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Product Development and International Trade

Gene M. Grossman
Elhanan Helpman

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ABSTRACT

We develop a multi-country, dynamic general equilibrium model of product innovation and international trade to study the creation of comparative advantage through research and development and the evolution of world trade over time. In our model, firms must incur resource costs to introduce new products and forward-looking potential producers conduct R&D and enter the product market whenever profit opportunities exist. Trade has both intra-industry and inter-industry components, and the different incentives that face agents in different countries for investment and savings decisions give rise to intertemporal trade. We derive results on the dynamics of trade patterns and trade volume, and on the temporal emergence of multinational corporations.

Gene M. Grossman
Woodrow Wilson School
Princeton University
Princeton, NJ 08544
609-452-4823

Elhanan Helpman
Department of Economics
Tel Aviv University
Ramat Aviv 69978
Tel Aviv, Israel
972-03-420-712

1. Introduction

International economists have long used static models of comparative advantage and (more recently) scale economies to great advantage in studying the pattern of international trade and the normative properties of trading equilibria. But increasingly, many issues of concern to theorists and casual observers alike are inherently dynamic in nature. Attention has focused on such topics as the creation of comparative advantage by technological innovation, the relationship between trade policy and economic growth, and the dynamic evolution of the volume and pattern of world trade. The static models of international trade must be extended if we are to deal with these new concerns.

In this paper, we develop a multi-country, dynamic, general equilibrium model of product innovation and international trade to study the creation of comparative advantage through research and development, and the evolution of world trade over time. Our model builds upon the static analyses of trade in differentiated products by Krugman (1979a, 1981), Dixit and Norman (1980), and Feenstra and Judd (1982), as well as the closed-economy dynamic model of product development studied by Judd (1985).

In our model, firms incur resource costs to introduce new products. Forward-looking potential producers conduct R&D and enter the product market whenever profit opportunities exist. New products substitute imperfectly for old, and prices, interest rates, and the pattern of trade evolve over time as more commodities become available for purchase. Trade has both intra-industry and inter-industry components, with the former governed by R&D expenditures and the latter by resource endowments. The incentives that face agents in different countries for investment and savings decisions give rise to intertemporal trade.

The approach adopted here differs in important respects from several recent studies of the dynamics of trade with product innovation, such as Krugman (1979b), Dollar (1986), Jensen and Thursby (1987) and Segerstrom et.al. (1987). These papers have provided useful insights into the evolution of the trade pattern. But all have been incomplete in important ways, be it due to the lack of explicit treatment of all general equilibrium interactions, the lack of explicit modelling of the economic factors that drive the rate of product innovation, or other features. We believe that in order to study the evolution of trade that is based on technological innovation it is necessary to develop models in which the process of innovation, the incentive to invest in R&D, and the interaction of these two with resource allocation, both temporal and intertemporal, are all treated explicitly.

The organization of the paper and some of the major results are as follows. In Section 2, we develop the model and derive the integrated equilibrium that would result in the absence of any international borders. Section 3 presents our investigation of the pattern of trade in a two-country, two-factor world, with the factors interpreted to be unskilled labor and human capital. If both product development and the production of differentiated products are more human-capital intensive activities than is the production of the outside good, then the human-capital rich country will be a net exporter of differentiated products and an importer of the outside good _at every moment in time_. This is true despite the fact that the human-capital rich country initially devotes more of its resources to R&D (as opposed to production), and despite the fact that trade is not balanced along the equilibrium path. The model predicts a rising share of trade in world GNP, at least when R&D is the most human-capital intensive of the three activities. Concerning intertemporal trade, we show that the human-capital rich country has both a greater incentive to invest and a greater incentive to save (per capita), the latter due to its declining relative factor income. Consequently, it appears that this country may run either a surplus or a deficit on current account in the dynamic equilibrium.

In Section 4, we introduce the possibility of multinational corporations in the manner of Helpman (1984) and Helpman and Krugman (1985, Chp. 12). We assume that headquarter services can be separated geographically from production activities, and that only the former must take place in the country in which a differentiated product has been developed. If headquarter services are more human-capital intensive than production, then the possibility of multinational activity expands the set of distributions of the world's factor endowment for which international exchange can reproduce the integrated world equilibrium. For certain compositions of factor endowments the model predicts the emergence of multinational enterprises at a particular point in time, which remain active thereafter. The extent of multinationality, as measured by output, employment of subsidiaries, or the number of multinational firms increases initially and as the world economy approaches the steady state.

2. A Dynamic Model of R&D

We consider an environment in which there are three activities: the production of a traditional commodity, the production of a continuum of varieties of a "modern" industrial product, and research and development (R&D) that leads to the acquisition of the know-how needed to produce new brands of the industrial good.

At every point in time there exists a given (measure of the) number of varieties that were developed in the past. Producers of these varieties engage in oligopolistic competition by setting prices. Given demands and costs, this process determines prices, outputs, and current operating profits. An entrepreneur who contemplates developing a new brand can calculate the future stream of operating profits that will be captured. He chooses to develop the brand only if the present value of this stream is at least as large as the cost of R&D. The competitive entry process leads to aggregate investment in R&D such that a brand's development cost is just equal to the present value of its future profits (unless no further products are developed).

As in Helpman and Krugman (1985), it proves convenient to solve for the "integrated world equilibrium", i.e., the equilibrium that would obtain in the absence of any international borders. Under conditions that give rise to factor price equalization, a world trading equilibrium reproduces the integrated equilibrium in its essential details. So properties of the latter equilibrium can be applied to the analysis of the former. For this reason the following discussion deals first with the integrated economy.

A. Consumers

Infinitely lived consumers maximize total lifetime utility. The representative consumer has a time-separable intertemporal utility function

$$U = \int_0^\infty e^{-\rho t} \log u(\cdot)dt, \tag{1}$$

where ρ is the constant subjective discount rate and $u(\cdot)$ is an instantaneous sub-utility function. We adopt a particular form of $u(\cdot)$,

$$u = \left[\int_0^n c_x(i)^\alpha di\right]^{s_x/\alpha} c_y^{1-s_x} \quad ; \quad \alpha,\ s_x \in (0,1), \tag{2}$$

where $c_x(i)$ is consumption of differentiated product i, c_y is consumption of the competitive good, and n is the (measure of the) number of available varieties. We note that this form implies constant expenditure shares s_x and $1-s_x$ on commodity classes x and y, and a constant elasticity of substitution between any two differentiated products of $\sigma = 1/(1-\alpha) > 1$.

