text in Section 1.3

$$\begin{aligned} w^*_{ijt} &= \mu^*_{ijt} mrpl_{ijt} \\ \mu^*_{ijt} &= \frac{\varepsilon_{ijt}}{\varepsilon_{ijt} + 1} \\ \varepsilon^*_{ijt} &= \left[s_{ijt}\frac{1}{\theta} + \left(1 - s_{ijt}\right)\frac{1}{\eta}\right]^{-1} \end{aligned}$$

E.3 Proof of Proposition 1.1

- Collect terms in the labor supply curve that are common to all firms in the market, $\boldsymbol{x}_j := \boldsymbol{w}_j^{\theta-\eta}\boldsymbol{W}^{-\theta}\boldsymbol{N}$.
- We have the following conditions

$$mrpl_{ij} = \widetilde{\alpha}\widetilde{z}_{ij}n_{ij}^{\widetilde{\alpha}-1}$$
$$n_{ij} = w_{ij}^{\eta} \times \boldsymbol{x}_j$$
$$w_{ij} = \mu\left(s_{ij}\right) mrpl_{ij}$$

- Substituting the labor supply curve into the $mrpl_{ij}$ definition, and then the pricing condition into the labor supply curve for w_{ij} we have

$$mrpl_{ij} = \left[\widetilde{\alpha}\widetilde{z}_{ij}\mu\left(s_{ij}\right)^{-\eta(1-\widetilde{\alpha})}\boldsymbol{x}_j^{\widetilde{\alpha}-1}\right] \times mrpl_{ij}^{-\eta(1-\widetilde{\alpha})}$$
$$mrpl_{ij} = \left[\widetilde{\alpha}\widetilde{z}_{ij}\mu\left(s_{ij}\right)^{-\eta(1-\widetilde{\alpha})}\boldsymbol{x}_j^{\widetilde{\alpha}-1}\right]^{\frac{1}{1+\eta(1-\widetilde{\alpha})}}$$

- Substiting this back into the optmiality condition:

$$w_{ij} = \left[\mu\left(s_{ij}\right)\widetilde{\alpha}\widetilde{z}_{ij}\right]^{\frac{1}{1+\eta(1-\widetilde{\alpha})}} \times \boldsymbol{x}_j^{-\frac{1-\widetilde{\alpha}}{1+\eta(1-\widetilde{\alpha})}} = \left[\mu\left(s_{ij}\right)\widetilde{z}_{ij}\right]^{\frac{1}{1+\eta(1-\widetilde{\alpha})}} \times g\left(\boldsymbol{x}_j\right)$$

- The definition of the payroll share, combined with the labor supply curve gives

$$s_{ij} = \frac{w_{ij}n_{ij}}{\sum_{k\in j} w_{kj}n_{kj}} = \frac{w_{ij}\left(\frac{w_{ij}}{\boldsymbol{w}_j}\right)^{\eta}\left(\frac{\boldsymbol{w}_j}{\boldsymbol{W}}\right)^{\theta}\boldsymbol{N}}{\sum_{k\in j} w_{kj}\left(\frac{w_{kj}}{\boldsymbol{w}_j}\right)^{\eta}\left(\frac{\boldsymbol{w}_j}{\boldsymbol{W}}\right)^{\theta}\boldsymbol{N}} = \frac{w_{ij}^{\eta+1}}{\sum_{k\in j} w_{kj}^{\eta+1}} = \frac{w_{ij}^{\eta+1}}{\sum_{k\in j} w_{kj}^{\eta+1}}.$$

- Under the above expression for w_{ij}:

$$s_{ij} = \frac{\left[\mu\left(s_{ij}\right)\widetilde{z}_{ij}\right]^{\frac{\eta+1}{1+\eta(1-\widetilde{\alpha})}}}{\sum_{k\in j}\left[\mu\left(s_{kj}\right)\widetilde{z}_{kj}\right]^{\frac{\eta+1}{1+\eta(1-\widetilde{\alpha})}}}.$$

- Now recall that $\widetilde{z}_{ij}$ is the firm productivity under the firm's optimal capital decision, and $\widetilde{\alpha}$is the corresponding exponent:

$$y_i = z_i\left(k^*\left(n_i, z_i, R\right)^{1-\gamma} n_i^{\gamma}\right)^{\alpha} = \widetilde{z}_i n_i^{\widetilde{\alpha}}$$
$$\widetilde{z}_i = \left[1-(1-\gamma)\alpha\right] z_i^{\frac{1}{1-(1-\gamma)\alpha}}\left(\frac{(1-\gamma)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}}$$
$$\widetilde{\alpha} = \frac{\gamma\alpha}{1-(1-\gamma)\alpha}$$

- Substituting these in

$$s_{ij} = \frac{\left[\mu\left(s_{ij}\right)^{1-(1-\gamma)\alpha} z_{ij}\right]^{\frac{\eta+1}{1-(1-\gamma)\alpha+\eta(1-\alpha)}}}{\sum_{k\in j}\left[\mu\left(s_{kj}\right)^{1-(1-\gamma)\alpha} z_{kj}\right]^{\frac{\eta+1}{1-(1-\gamma)\alpha+\eta(1-\alpha)}}}$$

- This is the expression in Proposition 1.1, which holds for all firms i in market j, and is independent of aggregates.

- Note that in the limit as $\gamma \to 1$, then we can check that we obtain the no-capital expression from above

$$s_{ij} = \frac{\left[\mu\left(s_{ij}\right) z_{ij}\right]^{\frac{\eta+1}{1+\eta(1-\alpha)}}}{\sum_{k\in j}\left[\mu\left(s_{kj}\right) z_{kj}\right]^{\frac{\eta+1}{1+\eta(1-\alpha)}}}$$

- Additionally in the limit with $\alpha \to 1$, we have

$$s_{ij} = \frac{\left[\mu\left(s_{ij}\right) z_{ij}\right]^{\eta+1}}{\sum_{k\in j}\left[\mu\left(s_{kj}\right) z_{kj}\right]^{\eta+1}}$$

- In the limit $\alpha \to 1$ with $\gamma < 1$:

$$s_{ij} = \frac{\left[\mu\left(s_{ij}\right)^{\gamma} z_{ij}\right]^{\frac{\eta+1}{\eta+\gamma}}}{\sum_{k\in j}\left[\mu\left(s_{kj}\right)^{\gamma} z_{kj}\right]^{\frac{\eta+1}{\eta+\gamma}}}$$

E.4 Proof of Proposition 1.2

- We proceed in three steps.
- First, consider an economy with a single nest, with a single elasticity of substitution η, and consider the case of labor as the only input into production with decreasing returns $\alpha \in (0, 1]$.
- Our starting point is the following set of equations, where we can take the markdown as exogenous. These describe firm level (i) output, (ii) labor supply, (iii) labor demand optimality, (iv) marginal revenue product:

$$\begin{aligned} y_i &= z_i n_i^{\alpha} \\ n_i &= \left(\frac{w_i}{\boldsymbol{w}}\right)^{\eta} \boldsymbol{n} \\ w_i &= \mu_i mrpl_i \\ mrpl_i &= \alpha z_i n_i^{\alpha-1} \end{aligned}$$

- We then have two aggregation conditions: (i) output, (ii) wage index

$$\begin{aligned} \boldsymbol{y} &= \int y_i di \\ \boldsymbol{w} &= \left[\int w_i^{1+\eta} di\right]^{\frac{1}{1+\eta}} \end{aligned}$$

- This set of 6 equations are our inputs to the following claim.
- **Claim** - *The aggregates* $\{\boldsymbol{y}, \boldsymbol{w}, \boldsymbol{n}\}$ *can be written:*

$$\begin{aligned} \boldsymbol{y} &= \omega z \boldsymbol{n}^{\alpha} \\ \boldsymbol{w} &= \mu \alpha z \boldsymbol{n}^{\alpha-1} \end{aligned}$$

where

$$z = \left[\int z_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} di \right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}$$

$$\mu = \left[\int \left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} di \right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}$$

$$\omega = \int \left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_i}{\mu} \right)^{\frac{\eta\alpha}{1+\eta(1-\alpha)}} di$$

- **Proof**
- With decreasing returns to scale, we first solve out for the marginal revenue product of labor. Note that here we only multiply and divide by z:

$$mrpl_i = \alpha z_i n_i^{\alpha-1}$$

$$mrpl_i = \alpha z_i \left(\left(\frac{w_i}{\boldsymbol{w}} \right)^{\eta} \boldsymbol{n} \right)^{\alpha-1}$$

$$mrpl_i = \left(\frac{z_i}{z} \right) w_i^{\eta(\alpha-1)} \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle \boldsymbol{w}^{\eta(1-\alpha)}$$

$$mrpl_i = \left(\frac{z_i}{z} \right) \mu_i^{\eta(\alpha-1)} mrpl_i^{\eta(\alpha-1)} \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle \boldsymbol{w}^{\eta(1-\alpha)}$$

$$mrpl_i = \left(\frac{z_i}{z} \right)^{\frac{1}{1+\eta(1-\alpha)}} \mu_i^{-\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}} \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle^{\frac{1}{1+\eta(1-\alpha)}} \boldsymbol{w}^{\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}}$$

- We can check that in the case of $\alpha = 1$, then $mrpl_i = z_i$.
- Using this in the wage

$$w_i = \mu_i mrpl_i$$

$$w_i = \left(\frac{z_i}{z} \right)^{\frac{1}{1+\eta(1-\alpha)}} \mu_i^{\frac{1}{1+\eta(1-\alpha)}} \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle^{\frac{1}{1+\eta(1-\alpha)}} \boldsymbol{w}^{\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}}$$

- Now aggregating:

$$\boldsymbol{w} = \left[\int w_i^{1+\eta} di \right]^{\frac{1}{1+\eta}}$$

$$\boldsymbol{w} = \left[\int \left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} di \right]^{\frac{1}{1+\eta}} \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle^{\frac{1}{1+\eta(1-\alpha)}} \boldsymbol{w}^{\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}}$$

$$\boldsymbol{w}^{\frac{1}{1+\eta(1-\alpha)}} = \left[\int \left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} di \right]^{\frac{1}{1+\eta}} \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle^{\frac{1}{1+\eta(1-\alpha)}}$$

$$\boldsymbol{w} = \left[\int \left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} di \right]^{\frac{1+\eta(1-\alpha)}{1+\eta}} \times \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle$$

$$\boldsymbol{w} = \mu \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle$$

- This delivers the first result. Note that if $\alpha = 1$, then

$$\mu = \left[\int \left(\frac{z_i}{z} \right)^{1+\eta} \mu_i^{1+\eta} di \right]^{\frac{1}{1+\eta}}$$

- Now turning to firm output, under the labor supply curve and labor demand:

$$
\begin{aligned}
y_i &= z_i n_i^{\alpha} \\
y_i &= z_i \left(\left(\frac{w_i}{\boldsymbol{w}} \right)^{\eta} \boldsymbol{n} \right)^{\alpha} \\
y_i &= z_i \left(\left(\frac{\mu_i mrpl_i}{\boldsymbol{w}} \right)^{\eta} \boldsymbol{n} \right)^{\alpha} \\
y_i &= z_i \mu_i^{\alpha\eta} mrpl_i^{\alpha\eta} \left(\frac{1}{\boldsymbol{w}} \right)^{\alpha\eta} \boldsymbol{n}^{\alpha}
\end{aligned}
$$

- Using the previous expression for $mrpl_i$:

$$
\begin{aligned}
y_i &= z_i \mu_i^{\alpha\eta} \left\{ \left(\frac{z_i}{z} \right)^{\frac{1}{1+\eta(1-\alpha)}} \mu_i^{-\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}} \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle^{\frac{1}{1+\eta(1-\alpha)}} \boldsymbol{w}^{\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}} \right\}^{\alpha\eta} \left(\frac{1}{\boldsymbol{w}} \right)^{\alpha\eta} \boldsymbol{n}^{\alpha} \\
y_i &= z \left[\left(\frac{z_i}{z} \right)^{1+\frac{\alpha\eta}{1+\eta(1-\alpha)}} \mu_i^{\alpha\eta\left(1-\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}\right)} \left\{ \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle \boldsymbol{w}^{\eta(1-\alpha)} \right\}^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left(\frac{1}{\boldsymbol{w}} \right)^{\alpha\eta} \right] \boldsymbol{n}^{\alpha} \\
y_i &= z \left[\left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left\{ \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle \boldsymbol{w}^{\eta(1-\alpha)} \right\}^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left(\frac{1}{\boldsymbol{w}} \right)^{\alpha\eta} \right] \boldsymbol{n}^{\alpha}
\end{aligned}
$$

Given that we have shown that $\boldsymbol{w} = \mu \alpha z \boldsymbol{n}^{\alpha-1}$, we can use this to simplify $\{\cdot\}$:

$$
\begin{aligned}
y_i &= z \left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left\{ \left(\frac{\boldsymbol{w}}{\mu} \right) \boldsymbol{w}^{\eta(1-\alpha)} \right\}^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left(\frac{1}{\boldsymbol{w}} \right)^{\alpha\eta} \boldsymbol{n}^{\alpha} \\
y_i &= z \left[\left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_i}{\mu} \right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \right] \boldsymbol{n}^{\alpha}
\end{aligned}
$$

- Then aggregating:

$$
\begin{aligned}
\boldsymbol{y} &= \int y_i di \\
\boldsymbol{y} &= \left[\int \left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_i}{\mu} \right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} di \right] \times \langle z \boldsymbol{n}^{\alpha} \rangle \\
\boldsymbol{y} &= \omega \times z \boldsymbol{n}^{\alpha}
\end{aligned}
$$

- This delivers the second result. Note that if $\alpha = 1$, then

$$
\omega = \int \left(\frac{z_i}{z} \right)^{1+\eta} \left(\frac{\mu_i}{\mu} \right)^{\eta} di
$$

- We stil need to show that the productivity term z is correct. Notice that up to this point these derivations would hold under any z. We pin down z, by requiring that if there are no distortions ($\mu_i = 1$ for all firms), then the aggregate markdown is also $\mu = 1$. This requires:

$$
\begin{aligned}
1 &= \left[\int \left(\frac{z_i}{z} \right)^{1+\eta} di \right]^{\frac{1}{1+\eta}} \\
z &= \left[\int z_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} di \right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}
\end{aligned}
$$

which also implies that in an undistorted economy, since $\boldsymbol{\omega} = 1$, then output is simply $\boldsymbol{y} = z\boldsymbol{n}^{\alpha}$.

