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TOLERANCE AND COMPROMISE IN SOCIAL NETWORKS

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ABSTRACT

In this paper, individuals are characterized by their identity — an ideal code of conduct — and by a level of tolerance for behaviors that differ from their own ideal. Individuals first choose their behavior, then form social networks. This paper studies the possibility of compromise, i.e. individuals choosing a behavior different from their ideal point, in order to be accepted by others, to "belong." I first show that when tolerance levels are the same in society, compromise is impossible: individuals all choose their preferred behavior and form friendships only with others whose ideal point belong to their tolerance window. In contrast, I show that heterogeneity in tolerance allows for compromise in equilibrium. Moreover, if identity and tolerance are independently distributed, any equilibrium involves some compromise.

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1. Motivation

Bernard Crick defines tolerance "as the degree to which we accept things of which we disapprove" (Crick (1963, rep. 1971)). It is the ability or willingness to withstandsomething, in particular the existence of opinions or behavior that one does not necessarily agree with. The literature on homophily has long highlighted the role of preferences in explaining why people associate with and bond more with others who are similar to them (see McPherson, Smith-Lovin, and Cook (2001) for a review). People prefer interacting with similar others.

However, there is an important distinction to be made between individuals caring about the innate *identity* of their friends – their type – versus caring about the *conduct* of their friends – their behavior or adopted identity. In some settings, preferences depend on others' types such as their religion, ethnicity, sexual orientation (see Currarini, Jackson, and Pin (2009), Currarini, Matheson, and Vega-Redondo (2016)). But in other settings, individuals may not actually care about others' true identity but care about others' behavior: whether they act religious, whether they dress conservatively, whether they appear gay or even how 'white' they act (see Austen-Smith and Fryer (2005), Carvalho (2013), Berman (2000) and Lagunoff (2001)). This is the type of preference that this paper studies.

In this model, individuals interact in social networks. A person's utility depends on her own conduct but also the conduct of others in her network. Individuals differ in their ideal behavior, their true *identity*, and potentially in their *tolerance* for conduct that differs from that ideal point. Prior to forming their social network, individuals choose their behavior and can therefore *compromise* in order to "fit in." For instance, a religious person might decide not to display any religious symbol to be accepted as a friend by a less religious person, or a non-religious person may sometime go to church to please a friend. To adopt a conduct that differs of one's innate identity comes at the cost of cognitive dissonance. The only motive for compromising in this model is to make friends, i.e. to belong.

I show that there are strict limits to compromise, and that these limits are strictly decreasing in the tolerance of the most intolerant members of society.

At the extreme, I show that if all individuals have the same tolerance levels, then compromise is impossible in equilibrium. Everyone chooses their preferred code of conduct. Individuals form links with each other if and only if they tolerate each other's ideal points. The intuition for this inability to compromise is simple. When tolerance levels are symmetric, if one individual needs to compromise to be friends with another then the latter needs to compromise as well. Since compromise is costly, individuals have the incentive to do the least possible in order to be accepted. But this implies that their friendship is not very valuable to others, who then have little incentive to compromise themselves.

Heterogeneity is needed for compromise to happen. I show that introducing more intolerant individuals allows the possibility of compromise in equilibrium. With differences in tolerance levels, tolerant individuals may value the friendship of relatively intolerant individuals, even if the latter do not compromise, and therefore the former may unilateraly compromise. What is more, this paper proves that if compromise and tolerance are independently distributed, then there *must* be compromise in equilibrium. Relatively tolerant individuals compromise for relatively intolerant ones. Naturally, the joint distribution of tolerance levels and identities matters. If more extreme identities are less tolerant, reciprocated compromise is not possible and behaviors tend to be polarized. In contrast, more tolerance at the extreme encourages a more connected society.

Finally, I contend that this model can also be applied to other settings. For instance, political compromises and alliances between politicians can be important (Levy (2004)). It could be applied to the choice of technological standards and the formation of trade networks. Countries could be endowed with different initial technologies and the complementarities between their technologies could decrease in the distance between their standard while the cost of modifying a technology could be proportional to the extent to which it needs to be modified.

The next Section discusses the related literature. Section 3 formalizes the model described above. Simple examples in Section 4 provide the intuition for the main results. The latter are presented in Section 5. Section 6 discusses some implications of the results and Section 7 concludes.

2. Literature

This work pertains to the general framework of Akerlof and Kranton (2000), where 1. people have identity-based payoffs derived from their own actions, 2. people have identity-based payoffs derived from others' actions, and 3. individuals can modify their identity at some cost[1]. Within this framework, this paper provides a model where an endogenous social network may provide incentives for individuals to compromise their identity.

Also related is Cervellati, Esteban, and Kranich (2010). In their work, as in this paper, moral judgments befall others as well as oneself. However, individuals value the esteem that they receive from others, and deviations of their observed behavior from a norm of morally appropriate behavior influences esteem. In contrast, in this paper, individuals value all behaviors according to how they conform or depart from their own ideals.

Homophily in social networks has been amply documented empirically (see Marmaros and Sacerdote (2006) for a recent example, and McPherson, Smith-Lovin, and Cook (2001) for an overview).[2] This paper contributes to a growing theoretical literature on homophily and the formation of friendship networks (Currarini, Jackson, and Pin (2009), Currarini, Matheson, and Vega-Redondo (2016), Jackson (2019)). Like them, this paper assumes that individuals prefer to associate with similar others. The difference is that this paper assumes that individuals care about others' conduct, as opposed to their identity.

This paper also speaks to the literature on diversity and social capital (see Putnam (2000), Dasgupta and Serageldin (1999) and Portes and Vickstrom (2011)). According to Putnam (2000), there is an important distinction between bridging (inclusive) and bonding (exclusive) social capital. Bonding social capital networks are inward-looking and tend to reinforce exclusive identities and homogenous groups.

[1]In this paper, individuals cannot change their innate identities but we can think of their behavior as an adopted identity.

[2]"Birds of a feather flock together " is attributed to Burton (1927, rep. 1651), but scholars have described the pattern starting in the antiquity: "we love those who are like themselves " (Aristotle (1934)), or "similarity begets friendship" (Plato (1968)).

On the other hand, bridging social capital networks are outward-looking and include people across "diverse social cleavages." This paper adds to this literature the importance of intolerant individual as "bridging" agents.

3. Premises of the model

Individuals and Preferences:

Consider a population I consisting in a mass of size 1 of individuals i distributed over the interval $[0,1]$. Each individual has an ideal point $\iota_i \in [0,1]$, her *identity*. This identity represents the person's ideal code of conduct and is immovable. In contrast, individuals select a code of conduct, a *behavior*, a_i in the Euclidean space. As will be described soon, individuals then form their social network.

