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ABSTRACT

Corporate versus pass-through status trades off benefits (perpetual identity, limited liability, public trading, earnings retention) against tax wedges, estimated from U.S. taxes on corporate profits, dividends, and partnership income. In regressions, C-corporate economic shares decline with the wedge and exhibit negative trends related to legal changes for LLCs. A calibrated model, fit to TFP and corporate shares, implies that, for 1958-2013, the declining wedge and gap between corporate and pass-through productivity contributed 0.37% per year of a TFP growth rate of 1.09%. From 1994 to 2004, the falling productivity gap contributed 0.77% per year to the TFP growth of 2.00%.

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In an assessment of the 2017 U.S. tax reform, Barro and Furman (2018) focused on the incentives of businesses to invest in capital within a given legal form of organization, which comprised C-corporations and pass-through businesses. In addition, the analysis considered the impact of differential taxation of C-corporations versus pass-throughs on the choice of legal form. Reductions in the relevant tax wedge, as in the 2017 reform, raised the frequency of C-corporate ownership. Moreover, if there are typically productivity advantages associated with C-corporate form—as must be the case if many businesses chose this status despite the often large tax penalty—then shifts in ownership form affect overall productivity. This paper assesses the effects of business taxation and legal changes on choices of legal form and, thereby, on productivity.

I. Previous Research

A. Basic framework

The seminal paper on choices of C-corporate versus pass-through status is Mackie-Mason and Gordon (1997). That study emphasized tax effects on the C-corporate share of economic activity. Other empirical research in this area includes Goolsbee (1998), Goolsbee (2004), and Prisinzano and Pearce (2018).

The framework, following Mackie-Mason and Gordon (1997), assumes that firm i has output (or productivity) $Y_c(i)>0$ in corporate (meaning C-corporate) form and $Y_p(i)>0$ in non-corporate (pass-through) form. The respective tax rates, taken here as proportionate to output, are $\tau_c<1$ and $\tau_p<1$. Negative tax rates, constituting subsidies, can be admitted. The tax rates for each legal form are assumed to be the same for all firms, but a dispersion of tax rates can be admitted. Firm i opts for corporate form if

(1) $$(1-\tau_c)Y_c(i)\geq\left(1-\tau_p\right)Y_p(i).$$

This condition is analogous to the one used by Mackie-Mason and Gordon (1997, equations [1] and [2]).

We can rewrite equation (1) as

(2) $$y(i) \equiv \log\left(\frac{Y_c(i)}{Y_p(i)}\right) \geq \log\left(\frac{1-\tau_p}{1-\tau_c}\right) \equiv \tau.$$

The term on the far right is the tax wedge, τ, which is the tax penalty for being corporate rather than pass-through. Equation (2) says that if this wedge is positive, a business has to enjoy at least the offsetting proportionate productivity advantage, *y(i)*, in order to opt for corporate form. If the magnitudes of τ_c and τ_p are much less than one, then $\tau \approx \tau_c - \tau_p$. Generally, τ is increasing in τ_c and decreasing in τ_p.

If tax rates are the same for all firms, the determinant of choices of legal form is the frequency distribution of the proportionate productivity advantage, *y(i)*. In the overall population of firms, the fraction opting to be corporate is one minus the cumulative density of *y(i)* evaluated at the cutoff τ. More generally, the distribution of *τ(i)* also matters.

B. Comments on the setup

The framework treats a firm's potential outputs, $Y_c(i)$ and $Y_p(i)$, as dependent only on the choice of legal form of organization. We could extend the model to include variable factor inputs, such as labor and capital chosen by corporate and pass-through businesses.[1] The tax rates τ_c and τ_p would then have the usual effects on quantities of inputs demanded. If taxes are levied on net business income with labor payments fully expensed, then the tax rates would not directly distort the margins associated with labor input. Similarly, if capital outlays were fully expensed

[1]See, for example, Gravelle and Kotlikoff (1989).

(with loss realizations fully allowed), there would be no direct distortion on the margins associated with capital input (as in King and Fullerton [1984]).

In the present setup, the only distortion comes from the difference in the tax rates levied on corporations and pass-through businesses; that is, the tax wedge, τ, given in equation (2). In this setting, the first-best outcome corresponds to $\tau=0$, because distortions arise only when the two organizational forms are taxed at different rates. The levels of taxation—τ_c and τ_p—are not distorting as long as the tax rates are equal. In particular, the government can raise more revenue in a non-distorting way by raising both tax rates while keeping the wedge, τ, unchanged. This property means that the framework is not amenable to usual optimal-tax analysis, which requires an extension to allow for distortions from levels of tax rates. However, that extension would leave the distortion associated with differential taxation, and that effect is likely to be largely separable from those involving levels of tax rates. Hence, it seems desirable to retain the framework of Mackie-Mason and Gordon (1997), which abstracts from choices of factor inputs and in which only differential taxation matters.

II. The Tax Wedge

To implement equation (2) for U.S. business data, we need a time series for the tax wedge, τ, which depends on the tax rates τ_c on C-corporations and τ_p on pass-through businesses. Conceptually, we think of a business as choosing whether to have a block of income, $Y_c(i)$, taxed on a C-corporate basis or else have a corresponding block, $Y_p(i)$, taxed on a pass-through basis. Although this choice involves a discrete amount of income accruing in one form or the other, the income in each case is "marginal" with respect to other forms of income that owners have. For example, owners of a C-corporation (shareholders) typically have labor income and other types of asset income. Similarly, in most cases, owners of a pass-through business have significant income in other forms. For this reason, the relevant tax rates τ_c and τ_p correspond more closely to marginal than to average rates.

A. C-corporate tax rate

We consider two aspects of U.S. taxation of C-corporate income: the federal corporate profits tax and the federal tax on dividends.[2] The rate τ_{prof} in Figure 1 is the top federal rate on C-corporate profits. The use of the top rate ignores the graduation in the tax schedule that prevailed from 1937 to 2017. However, we find from IRS, *Statistics of Income, Corporation Income Tax Returns* that the average marginal tax rate (AMTR) on C-corporate taxable income (where individual marginal rates are weighted by shares in C-corporate taxable income) is close to the top rate, at least from 1958 to 2013. For example, in 1958, when the top rate was 52%, the AMTR was 50.6%, and in 2013, when the top rate was 35%, the AMTR was 34.7%.[3]

We also factor in the double-taxation of C-corporate income, whereby owners are taxed in a second stage on dividends and capital gains. One familiar point is that dividend payouts and, hence, owners' dividend taxes can be deferred by corporate retention of earnings (an option unavailable for pass-through businesses). This corporate retention leads to increases in stock prices, which result in capital-gains taxes if owners choose to realize their gains. However, in a reasonable baseline setting, retained earnings affect the timing of dividends but not their present

[2]A full analysis would also consider the deductions from corporate taxable income associated with depreciation allowances, expensing, credits, and bond interest. These elements are considered in Barro and Furman (2018) and in many other papers. However, because these features of the tax law have been the same for C-corporate and pass-through businesses, they may not have major consequences for the choice of legal form.

[3]We are neglecting the taxation of corporate profits by state governments. Our analysis indicates that the average state tax rate on C-corporate profits (after factoring in the deductibility of these levies for federal purposes) was between 0.04 and 0.05 from 1979 to 2017, compared to the top federal rate between 0.34 and 0.46. Goolsbee (2004) has examined corporate-profits taxes across states.

value. Or, to put it alternatively, the present value of capital gains and losses created by retentions equals zero.[4] In this context, we can neglect capital-gains taxes as an approximation.[5]

Another well-known point is that double-taxation of C-corporate income can be mitigated or eliminated by replacing dividends with stock repurchases. Despite this option, C-corporations typically choose to pay dividends, and the tax rate on these payments should enter into the computed C-corporate tax rate. This calculation has to consider the tax status of shareholders with regard to dividend income. In this context, Rosenthal and Austin (2016, Figure I) document a large and increasing share of U.S. corporate stock held by entities that have zero or low tax rates, including retirement plans, non-profits, and foreigners, whom they treat as non-taxable. Our analysis assumes instead that the fraction of foreign holdings held in taxable form equals that of domestic holdings. In that case (when we also use data from Poterba [2004, Table 1]), we get the series for the estimated fraction of U.S. corporate stock held in taxable form as the red graph in Figure 2. This fraction declined from 88% in 1958 to 30% in 2015. We assume that this share for ownership of corporate stock equals the share of dividends accruing to taxable entities.

We constructed a dividend-income-weighted average marginal federal income-tax rate on dividends (or dividend AMTR), a concept that parallels the one used in previous research for the labor-income-weighted average marginal tax rate (described in Barro and Sahasakul [1983] and Barro and Redlick [2011]). Before the sharp cut in the qualified-dividend tax rate in 2003, the

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[4]These results apply if a corporation's expected rate of return on retained earnings equals the discount rate used to price the corporation's stock. We also need a transversality or no-Ponzi condition, which requires the discount rate, asymptotically, to exceed the expected growth rate of dividends. This last condition parallels one needed for Ricardian equivalence—that is, so that government borrowing does not affect the present value of taxes that have to be collected.

[5]We are neglecting here any tax advantage from the deferral of dividend payments. However, this advantage may not exist, because returns on funds held within a C-corporation are taxed at the rate on C-corporate profits, whereas those held by owners are taxed at the individual income tax rate.

dividend AMTR, shown by the blue graph in Figure 2,[6] is higher than the labor-income-weighted AMTR, which appears in Figure 3 (red graph).

To measure the dividend tax rate, τ_{div}, we multiplied the fraction of corporate stock held in taxable form (red graph in Figure 2) by the dividend AMTR (blue graph) to get the green graph.[7] In the theory, the contribution of C-corporate taxation to the tax wedge, τ, in equation (2) should enter as $\log(1-\tau_c) = \log(1-\tau_{prof}) + \log(1-\tau_{div})$, where τ_{prof} is from Figure 1 and τ_{div} is the green graph from Figure 2.

B. Pass-through tax rate

The pass-through entities that we focus on are S-corporations and partnerships. Sole proprietorships are not covered in our main data. Incomes from S-corporations and partnerships mostly flows through to owners and are reported by individuals on IRS form 1040, Schedule E. By using information provided by Dan Feenberg from the National Bureau of Economic Research's TAXSIM program, we were able to measure the Schedule E income-weighted average marginal tax rate (AMTR) from 1962 to 2012. This calculation first computes the additional federal tax on a sampled return generated by a small hypothetical increment to Schedule E income, holding constant other income and deductions.[8] The individual marginal tax rates are then averaged using Schedule E income as weights. Note that the income reported on Schedule E excludes portfolio income passed through to partners. To get close to the incomes

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[6]The data for calculating the dividend AMTR from 1960 to 2012 were provided by Dan Feenberg, based on the National Bureau of Economic Research's TAXSIM program. The value for 2013 was unavailable and was assumed to equal that for 2012. Values before 1960 were estimated by Tatjana Kleinberg, using issues of IRS, *Statistics of Income, Individual Income Tax Returns*. A similar measure of the dividend AMTR (but including state income taxes) appears in Poterba (2004, Table 1).
[7]Our assumption in this calculation is that taxable foreign stock holdings face the same marginal tax rate on dividends as that on domestic holdings.
[8]The assumption is that the additional income is "earned income," which matters for the top income-tax rate from 1971 to 1981. The increment to taxes includes self-employment tax.

derived from ownership of S-corporations and partnerships, we excluded amounts reported on Schedule E from rents, royalties, and estates & trusts. The resulting Schedule E AMTR is shown as the blue graph in Figure 3.[9]

Figure 3 also shows two series that serve as comparisons for the Schedule E AMTR. The green graph is the top marginal federal income-tax-rate on earned income, which is distinguished from ordinary income for 1971-1981. The red graph, mentioned before, is the labor-income weighted federal AMTR. This series comes from Barro and Sahasakul (1983), Barro and Redlick (2011), and the Tax Policy Center. Related marginal-tax-rate data have been used in macroeconomic research by Mertens and Ravn (2013) and Mertens and Montiel Olea (2018).

The AMTR weighted by Schedule E income (blue graph in Figure 3) is notably higher than that weighted by labor income (red graph). From 1962 to 2012, the Schedule E AMTR averaged 0.34, while that based on labor income averaged 0.24. This pattern arises particularly because Schedule E income is concentrated in returns that have relatively high overall taxable income and, therefore, face relatively high marginal tax rates. However, the Schedule E AMTR is also markedly lower than the top marginal tax rate on earned income until the top rate fell sharply after 1970. For example, in 1962, when the top rate was 91%, the Schedule E AMTR was only 37%, and in 1967, when the top rate was 70%, the Schedule E AMTR was only 36%. In recent years, the Schedule E AMTR is close to the top rate—values for 2012 were 35% for the top rate and 32% for the Schedule E AMTR. (Note that, unlike the top-rate variable, the Schedule E AMTR includes self-employment tax.)

Figure 4 combines the various components to get an overall tax wedge associated with the choice between C-corporate and pass-through legal status. The blue graph corresponds to the

[9]Because of missing data, our empirical analysis assumes that the Schedule E AMTR for 1958-61 equals the value for 1962 and the value for 2013 equals that for 2012.

C-corporate profits tax, τ_{prof} (from Figure 1), the red graph to the dividend tax rate, τ_{div} (Figure 2), and the green graph to the Schedule E tax rate, τ_p (Figure 3). The black graph shows the overall tax wedge, given from equation (2) by $\tau = \log(1- \tau_p) - \log(1-\tau_{prof}) - \log(1- \tau_{div})$. The overall wedge, τ, peaked at 0.77 in 1961, then fell gradually to reach 0.090 in 2013. The average from 1958 to 2013 was 0.37.