- This also implies that in the expression for μ and the expression for ω, the productivity terms are well-defined weights:

$$\mu = \left[\int \left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} di\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}} \quad , \quad \int \left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} di = 1$$
$$\omega = \int \left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_i}{\mu}\right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} di \quad , \quad \int \left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} di = 1$$

- Using the above results we can now turn to the nested economy.
- **Inner nest**
  - We take the same approach as above, starting with the isomorphic 6 conditions at the firm level and now those expressing *market level* aggregates:
  - **Firm conditions**: We have 4 conditions representing firm output, labor supply, labor demand, and the marginal revenue product of labor:

$$y_{ij} = z_{ij} n_{ij}^{\alpha}$$
$$n_{ij} = \left(\frac{w_{ij}}{\boldsymbol{w}_j}\right)^{\eta} \boldsymbol{n}_j$$
$$w_{ij} = \mu_{ij} mrpl_{ij}$$
$$mrpl_{ij} = \alpha z_{ij} n_{ij}^{\alpha-1}$$

  - **Market aggregates**: We then have two aggregation conditions: (i) market output, (ii) market wage index

$$\boldsymbol{y}_j = \sum_{i \in j} y_{ij}$$
$$\boldsymbol{w}_j = \left[\sum_{i \in j} w_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

  - Following the same steps as above, it is clear that we can show that:
  - **Claim** - *The market aggregates* $\left\{\boldsymbol{y}_j, \boldsymbol{w}_j, \boldsymbol{n}_j\right\}$ *can be written:*

$$\boldsymbol{y}_j = \omega_j \boldsymbol{z}_j \boldsymbol{n}_j^{\alpha}$$
$$\boldsymbol{w}_j = \mu_j \alpha z_j \boldsymbol{n}_j^{\alpha-1}$$

where

$$\boldsymbol{z}_j = \left[\sum_{i \in j} z_{ij}^{\frac{1+\eta}{1+\eta(1-\alpha)}}\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}$$
$$\boldsymbol{\mu}_j = \left[\sum_{i \in j} \left(\frac{z_{ij}}{z_j}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_{ij}^{\frac{1+\eta}{1+\eta(1-\alpha)}}\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}$$
$$\boldsymbol{\omega}_j = \sum_{i \in j} \left(\frac{z_i}{\boldsymbol{z}_j}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_i}{\boldsymbol{\mu}}\right)^{\frac{\eta\alpha}{1+\eta(1-\alpha)}}$$

- **Outer nest**
  - The solution to the inner nests allows us to establish a similar set of 6 conditions

- **Market level**: We have 3 conditions representing market output, aggregate labor supply, market labor demand:

$$\boldsymbol{y}_j = \omega_j \boldsymbol{z}_j \boldsymbol{n}_j^{\alpha}$$
$$\boldsymbol{n}_j = \left(\frac{\boldsymbol{w}_j}{\boldsymbol{W}}\right)^{\theta} \boldsymbol{N}$$
$$\boldsymbol{w}_j = \mu_j \alpha z_j \boldsymbol{n}_j^{\alpha-1}$$

- **Economy aggregates**: We have two aggregation conditions: (i) aggregate output, (ii) aggregate wage index

$$\boldsymbol{Y} = \int \boldsymbol{y}_j dj$$
$$\boldsymbol{W} = \left[\int \boldsymbol{w}_j^{1+\theta} dj\right]^{\frac{1}{1+\theta}}$$

- Following the above steps again, we can obtain:

$$\boldsymbol{Y} = \boldsymbol{\Omega} \boldsymbol{Z} \boldsymbol{N}^{\alpha}$$
$$\boldsymbol{W} = \mu \alpha \boldsymbol{Z} \boldsymbol{N}^{\alpha-1}$$

where

$$\boldsymbol{Z} = \left[\int \boldsymbol{z}_j^{\frac{1+\theta}{1+\theta(1-\alpha)}}\right]^{\frac{1+\theta(1-\alpha)}{1+\theta}}$$
$$\boldsymbol{\mu} = \left[\int \left(\frac{\boldsymbol{z}_j}{\boldsymbol{Z}}\right)^{\frac{1+\theta}{1+\theta(1-\alpha)}} \boldsymbol{\mu}_j^{\frac{1+\theta}{1+\theta(1-\alpha)}} dj\right]^{\frac{1+\theta(1-\alpha)}{1+\theta}}$$
$$\boldsymbol{\Omega} = \int \left(\frac{\boldsymbol{z}_j}{\boldsymbol{Z}}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\boldsymbol{\mu}_j}{\boldsymbol{\mu}}\right)^{\frac{\eta\alpha}{1+\eta(1-\alpha)}} \boldsymbol{\omega}_j dj$$

- This delivers the main expressions in Proposition 1.2, in the case of an economy without capital and decreasing returns to labor.

E.4.1 Adding capital

- The value-added production function of the firm in our model is

$$y_i = z_i \left(k_i^{1-\gamma} n_i^{\gamma}\right)^{\alpha}$$

- The optimal choice of capital solves

$$k_i^* (z_i, n_i, R) = \arg\max_{k_i} z_i \left(k_i^{1-\gamma} n_i^{\gamma}\right)^{\alpha} - Rk_i$$
$$k_i^* (z_i, n_i, R) = \left(\frac{(1-\gamma)\alpha z_i}{R}\right)^{\frac{1}{1-(1-\gamma)\alpha}} n_i^{\frac{\gamma\alpha}{1-(1-\gamma)\alpha}}$$

- We can substitute this back into output to obtain:

$$y_i = z_i^{\frac{1}{1-(1-\gamma)\alpha}} \left(\frac{(1-\gamma)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} n_i^{\frac{\gamma\alpha}{1-(1-\gamma)\alpha}}$$

- Note that in terms of factor payment shares, capital is competitively priced:

$$Rk_i = \alpha(1-\gamma) y_i$$

- Combining these, profits are

$$\pi_i = y_i - Rk_i - w_i n_i$$
$$\pi_i = [1-\alpha(1-\gamma)] y_i - w_i n_i$$
$$\pi_i = [1-\alpha(1-\gamma)] z_i^{\frac{1}{1-(1-\gamma)\alpha}} \left(\frac{(1-\gamma)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} n_i^{\frac{\gamma\alpha}{1-(1-\gamma)\alpha}} - w_i n_i$$

- We can write this as

$$\pi_i = \widetilde{y}_i - w_i n_i$$
$$\widetilde{y}_i = \widetilde{z}_i n_i^{\widetilde{\alpha}}$$
$$\widetilde{z}_i = [1-\alpha(1-\gamma)] z_i^{\frac{1}{1-(1-\gamma)\alpha}} \left(\frac{(1-\gamma)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}}$$
$$\widetilde{\alpha} = \frac{\gamma\alpha}{1-(1-\gamma)\alpha}$$

- Note that this implies that

$$\widetilde{y}_i = y_i - Rk_i = [1-\gamma(1-\alpha)] y_i$$
$$y_i = \left[\frac{1}{1-\gamma(1-\alpha)}\right] \widetilde{y}_i$$

- It should therefore be clear that what we have obtained so far in our aggregation results implies the following:

1. **Market level** - At the market level, define $\widetilde{\boldsymbol{y}}_j = \sum_{i\in j} \widetilde{y}_{ij}$, then

$$\widetilde{\boldsymbol{y}}_j = \boldsymbol{\omega}_j \widetilde{z}_j \boldsymbol{n}_j^{\widetilde{\alpha}}$$
$$\boldsymbol{w}_j = \boldsymbol{\mu}_j \widetilde{\boldsymbol{z}}_j \widetilde{\alpha} \boldsymbol{n}_j^{\widetilde{\alpha}-1}$$

where $\left\{\boldsymbol{\omega}_j, \boldsymbol{\mu}_j\right\}$ are as before, except with $\widetilde{\alpha}$ in place of α, and we define $\widetilde{\boldsymbol{z}}_j$ as:

$$\widetilde{\boldsymbol{z}}_j = \left[\sum_{i\in j} \widetilde{z}_{ij}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}}$$
$$\boldsymbol{\mu}_j = \left[\sum_{i\in j} \left(\frac{\widetilde{z}_{ij}}{\widetilde{\boldsymbol{z}}_j}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}} \mu_{ij}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}}$$
$$\boldsymbol{\omega}_j = \sum_{i\in j} \left(\frac{\widetilde{z}_{ij}}{\widetilde{\boldsymbol{z}}_j}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}} \left(\frac{\mu_i}{\boldsymbol{\mu}}\right)^{\frac{\eta\widetilde{\alpha}}{1+\eta(1-\widetilde{\alpha})}}$$

2. **Aggregate level** - At the aggregate level, define $\widetilde{\boldsymbol{Y}} = \int \widetilde{y}_j dj$, then

$$\widetilde{\boldsymbol{Y}} = \boldsymbol{\Omega} \widetilde{\boldsymbol{Z}} \boldsymbol{N}^{\widetilde{\alpha}}$$
$$\boldsymbol{W} = \mu \widetilde{\alpha} \widetilde{\boldsymbol{Z}} \boldsymbol{N}^{\widetilde{\alpha}-1}$$

where $\{\boldsymbol{\Omega}, \boldsymbol{\mu}\}$ are as before, except with $\widetilde{\alpha}$ in place of α, and we define $\widetilde{\boldsymbol{Z}}$ as:

$$\widetilde{\boldsymbol{Z}} = \left[\int \widetilde{\boldsymbol{z}}_j^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}}\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}$$

$$\boldsymbol{\mu} = \left[\int \left(\frac{\widetilde{\boldsymbol{z}}_j}{\widetilde{\boldsymbol{Z}}}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \boldsymbol{\mu}_j^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}$$

$$\boldsymbol{\Omega} = \int \left(\frac{\widetilde{\boldsymbol{z}}_j}{\widetilde{\boldsymbol{Z}}}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}} \left(\frac{\boldsymbol{\mu}_j}{\boldsymbol{\mu}}\right)^{\frac{\eta\widetilde{\alpha}}{1+\eta(1-\widetilde{\alpha})}} \boldsymbol{\omega}_j dj$$

- Now observe that when we aggregate capital

$$K = \int \sum_{i\in j} k_i dj$$

$$K = \int \sum_{i\in j} \left(\frac{\alpha(1-\gamma) y_i}{R}\right) dj$$

$$RK = \alpha(1-\gamma) \int \sum_{i\in j} y_i dj$$

$$RK = \alpha(1-\gamma)\boldsymbol{Y} \quad (*)$$

- Now note that

$$\boldsymbol{Y} = \int \sum_{i\in j} y_{ij} dj = \int \sum_{i\in j} \left[\frac{1}{1-\gamma(1-\alpha)} \widetilde{y}_{ij}\right] dj = \frac{1}{1-\gamma(1-\alpha)} \int \sum_{i\in j} \widetilde{y}_{ij} dj = \frac{1}{1-\gamma(1-\alpha)} \widetilde{\boldsymbol{Y}}$$

- Substituting the aggregate output expression $\widetilde{\boldsymbol{Y}} = \boldsymbol{\Omega}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}}$, into the aggregate labor demand condition:

$$\boldsymbol{W} = \mu\widetilde{\alpha}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}-1}$$

$$\boldsymbol{W} = \mu\widetilde{\alpha}\left(\frac{\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}}}{\boldsymbol{N}}\right)$$

$$\boldsymbol{W} = \left(\frac{\mu}{\boldsymbol{\Omega}}\right)\widetilde{\alpha}\left(\frac{\widetilde{\boldsymbol{Y}}}{\boldsymbol{N}}\right)$$

$$\boldsymbol{W} = \left(\frac{\mu}{\boldsymbol{\Omega}}\right)\left(\frac{\gamma\alpha}{1-(1-\gamma)\alpha}\right)\left(\frac{[1-\gamma(1-\alpha)]\boldsymbol{Y}}{\boldsymbol{N}}\right)$$

$$\boldsymbol{W} = \gamma\alpha\left(\frac{\mu}{\boldsymbol{\Omega}}\right)\frac{\boldsymbol{Y}}{\boldsymbol{N}} \quad (**)$$

- The equations $(*)$ and $(**)$ describe aggregate factor demand for capital and labor and appear in Proposition 1.2.
- The steady-state resource constraint is $\boldsymbol{C} = \boldsymbol{Y} - \delta K$, which requires no proof, and the steady-state Euler equation is

$1 = \beta [R + (1-\delta)]$, these can be combined with optimal capital demand to yield:

$$\begin{aligned}
\boldsymbol{C} &= \boldsymbol{Y} - \delta K \\
\boldsymbol{C} &= \boldsymbol{Y} - \frac{\delta}{R} RK \\
\boldsymbol{C} &= \left[1 - \frac{\delta}{R}(1-\gamma)\alpha\right] \boldsymbol{Y} \\
\boldsymbol{C} &= \left[1 - \frac{\delta}{R}(1-\gamma)\alpha\right] \frac{\widetilde{\boldsymbol{Y}}}{1-\gamma(1-\alpha)} \\
\boldsymbol{C} &= \left[1 - \frac{\beta\delta}{1-\beta(1-\delta)}(1-\gamma)\alpha\right] \frac{\widetilde{\boldsymbol{Y}}}{1-\gamma(1-\alpha)}
\end{aligned}$$

where we denote the constant s_C, which is the consumption share of output. These expressions appear in the main text.

- This implies that given $\boldsymbol{\mu}$, and $\boldsymbol{\Omega}$, we can solve for equilibrium $\left\{\widetilde{\boldsymbol{Y}}, \boldsymbol{W}, \boldsymbol{N}, \boldsymbol{C}\right\}$ from

$$\begin{aligned}
\boldsymbol{W} &= \boldsymbol{\mu}\widetilde{\alpha}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}-1} \\
\boldsymbol{W} &= \frac{U_N(\boldsymbol{C}, \boldsymbol{N})}{U_C(\boldsymbol{C}, \boldsymbol{N})} \\
\boldsymbol{C} &= s_C \frac{\widetilde{\boldsymbol{Y}}}{1-\gamma(1-\alpha)} \\
\widetilde{\boldsymbol{Y}} &= \boldsymbol{\Omega}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}}
\end{aligned}$$

- In the case of $U_N/U_C = \boldsymbol{N}^{\varphi}\boldsymbol{C}^{-\sigma}$, as is the case under GHH ($\sigma = 0$) or CRRA preferences ($\sigma \geq 1$), then all aggregates can be solved in closed form using the following equations from top to bottom:

$$\begin{aligned}
\boldsymbol{N} &= \left[\left(\frac{s_C}{1-\gamma(1-\alpha)}\boldsymbol{\Omega}\right)^{-\sigma\varphi} (\widetilde{\alpha}\boldsymbol{\mu})^{\varphi}\, \widetilde{\boldsymbol{Z}}^{(1-\sigma)\varphi}\right]^{\frac{1}{1+\varphi(1-\widetilde{\alpha})+\sigma\varphi\widetilde{\alpha}}} \\
\boldsymbol{W} &= \boldsymbol{\mu}\widetilde{\alpha}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}-1} \\
\widetilde{\boldsymbol{Y}} &= \boldsymbol{\Omega}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}} \\
\boldsymbol{C} &= \frac{s_c}{1-\gamma(1-\alpha)}\widetilde{\boldsymbol{Y}} \\
\boldsymbol{Y} &= \frac{1}{1-(1-\gamma)\alpha}\widetilde{\boldsymbol{Y}} \\
R &= \frac{1}{\beta} - (1-\delta) \\
K &= \frac{(1-\gamma)\alpha}{R}\boldsymbol{Y}
\end{aligned}$$

- Note that in the case of no wealth-effects on labor supply $\sigma = 0$, and we have the result, cited in the text, that the equilibrium aggregate employment and wage are independent of $\boldsymbol{\Omega}$,

$$\boldsymbol{N} = \left[\widetilde{\alpha}\boldsymbol{\mu}\widetilde{\boldsymbol{Z}}\right]^{\frac{\varphi}{1+\varphi(1-\widetilde{\alpha})}} \quad , \quad \boldsymbol{W} = \boldsymbol{\mu}\widetilde{\alpha}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}-1}$$

E.4.2 Production function

- In the main tet of the paper we provide the above conditions, but instead with the output as follows, which we now derive.

$$\boldsymbol{Y} = \boldsymbol{\Omega}^{1-(1-\gamma)\alpha}\boldsymbol{Z}\left(K^{1-\gamma}\boldsymbol{N}^{\gamma}\right)^{\alpha}$$

where we also need to specify $\boldsymbol{Z}$.