Individuals value friendships, but also care about both their own conduct and the behavior of the members of their social network. They judge all behaviors in comparison with their identity. I assume that an individual's utility strictly decreases in the Euclidean distance of behaviors from her ideal point. Individual i derives a utility $v_i\left(d(\iota_i, a_j)\right)$ from a link to an individual j with behavior a_j. The link has a strictly positive value when $\iota_i = a_j$ but its value strictly decreases as a_j differs from i's identity. Individuals have potentially heterogenous preferences regarding the benefits they derive from a link and their tolerance for behaviors that differ from their identity. Choosing a behavior that departs from one's own identity also comes at a cost, $g(d(\iota_i, a_i))$ strictly increasing in d.

Consider an individual i with ideal point ι_i, behavior a_i and links with individuals in S whose profile of behavior is given by $\mathbf{a}_S$. Her utility is expressed as:

$$u_i(a_i, \mathbf{a}_S) = \int_{j \in S} v_i\left(d(\iota_i, a_j)\right) - g(d(\iota_i, a_i)), \tag{1}$$

where $d(\iota, a)$ is the Euclidian distance between ι and a; v_i is continuous, strictly decreasing with $0 < v_i(0) \leq F$ for some finite F; and g is continuous, strictly increasing, and convex with $g(0) = 0$.

Note that this specification implies that utilities from friendships are additively separable.

Individual i is said to *compromise* if her chosen behavior differs from her ideal point $d(\iota_i, a_i) > 0$.

Define individual i's *tolerance level* t_i as the largest tolerable distance t, that is

$$t_i = \{max \ \ t \in \mathbb{R}_+ | v_i(t) \geq 0\}.$$

Person i if happy to have a person j in her social group (or, at least, does not mind), as long as j's behavior is within a distance t_i of i's ideal point: $d(\iota_i, a_j) \leq t_i$. An individual's tolerance level reflects both how much the individual values a friendship, and the extent to which she dislikes departures from her ideal behavior.

In this model, individuals are effectively characterized by two attributes: their *identity* ι_i or ideal code of conduct and their *tolerance level* t_i which represents the largest tolerable deviation from their ideal point. These are the two key attributes of individuals for the main results.

Let's define i's *tolerance window* as $\omega_i \equiv \{a \in \mathbb{R} | d(\iota_i, a) \leq t_i\}$ and say that a belongs to i's tolerance window if $a \in \omega_i$. Figure 1 illustrates these concepts.

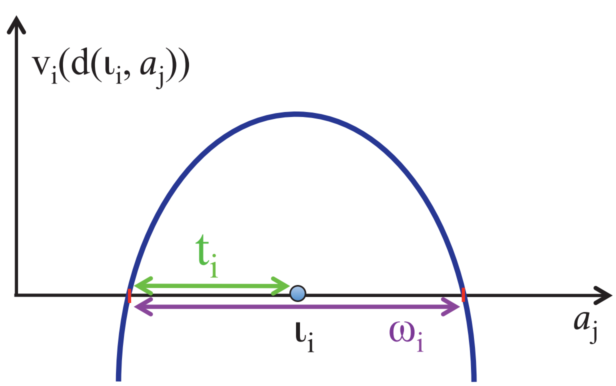

Figure 1. Tolerance

Timing:

This is a two-stage game. In a first stage, individuals choose their code of conduct (their behavior) and, in a second stage, they form their social network by choosing with whom to be friends. When people choose their lifestyles they have in mind this second-stage possibility, and may want to compromise in order to be accepted, to "belong."

This timing assumes that individuals are able to commit on a code of conduct or adopted identity. If people chose their behavior after having formed a social network, they would always choose their preferred behavior and compromise would not be possible.

Network Formation:

There is no cost of forming a link, and the benefits of a link are additive. As a result, network formation is trivial. Given a vector of behavior $\mathbf{a}$ in the population, an individual i is happy to form a link with an individual j if and only if $\mathbf{a}_j \in \omega_i$.

Following most of the network literature, I consider networks that are pairwise stable. Jackson and Wolinsky (1996) defined a network to be the *pairwise stable* if (i) no player would be better off if he or she severed one of his or her links, and (ii) no pair of players would both benefit from adding a link that is not in the network.

Assume that if both players are indifferent, they will form a link. Then, for any given profile of action $\mathbf{a}$, there is a unique pairwise stable graph G so that i and j have a link $g_{ij} = 1$ if and only if $\mathbf{a}_j \in \omega_i$ and $\mathbf{a}_i \in \omega_j$.

4. Examples

This section provides some intuition about the main results through simple examples with a discrete number of individuals. For these examples, I consider a discrete version of the model and assume the following linear payoffs:

$$u_i(a_i, \mathbf{a}_{S(i)}) = \sum_{j \in S(i)} [F_i - b_i d(\iota_i, a_j)] - g d(\iota_i, a_i), \qquad g \geq 0; \tag{2}$$

where F_i represents the intrinsic value of a friendship for i while b_i captures her aversion for behaviors that do not correspond to i's ideal. In this case, i's tolerance level is given by

$$t_i = \frac{F_i}{b_i}.$$

Observe that this formulation captures well the fact that one's tolerance depends on both the benefit that she derives from a friendship and her dislike of differences.

The more someone has to gain from social connections, the more she is willing to befriend individuals who differ from her ideal point.

4.1. **No Compromise with Homogeneity.**

Assume that $t_i = t$ for all i. When tolerance levels are symmetric, then either both individuals' identities belong to the other's tolerance window $\iota_i \in \omega_j$ and $\iota_j \in \omega_i$ or both individuals' identities lie outside of the other's tolerance window $\iota_i \notin \omega_j$ and $\iota_j \notin \omega_i$.

In the first panel of Figure 2, i and j are sufficiently tolerant or sufficiently similar that their ideal conducts already belong to the other's tolerance window. They therefore have no incentive to compromise, and can be friends in spite of their differences.

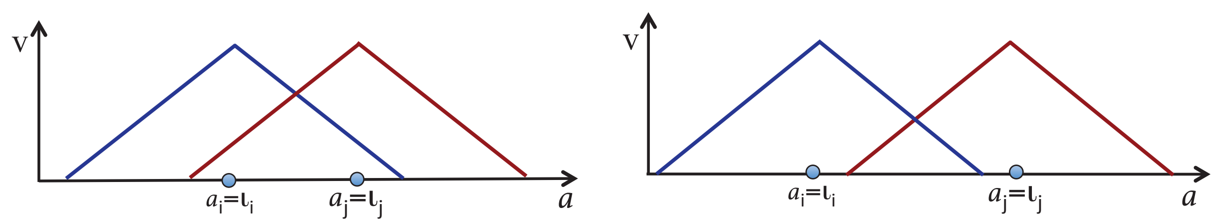

FIGURE 2. A. i AND j ARE NOT FRIENDS; B. i AND j ARE FRIENDS

In contrast, the second panel of Figure 2 illustrates a situation where i and j do not belong to each other's windows though their tolerance windows do overlap. The only way for them to become friends is for *both* to compromise. Since there is no incentive to unilaterally compromise, there is clearly an equilibrium without compromise. What this paper shows is that this equilibrium is unique. Since compromise is costly, individuals have an incentive to "minimally compromise": do the least possible in order to be accepted. But this implies that their friendship is not valuable to others who then have little incentive to compromise themselves. Hence, two individuals cannot compromise for each other.