C. A short U.S. legal history related to benefits of corporations and pass-throughs

We provide a list of productivity benefits associated with corporate legal ownership. We include historical context on how U.S. legal changes have shifted these benefits when compared to those arising from pass-through alternatives.

1. A corporation is a distinct and perpetual legal entity, the structure of which—unlike the typical partnership—is not compromised substantially by the departure of its owner(s). Schwartz (2012) and Stout (2019) discuss corporate perpetual existence and make a strong case for its importance in preventing the fractionalization of capital and thereby promoting economic development.[10]

2. C-corporations offer the potential for convenient public trading of shares, typically on organized markets. This public trading is important for the raising of capital and for gaining information from market prices. Starting in 1981 with Apache Petroleum, public trading became available for a narrow range of business for limited partners in the form of master limited partnerships (MLPs) and publicly traded partnerships (PTPs). Subsequently, these types of ownership were available mostly for companies operating in the energy sector, due to rules

[10]Kuran (2004) emphasizes this feature in his analysis of why the lack of corporate law hindered economic development in Muslim countries after the industrial revolution.

on a company's sources of income specified in a 1987 federal law. However, some financial firms, such as Blackstone, qualified for MLP status. (Blackstone recently shifted to C-corporate form, following the 2017 tax reform and the apparently successful transformation of KKR into a C-corporation in 2018.[11])

3. C-corporations provide owners with limited liability, a status that applies also to pass-through S-corporations, created in 1958. However, S-corporations have major limitations on numbers and types of shareholders—although the allowable number increased substantially over time from the original 10 to the current 100.[12] As an example of restrictions, S-corporate shareholders have to be U.S. entities and cannot be corporations or partnerships. In addition, there can be only one class of stock with respect to rights to distributions and liquidation proceeds. The major pass-through alternative to the S-corporation is the partnership, which does not feature the restrictions on ownership that apply to S-corporations. The partnership form has a long history, but a key innovation was the invention of the limited liability company (LLC) in Wyoming in 1977. LLCs, which offer limited liability for owners,[13] are regulated at the state level and became popular after an IRS ruling in September 1988 that LLCs could be taxed as pass-through partnerships. Another landmark is the presence by 1996 of LLC laws in all 50 states. For a discussion of the history of LLCs, see Hamill (2005). Although an LLC is not formally a partnership, it effectively functions that way—

[11]See *Institutional Investor*, July 19, 2018.
[12]The number rose to 15 in 1976, 25 in 1981, 35 in 1983, 75 in 1997, and 100 in 2004. Additionally, the effective number of shareholders was expanded by the treatment of some family units as constituting a single shareholder. See Sicular (2014).
[13]In contrast, limited partnerships have limited liability for limited partners but not for the general partner(s), who actively manage the business and have signing authority.

and the IRS data on "partnerships" include most LLCs.[14] The growth in the share of partnerships in limited-liability form—more specifically, in LLCs—has been dramatic since the mid-1990s. DeBacker and Prisinzano (2015, Figure 9) report that, from 1996 to 2011, the share of partnerships with limited liability rose from 30% to 80%, and the share that were LLCs went from 10% to 64%.

4. The retention of earnings is permissible in C-corporations. This retention may be useful for financing of investment and for deferring taxes on dividends. Partnerships and S-corporations can also retain earnings, but the owners are taxed as though all the earnings had been distributed.

5. C-corporations and pass-through businesses have numerous differences in regulations, filing rules, requirements to hold regular meetings, and government supervision.

Overall, U.S. legal changes over recent decades have mostly favored pass-through alternatives to C-corporations—notable here are the invention of the S-corporation in 1958, the IRS tax ruling in 1988 that allowed LLCs to be taxed as pass-through partnerships, and the adoptions by all 50 states of LLC laws by 1996.

III. C-Corporate Shares of Economic Activity

We have several empirical measures of the C-corporate share of businesses' economic activity, based on publicly available IRS data and mostly from 1958 to 2013.[15] Figures 5-7 apply to stock measures of business economic activity—net capital stocks, equity (book value),

[14]A multiple-member LLC that opts for partnership (rather than C-corporate) tax status files the usual partnership information return, Form 1065.

[15]The data we use exclude economic activity by governments, non-profits, real estate investment trusts (REITs), and regulated investment companies (RICs).

and gross assets. These variables are available for C-corporations, S-corporations, and partnerships, but not for sole proprietorships. The partnership numbers on net capital stocks and equity were interpolated for part of the sample, based on data available from the IRS every two years from 1959 to 1975 and annually for 1977-1982 and 1988-2013.

Figure 5 has shares of business net capital stocks. The C-corporate share was 0.95 in 1958 and trended downward to 0.53 in 2013. The main offsetting increase was in the partnership share, which went from 0.04 in 1958 to 0.40 in 2013. Legal changes noted before, especially for LLCs, likely explain much of this trend. The share for S-corporations was 0.004 in 1958 (the first year of existence), rose to 0.025 in 1986, then jumped upward to 0.074 by 1999. This share then fell to 0.067 in 2013, probably because of increased competition from LLCs.

Figure 6 has shares of business equity (book value). The trends are similar to those for net capital stocks, although the share of C-corporations in equity at 0.69 in 2013 exceeds that for net capital stocks. Correspondingly, the partnership share of equity in 2013 was 0.29. There is also more of an indication that S-corporations are being driven out of the market, with the share down to 0.024 in 2013, compared to a peak of 0.038 in 1990. Eventually, the attractiveness of the LLC may make the S-corporation obsolete.

Figure 7 has shares for business gross assets. This concept of C-corporate share was used by Mackie-Mason and Gordon (1997). The trend in business gross assets is similar to those for net capital stocks and equity, but the C-corporate share of gross assets has not declined as much—the share in 2013 was 0.75, whereas that for partnerships was 0.22. The S-corporation share of gross assets in 2013 was 0.033, compared to 0.037 in 1990.

Figure 8 applies to positive net income (excluding businesses with losses), which is a flow measure of economic activity. In this case, data are available for sole proprietorships.

Variants of these data were used by Mackie-Mason and Gordon (1997) and Prisinzano and Pearce (2018). A serious problem with these data, noted by Prisinzano and Pearce (2018, Section 2.1), is double-counting—arising especially because partnerships are owned partly by corporations and other partnerships. The share numbers are also highly volatile because of strong sensitivity of the various forms of net income to the business cycle (and the volatility is even more pronounced when businesses with losses are included). These difficulties cause us to discount the results for net income compared to those obtained with our other concepts. However, it is worth noting that recent trends in Figure 8 are similar to those in Figures 5-7. From 1979 to 2013, the C-corporate share of positive net income fell from 0.71 to 0.43, the partnership share rose from 0.09 to 0.31, the S-corporate share increased from 0.02 to 0.15 (but has been flat since 2001), and the sole proprietor share fell from 0.18 to 0.11.

IV. Formal Model of Choice of Business Legal Form

We now describe our formal model of choice of business legal form. The analysis emphasizes the frequency distribution of the corporate productivity advantage, $y \equiv \log(\frac{Y_c}{Y_p})$, which appears in equation (2) (where we now drop the index i). At a point in time, the distribution of y depends on the legal/regulatory framework, which applies to C-corporations and pass-through alternatives. Over time, changes in laws and regulations can shift the distribution of y. Implicitly, we are also holding constant the structure of production across sectors. Changes in this composition can affect the distribution of y. For example, benefits from corporate form may be more useful in some types of business—such as those with larger scale benefits or greater dependence on credit—than in others.

We assume that $log(Y_c)$ and $log(Y_p)$ are distributed bivariate normal with respective means and standard deviations of μ_c, σ_c, μ_p, and σ_p and a correlation coefficient between the two random variables of ρ. This specification implies that $y \equiv \log(\frac{Y_c}{Y_p})$ is distributed normally with mean μ=μ_c- μ_p and variance $\sigma^2 = \sigma_c^2 + \sigma_p^2 - 2\rho\sigma_c\sigma_p$. The fraction of firms that opt to be pass-through is the cumulative normal value for y at the cutoff τ, and the fraction corporate is one minus this cumulative normal value.[16]

The overall level of output, Y, is given by

(3) $$Y = prob(y \geq \tau) \cdot E(Y_c|y \geq \tau) + prob(y < \tau) \cdot E\left(Y_p \middle| y < \tau\right),$$

where, on the right, the first term is the level of output in the corporate sector, and the second term is the level of output in the pass-through sector. The corporate share of output is the ratio of the first term on the right to Y.

To make the calculations in equation (3), we have to derive the expectations of Y_c, conditional on $y \equiv \log(\frac{Y_c}{Y_p}) \geq \tau$, and of Y_p, conditional on $y < \tau$. The appendix shows that the expectation of Y_c, conditional on y, is given by:

(4) $$E(Y_c|\text{y}) = \exp\{\mu_c + \left(\frac{1}{\sigma^2}\right) \cdot \left[\sigma_c \cdot \left(\sigma_c - \rho\sigma_p\right) \cdot (y - \mu) + 0.5 \cdot (1 - \rho^2)\sigma_c^2\sigma_p^2\right]\}.$$

Equation (4) says that $E(Y_c|\text{y})$ effectively emerges from a regression of Y_c on y. Using equation (4), the appendix shows that corporate output is given by:

(5) $$prob(y \geq \tau) \cdot E(Y_c|y \geq \tau) = [\exp(\mu_c + 0.5\sigma_c^2)] \cdot [1 - \Phi(\tau')],$$

where $\tau' = \left(\frac{1}{\sigma}\right)\left[\tau - \mu - \sigma_c\left(\sigma_c - \rho\sigma_p\right)\right]$ and $\Phi(\cdot)$ is the cumulative standard normal density. The formula for pass-through output is analogous, except that the parameters related to c and p are switched, and τ is replaced by $-\tau$ in the expression for τ'.

[16]The fraction corporate is $1-\Phi[(\tau-\mu)/\sigma]$, where $\Phi(\cdot)$ is the cumulative standard normal density.

The quantitative results depend on the five parameters μ_c, σ_c, μ_p, σ_p, and ρ. We calibrate the model by specifying values of these parameters. One reason for carrying out this calibration is to construct reasonable ranges of values for coefficients that arise in the regression analysis. A more important reason is that the calibrated model allows inferences on how productivity responds to changes in τ or to shifts in the underlying parameters—especially μ_c and μ_p, which reflect the average productivity of the alternative legal forms. The difference μ_c- μ_p represents the typical advantage for corporate form. We think that the main legal changes in recent decades reduced this advantage.

The baseline specification assumes that the two standard deviations, σ_c and σ_p, are equal. To pin down these standard deviations, we use data on the dispersion of U.S. plant-level productivity. Foster, Haltiwanger, and Syverson (2008) use a pooled sample of data from the U.S. Census of Manufacturers for 1977, 1982, 1987, 1992, and 1997. From this broad data set, they select information on 11 goods that are sufficiently homogeneous so that quantities (and, hence, physical productivity) can be directly calculated across plants.[17] Based on these data, they report (in their Table 1) a standard deviation of 0.26 for the log of physical total factor productivity (TFPQ). Hsieh and Klenow (2009, Table II) use the same underlying data to compute the dispersion of the log of revenue-based total factor productivity (TFPR) for a much broader array of manufacturing plants. They find that this standard deviation was 0.49 in 1997 and slightly lower before that.[18] Since the findings from Foster, Haltiwanger, and Syverson's 11 homogeneous products would not be representative even of the manufacturing sector, we use

[17]The products are boxes, bread, carbon black, coffee, concrete, flooring, gasoline, block ice, processed ice, plywood, and sugar.
[18]The values are 0.45 in 1977 and 0.41 in 1987. They report (in their Table I) substantially higher standard deviations for TFPQ, around 0.8. However, their TFPQ values are not actually observed—they are inferred from the TFPR values given an assumed elasticity of substitution between alternative products in final demand. Foster, Haltiwanger, and Syverson (2008, Table 1) calculate a TFPR value, 0.22, that is only slightly below their TFPQ value.

Hsieh and Klenow's results to set the common standard deviation at $\sigma_c=\sigma_p=0.5$ in the main calibration of our model. We discuss later how our results change when we assume different values for σ_c and σ_p.

In the absence of direct evidence, we assume that the correlation coefficient, ρ, between $log(Y_c)$ and $log(Y_p)$ is positive and set it to 0.25 in the main analysis. As discussed later, the results are not very sensitive to variations in ρ.

The remaining parameters, μ_c and μ_p, relate to the mean productivity for corporate and pass-through status. We first use a normalization for the level of output, Y (which has arbitrary units). We normalize so that the peak of Y, which corresponds to $\tau=0$, equals 1.0 in an arbitrary benchmark year, which we take to be 1978 (the first year of the regression sample). We then determine μ_c and μ_p for 1978 so that the calibrated model matches the observed C-corporate share of economic activity, which we base on net capital stock, shown by the blue graph in Figure 5. In the benchmark year, 1978, the C-corporate share was 0.85, and the tax wedge, τ, equaled 0.48. The condition that the model match the C-corporate share in 1978 (along with the normalization for the level of Y in 1978) turns out to require μ_c=-0.137 and μ_p=-1.032. The large gap between μ_c and μ_p allows the model's predicted corporate share to be high, 0.85, despite the large tax wedge, τ=0.48.