- First we need to go remove the optimal capital decisions encoded in $\widetilde{z}_{ij}$ and consequently $\widetilde{z}_j$ and $\widetilde{\boldsymbol{Z}}$.
- Recall that

$$\widetilde{z}_{ij} = (1-(1-\gamma)\alpha) z_{ij}^{\frac{1}{1-(1-\gamma)\alpha}} \left(\frac{(1-\gamma)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} , \widetilde{z}_j = \left[\sum_{i\in j} \widetilde{z}_{ij}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}} , \widetilde{\boldsymbol{Z}} = \left[\int \widetilde{z}_j^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}$$

- Substituting $\widetilde{z}_{ij}$ into $\widetilde{z}_j$, we can define market productivity as follows, which gives the following implication:

$$\boldsymbol{z}_j := \left[\sum_{i\in j} z_{ij}^{\frac{1+\eta}{1-(1-\gamma)\alpha+\eta(1-\alpha)}}\right]^{\frac{1-(1-\gamma)\alpha+\eta(1-\alpha)}{1+\eta}} \implies \widetilde{z}_j = (1-(1-\gamma)\alpha) z_j^{\frac{1}{1-(1-\gamma)\alpha}} \left(\frac{(1-\gamma)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}}$$

- We can do the same for market and aggregate productivity:

$$\boldsymbol{Z} := \left[\int z_j^{\frac{1+\theta}{1-(1-\gamma)\alpha+\theta(1-\alpha)}} dj\right]^{\frac{1-(1-\gamma)\alpha+\theta(1-\alpha)}{1+\theta}} \implies \widetilde{\boldsymbol{Z}} = (1-(1-\gamma)\alpha) \boldsymbol{Z}^{\frac{1}{1-(1-\gamma)\alpha}} \left(\frac{(1-\gamma)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}}$$

- Note that this implies that the relationship between $\boldsymbol{Z}$ and $\widetilde{\boldsymbol{Z}}$ given in the text

$$\boldsymbol{Z} = \left[\frac{R}{(1-\gamma)\alpha}\right]^{(1-\gamma)\alpha} \left[\frac{\widetilde{\boldsymbol{Z}}}{1-(1-\gamma)\alpha}\right]^{1-(1-\gamma)\alpha}$$

- Now return to the aggregate production function $\widetilde{\boldsymbol{Y}} = \boldsymbol{\Omega}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}}$ and substitute in (i) the definitions of $\widetilde{\alpha}$, (ii) $\widetilde{\boldsymbol{Z}}$, and, (iii) $\widetilde{\boldsymbol{Y}} = [1-(1-\gamma)\alpha]\boldsymbol{Y}$, (iv) the aggregate capital demand condition $R = (1-\gamma)\alpha(\boldsymbol{Y}/K)$, thsi gives the expression in Proposition 1.2:

$$\begin{aligned}
\widetilde{\boldsymbol{Y}} &= \boldsymbol{\Omega}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\frac{\gamma\alpha}{1-(1-\gamma)\alpha}} \\
\widetilde{\boldsymbol{Y}} &= \boldsymbol{\Omega}(1-(1-\gamma)\alpha) \boldsymbol{Z}^{\frac{1}{1-(1-\gamma)\alpha}} \left(\frac{(1-\gamma)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} \boldsymbol{N}^{\frac{\gamma\alpha}{1-(1-\gamma)\alpha}} \\
\boldsymbol{Y} &= \boldsymbol{\Omega}\boldsymbol{Z}^{\frac{1}{1-(1-\gamma)\alpha}} \left(\frac{K}{\boldsymbol{Y}}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} \boldsymbol{N}^{\frac{\gamma\alpha}{1-(1-\gamma)\alpha}} \\
\boldsymbol{Y} &= \boldsymbol{\Omega}^{1-(1-\gamma)\alpha}\boldsymbol{Z}\left(K^{1-\gamma}\boldsymbol{N}^{\gamma}\right)^{\alpha}
\end{aligned}$$

where we have the productivity terms which are described in the footnote of Proposition 1.2:

$$\boldsymbol{z}_j = \left[\sum_{i\in j} z_{ij}^{\frac{1+\eta}{1-(1-\gamma)\alpha+\eta(1-\alpha)}}\right]^{\frac{1-(1-\gamma)\alpha+\eta(1-\alpha)}{1+\eta}} , \quad \boldsymbol{Z} = \left[\int z_j^{\frac{1+\theta}{1-(1-\gamma)\alpha+\theta(1-\alpha)}} dj\right]^{\frac{1-(1-\gamma)\alpha+\theta(1-\alpha)}{1+\theta}}$$

E.5 Proof of Proposition 1.3 - Labor share and concentration

- Rearranging the above conditions immediately gives the labor share

$$LS := \frac{\int \sum_{i\in j} w_{ij}n_{ij}dj}{\int \sum_{i\in j} y_{ij}dj} = \frac{\boldsymbol{W}\boldsymbol{N}}{\boldsymbol{Y}} = \gamma\alpha\left(\frac{\boldsymbol{\mu}}{\boldsymbol{\Omega}}\right)$$

- Since $\boldsymbol{\mu}$ and $\boldsymbol{\Omega}$ are independent of the supply block of the model, then they are independent of aggregates and preferences, and so is the labor share. That is, we only have to solve market equilibria to compute the labor share.
- We now show that the expression linking the labor share and aggregate HHI^{wn} holds.

- We can use 'tilde' objects and then scale up at the end:

$$\widetilde{ls}_{ij} = \frac{w_{ij}n_{ij}}{\widetilde{y}_{ij}} = \frac{w_{ij}n_{ij}}{\widetilde{z}_{ij}n_{ij}^{\widetilde{\alpha}}} = \widetilde{\alpha}\frac{w_{ij}}{\widetilde{\alpha}\widetilde{z}_{ij}n_{ij}^{\widetilde{\alpha}-1}} = \widetilde{\alpha}\frac{w_{ij}}{mrpl_{ij}} = \widetilde{\alpha}\mu_{ij}$$

- Therefore the inverse of the labor share is:

$$\widetilde{ls}_{ij}^{-1} = \frac{1}{\widetilde{\alpha}}\mu_{ij}^{-1}$$

- Note that from our earlier expression for the markdown:

$$\mu_{ij}^{-1} = s_{ijt}\left(\frac{1}{\theta} - \frac{1}{\eta}\right) + \frac{1}{\eta} + 1 = \frac{\eta+1}{\eta} + s_{ijt}\left(\frac{\theta+1}{\theta} - \frac{\eta+1}{\eta}\right)$$

- Now the *market* inverse labor share is

$$\widetilde{ls}_{ij}^{-1} = \frac{\sum_{i\in j}\widetilde{y}_{ij}}{\sum_{i\in j}w_{ij}n_{ij}} = \sum_{i\in j}\frac{w_{ij}n_{ij}}{\sum_{i\in j}w_{ij}n_{ij}}\frac{\widetilde{y}_{ij}}{w_{ij}n_{ij}} = \sum_{i\in j}s_{ij}\widetilde{ls}_{ij}^{-1}$$

  which using the above, gives:

$$\widetilde{ls}_{ij}^{-1} = \frac{1}{\widetilde{\alpha}}\left[\left(1 - hhi_j\right)\frac{\eta+1}{\eta} + hhi_j\frac{\theta+1}{\theta}\right]$$

  where $hhi_j = \sum_{i\in j} s_{ij}^2$.

- Then the aggregate labor share is as follows, where we use our definiton $HHI = \int s_j hhi_j dj$:

$$\begin{aligned}
\widetilde{LS} &= \left[\frac{\int\sum_{i\in j}y_{ij}dj}{\int\sum_{i\in j}w_{ij}n_{ij}dj}\right]^{-1} \\
&= \left[\int s_j\widetilde{ls}_j^{-1}dj\right]^{-1} \\
\widetilde{LS} &= \widetilde{\alpha}\left[HHI\left(\frac{\theta}{\theta+1}\right)^{-1} + (1-HHI)\left(\frac{\eta}{\eta+1}\right)^{-1}\right]^{-1}
\end{aligned}$$

- Now note that

$$\boldsymbol{Y} = \frac{1}{1-(1-\gamma)\alpha}\widetilde{\boldsymbol{Y}}$$

  therefore

$$LS = \frac{\boldsymbol{WN}}{\boldsymbol{Y}} = \frac{\boldsymbol{WN}}{\frac{1}{1-(1-\gamma)\alpha}\widetilde{\boldsymbol{Y}}} = \left[1-(1-\gamma)\alpha\right]\widetilde{LS},$$

  so under $\widetilde{\alpha} = \alpha\gamma/\left(1-(1-\gamma)\alpha\right)$ we have

$$LS = \alpha\gamma\left[HHI\left(\frac{\theta}{\theta+1}\right)^{-1} + (1-HHI)\left(\frac{\eta}{\eta+1}\right)^{-1}\right]^{-1}$$

- This is the expression in Proposition 1.3.
- This also established the following claims: (i) the moments LS and HHI are independent of aggregate preferences and shifters in labor supply and productivity which we study below $\widetilde{A}$ and $\overline{\varphi}$, since the capital share is $KS = \alpha(1-\gamma)$, then this is also independent of preferences, therefore (ii) we can estimate the model without specifying aggregate preferences.

E.6 Scaling the economy

- Here we prove our claim in the text that we can choose scaling parameters for productivity and labor supply that can always be chosen to match average firm size and average worker pay without affecting any of the other moments of the economy.
- Suppose we add two constants to the above economy $\widetilde{A}$ and $\overline{\varphi}$ such that the firm production function after optimizing out capital and the aggregate labor supply curve are:

$$\widetilde{y}_{ij} = \widetilde{A}\widetilde{z}_{ij}n_{ij}^{\widetilde{\alpha}}$$
$$\boldsymbol{W} = -\frac{U_N\left(\boldsymbol{C},\boldsymbol{N}\right)}{U_C\left(\boldsymbol{C},\boldsymbol{N}\right)} = \overline{\varphi}^{-1/\varphi}\boldsymbol{N}^{1/\varphi}\boldsymbol{C}^{\sigma}$$

- We claim that we can *always* choose these constants to match two moments of the data: average firm size, and average worker pay.
- Note that the above labor supply curve obtains under either CRRA or GHH preferences, where GHH preferences are the case of $\sigma = 0$:

$$U(\boldsymbol{C},\boldsymbol{N}) = \frac{\boldsymbol{C}^{1-\sigma}}{1-\sigma} - \overline{\varphi}^{-1/\varphi}\frac{\boldsymbol{N}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \quad , \quad U\left(\boldsymbol{C},\boldsymbol{N}\right) = u\left(\boldsymbol{C} - \overline{\varphi}^{-1/\varphi}\frac{\boldsymbol{N}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)$$

- Note that this implies that the changes to the set of equilibrium conditions are that aggregate output, labor demand and labor supply are:

$$\widetilde{\boldsymbol{Y}} = \boldsymbol{\Omega}\widetilde{A}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}}$$
$$\boldsymbol{W} = \boldsymbol{\mu}\widetilde{\alpha}\widetilde{A}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}-1}$$
$$\boldsymbol{N} = \overline{\varphi}\boldsymbol{W}^{\varphi}\boldsymbol{C}^{-\sigma\varphi}$$

- Combined these imply the same expression for the labor share:

$$\boldsymbol{W}\boldsymbol{N} = \left(\frac{\boldsymbol{\mu}}{\boldsymbol{\Omega}}\right)\widetilde{\alpha}\widetilde{\boldsymbol{Y}}$$

which gives

$$\widetilde{\boldsymbol{Y}} = \frac{\boldsymbol{W}\boldsymbol{N}}{\left(\frac{\boldsymbol{\mu}}{\boldsymbol{\Omega}}\right)\widetilde{\alpha}}$$

- Consider the following two moments: (i) Average firm size $(AveFirmSize)$, (ii) Average worker pay $(AveWorkerPay)$
- The average firm size in the economy is

$$AveFirmSize := \frac{\int\sum_{i\in j} n_{ij}dj}{\int M_j dj} = \frac{\int\sum_{i\in j}\left(\frac{w_{ij}}{\boldsymbol{w}_j}\right)^{\eta}\left(\frac{\boldsymbol{w}_j}{\boldsymbol{W}}\right)^{\theta}\boldsymbol{N}dj}{\int M_j dj}$$

- Define the following:

$$\nu_{ij} := \left(\frac{w_{ij}}{\boldsymbol{w}_j}\right)^{\eta}\left(\frac{\boldsymbol{w}_j}{\boldsymbol{W}}\right)^{\theta}$$

- Using this

$$AveFirmSize = \frac{\int\sum_{i\in j}\nu_{ij}dj}{\int M_j dj}\boldsymbol{N}$$

- Denote $\boldsymbol{\nu} = \int\sum_{i\in j}\nu_{ij}dj$.