In both cases, all individuals choose their preferred actions and are friends only if they belong to each other's tolerance window.

4.2. **Heterogeneity Enables Compromise.**

To see how heterogeneity in tolerance levels enables compromise, take two individuals j and k who differ in tolerance levels. If j is more tolerant than k, a situation where $\iota_k \in \omega_j$ but $\iota_j \notin \omega_k$ is possible. This is illustrated in the left panel of Figure 3. Person j values a link to k even if k does not compromise. If such a link is worth enough to j to compensate her for the disutility from compromising and becoming acceptable to k, a link will be formed. If she compromises, j would clearly choose the smallest compromise needed to be friends with k: the action a_j in ω_k that is the closest possible to ι_j as shown in the left panel of Figure 3. Hence, j compromises and befriends k if

$$F_j - b_j|\iota_j - \iota_k| - g|\iota_j - a_j| \geq 0. \tag{3}$$

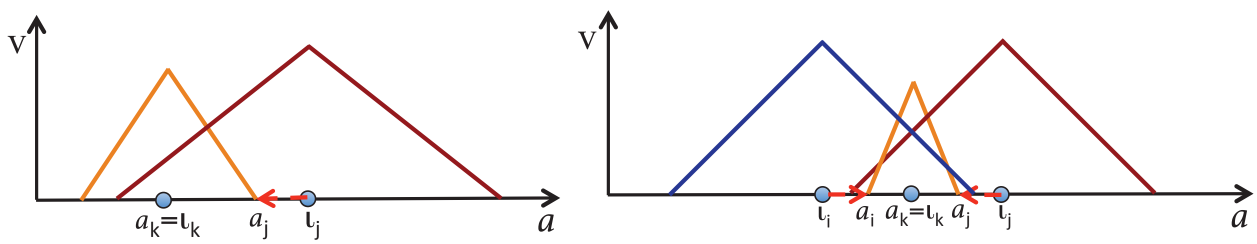

FIGURE 3. COMPROMISING FOR AN INTOLERANT PERSON

It follows naturally that the presence of less-tolerant individuals can allow more tolerant individuals to become friends. To show this, consider the example in Figure 2 where i and j have the same tolerance levels and add a more intolerant individual k between them so that $\omega_k \subseteq \omega_i$, $w_k \subseteq \omega_j$, $\iota_i \notin \omega_j$ and $\iota_j \notin \omega_i$, as in Figure 3. If i compromises to be acceptable to k, she is attractive to j as well and vice versa. Let ℓ_k and r_k be, respectively, the left and the right extremities of k's tolerance window. There is an equilibrium where $a_i = \ell_k$, $a_k = \iota_k$ and $a_j = r_k$ and all three individuals are friends if the following two inequalities hold:

$$[F_i - b_i|\iota_k - \iota_i|] + [F_i - b_i|a_j - \iota_i|] \geq g|\iota_i - \ell_k| \ \ \&$$
$$F_i - b_i|\iota_k - \iota_i| \geq g|\ell_k - \ell_j|.$$

The first inequality requires the overall value of the compromise to be positive: the value of the friendships with j and k exceeds the cost of compromise. In addition, the second inequality guarantees that i prefers choosing ℓ_k and being friends with both i and j, rather than choosing the left extremity of j's tolerance window, ℓ_j, and being friends only with j. Both these constraints are satisfied for a sufficiently low cost of compromise, g. This equilibrium is illustrated in the right panel of Figure 3.

Example: Assume that i and j have ideal positions $\iota_i = 0.2$ and $\iota_j = 0.8$, and are otherwise symmetric with $b_i = b_j = 1$ and $F_i = F_j = 0.5$, while k, who has an ideal position in between, $\iota_k = 0.5$, is less tolerant $b_k = 5$ and $F_k = 0.5$. With respect to their own actions, they all have the same disutility from deviating from their ideal point $g = 1.1$. Interestingly, it can be checked that i would not compromise for k alone, but $a_i = \ell_k = 0.4$, $a_k = \iota_k$ and $a_j = r_k = 0.6$ is an equilibrium.

4.3. **Compromises builds on Compromise.**

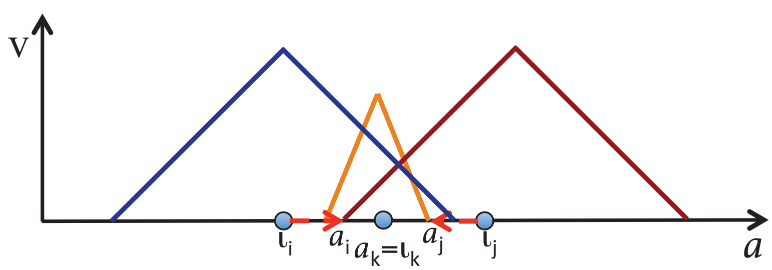

Figure 4. Compromise builds on compromise

As we have just seen, and as will be shown more generally below, compromise originates with a individual who compromises to become friends with a less-tolerant person. But it is worth noting that further compromise can be built from that initial effort. Indeed, other high-tolerance individuals may compromise for the initial compromiser.

This is illustrated in Figure 4, where i and j are high-tolerance, while k is a low-tolerance person. In this example, k does not compromise, i compromises for k but j compromises for i who is now valuable to j. A complete network is achieved.

5. Main Results

After these illustrative examples, we return to our most general setup: a continuum of agents with preferences represented by (1) and tolerance levels that can take values in $[\underline{t}, \bar{t}]$ with $\bar{t} \geq \underline{t} > 0$.

5.1. The Limits to Compromise.

This section characterizes the limits to compromise. Before proceeding, I will introduce some definitions that are useful for the proof and the rest of the paper. Denote as $I_i \equiv \{a \in \mathbb{R} | d(\iota_i, a) < t\}$ the interior of i's tolerance window. We shall say that:

Definition **1.** j is *valuable* to i if $a_j \in I_i$.

Definition **2.** i *compromises for* j if $d(\iota_i, \iota_j) > t_j \geq d(a_i, \iota_j)$.

Definition **3.** i *minimally compromises for* j if i compromises for j and $a_i = argmin_{a_i \in \omega_j} d(\iota_i, a_i)$.

That is, an individual i is said to *compromise for* another one j if her ideal point is outside of j's tolerance window while her chosen behavior is inside of j's window. She is said to *minimally* compromise for j if she compromises for j while deviating as little as possible from her ideal behavior. We now turn to our first proposition.

Proposition **1.** *An individual i with tolerance t_i never compromises by more than $t_i - \underline{t}$ in equilibrium.*

The proof of this proposition is in the Appendix. I'll outline its underlying argument after presenting four direct implications of Proposition 1. The first corollary of this proposition sets an upper limit to the compromise that can be observed in a society.

