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ABSTRACT

We consider an economy in which investors believe dividend growth is predictable, when in reality it is not. We show that these beliefs lead to excess volatility and return predictability. We also show that these beliefs are rational in the face of evidence on dividend growth. We apply this framework to explaining the value premium, predictability of bond returns, and the violation of uncovered interest rate parity.

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1 Introduction

Why is aggregate stock price volatility so high? Starting with Shiller (1981) and Campbell and Shiller (1988), an influential literature shows that stock market volatility is too large to arise from rational expectations of future dividends. In response, the literature has proposed several explanations that maintain the notion of the rational investor. The "excess" volatility could arise from time-varying discount rates, which could in turn be driven by time-varying volatility of dividends (Bansal and Yaron, 2004; Calvet and Fisher, 2007; Lettau et al., 2008), or time-varying risk aversion Campbell and Cochrane (1999). Stock price volatility could also arise from time-varying forecasts of the occurrence or impact of rare events (Gabaix, 2012; Wachter, 2013).

These rational expectation models have considerable appeal. They hold out the promise that asset pricing puzzles can be solved in a world that is complicated and unpredictable, yet well-understood by investors. The careful development of these models have led to specific tests, which have not always worked in the models' favor. Problems include counterfactual predictions for the term structure of dividend claims (Binsbergen et al., 2012; Lettau and Wachter, 2007), interest rates (Backus et al., 2014) and variance risk (Dew-Becker et al., 2017). Some models do not extend well to economies where agents in aggregate can transfer resources across states and time (Lettau and Uhlig, 2000; Kaltenbrunner and Lochstoer, 2010). Finally, because these models are rational, risk premia must ultimately represent a return for bearing risk. Empirical studies have looked for this relation and failed to find it.(Duffee, 2005; Moreira and Muir, 2017).[1]

This paper proposes a model for stock return volatility that does not assume rational expectations. This is not the same as assuming investors are irrational, it simply

[1]Models with time-varying rare events would seem to hold out the best hope, among rational models, for disentangling the relation between risk and return. However, requiring infinitely precise knowledge of a difficult-to-measure time-varying quantity does not seem like a victory for rational expectations.

means that investors have a biased prior on the data generating process. Inspired by the literature on behavioral finance (Barberis et al., 2003; Shiller, 2003; Hirshleifer, 2015), we motivate beliefs based on psychological studies. We take as motivation the classic animal learning study of Skinner (1948). In Skinner's study, hungry pigeons were presented food at regular intervals. Most of the pigeons developed bizarre habits of behavior, the reason for which is that they happened to have displayed that specific behavior when the food was offered.

What do these pigeons have to do with investors? While the pigeons' associations between behavior and food may seem ridiculous, their behavior illustrates a tendency to create structure out of randomness. The strong tendency to find structure where none exists characterizes human subjects as well, both in the laboratory and real-world situations (Bar-Hillel and Wagenaar, 1991). It persists even when subjects are trained to know what is random and what is not (Neuringer, 1986).

In our base case, we assume (for simplicity) that investors are risk-neutral. They believe they can forecast dividend growth using a persistent signal, though dividend growth is in fact iid. Like the pigeons they believe events can be forecasted (dividend growth, as opposed to food) even when they are completely random. We show that this condition itself is sufficient to generate excess volatility and return predictability seen in the data. Prices embed the incorrect beliefs about dividend growth, and thus are excessively volatile. Moreover, prices revert to more correct values as the expected growth fails to materialize, generating excess returns that appear to vary over time. However, in this risk-neutral environment, true risk premia are always equal to zero.

A slightly generalized model can produce an unconditional equity premium assuming investors have time-additive CRRA utility and rare disasters that occur with constant probability(Barro, 2006; Rietz, 1988). In such a model, there is no time series relation between risk and return. Moreover, time-additive CRRA utility implies flat term unconditional term structures of equity and interest rates, rather than a counterfactual upward-sloping term structure of equities and a downward-sloping term structure of interest rates.

Finally, we extend the model to address other asset pricing puzzles. A longstanding puzzle is the high abnormal returns on value stocks (Fama and French, 1992). We show that the same mechanism that explains excess volatility in the time series can explain this abnormal performance. We show that a belief in an excessive amount of interest rate predictability can explain the ability of the yield spread to forecast excess returns (Campbell and Shiller, 1991). We apply the model to forecastability of exchange rates, and shows it accounts for the failure of uncovered interest rate parity and the forward premium puzzle.

A well-developed literature explores the potential for deviations from a full-information rational-expectations benchmark to explain asset pricing anomalies. Some early work on this subject focused on Bayesian learning with incorrect priors (Timmermann, 1993; Veronesi, 1999; Lewellen and Shanken, 2002). Such a setting does induce excess volatility and predictability, but effects eventually dissipate.[2] Other early work introduced ad hoc specifications of investor irrationality (Barsky and De Long, 1993; Cecchetti et al., 2000) to address excess stock market volatility. More recent work motivates subjective beliefs in various ways. One motivation is that they arise as a worst-case scenario under ambiguity aversion (Bidder and Dew-Becker, 2016; Hansen and Sargent, 2010). Investors may be over-confident in their own signals relative to others (Scheinkman and Xiong, 2003; Dumas et al., 2009).[3] They may form expectations based on an intuitive but incorrect model of autocorrelated growth rates (Fuster et al., 2010). They may incorrectly extrapolate from past data (Barberis et al., 2015; Hirshleifer et al., 2015;

[2]More recently, Collin-Dufresne et al. (2016) combine learning with recursive utility to explain the equity premium.

[3]Scheinkman and Xiong (2003) focus on the interaction of overconfidence and short-sale constraints, which can create asset bubbles. In this paper, we assume a representative agent; however our model could serve as a motivation for heterogeneous beliefs, which, in the presence of short-sale constraints, would lead to yet greater volatility.

Adam et al., 2017; Jin and Sui, 2018; Nagel and Xu, 2018).

Our contribution relative to this literature is to quantitatively explain a number of seemingly unrelated asset pricing anomalies (e.g. stock return predictability, stock return volatility, the value premium, the success of the value-minus-growth factor, the failure of the expectations hypothesis and of uncovered interest rate parity) with a single behavioral mechanism that has a long-established psychological foundation. Note that our mechanism is consistent with many behavioral models that have already been proposed. As we will argue, the precise form of the investor's expectation does not matter so much as the fact that the expectation varies over time and over assets, indicating that investors think they know more than they in fact do. Regardless of the form of the expectation, it is embedded into the asset price, creating the myriad of anomalies described above, which together are very difficult to explain in a fully rational model.

2 Model

Consider an infinite-horizon discrete time economy with risk-neutral investors. Let D_t denote the aggregate dividend at time t, and $d_t = \log D_t$. Assume that investors believe

$$\Delta d_{t+1} = x_t + u_{t+1}, \tag{1}$$

where

$$x_{t+1} = \phi x_t + v_{t+1}, \tag{2}$$

and

$$\begin{bmatrix} u_t \\ v_t \end{bmatrix} \overset{iid}{\sim} N\left(0, \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}\right) \tag{3}$$

Assume $0 < \phi < 1$, so that dividend growth is stationary and positively autocorrelated. The assumption that realized and expected dividends are uncorrelated is for convenience.[4]

Under risk neutrality and assuming a discount factor δ, the absence of arbitrage implies that the value today of a dividend paid an integer $n \geq 0$ periods in the future is

$$P_{nt} = E_t^*[\delta^n D_{t+n}], \tag{4}$$

where we use the notation E^* to denote the expectations of investors. The law of iterated expectations then implies the following recursion for (4):

$$P_{nt} = E_t^*[\delta P_{n-1,t+1}], \quad n \geq 1, \tag{5}$$

with boundary condition $P_{0t} = D_t$. The asset priced in (4) is an "equity strip" (see Lettau and Wachter (2007)), analogous to a zero-coupon bond.

Equations (1–3) define a Markov structure for dividend growth, so if we divide both sides of (4) by D_t, we obtain a function of x_t. Let

$$F_n(x_t) = \frac{P_{nt}}{D_t}. \tag{6}$$

The recursion (5) pins down the functions $F_n(\cdot)$:

$$F_n(x_t) = E_t^*\left[\delta F_{n-1}(x_{t+1})\frac{D_{t+1}}{D_t}\right] \tag{7}$$

with boundary condition $F_0(x_t) = 1$. The solution is

$$F_n(x_t) = e^{a_n + b_n x_t}, \tag{8}$$

[4]Dividend data alone is not sufficient to identify the correlation in (3).

where the coefficients are defined recursively as

$$
\begin{aligned}
a_n &= a_{n-1} + \frac{1}{2} b_{n-1}^2 \sigma_v^2 + \frac{1}{2}\sigma_u^2 + \log\delta \\
b_n &= b_{n-1}\phi + 1.
\end{aligned} \tag{9}
$$

with boundary conditions $a_0 = b_0 = 0$. The recursion for b_n has the well-known solution

$$b_n = \frac{1-\phi^n}{1-\phi}. \tag{10}$$

The price-dividend ratio on the aggregate stock market is a sum of these claims:

$$\frac{P_t}{D_t} = \sum_{n=1}^{\infty} \frac{P_{nt}}{D_t} = \sum_{n=1}^{\infty} F_n(x_t). \tag{11}$$

Defining R_t^m as the net return on the aggregate market and $R_{n,t+1}$ as that on the n-period equity strip, it follows from (11) that

$$R_{t+1}^m \equiv \frac{P_{t+1} - P_t + D_{t+1}}{P_t} = \sum_{n=1}^{\infty} \left(\frac{P_{nt}}{\sum_{k=1}^{\infty} P_{kt}} \right) R_{n,t+1}. \tag{12}$$

Namely, the market return is a weighted average of the returns on the equity strips.[5] The weights depend on the value of x_t (an increase in x_t shifts the weight toward high-maturity claims), but this effect is second-order under our distributional assumptions. It will also be useful, in what follows, to note that an alternative characterization of prices and of $F_n(x_t)$, following directly from the recursion (7), is

$$\frac{P_{nt}}{D_t} = F_n(x_t) = E_t^* \left[\delta^n e^{\sum_{s=1}^{n} \Delta d_{t+s}} \right] \tag{13}$$

[5] The intermediate steps in this calculation are as follows:

$$R_{t+1}^m = \frac{\sum_{n=1}^{\infty} P_{n,t+1} + D_{t+1}}{\sum_{n=1}^{\infty} P_{nt}} - 1 = \frac{\sum_{n=1}^{\infty} P_{n-1,t+1}}{\sum_{n=1}^{\infty} P_{nt}} - 1 = \sum_{n=1}^{\infty} \left(\frac{P_{nt}}{\sum_{k=1}^{\infty} P_{kt}} \right) \left(\frac{P_{n-1,t+1}}{P_{nt}} - 1 \right).$$

Up until now, the assumption of risk-neutral investors is without loss of generality, as we simply could have considered E^* as the risk-neutral expectation, which generates prices even when investors are risk averse. The assumption of risk neutrality enters when we consider the physical distribution, necessary for calculating realized returns. We focus on returns on an equity strip because it makes the calculations easier and, because of (12), the intuition caries over to the market. The return on the equity strip with maturity n is given by

$$\begin{aligned} 1 + R_{n,t+1} &= \frac{P_{n-1,t+1}}{P_{nt}} \\ &= \frac{F_{n-1}(x_{t+1})}{F_n(x_t)} \frac{D_{t+1}}{D_t}. \end{aligned} \tag{14}$$

Suppose first that the investor's beliefs match reality, so that (1–3) represent the physical process for dividends. Substituting (1) and (8) into (14), we find

$$\begin{aligned} \log(1 + R^*_{n,t+1}) &= a_{n-1} - a_n + b_{n-1}x_{t+1} - b_n x_t + x_t + u_{t+1} \\ &= a_{n-1} - a_n + (b_{n-1}\phi - b_n + 1)x_t + b_{n-1}v_{t+1} + u_{t+1}, \end{aligned}$$

where we use R^* to denote returns when the physical distribution matches the subjective one. Substituting from (9) implies that

$$\log(1 + R^*_{n,t+1}) = a_{n-1} - a_n + b_{n-1}v_{t+1} + u_{t+1}. \tag{15}$$

When dividend growth is in fact predictable, returns are iid. Prices incorporate all available information, and so any innovation to returns must come from an innovation to expected dividend growth represented by v_{t+1}, or an innovation to dividend growth itself, represented by u_{t+1}. Furthermore, (9) implies $E^*[R^*_t] = \delta^{-1}$, namely there is zero risk premium, as must be the case because investors are risk neutral.