- The average worker pay in the economy is

$$
\begin{aligned}
AveWorkerPay &= \frac{\int \sum_{i\in j} w_{ij} n_{ij} dj}{\int \sum_{i\in j} n_{ij} dj} \\
&= \frac{\boldsymbol{W}\boldsymbol{N}}{AveFirmSize \times \int M_j dj} \\
AveWorkerPay &= \frac{\boldsymbol{W}}{\boldsymbol{\nu}}
\end{aligned}
$$

- We can re-write these:

$$
\begin{aligned}
\boldsymbol{N} &= AveFirmSize \times \frac{\int M_j dj}{\boldsymbol{\nu}} \\
\boldsymbol{W} &= AveWorkerPay \times \boldsymbol{\nu}
\end{aligned}
$$

- **Claim** - The aggregate $\boldsymbol{\nu}$ is independent of all other aggregates including the shifters $\widetilde{A}$ and $\overline{\varphi}$
  - Using our above results, but now including the shifter terms:

$$
\begin{aligned}
\left(\frac{w_{ij}}{\boldsymbol{w}_j}\right) &= \frac{\mu_{ij}\widetilde{\alpha}\widetilde{z}_{ij}\widetilde{A}n_{ij}^{\widetilde{\alpha}-1}}{\boldsymbol{\mu}_j\widetilde{\alpha}\widetilde{z}_j\widetilde{A}\boldsymbol{n}_j^{\widetilde{\alpha}-1}} \\
\left(\frac{w_{ij}}{\boldsymbol{w}_j}\right) &= \left(\frac{\mu_{ij}\widetilde{z}_{ij}}{\boldsymbol{\mu}_j\widetilde{z}_j}\right)^{\frac{1}{1+\eta(1-\widetilde{\alpha})}}
\end{aligned}
$$

  and using a similar approach at the market level:

$$
\left(\frac{\boldsymbol{w}_j}{\boldsymbol{W}}\right) = \left(\frac{\boldsymbol{\mu}_j\widetilde{\boldsymbol{z}}_j}{\boldsymbol{\mu}\widetilde{\boldsymbol{Z}}}\right)^{\frac{1}{1+\theta(1-\widetilde{\alpha})}}
$$

  - Therefore we have the following equation which implies that ν_{ij} is independent of aggregates including the shift parameters $\widetilde{A}$ and $\overline{\varphi}$

$$
\nu_{ij} = \left(\frac{\mu_{ij}\widetilde{z}_{ij}}{\boldsymbol{\mu}_j\widetilde{z}_j}\right)^{\frac{1}{1+\eta(1-\widetilde{\alpha})}} \left(\frac{\boldsymbol{\mu}_j\widetilde{\boldsymbol{z}}_j}{\boldsymbol{\mu}\widetilde{\boldsymbol{Z}}}\right)^{\frac{1}{1+\theta(1-\widetilde{\alpha})}}
$$

- We can then write the following system of equations, which we describe below

$$
\begin{aligned}
\boldsymbol{N} &= AveFirmSize \times \frac{\int M_j dj}{\boldsymbol{\nu}} \\
\boldsymbol{W} &= AveWorkerPay \times \boldsymbol{\nu} \\
\widetilde{\boldsymbol{Y}} &= \frac{\boldsymbol{W}\boldsymbol{N}}{\left(\frac{\boldsymbol{\mu}}{\boldsymbol{\Omega}}\right)\widetilde{\alpha}} \\
\boldsymbol{Y} &= \frac{\widetilde{\boldsymbol{Y}}}{1-(1-\gamma)\alpha} \\
K &= \frac{(1-\gamma)\alpha\boldsymbol{Y}}{R} \\
\boldsymbol{C} &= \boldsymbol{Y} - \delta K
\end{aligned}
$$

- Starting with a solution of the market equilibria, we can obtain $\{\boldsymbol{\nu}, \boldsymbol{\Omega}, \boldsymbol{\mu}\}$ which we have proven are independent of aggregates and shifters $\left\{\widetilde{A}, \overline{\varphi}\right\}$. Then given data on $AveFirmSize^{Data}$ and $AveWorkerPay^{Data}$, we can put data into the first two expressions above, and use these to compute $\boldsymbol{N}$ and $\boldsymbol{W}$. Given these we can use the remaining equations to compute all other aggregate quantities $\{\boldsymbol{Y}, \boldsymbol{C}, K\}$. What remains is to choose $\widetilde{A}$ and $\overline{\varphi}$ to be consistent with these aggregates. For this we use the two conditions that have not been used above by themselves: output and labor supply

$$\widetilde{\boldsymbol{Y}} = \boldsymbol{\Omega}\widetilde{A}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}} \quad , \quad \boldsymbol{N} = \overline{\varphi}\boldsymbol{W}^{\varphi}\boldsymbol{C}^{-\sigma\varphi},$$

which imply that

$$\widetilde{A} = \frac{\widetilde{\boldsymbol{Y}}}{\boldsymbol{\Omega}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}}} \quad , \quad \overline{\varphi} = \frac{\boldsymbol{N}}{\boldsymbol{W}^{\varphi}\boldsymbol{C}^{-\sigma\varphi}}.$$

- Proceeding in this way we can *always* choose shifters $\widetilde{A}$ and $\overline{\varphi}$ in order to match data on average firm size and average worker pay.
- Once these are pinned down, then the system of equilibrium conditions without the moment conditions can be solved. Going from top to bottom, the equilibrium is computed:

$$\begin{aligned}
\boldsymbol{N} &= \left[\overline{\varphi}\left(\frac{s_C}{1-\gamma\left(1-\alpha\right)}\boldsymbol{\Omega}\right)^{-\sigma\varphi}\left(\tilde{\alpha}\boldsymbol{\mu}\right)^{\varphi}\left(\widetilde{A}\widetilde{\boldsymbol{Z}}\right)^{(1-\sigma)\varphi}\right]^{\frac{1}{1+\varphi(1-\tilde{\alpha})+\sigma\varphi\tilde{\alpha}}} \\
\boldsymbol{W} &= \boldsymbol{\mu}\tilde{\alpha}\widetilde{A}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\tilde{\alpha}-1} \\
\widetilde{\boldsymbol{Y}} &= \boldsymbol{\Omega}\widetilde{A}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}} \\
\boldsymbol{C} &= \frac{s_C}{1-\gamma\left(1-\alpha\right)}\widetilde{\boldsymbol{Y}} \\
\boldsymbol{Y} &= \frac{1}{1-\left(1-\gamma\right)\alpha}\widetilde{\boldsymbol{Y}} \\
R &= \frac{1}{\beta}-\left(1-\delta\right) \\
K &= \frac{\left(1-\gamma\right)\alpha}{R}\boldsymbol{Y}
\end{aligned}$$

E.7 Reduced form labor supply elasticities

- We derive the expression linking reduced form and structural labor supply elasticities in Section 2.1
- Conisder the labor supply curve to the firm

$$n_{ijt} = \left(\frac{w_{ijt}}{\boldsymbol{w}_{jt}}\right)^{\eta}\left(\frac{\boldsymbol{w}_{jt}}{\boldsymbol{W}_t}\right)^{\theta}\boldsymbol{N}_t$$

- Consider a first order approximation around the Nash equilibrium following *any* change to firms in the market

$$\Delta\log n_{ijt} = \eta\Delta\log w_{ijt} + \left(\theta-\eta\right)\sum_{k\in j}\left.\frac{\partial\log\boldsymbol{w}_{jt}}{\partial\log w_{kjt}}\right|_{n^*_{-kjt}}\Delta\log w_{kjt}$$

$$\Delta\log n_{ijt} = \eta\Delta\log w_{ijt} + \left(\theta-\eta\right)\sum_{k\in j}\left.\frac{\partial\log\boldsymbol{w}_{jt}}{\partial\log w_{kjt}}\right|_{n^*_{-kjt}}\Delta\log w_{kjt}$$

- Consider the labor supply curve to the firm

$$\begin{aligned}
w_{ijt} &= \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{1}{\eta}}\left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{1}{\theta}}\boldsymbol{W}_t \\
\log w_{ijt} &= \frac{1}{\eta}\log n_{ijt} + \left(\frac{1}{\theta}-\frac{1}{\eta}\right)\log\boldsymbol{n}_{jt} + \text{Aggregates}
\end{aligned}$$

- Consider a first order approximation around the Nash equilibrium, denoted by asterisks, following *any* change to firms in the market

$$\Delta \log w_{ijt} = \frac{1}{\eta}\Delta \log n_{ijt} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) \sum_{k\in j} \frac{\partial \log \boldsymbol{n}_{jt}}{\partial \log n_{kjt}}\bigg|_{n^*_{-kjt}} \Delta \log n_{kjt}$$

$$\Delta \log w_{ijt} = \left(\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) \frac{\partial \log \boldsymbol{n}_{jt}}{\partial \log n_{ijt}}\bigg|_{n^*_{ijt}}\right) \Delta \log n_{ijt} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) \sum_{k\neq i} \frac{\partial \log \boldsymbol{n}_{jt}}{\partial \log n_{kjt}}\bigg|_{n^*_{-kjt}} \Delta \log n_{kjt}$$

$$\Delta \log w_{ijt} = \left(\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s^*_{ijt}\right) \Delta \log n_{ijt} + \left(\frac{\eta-\theta}{\theta\eta}\right) \sum_{k\neq i} s^*_{kjt} \Delta \log n_{kjt}$$

- The definition of the *reduced form labor supply elasticity* in the text is

$$\epsilon_{ijt} = \frac{\Delta \log n_{ijt}}{\Delta \log w_{ijt}}$$

- Using the above approximation:

$$\epsilon_{ijt} = \frac{\Delta \log n_{ijt}}{\left(\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s^*_{ijt}\right) \Delta \log n_{ijt} + \left(\frac{\eta-\theta}{\theta\eta}\right) \sum_{k\neq i} s^*_{kjt} \Delta \log n_{kjt}}$$

$$\epsilon_{ijt} = \frac{1}{\left(\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s^*_{ijt}\right) + \left(\frac{\eta-\theta}{\theta\eta}\right) \left\{\sum_{k\neq i} s^*_{kjt} \frac{\Delta \log n_{kjt}}{\Delta \log n_{ijt}}\right\}}$$

$$\epsilon_{ijt} = \frac{\left(\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s^*_{ijt}\right)^{-1}}{1 + \left(\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s^*_{ijt}\right)^{-1} \left(\frac{\eta-\theta}{\theta\eta}\right) \left\{\sum_{k\neq i} s^*_{kjt} \frac{\Delta \log n_{kjt}}{\Delta \log n_{ijt}}\right\}}$$

$$\epsilon_{ijt} = \left\langle \frac{1}{1 + \varepsilon^*_{ijt} \left(\frac{\eta-\theta}{\theta\eta}\right) \left\{\sum_{k\neq i} s^*_{kjt} \frac{\Delta \log n_{kjt}}{\Delta \log n_{ijt}}\right\}} \right\rangle \varepsilon^*_{ijt}$$

- This gives the expression in the text.

E.8 Pass-through expression

- We derive the pass-through expression that appears in Section 3.1
- Consider a firm's Nash equilibrium wage

$$w^*_{ijt} = \mu^*_{ijt} mrpl_{ijt}$$

- Here we have

$$mrpl_{ijt} = \widetilde{\alpha}\widetilde{z}_{ijt} n_{ijt}^{\widetilde{\alpha}-1}$$

$$mrpl_{ijt} = \widetilde{\alpha}\frac{\widetilde{y}_{ijt}}{n_{ijt}}$$

$$mrpl_{ijt} = \widetilde{\alpha}\left[1-(1-\gamma)\alpha\right]\frac{y_{ijt}}{n_{ijt}}$$

$$mrpl_{ijt} = \widetilde{\alpha}\left[1-(1-\gamma)\alpha\right] vapw_{ijt}$$

- Where $vapw_{ijt}$ is *value-added per worker*. Using this we have the equilibruim system

$$\log w^*_{ijt} = \log vapw_{ijt} + \log \mu^*_{ijt} + \text{Constant.} \quad \text{for all } i \in j$$

- In each labor market, the equilibrium markdown of a firm is a function of its share s_{ijt}, which we can write $s_{ijt} =$

$\left(w_{ijt}/\boldsymbol{w}_{jt}\right)^{\eta+1}$. Therefore we can write $\mu_{ijt} = \mu^*\left(w^*_{ijt}, w_{-ijt}\right)$ as a function of the firms wage and competitor wages.

- Consider a perturbation of any firm in the market's $vapw_{kjt}$, then to a first order around the Nash equilibrium:

$$d\log w_{ijt} = d\log vapw_{ijt} + \left.\frac{\partial \log \mu\left(w_{ij}, w^*_{-ijt}\right)}{\partial \log w_{ij}}\right|_{w^*_{ijt}} d\log w_{ijt} + \sum_{k\neq i} \left.\frac{\partial \log \mu\left(w_{ij}, w^*_{-kjt}\right)}{\partial \log w_{kj}}\right|_{w^*_{ijt}} d\log w_{kjt}$$

- Denote these elasticities m_{ii} and m_{ik}, then we have

$$d\log w_{ijt} = \frac{1}{1-m_{ii}} d\log vapw_{ijt} + \frac{1}{1-m_{ii}} \sum_{k\neq i} m_{ik} d\log w_{kjt}$$

- We can compute these elasticities, computing the following one by one:

$$\frac{\partial \log \mu_{ij}}{\partial \log w_{ij}} = \frac{\partial \log \mu_{ij}}{\partial \log \varepsilon_{ij}} \frac{\partial \log \varepsilon_{ij}}{\partial \log s_{ij}} \frac{\partial \log s_{ij}}{\partial \log w_{ij}}$$

1. The elasticity of the markdown with respect to the labor supply elasticity is

$$\frac{\partial \log \mu_{ij}}{\partial \log \varepsilon_{ij}} = \frac{\mu_{ij}}{\varepsilon_{ij}}$$

2. The elasticity of the elasticity with respect to the payroll share is

$$\frac{\partial \log \varepsilon_{ij}}{\partial \log s_{ij}} = -\left(\frac{\eta-\theta}{\theta\eta}\right) \varepsilon_{ij} s_{ij}$$

3. The elasticity of the payroll share with respect to the wage is

$$\frac{\partial \log s_{ij}}{\partial \log w_{ij}} = (1+\eta)\left(1-s_{ij}\right)$$

- Combined these give

$$\frac{\partial \log \mu_{ij}}{\partial \log w_{ij}} = -\mu_{ij}\left(\frac{\eta-\theta}{\theta\eta}\right)(1+\eta) s_{ij}\left(1-s_{ij}\right)$$

- We can also write the markdown only in terms of shares

$$\mu_{ij} = \frac{\theta\eta}{\left(1-s_{ij}\right)\theta + s_{ij}\eta + \theta\eta}$$

- Substituting this into the above:

$$m_{ii} = \frac{\partial \log \mu_{ij}}{\partial \log w_{ij}} = -\frac{(\eta-\theta)(1+\eta) s_{ij}\left(1-s_{ij}\right)}{\left(1-s_{ij}\right)\theta + s_{ij}\eta + \theta\eta}$$

- We can also compute the elasticity of the firms' markdown with respect to *competitor* wages. Proceeding as above

$$\begin{aligned} m_{ik} = \frac{\partial \log \mu_{ij}}{\partial \log w_{kj}} &= \frac{\partial \log \mu_{ij}}{\partial \log \varepsilon_{ij}} \frac{\partial \log \varepsilon_{ij}}{\partial \log s_{ij}} \left\langle \frac{\partial \log s_{ij}}{\partial \log w_{kj}} \right\rangle \\ \frac{\partial \log \mu_{ij}}{\partial \log w_{kj}} &= \frac{\mu_{ij}}{\varepsilon_{ij}} \left\{-\left(\frac{\eta-\theta}{\theta\eta}\right)\varepsilon_{ij}s_{ij}\right\} \left\langle -(1+\eta) s_{kj}\right\rangle \\ \frac{\partial \log \mu_{ij}}{\partial \log w_{kj}} &= \mu_{ij}\left(\frac{\eta-\theta}{\theta\eta}\right)(1+\eta) s_{ij} \left\langle s_{kj}\right\rangle \end{aligned}$$