The consumer's maximization problem can be solved in two stages. First we find $\{c_x(i)\}$ and c_y to maximize $u(\cdot)$ given total expenditure at time t, E(t), prices, and the available brands. Then we solve for the time pattern of expenditures that maximizes U. The solution to the first stage gives instantaneous demand functions[1]

$$c_x(i) = s_x E \frac{p(i)^{-\sigma}}{\int_0^{n(t)} p(j)^{1-\sigma} dj} \tag{3}$$

and $c_y = (1-s_x)E/p_y$, where p(i) is the price of the differentiated product i and p_y is the price of the traditional good y.

In maximizing U, the consumer must satisfy an intertemporal budget constraint. We assume that the consumer can borrow or lend freely on a capital market with instantaneous rate of interest $\dot{R}(\tau)$. Then the budget constraint is

$$\int_t^{\infty} e^{-[R(\tau)-R(t)]} E(\tau) d\tau = \int_t^{\infty} e^{-[R(\tau)-R(t)]} I(\tau) d\tau + A(t), \tag{4}$$

where $I(\tau)$ is the consumer's factor income in period τ, A(t) is the value of his accumulated assets at t, with A(0)=0, and $R(\tau)$ is the cumulative interest

1. See Helpman and Krugman (1985, Chap. 6) for more details.

factor through time τ. Then, substituting (3) into (2) and the result into (1), the first-order condition for maximizing U subject to (4) at t=0 implies[2]

$$\frac{\dot{E}}{E} = \dot{R} - \rho. \tag{5}$$

B. Producers

Costs of manufacturing industrial products comprise two parts, fixed development costs and variable production costs. It is assumed that production takes place under constant returns to scale and that the input requirements for R&D do not vary with the number of innovating firms. Let $\phi_x(w_f)$ be the unit cost in production and $\phi_n(w_f)$ the cost of developing a brand, where w_f is a vector of input prices.[3] These costs are the same for all brands, regardless of whether the variety has previously been introduced by another entrepreneur or not. Then $\phi_n(\cdot)$ is the fixed cost and $\phi_x(\cdot)$ is the average and marginal variable cost for all firms in this sector.

The number of potential products is infinite. Therefore, it will never be rational for an entrepreneur to develop an already existing brand, and each innovator enjoys monopoly power in the production of his particular variety for the indefinite future.

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2. This can be seen as follows. The indirect utility function derived from (2) has the form $v[p(t), E(t)] = E(t)f(p(t))$. Then $\log u(\cdot) = \log E(t) + \log f(p(t))$, and the first-order condition for maximization of (1) implies $e^{-\rho t}/E(t) = \zeta e^{-R(t)}$, where ζ is the time-independent Lagrange multiplier associated with the budget constraint in (4).

3. An implicit assumption here is that product development does not require finite time. We could relax this assumption without substantially affecting the structure of the model.

A producer of an existing brand faces at time t a measure $n(t)$ of competitors who have developed products in the past. He also faces a given aggregate expenditure level $E(t)$ and the pricing policy of the competitors. He chooses the price of his brand so as to maximize operating profits; namely, revenue minus production costs, using the demand function given in (3). As is well known, this results in fixed markup pricing over unit production costs. Since all producers are symmetrical, we consider the symmetric equilibrium. In this equilibrium output per variety $x(i)=x$ and prices $p(i)=p$ for all $i \in [0,n(t)]$ satisfy

$$x = s_x E/pn, \tag{6}$$

$$\alpha p = \phi_x(w_f). \tag{7}$$

The resulting operating profits per variety are

$$\pi = (1 - \alpha)s_x E/n. \tag{8}$$

An entrepreneur has perfect foresight regarding the evolution of spending E and the number of firms n. Therefore, using (8) he has perfect foresight regarding profits per variety. In an equilibrium the present value of these profits cannot exceed current R&D costs. Hence, if at time t there is positive but finite investment in product development, each new variety breaks even; i.e.,

$$\int_t^\infty e^{-[R(\tau)-R(t)]}\pi(\tau)d\tau = \phi_n[w_f(t)]. \tag{9}$$

We normalize nominal prices so that

$$1 = \phi_n[w_f(t)] \text{ for all } t. \tag{10}$$

With this choice of numeraire, (9) implies that the instantaneous interest rate is equal to operating profits; i.e.,

$$\pi(t) = \dot{R}(t). \tag{11}$$

The traditional good also is produced subject to constant returns to scale. Its unit cost function is $\phi_y(w_f)$. Therefore its price, which equals marginal cost, satisfies

$$p_y = \phi_y(w_f). \tag{12}$$

Equations (7), (10) and (12) describe the equilibrium relationships between product and factor prices.

C. Integrated Equilibrium

First, substitute (8) and (11) into (5) to obtain

$$\frac{\dot{E}}{E} = (1 - \alpha)s_x \frac{E}{n} - \rho. \tag{13}$$

This is the first differential equation that will be used to describe the evolution of the integrated economy over time. It shows the rate of change of spending as a function of spending and the number of available varieties. The next step is to derive a differential equation for changes in the number of available brands; i.e., an investment equation.

Let $a_z(w_f)$ be the (column vector) gradient of the unit cost function $\phi_z(w_f)$, $z = n,x,y$. Then $a_z(w_f)$ represents the employment vector per unit output at factor prices w_f, and the factor market clearing condition is

$$a_n(w_f)\dot{n} + a_x(w_f)X + a_y(w_f)Y = V, \tag{14}$$

where

$$X = nx \tag{15}$$

is the aggregate output of industrial products, Y is the output of traditional products and V is the vector of available inputs. The pricing equations (7), (10) and (12) together with (14) imply that the total reward to factors of production can be written as $\Pi(1, \alpha p, p_y; V)$, where $\Pi(\cdot)$ has the usual properties of a GNP function. In particular, if it is differentiable in the first three arguments, the first partial derivative is equal to $\dot{n}$, the second to X, and the third to Y (see Helpman (1984) and Flam and Helpman (1987)). If it is not, the vectors of supplied outputs consist of the set of gradients with respect to the first three arguments. Thus, with differentiability of $\Pi(\cdot)$ the commodity market clearing conditions can be written as

$$s_x E/p = \Pi_2(1, \alpha p, p_y; V), \tag{16'}$$

$$(1 - s_x)E/p_y = \Pi_3(1, \alpha p, p_y; V), \tag{17'}$$

$$\dot{n} = \Pi_1(1,\ \alpha p,\ p_y;\ V).$$

These conditions provide a solution for the equilibrium prices (p, p_y) and the development of new products $\dot{n}$ as functions of the expenditure level, E. The equilibrium factor rewards as functions of E can then be derived by observing that factor rewards are equal to the gradient of $\Pi(\cdot)$ with respect to V.