Assume however, that investors' beliefs do not match reality. The physical process for dividends is not (1–3), but rather

$$\Delta d_{t+1} = u_{t+1}. \tag{16}$$

For simplicity, we assume investors are correct about the evolution of the state variable x_t, namely (2) represents the physical process. Additional effects could arise from incorrect beliefs concerning the persistence of x_t. For simplicity, we do not consider these here.[6]

Prices reflect agents' (incorrect) beliefs and are given by (8) and (9). These prices are identical under both correct and incorrect beliefs and, because they accurately represent some form of beliefs, are arbitrage-free. However, consider returns:

$$\begin{aligned} \log(1 + R_{n,t+1}) &= \log\left(\frac{F_{n-1}(x_{t+1})}{F_n(x_t)}\frac{D_{t+1}}{D_t}\right) && (17) \\ &= a_{n-1} - a_n + b_{n-1}x_{t+1} - b_n x_t + u_{t+1} && (18) \\ &= a_{n-1} - a_n + b_{n-1}(\phi x_t + v_{t+1}) - b_n x_t + u_{t+1}. && (19) \end{aligned}$$

Thus,

$$\log(1 + R_{n,t+1}) = a_{n-1} - a_n - x_t + b_{n-1}v_{t+1} + u_{t+1}, \tag{20}$$

[6] Cochrane (2008) argues that dividend growth is in fact unpredictable. The strength of the predictability in the data depends on how dividends are measured, a point made by van Binsbergen and Koijen (2010), Larrain and Yogo (2008). Dividend predictability, to the extent it exists, appears to be transient (Lettau and Ludvigson, 2005; Li and Wang, 2018). While we focus, for clarity, on the case in which investors believe there is no persistence in dividend growth, what matters for our mechanism is that investors overestimate the persistence of expected dividend growth. In recent work, de la O and Myers (2018) offer direct evidence that stock prices are driven primarily by investors' expectations of cash flows.

and, under the physical expectation,

$$\log E_t \left[1 + R_{n,t+1}\right] = -\log \delta - x_t.$$

Unlike the case where investors' beliefs are correct (see Eq.15) excess returns are predictable. When x_t is high, prices are high and future returns are low.

Equation (20) shows that superstition on the part of investors leads to return predictability. It also leads to return volatility. It is again useful to contrast superstition with rational (i.e. correct) beliefs. When the physical and subjective distributions coincide,

$$\mathrm{Var}(\log(1 + R^*_{nt})) = b^2_{n-1}\sigma^2_v + \sigma^2_u, \tag{21}$$

whereas

$$\mathrm{Var}(\log(1 + R_{nt})) = \sigma^2_x + b^2_{n-1}\sigma^2_v + \sigma^2_u, \tag{22}$$

where

$$\sigma^2_x \equiv \frac{\sigma^2_v}{1 - \phi^2}.$$

At first glance, it appears that return volatility arises from the term σ^2_x, because this is the source of predictability. Also, this is missing in the case of rationality. However, the link between superstition and volatility is more subtle. In fact, almost all of the volatility arises, in both cases, from the σ^2_v term: as discussed in the next paragraph, this term is an order of magnitude bigger than the others. It appears in both the rational and superstition cases, and in both cases it represents changes in investors subjective expectations about dividend growth. In the rational case, however, these expectations coincide with the true distribution. In the case with superstition, it will appear, ex post, as a time-varying discount rate. Volatility is similar in both cases; in one case it is accompanied by predictable dividends (counterfactually) whereas in the other it is accompanied by predictable returns.

We now return to the question of the volatility decomposition in (22). In the paragraph above, we claimed that nearly all the volatility in returns arises from the volatility in expected dividends, as represented by $b_{n-1}^2\sigma_v^2$. We now explain why this is so. First note that σ_u^2 is the volatility of realized dividends. This 0.07^2 per annum in postwar data. On the other hand, the volatility of shocks to x_t, σ_v, and the unconditional volatility of x_t, σ_x, are unobserved. To understand the magnitude of the remaining terms, we turn to the prices of dividend claims, normalized by current dividends. These are denoted by $F_n(x_t)$ and given in (8) and (9).

Recall that the price-dividend ratio on the market is a sum of these component price-dividend ratios. Furthermore, even if the persistence ϕ is high, decay is geometric, and so for n sufficiently large, $b_n \approx (1-\phi)^{-1}$. If we let σ_{pd}^2 be the variance of the log price-dividend ratio on the market, roughly speaking,[7]

$$\sigma_{pd}^2 \equiv \lim_{n\to\infty} \mathrm{Var}(\log F_n(x_t)) = \frac{\sigma_x^2}{(1-\phi)^2}$$

Then, for long-maturity equity strips (which, due to the properties of geometric decay, best represents the return on the market) the decomposition (22) takes the form

$$\begin{aligned} \lim_{n\to\infty} \mathrm{Var}(\log(1+R_{nt})) &= \sigma_x^2 + \frac{\sigma_v^2}{(1-\phi)^2} + \sigma_u^2 \\ &\approx (1-\phi)^2\sigma_{pd}^2 + (1-\phi^2)\sigma_{pd}^2 + \sigma_u^2. \end{aligned} \tag{23}$$

While $\sigma_u \approx 0.07$, $\sigma_{pd} \approx 0.42$. The persistence ϕ will equal the persistence of the price-dividend ratio. At $\phi = 0.92$, the first term in (23) equals $(0.08 \times 0.42)^2$, whereas the second term equals $(0.39 \times 0.42)^2$. The second term, representing the effect of

[7]Note that the log price-dividend ratio equals

$$pd = \log \sum_{n=1}^{\infty} F_n(x_t) \approx \sum_{n=1}^{\infty} a_n + b_n x_t = a^* + b^* x_t.$$

Because of geometric decay, $b^* \approx (1-\phi)^{-1}$.

innovations to x_t is thus roughly 25 times larger than the term representing x_t itself, and roughly 5 times larger than the term representing dividend volatility.[8] Finally note that these terms add up to $(0.18)^2$, thus (roughly) accounting for the annual volatility in stock returns.

This accounting exercise suggests that this simple model can explain return volatility, predictability in excess returns, together with the lack of predictability in dividends. As yet, it has nothing to say about the equity premium. Below, we address this lack, and perform a more formal calibration exercise.

3 Model with IID Disasters

We now show that a realistic equity premium can be incorporated into the model above. Assume a representative agent who maximizes a time-additive utility function with constant relative risk aversion:

$$E\sum_{t=0}^{\infty}\delta^t\frac{C_t^{1-\gamma}-1}{1-\gamma},$$

where γ is relative risk aversion and δ remains the time discount factor. The agent holds the following beliefs about the consumption and dividend growth processes:

$$\Delta c_{t+1}=\mu+u_{t+1}+w_{t+1}, \tag{24}$$

$$\Delta d_{t+1}=\mu+x_t+u_{t+1}+w_{t+1}, \tag{25}$$

[8]This will also be true in a rational model with prices driven by discount rate variation. Most of the variation in realized returns comes from *innovations* in the discount rate, which are unpredictable. Very little comes from the variation in the discount rate itself.

where x_t is as in (2) above, with shocks u_{t+1} and v_{t+1} distributed as in (3). We further assume, following Barro (2006), that

$$w_t \overset{iid}{\sim} \begin{cases} \xi & \text{probability } = p \\ 0 & \text{probability } = 1-p \end{cases} \tag{26}$$

where ξ is a constant and w_t is independent of u_t and v_t.

In equilibrium, the aggregate market and the riskfree rate are priced using the representative investor's Euler equation. That is, if we let P_{nt} be the price of an n period ahead equity strip, then P_{nt} satisfies the recursion

$$P_{nt} = E_t^*\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} P_{n-1,t+1}\right],$$

where E^* denote expectations taken with respect to the subjective distribution, and where $P_{0t} = D_t$. Defining $F_n(x_t) = P_{nt}/D_t$, as in the previous section, we have

$$F_n(x_t) = E_t^*\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} F_{n-1}(x_{t+1})\frac{D_{t+1}}{D_t}\right] \tag{27}$$

with boundary condition $F_0(x_t) = 1$. The solution is again

$$F_n(x_t) = e^{a_n + b_n x_t}, \tag{28}$$

where a_n follows the modified recursion

$$a_n = a_{n-1} + \log\delta + (1-\gamma)\mu + \frac{1}{2}b_{n-1}^2\sigma_v^2 + \frac{1}{2}(1-\gamma)^2\sigma_u^2 + \log(pe^{(1-\gamma)\xi} + (1-p)) \tag{29}$$

with $a_0 = 0$. The recursion for b_n is the same, and so $b_n = (1-\phi^n)/(1-\phi)$ still holds.

The riskfree asset is also priced using the investor's Euler equation. Let R_f be the one-period riskfree rate. Then:

$$E^* \left[\delta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 + R_f) \right] = 1,$$

implying

$$\log(1 + R^f) = -\log \delta + \gamma\mu - \frac{1}{2}\gamma^2 \sigma_u^2 - \log(pe^{-\gamma\xi} + (1 - p)). \tag{30}$$

We assume that the investor has correct beliefs about the consumption distribution (24). Moreover, the investor correctly assumes that dividends are equally subject to disasters as are consumption. However, the investor believes that dividends are predictable, when in reality they are not. We parsimoniously capture these assumptions by setting the physical distribution of Δd_{t+1} equal to Δc_{t+1}.

Defining $R_{n,t+1}$, as in the previous section, as the return on the n-period dividend claim:

$$\begin{aligned} \log(1 + R_{n,t+1}) &= \log \left(\frac{F_{n-1}(x_{t+1})}{F_n(x_t)} \frac{D_{t+1}}{D_t} \right) \\ &= a_{n-1} - a_n + b_{n-1}x_{t+1} - b_n x_t + \mu + u_{t+1} + w_{t+1} \\ &= a_{n-1} - a_n + \mu - x_t + b_{n-1}v_{t+1} + u_{t+1} + w_{t+1}. \end{aligned}$$

We therefore have, under the physical measure,

$$\log E_t \left[1 + R_{n,t+1} \right] = a_{n-1} - a_n + \mu - x_t + \frac{1}{2}b_{n-1}^2 \sigma_v^2 + \frac{1}{2}\sigma_u^2 + \log(pe^{\xi} + (1 - p)),$$

and, for the expected excess return under the physical measure:

$$\log E_t\left[(1+R_{n,t+1})/(1+R^f)\right] = -x_t + \gamma\sigma_u^2 + \log(pe^{\xi} + (1-p)) + \log(pe^{-\gamma\xi} + (1-p)) - \log(pe^{(1-\gamma)\xi} + (1-p)).$$

For small p (or, as the time interval shrinks):

$$\log E_t\left[(1+R_{n,t+1})/(1+R^f)\right] \approx -x_t + \gamma\sigma_u^2 - p(1-e^{-\gamma\xi})(1-e^{\xi}), \tag{31}$$

where we have used, e.g., $\log(pe^{\xi}+(1-p)) = \log(1+p(e^{\xi}-1)) \approx p(e^{\xi}-1)$. The expected excess return has its usual unconditional component, $\gamma\sigma_u^2 - p(1-e^{-\gamma\xi})(1-e^{\xi})$, the first term of which represents the normal risk, and the second term of which represents the risk of disasters. This term captures the negative covariance between returns and marginal utility during disaster periods. These components represent a risk premium, namely a return to bearing the risk of equity, which might go down during a disaster. The first term, x_t, does not represent a return to bearing risk, but rather is mispricing.[9]

Note that our assumption that the agent correctly assesses disaster risk is to discipline the model. We would find nearly the same equity premium if the agent overly assessed disaster risk; i.e. was pessimistic. If the values here represent an optimistic assessment of disaster risk (namely, disasters should have occurred with probability greater than 2%), then that simply implies that we were lucky and that the equity premium is not as much of a puzzle as believed. Also, allowing the agent to believe consumption growth is forecastable would also not affect our results; however we believe this is less of a plausible assumption. As discussed above, the literature shows less predictability in consumption growth than in dividend growth. As we show below,

[9] As described in the previous section, the variance of x_t is relatively small. Thus the wedge between the unconditional expectation of (31) and the true unconditional equity premium is small as well.

beliefs in favor of dividend growth predictability are reasonable (though not required) given the data.