- Then note that we have the following relationship between the two elasticities:

$$\sum_{k\neq i}\frac{\partial\log\mu_{ij}}{\partial\log w_{kj}}=\mu_{ij}\left(\frac{\eta-\theta}{\theta\eta}\right)(1+\eta)\,s_{ij}\left\langle\sum_{k\neq i}s_{kj}\right\rangle$$
$$=\mu_{ij}\left(\frac{\eta-\theta}{\theta\eta}\right)(1+\eta)\,s_{ij}\left\langle 1-s_{ij}\right\rangle$$
$$\sum_{k\neq i}m_{ik}=-m_{ii}$$

- Using this in the pass-through equilibrium expression:

$$d\log w_{ijt}=\frac{1}{1-m_{ii}}d\log vapw_{ijt}+\frac{1}{1-m_{ii}}\sum_{k\neq i}m_{ik}d\log w_{kjt}$$
$$d\log w_{ijt}=\frac{1}{1-m_{ii}}d\log vapw_{ijt}+\frac{\sum_{l\neq i}m_{il}}{1-m_{ii}}\sum_{k\neq i}\frac{m_{ik}}{\sum_{l\neq i}m_{il}}d\log w_{kjt}$$
$$d\log w_{ijt}=\frac{1}{1-m_{ii}}d\log vapw_{ijt}+\frac{-m_{ii}}{1-m_{ii}}\sum_{k\neq i}\frac{m_{ik}}{\sum_{l\neq i}m_{il}}d\log w_{kjt}$$

- Now note that

$$\frac{m_{ik}}{\sum_{l\neq i}m_{il}}=\frac{\mu_{ij}\left(\frac{\eta-\theta}{\theta\eta}\right)(1+\eta)\,s_{ij}s_{kj}}{\sum_{l\neq i}\mu_{ij}\left(\frac{\eta-\theta}{\theta\eta}\right)(1+\eta)\,s_{ij}s_{lj}}$$
$$\frac{m_{ik}}{\sum_{l\neq i}m_{il}}=\frac{s_{kj}}{\sum_{l\neq i}s_{lj}}=\frac{s_{kj}}{1-s_{ij}}$$

- Therefore we have

$$d\log w_{ijt}=\frac{1}{1-m_{ii}}d\log vapw_{ijt}+\frac{-m_{ii}}{1-m_{ii}}\sum_{k\neq i}\frac{s_{kj}}{1-s_{ij}}d\log w_{kjt}$$

- Now define $\Omega_{ii}=1/\left(1-m_{ii}\right)$. Using this:

$$d\log w_{ijt}=\Omega_{ii}d\log vapw_{ijt}+(1-\Omega_{ii})\sum_{k\neq i}\frac{s_{kj}}{1-s_{ij}}d\log w_{kjt}$$

- Using the expression for m_{ii} we can obtain an expression for Ω_{ii}:

$$m_{ii}=-\frac{(\eta-\theta)\,(1+\eta)\,s_{ij}\left(1-s_{ij}\right)}{\left(1-s_{ij}\right)\theta+s_{ij}\eta+\theta\eta}$$
$$\Omega_{ii}=\frac{s_{ij}\,(\eta-\theta)+\theta\,(\eta+1)}{\left[1+(1+\eta)\left(1-s_{ij}\right)\right]s_{ij}\,(\eta-\theta)+\theta\,(\eta+1)}$$

- This gives the expression in Section 3.1 of the main text.

E.9 Corporate taxes

- Consider a single firm i, and assumes constant returns to scale.
- Let the corporate tax rate be given by τ_C . Suppose that the firm can deduct some portion of its capital expenses λ_K. This corresponds to the fraction of capital expenses that are financed by long-term debt.

- Accounting profits of the firm, on which taxes are based, are given by

$$\pi_i^A = z_i k_i^{1-\gamma} n_i^\gamma - w_i n_i - \underbrace{\lambda_K R k_i}_{\text{Interest expense}}$$

- The economic profits of the firm are

$$\pi_i^E = z_i k_i^{1-\gamma} n_i^\gamma - w_i n_i - R k_i$$

- The after tax profits are given by

$$\pi_i = \pi_i^E - \tau_C \pi_i^A$$

- This gives, the following which reflects the idea that on net the firm pays corporate taxes on its total economic profits, and then is reimbursed for the taxes paid on capital financed by debt.

$$\pi_i = (1-\tau_C)\left[z_i k_i^{1-\gamma} n_i^\gamma - w_i n_i - R k_i\right] + \tau_C \lambda_K R k_i$$

- Dividing by $(1-\tau_C)$, the firm maximizes

$$\frac{\pi_i}{1-\tau_C} = z_i k_i^{1-\gamma} n_i^\gamma - w_i n_i - \underbrace{\left(\frac{1-\tau_C\lambda_K}{1-\tau_C}\right)}_{\text{Term is }>1} R k_i$$

- The effective rental rate of capital $\widetilde{R}(R,\tau_C,\lambda_K)$, that the firm faces is now *higher* than R due to the fact that not all of capital expenses can be deducted, while all of its labor expenses can. This causes the firm to take on a sub-optimal amount of capital. This lowers the marginal revenue product of other factors, including labor. If the firm could deduct *all* of its capital costs, then $\lambda_K = 1$, and the firm's input decisions are undistorted.
- Substituting in the firm's capital decision into their production function gives

$$\frac{\pi_i}{1-\tau_C} = \widetilde{z}(z_i,R,\tau_C,\lambda_K)\, n_i - w_i n_i$$

$$\widetilde{z}(z_i,R,\tau_C,\lambda_K) = \gamma z_i^{\frac{1}{\gamma}} \left(\frac{1-\gamma}{\widetilde{R}(R,\tau_C,\lambda_K)}\right)^{\frac{1-\gamma}{\gamma}} = \gamma z_i^{\frac{1}{\gamma}} \left(\frac{1-\tau_C}{1-\tau_C\lambda_K}\frac{1-\gamma}{R}\right)^{\frac{1-\gamma}{\gamma}}$$

- The marginal product of labor $\widetilde{z}_i$ is now *lower due to* the presence of corporate taxes and deductibility of interest payments on debt.

E.10 Mergers - Proposition 3.1

- In this section we prove the claims in Proposition 3.1. These are listed in a different order in Proposition 3.1, but here listed in the order that they are proved:
  1. Following a merger, the markdowns at the merged firms are equalized and depend on the total market share, $\mu_{1j} = \mu_{2j} = \mu\left(s_{1j} + s_{2j}\right)$.
  2. Under either monopsony limit a merger has no effect on any labor market variables.
  3. The individual shares s_{ij} of all non-merging firms increase. Therefore the total market share of merging firms falls.
  4. The wage index of non-merging firms <u>decreases</u> and employment index <u>increases</u>.
  5. Market wage $\boldsymbol{w}_j$ and employment $\boldsymbol{n}_j$ decline, so total market pay $\boldsymbol{w}_j\boldsymbol{n}_j = \sum_{i\in j} w_{ij}n_{ij}$ declines.
  6. The wages of both merging firms w_{1j} and w_{2j} decline. The wage index of merging firms <u>decreases</u> and employment index <u>decreases</u>.
- Parts 1 and 2 we prove under decreasing returns to scale. The remainder we establish under constant returns to scale. The proof of Part 3 is the most involved, and remaining parts follow from Part 3 in a straight-forward manner.

E.10.1 Markdowns

- Throughout we assume $M_j \geq 3$, and assign $i = 1$ and $i' = 2$ to the two merging firms.
- A merged firm chooses employment at both firms to maximize profits, where without loss of generality for this proof we can consider the case of a production function $f(\cdot)$ that already incorporates the (competitive) intermediate and capital choices

$$\max_{n_{1j}, n_{2j}} \left[f\left(z_{1j}, n_{1j}\right) - w\left(n_{1j}, \boldsymbol{n}_{-1j}\right) n_{1j} \right] + \left[f\left(z_{2j}, n_{2j}\right) - w\left(n_{2j}, \boldsymbol{n}_{-2j}\right) n_{2j} \right]$$

- When taking the first order condition, the firm understands that n_{2j} appears in $\boldsymbol{n}_{-1j}$ and vice versa.
- The first order condition for n_{1j} is as follows, where we use $mrpl_{1j} = f_n\left(z_{1j}, n_{1j}\right)$ to denote the marginal revenue produt of labor

$$\left(mrpl_{1j} - \frac{\partial w_{2j}}{\partial n_{1j}} n_{2j} \right) = \frac{\partial w_{1j}}{\partial n_{1j}} n_{1j} + w_{1j}$$

- Written this way we can see that in understanding that increasing n_{1j} increases the wage at firm 2, maps into an effective reduction in productivity at firm 1.
- Recall that

$$w_{1j} = n_{1j}^{\frac{1}{\eta}} \boldsymbol{n}_j^{\frac{1}{\theta} - \frac{1}{\eta}} \boldsymbol{X}$$
$$w_{2j} = n_{2j}^{\frac{1}{\eta}} \boldsymbol{n}_j^{\frac{1}{\theta} - \frac{1}{\eta}} \boldsymbol{X}$$

- Using this expression

$$mrpl_{1j} - w_{1j} = \left(\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta} \right) s_{1j} \right) w_{1j} + \left(\frac{1}{\theta} - \frac{1}{\eta} \right) s_{1j} \frac{w_{2j} n_{2j}}{n_{1j}}$$
$$mrpl_{1j} - w_{1j} = \left(\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta} \right) s_{1j} \right) w_{1j} + \left(\frac{1}{\theta} - \frac{1}{\eta} \right) s_{2j} w_{1j}$$
$$mrpl_{1j} - w_{1j} = \left[\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta} \right) \left(s_{1j} + s_{2j} \right) \right] w_{1j}$$

- Therefore $w_{1j} = \mu\left(s_{1j} + s_{2j}\right) mrpl_{1j}$. Note that the same algebra can be applied to firm 2. Therefore this establishes the first result:

$$\mu'_{1j} = \mu'_{2j} = \mu\left(s'_{1j} + s'_{2j}\right)$$

- Note that the above algebra generalizes in a straight-forward way to the case of an arbitrary set of firms merging. Let the set of merging firms be A, then

$$\mu'_{ij} = \mu\left(\boldsymbol{s}'_{jA}\right) \quad \boldsymbol{s}'_{jA} = \sum_{i \in A} s'_{ij}.$$

E.10.2 No effect of mergers in monopsony

- Consider the above problem of the merged firm in a monopsonistically competitive labor market

$$\max_{n_{1j}, n_{2j}} \left[f\left(z_{1j}, n_{1j}\right) - w\left(n_{1j}\right) n_{1j} \right] + \left[f\left(z_{2j}, n_{2j}\right) - w\left(n_{2j}\right) n_{2j} \right]$$

- Here the wage depends on $\boldsymbol{n}_j$ but since the firm is infintessimal, it does not internalize its effect on $\boldsymbol{n}_j$.
- The first order condition for firm 1 employment is:

$$mrpl_{1j} = w'\left(n_{1j}\right) n_{1j} + n_{1j}$$

- This is identical to the first order condition of firm 1 in the pre-merger economy. Therefore there is no effect at all on employment and wages.

E.10.3 Shares of all non-merging firms increase. Therefore the combined share of merging firms falls.

Groups - A useful concept is that of a grouping within a market. Split the firms in the market into those that merge $i \in A$, and those that don't merge $i \in B$.

- Define the group-level employment and wage indexes:

$$\boldsymbol{n}_{jG} = \left[\sum_{i \in G} n_{ij}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}} \quad , \quad \boldsymbol{w}_{jG} = \left[\sum_{i \in G} w_{ij}^{\eta+1}\right]^{\frac{1}{\eta+1}} \quad , \quad G \in \{A, B\}$$

- It is straight-forward to use these definitions to show that the market indices are

$$\boldsymbol{n}_j = \left[\boldsymbol{n}_{jA}^{\frac{\eta+1}{\eta}} + \boldsymbol{n}_{jB}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}} \quad , \quad \boldsymbol{w}_j = \left[\boldsymbol{w}_{jA}^{\eta+1} + \boldsymbol{w}_{jB}^{\eta+1}\right]^{\frac{1}{\eta+1}}$$

- These can then be used to derive *group level* supply curves and share relationships:

$$\boldsymbol{n}_{jG} = \left(\frac{\boldsymbol{w}_{jG}}{\boldsymbol{w}_j}\right)^{\eta} \boldsymbol{n}_j \ , \ \boldsymbol{w}_{jG}\boldsymbol{n}_{jG} = \sum_{i \in G} w_{ij}n_{ij} \ , \ \boldsymbol{s}_{jG} := \frac{\sum_{i \in G} w_{ij}n_{ij}}{\sum_{i \in j} w_{ij}n_{ij}} = \sum_{i \in G} s_{ij} = \left(\frac{\boldsymbol{w}_{jG}}{\boldsymbol{w}_j}\right)^{\eta+1} = \left(\frac{\boldsymbol{n}_{jG}}{\boldsymbol{n}_j}\right)^{\frac{\eta+1}{\eta}}$$

- For individual firms, then we can allocate labor relative to the group, and derive a *relative share* $\widetilde{s}_{iG}$ of group wages, which we can show is equal to overall market share divided by group market share.

$$n_{ij} = \left(\frac{w_{ij}}{\boldsymbol{w}_{jG}}\right)^{\eta} \boldsymbol{n}_{jG} \ , \ \widetilde{s}_{iG} := \frac{w_{ij}n_{ij}}{\sum_{i \in G} w_{ij}n_{ij}} = \left(\frac{w_{ij}}{\boldsymbol{w}_{jG}}\right)^{\eta+1} = \frac{s_{ij}}{\boldsymbol{s}_{jG}}$$

Lemmas - We can use these definitions to establish three Lemmas that will be useful in proving the remaining content of the proposition. Proofs for each Lemma is at the end of this appendix.