For other years, we back out the values of μ_c and μ_p by requiring the calibrated model to match, first, the year's C-corporate share of net capital stock and, second, the year's overall level of business productivity (TFP), as given by Fernald's (2019) data.[19] The time series for TFP is given by the blue graph in Figure 9. The values of μ_c and μ_p are given, respectively, by the red and green graphs in the figure. Loosely speaking, the gap between μ_c and μ_p is set to match the

[19]We use the utilization-adjusted quarterly data to gauge TFP at the end of each year. The level of the series is normalized so that the calibrated model's TFP in 1978 equals 1.0 when the tax wedge, τ, equals zero.

C-corporate share of business net capital stock (blue graph in Figure 5), given the value of the tax wedge, τ. The overall levels of μ_c and μ_p are chosen to generate the observed TFP level, given again the value of τ. Note that the estimated gap between μ_c and μ_p falls in most years after 1958. The gap closes especially rapidly between 1994 and 2004, likely reflecting the spread of LLCs. By 2009, the gap is nearly closed, and thereafter remains roughly constant.

Figures 10-13 shows the workings of the model, given the calibrated parameters. The graphs show the effects on a designated outcome from a change in τ while holding constant these parameters. Two graphs are displayed in each case, one corresponding to 1978 (the start date of our regression sample) and the other to 2013 (the final date of the sample).

Figure 10 shows that the fraction of firms opting to be corporate declines monotonically with the tax wedge, τ. We focus on values of τ between 0.1 and 0.5—this range applies in Figure 4 to our main regression sample, from 1978 to 2013. In this range, for the 1978 calibration, the corporate share of numbers is between 0.74 and 0.90. For the 2013 calibration, the corresponding range is between 0.24 and 0.48. The much lower corporate share, at each τ, for 2013 reflects the much smaller gap between μ_c and μ_p (Figure 9). Since it is unclear that the number of firms is an empirically meaningful concept, we focus our analysis on results related to corporate and total output.

Figure 11 shows the relation of overall output (economy-wide productivity) to the tax wedge, τ, again for the calibrations based on 1978 and 2013. Because the only distortion in the model is this tax wedge, the maximum of output occurs in each year at $\tau=0$ (with the peak for 1978 normalized to equal 1.0). The marginal effect of τ on productivity is positive when $\tau<0$ and negative when $\tau>0$.

Note that we are using the revealed preference of business owners to infer the effects of the tax wedge, τ, on overall output and productivity. Specifically, when $\tau>0$, a firm opts to be corporate only if the productivity advantage associated with corporate form is sufficient to justify the tax penalty. Moreover, a firm at the margin must have a productivity advantage that exactly compensates for the tax penalty.

Figure 12 shows the relation of the corporate share of output to the tax wedge, τ. At a given τ and for either calibration (based on 1978 or 2013), the corporate output share exceeds the corporate share of numbers, shown in Figure 10. This pattern applies because, when $\mu_c>\mu_p$, the typical corporate firm is more productive that the typical pass-through firm. The results for the corporate output share in Figure 12 can be matched with data on the C-corporate share of economic activity, such as net capital stock. The calibrated model for 1978 implies that the corporate output share is between 0.85 and 0.94 when τ is between 0.1 and 0.5, whereas that calibrated for 2013 has a corresponding range for corporate output share between 0.31 and 0.51.

Figure 13 shows the marginal effect of the tax wedge, τ, on the corporate share of output. Consistent with Figure 12, this marginal effect is negative throughout. Quantitatively, for the 1978 calibration, the marginal effect in Figure 13 is between -0.16 and -0.28 when τ is between 0.1 and 0.5. For the model calibrated to 2013, the corresponding range is between -0.47 and -0.50. These marginal effects should correspond to regression coefficients in linear relations between the C-corporate share of economic activity and τ. The calibrated model implies that these coefficients should be in a range of roughly -0.2 to -0.5, and the magnitudes of regression coefficients found empirically turn out to accord reasonably well with this prediction.

The results suggest that, for a given set of underlying parameters—corresponding to a given graph in Figure 13—the relationship between the C-corporate share and τ is roughly linear;

that is, the slopes connecting τ to the share do not vary much with τ. However, the slopes are higher in magnitude for the 2013 curve than for the 1978 curve; that is, the size of the slope is larger when the average productivity of pass-through organization, represented by μ_p, is close to that for corporate form, μ_c. In the regression analysis, we do not find clear evidence concerning this result.

Figures 10-13 apply when the dispersion of the underlying C-corporate and pass-through productivities, determined by σ_c, σ_p, and ρ, take on the values assumed, but the tax wedge, τ, is the same for all entities. Dispersion in the tax wedge would also affect the results. Specifically, we have computed the standard deviation of the cross-section of individual marginal tax rates that underlie the computation of the Schedule E AMTR (blue graph in Figure 3). This standard deviation has fallen over time, from 0.18 in 1962 to 0.07 in 2012. If this variation were independent of that in the underlying productivities, there would effectively be an addition to the amount of dispersion. However, with the baseline values of $\sigma_c=\sigma_p=0.5$, the added amount of dispersion due to the variations in τ would be minor. Moreover, the results are isomorphic to those derived when there is no cross-sectional variation in tax rates but where the values of σ_c and σ_p are higher than those assumed in the baseline case.

V. Regressions

A. Regression framework

The regression analysis relates the C-corporate shares of net capital stock (Figure 5), equity (Figure 6), gross assets (Figure 7), and positive net income (Figure 8) to the tax wedge, τ. In each case, we consider C-corporate amounts expressed as a ratio to the total for C-corporations, S-corporations, and partnerships (but excluding sole proprietorships). The overall federal tax wedge, based on equation (2), is $\tau = \log(1-\tau_p) - \log(1-\tau_{prof}) - \log(1-\tau_{div})$, where "p" refers to pass-through income (reported on Schedule E), "prof" to C-corporate profits, and "div" to dividends. The main regressions separate this wedge into two parts, the first associated with C-corporations, $log(1-\tau_c)$ = $\log(1-\tau_{prof}) + \log(1-\tau_{div})$, and the second with pass-throughs, $\log(1-\tau_p)$. We test whether the estimated coefficients on $\log(1-\tau_p)$ and $\log(1-\tau_c)$ are equal in magnitude with opposite signs. We also test whether unrestricted estimates of coefficients on $\log(1-\tau_{prof})$ and $\log(1-\tau_{div})$ are equal.

Level regressions for C-corporate shares, as implemented in Mackie-Mason and Gordon (1997, Table III), are probably not meaningful. Specifically, as is evident from Figures 5-8, the C-corporate share measures have strong persistence and may be non-stationary. This problem was noted by Prisinzano and Pearce (2018, Tables IV and V), who emphasized regressions with annual first-differences (for dependent variables based on C-corporate shares of net income). However, this specification is likely to be heavily influenced by measurement error, particularly because the timing between changes in the tax system and changes in C-corporate shares are not well determined.

Given these concerns, our empirical analysis relies on long-difference estimation. We emphasize results with 20-year changes in C-corporate shares and the tax-rate variables, but results are similar for 15-year changes. Because this procedure creates or intensifies serial dependence in the error terms for the overlapping data, we use the Newey-West procedure with a bandwidth of 20 or more years to construct standard errors of the estimated coefficients.[20] Our sample in the context of 20-year changes is 1978-2013 because the data on C-corporate shares start in 1958.

[20]An analogous procedure was implemented by Montamat and Stock (2018, Table 1).

In principle, we would like to isolate variables, such as changes in the legal/regulatory environment and shifts in the composition of production, that influence the relative attractiveness of C-corporate and pass-through forms. With regard to important legal changes, the one in 1958 that created the S-corporation predates the start of our sample. Otherwise, we think that the most significant changes are the IRS ruling in late 1988 that allowed LLCs to be taxed as partnerships and the adoption of LLC laws in all 50 states by 1996. To account for these changes, we estimate one trend (intercept) coefficient from 1978 to 1988, another from 1989 to 1996, and a third from 1997 to 2013.

We have IRS data on the division of C-corporate and pass-through gross assets and positive net income into eight sectors: agriculture, construction, finance/insurance/real estate or FIRE, manufacturing, mining, services, trade, and transportation. The pass-through category covers S-corporations and partnerships. Data are unavailable on the composition of net capital stocks and equity.

Figure 14 shows the composition of gross assets by sector at the beginning and end of the data sample, 1958 and 2013. Figure 15 has the comparable data for positive net income. The main pattern is the shift toward FIRE and especially away from manufacturing and transportation. FIRE rises from 53% to 72% of gross assets and from 18% to 43% of positive net income, while manufacturing and transportation combined fall from 33% to 17% of gross assets and from 51% to 27% of positive net income.

The eight sectors differ in C-corporate shares, as shown for 1958, 1985, and 2013 in Figures 16 and 17. For example, on average for gross assets, the highest starting from the top are manufacturing, transportation, FIRE, trade, and mining, and the lowest starting from the bottom are agriculture, construction, and services. Given these differences, exogenous changes in the composition of assets or positive net income across the sectors might explain part of the movement in overall C-corporate shares. In practice, however, the dominant changes in shares were toward FIRE and away from manufacturing and transportation. Because these sectors are all comparatively high in average C-corporate shares, the compositional changes among these sectors turn out to explain almost none of the changes in overall C-corporate shares. That is, the changes are mainly within sectors—all eight had lower C-corporate shares in 2013 than in 1958, as shown in Figures 16 and 17. Given these patterns, it is not surprising that compositional-change variables lack explanatory power in the regressions for changes in overall C-corporate shares.

B. Regression results

Table 1 has regressions where the dependent variable is the 20-year difference of the C-corporate share of net business capital stocks (column 1), equity or book value (column 2), gross assets (column 3), and positive net income (column 4). The sample period is 1978 to 2013 (dictated by the availability of data beginning in 1958). These regressions have the three intercept terms noted before (which pick up trends in levels) and the 20-year changes in the two tax-rate variables, $log(1-\tau_c)$ and $log(1-\tau_p)$. The compositional change variables, which turn out to be unimportant, are excluded in these results.

The estimated coefficients on $log(1-\tau_c)$ are all positive, as predicted; that is, the estimated effects of τ_c on the C-corporate shares are negative. These estimated coefficients are statistically significantly different from zero at less than the 5% level for net capital stock, equity, and gross assets (columns 1-3) but not for positive net income (column 4). The point estimates of coefficients in the first three cases range between 0.24 and 0.53.

The estimated coefficients on $log(1-\tau_p)$ are all negative, as predicted; that is, the estimated effects of τ_p on the C-corporate shares are positive. These estimated coefficients are again statistically significantly different from zero at least at the 5% level for net capital stock, equity, and gross assets (columns 1-3) but not for positive net income. The magnitudes of the estimated coefficients range from 0.34 to 0.56.

As noted, the C-corporate part of the tax wedge combines effects from federal taxation of C-corporate profits and dividends: $log(1-\tau_c) = log(1-\tau_{prof}) + log(1-\tau_{div})$. As shown in Table 1, the hypothesis of equal coefficients on these two parts is accepted at a high p-value for net capital stock, equity, and gross assets (columns 1-3) but not for positive net income.[21]

The model also implies that the coefficients of the tax variables $log(1-\tau_c)$ and $log(1-\tau_p)$ should be of equal magnitude and opposite sign. As shown by the p-values in Table 1, this hypothesis is rejected at less than the 1% critical level for net capital stock and gross assets and at a p-value of 0.06 for positive net income. For equity, the hypothesis is accepted with a p-value of 0.41. Thus, some of the econometric results deviate from the precise theoretical restriction—possibly because the empirical measures of tax rates are imperfect.

The magnitudes of the estimated tax-rate coefficients can be compared with those predicted by the calibrated theoretical model. Since the results for positive net income appear unreliable—possibly due to the data problems noted earlier—we focus on the estimates for the first three columns of Table 1. In this case, the average of the estimated coefficients on $log(1-\tau_c)$ is 0.34, whereas that on $log(1-\tau_p)$ is -0.46. The magnitudes of these coefficients are in the ballpark of those predicted by the calibrated model. As noted before, the marginal tax-wedge

[21]The fits of the regressions deteriorate if we use the dividend tax rate in Figure 2 that is not adjusted for the share accruing to taxable entities. For example, for net capital stock (Table 2, column 1), the estimated tax coefficients become 0.148 (s.e.=0.057) on $\log(1-\tau_c)$ and -0.357 (0.152) on $\log(1-\tau_p)$, and the standard error of the regression rises from 0.0158 to 0.0185.

effects in Figure 13 range between -0.2 and -0.5 when τ is in the range from 0.1 to 0.5 that applies to the regression sample, 1978 to 2013.

We checked whether the coefficients on the tax-rate variables were different in magnitude in the former part of the sample, say up to 1996, compared to that afterwards. The results from the model in Figure 13 suggested that the responsiveness of the corporate share to the tax-rate variables would be larger in size in the latter period, for which the average productivity of pass-through status was close to that for corporate status (Figure 9). The results here are mixed. For equity and gross assets, the magnitudes of the estimated coefficients are larger in the latter period, as expected, but for net capital stock and positive net income, the opposite pattern applies.

The results in Table 1 gauge the pass-through tax rate, τ_p, by the AMTR on Schedule E income. However, as is clear from Figure 3, this tax-rate variable is positively correlated with the more standard AMTR based on labor income. If we measure τ_p by the standard AMTR, the fits of the regressions deteriorate but the qualitative results remain. For example, for net capital stock in Table 1, column 1, the estimated tax coefficients become 0.175 (s.e.=0.050) on $\log(1-\tau_c)$ and -0.747 (0.254) on $\log(1-\tau_p)$, and the standard error of the regression rises from 0.0158 to 0.0181.