If the differentiability condition is not satisfied, one can use (7), (10), (12) and (14) directly, together with

$$s_x E/p = X, \tag{16}$$

$$(1 - s_x)E/p_y = Y, \tag{17}$$

in order to derive these equilibrium functional relationships, including the output levels X and Y as functions of E. We denote these functional relationships by $w_f(E)$, $p(E)$, $p_y(E)$, $X(E)$, $Y(E)$, and

$$\dot{n} = \nu(E). \tag{18}$$

Equations (13) and (18) constitute an autonomous system of differential equations. They apply whenever the implied rate of product development is non-negative. Global stability requires the function $\nu(\cdot)$ to be declining in E whenever $\nu(E)>0$. For now, we simply assume that this condition is satisfied.[4]

The phase diagram for the system is depicted in Figure 1. From (13), we see that the $\dot{E}=0$ schedule is an upward-sloping line in (E,n) space with slope

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4. For the special case that we consider in the next subsection, we establish that the stability condition is always satisfied.

given by $\rho/(1-\sigma)s_x$. Equation (18) implies that $\dot{n}=0$ for some particular value of E, which we denote by $\bar{E}$. The horizontal line in the figure depicts points at which there is no product development. Note that there are can be no equilibrium of the economy above this line, because this would require negative product development, which of course is not feasible.[5] The relevant regions in the figure are those on or below the horizontal line.

Point S in the figure represents the steady state. For $n(0)< \bar{n}$, there is a single trajectory that leads to point S, represented by the dashed path. This is, in fact, the unique equilibrium trajectory for $n(0)< \bar{n}$. For initial points below this trajectory, expenditure approaches zero as time progresses, which violates the conditions for consumer optimization. For initial points above the path, the conditions for profit maximization ultimately are violated. To see this, note that any such trajectory hits the horizontal line along which $\dot{n}=0$ at a point such as S' to the left of S. With $E=\bar{E}$, it remains there ever after. But the constancy of expenditure implies, from (5), that $\dot{R}=\rho$. Since S' is above the upward sloping ray, operating profits $\pi = (1-\alpha)s_x\bar{E}/n$ are larger than ρ, and hence the interest rate. This means that the present value of operating profits exceeds the cost of R&D, making it profitable to develop new

5. Suppose that $\dot{n}=0$, and consider the system of equilibrium conditions comprising (7), (10), (12), (14), (16) and (17). These are 5+k equations in 5+k unknowns, where k is the number of factors of production. The unknowns in this system are the k factor rewards, two prices for final consumer goods, two aggregate output levels for the final-goods sectors, and the expenditure level. Naturally, the solution for the expenditure level in this system is $\bar{E}$. Thus, $\dot{n}=0$ in equilibrium implies $E=\bar{E}$, and the system can only be on or below the horizontal line in the figure.

products at S'. Therefore, trajectories that hit the horizontal line to the left of S are not consistent with long run equilibrium.[6]

D. Special Case

We now consider a special case that will be used to discuss trade issues. There are only two factors of production -- unskilled and skilled labor -- and there are fixed input-output coefficients. Hence,

$$V = \begin{bmatrix} L \\ H \end{bmatrix}, \quad w_f = \begin{bmatrix} w \\ r \end{bmatrix} \quad \text{and} \quad a_z(w_f) = \begin{bmatrix} a_{Lz} \\ a_{Hz} \end{bmatrix} \quad \text{for} \quad z = n,x,y,$$

where L stands for unskilled labor and H stands for human capital, which is our measure of skilled labor. We assume that the traditional sector is the least human-capital intensive and that the overall human capital-to-labor ratio satisfies $a_{Hy}/a_{Ly} < H/L < a_{Hj}/a_{Lj}$ for j=x,n. The latter assumption is required to ensure full employment.

The comparative statics analysis of (7), (10), (12), (14), (16) and (17) that is executed in the Appendix shows that the function $\upsilon(E)$ is declining for this case. Therefore the dynamic path is as described in Figure 1; the number of available brands and expenditure are increasing over time. This in turn,

6. If the initial number of products exceeds $\bar{n}$, the economy settles immediately at a stationary state, with the number of varieties and all real magnitudes forever constant. In what follows, attention is focused on the case in which the initial number of products is smaller than $\bar{n}$, and in particular on the case in which it is zero.

implies that the level of R&D activity is declining through time, as well as the following price and quantity dynamics (see Appendix for a proof):[7]

Proposition 1:

(a) $\dot{w}/w > \dot{p}_y/p_y > \dot{p}/p > \dot{r}/r$;

(b) $\dot{p}_y > 0$;

(c) $\dot{r} < 0$;

(d) $\dot{p} > 0$ if and only if $a_{Hn}/a_{Ln} > a_{Hx}/a_{Lx}$;

(e) $\partial\dot{n}/\partial t < 0$;

(f) $\dot{X} > 0$;

(g) $\dot{Y} < 0$ if and only if $a_{Hn}/a_{Ln} > a_{Hx}/a_{Lx}$;

(h) $\dot{E} > 0$.

Hence, the real wage (of unskilled labor) is rising and the real reward to human capital is falling through time. This statement refers, however, only to the standard method of measuring real incomes. Since in this type of an

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7. In deriving our comparative statics results, we have assumed that full employment of both factors obtains all along the equilibrium trajectory. As is well known, even in static models, full employment is not guaranteed for fixed-coefficient production functions. In the steady state of our dynamic system we have an essentially-static two-sector model (because R&D is zero) with a piecewise-linear, kinked transformation curve. Full employment then requires restrictions on the parameters of the utility function so that the slope of the indifference curve at the kink, adjusted for the degree of monopoly power $1/\alpha$, falls between the slopes of the flat portions of the transformation curve. Put differently, full employment obtains in the steady state if for $E=\bar{E}$ the solution to the system of equations (7), (10), (12), (14), (16), (17) and (18) yields non-negative factor rewards. We also require non-negative values of the factor rewards when the system is solved with $E=E(0)$. These two conditions at the endpoints ensure full employment and non-negative factor rewards along the entire equilibrium trajectory, since the wage rate is rising and the reward to human capital is falling whenever there is full employment (see (a), (b) and (c) of Proposition 1). If these conditions are not met, however, there may be unemployment of unskilled labor during an initial phase of the dynamic equilibrium, or unemployment of human capital during an ultimate stage, or both.

environment variety is valued per-se (see Helpman and Krugman (1985, Chp. 6)) and the available variety increases over time, real incomes of unskilled workers necessarily increase but real incomes of skilled workers need not decline.[8] The product-development sector contracts while the production of industrial goods expands. The traditional sector contracts if and only if R&D is more human-capital intensive than production of industrial goods. This completes the description of the integrated world equilibrium.