Corollary **1.** *Individual compromise cannot exceed $T = \bar{t} - \underline{t}$ in equilibrium.*

Another straightforward but powerful corollary of Proposition 1 is that there cannot be any compromise when all individuals have the same tolerance levels (though the underlying utility functions could differ). Only individuals who already belong to each other's tolerance window can be friends in equilibrium. This result generalizes the example of Section 4.1.

Corollary **2.** *If all individuals have the same tolerance, $t_i = t$ for all i,*
[1] *compromise is not possible in equilibrium,*
[2] *i and j are friends if and only if $i \in w_j$ and $j \in \omega_i$.*

It also follows directly from Proposition 1 that the most intolerant individuals in society never compromise.

Corollary **3.** *The least tolerant type never compromises in equilibrium.*

Finally, bounds on compromise imply a maximal distance between the ideal points of any two linked individuals.

Corollary **4.** *In equilibrium, $|\iota_i - \iota_j| \leq t_i + t_j - \underline{t}$ for all pair $ij \in G$.*

Indeed, it follows from Proposition 1 that $d(\iota_i, a_i) \leq t_i - \underline{t}$ and $d(\iota_j, a_j) \leq t_j - \underline{t}$. If i and j are friends $d(\iota_j, a_i) \leq t_j$ and $d(\iota_i, a_j) \leq t_i$. Hence, $d(\iota_i, \iota_j) \leq t_i + t_j - \underline{t}$.

The intuition for the proof of Proposition 1 is simple. Take homogeneous individuals and assume that the claim is wrong: an individual compromises. That person must minimally compromise for a set of valuable individuals, otherwise she would benefit from compromising a little bit less. Now, take the individuals in that set. I show that they themselves must be compromising, and therefore must minimally compromise for a different set of individuals valuable to them. Proceeding in this manner gives us sequences of compromising individuals. The proof shows that along these sequences, compromise must be ever-expanding and cannot converge. This means that, along the sequence, compromise will at some point reach a level such that individuals would be better off compromising less. The argument with heterogenous individuals is similar. I show that if one individual compromises by more than $T = \bar{t} - \underline{t}$, then it would imply the existence of sequences of individuals $m = 1, 2, ...$ who compromise and $d(\iota_m, a_m) - t_m$ would be ever-increasing along these sequences. Again, there must be a point along any of these sequences where compromise becomes prohibitive and we reach a contradiction.

5.2. **How Heterogeneity Helps Compromise.**

The previous section showed that compromise is bounded. At the extreme, we proved that no compromise is possible with homogenous tolerance levels. In contrast, the examples of Sections 4.2 and 4.3 demonstrated that heterogeneity in tolerance levels makes compromise possible. This section goes further. Proposition 2 proves that if identities and tolerance levels are independently distributed, and very small deviation from one's ideal point are costless, then there is compromise in *any* equilibrium. Assume:
$[I]$ Identities ι_i and tolerance levels t_i are independently distributed with non-degenerate interval support

Proposition **2.** *If* $[I]$ *holds and* $g'(0) = 0$*, then there must be compromise in equilibrium.*

The proof (in the Appendix) builds on the intuition behind the example in Section 4.2. If there were no compromise in equilibrium, we could always find some relatively tolerant individuals at an extremity who would have incentives to unilaterally compromise. This is because, while almost costless, a little compromise allows a relatively tolerant person to become acceptable to a positive mass of more intolerant individuals that she values, without loosing any friendships. Hence, there will be compromise in equilibrium.

Assumption $[I]$ requires that tolerance levels and ideal positions be unrelated. This independence assumption could certainly be weakened, but a mix of types at the extremities is crucial for the result in Proposition 2.

Now it is obvious that, in general, there will be many equilibria. A relatively tolerant individual may compromise toward the center or the extreme depending on the behavior of others. This means that adopted identities in the population could change rapidly, from moderate to extreme positions for instance, without much change in people's innate identities. Characterizing the set of equilibria would be difficult, though Proposition 1 and its corollaries help by limiting the range of possible behaviors. Proposition 3 helps us further by identifying the necessity of a *bridge* person in between any two individuals who compromise for each other.

Definition **4.** Say that i and j *reciprocally compromise* if i compromises for j and j compromises for i.

Proposition **3.** *If i and j ($\iota_i \leq \iota_j$) reciprocally compromise, then there be some individual k in between ($\iota_i < \iota_k < \iota_j$) such that $\iota_k - t_k$ or $\iota_k + t_k$ in $\Omega_i \cap \Omega_j$.*

Proposition 3 tells us that in between any pair of individuals who reciprocally compromise, there must be a *bridge*: an individual with an edge to his tolerance window at the intersection of the pair's tolerance windows. This bridge individual is necessarily strictly less tolerant than the most tolerant of the pair. It follows that for two agents of the same tolerance level to compromise for each other, we need a more intolerant person to serve as a *bridge* between them.

5.3. **Tolerance and Extremism.** In this section, I investigate what happens when, in contrast to [I], tolerance and ideal points are systematically related. Specifically, I assume that there is more intolerance at the extremes. Formally, assume: [T] There exists a deterministic mapping $M : [0, 1] \to \mathbb{R}_+$ from identity to tolerance, and M is single-peaked.

Proposition **4.** *Under [T], reciprocal compromise is not possible in equilibrium.*

The impossibility of reciprocal compromise implies that if two individuals are friends, one person's ideal point must lie within the other one's tolerance window.

Corollary **5.** *If $ij \in G$ and $t_i \geq t_j$ then $\iota_j \in \omega_i$.*

It follows that more tolerant individuals compromise toward less-tolerant individuals.

If the mapping from identity to tolerance is continuous and strictly concave then there is a unique i, t and j such that $t_i = t_j = t$ and $\iota_j = \iota_i + t$. All individuals at or to the left of ι_i can only compromise to the left, $a_{i'} \leq \iota_{i'}$ for all i' such that $\iota_{i'} \leq \iota_i$. Similarly all individuals at or to the right of ι_j can only compromise to the right, $a_{j'} \geq \iota_{j'}$ for all j' such that $\iota_{j'} \geq \iota_j$. Behaviors will be more polarized than identities. In this sense, intolerance at the extremes leads to greater polarization.

6. Discussion

6.1. **Welfare.** It should not come as a surprise that compromise can be suboptimal in equilibrium.

Take two individuals i and j with the same tolerance level, $t_i = t_j = t$. While Corollary 5.1 tells us that no compromise is possible in equilibrium, it is easy to show that compromise could benefit them both.

A necessary condition for compromise between two individuals i and j to be optimal is that, for some behaviors, the gain in i's utility as j moves toward her must be higher than i's loss as i moves away from her ideal position, and the same for j.

With linear payoffs as in (2), this is rather unlikely, as it requires individuals to be more sensitive to the behavior of others than to one's own behavior as they move away from one's ideal point (b_i to be higher than g). Although some people are stricter with others than with themselves – finding unacceptable behavior in others that they themselves engage in – it may not be the majority.