4 Data and Calibration Results

4.1 Data

We use the value-weighted CRSP index to represent the market. We compute an annual dividend by taking monthly dividends and summing. The dividend-price-ratio of the market is the trailing one year aggregate dividend divided by the ex-dividend price of the market. We use 3-month Treasury bill returns to proxy for the riskfree rate. We use the CPI index to go from nominal returns and dividend growth to real returns and dividend growth. The full sample for this study ranges from 1927 to 2017, and the post-war subsample ranges from 1948 to 2017. All data are annual.

4.2 Parameters Values

Table 1 shows the parameter choices for our simulations, done at an annual frequency. We choose σ_u to be the volatility of log real dividend growth in the data. We choose $\phi = 0.95$ to match the observed first-order autocorrelation in the log dividend price ratio in postwar data. The discount factor δ, provided it is within a reasonable range and high enough to ensure convergence, has a second-order effect on the results. We choose $\delta = 0.97$, which is consistent with a low riskfree rate, and still allows for convergence of the infinite sum (11). For the risk-neutral model, the remaining parameter is σ_v, which we choose to be 0.01. This generates the correct volatility of the price-dividend ratio under risk neutrality.

For the model with disaster risk, we follow Barro (2006) and choose risk aversion γ to be 3, the average growth rate of consumption μ to be 2%, the annual disaster probability p to be 2%, and the size of the disaster to be 33%. We set the time-discount factor δ to match the average return on the riskfree asset, which we set at the average annual (real) return on three-month Treasury bills.

4.3 Results

We simulate 4000 samples of either 91 years of data (to represent the 1927–2017 sample) or 70 years of data (to represent the 1948 to 2017 sample). We report three types of results: the results for the risk neutral model with the longer simulation, the results for the disaster model, with the longer simulation, and the results from the disaster model for the shorter simulation, in which we consider only samples with no disasters. Reporting the risk-neutral results for the shorter simulations would be repetitive, as the only difference is in the degree of small-sample bias of some of the statistics.

Table 2 show the results for the 1927–2017 sample, and compare these to the model. This comparison confirms the informal analysis in Section 2: the model can simultaneously match the standard-deviation of returns, of the price-dividend ratio, of dividend growth. The model fits the slight negative autocorrelation of annual returns. However, the model, by construction does not fit the slight autocorrelation in dividend growth, which is 20% at an annual horizon.

Including rare disasters in the model, which account for a high equity premium and low riskfree rate, have little impact on the second moments. While there is a slight reduction in the standard deviation of the divided-price ratio (due to the duration effect; the equity premium causes a down-weighting of long-horizon claims which are the most sensitive to changes in expectations), the data value remains well-within the 10% confidence bounds. Table 3 show similar results for the postwar sample.

Table 4 reports reports from regressions of excess returns on the price-dividend ratio. The first panel replicates the well-known result that excess returns are indeed predictable by the dividend-price ratio. This result also holds in post-war data (Table 5). Coefficients are statistically significant at nearly all horizons, with R^2 statistics increasing from 2% to 28%. Table 6 shows that, in contrast, dividend growth is significantly forecastable only at the 1-year horizon.[10] This forecastability is transient, in that the R^2 statistics do not increase with horizon. Even this short-horizon effect becomes insignificant in post-war data (Table 7).

These tables also show simulations from the model. Note that by simulating the series of the correct length under a model that captures the correlational structure of the data, we capture the source of bias in the model that is also in the data (Stambaugh, 1999). The superstitious investor model captures the correct magnitude of return predictability, and the lack of dividend growth predictability. The amount of dividend growth predictability in the data (with the exception of the shortest horizon in the 1927–2017 series) can easily be accounted for by finite-sample noise. The superstitious investor model captures the economically and statistically significant predictability. However, returns are not *too* predictable in the model; the data coefficients lie within the confidence intervals. It is not easy to take advantage of the superstitious agent because there is a sense in which he is correct.

Finally, Figure 1 shows a time series plot of the level of prices and the level of dividends, post-1926. On the figure, the level of dividends is multiplied by a constant so the average level of the series are the same. Consistent with the superstition model, but inconsistent with a model in which investors have correct beliefs, deviations from the mean of the price-dividend ratio are usually followed by adjustments in prices, rather than adjustments in dividends. This is a graphical illustration of the results in Table 4. Figure 2 shows that this effect holds more dramatically in postwar data.

[10] This and earlier statements of significance are under the assumption of a single test. Accounting for multiple comparisons would likely further decrease the significance of dividend growth predictability.

5 A Bayesian view of dividend predictability

A possible objection to the model in Section 2 is that, over time, investors would learn that dividends are in fact unpredictable. If investors did learn the correct distribution, prices would remain volatile, but return predictability would dissipate. In this section, we confront the hypothesized beliefs with data. We consider an investor whose prior beliefs include the possibility of dividend growth predictability. The agent updates these beliefs given the historical time series, seen through the lens of the likelihood implied by (1–3). Our evidence speaks to the difficulty of learning the true process for dividend growth.

We assume, as in Section 2, the agent believes that dividend growth contains a predictable component. Should this predictable component exist, it follows from the reasoning in Section 2 that it should be captured by the price-dividend ratio.[11] The agent therefore considers the predictive system:

$$\Delta d_{t+1} = \beta \hat{x}_t + u_{t+1} \tag{32}$$

$$\hat{x}_{t+1} = \hat{\phi} \hat{x}_t + \hat{v}_{t+1}, \tag{33}$$

where $\hat{x}_t = p_t - d_t$, the log price-dividend ratio, and where

$$\begin{bmatrix} u_t \\ \hat{v}_t \end{bmatrix} \overset{iid}{\sim} N\left(0, \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & \hat{\sigma}_v^2 \end{bmatrix}\right). \tag{34}$$

We refer to the predictor variable as $\hat{x}_t$ in contrast to x_t. Up to linearization error, the assumptions in Section 2 imply that $\hat{x}$ and x differ only by a scale factor, approximately equal to $1/(1-\phi)$. For convenience, we de-mean both variables.[12]

[11]To the extent that the price-dividend ratio fails to capture this component, we are biased against finding dividend growth predictability, and therefore proving the beliefs to be less justifiable than otherwise.

[12]De-meaning the variables simplifies the analysis, and only affects the conclusions through a

Under conditions described in Appendix A, it suffices to consider a prior on the parameters of the dividend process and the marginal likelihood for the dividend process, taking observations on $\hat{x}_t$ as given. That is, the time-series regression (32) for dividend growth is, in this case, equivalent to standard OLS in which the regressor is strictly exogenous.

We assume a prior inverse-gamma distribution for σ_u^2 and, conditional on σ_u^2, a normal distribution for the predictive coefficient β:

$$\beta \,|\, \sigma_u \sim N(\beta_0, g^{-1}\sigma_u^2\Lambda_0^{-1}) \quad (35)$$

$$\sigma_u^2 \sim IG(a_0, b_0). \quad (36)$$

We set parameters a_0 and b_0 so that the prior on σ_u^2 is diffuse.[13] Equation 36 implies a conjugate prior on β (Zellner, 1996). As explained below, Λ_0 is a scale factor that will allow us to interpet g as indexing the strength of the prior.

Given the priors (35) and (36), and the likelihood defined by (32–34), the agent forms a posterior. Let $\hat{\mathbf{x}}_t = \{\hat{x}_0, \ldots, \hat{x}_t\}$, namely the set of observations on $\hat{x}_s$, up to and including time t. Let $\mathbf{y}_t = \{\Delta d_1, \ldots \Delta d_t\}$ be the dividend growth observations up to and including time t. The agent calculates

$$p(\beta, \sigma_u \,|\, \hat{\mathbf{x}}_t, \mathbf{y}_t) \propto \mathcal{L}(\mathbf{y}_t \,|\, \hat{\mathbf{x}}_t, \beta, \sigma_u) p(\beta, \sigma_u), \quad (37)$$

where $p(\beta, \sigma_u)$ is the prior specified in (35) and (36) and $\mathcal{L}(\mathbf{y}_t \,|\, \hat{\mathbf{x}}_t, \beta, \sigma_u)$ is the likelihood of observing the dividend growth data given the predictor variable and the parameters.

We fix time T as the last data point observed. We stack the observations on $\hat{x}_t$ and

degree-of-freedom adjustment that becomes negligible as the same size grows.

[13] Because our focus will be on the posterior mean of β, these play no further role in our analysis.

Δd_t into vectors:

$$Y = \begin{bmatrix} \Delta d_1 \\ \vdots \\ \Delta d_T \end{bmatrix}, \qquad X = \begin{bmatrix} \hat{x}_0 \\ \vdots \\ \hat{x}_{T-1} \end{bmatrix}.$$

Note that the OLS estimate of β equals

$$\hat{\beta} = (X^\top X)^{-1} X^\top Y,$$

and that (32) implies

$$Y = \beta X + U,$$

where $U \sim N(0, \sigma_u^2 I)$, and I is the $T \times T$ identity matrix. It follows that the posterior (37) is given by

$$p(\beta, \sigma_u \,|, \hat{\mathbf{x}}_T, \mathbf{y}_T) \propto \sigma_u^{-n} \exp\left\{ -\frac{1}{2\sigma_u}(Y - X\beta)^\top (Y - X\beta) \right\} \sigma_u^{-1} \exp\left\{ -\frac{g\Lambda_0(\beta - \beta_0)^2}{2\sigma_u^2} \right\}$$

where $\propto$ means up to a proportionality factor that does not depend on β and σ_u. Completing the square implies

$$p(\beta, \sigma_u \,|, \hat{\mathbf{x}}_T, \mathbf{y}_T) \propto \sigma_u^{-1} \exp\left\{ -\frac{(X^\top X + g\Lambda_0)(\beta - \bar{\beta})^2}{2\sigma_u^2} \right\} \times p(\sigma_u \,|\, \hat{\mathbf{x}}_T, \mathbf{y}_T), \qquad (38)$$

where

$$\begin{aligned} \bar{\beta} &= (g\Lambda_0 + X^\top X)^{-1}(g\Lambda_0 \beta_0 + X^\top Y) \\ &= (g\Lambda_0 + X^\top X)^{-1}(g\Lambda_0 \beta_0 + (X^\top X)\hat{\beta}), \end{aligned}$$

and where $p(\sigma_u \,|\, \hat{\mathbf{x}}_T, \mathbf{y}_T)$ is a term that does not depend on β and is therefore the marginal posterior of σ_u (see (Zellner, 1996, Chapter 8) for more detail). It is clear from (38) that the posterior of β conditional on σ_u is multivariate normal with posterior mean $\bar{\beta}$. Note also that $\bar{\beta}$ is a weighted average between the prior mean β_0 and the sample mean $\hat{\beta}$, with the weights determined by the precisions of the prior and of the sample respectively.