3. The Pattern of Trade in a Two-Country World

We suppose now that the world consists of two countries, labelled "A" and "B". The two countries share common tastes and technologies identical to those specified for the integrated economy. We allow for the existence of integrated world commodity and financial markets, but assume that factor services and "blueprints" are not tradable. In this section we assume as well that an entrepreneur cannot establish production facilities offshore; we relax this assumption to allow for the emergence of multinational corporations in the next section. We ask first whether the trade equilibrium can reproduce the integrated equilibrium described in subsection D of Section 2. Then, for those cross-country divisions of H and L that are consistent with factor price

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8. The temporal indirect utility function of a representative agent is calculated to be

$$\text{con.} + s_x(\alpha^{-1} - 1)\log n + \log I - [s_x \log p + (1 - s_x)\log p_y],$$

where I is his income. The last two terms represent the usual real income component, where the last term represents the deflator. It is clear from part (a) of Proposition 1 that this real income component is rising for unskilled workers and falling for skilled workers. However, apart from this component, there exists the term with n, which represents the love-of-variety effect. This real income component is rising as a result of expanding variety.

equalization everywhere along the equilibrium path, we derive the properties of the trade equilibrium.

Consider Figure 2. The dimensions of the rectangle in the figure represent the worldwide factor endowments, with the division of these endowments between countries represented by a point such as E in the interior of the rectangle. For concreteness, we suppose that country A is the relatively human capital rich country; i.e., $H_a/L_a > H_b/L_b$.

At time t=0, the resource allocation of the integrated equilibrium is found by substituting E(0) and n(0)=0 from the equilibrium trajectory into (7), (10), (12), (14), (16), (17) and (18) and solving for $\dot{n}(0)$, X(0), and Y(0).[9] Let points Q^o and C^o in the figure represent this allocation, where vector AQ^o is employed in R&D, Q^oC^o is employed in the X-sector, and C^oB is employed in the Y-sector (and the slopes of these vectors correspond to the factor proportions required in each of these activities). The allocation of the integrated equilibrium can be achieved in a two-country world so long as it is possible to decompose the industry employment vectors into non-negative components for each country that exhaust their separate endowments. In the figure, this is accomplished with employment vectors AM^o ($=AP^o + P^oM^o$) and M^oE is country A, and vectors EZ^o ($=M^oN^o + N^oC^o$) and Z^oB in country B. Evidently, the feasibility of such a decomposition requires that point E be in the interior of the triangle AC^oB. A sufficient condition for this is $a_{Hn}/a_{Ln} > H_a/L_a > H_b/L_b > a_{Hy}/a_{Ly}$.

9. The system of equations that determines resource allocations, commodity prices and factor rewards yields a solution for X(0) that is strictly positive when E=E(0), despite the fact that n(0)=0. Strictly positive consumption of both classes of goods is dictated by the Cobb-Douglas form of the sub-utility function. It requires, of course, that $x(t) \to +\infty$ as $t \to 0$ from above. Although our model breaks down at t=0, it is perfectly well behaved in the limit as t approaches zero from above. Therefore, we feel justified in ignoring the technical problems that arise at time zero.

In the steady state, R&D ceases, and all resources are devoted to production. Let point C^{∞} represent the allocation of resources to the two productive sectors in the steady-state equilibrium of the integrated economy. In the diagram, we depict the case where R&D is more human-capital intensive than production of the differentiated products. In any event, the allocation at point C^{∞} can be decomposed into feasible allocations for the two countries provided that $a_{Hx}/a_{Lx} > H_a/L_a > H_b/L_b > a_{Hy}/a_{Ly}$. In the figure, this decomposition is achieved by allocating in country A the vector of factors AM^{∞} to the production of x-sector goods, and the vector $M^{\infty}E$ to the production of y.[10] Finally, consider allocations at times between t=0 and t=∞. Points C^1 and C^2 represent sectoral allocations for the equilibrium of the integrated economy. Each such point can be viewed as an allocation of some factors to industry y and some factors to the combined activity of development and production of x-sector goods. The latter composite activity requires factors in proportions intermediate to the requirements for the two component activities. It follows that, if it is feasible to decompose the employment vectors of the integrated economy corresponding to the initial allocation and the steady-state allocation, then it will also be possible to do so for all times between these extremes. A sufficient condition for factor price equalization to obtain all along the path of the trade equilibrium is that the human capital-to-labor ratios of the two countries be bounded by the factor intensities of (i) the less human-capital intensive activity among R&D and

10. We must show further that this proposed allocation of resources to the production of industrial goods in each country is consistent with the number of products previously developed there, since outputs of all varieties are equal in the integrated equilibrium. We establish below that this condition is indeed satisfied for the proposed decomposition.

production of sector-x products and (ii) production of good y. For the remainder of this section, we shall assume that this condition is satisfied.

At an arbitrary point in time, the full employment conditions for a single country can be represented with the help of (14)-(17) as:

$$L_i = a_{Ly}Y_i + a_{Lx}\frac{s_x E}{np} n_i + a_{Ln}\dot{n}_i, \qquad (19)$$

$$H_i = a_{Hy}Y_i + a_{Hx}\frac{s_x E}{np} n_i + a_{Hn}\dot{n}_i, \qquad (20)$$

for $i = a,b$. Combining these two equations and eliminating Y_i, we have

$$\dot{n}_i + b(t)n_i = k_i,$$

where

$$k_i = \frac{L_i(H_i/L_i - a_{Hy}/a_{Ly})}{a_{Ln}(a_{Hn}/a_{Ln} - a_{Hy}/a_{Ly})}, \text{ for } i = a,b,$$

$$b(t) = \frac{a_{Lx}(a_{Hx}/a_{Lx} - a_{Hy}/a_{Ly})}{a_{Ln}(a_{Hn}/a_{Ln} - a_{Hy}/a_{Ly})}\ \frac{s_x E(t)}{n(t)p(t)}.$$

and the functions E(t), p(t) and n(t) are taken from the integrated world equilibrium. This differential equation can be solved explicitly, which gives:

$$n_i(t) = k_i \int_0^t e^{-\int_z^t b(\tau)d\tau} dz. \tag{21}$$

In writing (21), we have set $n_i(0) = 0$.