The regressions deteriorate more sharply if we measure τ_p by the top individual tax rate on earned income (Figure 3). For example, for net capital stock in Table 1, column 1, the estimated tax coefficients have the "wrong" signs: -0.039 (s.e.=0.059) on $\log(1-\tau_c)$ and 0.020 (0.013) on $\log(1-\tau_p)$, and the standard error of the regression rises from 0.0158 to 0.0205. These results suggest that the high top individual tax rates that prevailed before 1987 did not influence choices between C-corporate and pass-through legal form. This result makes sense because the large gap between the top tax rate and the Schedule E AMTR in this period (Figure 3) indicates that little pass-through income actually faced these high marginal tax rates.

The intercept terms apply in Table 1 to the periods 1978-1988, 1989-1996, and 1997-2013. The estimated coefficients on these intercepts are significantly negative at the 1% level, except for net income in the 1978-1988 period. These results accord with the patterns shown in Figures 5-8, although the negative trend for equity seems to set in only around 1980. Quantitatively, the estimated trends in Table 1 are around -1 percentage point per year. The magnitude is significantly higher in all cases for the last period, 1997-2013, than for the previous two. This result likely reflects the broad availability of the LLC legal form, which was recognized in all 50 states by 1996.

The tax changes from 1958 to 2013 imply a substantial overall drop in the tax wedge, τ (Figure 4). Thus, this tax effect goes against the estimated trend coefficients (intercepts), which imply declining C-corporate shares of economic activity, consistent with Figures 5-8. On their own, the tax changes from 1958 to 2013 should have increased C-corporate shares of economic activity.

The 20-year difference estimates shown in Table 1 calculate the standard errors of the estimated coefficients by the Newey-West procedure with a bandwidth of 20 years. The results change little if the bandwidth is raised to 25 years (to allow for serial dependence in the error term independently from that created by the overlapping data). For example, with a 20-year bandwidth, the estimated standard errors on the two estimated tax coefficients in Table 1, column 1, were 0.049 and 0.093. These values change with a 25-year bandwidth to 0.044 and 0.088, respectively.

The results shown in Table 1 do not change greatly if the long differencing applies to 15 years, rather than 20. For the 15-year regressions (for samples from 1978 to 2013 with 20-year bandwidths), the estimated coefficients on the tax variables, $log(1\text{-}\tau_c)$ and $log(1\text{-}\tau_p)$, are, respectively: 0.241 (s.e.=0.056) and -0.404 (0.075) for net capital stock, 0.344 (0.084) and -0.404 (0.047) for equity, 0.210 (0.027) and -0.269 (0.019) for gross assets, and 0.048 (0.086) and -0.339 (0.113) for positive net income.

In contrast, a differencing interval much shorter than 15 years tends to generate estimated tax coefficients that often differ insignificantly from zero and sometimes have the "wrong" signs. For example, with 10-year differencing, a 15-year bandwidth, and a sample from 1973 to 2013, the estimated coefficients on the tax variables, $log(1\text{-}\tau_c)$ and $log(1\text{-}\tau_p)$, are, respectively: 0.017 (s.e.=0.104) and -0.282 (0.127) for net capital stock, -0.010 (0.094) and -0.111 (0.095) for equity, 0.031 (0.042) and -0.145 (0.045) for gross assets, and -0.407 (0.304) and 0.005 (0.335) for positive net income. The inference is that the association between changes in the tax variables and changes in C-corporate shares of economic activity are not well determined over short periods. However, the long-run association—over 15 or more years—is reasonably well pinned down.

As a general statement, we think that the regression results support the conceptual framework and the calibration discussed earlier. Therefore, we think it reasonable to now apply the calibrated model to assess implications for productivity.

VI. Productivity Effects

The calibrated model has implications for the time path of productivity. Our baseline application uses the values of μ_c and μ_p shown in Figure 9; the parameter values $\sigma_c=\sigma_p=0.5$ and $\rho=0.25$; the tax wedge, τ, given by the black graph in Figure 4; and the corporate share of output gauged by the C-corporate share of net capital stock in Figure 5. The blue graph in Figure 18 shows the log of TFP expressed as a deviation from its value in 1958. (The level of TFP is the blue graph in Figure 9.) The implied average TFP growth rate from 1958 to 2013 was 1.09% per year.

We use the calibrated model to compute three counter-factual time series. The first scenario, corresponding to the red graph in Figure 18, gives the log of TFP (relative to that in 1958) that would have arisen if, instead of mostly falling over time, μ_c-μ_p had remained fixed at its value in 1958 (see Figure 9). In this case, μ_c and τ are maintained at their values shown in Figures 9 and 4, and μ_p moves along with μ_c.

The second scenario, corresponding to the green graph in Figure 18, computes the log of TFP assuming that the tax wedge, τ, instead of mostly declining over time, had remained constant at its value in 1958 (Figure 4). In this setting, μ_c and μ_p are maintained at their values shown in Figure 9.

The third scenario, given by the black graph in Figure 18, assumes that μ_c-μ_p and τ are kept fixed at their respective values in 1958. In this case, μ_c is maintained at the values shown in Figure 9, and μ_p moves along with μ_c.

The red graph in Figure 18 implies that, over the full period from 1958 to 2013, the effect of the fall in μ_c-μ_p is a rise in TFP by 0.33% per year, compared to the total of 1.09% per year. However, the effects are very different in the early and late parts of the sample. From 1958 to the mid 1980s, the substantial rise in μ_p (Figure 9) did not contribute to rising TFP. The reason is the tax penalty, τ, for being corporate (Figure 4), which creates a distortion in the sense that the public return from being corporate rather than pass-through exceeds the private return. In the years up to 1986, when τ is very high—between 0.37 and 0.77—this distortion turns out to be large enough so that overall output falls slightly or rises only slightly when μ_p increases for given μ_c and τ (despite the direct positive impact of a rise in μ_p on productivity). By 1987, τ had fallen enough (below 0.30), so that increases in μ_p contribute to higher TFP. The effect from the shrinking gap between μ_c and μ_p is particularly large from 1994 to 2004, when the growth rate of TFP is unusually high—2.00% per year. In this period, the contribution from the falling μ_c-μ_p is 0.77% per year. This effect lines up reasonably well with the legal changes that made LLCs attractive alternatives to C- or S-corporations or to existing forms of partnerships. That is, the results suggest that the invention of the LLC was an important form of technical progress.

The green graph in Figure 18 gives the results from the mostly falling tax wedge, τ. In this case, the contribution to TFP growth is most important in the early period, when τ and the gap μ_c-μ_p are particularly high. From 1958 to 1987, the average TFP growth rate was 1.24% per year, and the contribution from the mostly declining τ was 0.41% per year. Over the full period from 1958 to 2013, the contribution from the reductions in τ was 0.21% per year, compared to the total of 1.09% per year.

The black graph in Figure 18 shows the combined contribution from the declines in μ_c-μ_p and τ. (Because of the important interactions between the two components, the effects are not additive.) From 1958 to 2013, the combined contribution to TFP growth was 0.37% per year, compared to the total of 1.09% per year.

VII. Alternative Parameter Values

The results for TFP growth described in the previous section apply when the mean parameters μ_c and μ_p are given in Figure 9 and the other parameters are set at $\sigma_c=\sigma_p=0.5$ and $\rho=0.25$. We now assess how the results change with different settings for σ_c, σ_p, and ρ. Table 2 considers four cases: $\sigma_c=\sigma_p=0.25$, $\rho=0.25$; σ_c=0.25, $\sigma_p=0.5$, $\rho=0.25$; σ_c=0.5, $\sigma_p=0.25$, $\rho=0.25$; and $\sigma_c=\sigma_p=0.5$, $\rho=0.5$.

Among the four alternatives, the results on contributions to TFP growth from 1958 to 2013 change substantially only under the first case, where σ_c and σ_p are each lowered to 0.25. For the setting where the gap between μ_c and μ_p and the tax wedge, τ, are both held fixed at their values in 1958, the result is that the joint contribution to TFP growth from these two forces averaged 0.20% per year, rather than the 0.37% per year found in the baseline. For the μ_c-μ_p gap on its own, the contribution is now 0.13% per year, compared to 0.32% per year in the baseline; and for τ on its own, the contribution is also 0.13% year, compared to 0.21% per year in the baseline.

The change in the correlation coefficient, ρ, from 0.25 to 0.5 has only a small impact on the results. For the setting where the gap between μ_c and μ_p and the tax wedge, τ, are held fixed at their respective values in 1958, the result (when $\sigma_c=\sigma_p=0.5$) is that the joint contribution to TFP growth from the two forces averaged 0.31% per year, rather than the 0.37% per year found in the baseline. For the μ_c-μ_p gap on its own, the contribution is now 0.26% per year, compared to 0.32% per year in the baseline; and for τ on its own, the contribution is now 0.24% year, compared to 0.21% per year in the baseline.

Our conclusion is that the broad nature of the results about contributions to TFP growth is robust to substantial changes in the underlying distribution parameters. The main change arises when the underlying standard deviations, σ_c and σ_p, are much lower than 0.5. Recall that his number came from the observed dispersion of productivity of U.S. manufacturing firms, as reported by Hsieh and Klenow (2009). Since standard deviations as low as 0.25 seem unrealistic, we are inclined to stick with the results on contributions to TFP growth found in our baseline specification.

VIII. Concluding Observations

We dealt theoretically and empirically with the relation between tax rates and the composition of U.S. business economic activity between C-corporate and pass-through forms. The main federal tax wedge, τ, that we measured since 1958 involves the tax rate on C-corporate profits, the effective tax rate on dividends, and the pass-through tax rate, gauged by the average marginal tax rate on Schedule E income. Our regression estimates from 1978 to 2013 imply that a fall in the tax wedge raises the C-corporate share of economic activity.

Despite the overall decline in the tax wedge, the measures of C-corporate share of economic activity exhibit downward trends at least since the 1970s. We attributed these trends particularly to legal changes that favored pass-through forms, notably LLCs. We gauged these effects by aligning intercept terms in the regressions with the dates of the principal changes that affected the legal status of the LLC form of business.

The calibrated model provides estimates of the contribution to business TFP growth from the mostly falling gap between underlying C-corporate and pass-through productivity (due especially to legal changes that favored pass-throughs) and from the mostly declining tax wedge. In our baseline analysis, the combined contribution to TFP growth from 1958 to 2013 was 0.37% per year, compared to the total growth rate of 1.09% per year. The contribution from the declining gap between C-corporate and pass-through productivity was negligible up to the mid 1980s but became important thereafter. The growth contribution was especially large—0.77% per year—in the period from 1994 to 2004 when the observed TFP growth rate was unusually high—2.00% per year.

We are currently extending the empirical research internationally. We are collecting data on the tax code and measures of business activity by legal form for several high-income countries. The resulting panel data will allow us to replicate and further develop the empirical results in this paper.

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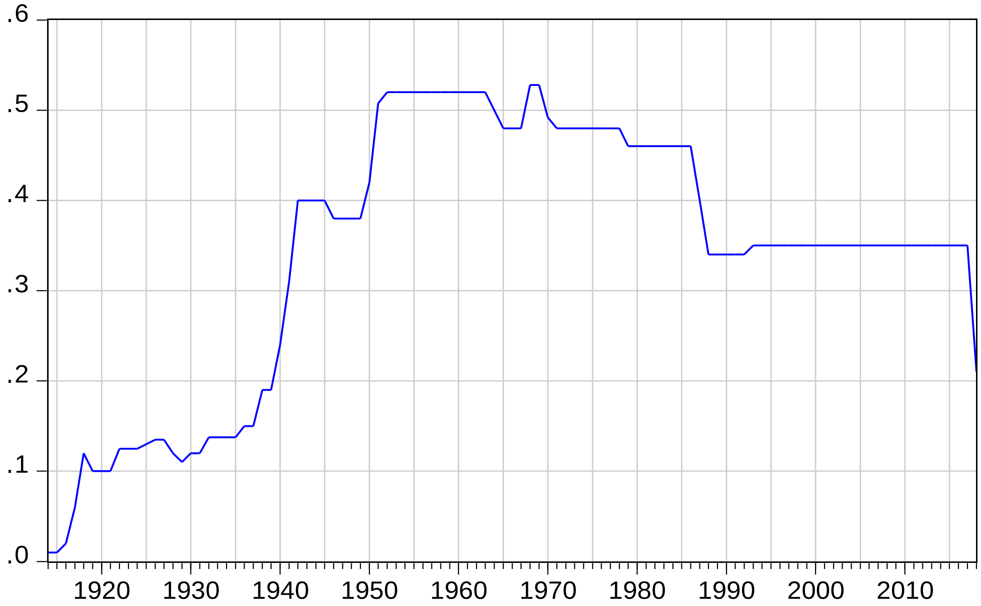
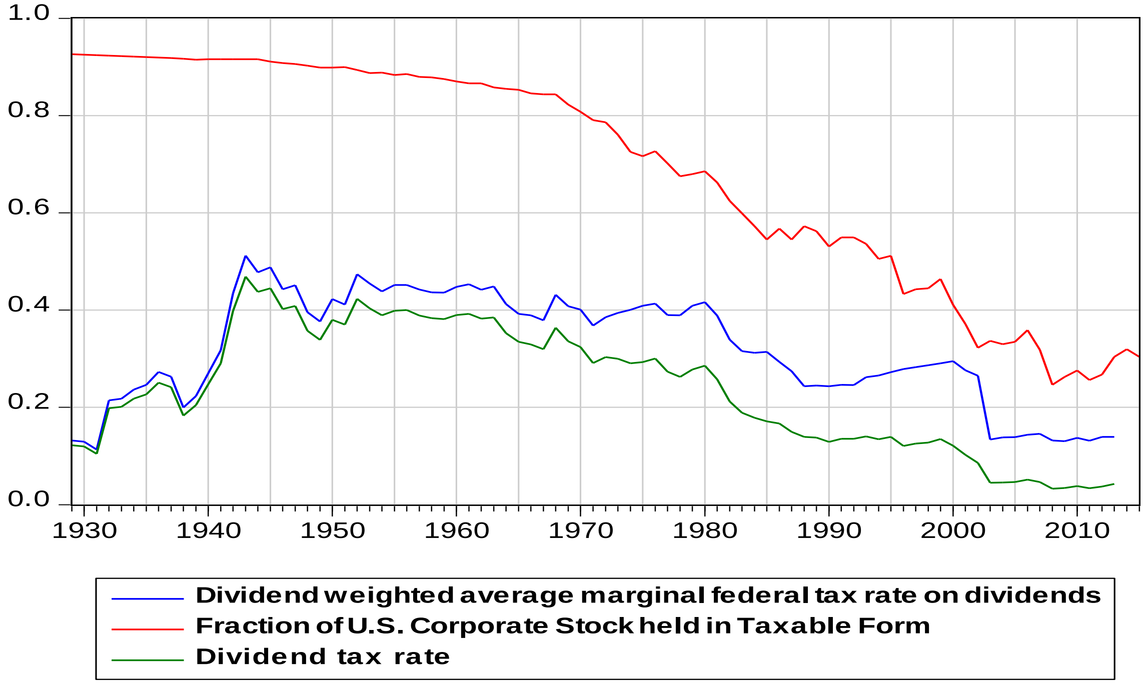
Figure 1 Top Federal Tax Rate on C-Corporate Profits



Note: Data on the top federal marginal tax rate on C-corporate profits are in IRS, *Statistics of Income Bulletin*, Fall 2003, and in recent issues of IRS *Statistics of Income: Corporation Income Tax Returns*.