- **Lemma 1 -** *Consider some change in a market that* ***directly*** *effects some group of firms $i \in A$. Then the shares of* ***all other*** *firms $i \in B = \mathcal{I} \backslash A$, change in the same direction.* ***(Proof at the end of this appendix)***
- **Lemma 2** - *Merging firms* ***(Proof at the end of this appendix)***
  1. *In terms of wage changes:*$\Delta \log w_1 > \Delta \log w_2$
  2. *The relative share of the most productive of the merging firms increases* $\widetilde{s}'_{1A} > \widetilde{s}_{1A}$.
- **Lemma 3** - *For non-merging firms, if $s'_{ij} > s_{ij}$ then $n'_{ij} > n_{ij}$.* ***(Proof at the end of this appendix)***

We are now ready to prove the proposition: *The shares of all non-merging firms increase.*

- Applying **Lemma 1**, we know that the shares of non-merging firms either (i) *all decrease*, or (ii) *all increase*. We proceed by contradiction. **Suppose**: All non-merging firms' shares *decrease*: $s'_{ij} < s_{ij}$ for all $i \in B$.
  1. Since all non-merging firms' shares decrease then $\boldsymbol{s}'_{jB} < \boldsymbol{s}_{jB}$. Since $\boldsymbol{s}_{jA} + \boldsymbol{s}_{jB} = 1$, then the total share of merging firms increases: $\boldsymbol{s}'_{jA} > \boldsymbol{s}_{jA}$. From **Lemma 2.2** we know that the relative share of the most productive merging firm increases: $\widetilde{s}'_{1A} > \widetilde{s}_{1A}$. Since $s_{1j} = \widetilde{s}_{1A}\boldsymbol{s}_{jA}$, and $\boldsymbol{s}_{jA}$ increases, then $s'_{1j} > s_{1j}$ $(*)$. Since $\boldsymbol{s}'_{jA} > \boldsymbol{s}_{jA}$, then by definition

$$s'_{1j} + s'_{2j} > s_{1j} + s_{2j}$$

therefore

$$\mu\left(s'_{1j} + s'_{2j}\right) z_{1j} < \mu\left(s_{1j} + s_{2j}\right) z_{1j}$$
$$w'_{1j} < w_{1j} \quad (**)$$

Combined $(*)$ and $(**)$imply that firm 1's wage is falling, while its share is increasing. Since $s_{ij} = \left(w_{ij}/\boldsymbol{w}_j\right)^{1+\eta}$, this requires the market wage to be falling: $\boldsymbol{w}_j' < \boldsymbol{w}_j$ (#).

2. By our supposition, all non-merging firms shares decrease, $s_{ij}' < s_{ij}$, which since $w_{ij}' = \mu\left(s_{ij}'\right) z_{ij}$, implies that $w_{ij}' > w_{ij}$ for all non-merging firms. But since $s_{ij} = \left(w_{ij}/\boldsymbol{w}_j\right)^{1+\eta}$, then if $s_{ij}' < s_{ij}$ and $w_{ij}' > w_{ij}$, then it must be that $\boldsymbol{w}_j' > \boldsymbol{w}_j$ (##).

- **Contradiction**. The market wage can not be increasing (#) and decreasing (##).
- Therefore all non-merging firms' shares *increase.* It is then immediate that the *combined* share of the merging firms decrease: $\boldsymbol{s}_{jA}' < \boldsymbol{s}_{jA}$.

E.10.4 Wage index of non-merging firms $\boldsymbol{w}_{jB}$ decreases, and employment index $\boldsymbol{n}_{jB}$ increases

Consider a non-merging firm $i \in B$. Since z_{ij} is fixed, and by the above $s_{ij}' > s_{ij}$, then $\mu\left(s_{ij}'\right) < \mu\left(s_{ij}\right)$, so $w_{ij}' < w_{ij}$. Since $\boldsymbol{w}_{jB}^{1+\eta} = \sum_{i\in B} w_{ij}^{1+\eta}$, then the wage index of non-merging firms decreases: $\boldsymbol{w}_{jB}' < \boldsymbol{w}_{jB}$. From **Lemma 3**, since $s_{ij}' > s_{ij}$, then $n_{ij}' > n_{ij}$. Since $\boldsymbol{n}_{jB}^{(\eta+1)/\eta} = \sum_{i\in B} n_{ij}^{(\eta+1)/\eta}$, then $\boldsymbol{n}_{jB}' > \boldsymbol{n}_{jB}$.

E.10.5 Market wage $\boldsymbol{w}_j$ and market employment $\boldsymbol{n}_j$ both decrease

Since for non-merging firms their share is increasing $s_{ij}' > s_{ij}$ while their wages are falling $w_{ij}' < w_{ij}$, and $s_{ij} = \left(w_{ij}/\boldsymbol{w}_j\right)^{1+\eta}$, then it must be that the market wage is falling: $\boldsymbol{w}_j' > \boldsymbol{w}_j$. Since $\boldsymbol{w}_j' < \boldsymbol{w}_j$, then by market labor supply $\boldsymbol{n}_j' < \boldsymbol{n}_j$.

E.10.6 The wages of both merging firms w_{1j} and w_{2j} fall. The merging firms' index $\boldsymbol{w}_{jA}$ and employment index $\boldsymbol{n}_{jA}$ falls.

- From **Lemma 2.1**, we know that $\Delta \log w_1 > \Delta \log w_2$.
  - Suppose that $w_{2j}' > w_{2j}$. Then the above implies that $w_1' > w_1$. As $\boldsymbol{w}_j' < \boldsymbol{w}_j$, while the merging firms' wages are increasing then both merging firms' shares increase: $s_{ij} = \left(w_{ij}/\boldsymbol{w}_j\right)^{1+\eta}$. This would imply that $\boldsymbol{s}_{jA}' > \boldsymbol{s}_{jA}$. **Contradiction**. (Since we have already shown that the total share of merging firms decreases). Therefore $w_{2j}' < w_{2j}$.
  - Suppose that $w_{1j}' > w_{1j}$, this requires $\mu\left(s_1' + s_2'\right) > \mu\left(s_1\right)$, which requires that $s_1' + s_2' < s_1$. This requires $s_1' < s_1$. But we have shown that$\boldsymbol{w}_j' < \boldsymbol{w}_j$, so if $w_{1j}' > w_{1j}$, then $s_{1j}' > s_{1j}$. **Contradiction**. Therefore $w_{1j}' < w_{1j}$.
- Therefore $w_{1j}' < w_{1j}$ and $w_{2j}' < w_{2j}$. Since both firms' wages fall, then $\boldsymbol{w}_{jA}' < \boldsymbol{w}_{jA}$. Since the market employment index $\boldsymbol{n}_j' < \boldsymbol{n}_j$, but the employment index of non-merging firms increases $\boldsymbol{n}_{jB}' > \boldsymbol{n}_{jB}$, then it must be that $\boldsymbol{n}_{jA}' < \boldsymbol{n}_{jA}$.

E.10.7 Proofs of Lemmas

Lemma 1 - *Consider some change in a market that* ***directly*** *effects some group of firms $i \in A$. Then the shares of* ***all other*** *firms $i \in B = \mathcal{I}\backslash A$, change in the same direction.*

- **Proof**: Suppose not. Then there are two firms $i, k \in B$ such that $s_{ij}' > s_{ij}$ and $s_{kj}' < s_{kj}$.
- For firm i, since $s_{ij}' > s_{ij}$, then $\mu\left(s_{ij}'\right) < \mu\left(s_{ij}\right)$, so $w_{ij}' < w_{ij}$. From $s_{ij} = \left(w_{ij}/\boldsymbol{w}_j\right)^{1+\eta}$, the only way that $s_{ij}' > s_{ij}$ while $w_{ij}' < w_{ij}$ is if the market wage decreased: $\boldsymbol{w}_j' < \boldsymbol{w}_j$.
- For firm k, arguing the opposite implies $\boldsymbol{w}_j' > \boldsymbol{w}_j$. This is a contradiction: $\boldsymbol{w}_j$ can not have increased and decreased □.

Lemma 2 - *Merging firms* ***(Proof at the end of this appendix)***

1. *In terms of wage changes:* $\Delta \log w_1 > \Delta \log w_2$

   - Since both firms' productivity is constant and both have the same markdown post-merger:

$$\Delta \log w_{1j} - \Delta \log w_{2j} = \underbrace{\log \mu\left(s_{2j}\right) - \log \mu\left(s_{1j}\right) > 0}_{\text{Since } z_{1j} > z_{2j} \text{then } \mu\left(s_{1j}\right) < \mu\left(s_{2j}\right)}$$

2. *The relative share of the most productive of the merging firms increases* $\widetilde{s}'_{1A} > \widetilde{s}_{1A}$.

   - Since $\mu\left(s_1\right) < \mu\left(s_2\right)$, then

$$\frac{w'_1}{w_1} > \frac{w'_2}{w_2} \quad \Longrightarrow \quad \frac{w'_2}{w'_1} < \frac{w_2}{w_1}$$

   - Manipulating both sides

$$\frac{1}{1+\left(\frac{w'_2}{w'_1}\right)^{1+\eta}} > \frac{1}{1+\left(\frac{w_2}{w_1}\right)^{1+\eta}}$$
$$\frac{w_1'^{1+\eta}}{w_1'^{1+\eta}+w_2'^{1+\eta}} > \frac{w_1^{1+\eta}}{w_1^{1+\eta}+w_2^{1+\eta}}$$
$$\left(\frac{w'_1}{\boldsymbol{w}_A}\right)^{1+\eta} > \left(\frac{w_1}{\boldsymbol{w}_A}\right)^{1+\eta}$$
$$\widetilde{s}'_{1A} > \widetilde{s}_{1A}$$

Lemma 3 - *For non-merging firms, if* $s'_{ij} > s_{ij}$ *then* $n'_{ij} > n_{ij}$.

- **Proof**: Firm profit is

$$\pi_{ij} = z_{ij}n_{ij} - w_{ij}n_{ij} = z_{ij}n_{ij} - \left(n_{ij}^{\frac{1}{\eta}}\boldsymbol{n}_j^{\frac{1}{\theta}-\frac{1}{\eta}}X\right)n_{ij}$$

- First order condition for non-merging firms

$$z_{ij} - w_{ij} = \left(\frac{1}{\eta}n_{ij}^{\frac{1}{\eta}-1}\boldsymbol{n}_j^{\frac{1}{\theta}-\frac{1}{\eta}}X + \left(\frac{1}{\theta}-\frac{1}{\eta}\right)n_{ij}^{\frac{1}{\eta}}\boldsymbol{n}_j^{\frac{1}{\theta}-\frac{1}{\eta}-1}X\frac{\partial \boldsymbol{n}_j}{\partial n_{ij}}\right)n_{ij}$$
$$z_{ij} - w_{ij} = \frac{1}{\eta}w_{ij} - \left(\frac{1}{\theta}-\frac{1}{\eta}\right)n_{ij}^{\frac{1}{\eta}}\boldsymbol{n}_j^{\frac{1}{\theta}-\frac{1}{\eta}}X\left(\frac{\partial \boldsymbol{n}_j}{\partial n_{ij}}\frac{n_{ij}}{\boldsymbol{n}_j}\right)$$
$$z_{ij} - \left(\frac{\eta+1}{\eta}\right)w_{ij} = \left(\frac{1}{\theta}-\frac{1}{\eta}\right)\left(n_{ij}^{\frac{1}{\eta}}\boldsymbol{n}_j^{\frac{1}{\theta}-\frac{1}{\eta}}X\right)s_{ij}$$
$$\frac{\eta}{\eta+1}z_{ij} - w_{ij} = \frac{\eta}{\eta+1}\left(\frac{1}{\theta}-\frac{1}{\eta}\right)n_{ij}^{\frac{1}{\eta}}\boldsymbol{n}_j^{\frac{1}{\theta}-\frac{1}{\eta}}Xs_{ij}$$

- Now use the fact that $s_{ij} = \left(n_{ij}/\boldsymbol{n}_j\right)^{\frac{\eta+1}{\eta}}$, which implied that $\boldsymbol{n}_j = n_{ij}s_{ij}^{-\frac{\eta}{\eta+1}}$.

$$\frac{\eta}{\eta+1}z_{ij} - w_{ij} = \frac{\eta}{\eta+1}\left(\frac{1}{\theta}-\frac{1}{\eta}\right)n_{ij}^{\frac{1}{\eta}}\left(n_{ij}s_{ij}^{-\frac{\eta}{\eta+1}}\right)^{\frac{1}{\theta}-\frac{1}{\eta}}Xs_{ij}$$
$$\frac{\eta}{\eta+1}z_{ij} - w_{ij} = \frac{\eta}{\eta+1}\left(\frac{1}{\theta}-\frac{1}{\eta}\right)n_{ij}^{\frac{1}{\theta}}s_{ij}^{-\frac{\eta}{\eta+1}\left(\frac{1}{\theta}-\frac{1}{\eta}\right)+1}X$$
$$\left[\frac{\eta}{\eta+1}z_{ij} - w_{ij}\right]s_{ij}^{\frac{\eta}{\eta+1}\left(\frac{1}{\theta}-\frac{1}{\eta}\right)-1} = \frac{\eta}{\eta+1}\left(\frac{1}{\theta}-\frac{1}{\eta}\right)Xn_{ij}^{\frac{1}{\theta}}$$

- We can substitute in the wage given our closed form expression for $\mu\left(s_{ij}\right)$:

$$
\begin{aligned}
\left[\frac{\eta}{\eta+1}z_{ij} - \mu\left(s_{ij}\right)z_{ij}\right] s_{ij}^{\frac{\eta}{\eta+1}\left(\frac{\eta-\theta}{\theta\eta}\right)-1} &= \frac{\eta}{\eta+1}\left(\frac{\eta-\theta}{\theta\eta}\right) X n_{ij}^{\frac{1}{\theta}} \\
z_{ij}\left[\frac{\eta}{\eta+1} - \frac{\eta}{\eta+1}\frac{1}{1+\frac{\eta}{\eta+1}\left(\frac{\eta-\theta}{\theta\eta}\right)s_{ij}}\right] s_{ij}^{\frac{\eta}{\eta+1}\left(\frac{\eta-\theta}{\theta\eta}\right)-1} &= \frac{\eta}{\eta+1}\left(\frac{\eta-\theta}{\theta\eta}\right) X n_{ij}^{\frac{1}{\theta}} \\
z_{ij}\left[1 - \frac{1}{1+\frac{\eta}{\eta+1}\left(\frac{\eta-\theta}{\theta\eta}\right)s_{ij}}\right] s_{ij}^{\frac{\eta}{\eta+1}\left(\frac{\eta-\theta}{\theta\eta}\right)-1} &= \left(\frac{\eta-\theta}{\theta\eta}\right) X n_{ij}^{\frac{1}{\theta}} \\
z_{ij}\left[\frac{\frac{\eta}{\eta+1}\left(\frac{\eta-\theta}{\theta\eta}\right)s_{ij}}{1+\frac{\eta}{\eta+1}\left(\frac{\eta-\theta}{\theta\eta}\right)s_{ij}}\right] s_{ij}^{\frac{\eta}{\eta+1}\left(\frac{\eta-\theta}{\theta\eta}\right)-1} &= \left(\frac{\eta-\theta}{\theta\eta}\right) X n_{ij}^{\frac{1}{\theta}} \qquad (\#)
\end{aligned}
$$

- **Sufficient** - *If the LHS is increasing in s_{ij}, then the RHS is increasing in n_{ij}. Since we have already shown that non-merging firms' shares increase, then n_{ij} increases.*
- Note that $z_{ij} > 0$, and the remainder of the LHS takes the form of a function $f\left(s\right) = \frac{as}{1+as}s^{a-1}$, $a > 0$.
- Then

$$
f'\left(s\right) = \frac{as^{a-1}}{1+as}\left[\frac{1}{1+as} + \left(a-1\right)\right]
$$

- The first term is positive, and the second term implies that $f'\left(s\right) > 0$, if $s\left(1-a\right) < 1$.
- Sufficient conditions are $a > 0$, and $s \in [0,1]$. Since s_{ij} is a share, then $s_{ij} \in [0,1]$. And $a = \frac{\eta}{\eta+1}\left(\frac{\eta-\theta}{\theta\eta}\right) > 0$, since $\eta > \theta$.