However, one expects the cost of deviating from one's ideal point to be convex. In this case, it is easy to construct examples where compromise would be optimal even if one's deviations from one's own ideal point by oneself are no less costly than others' deviations from that point.

Example: Take the discrete case of 2 individuals i and j. Assume their preferences to be:

$$u_k(a_k, \mathbf{a}_S) = \sum_{l \in S} [F - bd(\iota_k, a_l)^2] - gd(\iota_k, a_k)^2, \qquad g \geq b > 0. \tag{4}$$

Let $\lambda = g/b (\geq 1)$. It is easy to show that the Pareto optimum is given by

$$a_i^* = \frac{\iota_j + \lambda \iota_i}{1 + \lambda} \quad \& \quad a_j^* = \frac{\iota_i + \lambda \iota_j}{1 + \lambda}$$

when $d(\iota_i, \iota_j) < \sqrt{\frac{1+\lambda}{\lambda}} t$. In particular, if $b = g$ then meeting in the middle is optimal for i and j as long as $d(\iota_i, \iota_j) < \sqrt{2} t$.

6.2. **Non-monotonicity of payoffs in $\underline{t}$.** We can build on the previous example to see that the payoffs to the tolerant individuals in a society are non-monotonic in the tolerance of more intolerant individuals. Consider two individuals, i and j, whose identities lie just outside of each other's tolerance window, but for whom reciprocal compromise would be optimal. Now, introduce a relatively more intolerant person, k, in between these two.

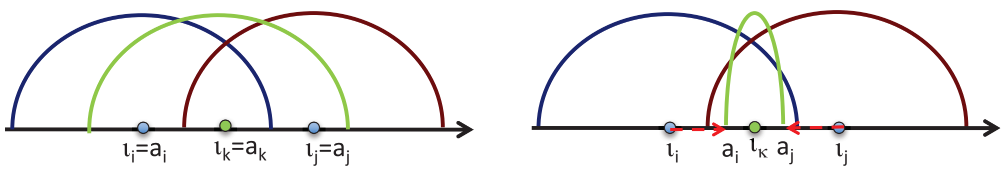

Figure 5. Non-monotonicity of payoff in $\underline{t}$

If k is almost as tolerant as i and j, as illustrated in the left panel of Figure 5, no one compromises. Individuals i and k are within each other's tolerance window and so are friends. The same is true for j and k. As we reduce k's tolerance level, we first reach a point where k's tolerance window lies just outside of i and k's ideal point. If compromise is not too costly, i and j will minimally compromise for k, and further reductions in k's tolerance level *decreases* i and j's payoff. However, if we keep on making k less tolerant, then at some point k's entire tolerance window will lie within i and j. At that point, compromising for k allows i and j to be friends with each other (as shown on the right of Figure 5). By being more intolerant, k brings the Nash equilibrium closer to the Pareto optimum and *increases* their utility.

This is illustrated in Figure 6 for an example where $\iota_i = 0.1$, $\iota_k = 0.5$, $\iota_j = 0.9$ and preferences are given by (4) with $F = 0.5$ and $g = 1$. Assume that $b_i = b_j = 1$ which correspond to a tolerance of $\bar{t} = 0.7$ for i and j. Steadily decreasing b_k from high values to 1 corresponds to an increasing tolerance level for k. This increase in tolerance for k first hurts i and j, until the point where they are no longer able to be friends. Then further increases in the tolerance of k benefits them.

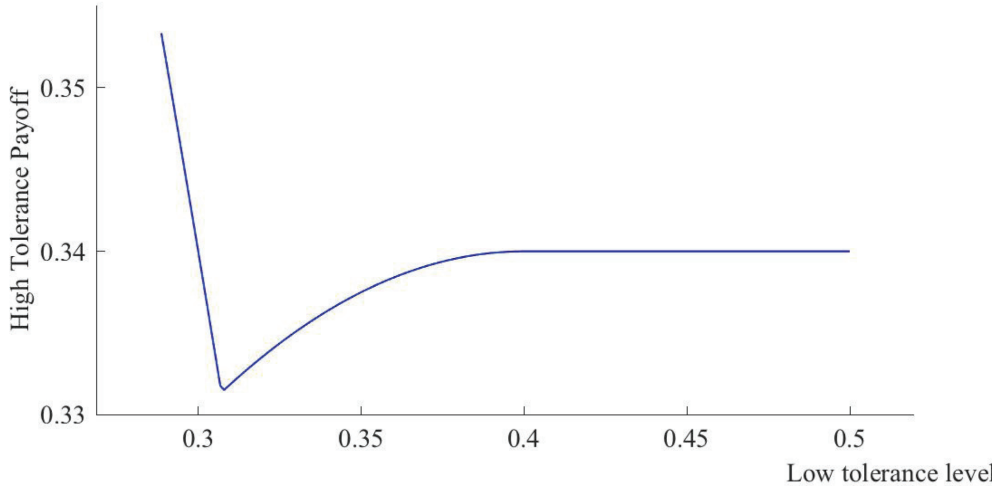

Figure 6. Non-monotonicity of payoff in $\underline{t}$

6.3. **Uncertainty.** One might think that some uncertainty about the other's tolerance level would help individuals compromise for each other. However, it is not clear how much it does help. At least, the same logic seen thus far applies.

Consider two individuals i and j with identities $\iota_i < \iota_j$ (such as the two individuals of Section 6.1). Suppose now that i and j's tolerance levels t_i and t_j are private information but known to be drawn from a distribution on $[\underline{t}, \bar{t}]$.

Observation **1.** *If i and j's tolerance levels $t_i, t_j \in [\underline{t}, \bar{t}]$ are private information, then no compromise arise if $\bar{t} < d(\iota_i, \iota_j)$ or $\underline{t} \geq d(\iota_i, \iota_j)$. Compromise can occur if $\underline{t} < \iota_j - \iota_i < \bar{t}$.*

Being uncertain about the tolerance of the other does not, by itself, allow individuals to compromise for each other. Observation 1 argues that, if $\bar{t} < d(\iota_i, \iota_j)$, compromise is impossible, even with uncertainty regarding the other person's tolerance level. The intuition is similar to the homogenous case (the proof is in Appendix). Because bilateral compromise is needed, i and j are unable to become friends.

If $\underline{t} \geq d(\iota_i, \iota_j)$ then i and j will be friends and do not need compromise to become friends. Only if $\bar{t} > d(\iota_i, \iota_j)$ and $\underline{t} < d(\iota_i, \iota_j)$, can we get i and j to compromise. The logic is similar to the role of heterogeneity in Section 5.2. Compromise is sparked by the willingness of a relatively high tolerance type to compromise for a relatively low tolerance type, even if the latter does not compromise.

6.4. **Compromise and population density.** Another interesting question is what happens to compromise when populations increase or new technology – such as social networks – allow us to connect with new individuals? This section shows that increasing the density of the population has a non-monotonic effect on compromise. To see this, consider the following example.