An important conclusion emerges from equation (21): the ratio of the numbers of differentiated goods produced in either country is constant for all t. We see that $n_a(t)/n_b(t) = k_a/k_b$. Then the ratio of R&D activity in the two countries, $\dot{n}_a/\dot{n}_b$, also is constant and equal to k_a/k_b, as is the ratio of the total outputs of x-sector goods, X_a/X_b.[11]

These features of the two-country equilibrium can also be seen from Figure 2. Recall that the points M^o, M^1, M^2, and M^∞ represent allocations of factors in Country A to the composite activity of R&D and the production of differentiated goods in the two-country equilibrium. These points all lie on a straight line through E with slope a_{Hy}/a_{Ly}. We can further decompose these allocations into vectors of factors employed in the component industries. For example, at time 1 (corresponding to global allocation C^1), country A employs the vector of factors AP^1 in R&D and the vector P^1M^1 in the x-sector, while country B employs M^1N^1 in R&D and N^1C^1 in the x-sector. The corresponding points for time 2 are shown in Figure 3, where we have enlarged the relevant portion of Figure 2. In Figure 3, the triangles AP^1M^1 and $M^1N^1C^1$ are similar triangles, as are the triangles AP^2M^2 and $M^2N^2C^2$. Thus, at each moment in time, the ratio of investments in R&D in the two countries equals the ratio of

11. We note that x(t) is common to goods produced in both countries, because factor price equalization implies equal prices of the different differentiated goods, and thus equal amounts of these goods are demanded by consumers. Since $X_i(t)=n_i(t)x(t)$, the last statement follows.

their total outputs of differentiated products. Finally, because M^1M^2 is parallel to C^1C^2, both of these ratios must remain constant through time.

We are now prepared to investigate the evolution of the pattern of trade in the two-country equilibrium. Consider first the direction of trade in good y at some arbitrary time t. From equations (19) and (20) we can solve for the outputs of good y in each country. The ratio of these outputs is given by

$$\frac{Y_a(t)}{Y_b(t)} = \frac{L_a[h_c(t) - H_a/L_a]}{L_b[h_c(t) - H_b/L_b]} .$$

where $h_c(t)$ is the human capital-to-labor ratio in the composite activity of R&D and production of good x. Since h_c is a weighted average of the human-capital intensities of the two component activities, it is bounded by $h_n \equiv a_{Hn}/a_{Ln}$ and $h_x \equiv a_{Hx}/a_{Lx}$. But each of these exceeds $h_i \equiv H_i/L_i$ under the conditions needed for factor price equalization, so $h_c(t)-H_b/L_b > h_c(t)-H_a/L_a > 0$. It follows that $Y_a(t)/Y_b(t) < L_a/L_b$.

Next we calculate the ratio of demands for good y. In each country, expenditure on good y is a constant fraction of total expenditure. Since consumers in both countries face the same price for the good, the ratio of aggregate demands is equal to the ratio of total expenditures. Now, from (5), $E_i(t)=E_i(0)e^{R(t)-\rho t}$, so $E_a(t)/E_b(t) = E_a(0)/E_b(0)$. This ratio is, in turn, equal to the ratio of initial wealth levels; i.e.,

$$\frac{E_a(0)}{E_b(0)} = \frac{L_a}{L_b} \frac{\int_0^\infty e^{-R(t)}[w(t) + r(t)h_a]dt}{\int_0^\infty e^{-R(t)}[w(t) + r(t)h_b]dt}. \tag{22}$$

Note that the ratio of initial wealth levels on the right-hand side of (22) includes only factor incomes, because initial asset holdings are zero and assets acquired along the path earn no excess returns.

Equation (22) and $h_a/h_b > 1$ together imply $E_a(0)/E_b(0) > L_a/L_b$. Thus, the ratio of demands for good y, $c_{ya}(t)/c_{yb}(t)$, also exceeds L_a/L_b. Since $Y_a(t)/Y_b(t) < L_a/L_b$, it follows immediately that $c_{ya}(t)/c_{yb}(t) > Y_a(t)/Y_b(t)$. But $c_{ya}(t)+c_{yb}(t) = Y_a(t)+Y_b(t)$ by market clearing, implying $c_{ya}(t) > Y_a(t)$; i.e.,

Proposition 2: The human-capital rich country imports the labor intensive traditional good y at every moment in time.

It is not surprising, of course, that factor endowments should play a major role in determining the pattern of trade in good y. What is surprising, perhaps, is that neither the diversion of resources to R&D, nor the existence of aggregate trade imbalances can upset the strong prediction of the Heckscher-Ohlin theorem at any point along the equilibrium path.

We establish a similar result for trade in differentiated products. Each differentiated product is manufactured in only one country, yet each is consumed world-wide, so the pattern of trade in the individual products is clear-cut. The existence of such intra-inudstry trade features prominently in the static models of trade with increasing returns to scale. We focus here on the pattern of trade for the sector as a whole. We have already shown that X_a/X_b is constant over time. So too is C_{xa}/C_{xb}, where $C_{xi}=nc_{xi}$. This ratio, like that for consumption of good y, equals the ratio of initial wealth levels in the two countries. Now if $C_{xa}/C_{xb} > X_a/X_b$, this would imply that Country A imports both goods for all t. But such an outcome would violate the (aggregate) intertemporal budget constraint (4). We conclude, therefore, that $C_{xa}/C_{xb} < X_a/X_b$; i.e.,

Proposition 3: The human-capital rich country is a net exporter of differentiated products at every moment in time.

Next we consider the volume of trade, which is defined as the sum of exports across countries and industries. In our case, it is given by

$$VT = p_y(Y_b - s_b Y) + s_a p X_b + s_b p X_a,$$

where s_i is the share of country i in spending and $X_i = n_i x$ is country i's output of manufactures. Dividing by world spending E and rearranging, we obtain

$$\frac{VT}{E} = (1 - s_x)\left(\frac{Y_b}{Y} - s_b\right) + s_x\left(s_a \frac{X_b}{X} + s_b \frac{X_a}{X}\right), \tag{23}$$

where X and Y are the output levels of the world economy. The second term on the right-hand side is constant on the dynamic trajectory. The first term changes as a result of shifts in country b's share of output of y-goods. When R&D is human-capital intensive relative to production of differentiated products, Y_b/Y rises through time and the volume of trade rises faster than spending. In addition, due to declining investment, the ratio of world spending to world GNP increases over time. Hence,

Proposition 4: If product development is human-capital intensive relative to production of differentiated products, the volume of world trade grows faster than world spending and GNP.

Finally, we consider the pattern of intertemporal trade. We define aggregate savings in country i as the difference between total income and total expenditure. These savings are used to accumulate assets, where z_i represents the accumulated stock of assets in country i. We may think of these as being ownership shares in firms, in which case current account imbalances give rise to foreign equity ownership. Or we may think instead of international trade taking place in short-term bonds, with all firms owned by local residents. The two forms of portfolio trade are equivalent here, as is clear from the fact that the profit rate equals the instantaneous interest rate (see (11)). With either interpretation, the instantaneous current account surplus for country i is given by

$$\dot{z}_i(t) - \dot{n}_i(t) = w(t)L_i + r(t)K_i + \pi(t)z_i(t) - E_i(t) - \dot{n}_i(t).$$

Of course, $z_a(t) + z_b(t) = n_a(t) + n_b(t)$ at all points in time and the two current accounts sum to zero.