If we, ex post, set $\Lambda_0 = X^\top X$, then g corresponds to the weight on β_0 as a percent of the weight on $\hat{\beta}$, so that $g = 0.1$ implies that the prior receives 1/10 of the weight of the sample, and $g = 0.01$ means it receives 1/100 of the weight. Because we are asking whether, if the agent had the beliefs we attribute to her in Section 2, her beliefs would change, we set the prior mean to equal $1/(1-\phi) = 0.05$. For comparability with Tables 4–7, which show regressions on the dividend-price ratio, Figure 3 shows the negative of the posterior mean of β. We consider an informative prior, with $g = 0.10$, and a diffuse prior, with $g = 0.01$.

Figure 3 shows that the agent does indeed revise her prior beliefs, at least at first. She revises it to imply more, not less predictability of dividend growth. Indeed, from the 1930s to the 1970s, it appears that dividend growth was more predictable than later in the sample. Only when nearly the full sample is used, namely around 2000, does the posterior mean converge to the sample estimate, which happens to be close to, though implying slightly more predictability than, the prior. Note that the convergence implies that the prior does not matter when the full sample is used.

Thus an agent, viewing the evidence on annual dividend growth rates in isolation, would be justified in maintaining a belief that dividend growth rates are predictable. This agent, however, is not fully rational. He incorrectly extrapolates the predictability from the one-year horizon to long horizons. Moreover, he fails to notice that excess returns are also predictable.

6 Extensions

Cochrane (2011) notes that predictability, both in the time series and in the cross-section, appears to be ubiquitous. He attributes this predictability to variation in discount rates across time and across assets. He notes that, within a no-arbitrage setting (like the current paper), discount-rate based explanations of phenomena and belief-based explanations are isomorphic (see Harrison and Kreps (1979)). However, the fact that one can be mapped into the other does not necessarily make them equally good explanations, as the discount-rate equivalent of a belief-based model might be complicated (and likewise, for a belief-based equivalent of an discount-rate explanation). Furthermore, time-varying discount rates are ideally viewed as an endogenous outcome of an economic model. In most models, time-varying discount rates are tied to time-varying risk, providing testable implications discussed in the introduction. Discount rates might also vary because risk aversion varies, or a rare event probability varies; yet this would suggest a co-movement in measures of discount rates, which is absent in the data Lettau and Wachter (2011).[14]

On the other hand, if investors display superstitious behavior about aggregate market dividends, it is natural to assume that this behavior could be seen in other asset classes, and would produce the kind of (ex post) predictability seen in the data. We give specific parametric examples below.

6.1 The value premium

Fama and French (1992) show that stocks with high ratios of book equity to market equity (value stocks) exhibit significantly higher excess returns than those with low ratios (growth stocks). Moreover, market betas line up in roughly the opposite direction of the expected returns, as do standard deviations. Thus standard risk-based stories fail to account for the observed value premium. Here, we show that a simple extension

[14]Differential time-varying exposure to rare events offers another potential route for unifying this evidence (Gabaix, 2012). Moreover, existing models of time-varying risk aversion or rare events do not in practice entirely break the link between first and second moments.

to the model presented in Section 2 naturally accounts for this finding.[15]

As is well known, the value premium result extends to ratios of other fundamentals to price, such as earnings to price (see, e.g. Lettau and Wachter (2007)). What appears to be important is having price in the denominator and a plausible non-price scaling variable in the numerator. For our model, the most natural scaling variable is payouts, namely dividends, though these could be connected, through a standard production framework, to book value. We focus on the earnings-to-price ratio in the data because dividends are to some extent arbitrary.

Assume n risky assets. Let D_{jt} denote the time-t dividend, and Δd_{jt} log dividend growth, for stock j, where $j = 1, ..., n$. Investors believe that dividend growth is predictable, as before. However, besides a component that effects all firms in the same way, there is a second component that effects firms differentially. That is,

$$\Delta d_{j,t+1} = x_t + \beta_{z,j} z_t + u_{j,t+1}, \tag{39}$$

where

$$x_{t+1} = \phi_x x_t + v_{x,t+1} \tag{40}$$

$$z_{t+1} = \phi_z z_t + v_{z,t+1}. \tag{41}$$

We assume the shocks $u_{j,t+1}$ (for $j = 1, \ldots, n$), $v_{x,t+1}$, and $v_{z,t+1}$, are normally distributed, independent of one another, and independent over time, with variances σ_u^2 ($\forall j$), σ_{vx}^2, and σ_{vz}^2 respectively.

Equation 39 indicates that subjective expectations are driven by x_t and z_t. Firms are affected by x_t in the same way, while they are differentially affected by z_t. So that

[15] Other approaches to explaining the value premium that rely on subjective expectations include Barberis et al. (1998); Daniel et al. (2001); Alti and Tetlock (2014); Tsai and Wachter (2016); Bordalo et al. (2017). La Porta (1996) finds direct evidence for the role of incorrect expectations in the observed value premium.

x_t has the interpretation of expected dividend growth in the aggregate, we assume that $\sum_j \beta_{z,j} = 0$.

We assume risk-neutral investors with discount rate δ. Let P_t^j denote the price of stock j. As in Section 2,

$$P_t^j = \sum_{n=1}^{\infty} P_{n,t}^j,$$

where $P_{n,t}^j$ is the price of the n-period dividend strip for stock j. Prices $P_{n,t}^j$ satisfy a recursion analogous to (7). Conjecture that the solution takes the form

$$\frac{P_{n,t}^j}{D_{j,t}} = F_n^j(x_t, z_t) = e^{a_{j,n} + b_{x,n} x_t + \beta_{z,j} b_{z,n} z_t}. \tag{42}$$

Using a recursion analogous to (7), we find the difference equations

$$\begin{aligned} a_{j,n} &= a_{j,n-1} + \frac{1}{2} b_{x,n-1}^2 \sigma_{vx}^2 + \frac{1}{2} \beta_{z,j}^2 b_{z,n-1}^2 \sigma_{vz}^2 + \frac{1}{2} \sigma_u^2 + \log \delta \\ b_{x,n} &= b_{x,n-1} \phi_x + 1 \\ b_{z,n} &= b_{z,n-1} \phi_z + 1, \end{aligned} \tag{43}$$

with $P_{0,t}^j / D_{jt} = 1$ implying boundary conditions $a_{j,0} = b_{x,0} = b_{z,0} = 0$. Thus

$$\begin{aligned} b_{x,n} &= \frac{1 - \phi_x^n}{1 - \phi_x} \\ b_{z,n} &= \frac{1 - \phi_z^n}{1 - \phi_z}. \end{aligned}$$

It is also useful to note that:

$$\frac{P_{n,t}^j}{D_{j,t}} = E_t^* \left[\delta^n e^{\sum_{s=1}^n \Delta d_{j,t+s}} \right] = \exp\{a_{j,n} + b_{x,n} x_t + \beta_{z,j} b_{z,n} z_t\}, \tag{44}$$

with $a_{j,n}$, $b_{x,n}$, $b_{z,n}$ as above.

Equation 42 implies a cross-section of scaled-price ratios as long as there is a cross-section of exposures $\beta_{z,j}$. Following the empirical literature, we refer to stocks with high price ratios as growth and those with low price ratios as value. For example, if $z_t > 0$, then growth stocks will have high β_{zj} and value stocks will have low β_{zj}. On the other hand, if $z_t < 0$, the reverse pattern will be the case.[16] Note that all that is required to produce a spread in price-dividend ratios is variation in the loadings $\beta_{z,j}$. Value stocks need not be pre-assigned some $\beta_{z,j}$.[17]

We assume, as in Section 2, that dividend growth is in fact unpredictable. We define the market portfolio to be the weighted average of the individual assets. We take this simple model to the data.[18] Table 8 reports means of portfolios formed on earnings-to-price ratios in postwar data, and in simulations from the model. In historical data, firms are sorted into quintiles based on earnings-to-price ratios (details can be found on Kenneth French's website). In the data, value firms (those with high earnings-to-price ratios) have high expected returns relative to growth firms. Except for the extreme value quintile, they have lower standard deviations and lower betas with respect to the market. Thus the Capital Asset Pricing Model does not explain the spread in expected returns, and abnormal returns are large.

Table 8 also reports means from simulations in the model. True risk premia in the model equal zero. However measured risk premia do not. Just as in the data, the higher is the earnings-to-price ratio, the higher the return, with the difference in the model being 3%. All of this is abnormal return because risk premia equal zero.

Why does the model produce a spread in returns? In the spirit of Cohen et al. (2003), consider the value spread, defined by as the difference in dividend-to-price

[16] We disregard for the moment the Jensen's inequality adjustments in the $a_{j,n}$ terms. In our calibration, these are small.

[17] While we focus on a common component in subjective beliefs on which stocks load differentially, a reasonable extension would be to allow for firm-specific components. Firm-specific differences in beliefs would produce a long-run reversal effect distinct from an over-reaction mechanism (Daniel et al., 1998; Hong and Stein, 1999).

[18] The persistences $\phi_x = \phi_z = 0.85$. The volatilities $\sigma_{vx} = \sigma_{vz} = 2.5\%$, while $\sigma_u = 20\%$. $\log \delta = -5.7\%$ so that prices converge. The loading on z_t, β_{jz} ranges from -1 to 1.

ratios between value and growth stocks. The value spread in the model is given by

$$\log \frac{D_{j,t}}{P^j_{n,t}} - \log \frac{D_{k,t}}{P^k_{n,t}} = -\frac{1}{2}(\beta^2_{z,j} - \beta^2_{z,k})\sigma^2_{vz} \sum_{s=1}^{n-1} b^2_{z,s} - (\beta_{z,j} - \beta_{z,k}) b_{z,n} z_t, \qquad (45)$$

with $\beta_{z,j} < 0$ and $\beta_{z,k} > 0$ when $z_t > 0$, and the signs reversed when $z_t < 0$.[19] When $z_t > 0$, firms that have the highest valuations relative to their dividends (and thus the lowest dividend-price ratios) have the greatest loadings on z_t. Note that the value spread in the model is perfectly correlated with the perceived differential forecast z_t.

Then realized returns on the n-period dividend strip for stock j equal:

$$\begin{aligned}
\log(1 + R^j_{n,t+1}) &= \log P^j_{n-1,t+1} - \log P^j_{n,t} \\
&= a_{j,n-1} - a_{j,n} + b_{x,n-1} x_{t+1} - b_{x,n} x_t + \beta_{z,j}(b_{z,n-1} z_{t+1} - b_{z,n} z_t) + u_{j,t+1} \\
&= a_{j,n-1} - a_{j,n} - x_t - \beta_{z,j} z_t + b_{x,n-1} v_{x,t+1} + \beta_{z,j} b_{z,n-1} v_{z,t+1} + u_{j,t+1}
\end{aligned}$$

Consider the differential return between a value stock j and growth stock k.

$$\begin{aligned}
\log(1 + R^j_{n,t+1}) - \log(1 + R^k_{n,t+1}) = (a_{j,n-1} - a_{j,n} - (a_{k,n-1} - a_{k,n})) - (\beta_{z,j} - \beta_{z,k}) z_t + \\
(\beta_{z,j} - \beta_{z,k}) b_{z,n-1} v_{z,t+1} + u_{j,t+1} - u_{k,t+1}.
\end{aligned}$$

The preceding argument implies that, for $z_t > 0$, we have $\beta_{z,k} > \beta_{z,j}$.[20] Note that

$$\log E_t\left[1 + R^j_{n,t+1}\right] - \log E_t\left[1 + R^k_{n,t+1}\right] = (\beta_{z,k} - \beta_{z,j}) z_t \qquad (46)$$

Because $z_t > 0$, value stocks appear to offer a premium over growth stocks. It follows that the value factor always has a positive average return. The model not only predicts that value stocks have higher average returns, but that the degree of the mispricing is

[19] Again, we ignore for the purpose of discussion, the second-order Jensen's inequality term, which is the first term in (45).

[20] Note that for $z_t < 0$, we would have the opposite inequality.

perfectly correlated with the value spread, as Cohen et al. (2003) show.