Take two individuals located at 0 and 1 with the following preference:

$$u_i(a_i, \mathbf{a}_{S(i)}) = \sum_{j \in S(i)} [F - bd(\iota_i, a_j)] - gd(\iota_i, a_i),$$

where $g > 0$ but is small. Their tolerance level is $\bar{t} = F/b < 1/2$.

Now, introduce on the interval some relatively more intolerant individuals with a level of tolerance $\underline{t} < \bar{t}$. I assume that these intolerant individuals are equally spaced in terms of identities: they are located at $\frac{1}{2^n}$, $\frac{2}{2^n}$,... $\frac{2^n-1}{2^n}$ for increasing values of $n \in \{1, 2, ..\}$ raising the density of the population.

We know from Corollary 3 that the relatively intolerant individuals will not compromise. Consider individual i with identity $\iota_i = 0$. Clearly, if she compromises at all, she must minimally compromise for a low-tolerance person: $a_i = 0$ or $a_i = \frac{k}{2^n} - \underline{t}$ for $k \in \{1, ...2^n - 1\}$.

If $k \geq 1$ (strictly so if $\frac{1}{2^n} < \underline{t}$), it must be that the additional benefit from compromising for k compared to $k-1$ dominates the cost, while it would not be the case at $k+1$.

$$\begin{aligned} \frac{g/b}{2^n} &\leq \bar{t} - \frac{k}{2^n} \\ \frac{g/b}{2^n} &> \bar{t} - \frac{k+1}{2^n}. \end{aligned}$$

Clearly $\bar{t} - \frac{g/b}{2^n}$ increasing with n. This force, for small g, promotes compromises as the density of the population n increases.

On the other hand, it is easy to see that $a_i \leq \frac{1}{2^n} + \underline{t}$ must be true. Otherwise, it means that, by compromising minimally for $\frac{k}{2^n}$, i compromises so much that she misses out on a friendship with $\frac{1}{2^n}$. By instead compromising minimally for $\frac{k-1}{2^n}$, she would have to compromise less, have as many friends and these would be more valuable to her. Hence, if i compromises, she compromises minimally for $\frac{k^*}{2^n} \leq \frac{1}{2^n} + 2\underline{t}$. As n increases, the right-hand side of this inequality decreases, making this constraint more likely to bind. This tends to reduce compromise.

As n increases, these two effects play a role. Figure 7 illustrates an example where going from $n = 1$ to $n = 2$ encourages i (and j) to compromise (g is assumed to be small). However at $n = 3$, i would have a friend without compromising and therefore return to her ideal behaviors. At a higher population density, $n = 4$, i now can compromise in order to make two friends. This example shows how non-monotonic compromise can be as the density of the population increases or as new communication technology emerges.

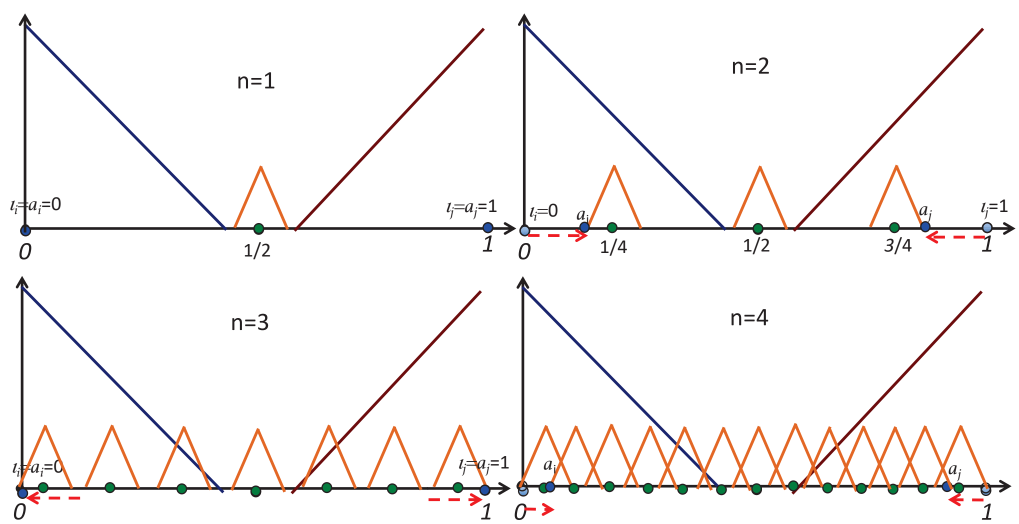

Figure 7. Compromise and density

This finding is related toRosenblat and Mobius (2004), who show that decreasing costs of communication allows heterogeneous agents to segregate along special interests rather than by geography.

6.5. **Externalities.** To be sure, the assumption that individuals only care about the behavior of individuals with whom they are linked is strong. We are affected by others' behavior even if we do not interact frequently with them. However, the no-externality assumption can easily be relaxed to allow individuals to care about everyone's behavior, but more strongly about people to whom they are linked. The results would not be affected.

7. Conclusion

This paper studies compromise in a model of social network formation. Individuals identities characterizes their preferred conduct for themselves and for others. People derive utility from links to others whose conduct is within their *tolerance windows*. Individuals first choose their conduct, and then form their social networks. They may choose to compromise in order to "fit in" and be acceptable to others.

I show that compromise is strictly limited and that the bounds to compromise decrease in the tolerance level of the most intolerant. When all individuals have the same tolerance level, there cannot be any compromise in equilibrium. In contrast, with heterogeneity, any equilibrium has to exhibit some compromise if tolerance and identity are independently distributed. I also show the key role that relatively intolerant *bridge* individuals play in compromise.

I further demonstrate that welfare and compromise are non-monotonic in the lowest level of tolerance in society, and in new opportunities for friendships due to population growth or social media.

The emphasis of my research on innate and adopted identity has implications for the measure of diversity and tolerance in a society. Looking at the identity of the members of a person's social network (for instance, counting the number of gay friends that one has) overestimates the actual tolerance exhibited by that person. The distance between a person's *identity* and her friends' *behavior* would be more informative of her level of tolerance.