Figure 2

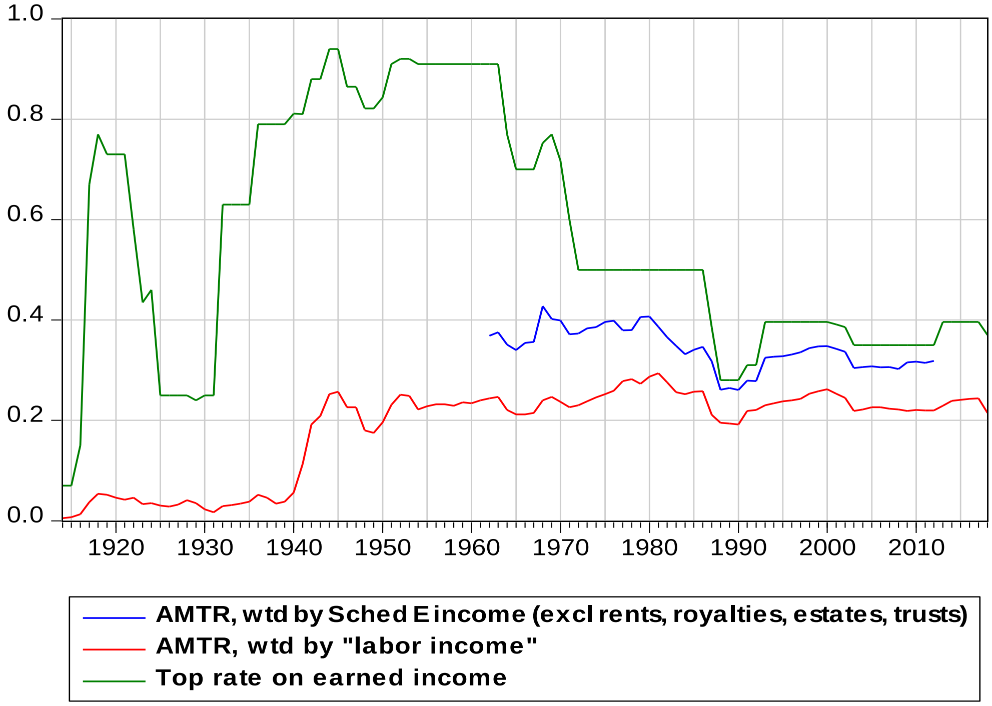
Federal Tax Rates on Dividends



Note: The dividend weighted average marginal federal dividend tax rate in the blue graph was provided for 1960-2012 by Dan Feenberg, using the TAXSIM program of the National Bureau of Economic Research. (Qualified dividends are used since 2003.) The value for 2013 was unavailable and was assumed to equal that for 2012. Values before 1960 are estimates based on issues of IRS, *Statistics of Income, Individual Income Tax Returns*. The fraction of U.S. corporate stock held in taxable form in the red graph is from Rosenthal and Austin (2016, Figure I) and Poterba (2004, Table 1). We measure the dividend tax rate, τ_{div} (green graph), as the product of the values in the blue and red graphs.

Figure 3

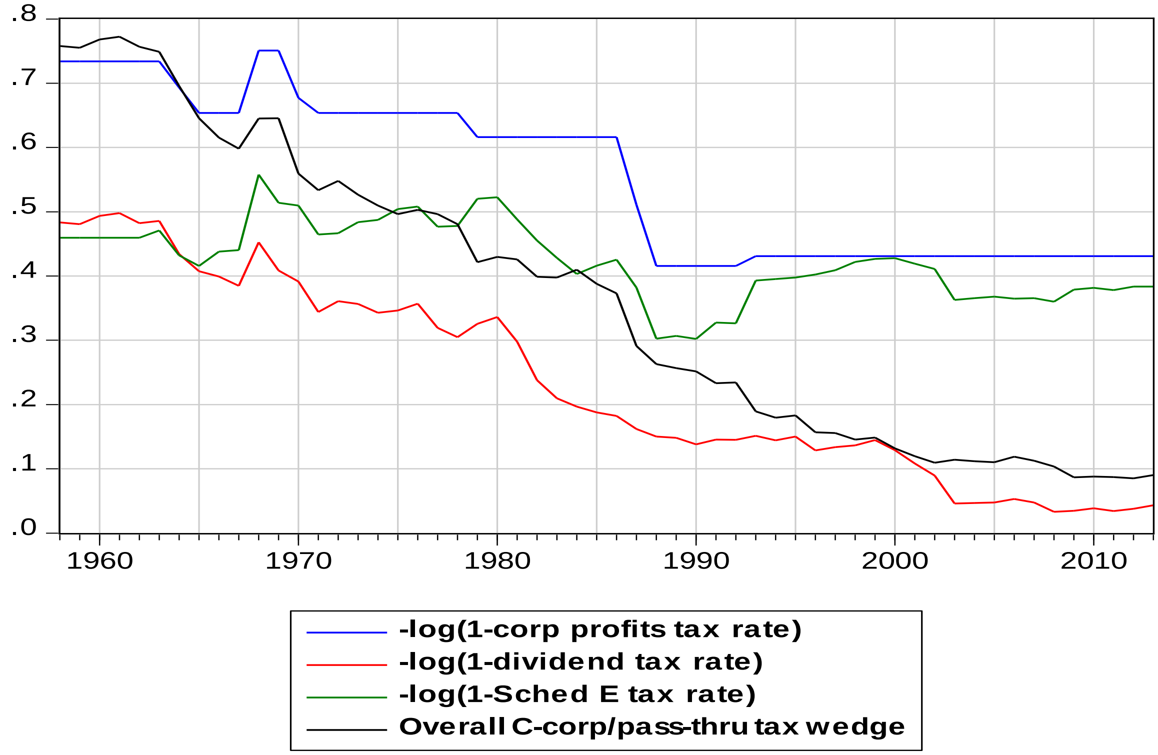
Federal Average Marginal Individual Income-Tax Rates



Note: The blue graph is the income-weighted federal average marginal tax rate based on Schedule E income (exclusive of rents, royalties, estates, trusts). The red graph is the corresponding federal AMTR based on a broad concept of labor income. The green graph is the top federal marginal rate on earned income (distinguished from ordinary income for 1971-1981). The data for calculating the Schedule E-income weighted average marginal federal tax rate were provided for 1962-2012 by Dan Feenberg, using the TAXSIM program of the National Bureau of Economic Research. The AMTR weighted by labor income is from Barro and Sahasakul (1983), Barro and Redlick (2011), and the Tax Policy Center.

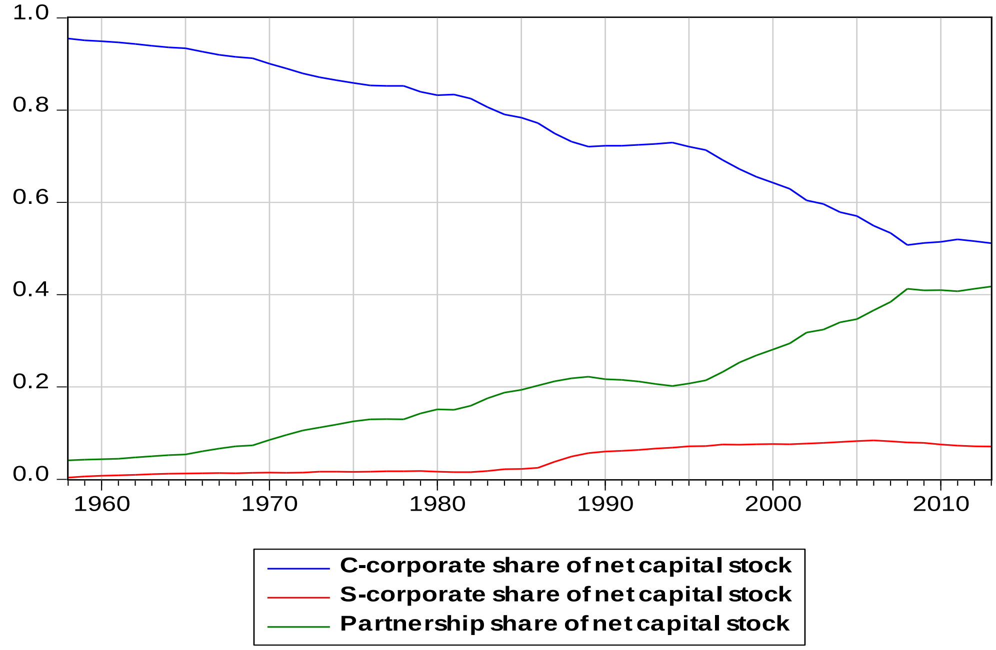
Figure 4

Federal Income-Tax Wedge



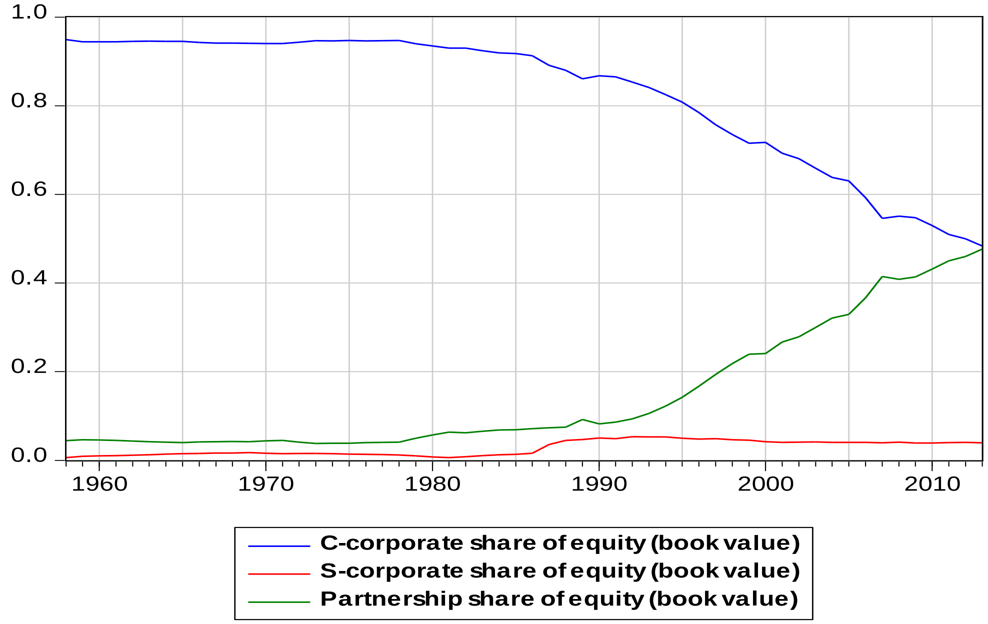
Note: The top federal tax rate on C-corporate profits is from Figure 1, the federal dividend tax rate is from Figure 2 (green graph), and the federal average marginal tax rate for Schedule E income is from Figure 3 (blue graph). (In the blue graph, the values for 1958-1961 are assumed to equal that for 1962 and the value for 2013 is assumed to equal that for 2012.) The overall federal tax wedge for C-corporate versus pass-through status, indicated by the black graph, equals the blue graph plus the red graph minus the green graph.