Interestingly, however, the model reproduces the u-shape in volatilities. The extreme growth and value stocks are those with extreme β loadings in either direction, and hence are the most volatile. The model also reproduces the high market betas of growth stocks. Growth stocks are especially sensitive to changes in x_t because of the effect of duration, and x_t is what drives the market portfolio. The model cannot, however, reproduce all the results in the data. Besides the fact that average returns are too low (this, however, is by construction and could be altered in the same way as in Section 2), the model also produces a high-minus-low portfolio that is too volatile.

An important aspect of the value premium in that, while it cannot be explained by conventional risk measures, it can be explained by factor loadings on the HML (high-minus-low) factor. However, this too can be replicated in the model. When we run time-series regressions using the market return and the HML return (the return on the value portfolio minus the return on the growth portfolio), we find zero abnormal return, just as in the data. The reason is that the value-minus-growth return is almost perfectly correlated with innovations in z_t. Note that the factor loading on z_t innovations is proportional to $\beta_{z,j}$. Expected returns are also proportional to this factor loading.

To summarize, a time-series factor in expected dividend growth can lead to a cross-sectional factor if firms have different loadings. This will always produce a spread in ratios of prices to dividends (or, in a richer model, prices-to-earnings or prices-to-book value). Moreover, subsequent returns will go in the opposite direction, as the (incorrectly) predicted aggregate dividend growth fails to materialize. Note that the link to time-series variation in z_t is important; it would not be sufficient to have, for example, a model in which some investors are excessively optimistic about some stocks and pessimistic about others. Such a static model would neither explain the association with the value spread and the value return noted by Cohen et al. (2003), nor the ability of the HML factor to account for the value premium, as shown by Fama and French

(1993). Both of these facts are naturally accounted for in the dynamic model shown above.

6.2 Violations of the expectations hypothesis of interest rates

We now apply these ideas to the pricing of Treasury bonds. Assume that investors believe that the continuously-compounded short-term interest rate r_t follows a first-order autoregressive process, so that

$$\Delta r_{t+1} = (\phi - 1)(r_t - \bar{r}) + v_{t+1} \tag{47}$$

where $\Delta r_{t+1} = r_{t+1} - r_t$, $|\phi| < 1$, $\bar{r}$ is the unconditional mean of r_t, and $v_{t+1} \overset{iid}{\sim} N(0, \sigma_v^2)$. Note that ϕ is the first-order autocorrelation of r_t.[21]

As with dividend growth, investors believe that changes in interest rates are more forecastable than they are in reality. That is, while (47) represent beliefs, the true process is governed by

$$\Delta r_{t+1} = (\zeta - 1)(r_t - \bar{r}) + v_{t+1}, \tag{48}$$

with

$$|\zeta - 1| < |\phi - 1|. \tag{49}$$

We focus on the case where $\zeta, \phi \in [0, 1]$ so that (49) implies $\zeta > \phi$. In forecasting next

[21] The analysis in this section takes the short-term interest rate r_t as a given. Perhaps the simplest way to micro-found variation in this rate is to consider a risk-neutral investor with discount rate δ and an exogenous inflation process $\Delta\pi_{t+1}$ such that

$$\Delta\pi_{t+1} = \bar{\pi} + z_t + u_{t+1}$$

and

$$z_{t+1} = \phi z_t + v_{t+1},$$

with u_{t+1} and v_{t+1} distributed as in (3). The interest rate r_t then solves

$$E_t\left[\delta e^{-\Delta\pi_{t+1} + r_t}\right] = 1.$$

Under these assumptions, the analysis proceeds exactly as described.

period's interest rate, (49) implies that investors put more weight on previous values of the interest rate than they should. Alternatively stated, interest rates are closer to a random walk (they mean revert more slowly) in the data than investors believe ($\zeta > \eta$).

We consider risk-neutral pricing for bonds. The dynamics thus far define a discrete-time Vasicek (1977) model.[22] Let $B_n(r_t)$ denote the price of the n-period bond as a function of the riskfree rate between periods t and $t+1$. Then bond prices satisfy the recursion

$$B_n(r_t) = E_t^*\left[e^{-r_t} B_{n-1}(r_{t+1})\right], \tag{50}$$

with $B_0(r_t) = 1$ and $B_1(r_t) = e^{-r_t}$. It follows that

$$\log B_n(r_t) = -a_n - b_n r_t \tag{51}$$

with

$$\begin{aligned} a_n &= a_{n-1} + b_{n-1}(1-\phi)\bar{r} - \frac{1}{2} b_{n-1}^2 \sigma_v^2 \\ b_n &= 1 + b_{n-1}\phi \end{aligned} \tag{52}$$

and $a_0 = b_0 = 0$. Note that $a_1 = 0$ and $b_1 = 1$, so that $B_1(r_t) = e^{-r_t}$. The solution for b_n is again

$$b_n = \frac{1-\phi^n}{1-\phi}. \tag{53}$$

Defining the continuously compounded yield on the n-period bond as

$$y_{nt} = -\frac{1}{n} \log B_n(r_t)$$

[22] A substantial literature on latent factor models strongly rejects a single-factor model in favor of multi-factor alternatives (Dai and Singleton, 2002; Duffee, 2002). Piazzesi et al. (2015) show how subjective expectations can be incorporated into a model with richer dynamics. For the purpose of illustrating our mechanism, however, this simple model suffices.

It follows from (53) that the yield spread equals

$$y_{nt} - y_{1t} = \text{constant} + \left(\frac{1}{n} \frac{1-\phi^n}{1-\phi} - 1 \right) r_t$$

(recall that $y_{1t} = r_t$). The (continuously compounded) holding period return on the n-period bond is given by

$$r_{n,t+1} = \log B_{n-1}(r_{t+1}) - \log B_n(r_t)$$

(note that $r_{1,t+1} = r_t$). Substituting in for (51), (53), and for the physical evolution of r_t, (48), we find the following equation for continuously-compounded excess returns:

$$rx_{n,t+1} = r_{n,t+1} - r_{1,t+1} = \text{constant} + (\zeta - \phi) \frac{b_{n-1}}{1-(1/n)b_n} (y_{nt} - y_{1t}) + b_{n-1} v_{t+1}.$$

When $\zeta = \phi$, we recover the equilibrium with correct beliefs in which excess returns are unpredictable. However, when $\zeta > \phi$, the yield spread will predict excess returns with a positive sign, as in the data.

The economic intuition is similar to that of predictability in equity ratios. Yields fluctuate based on forecasts of future interest rates. Relatively high values of long-term yields indicate investor forecasts of rising short-term interest rates. Short-term rates are not as predictable as investors think, and on average, when the yield spread is high, interest rates fall relative to investor's expectations. As a result, an above-average yield spread forecasts positive excess returns on bonds.

The ability of the yield spread to forecast excess bond returns was first noted in the data by Campbell and Shiller (1991). According to the expectations hypothesis of interest rates, yields on long-term bonds should reflect forecasts of future short-term interest rates.[23] Indeed, the recursion (50) implies

$$y_{nt} = -\frac{1}{n} \log E_t^* \left[e^{\sum_{\tau=0}^{n-1} r_{t+\tau}} \right].$$

If investors correctly anticipate yields, then bond returns will be unpredictable. However, Campbell and Shiller (1991), Fama and Bliss (1987) and a large subsequent literature show that excess bond returns are strongly forecastable. We replicate this finding in Table 10, which reports coefficients from regressing bond returns on yield spreads using the Fama-Bliss data set for zero-coupon bonds.

As an illustrative calculation, we calibrate σ_v and ϕ to jointly match the volatility and first-order autocorrelations of yields. This implies $\sigma_v = 1.5\%$ per annum and an annual autocorrelation ζ of (roughly) 0.90. Given these parameters, $\phi = 0.45$ gives us roughly the amount of predictability in the data.

Table 10 shows results from historical data and from simulating 1000 samples of length 70 years. We run the regression

$$rx_{n,t+1} = \alpha_n + \beta_n (y_{nt} - y_{1t}) + \epsilon_{t+1}$$

for zero-coupon bonds for maturities ranging from 2 to 5 years. Bond excess returns are strongly predictable in both data and model.

Though there are aspects of the data that this one factor model cannot match (for example, yield spreads are less persistent then yields themselves), it offers a very simple explanation for a difficult feature of the data: namely, why investors appear to require, at some points in time, very different term premia for long-term bonds. In this model, the answer is that they do not require such premia, but rather, they do not know the correct process for interest rates.[24] Given that this process has itself been a matter of

[23] There are slight differences depending on whether this hypothesis is articulated in logs or levels (Campbell, 1986).

[24] Recent work has assembled direct evidence in favor for this hypothesis. Cieslak (2018) shows that

long debate in the finance profession, this seems like a relatively weak assumption.

6.3 Uncovered interest rate parity

While a full account of the behavior of currencies and international interest rates is far outside the scope of this paper, we offer a simple extension of the previous ideas to the forward premium anomaly, otherwise known as the failure of uncovered interest rate parity.[25]

Let S_t be the exchange rate in units of foreign currency per U.S. dollar. Let R_{t+1} be the nominal interest rate in the U.S. available between times t and $t+1$, and $\tilde{R}_{t+1}$ be the nominal interest rate in the foreign country, denominated in that country's currency. Let F_t be the forward price of the foreign currency. That is, at time t, one dollar can be converted into F_t units of the foreign currency at time $t+1$. We consider nominal rates, and assume no risk of sovereign default, so that R_{t+1} and $\tilde{R}_{t+1}$ are known at time t.

Risk-neutral pricing for the U.S. investor requires that expected rates of return be equal when computed with respect to the investor's probability distribution:

$$E_t^*\left[1 + R_{t+1} - \frac{S_t}{S_{t+1}}(1 + \tilde{R}_{t+1})\right] = 0. \tag{54}$$

That is, returns from investing risk-free in the U.S. should be equal, in expectation, to investing in the foreign country's riskfree rate. Of course, one first needs to convert

survey errors are forecastable, and that the forecastable component predicts excess return on bonds, in the decreasing pattern shown in Table 10. Cieslak (2018) and Piazzesi et al. (2015) show that, over the 1980–2010 period, featuring a long decline in interest rates, survey expectations were systematically above the expectations formed using a model estimated on the entire period. Consistent with (49), it appears that investors kept expecting a reversion to higher interest rates (relatively speaking), despite years of evidence that such a reversion failed to occur.

[25] The potential for distorted beliefs to resolve exchange rate puzzles has also been noted by Gourinchas and Tornell (2004) and Burnside et al. (2011). The field of international finance offers a rich array of puzzles in which mis-specified beliefs could play a role, as shown in Dumas et al. (2017). Frankel and Froot (1987) offer survey evidence that is consistent with the explanation we propose here.

U.S. dollars into the foreign currency, and then back again in the following period.[26] Equation 54 can be rewritten as:

$$\frac{1+\tilde{R}_{t+1}}{1+R_{t+1}} = \left(E_t^*\left[\frac{S_t}{S_{t+1}}\right]\right)^{-1}. \tag{55}$$

Furthermore, no-arbitrage implies *covered* interest rate parity. That is, investing at the U.S. interest rate must equal buying the foreign currency today, investing at the foreign country's interest rate, and then converting back via a forward contract:

$$1+R_{t+1} = \frac{S_t}{F_t}(1+\tilde{R}_{t+1}). \tag{56}$$

Combining (55) and (56) implies that the so-called *forward discount* F_t/S_t is related to appreciation (or depreciation) in the exchange rate via the expectation:

$$\frac{F_t}{S_t} = \left(E_t^*\left[\frac{S_t}{S_{t+1}}\right]\right)^{-1}. \tag{57}$$

That is, high forward discounts indicate investors expect appreciation of the currency.

Equations 55 and 57, each constituting uncovered interest rate parity, have been extensively tested and found to fail in the time series and in the cross section of currencies. For example, Lustig et al. (2011) sort currencies on the basis of the left-hand-side of (57) and then compute subsequent changes in exchange rates. Contrary to (57), they find no relation between a high forward discount and future appreciation of the currency. Nor is there a relation between the interest rate differential (55) and future appreciation. What they do find is a relation between the forward discount and excess returns on the foreign currency.