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Appendix: Proofs

Lemma **1.** *If i compromises in equilibrium then the largest set of individuals who are valuable to i and for whom i minimally compromises, X_i, contains a positive mass of individuals.*

Proof If i compromises, $d(\iota_i, a_i) > 0$, then there must be a positive mass of individuals who are valuable to i (in the sense of Definition 1) for whom i minimally compromises (in the sense of Definition 3). To prove this claim, assume that there was no such set. Since compromise is costly, there then would exist a small $\varepsilon > 0$ so that i can improve her utility by bringing her behavior a_i closer to her ideal point by ε while keeping all links to any valuable individual j (for which $a_j \in I_i$) and deleting all others. ■

Lemma **2.** *If i minimally compromises for j, then $d(\iota_j, a_i) = t_j$ and $d(\iota_i, \iota_j) = d(\iota_i, a_i) + t_j$.*

Proof By definition, if i compromises minimally for j, then $d(\iota_i, \iota_j) > t_j$ and $a_i = argmin_{a_i \in \omega_j} d(\iota_i, a_i)$. The claim follows directly from these two facts. ■

Lemma **3.** *If i minimally compromises for j and j is valuable to i, $d(\iota_j, a_j) - t_j > d(\iota_i, a_i) - t_i$.*

Proof Indeed, if i minimally compromises for j, Lemma 2 tells us that

$$d(\iota_i, \iota_j) = d(\iota_i, a_i) + t_j,$$

while $a_j \in I_i$ means that

$$d(\iota_i, \iota_j) < d(\iota_j, a_j) + t_i.$$

These two inequalities imply that

$$d(\iota_i, a_i) - t_i < d(\iota_j, a_j) - t_j. \tag{5}$$

■

Proof of Proposition 1. Assume that the proposition does not hold, so that there is an equilibrium vector of behaviors **a** and an individual i whose behavior differs from her ideal by more than $t_i - \underline{t}$: $d(\iota_i, a_i) > t_i - \underline{t}$.

Step 1. We first prove that this implies that there are sequences of compromising individuals $\{x_m\}$ for $m \in \{1, 2, ...\}$ originating in i ($x_1 = i$) such that x_m minimally compromises for x_{m+1} and x_{m+1} is valuable to x_m. Along this sequence, let's denote x_m's ideal point ι_m, her tolerance level t_m, and her choice of action a_m.

The first thing to notice is that $d(\iota_m, a_m) > t_m - \underline{t}$ (something that we assumed for $x_1 = i$) means that x_m compromises (since $t_m - \underline{t} \geq 0$). Lemma 1 tells us this implies the existence of a non-empty associated set X_m that is the largest set of individuals who are valuable to m and for whom m minimally compromises. It follows that we can select any element of X_m as the next individual, x_{m+1}, in the sequence. It remains to show that x_{m+1} too compromises.

Lemma 3 tells us that $d(\iota_m, a_m) > t_m - \underline{t}$ implies $d(\iota_{m+1}, a_{m+1}) > t_{m+1} - \underline{t}$ for $x_{m+1} \in X_m$. Since $t_{m+1} - \underline{t} \geq 0$, x_{m+1} compromises.

Step 2. Denote as $\mathcal{S}_i$ the set of all the sequences identified in Step 1 that originate in i. Lemma 3 tells us that $d(\iota_m, a_m) - t_m$ strictly increases along any sequence in $\mathcal{S}_i$.

Now assume that, along one of the sequences in S_i, $\{x_m\}_{m=1,2...}$, the distance $d(\iota_m, a_m) - t_m$ does not converge. Then there would exist n so that $g(d(\iota_n, a_n)) > v_n(0)$. Even if compromising to a_n allowed x_n to become friend with everyone and if everyone chose her favorite behavior, it would not be worth such compromise.

This means that $d(\iota_m, a_m) - t_m$ needs to converge along every sequence in $\mathcal{S}_i$. Pick one of these sequence originating in i: $\{x_1, x_2, ...\} \in \mathcal{S}_i$. For each individual x_m, let X_m denote the largest associated set of individuals who are valuable to x_m and for whom x_m minimally compromises and let μ_m denote the density of this associated set. For any $\varepsilon > 0$ there exists n so that $(d(\iota_y, a_y) - t_y) - (d(x_n, a_n) - t_n) < \varepsilon$ or

$$t_n + d(\iota_y, a_y) - t_y - d(\iota_n, a_n) < \varepsilon \tag{6}$$

for all $y \in X_n$. Using the facts that $d(\iota_y, a_y) \geq d(\iota_n, \iota_y) - d(\iota_n, a_y)$ (since y compromises for x_n) and that $d(\iota_n, \iota_y) = d(\iota_n, a_n) + t_y$ (since x_n minimally compromises for y) in (6), we get that, for any $y \in X_n$,

$$t_n - d(\iota_n, a_y) < \varepsilon.$$

It follows that

$$v_n(d(\iota_n, a_y)) < v_n(t_n - \varepsilon) \;\; \forall y \in X_n, \tag{7}$$

where v_n represent the utility Let $\eta > 0$ be the smallest compromise along the sequence $\{x_m\}_{m=1,2...}$. We can pick ε to be such that $v_n(t_n - \varepsilon) < g'(\eta)$. In which case

$$g'(d(\iota_n, a_n)) > \mu_n v_n(t_n - \varepsilon),$$

and x_n would strictly increase her utility by choosing a behavior slightly closer to her ideal point. ■

Proof of Proposition 2. Assume not. This implies that $a_i = \iota_i$ for all i. However pick an individual i located at one extreme $\iota_i = 0$ with tolerance $t_0 > \underline{t}$. Let F denote the distribution of ideal positions and G the distribution of tolerance levels. If i does not compromise, he will be friends with all individuals j in his tolerance window $[0, t_0]$ who tolerate him $t_j \geq \iota_j$: a proportion $F(t_0)[1-G(\iota_j)]$. If he chooses a code of conduct $a_i = \varepsilon > 0$ instead, the individuals j in his tolerance window $[0, t_0]$ who tolerate him are now such that $t_j \geq \iota_j - \varepsilon$: a proportion $F(t_0)[1 - G(\iota_j - \varepsilon)]$. Hence, the gain from compromising is given by

$$\int_0^{t_0} v_i(\iota_j)[G(\iota_j) - G(\iota_j - \varepsilon)]F(j)$$

while the cost is $g(\varepsilon) - g(0) = g(\varepsilon)$. Since g is continuous and $g'(0) = 0$, it must be that for a sufficiently small $\varepsilon > 0$

$$\int_0^{t_0} v_i(\iota_j)[G(\iota_j) - G(\iota_j - \varepsilon)]F(j) > g(\varepsilon).$$

Hence, compromise must arise in equilibrium. ■

Lemma 4. *If $ij \in G$ then a_i (and a_j) $\in \Omega_i \cap \Omega_j$.*

Proof For j to accept i's friendship it must be that $a_i \in \Omega_j$. $a_i \in \Omega_i$ follows directly from Proposition 1. ■

Proof of Proposition 3. Assume that the proposition is not true. There must then exist individuals i and j (with $\iota_i < \iota_j$) who reciprocally compromise but no intermediary individual k with $\iota_i < \iota_k < \iota_j$ and an extremity, either $(\iota_k - t_k)$ or $(\iota_k + t_k)$, in $(\Omega_i \cap \Omega_j)$.

Since i compromises, there must be a non-empty set X_i of valuable individuals for whom i minimally compromises. For any $k \in X_i$, $a_i = \iota_k - t_k$ so that Lemma 4 implies $(\iota_k - t_k) \in (\Omega_i \cap \Omega_j)$. If $\iota_k \in (\iota_i, \iota_j)$, we have a contradiction. Hence, it must be that $\iota_k \notin (\iota_i, \iota_j)$. And since Lemma 4 tells us that $a_k \in \Omega_i$, it means that k compromises for i. $\Omega_i \cap \Omega_k \subset \Omega_i \cap \Omega_j$.