Figure 5 Shares of Business Net Capital Stock



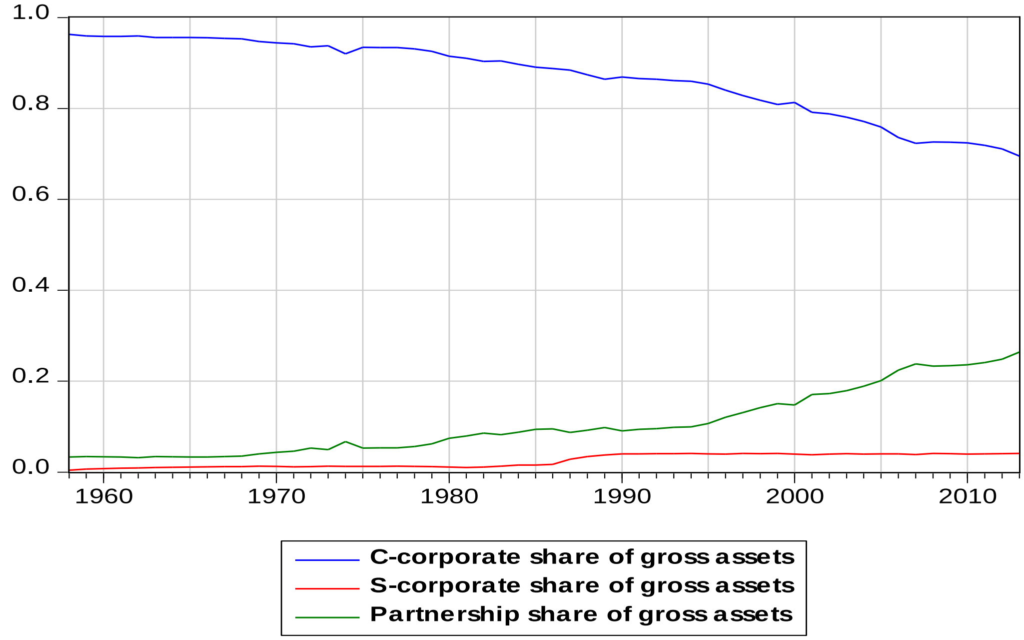
Note: The underlying data on business capital stocks net of depreciation are from various IRS sources, noted in the references. Data for sole proprietorships are unavailable. The partnership numbers are interpolated based on data available every two years from 1959 to 1975 and annually for 1977-1982 and 1988-2013.

Figure 6 Shares of Business Equity (Book Value)



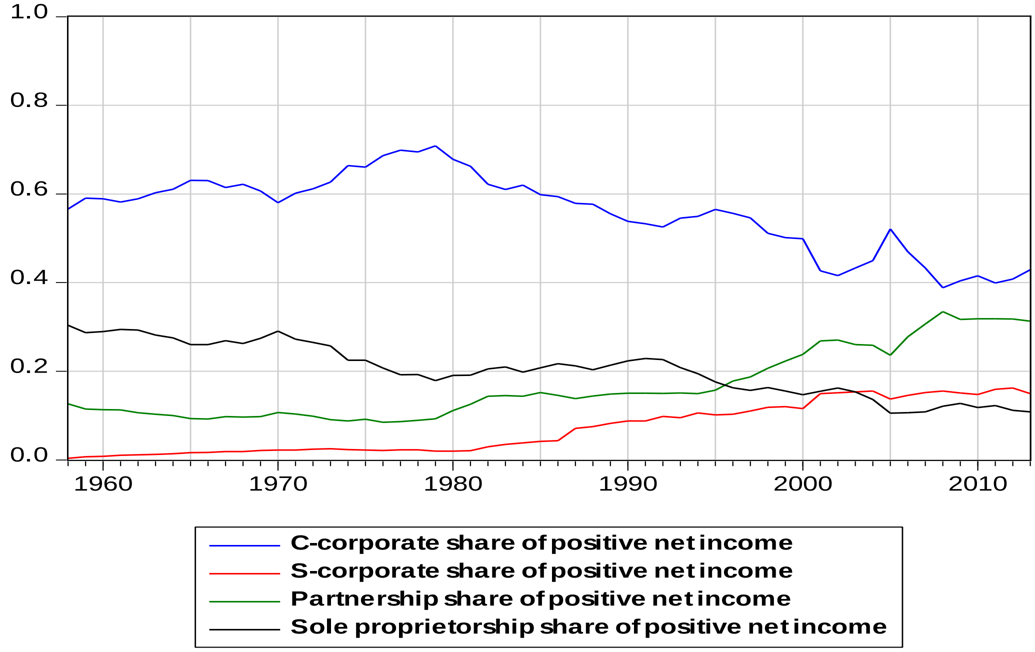
Note: The underlying data on business equity (book value) are from various IRS sources, noted in the references. Data for sole proprietorships are unavailable. The partnership numbers are interpolated based on data available every two years from 1959 to 1975 and annually for 1977-1982 and 1988-2013.

Figure 7 Shares of Business Gross Assets



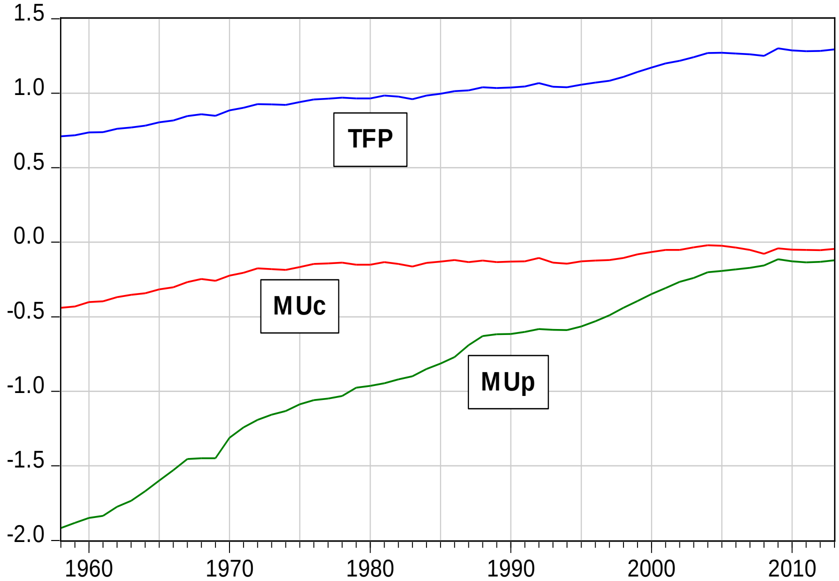
Note: The underlying data on business gross assets are from various IRS sources, noted in the references. Data for sole proprietorships are unavailable.

Figure 8 Shares of Business Positive Net Income



Note: The underlying data on business positive net income are from various IRS sources, noted in the references.

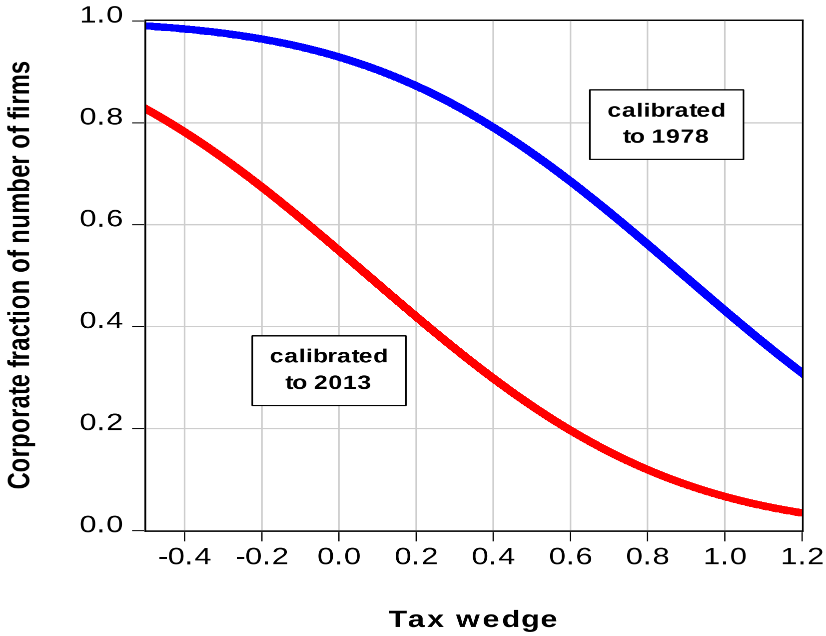
Figure 9 TFP and Calibrated Productivity Parameters



Notes: The TFP series comes from the utilization-adjusted quarterly data on total factor productivity given in Fernald (2019). This series is used to gauge the level of output, y, in the model developed in Section IV. The level of TFP is normalized so that the model's implied level of y in 1978 would equal 1.0 when the tax wedge, τ, is set to zero.

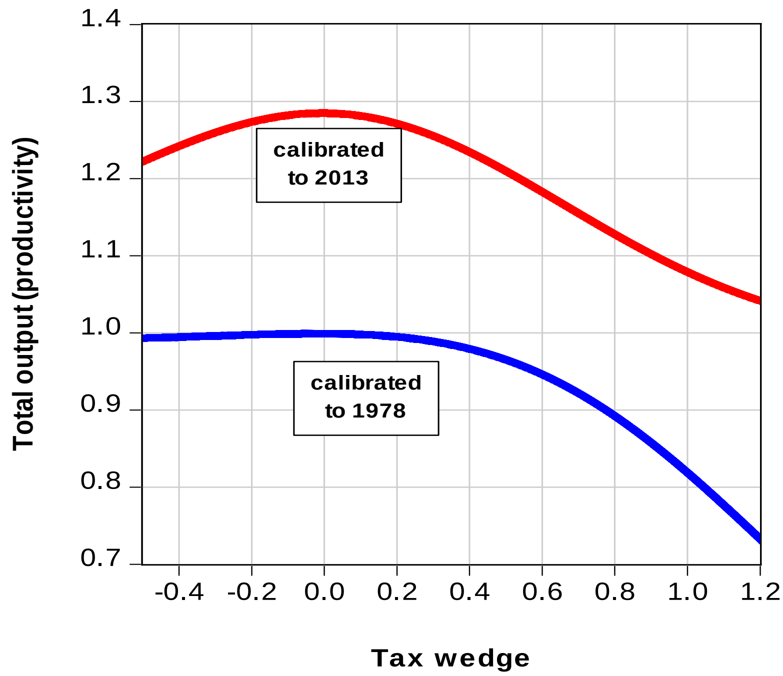
The parameters μ_c and μ_p refer in the model to the means of the logs of productivity under corporate and non-corporate ownership, respectively. The values of μ_c and μ_p for each year are backed out from the model, given the observed values of TFP; the tax wedge, τ, from the black graph in Figure 4; and the C-corporate share of net capital stock from the blue graph in Figure 5 (used to gauge the corporate share of output in the model). The other distributional parameters in the model are set at σ_c=σ_p=0.5 and ρ=0.25.

Figure 10 Corporate Fraction of Numbers of Firms as Function of Tax Wedge, τ



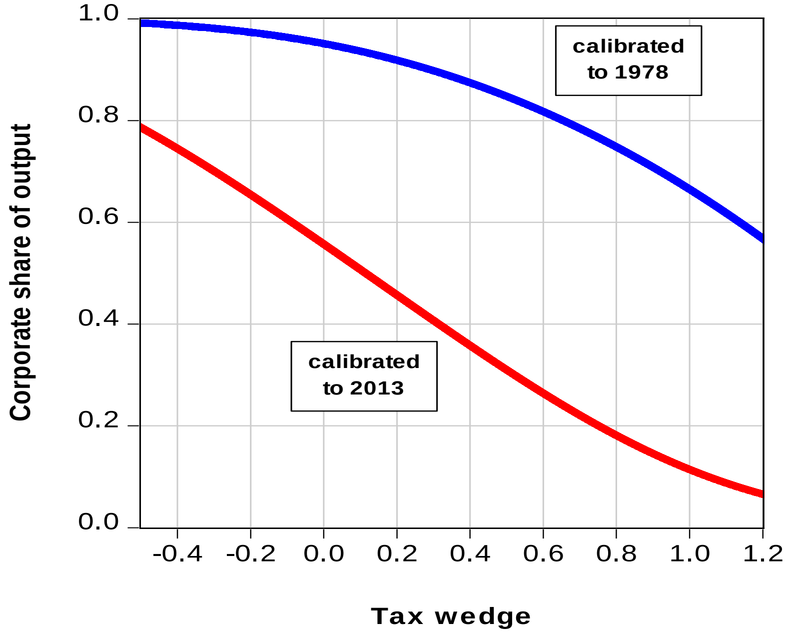
Note: This graph uses the baseline parameter values $\sigma_c = \sigma_p = 0.5$ and $\rho = 0.25$. The values of μ_c and μ_p from Figure 9 are, respectively, -0.137 and -1.032 for 1978 and -0.045 and -0.121 for 2013. The corporate share of numbers of firms declines monotonically with the tax wedge, τ, given in equation (2). This share approaches 1 as τ approaches $-\infty$ (as τ_p approaches 1) and approaches 0 as τ approaches ∞ (as τ_c approaches 1).

Figure 11 Total Output (Productivity) as Function of Tax Wedge, τ



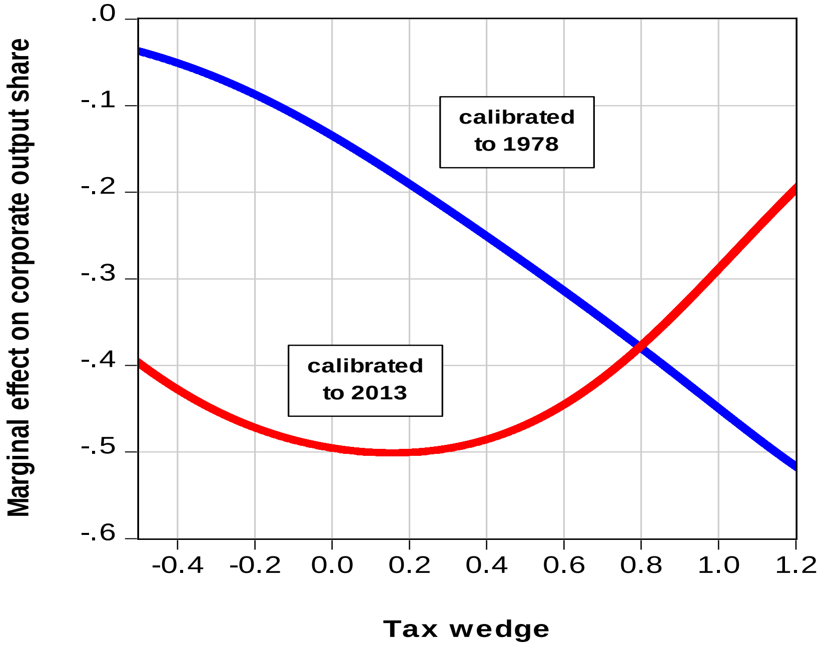
Note: See the note to Figure 10. Total output (productivity) peaks at a tax wedge, τ, of 0. This peak value is normalized to equal 1.0 for 1978. Total output falls with τ when $\tau > 0$ and rises with τ when $\tau < 0$.