[26] The risk-neutral investor cares about expected returns being equated in real terms. Thus, if $\Delta\pi_{t+1}$ is log inflation between t and $t+1$ in the U.S., (54) should have, inside the square brackets, $e^{\Delta-\pi_{t+1}}$. By using (54), we effectively assume (for simplicity) that U.S. inflation is uncorrelated with innovations in the foreign exchange rate.

Specifically, define the continuously-compounded excess return on the currency as

$$rx_{t+1} = \log\left(\frac{S_t}{S_{t+1}}(1+\tilde{R}_{t+1})\right) - \log(1+R_{t+1}). \tag{58}$$

Lustig et al. (2011) show sorting on the forward discount produces a large spread in excess currency returns in the next period. We show their results in the data row of Table 11. The currencies with the lowest forward discount have a subsequent excess currency return of -3%, while those with the highest have a return of 6%. However, their volatilities are approximately the same. This result parallels a time-series finding that high forward discounts (equivalently, high interest rate differentials), predict high excess returns on the currency (Backus et al., 2001).

To understand these results, we consider a very simple model for the exchange rate. Consider a set of countries indexed by j, $j = 1, \ldots, n$. Let $s_{jt} = \log S_{jt}$, and $\Delta s_{j,t+1} = s_{j,t+1} - s_{jt}$. Assume that

$$\Delta s_{j,t+1} = x_{jt} + \sigma_j u_{j,t+1}, \tag{59}$$

for some random variable x_{jt}, with $u_{j,t+1} \overset{iid}{\sim} N(0,1)$. It follows from (57) that the log of the forward discount $f_{jt} - s_{jt} = \log(F_{jt}/S_{jt})$ equals

$$f_{jt} - s_{jt} = x_{jt} - \frac{1}{2}\sigma_j^2 \tag{60}$$

Define continuously-compounded returns $\tilde{r}_{j,t+1} = \log(1+\tilde{R}_{j,t+1})$ and $r_{t+1} = \log(1+$

R_{t+1}). Consider the excess return on the foreign currency, defined in (58).

$$
\begin{aligned}
rx_{j,t+1} &= s_{jt} - s_{j,t+1} + \tilde{r}_{j,t+1} - r_{j,t+1} \\
&= -\Delta s_{j,t+1} + f_{jt} - s_{jt} && (61) \\
&= -x_{jt} - \sigma_j u_{j,t+1} + x_{jt} - \frac{1}{2}\sigma_j^2 && (62) \\
&= -\sigma_j u_{j,t+1} - \frac{1}{2}\sigma_j^2, && (63)
\end{aligned}
$$

where (61) follows from (56), and (62) imposes equality between the subjective and physical distribution, and uses (59). Thus continuously compounded excess returns are unpredictable. In the cross-section, $E[rx_{j,t+1}]$ depend only σ_j^2, a Jensen's inequality effect. On the other hand, exchange rates should be predictable by the forward discount, as is clear from comparing (60) and (59). This predictability is absent in the data.

Suppose instead that the true process for exchange rates is a random walk:

$$\Delta s_{j,t+1} = \sigma_j u_{j,t+1}.$$

In this case,

$$
\begin{aligned}
rx_{j,t+1} &= -\Delta s_{j,t+1} + f_{jt} - s_{jt} && (64) \\
&= -\sigma_j u_{j,t+1} + f_{jt} - s_{jt} && (65)
\end{aligned}
$$

where $u_{j,t+1}$ is an iid shock. Clearly excess returns on currencies will be forecastable, both in the time series and the cross-section, by the forward discount. Table 11 shows that indeed this is the case, and that the model replicates the magnitude of the cross-sectional relation.[27]

[27]The results of Lustig et al. (2011) indicate an HML-type factor in currency premia. To capture this common factor, one could proceed as in Section 6.1 and model differential loadings on a common forecast.

Note that, unlike previous results, the one-period nature of the forward-premium regressions implies that we need not specify a process for x_{jt}. Bringing in term structure information, such as in Lustig et al. (2018), would help pin down such a process. We leave such extensions to future work.

7 Conclusion

Like the pigeons in Skinner's classic (1948) experiment, investors discover meaning in randomness. In this paper, we have shown that this simple insight has far-reaching consequences for asset pricing. An asset price is today's forecast of the future outcome of a random process, such as a company's dividend, or a country's exchange rate. Any information investors *think* they have about this future outcome will be in today's price. And yet if the process in question is not in fact forecastable, the price will adjust to meet reality, rather than reality adjusting to meet the price. We have shown, in four distinct settings, that the former is what occurs. For the aggregate stock market, prices have adjusted to meet dividends. For the cross-section of stocks, those with high prices relative to earnings see their prices fall. Long-term bond prices adjust to meet stable short-term interest rates, rather than the other way around. Forward prices of currencies adjust to meet spot prices.

A difficult and interesting question is how investors form their expectations. We have shown that, regardless of the specifics of this process, a tendency to find structure in randomness leaves a signature pattern in asset prices, one which we can observe in a strikingly consistent way.

Appendix

A Bayesian analysis of predictive regressions

Consider the predictive system

$$y_{t+1} = \beta_0 + \beta x_t + u_{t+1} \tag{A.1}$$

$$x_{t+1} = \phi_0 + \phi x_t + v_{t+1} \tag{A.2}$$

The agent observes $x_0, \ldots, x_T$ and $y_1, \ldots, y_T$, perhaps because y_t represents a return or a growth rate (and so one observation is lost relative to x_t). We assume

$$\begin{bmatrix} u_t \\ v_t \end{bmatrix} \overset{iid}{\sim} N(0, \Sigma) \tag{A.3}$$

for a positive-semidefinite matrix Σ, representing the variance-covariance matrix.

Define

$$B = \begin{bmatrix} \beta_0 & \phi_0 \\ \beta & \phi \end{bmatrix}$$

and $\mathbf{x}_t, \mathbf{y}_t$, analogously to Section 5. Let $\mathcal{L}$ denote the joint likelihood of the data. The agent forms the posterior

$$p(B, \Sigma \,|\, \mathbf{x}_T, \mathbf{y}_T) \propto \mathcal{L}(\mathbf{x}_T, \mathbf{y}_T | B, \Sigma) p(B, \Sigma), \tag{A.4}$$

where $\propto$ denotes up to a factor that does not depend on B and Σ.

We make the following assumptions, to reduce the problem to the one considered in Section 5.

Assumption 1. *The matrix Σ is diagonal.*

Assumption 2. *The parameters β_0, β_1 and σ_u are independent, under the prior, of*

ϕ_0, ϕ_1 *and* σ_v*, where* σ_u^2 *is the first, and* σ_v^2 *the second, diagonal element of* Σ*.*

As shown below, these assumptions guarantee strict exogeneity of x_t in relation to y_t. If these assumptions hold approximately, i.e. if the contemporaneous correlation between y_{t+1} and x_{t+1} is small, then it is likely that inference will not be strongly effected. See Wachter and Warusawitharana (2015) for the analysis when these don't hold, as is the case when y_t represents stock returns.

We now show the marginal posterior for β_0, β and σ_u reduces to (37). Define $l(x_{t+1}, y_{t+1} \,|\, x_t, B, \Sigma)$ as the likelihood of the time-t observation. Note that (A.1–A.3) imply

$$l(x_{t+1}, y_{t+1} \,|\, x_t, B, \Sigma) = l(x_{t+1}, y_{t+1} \,|\, \mathbf{x}_t, \mathbf{y}_t B, \Sigma).$$

Conditional probability calculations imply

$$\mathcal{L}(\mathbf{x}_T, \mathbf{y}_T | B, \Sigma) = \prod_{t=0}^{T-1} l(x_{t+1}, y_{t+1} \,|\, x_t, B, \Sigma) l(x_0 \,|\, B, \Sigma),$$

where we use $l(x_0 \,|\, B, \Sigma)$ to denote the likelihood of the initial observation.

Assumption 1 implies that, conditional on x_t and on the parameters, y_{t+1} is independent of x_{t+1}. We can factor l as follows:

$$\begin{aligned} l(x_{t+1}, y_{t+1} \,|\, x_t, B, \Sigma) &= l(y_{t+1} \,|\, x_t, x_{t+1}, B, \Sigma) l(x_{t+1} \,|\, x_t, B, \Sigma) \\ &= l(y_{t+1} \,|\, x_t, B, \Sigma) l(x_{t+1} \,|\, x_t, B, \Sigma) \qquad \text{(A.5)} \\ &= l(y_{t+1} \,|\, x_t, \beta_0, \beta, \sigma_u) l(x_{t+1} \,|\, x_t, \phi_0, \phi_1, \sigma_v) \qquad \text{(A.6)} \end{aligned}$$

Note that (A.5) does indeed require Assumption 1. If this assumption does not hold, then realizations of x_{t+1} give additional information about the shocks u_{t+1}. Given (A.5), (A.6) follows from the form of (A.1) and the definition of Σ.

We apply Assumption 2 and (A.6) to find the following form of the posterior:

$$p(B, \Sigma \,|\, \mathbf{x}_T, \mathbf{y}_T) \propto \prod_{t=0}^{T-1} l(y_{t+1} \,|\, x_t, \beta_0, \beta, \sigma_u) \prod_{t=0}^{T-1} l(x_{t+1} \,|\, x_t, \phi_0, \phi, \sigma_v) l(x_0 \,|\, \phi_0, \phi_1, \sigma_v) \times p(\beta_0, \beta, \sigma_u) p(\phi_0, \phi, \sigma_v). \quad \text{(A.7)}$$

Furthermore,

$$p(B, \Sigma \,|\, \mathbf{x}_T, \mathbf{y}_T) = p(\beta_0, \beta, \sigma_u \,|\, \phi_0, \phi, \sigma_v, \mathbf{x}_T, \mathbf{y}_T) p(\phi_0, \phi, \sigma_v \,|\, \mathbf{x}_T, \mathbf{y}_T). \quad \text{(A.8)}$$

The right hand side of (A.7) factors into two terms, one of which depends on $(\beta_0, \beta, \sigma_u)$, and one of which depends on (ϕ_0, ϕ, σ_v). Thus we can write:

$$p(\phi_0, \phi, \sigma_v \,|\, \mathbf{x}_T, \mathbf{y}_T) \propto \prod_{t=0}^{T-1} l(x_{t+1} \,|\, x_t, \phi_0, \phi, \sigma_v) l(x_0 \,|\, \phi_0, \phi_1, \sigma_v) p(\phi_0, \phi, \sigma_v),$$

and, from (A.8),

$$p(\beta_0, \beta, \sigma_u \,|\, \mathbf{x}_T, \mathbf{y}_T) \propto \prod_{t=0}^{T-1} l(y_{t+1} \,|\, x_t, \beta_0, \beta, \sigma_u) p(\beta_0, \beta, \sigma_u).$$

This proves that (37) is the correct posterior.

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Table 1: Parameters Used in Simulations

Parameter	Risk Neutral	Disaster
Shock to realized log dividend growth σ_u	0.11	0.11
Shock to expected log dividend growth σ_v	0.01	0.01
Subjective persistence in expected log dividend growth ϕ	0.95	0.95
Time-discount factorδ	0.97	0.95
Expected dividend growth μ	0	0.02
Relative risk aversion γ	0	3.00
Disaster probability p	0	0.02
Disaster size $1 - e^{\xi}$	-	0.33

The table shows parameters used in the simulations. For the model with disasters, the agent has constant relative risk aversion with parameter γ. The physical distribution of aggregate consumption growth is the same as that of dividends growth and is not subject to bias. The model is simulated at an annual frequency.