F Estimation details and bias exercise

F.1 Distribution of firms across markets

We assume there are 5,000 markets. For computational reasons, we must cap the number of firms per market since the Pareto distribution has a fat tail. We set the cap equal to 200 firms per market. Our results are not sensitive to the number of markets or the cap on firms per market.

Tradeable firm distribution. Figure F1 (left) plots the distribution from which we draw the number of firms per market, M_j. The distribution is a mixture of a discrete mass point at $M_j = 1$ and a Pareto distribution over the support $M_j \in [2, \infty]$. The Pareto's shape, scale, and location parameters are set to minimize the distance with the first three moments of the tradeable firm distribution. The parameters, data moments, and simulated moments are in Table F1.

Economy-wide firm distribution. Figure F1 (right) plots the economy-wide distribution from which we draw the number of firms per market, M_j. The distribution is a mixture of a discrete mass point at $M_j = 1$ and a Pareto distribution over the support $M_j \in [2, \infty]$. The Pareto's shape, scale, and location parameters are set to minimize the distance with the first three moments of the economy-wide firm distribution. The parameters, data moments, and simulated moments are in Table F2.

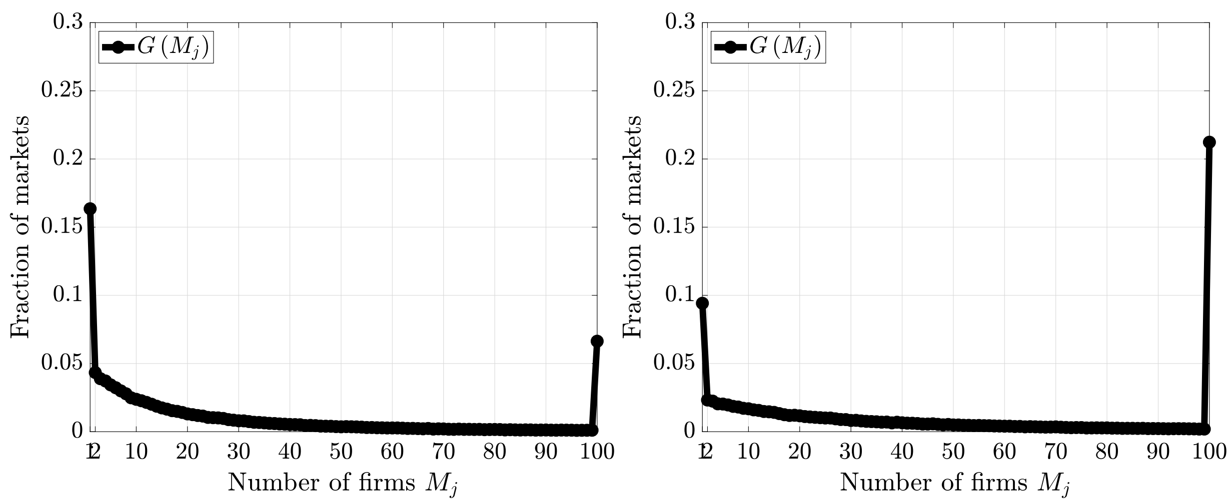

Figure F1: Distribution of the number of firms across sectors. Left: Tradeable industries, Right: all industries

Notes: Parameters given in Table F1 for tradeable and Table F2 for all industries.

A. Moments Distribution of firms M_j	Mean	Std. Dev	Skewnewss
Data (LBD 2014)	33.86	102.90	10.44
Model	33.80	102.94	22.90
B. Parameters **Mass at $M_j = 1$**	**Pareto Tail**	**Pareto Scale**	**Pareto Location**
0.16	0.52	18.74	2.00

Table F1: Distribution of firms across markets, $M_j \sim G(M_j)$, tradeable industries

A. Moments Distribution of firms M_j	Mean	Std. Dev	Skewnewss
Data (LBD 2014)	113.10	619.00	26.14
Model	113.14	618.82	36.08
B. Parameters Mass at $M_j = 1$	Pareto Tail	Pareto Scale	Pareto Location
0.09	0.71	38.36	2.00

Table F2: Distribution of firms across markets, $M_j \sim G(M_j)$, all industries

F.2 Tax Experiment Details

In each simulation of the model, we conduct a tax experiment where we simulate a common corporate tax change of $\Delta_\tau = \tau_C' - \tau_C = .01$, holding aggregate quantities fixed. We rerun our reduced-form regressions on the simulated data in order to recover average reduced-form labor supply elasticities as a function of wage-bill shares. These market-share-dependent reduced-form labor supply elasticities are the moments used to recover η and θ in Section 2. We describe the details of the exercise below:

1. Simulate the benchmark equilibrium for two periods (date $t = 0, 1$) without taxes. Treat these observations as 'data.' We must simulate two prior periods in order to define the lagged wage-bill share of the firm in the market.
2. C-corps in the model economy (recall there is a share ω_C of C-corps in all markets) have their taxes raised by 1 percentage point.
3. Simulate the 'post-shock' equilibrium, treat as date $t = 2$ 'data.'
4. Estimate the same reduced-form regressions as Section 2 using the $t = 0, 1, 2$ simulated data. Estimate the following regressions for each firm i in region j:

$$\log(n_{ijt}) = \alpha_i + \beta_n \tau_{Ct} + \gamma_{0s} s_{ijt-1} + \beta_{ns} \tau_{Ct} * s_{ijt-1} + \epsilon_{ijt}$$
$$\log(w_{ijt}) = \alpha_i + \beta_w \tau_{Ct} + \omega_{0s} s_{ijt-1} + \beta_{ws} \tau_{Ct} * s_{ijt-1} + u_{ijt}$$

5. Compute the employment and wage elasticities with respect to productivity, $\frac{d\log(n_{ijt})}{d\tau_{Ct}}$ and $\frac{d\log(w_{ijt})}{d\tau_{Ct}}$. Use these expressions to recover the average reduced-form labor supply elasticities using the formula:

$$\widehat{\epsilon}(s_{ijt-1}) = \frac{\beta_n + \beta_{ns} s_{ijt-1}}{\beta_w + \beta_{ws} s_{ijt-1}}$$

6. Use the recovered $\{\widehat{\epsilon}(s_{ijt-1}), s_{ijt-1}\}$ pairs as moments to recover η and θ.

F.3 Biases

To explore the difference between structural and reduced-form labor supply elasticities, we conduct a Monte Carlo exercise where we simulate a perfectly idiosyncratic shock and then compute reduced form elasticities. We average these across firms within payroll share bins and compare these to the structural labor supply elasticity implied by $\varepsilon(s_{ij}) = [\theta^{-1} s_{ij} + \eta^{-1}(1 - s_{ij})]^{-1}$. We repeat this exercise for 5,000 simulations and report the averages in Figure 6. We describe the details of the exercise below:

1. Simulate the benchmark equilibrium, treat as date $t = 1$ 'data.'
2. Randomly select 1 firms in each market and increase their productivity by 1% (20% or 50%), holding aggregates fixed (assuming partial equilibrium).
3. Simulate 'post-shock' partial equilibrium (industry competitors adjust but aggregates are held fixed), treat as date $t = 2$ 'data.'

4. Isolate firms with pre-shock wage-bill shares s_{ij} in bins with nodes $[.1, \ldots, .9]$. Within each bin, compute the mean of $\Delta \log(n_{ijt})/\Delta \log(w_{ijt})$ using the $t = 1, 2$ simulated data.

5. Figure 6 plots these values at the upper cutoff of these bins. For shares equal to 0 and 1, the solution is exact $\varepsilon(1) = \widehat{\varepsilon}(1) = \theta$, $\varepsilon(0) = \widehat{\varepsilon}(0) = \eta$.

F.4 Additional threats to consistency.

There are two additional threats to consistency of our simulations. (i) apportionment of state taxes across multi-state production units may mean that state corporate taxes do not affect firms within a state, and (ii) anticipation of tax changes. We discuss these issues in the context of prior analysis by Suárez Serrato and Zidar (2016) and Giroud and Rauh (2019).

First, Suárez Serrato and Zidar (2016) show that the impact of state corporate taxes on local economic activity is extremely similar for both (i) the statutory corporate taxes used here, and (ii) effective corporate taxes—i.e. 'business taxes'—carefully adjusted for apportionment weights.[52] Since establishment sales and company property values are not available to us, we cannot construct accurate apportionment weights and thus we focus on statutory tax rates compiled by Giroud and Rauh (2019). We only require similarly sized firms to face similarly sized shocks. The magnitude of the shock is not important for our identification of η and θ, instead it is their relative employment to wage adjustment that identifies η and θ.

Second, both Suárez Serrato and Zidar (2016) and Giroud and Rauh (2019) establish that including various aspects of changes to fiscal policy around corporate tax adjustments have negligible affects on their measured elasticities of local economic activity to state corporate taxes.[53] We interpret this as indirect evidence that the reforms are not paired with other predictable components of fiscal stimulus, such as unemployment insurance, which follow time-invariant threshold rules and are typically triggered in recessions (e.g. Mitman and Rabinovich (2019)).

G Discussion of empirical estimation and robustness

This section is divided into three parts. First, a discussion of how our empirical strategy relates to other papers in the literature. Second, including exit in our regressions and re-estimating the model under exit. Third, a set of robustness exercises around state-level omitted variables, non-wage compensation, variation in capital intensity, and an alternative approach using 'direct' elasticities.

G.1 Discussion

As discussed in Section 1.4, the model predicts that the labor supply elasticity faced by firms varies by their market share. If this relationship were known in the data, it would precisely pin down the elasticities of substitution of labor within and across sectors. Existing work estimating labor supply elasticities to firms has focused either on specific markets (e.g. (Webber, 2016) or in well identified responses to small experimental variations in wages (Dube, Jacobs, Naidu, and Suri, 2020; Dube, Cengiz, Lindner, and Zipperer, 2019). A contribution of this paper is to estimate a share-elasticity relationship through a novel quasi-natural experiment using a large cross-section of firms.

The intuition for our procedure is as follows. We first estimate the rate at which labor demand shocks *pass-through* to wages and employment and the reduced-form relationship between these labor supply elasticities and local labor market shares. We then invert this empirical relationship using our model to recover estimates of the structural parameters that control the relative substitutability of labor within and between markets. To identify how pass-through rates vary by market share, we compare

[52]See their discussion of Table A21, p.19 (emphasis added): "Column (6) of Table 5 and Appendix Table A21 show that using statutory state corporate tax rates in Equation 21 (instead of business tax rates τ_b) results in similar and significant estimates, indicating that *our measure of business tax rates is not crucial for the results*."

[53]Giroud and Rauh (2019) establish the plausible exogeneity of state-corporate tax changes. From a public finance perspective they study the effects of state corporate tax changes on employment and wages. Their focus is *within firm*, *across state* responses, and the reallocation of firm employment across states following tax changes. For an exhaustive description of these tax changes we point the interested reader to their paper.

how the firm responds to these labor demand shocks differentially across markets within the same state, but in which their shares of the labor market differ.

This procedure requires a shock to labor demand in order to trace out the labor supply curve. We use state corporate tax changes which constitute a shock to firm labor demand via their distortion of accounting profits relative to economic profits, shifting the marginal revenue product of labor.[54] Both Suárez Serrato and Zidar (2016) and Giroud and Rauh (2019) have studied the impact of state-level corporate tax shocks on local economic activity. We address three issues that may arise: (i) apportionment of state taxes across multi-state production units may mean that state corporate taxes do not affect firms within a state, (ii) taxes are anticipated, (iii) such shocks affect all firms in a region and so can only be used to identify θ.

First, Suárez Serrato and Zidar (2016) show that the impact of corporate taxes on local economic activity is extremely similar for both (i) the statutory corporate taxes that we use and (ii) effective corporate taxes adjusted for apportionment weights.[55] Since establishment sales and company property values are not available to us, we focus on statutory taxes rates compiled by Giroud and Rauh (2019) and based on Suárez Serrato and Zidar (2016) we do not adjust for the apportionment regime of the state.

Second, both Suárez Serrato and Zidar (2016) and Giroud and Rauh (2019) establish that the inclusion of other aspects of changes to fiscal policy around the corporate tax changes does not affect their measured elasticities of local economic activity to corporate taxes.[56]

Third, the fact that (i) only C-corps pay statutory corporate tax rates, (ii) the structure of our model and (iii) Monte Carlo exercises, provide support that we may infer η and θ from a shock that affects some but not all firms. We briefly discuss this in more detail.

G.2 Exits

G.2.1 Empirics

In Table G1, we estimate linear probability models of firm-market exit in year $t+1$ as a function of corporate taxes in year t. In column (1) and (2), we find economically insignificant results. This complements the work of Giroud and Rauh (2019), who aggregate plants at the firm-state level and study how the number of plants per C-Corp in a state responds to corporate tax changes. Since our relevant level of economic activity is at the firm-market level, and since we are interested in exits from the market entirely, we use a different approach. We regress whether a firm exits a firm-market entirely, instead of simply regressing the number of plants in the state on the tax change. Our results do not necessary contradict Giroud and Rauh (2019). Giroud and Rauh (2019)'s results imply that firms may adjust the number of plants in the state. Our results imply that firms do not appear to be exiting markets *entirely* in response to a corporate tax change.

G.2.2 Model re-estimation with exit

Our empirical results suggest that exits are not a threat to our exercise. Nonetheless, we show that our model estimates of η and θ are robust under the assumption that 5% of C-Corps exit. This is an extreme and counterfactually high exit response to corporate tax hikes. Table G2 reports the results. Our estimates of η and θ are similar to the baseline.

[54] We have not included corporate taxes in our benchmark model. We show that the mapping of our model to the data does not require us to take a stance on the transmission mechanism linking corporate taxes to productivity. Nevertheless, Appendix E.9 shows how corporate tax rates map to shocks to the marginal revenue productivity of labor in our framework.