Figure 12 Corporate Share of Output as Function of Tax Wedge, τ



Note: See the note to Figure 10. The corporate share of output declines monotonically with the tax wedge, τ. This share approaches 1 as τ approaches $-\infty$ and approaches 0 as τ approaches ∞.

Figure 13 Marginal Effect of τ on Corporate Output Share



Note: See the notes to Figures 10 and 12. The marginal effect of the tax wedge, τ, on the corporate output share is negative throughout.

Figure 14

Composition of Gross Assets by Sector

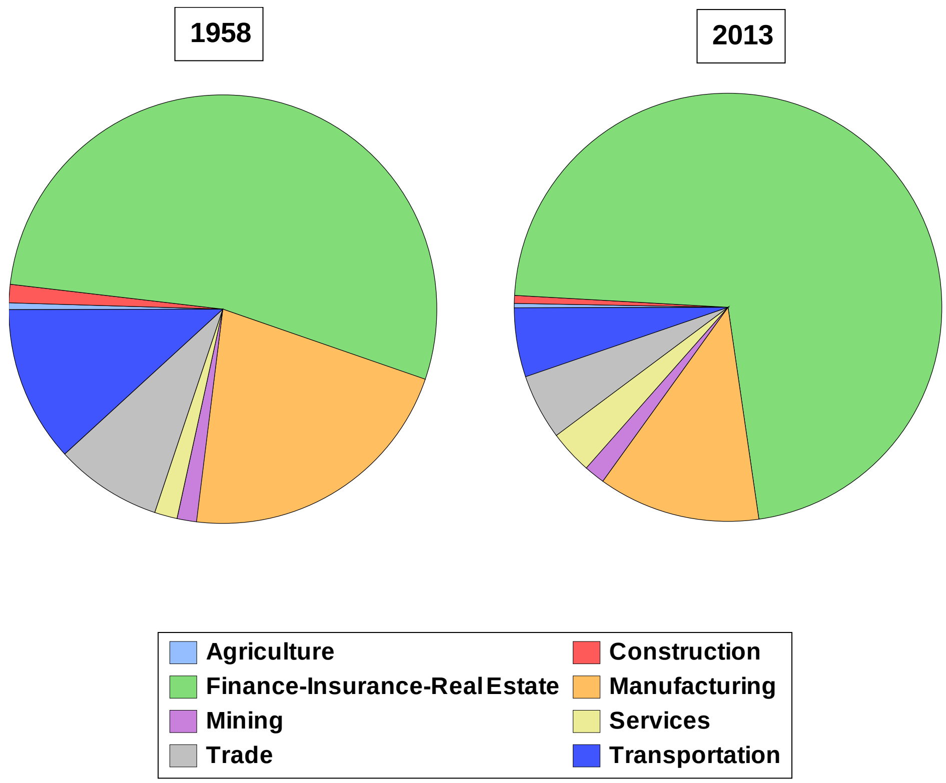


Figure 15

Composition of Positive Net Income by Sector

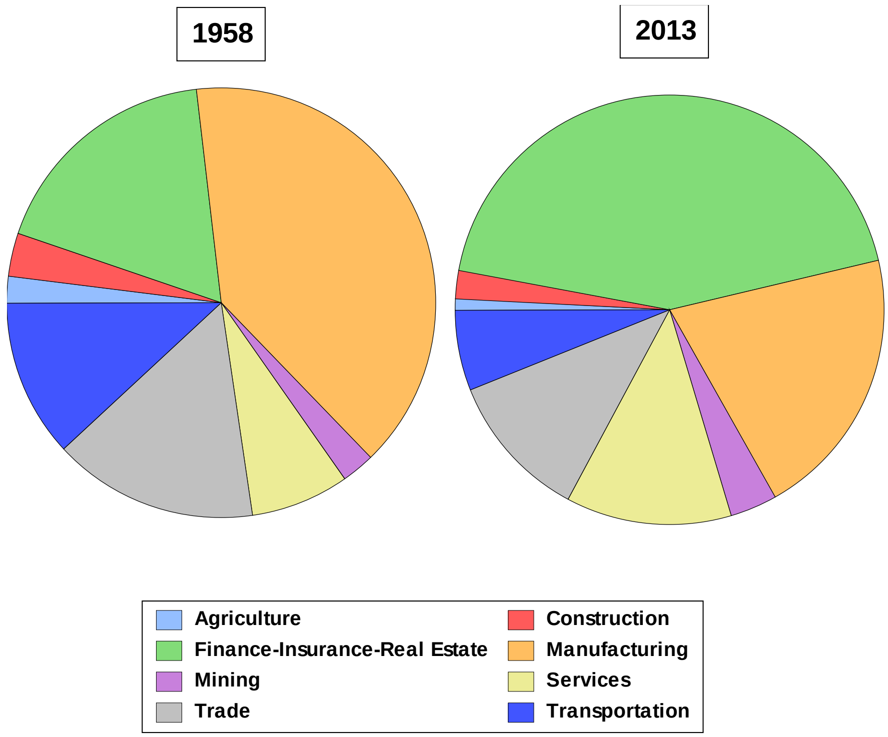
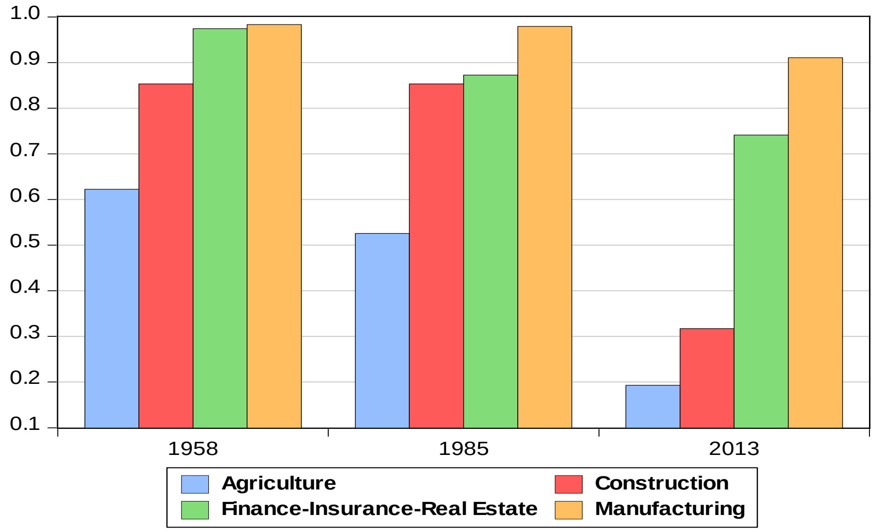


Figure 16 C-Corporate Shares of Gross Assets by Sector



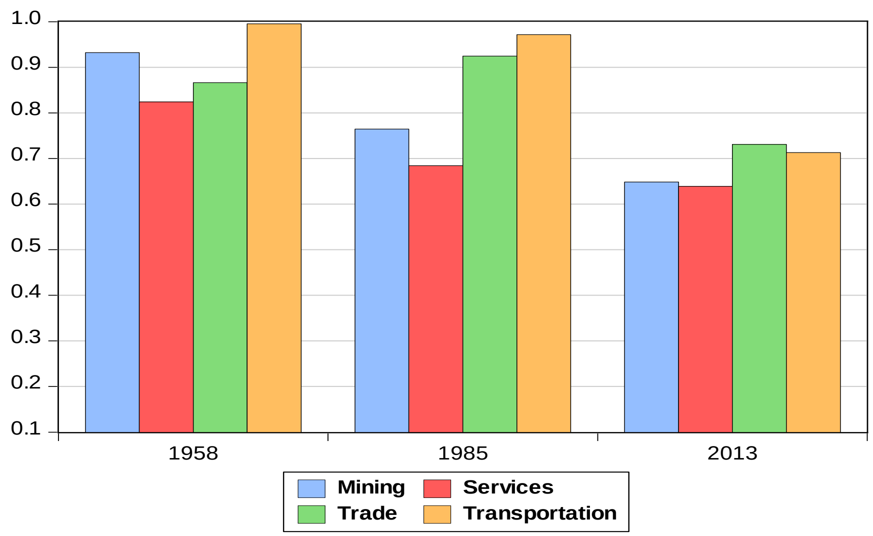
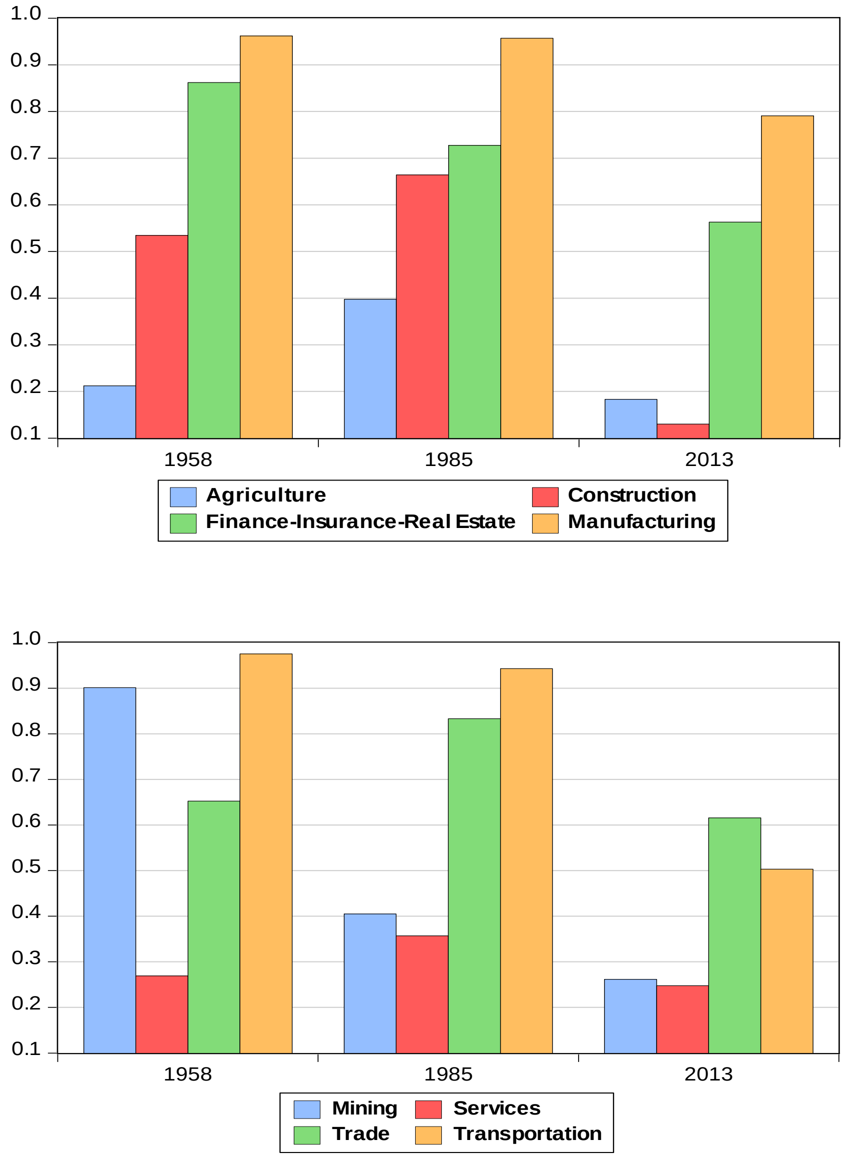


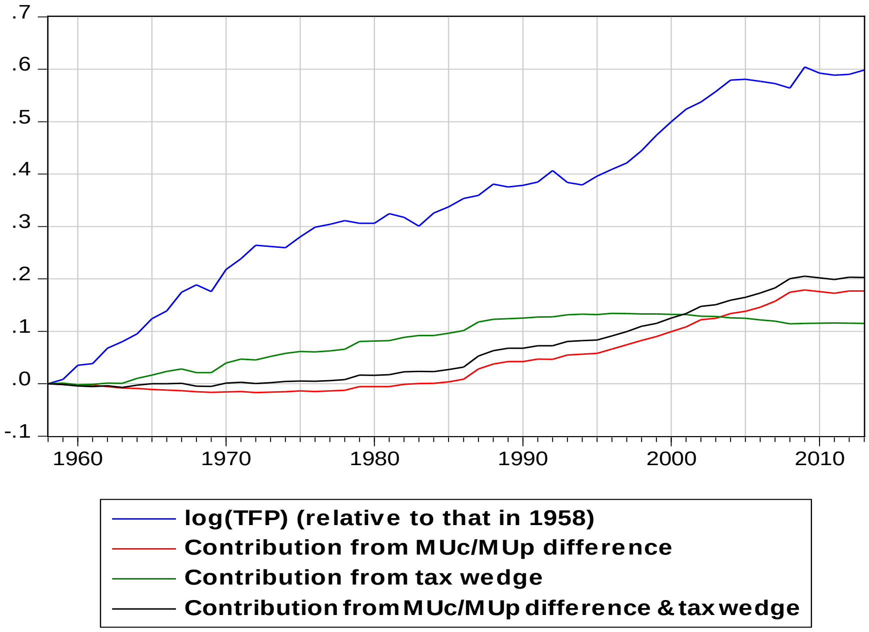
Figure 17 C-Corporate Shares of Positive Net Income by Sector



Note to Figures 14-17: The data refer to gross assets and positive net income for C-corporations, S-corporations, and partnerships. The breakdown for the eight indicated sectors are from Internal Revenue Service, *Statistics of Income: Corporation Income Tax Returns*, *Statistics of Income: Business Income Tax Returns*, *Statistics of Income Bulletin: Partnership Tax Returns*, and *Statistics of Income: Partnership Tax Returns*, various years, as noted in the references.