Table 2: Empirical and Simulated Moments for the Aggregate Market, Full Sample

	Data	Model: Risk Neutral			Model: Disaster		
	1927-2017	5	50	95	5	50	95
$\sigma(R^m)$	0.20	0.20	0.23	0.26	0.17	0.19	0.22
AC of R^m	-0.01	-0.19	-0.02	0.14	-0.18	-0.01	0.16
$\sigma(d-p)$	0.45	0.30	0.46	0.71	0.21	0.33	0.52
AC of $d-p$	0.88	0.80	0.91	0.96	0.80	0.91	0.96
$\sigma(\Delta d)$	0.11	0.10	0.11	0.12	0.10	0.12	0.14
AC of Δd	0.19	-0.18	-0.01	0.16	-0.19	-0.01	0.16
$E[R^m]$	0.09	0.00	0.03	0.06	0.03	0.06	0.09
$E[R^f]$	0.01	0.03	0.03	0.03	0.01	0.01	0.01

We simulate 4000 samples each consisting of 91 years of data from the model with risk-neutral investors, and the model with risk-averse investors and rare disasters. The table reports moments from the 1927–2017 sample (second column), and medians, 5th percentile values, and 95th percentile values (remaining columns). R^m denotes the net return on the market, $d-p$ the log dividend-price ratio,Δd log dividend growth, and R^f the riskfree rate. AC refers to the first-order autocorrelation and $\sigma(\cdot)$ the standard deviation. The model is simulated at an annual frequency.

Table 3: Empirical and Simulated Moments for the Aggregate Market, Post-war

	Data	Model: Disaster, No Realization		
	1948-2017	5	50	95
$\sigma(R^m)$	0.17	0.16	0.19	0.22
AC of R^m	-0.07	-0.22	-0.02	0.17
$\sigma(d-p)$	0.42	0.19	0.31	0.50
AC of $d-p$	0.92	0.76	0.90	0.96
$\sigma(\Delta d)$	0.07	0.10	0.11	0.12
AC of Δd	0.24	-0.21	-0.01	0.18
$E[R^m]$	0.09	0.04	0.07	0.10
$E[r_f]$	0.01	0.01	0.01	0.01

We simulate 4000 samples each consisting of 70 years of data from the model with rare disasters. We remove samples that contain disaster realizations. The table reports moments from the 1947–2017 sample (second column), and medians, 5th percentile values, and 95th percentile values (remaining three columns). R^m denotes the net return on the market, $d-p$ the log dividend-price ratio,Δd log dividend growth, and R^f the riskfree rate. AC refers to the first-order autocorrelation and $\sigma(\cdot)$ the standard deviation. The model is simulated at an annual frequency.

Table 4: Predictability of Stock Market Excess Return, Full Sample

	Horizon in Years					
	1	2	4	6	8	10
	Panel A: Data 1927-2017					
β	0.07	0.16	0.26	0.36	0.49	0.59
t-stat	[1.39]	[1.95]	[2.72]	[2.71]	[2.86]	[2.75]
R^2	0.02	0.06	0.10	0.16	0.25	0.28
	Panel B: Risk Neutral Model					
β	0.09	0.18	0.34	0.48	0.60	0.71
5th percentile	0.02	0.05	0.09	0.12	0.15	0.17
95th percentile	0.22	0.41	0.72	0.97	1.16	1.30
R^2	0.04	0.07	0.14	0.20	0.25	0.29
	Panel C: Disaster Model					
β	0.11	0.22	0.41	0.58	0.73	0.86
5th percentile	0.03	0.07	0.12	0.16	0.20	0.21
95th percentile	0.25	0.46	0.85	1.13	1.39	1.60
R^2	0.04	0.08	0.15	0.21	0.26	0.30

This table reports predictive coefficients and R^2-statistics from regressions of the form

$$\sum_{i=1}^{H} r^m_{t+i} - r^f_{t+i} = \beta_0 + \beta(d_t - p_t) + \epsilon_{t+H},$$

where $r^m_{t+i} = \log(1 + R^m_{t+i})$ is the continuously-compounded aggregate market return between $t + i - 1$ and $t + i$, $r^f_{t+i} = \log(1 + R^f_{t+i})$ is the continuously-compounded Treasury Bill return between $t+i-1$ and $t+i$, and $d_t - p_t = \log D_t/P_t$ is the aggregate dividend-price ratio. Panel A reports results from the 1927–2017 sample. Panel B and Panel C report medians and 5th and 95th percentile values from simulated data for predictive regressions, and medians for R^2-statistics as described in Table 2. For the data panel, t-statistics are adjusted for heteroskedasticity and autocorrelation.

Table 5: Predictability of Stock Market Excess Return, Post-war

	Horizon in Years					
	1	2	4	6	8	10
	Panel A: Data 1948-2017					
β	0.10	0.20	0.28	0.40	0.51	0.59
t-stat	[2.27]	[2.59]	[2.90]	[2.91]	[2.87]	[2.71]
R^2	0.07	0.13	0.16	0.22	0.27	0.31
	Panel B: Disaster Model No Realization					
β	0.12	0.24	0.44	0.62	0.77	0.89
5th percentile	0.03	0.06	0.11	0.13	0.16	0.16
95th percentile	0.30	0.55	0.95	1.28	1.53	1.74
R^2	0.05	0.09	0.18	0.24	0.30	0.34

This table reports predictive coefficients and R^2-statistics from regressions of the form

$$\sum_{i=1}^{H} r^m_{t+i} - r^f_{t+i} = \beta_0 + \beta(d_t - p_t) + \epsilon_{t+H},$$

where $r^m_{t+i} = \log(1 + R^m_{t+i})$ is the continuously-compounded aggregate market return between $t + i - 1$ and $t + i$, $r^f_{t+i} = \log(1 + R^f_{t+i})$ is the continuously-compounded Treasury Bill return between $t+i-1$ and $t+i$, and $d_t - p_t = \log D_t/P_t$ is the aggregate dividend-price ratio. Panel A reports results from the 1947–2017 sample. Panel B and Panel C report medians and 5th and 95th percentile values from simulated data for predictive regressions, and medians for R^2-statistics as described in Table 3. For the data panel, t-statistics are adjusted for heteroskedasticity and autocorrelation.

Table 6: Predictability of Aggregate Dividend Growth, Full Sample

	Horizon in Years					
	1	2	4	6	8	10
			Panel A: Data 1927-2017			
β	-0.07	-0.10	-0.12	-0.14	-0.14	-0.13
t-stat	[-2.14]	[-1.54]	[-1.72]	[-1.85]	[-1.45]	[-1.18]
R^2	0.09	0.07	0.06	0.07	0.08	0.07
			Panel B: Risk Neutral Model			
β	0.00	0.00	0.00	0.00	-0.00	-0.00
5th percentile	-0.04	-0.09	-0.17	-0.24	-0.31	-0.37
95th percentile	0.05	0.09	0.17	0.24	0.32	0.39
R^2	0.01	0.01	0.02	0.03	0.04	0.05
			Panel C: Disaster Model			
β	-0.00	-0.00	-0.01	-0.00	-0.00	-0.00
5th percentile	-0.07	-0.13	-0.26	-0.38	-0.48	-0.59
95th percentile	0.07	0.14	0.26	0.38	0.49	0.59
R^2	0.01	0.01	0.02	0.03	0.04	0.05

This table reports predictive coefficients and R^2-statistics from regressions of the form

$$\sum_{i=1}^{H} \Delta d_{t+i} = \beta_0 + \beta(d_t - p_t) + \epsilon_{t+H},$$

where Δd_{t+i} is the change in log aggregate dividends between $t+i-1$ and $t+i$ and $d_t - p_t = \log D_t/P_t$ is the aggregate dividend-price ratio. Panel A reports results from the 1927–2017 sample. Panel B and Panel C report medians and 5th and 95th percentile values from simulated data for predictive regressions, and medians for R^2-statistics as described in Table 2. For the data panel, t-statistics are adjusted for heteroskedasticity and autocorrelation.

Table 7: Predictability of Aggregate Dividend Growth, Post-war

	Horizon in Years					
	1	2	4	6	8	10
	Panel A: Data 1948-2017					
β	-0.01	-0.01	-0.04	-0.08	-0.09	-0.12
t-stat	[-0.59]	[-0.29]	[-0.72]	[-1.00]	[-0.83]	[-0.86]
R^2	0.01	0.00	0.01	0.04	0.05	0.06
	Panel B: Disaster Model No Realization					
β	-0.00	-0.00	-0.00	0.00	-0.01	-0.01
5th percentile	-0.07	-0.15	-0.29	-0.41	-0.52	-0.61
95th percentile	0.08	0.15	0.28	0.42	0.52	0.63
R^2	0.01	0.01	0.03	0.04	0.05	0.06

This table reports predictive coefficients and R^2-statistics from regressions of the form

$$\sum_{i=1}^{H} \Delta d_{t+i} = \beta_0 + \beta(d_t - p_t) + \epsilon_{t+H},$$

where Δd_{t+i} is the change in log aggregate dividends between $t+i-1$ and $t+i$ and $d_t - p_t = \log D_t/P_t$ is the aggregate dividend-price ratio. Panel A reports results from the 1947–2017 sample. Panel B and Panel C report medians and 5th and 95th percentile values from simulated data for predictive regressions, and medians for R^2-statistics as described in Table 3. For the data panel, t-statistics are adjusted for heteroskedasticity and autocorrelation.

Table 8: Return statistics for value and growth portfolios

	1 (Low)	2	3	4	5 (High)	5 - 1
			Panel A: Data 1952-2017			
$E[R]$	6.46	7.61	8.96	11.34	13.65	7.19
t-stat	[2.72]	[3.73]	[4.25]	[4.86]	[4.79]	[3.46]
$\sigma(R)$	19.29	16.60	17.13	18.97	23.17	16.87
α	-2.05	-0.05	1.20	2.96	3.77	5.82
t-stat	[-1.99]	[-0.09]	[1.59]	[2.74]	[2.72]	[2.58]
β_{mkt}	1.03	0.93	0.94	1.01	1.19	0.17
			Panel B: Model			
$E[R]$	-0.14	-0.14	0.39	1.37	2.67	2.83
$\sigma(R)$	21.63	17.65	16.19	17.00	19.51	25.18
α	-1.01	-1.01	-0.42	0.57	1.89	2.93
β_{mkt}	1.07	1.02	0.99	0.97	0.95	-0.12

Each year we form portfolios based on the earnings-to-price ratio and compute value-weighted portfolio returns over the subsequent year. Panel A reports the mean, standard deviation, CAPM alpha and beta with respect to the market in annual data from 1952 to 2017. Panel B reports the 50th percentiles of these statistics over 1000 simulations of length designed to match the data, each with 1000 stocks in the cross section.

Table 9: Performance relative to a two-factor model

	1 (Low)	2	3	4	5 (High)
			Panel A: Data 1952-2017		
α	0.27	0.08	-0.05	0.95	0.27
t-stat	[0.57]	[0.12]	[-0.09]	[1.47]	[0.57]
β_{mkt}	1.10	0.93	0.90	0.96	1.10
β_{hml}	-0.40	-0.02	0.22	0.35	0.60
			Panel B: Model		
α	0.49	-0.30	-0.48	-0.18	0.49
β_{mkt}	1.01	1.00	1.00	1.00	1.01
β_{hml}	-0.52	-0.24	0.02	0.26	0.48

Each year we form portfolios based on the earnings-to-price ratio and compute value-weighted portfolio returns over the subsequent year. Panel A reports coefficients generated from the regression $r_{i,t} = \alpha + \beta_{hml} hml_t + \beta_{mkt} mkt_t$ where $r_{i,t}$ is the portfolio return in excess of the riskfree rate, hml_t is the return on the 5th quintile (high) minus that of the 1st quintile (low), and mkt_t is the average excess return on all 5 portfolios. Panel B reports the 50th percentiles of those coefficients over 1000 simulated samples of length designed to match the data. Data are annual, from 1952 to 2017.