[55] See their discussion of Table A21, p.19 (emphasis added): "Column (6) of Table 5 and Appendix Table A21 show that using statutory state corporate tax rates in Equation 21 (instead of business tax rates τ_b) results in similar and significant estimates, indicating that *our measure of business tax rates is not crucial for the results*."

[56] Giroud and Rauh (2019) establish the plausible exogeneity of state-corporate tax changes. From a public finance perspective they study the effects of state corporate tax changes on employment and wages. Their focus is *within firm*, *across state* responses, and the reallocation of firm employment across states following tax changes. For an exhaustive description of these tax changes we point the interested reader to their paper.

	(1) Exit$_{ijt+1}$	(2) Exit$_{ijt+1}$
$\tau_{s(j)t}$	-0.000447	-0.000938
	(0.000556)	(0.000771)
	[0.000168]	[0.000194]
Fixed Effects	Market, Year	Firmid, Market, Year
R-squared	0.042	0.218
Observations	2.844e+06	2.844e+06

Table G1: Exit probability

Notes: According to Census requirements, the number of observations is rounded to the nearest 1,000. Standard errors in round parentheses $(\cdot)$ are clustered at State $\times$ Year level. Standard errors in square parentheses $[\cdot]$ are clustered at Market $\times$ Year level. Sample includes tradeable *C*-Corps from 1977 to 2011.

	η	θ
Benchmark	6.96	0.45
Exit rate 5%	8.17	0.43

Table G2: Re-estimation of the model assuming 5% of firms exit the market in response to a corporate tax hike of 1%.

G.3 Regression Robustness

We discuss robustness of our regression specifications and their implications for the relationship between market shares and reduced form labor supply elasticities. We consider (i) state-level omitted variables, (ii) compute direct elasticities at the firm level, (iii) account for systematic variation in non-wage compensation, (iv) account for systematic variation in capital intensity.

G.3.1 State-level omitted variables

Model estimation simply requires *consistent* auxiliary moments that can be simulated. The threat to *consistency* when we estimate equation 13 is that there are other forces moving employment and wages at the state-year level, e.g. tax cuts occur in boom years etc. To control for state-level responses, Giroud and Rauh (2019) include S-Corps as a control for *C*-Corps. Through the lens of our theory, S-Corps do not provide a suitable control group, since they respond to the treatment as well. Thus, the stable unit treatment value assumption (SUTVA) is violated. To alleviate concerns that our estimates are being driven by omitted state-year level variation, we include specifications which include both state$\times$year fixed effects as well as firmid$\times$market fixed effects. State-level corporate tax changes are subsumed in the fixed effects, and so we are only able to identify the interaction between corporate taxes and wage-bill shares. Table G3 illustrates our results. Comparing columns (1) through (4) in Table G3 to Table 1, we find very similar interactions between taxes and wage-bill shares for both date t and $t+1$ employment and wages. We view these results as suggestive evidence that omitted variables at the state-year level are unlikely to explain our results.

G.3.2 Direct elasticities

To provide further evidence that labor supply elasticities decline as a function of a firm's wage-bill share, we directly compute the ratio of wage changes to employment changes at the firm-level and we study their relationship to a firm's wage-bill share. To allow for perfect competition (non-zero employment change with zero wage change), we compute the inverse elasticity at the firm level $\frac{\Delta w_{ijt}}{\Delta n_{ijt}} \frac{n_{ijt}}{w_{ijt}}$ between year t and $t+1$. To measure an elasticity, we require a supply or demand shifter. We use corporate tax changes as demand shifters. These 'direct' elasticities include significant amounts of measurement error. In particular, the measurement error in the denominator results in many extreme outliers. We impose several criteria to deal with

	(1)	(2)	(3)	(4)
	$\log n_{ijt}$	$\log w_{ijt}$	$\log n_{ijt+1}$	$\log w_{ijt+1}$
s_{ijt-1}	0.894***	0.113***	0.723***	0.0959***
	(0.0206)	(0.00982)	(0.0202)	(0.0107)
	[0.0159]	[0.00898]	[0.0165]	[0.0100]
$\tau_{s(j)t} \times s_{ijt-1}$	0.0169***	0.00739***	0.0159***	0.00760***
	(0.00275)	(0.00127)	(0.00268)	(0.00133)
	[0.00199]	[0.00112]	[0.00208]	[0.00125]
Fixed effects	Y	Y	Y	Y
R-squared	0.897	0.791	0.876	0.736
Round N	2.844e+06	2.844e+06	2.844e+06	2.844e+06

Table G3: State-year fixed effects

Notes: All specifications include fixed effects for: (i) year, (ii) firmid×market, and (iii) state×year. According to Census requirements, the number of observations is rounded to the nearest 1,000.

this measurement error. First, we require an employment adjustment of at least ±5 employees.[57] Second, we only use tax changes of at least half of a percentage point $|\Delta\tau_{s(j)t}| > .5$. Third, we winsorize the dependent variable at the .5% level.[58] Fourth, to remove common state-year fluctuations in wages and employment, we include state-year fixed effects as well as firm-market fixed effects.

To isolate the size-dependent labor supply elasticity, we interact the corporate tax changes with the firm's wage-bill share. Because of the high-dimensional firm must adjust employment twice by at least ±5 employees between 1977 to 2011 in order for our fixed effects to be estimated. We run specifications of the following form:

$$[\epsilon^{Data}(s)]^{-1} = \frac{\Delta w_{ijt}}{\Delta n_{ijt}}\frac{n_{ijt}}{w_{ijt}} = \alpha_{ij} + \gamma_{s(j)t} + \beta_1 \mathbf{1}(s_{ijt-1} \in [.01, .05)) + \beta_2 \mathbf{1}(s_{ijt-1} \in [.05, 1]) \tag{G1}$$

$$+ \beta_3 \mathbf{1}(|\Delta\tau_{s(j)t}| > .5) \times \mathbf{1}(s_{ijt-1} \in [.01, .05)) + \beta_4 \mathbf{1}(|\Delta\tau_{s(j)t}| > .5) \times \mathbf{1}(s_{ijt-1} \in [.05, 1]) + \epsilon_{ijt}$$

Table H4 provides estimates of equation (H1). Column (1) shows that the inverse elasticity for firms with wage bill shares between 1% and 5% is significantly different from those whose wage-bill shares are less than 1%. Their inverse elasticity is .0476 percentage points greater. Based on our estimates in Table 2, if firms with a wage-bill share less than 1% have a labor supply elasticity of 2, then these estimates imply that firms with a wage bill share between 1% and 5% have a labor supply elasticity of roughly 1.83. For those with wage-bill shares greater than 5%, their inverse elasticity increases by 0.075 percentage points relative to those with a wage-bill share less than 1%. Based on our estimates in Table 2, if firms with a wage-bill share less than 1% have a labor supply elasticity of 2, then these estimates imply that firms with a wage bill share greater than 5% have a labor supply elasticity of roughly 1.74. Lastly, column (2) estimates the inverse labor supply elasticity using changes in employment and wages between t and $t+2$, while keeping the same and all other right-hand-side regressors in equation (H1) the same. We interpret column (2) as a long-run inverse labor supply elasticity. Relative to the omitted group of firms with shares less than 1%, we find that firms with wage-bill shares greater than 5% have an inverse elasticity that is 0.0959 percentage points greater. Based on our estimates in Table 2, if firms with a wage-bill share less than 1% have a labor supply elasticity of 2, then these estimates imply that firms with a wage bill share greater than 5% have a labor supply elasticity of roughly 1.68. These estimates are remarkably close to our linear regression estimates in Section 2.2. Figure H4 graphically depicts the inverse elasticity $[\hat{\epsilon}^{Data}(s)]^{-1} = \frac{\Delta w_{ijt}}{\Delta n_{ijt}}\frac{n_{ijt}}{w_{ijt}}$ relative to the omitted group of firms with wage-bill shares less than 1%.

[57] We also tried cutoffs of $\{3, 7, 10\}$ and our results are robust.

[58] We also tried winsorizing at the 1% and 5% levels and our results are robust.

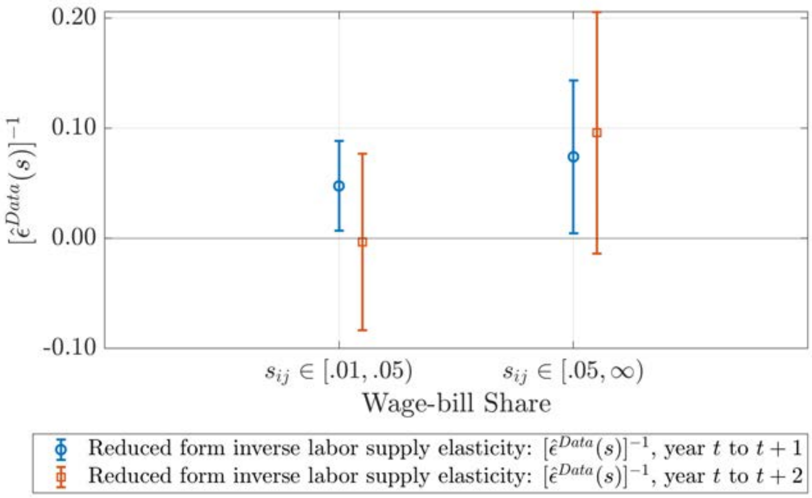

Figure G1: Inverse elasticities

Notes: Point estimates plotted from Table H4. See text for details.

G.3.3 Non-wage compensation

Can our results be attributed to non-wage benefits that vary by size? We argue no. We find that while benefits covary positively with wage-bill share, they cannot explain the magnitude of size-dependent markdowns we estimate in the data.

To bound the effect of benefits on our markdowns, we must measure the elasticity of benefits with respect to the wage-bill share of the firm. The Census of Manufacturers includes data on worker benefits in recent survey waves. We use the 2012 Census of Manufacturers to estimate how benefits per employee varies with local wage-bill shares. To mitigate spurious correlations between market-share and benefits, we include firm fixed effects. Thus our empirics compare within a firm, across plants, how local wage-bill shares covary with benefits. Table H3 includes our results. We find that a 1 percentage point increase in the wage bill share results in a 0.597% increase in benefits per worker. We repeat the exercise after winsorizing benefits per worker at the 1% level, and we find a very similar elasticity; thus, our low elasticity of benefit per worker with wage-bill share is not driven by outliers.

Figure H3 plots the elasticity of the model's markdowns as a function of the wage-bill share. For wage-bill shares of 1%, the elasticity of the wage-bill share is close to -6%. Thus non-wage benefits are too small to be responsible for our estimated markdown elasticities.

G.3.4 Capital Intensity

The Census of Manufacturers includes data on assets per employee.[59] We use the 2012 Census of Manufacturers to estimate how assets per employee varies with local wage-bill shares. We include firm (firmid) fixed effects to isolate within-firm, across-plant variation in the way assets per employee covaries with local wage-bill shares. Table H3 includes our results. We find that a 1 percentage point increase in the wage bill share results in a 0.176% increase in assets per employee. Our mean value of assets per employee in this sample of multi-plant firms is $332,500. Thus a 1 percentage point increase in wage-bill share corresponds to an increase in assets of $595 for the average firm. We view this as economically insignificant. Column (2) removes outliers by winsorizing the data at the 1% level. We find a slightly larger elasticity, however, we view these results as supportive evidence that our size-dependent labor supply elasticities cannot be explained by differential capital adjustment.

[59]We use beginning-of-year asset values.

	(1) Inverse elasticity t to $t+1$	**(2)** Inverse elasticity t to $t+2$
$1(\lvert\Delta\tau_{s(j)t}\rvert > .5) \times s_{ijt-1} \in [.01, .05)$	0.0476**	-0.00314
	(0.0208)	(0.0409)
	[0.0221]	[0.0421]
$1(\lvert\Delta\tau_{s(j)t}\rvert > .5) \times s_{ijt-1} \in [.05, 1]$	0.0741**	0.0959*
	(0.0354)	(0.0560)
	[0.0300]	[0.0517]
Fixed Effects	Yes	Yes
R-squared	0.257	0.225
N	722000	722000

Table G4: Estimation results for equation (H1)

Notes: All specifications include fixed effects for: (i) year, (ii) firmid×market, and (iii) state×year. According to Census requirements, the number of observations is rounded to the nearest 1,000. Standard errors in round parentheses $(\cdot)$ are clustered at State × Year level. Standard errors in square parentheses $[\cdot]$ are clustered at Market × Year level. Sample includes tradeable *C*-Corps from 1977 to 2011.

	(1) Log benefits per employee$_{ijt}$	(2) Log benefits per employee$_{ijt}$, 1% winsorized
s_{ijt}^{wn}	0.00597***	0.00565***
	(0.000261)	(0.000241)
R-squared	0.687	0.688
N	36000	36000

Table G5: Benefits per employee and wage-bill shares

Notes: All specifications include fixed effects for: (i) year, (ii) firmid , and (iii) market. According to Census requirements, the number of observations is rounded to the nearest 1,000. Standard errors clustered at the firmid level. Sample includes tradeable Census of Manufacturers firms in 2012.

	(1) Log assets per employee$_{ijt}$	(2) Log assets per employee$_{ijt}$, 1% winsorized
s_{ijt}^{wn}	0.00176**	0.00193***
	(0.000749)	(0.000693)
R-squared	0.639	0.638
Observations	36000	36000

Table G6: Assets per employee and wage-bill shares

Notes: All specifications include fixed effects for: (i) year, (ii) firmid , and (iii) market. According to Census requirements, the number of observations is rounded to the nearest 1,000. Standard errors clustered at the firmid level. Sample includes tradeable Census of Manufacturers firms in 2012.

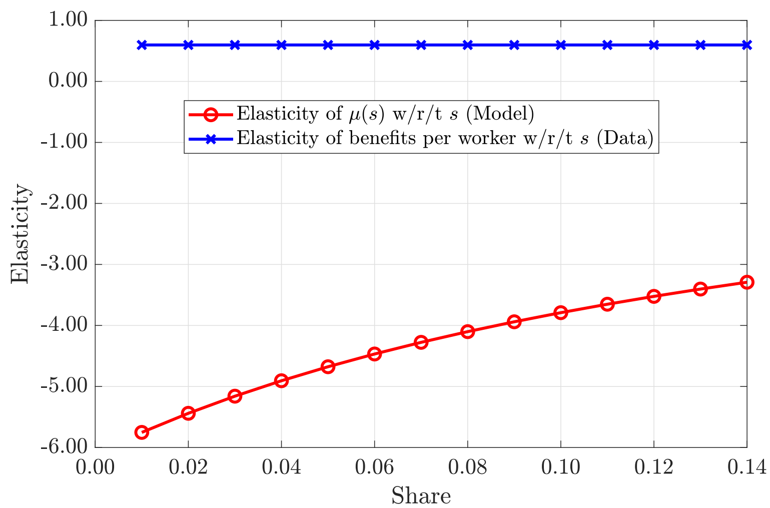

Figure G2: Non-wage benefits and wage-bill shares