There are two offsetting influences at work in determining the current account in our model. On the one hand, the human-capital rich country undertakes relatively more investment in product development than would be predicted based on its relative size alone. This excess of investment demand tends to create a current account deficit for this country. On the other hand, the reward to human capital is falling over time, while the wage rate of unskilled workers is rising, so that the human-capital rich country experiences a decline in its relative factor income. This effect alone should lead Country A to save a relatively greater share of its income, at least early on. For these reasons, it seems possible that the human-capital rich country may be running either a deficit or a surplus on its current account. We have not been able to establish any analytical results that prove otherwise.

4. Multinational Corporations

Our analysis of the trade equilibrium has relied upon the assumption that every brand has to be produced in the country in which it was developed. This requirement excludes the possibility of licensing and the existence of multinational corporations. Naturally, under the conditions of the previous section, entrepreneurs have no incentive to license and firms have no incentive to become multinational. Suppose, however, that R&D requires more human capital per unskilled labor than production of industrial goods, but that Country A's human capital-to-unskilled-labor ratio is larger than that of the industrial sector. In terms of Figure 2, this means that point E is above the ray AC^{∞}. Then the integrated equilibrium cannot be reproduced without licensing or the emergence of multinationals. In what follows we explore the latter possibility.

Following Helpman (1984), assume that production of a variety consists of two activities that can be decomposed, such that headquarter services can be located in one country while actual production takes place in another. For simplicity assume that headquarter services are produced with human capital and that production plants use these services and unskilled labor only.[12] Suppose also that headquarter services have to be produced in the country in which a brand was developed. Then the integrated equilibrium can be reproduced even when the endowment point E is above AC^{∞} in Figure 2. The resulting allocation

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12. It is easy to see how the analysis is modified when both activities require human capital and unskilled labor, as in Helpman and Krugman (1985, Chp. 12).

patterns are presented in Figure 4 for the case in which the extent of multinationality is minimal (see Helpman (1984) for a discussion of this assumption).

It is clear from the figure that up to time T_m at which C^{Tm} becomes the integrated equilibrium allocation there is no pressure for the formation of multinational corporations. However, immediately after this point in time equality of factor rewards cannot be maintained if both activities of industrial firms are concentrated in the same country. This exerts pressure for their separation, with the tendency to locate production in the potentially unskilled-labor cheap country; i.e., Country B. A suitable allocation is presented in the figure. At time T the integrated equilibrium allocation is described by point C^T. Its aggregate variables are reproduced by the following allocation in the trade equilibrium. Country A does not produce traditional goods y. It devotes AP^T resources to R&D, P^TD to production of industrial products by firms that are not multinational, and DE to the production of headquarter services by its multinationals. Country B devotes BC^T resources to the production of y-goods, C^TN^T to the production of industrial products by domes firms (which are not multinational), N^TM^T to R&D, and M^TE to production of x-goods in plants owned by Country A's multinational corporations.

It is clear from this description that starting with $t=T_m$ the extent of multinationality -- as measured by employment in subsidiaries or their output volume -- is increasing at least for some time; we have not been able to prove that it is increasing throughout. We can show, however, that the extent of multinationality also increases towards the steady state. The latter point is seen as follows. The condition of minimal foreign involvement implies that from time T_m Country A does not produce good y. Therefore, after that point in time, its factor market clearing conditions read (compare to the discussion of Figure 4):

$$a_{Ln}\dot{n}_a + a_{Lx}(n_a - m)x = L_a, \qquad (24)$$

$$a_{Hn}\dot{n}_a + a_{Hx}n_a x = H_a, \qquad (25)$$

where m is the number of products produced in foreign subsidiaries and x is output per firm, taken from the integrated world equilibrium.

From (25) we observe that output of x-products is increasing in country A if and only if R&D is declining. Since R&D approaches zero when the steady state is approached, $n_a x$ must be increasing close to the steady state. However, from (24) and (25) we obtain

$$mx = \frac{L_a}{a_{LX}h_n}(h_a - h_n) + \frac{1}{h_n}(h_n - h_x)n_a x,$$

which together with the previous result implies that the degree of multinationality increases close to the steady state (recall that $h_n > h_x$ is assumed in this section).

We have shown that the degree of multinationality -- as measured by the volume of output or employment in subsidiaries -- is increasing when the multinationals start to form and when the economy approaches a steady state. These results also can be extended to cover a third definition of the extent of multinationality: the number of products produced by subsidiaries. This number obviously is rising initially, when the multinationals start to form. That it is also rising close to the steady state we show by proving that x is declining close to the steady state. Since we have already shown that mx is rising, a declining x implies a rising m; i.e., an increasing number of products produced by subsidiaries.

The proof proceeds as follows. Since the dashed path in Figure 1 is above the $\dot{E}=0$ line, the ratio E/n is declining when the trajectory approaches point $(\tilde{n},\tilde{E})$. On the other hand, from part (d) of Proposition 1 we know that p is increasing as this point is approached, because in this section we require R&D to be more human-capital intensive than x. Therefore $x=s_x E/pn$ is declining close to the steady state. Hence,

<u>Proposition 5</u>: If differences in factor composition are wide enough and product development is human-capital intensive relative to production of differentiated products, then there exists a time period in which multinational corporations emerge. The degree of multinationality -- as measured by the number of products produced by subsidiaries, or their volume of output, or employment -- is rising initially and when the world economy approaches the steady state.

In closing this section we note two additional features of economies with multinational corporations. First, it can be shown that in comparison to economies with factor endowments that do not bring about the formation of multinationals but which have the same evolution of the number of products up to time T_m, in the world with multinationals after time T_m Country A has a smaller number of products and Country B a larger number than in the world without multinationals. Second, the pattern of trade described in the previous section need not hold in the presence of multinationals. It is clear that the pattern of trade in y-goods is the same as before. However, the pattern of trade in x-goods might change. This may come about because Country A exports headquarter services, and it may therefore end up importing x-goods, which it exports before the formation of multinational corporations.

5. Concluding Remark

We have extended a number of important results in international trade theory to a dynamic environment in which comparative advantage must be developed over time via the allocation of resources to research and development. In our model, R&D must take place prior to the production of any new variety of a differentiated product. This R&D is motivated by the stream of profits that accrues to the producer of a differentiated good, and is financed by savings that are endogenously determined. When R&D, production of differentiated products, and production of a homogenous good all require fixed input proportions of two primary factors of production, then the Heckscher-Ohlin pattern of trade is preserved all along the dynamic path of the trading equilibrium. This is true despite the fact that trade is not balanced along this path. We further establish that if product development is human-capital intensive relative to production of differentiated products, the volume of trade as a fraction of world GNP or world expenditure grows over time. Finally, we show that for certain endowments points, the emergence of multinational corporations is necessary for the preservation of factor price equalization, and that in these circumstances the extent of multinational activity generally rises over time.