Table 10: Moments of Bond Yields

	Maturity in Years				
	1	2	3	4	5
Panel A: Data 1952-2017					
β_n		1.61	2.13	2.39	2.50
t-stat		[2.92]	[3.51]	[3.81]	[3.60]
$\sigma(y_n)$	3.10	3.05	2.97	2.92	2.85
$AC(y_n)$	0.88	0.90	0.90	0.91	0.91
$\sigma(y_n - y_1)$		0.33	0.54	0.69	0.81
$AC(y_n - y_1)$		0.40	0.46	0.52	0.55
Panel B: Model					
β_n		1.45	1.29	1.17	1.08
$\sigma(y_n)$	2.80	2.03	1.54	1.22	1.00
$AC(y_n)$	0.85	0.85	0.85	0.85	0.85
$\sigma(y_n - y_1)$		0.77	1.26	1.58	1.80
$AC(y_n - y_1)$		0.85	0.85	0.85	0.85

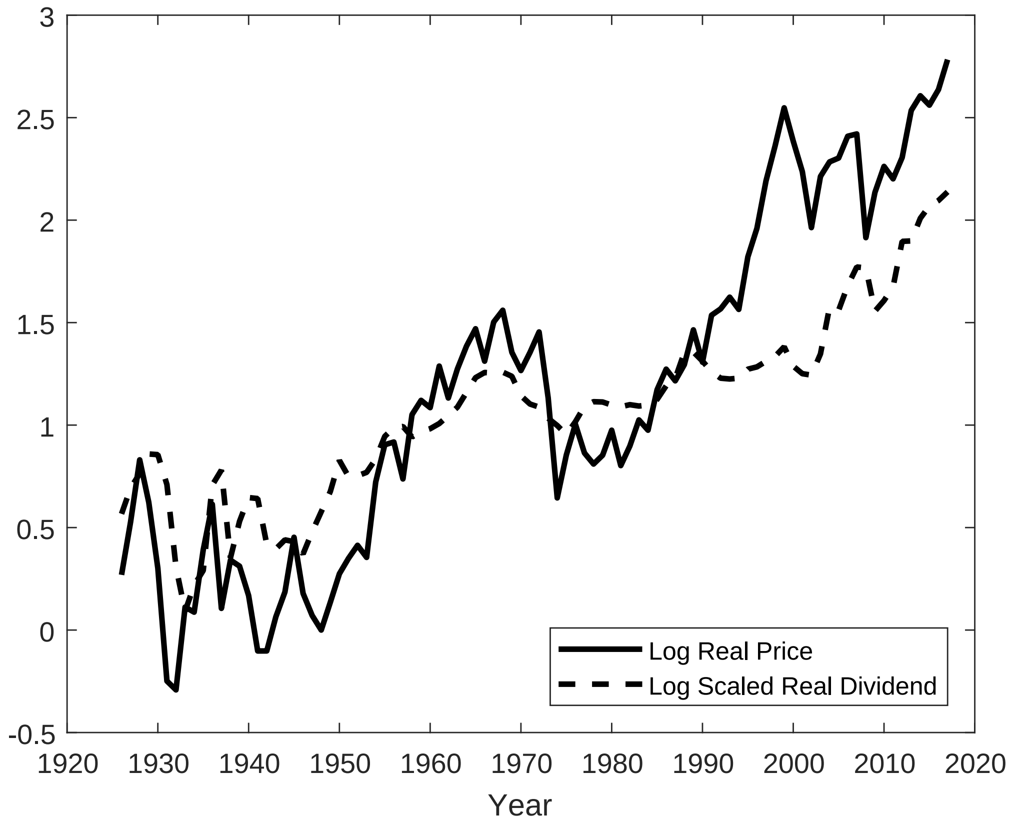
Panel A of the table reports the volatility and the first-order autocorrelation of zero-coupon bond yields and yields spread, as well as the regression coefficients β_n as in $rx_{n,t+1} = \alpha_n + \beta_n(y_{nt} - y_{1t}) + \epsilon_{t+1}$, where $rx_{n,t+1}$ is the return of n-year bond in excess of y_1 over period $t+1$. The t-statistics adjust for heteroskedasticity. Panel B report the percentiles of those moments computed over 1000 simulations, each with 66 years of length. Data are from 1952 to 2017.

Table 11: Moments of Portfolios Sorted on Forward Discount

	1 (Low)	2	3	4	5	6 (High)
Panel A: Data 1983-2008						
$\mu(rx)$	-2.92	0.02	1.40	3.66	3.54	5.90
$\sigma(rx)$	8.22	7.36	7.46	7.53	7.85	9.26
$\mu(f-s)$	-3.90	-1.30	-0.15	0.94	2.55	7.78
$\sigma(f-s)$	1.57	0.49	0.48	0.53	0.59	2.09
Panel B: Model						
$\mu(rx)$	-7.80	-4.66	-2.86	-1.15	0.71	3.80
$\sigma(rx)$	8.24	8.22	8.19	8.20	8.20	8.23
$\mu(f-s)$	-7.82	-4.71	-2.84	-1.16	0.71	3.81
$\sigma(f-s)$	1.07	0.85	0.79	0.79	0.85	1.07

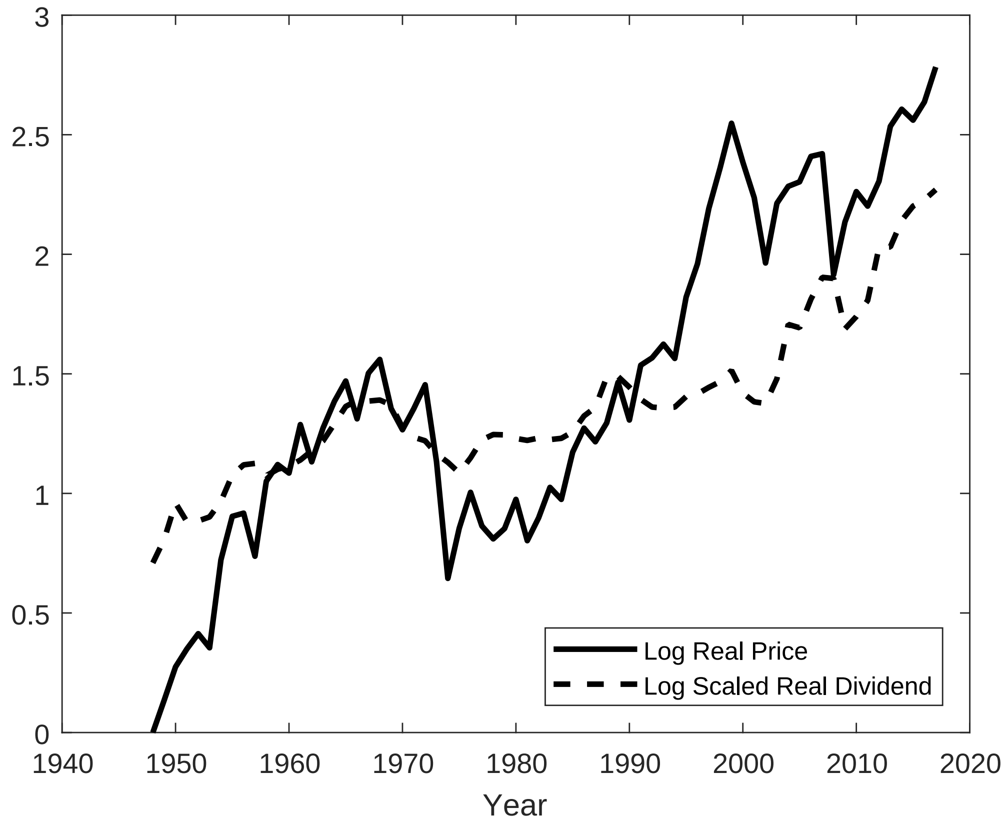
Panel A of the table reports means and standard deviations of average log excess currency returns rx and log forward discount $f-s$ within each of 6 currency portfolios formed on the forward discount. Data, from Lustig et al. (2011), are monthly, from 1983–2018. Panel B reports the 50th percentiles of those moments over 1000 simulations of the model, each with 293 monthly observations.

Figure 1: Log Real Dividend and Log Real Price Level of the Aggregate Market, Full Sample



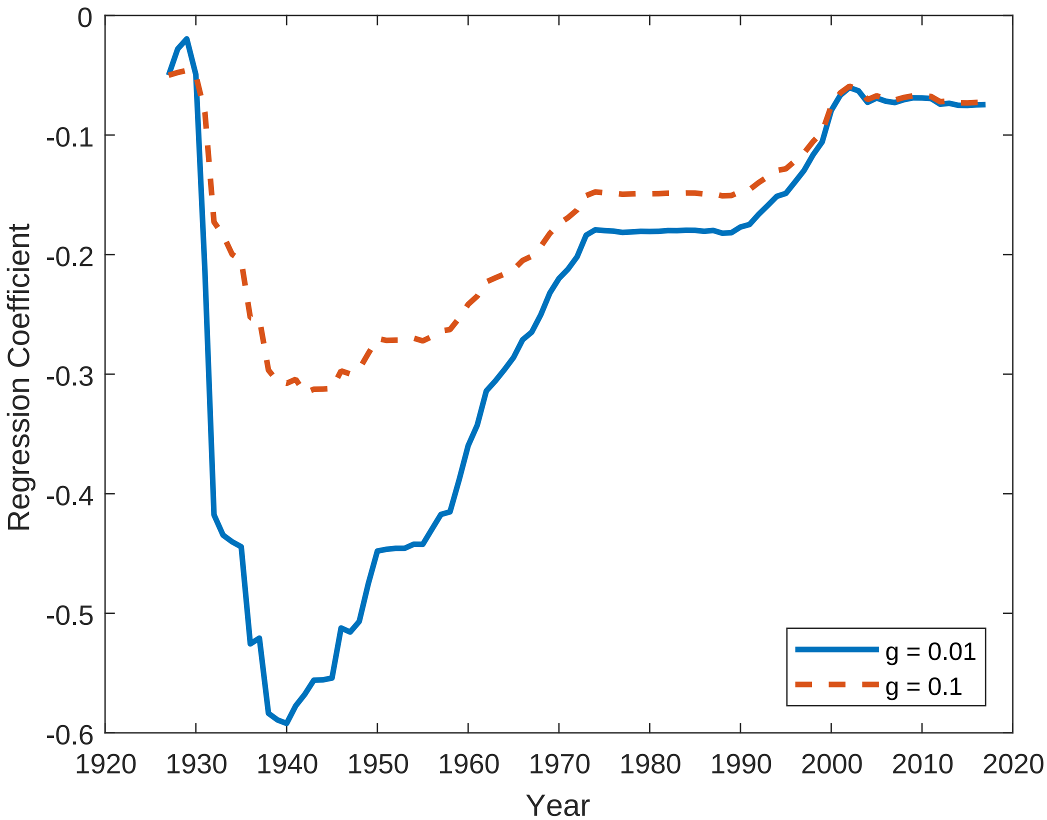
This figure plots the annual frequency log price level and log dividend of the US stock market in the post-1926 era. The log real dividend is multiplied by 27.43, the mean P/D ratio post-1926. The dividend and the price are adjusted for inflation.

Figure 2: Log Real Dividends and Log Real Price Level of the Aggregate market, Post-War



This figure plots the annual frequency log price level and log dividend of the US stock market in the post-1948 era. The log real dividend is multiplied by 27.43, the mean P/D ratio post-1948. The dividend and the price are adjusted for inflation.

Figure 3: Predicting dividend growth using the dividend-price ratio



This figure shows the posterior mean of the predictive coefficient in a regression of one-year ahead dividend growth on the dividend-price ratio. The posterior mean is calculated using Bayesian methods, assuming an informative prior, where g indexes the degree of informativeness. For each year in the sample, the agent uses all available data to form a posterior for the predictive coefficient. Data begin in 1927. A prior parameter of $g = 0.1$ implies that the prior mean of the coefficient receives a weight of 10% relative to the sample estimate, whereas a prior parameter of $g = 0.01$ implies that the prior mean receives a weight of 1%.