Figure 18

Contributions to TFP Growth



Notes: The blue graph is log(TFP)-log(TFP_{1958}), where TFP is total factor productivity from Figure 9. The other graphs are calculations from the model developed in Section IV. These calculations use the parameter values $\sigma_c = \sigma_p = 0.5$ and $\rho = 0.25$.

For the contribution from μ_c-μ_p, the model is used to estimate the output, $\tilde{y}$, that would have arisen each year if μ_c-μ_p were kept at its 1958 value, 1.478 (see Figure 9). Each year's tax wedge, τ, is kept at that shown by the black graph in Figure 4, μ_c is kept at the value shown in Figure 9, and μ_p is set to μ_c-1.478. The red graph shows log(TFP)-log($\tilde{y}$)—the contribution to log(TFP) from the change in μ_c-μ_p compared to its value in 1958.

For the tax-wedge calculation, the model is used to estimate the output, $\tilde{y}$, that would have arisen each year if τ were kept at its 1958 value, 0.7580 (see black graph in Figure 4). Each year's μ_c and μ_p are kept at those shown in Figure 9. The green graph shows log(TFP)-log($\tilde{y}$)—the contribution to log(TFP) from the change in τ compared to its value in 1958.

For the calculation for the joint contribution from the changes in μ_c-μ_p and τ, the model is used to estimate the output, $\tilde{y}$, that would have arisen each year if μ_c-μ_p and τ were kept at their 1958 values of 1.478 and 0.7580, respectively. Each year's μ_c is kept at that shown in Figure 9, and μ_p is set to μ_c-1.478. The black graph shows log(TFP)-log($\tilde{y}$)—the contribution to each year's log(TFP) from the changes in τ and μ_c-μ_p compared to their values in 1958.

Table 1

Regressions for C-Corporate Shares of Economic Activity, 1978-2013

	(1)	(2)	(3)	(4)
Dependent variable:	**C-Corp share net capital stock**	**C-Corp share equity (book value)**	**C-Corp share gross assets**	**C-Corp share positive net income**
Independent variables:				
Constant (trend), 1978-1988	-0.0103*** (0.0008)	-0.0096*** (0.0015)	-0.0064*** (0.0006)	-0.0037* (0.0021)
Constant (trend), 1989-1996	-0.0101*** (0.0006)	-0.0139*** (0.0022)	-0.0071*** (0.0007)	-0.0075*** (0.0013)
Constant (trend), 1997-2013	-0.0128*** (0.0004)	-0.0206*** (0.0013)	-0.0094*** (0.0005)	-0.0132*** (0.0015)
C-Corp federal tax rate, $\log(1-\tau_c)$	0.238*** (0.049)	0.534*** (0.091)	0.238*** (0.034)	0.105 (0.136)
Sched. E federal AMTR, $\log(1-\tau_p)$	-0.481*** (0.093)	-0.564*** (0.077)	-0.343*** (0.029)	-0.394 (0.278)
p-value for τ_{prof} and τ_{div}†	0.60	0.70	0.83	0.0000
p-value for τ_c and τ_p††	0.0001	0.41	0.0001	0.062
R-squared	0.84	0.95	0.92	0.85
s.e. of regression	0.0158	0.0314	0.0112	0.0392

***Significant at 1%, **significant at 5%, *significant at 10%.

†The C-corporate tax variable is $log(1-\tau_c) = log(1-\tau_{prof}) + log(1-\tau_{div})$, where prof refers to C-corporate profits and div to dividends. These tests are for equal coefficients on $log(1-\tau_{prof})$ and $log(1-\tau_{div})$, which have unrestricted estimated coefficients of, respectively, 0.229 (s.e.=0.046) and 0.295 (0.134) in col. 1, 0.501 (0.063) and 0.749 (0.326) in col. 2, 0.233 (0.019) and 0.267 (0.165) in col. 3, and 0.238 (0.103) and -0.766 (0.231) in col. 4.

††These tests are for the hypothesis that the coefficients on $log(1-\tau_c)$ and $log(1-\tau_p)$ are of equal magnitude with opposite signs.

Notes to Table 1: Variables in the regressions are 20-year differences. Sample periods are 1978-2013. Standard errors, shown in parentheses, are calculated from the Newey-West method with 20-year bandwidths. Dependent variables are Col. 1: C-corporate share of business net capital stocks (Figure 5), Col. 2: C-corporate share of business equity or book value (Figure 6), Col. 3: C-corporate share of business gross assets (Figure 7), and Col. 4: C-corporate share of business positive net income (Figure 8). The shares are calculated relative to business totals that comprise C-corporations, S-corporations, and partnerships (including LLCs), but excluding sole proprietorships. (Data on sole proprietor amounts are available only for business net income.) The top federal tax rate on C-corporate profits, τ_{prof}, and the federal AMTR for dividends, τ_{div}, are in Figures 1 and 2. The pass-through federal tax rate, τ_p, is gauged by the federal AMTR for Schedule E income (exclusive of rents, royalties, and estates & trusts) and is in Figure 3. The tax variables enter, as in equation (2), as $log(1\text{-}\tau_c) = log(1\text{-}\tau_{prof}) + log(1\text{-}\tau_{div})$ and $log(1\text{-}\tau_p)$. The constants indicate trend rates of change per year for the underlying level variables. The break points of 1989 and 1997 correspond to key historical legal events involving the role of LLCs (see the text). The first p-value is for a test that the coefficients on $log(1\text{-}\tau_{prof})$ and $log(1\text{-}\tau_{div})$ are equal—the estimated coefficients shown impose this restriction. The second p-value is for a test that the sum of the coefficients on $log(1\text{-}\tau_c)$ and $log(1\text{-}\tau_p)$ add to zero. These restrictions are implied by the model.

Table 2

Alternative Parameter Values

Year	μ_c	μ_p	σ_c	σ_p	ρ	τ	TFP	Corp share
I. μ_c and μ_p chosen to match observed TFP and corporate share								
1958	-0.440	-1.918	0.5	0.5	0.25	0.758	0.711	0.955
	-0.335	-1.478	0.25	0.25	0.25	0.758	0.711	0.955
	-0.342	-1.795	0.25	0.5	0.25	0.758	0.711	0.955
	-0.436	-1.703	0.5	0.25	0.25	0.758	0.711	0.955
	-0.436	-1.796	0.5	0.5	0.5	0.758	0.711	0.955
2013	-0.045	-0.121	0.5	0.5	0.25	0.090	1.294	0.512
	0.153	0.070	0.25	0.25	0.25	0.090	1.294	0.512
	0.085	-0.087	0.25	0.5	0.25	0.090	1.294	0.512
	-0.008	0.007	0.5	0.25	0.25	0.090	1.294	0.512
	-0.008	-0.087	0.5	0.5	0.5	0.090	1.294	0.512
II. μ_c-μ_p fixed at 1958 value (μ_c and τ as in part I for 2013)								
2013	-0.045	-1.523	0.5	0.5	0.25	0.090	1.084	0.994
	0.153	-0.990	0.25	0.25	0.25	0.090	1.202	1.000
	0.085	-1.368	0.25	0.5	0.25	0.090	1.123	0.997
	-0.008	-1.275	0.5	0.25	0.25	0.090	1.124	0.997
	-0.008	-1.368	0.5	0.5	0.5	0.090	1.124	0.997
III. τ fixed at 1958 value (μ_c and μ_p as in part I for 2013)								
2013	-0.045	-0.121	0.5	0.5	0.25	0.758	1.153	0.197
	0.153	0.070	0.25	0.25	0.25	0.758	1.205	0.004
	0.085	-0.087	0.25	0.5	0.25	0.758	1.133	0.132
	-0.008	0.007	0.5	0.25	0.25	0.758	1.133	0.133
	-0.008	-0.087	0.5	0.5	0.5	0.758	1.133	0.133
IV. μ_c-μ_p and τ fixed at 1958 values (μ_c as in part I for 2013)								
2013	-0.045	-1.523	0.5	0.5	0.25	0.758	1.056	0.955
	0.153	-0.990	0.25	0.25	0.25	0.758	1.159	0.955
	0.085	-1.368	0.25	0.5	0.25	0.758	1.090	0.955
	-0.008	-1.275	0.5	0.25	0.25	0.758	1.091	0.955
	-0.008	-1.368	0.5	0.5	0.5	0.758	1.091	0.955

Notes to Table 2: The analysis applies to the model discussed in Section IV of the text. The baseline specification assumes the parameter values $\sigma_c=\sigma_p=0.5$ and $\rho=0.25$, where σ_c and σ_p are the standard deviations for the log-normal distributions of, respectively, corporate and pass-through productivity and ρ is the correlation coefficient between the random variables. The results in Part I for 1958 and 2013 give the values of the respective mean parameters, μ_c and μ_p, for which the calibrated model delivers the overall TFP and corporate share of economic activity (net capital stock) that are shown, given the observed tax wedge, τ. The other results in Part I are comparable but correspond to four alternative specifications: $\sigma_c=\sigma_p=0.25$, $\rho=0.25$; $\sigma_c=0.25$, $\sigma_p=0.5$, $\rho=0.25$; $\sigma_c=0.5$, $\sigma_p=0.25$, $\rho=0.25$; and $\sigma_c=\sigma_p=0.5$, $\rho=0.5$.

Parts II-IV give results for counter-factual situations where σ_c-σ_p, τ, or both are held fixed in 2013 at their values in 1958.

Appendix

Derivation of Expectations of Corporate and Pass-Through Output

We start with the derivation of equation (3), which gives the conditional expectation of corporate output, Y_c. The setup is that $log(Y_c)$ and $log(Y_p)$ are bivariate normal with respective means and standard deviations of μ_c, σ_c, μ_p, and σ_p. The correlation coefficient between the two random variables is ρ. This specification implies that $y \equiv log(\frac{Y_c}{Y_p})$ is distributed normally with mean μ=μ_c- μ_p and variance $\sigma^2 = \sigma_c^2 + \sigma_p^2 - 2\rho\sigma_c\sigma_p$.

The distribution of $log(Y_c)$, conditional on y, is normal with respective mean and standard deviation of $\tilde{\mu}_c$=$\mu_c + \left(\frac{\sigma_c}{\sigma}\right) \cdot \left(\frac{\sigma_c - \rho\sigma_p}{\sigma}\right) \cdot (y - \mu)$ and $\tilde{\sigma}_c^2 = \sigma_c^2\sigma_p^2(1 - \rho^2)/\sigma^2$ (see Hogg and Craig [1965, pp. 102-104]). That is, $\left(\frac{\sigma_c}{\sigma}\right) \cdot \left(\frac{\sigma_c - \rho\sigma_p}{\sigma}\right)$ is the regression coefficient of $log(Y_c)$ on y. The expectation of Y_c conditional on y is $exp(\tilde{\mu}_c+0.5\cdot\tilde{\sigma}_c^2)$. The expectation of Y_c is then

$$Prob.(y \geq \tau) \cdot E(Y_c|y \geq \tau) = \frac{1}{\sigma\sqrt{2\pi}} \int_\tau^\infty \exp(\tilde{\mu}_c + 0.5 \cdot \tilde{\sigma}_c^2) \cdot \exp[\frac{-(y-\mu)^2}{2\sigma^2}]dy.$$

Using the expressions for $\tilde{\mu}_c$ and $\tilde{\sigma}_c^2$, the result can be written as:

$$Prob.(y \geq \tau) \cdot E(Y_c|y \geq \tau) = \frac{\exp(\mu_c + 0.5\sigma_c^2)}{\sigma\sqrt{2\pi}} \int_\tau^\infty \exp[(\frac{-1}{2\sigma^2}) \cdot [y - \mu - \sigma_c(\sigma_c - \rho\sigma_p)]^2 dy.$$

Finally, using the change of variable z=$[y - \mu - \sigma_c(\sigma_c - \rho\sigma_p)]/\sigma$, the lower limit of integration becomes $\tau' = \left(\frac{1}{\sigma}\right) \cdot [\tau - \mu - \sigma_c(\sigma_c - \rho\sigma_p)]$. We then get equation (4):

$$Prob.(y \geq \tau) \cdot E(Y_c|y \geq \tau) = [\exp(\mu_c + 0.5\sigma_c^2)] \cdot [1 - \Phi(\tau')],$$

where $\Phi(\cdot)$ is the cumulative standard normal density. We can also replace $1 - \Phi(\tau')$ by $\Phi(-\tau')$.

The result for pass-through output, $Prob.(y < \tau) \cdot E(Y_p | y < \tau)$, is analogous, with the parameters for c and p switched (including that μ is now μ_p-μ_c) and τ replaced by -τ in the expression for τ'.