The framework that we have developed is suitable for the study of additional issues. The analysis here excludes factor accumulation to focus attention sharply on growth due to product innovation. In future work, we hope to incorporate accumulation as an alternative vehicle for investment. This will allow us to consider the interactions between resource expansion and technological progress as sources of growth, and to derive the conditions under which there occurs everlasting growth in per capita income. We also plan to study the dynamic effects and efficacy of alternative commercial and industrial policies.

APPENDIX

We provide in this appendix a proof of Proposition 1 by explicitly calculating expressions for the co-movement of six variables with expenditure E. By substituting (16) and (17) into (14) and using the result together with (7), (10), and (12) for the special case considered in Section 2.D -- namely, two factors of production and constant coefficients -- we obtain the following system:

$$1 = a_{Ln}w + a_{Hn}r, \tag{A.1}$$

$$\alpha p = a_{Lx}w + a_{Hx}r, \tag{A.2}$$

$$p_y = a_{Ly}w + a_{Hy}r, \tag{A.3}$$

$$L = a_{Ln}\dot{n} + a_{Lx}s_xE/p + a_{Ly}(1-s_x)E/p_y, \tag{A.4}$$

$$H = a_{Hn}\dot{n} + a_{Hx}s_xE/p + a_{Hy}(1-s_x)E/p_y, \tag{A.5}$$

where w is the reward to unskilled labor and r is the reward to human capital. This system enables us to solve $(w,r,p,p_y,\dot{n})$ as functions of E. In what follows, we calculate the proportional rate of change of each one of these variables in response to a proportional change in expenditure of $\hat{E} \equiv dE/E = 1$; a 'hat' over a variable indicates a proportional rate of change. The following expressions use the standard notation; i.e., θ_{ij} is the share of input i in the cost of activity j and λ_{ij} is the share of factor i employed in activity j:

$$\hat{w} = \frac{1}{\Delta}\,\theta_{Hn}(\lambda_{Hn} - \lambda_{Ln}), \tag{A.6}$$

$$\hat{r} = \frac{1}{\Delta}\,\theta_{Ln}(\lambda_{Ln} - \lambda_{Hn}), \tag{A.7}$$

$$\hat{p} = \frac{1}{\Delta}\,(\theta_{Lx} - \theta_{Ln})\,(\lambda_{Hn} - \lambda_{Ln}), \tag{A.8}$$

$$\hat{p}_y = \frac{1}{\Delta}\,(\theta_{Ly} - \theta_{Ln})\,(\lambda_{Hn} - \lambda_{Ln}), \tag{A.9}$$

$$\hat{\dot{n}} = \frac{1}{\Delta}\,(\theta_{Ly} - \theta_{Lx})\,(\lambda_{Lx}\lambda_{Hy} - \lambda_{Ly}\lambda_{Hx}), \tag{A.10}$$

where

$$\Delta = (\theta_{Lx} - \theta_{Ln})(\lambda_{Lx}\lambda_{Hn} - \lambda_{Ln}\lambda_{Hx}) + (\theta_{Ly} - \theta_{Ln})(\lambda_{Ly}\lambda_{Hn} - \lambda_{Ln}\lambda_{Hy}). \tag{A.11}$$

From the definition of Δ, we have $\Delta > 0$, because $(\theta_{Li} - \theta_{Lj})$ is of the same sign as $(\lambda_{Li}\lambda_{Hj} - \lambda_{Lj}\lambda_{Hi})$, being positive when i is labor intensive relative to j and negative when i is human capital intensive relative to j. We have assumed $a_{Hy}/a_{Ly} < H/L < a_{Hi}/a_{Li}$, $i = x,n$, which implies:

$$\lambda_{Hy} - \lambda_{Ly} < 0, \tag{A.12a}$$

$$\lambda_{Hi} - \lambda_{Li} > 0 \text{ for } i = x,n. \tag{A.12b}$$

Condition (A.12b) proves parts (b) and (c) of Proposition 1, given part (h) that is proved in the text. From (A.6) and (A.9), we obtain

$$\hat{w} - \hat{p}_y = \frac{1}{\Delta}\,\theta_{Hy}(\lambda_{Hn} - \lambda_{Ln}); \tag{A.13}$$

from (A.8) and (A.9), we obtain

$$\hat{p}_y - \hat{p} = \frac{1}{\Delta}(\theta_{Ly} - \theta_{Lx})(\lambda_{Hn} - \lambda_{Ln}); \tag{A.14}$$

and from (A.7) and (A.8), we obtain

$$\hat{p} - \hat{r} = \frac{1}{\Delta}\theta_{Lx}(\lambda_{Hn} - \lambda_{Ln}). \tag{A.15}$$

Equations (A.13) - (A.15) together with (A.12b), part (h) and the assumption that y is labor intensive relative to x -- which implies $\theta_{Ly} > \theta_{Lx}$ -- prove part (a) of the proposition. Part (d) is a direct consequence of (h), (A.8) and (A.12b). Part (e) is a direct consequence of (A.10) and (h). Moreover, (A.10) proves that the function $\nu(E)$ is declining, because the right-hand side of (A.10) is negative.

From (16) and (17) we have:

$$\hat{X} = \hat{E} - \hat{p},$$

$$\hat{Y} = \hat{E} - \hat{p}_y.$$

Using these expressions, $\hat{E} = 1$, as well as (A.8), (A.9) and (A.11), we obtain:

$$\hat{X} = \frac{1}{\Delta}(\theta_{Ly} - \theta_{Lx})(\lambda_{Ly}\lambda_{Hn} - \lambda_{Ln}\lambda_{Hy}), \tag{A.16}$$

$$\hat{Y} = \frac{1}{\Delta}(\theta_{Ly} - \theta_{Lx})(\lambda_{Ln}\lambda_{Hx} - \lambda_{Lx}\lambda_{Hn}). \tag{A.17}$$

Equations (A.12a,b) together with the assumption that y is labor intensive relative to x imply that the right-hand side of (A.16) is positive. This together with part (h) proves part (f) of the proposition. From (A.17) it is evident that given that y is labor intensive relative to x, the right-hand side is positive if and only if n is labor intensive relative to x, which together with (h) proves part (g).