Using this logic repeatedly, we can show that there are sequences of compromising individuals $\{x_m\}$ for $m \in \{1, 2, ...\}$ originating in i, $x_1 = i$ (and we can do the same for j), such that x_m minimally compromises for x_{m+1} and x_{m+1} is valuable to x_m. Along this sequence, we denote x_m's ideal point ι_m, her tolerance level t_m and her choice of action a_m. If x_m for $m \geq 2$ compromises, then X_m is non empty (Lemma 1). Since $a_m \in \Omega_i \cap \Omega_j$, any $\ell \in X_m$ has an extremity in $\Omega_i \cap \Omega_j$ and therefore $\iota_\ell \notin (\iota_i, \iota_j)$. From Lemma 4 $a_\ell \in \Omega_m \cap \Omega_{m+1} \subset \Omega_i \cap \Omega_j$. Hence, $x_{m+1} \in X_m$ compromises for m.

We can then now apply the second Step of Proposition 1 to reach a contradiction.
■

Proof of Proposition 4. Assume that the claim does not hold so that there is a non empty set P of pairs of individuals (x, y), $\iota_x < \iota_y$, who engage in reciprocal compromise. Next, select a pair (i, j) in P according to the following criteria :

(a) either i or j has the lowest level of tolerance among all members of P;
(b) if multiple pairs satisfy (a), select among these pairs one where the most intolerant individual is closest to the extreme in the following sense: (i, j) minimizes $\delta(x, y)$ defined as follows

$$\delta(x,y) = \begin{cases} d(\iota_y, 1) & if\ t_y < t_x \\ d(\iota_y, 1) & if\ t_y = t_x\ and\ t_z \leq t_y \forall z\ s.t.\ \iota_z \geq \iota_y \\ d(0, \iota_x) & otherwise; \end{cases}$$

(c) if multiple pairs satisfy (a) and (b), select one of these pairs with the largest distance between the two individuals.

In what follows, assume that j is the least tolerant of the two. That is assume [J]: either $t_j < t_i$ or if $t_j = t_i$ then $t_k \leq t_j$ for all k with $\iota_k \geq \iota_y$. A symmetric argument applies to the case where i is the least tolerant.

Step 1. Since i compromises, there must be a non-empty set X_i of valuable individuals for whom i compromises. Take any $k \in X_i$. If $t_k > t_j$, then

$$d(\iota_i, \iota_k) = t_k + d(\iota_i, a_i) > t_j + d(\iota_i, a_i) \geq d(\iota_i, \iota_j)$$

where the first inequality follows from Definition 2 and the last inequality follows from the fact that i and j become friends and therefore a_i must be in j's tolerance window. But this means that $\iota_k > \iota_j$ while $t_k > t_j$: a contradiction to [T] if $t_j < t_i$ or to [J] if $t_j = t_i$. Now, if $t_k < t_j$ then [T] implies that $\iota_k > \iota_j$ and $d(\iota_i, \iota_k) > d(\iota_i, \iota_j)$. Since k is valuable to i, it must be compromising. But then $t_k < t_j$ contradicts the selection criterion (a). Hence, $\iota_k = \iota_j$ and $t_k = t_j$ for any $k \in X_i$ and any $k \in X_i$ compromises. Either $j \in X_i$ or j minimally compromises for i.

Step 2. Take any $k \in X_i \cup j$. Since k compromises, there is a non-empty set X_k of valuable individuals for whom k minimally compromises. Take any $l \in X_k$. First, we show that $\iota_l \in [\iota_i, \iota_k]$. Since j compromises toward i, $\iota_l < \iota_k$, and if $\iota_l < \iota_i$ then l and k would be engaged in reciprocal compromise with $d(\iota_l, \iota_k) > d(\iota_i, \iota_k)$, in contradiction with part (c) of the selection. Next, following the same logic as before, it must also be the case that $t_l \leq t_i$. Otherwise it would imply that l and k would be engaged in reciprocal compromise while $d(\iota_l, \iota_k) > d(\iota_i, \iota_k)$, in contradiction with part (c) of the selection. Hence, $\iota_l \in [\iota_i, \iota_k]$ and $t_i \geq t_l \geq t_k$ for all $l \in X_k$ (where the last inequality follows from [T]).

Step 3. Assume that $t_l < t_i$ for some $l \in X_k$. To be valuable to k, l must compromise (as $t_k \leq t_l$). Let X_l be the set of valuable individuals for whom l minimally compromises and $m \in X_l$. If $t_m < t_j$ then [T] implies that $d(\iota_l, \iota_m) > d(\iota_l, \iota_j)$ so that l and m would be engaged in reciprocal compromise while $t_m < t_j$: a contradiction of selection criterion (a). Hence, $t_m \geq t_j$. Since l is valuable to k, it implies that

$$d(\iota_l, \iota_m) = d(\iota_l, a_m) + t_m > d(\iota_l, \iota_j)$$

which implies that $\iota_m > \iota_j$. Since [T] means $t_m \leq t_j$, it must be that $t_m = t_j$. But then there is a pair of individuals (l, m), with the lowest-tolerance individual m located more at the extreme than the pair (i, j). This directly contradicts criterion (b) of the selection. It follows that $t_l = t_i$ for all $l \in X_k$.

Step 4. Since $t_l = t_i$ for all $l \in X_k$, $\iota_k = \iota_i$ and $d(\iota_i, a_k) = t_i$ and this for all $k \in X_i \cup j$. None of the individuals for whom i compromises has an action in the interior of i's tolerance window: a contradiction. ■

Proof of Observation 1. Assume not. Then there must be some type of individual that compromises. Among the types who compromise, select i with type t_i be the agent (or one of the agents) with the largest compromise minus her tolerance $d(a_i, \iota_i) - t_i$. The same logic as before applies. Since i compromises, she must be minimally compromising for some type of agent j, $d(a_i, \iota_j) = t_j$ and that type of agent must be valuable to i, $d(a_j, \iota_i) < t_i$. This implies that $t_j + d(a_j, \iota_i) < t_i + d(a_i, \iota_j)$. Since $\iota_i \leq a_i, a_j \leq \iota_j$, $d(a_j, \iota_i) = d(\iota_j, \iota_i) - d(a_j, \iota_j)$ and $d(a_i, \iota_j) = d(\iota_j, \iota_i) - d(a_i, \iota_i)$. Using these equalities in the previous inequality yields $t_j + d(\iota_j, \iota_i) - d(a_j, \iota_j) < t_i + d(\iota_j, \iota_i) - d(a_i, \iota_i)$. Rewriting the latter gives $d(a_i, \iota_i) - t_i < d(a_j, \iota_j) - t_j$. Since j must compromise as well to be valuable, this contradicts the selection of i. ■