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Figure 1

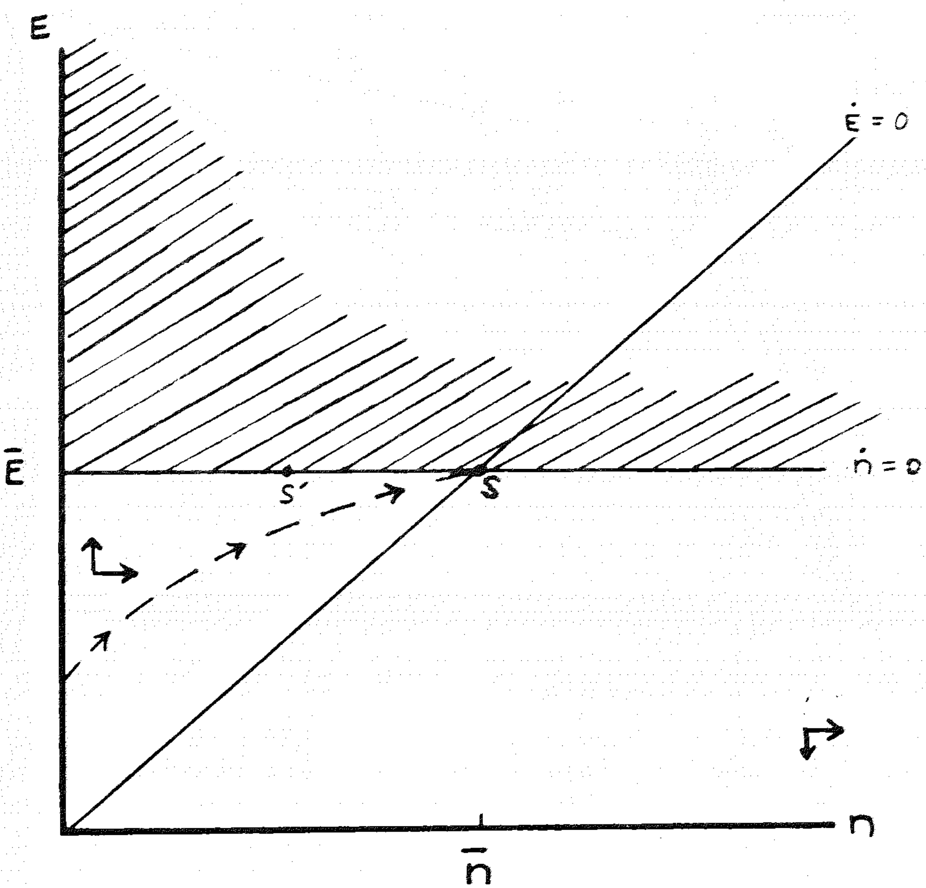

Figure 2

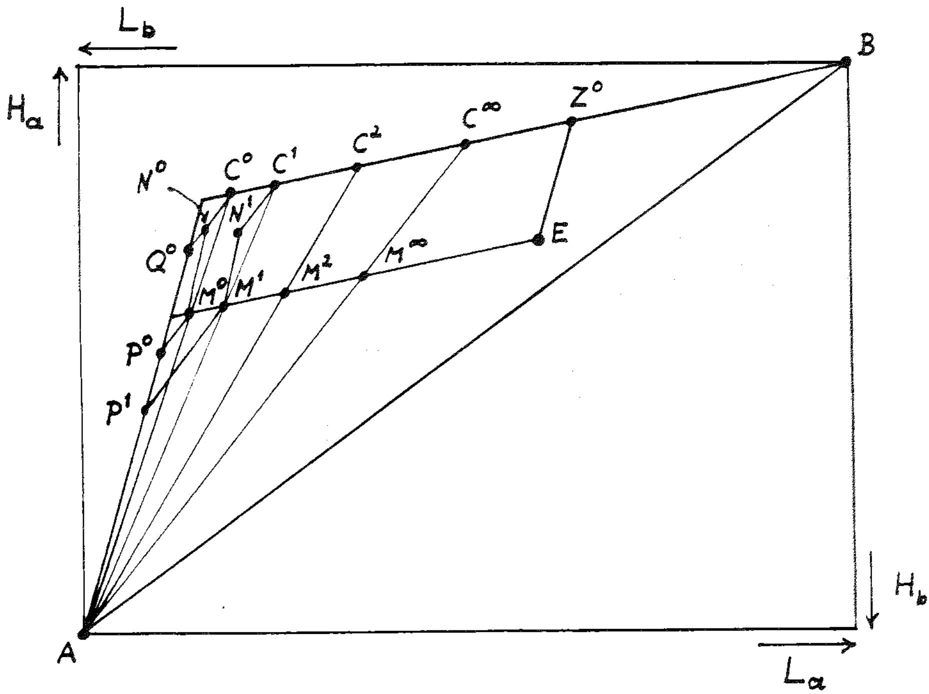

Figure 3

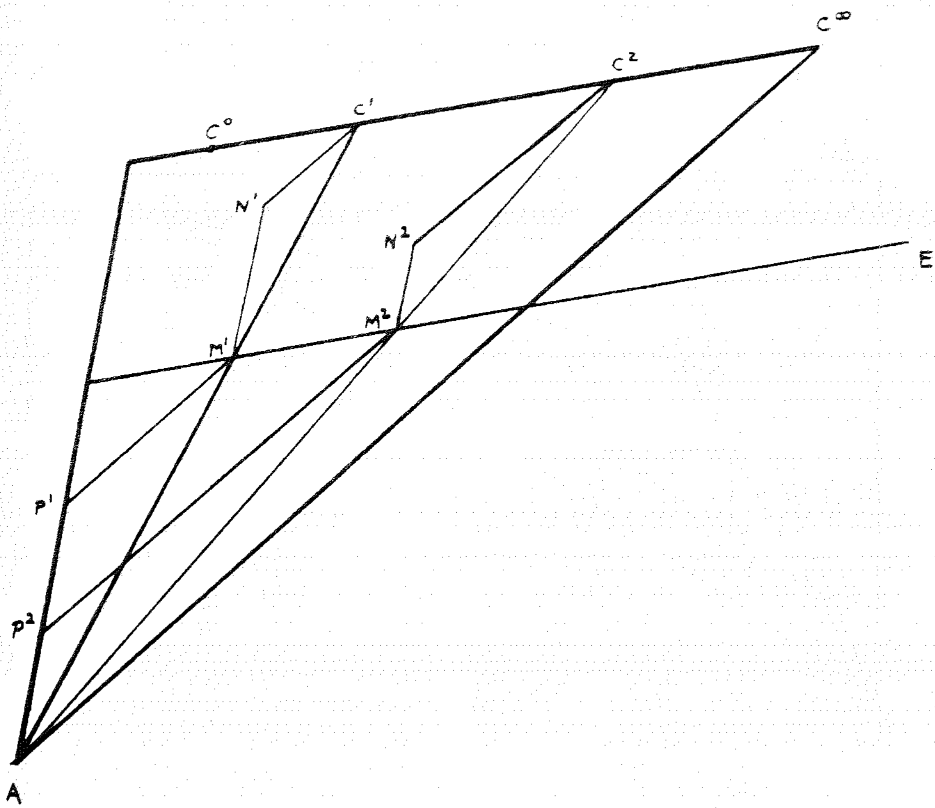

Figure 4