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ABSTRACT

This paper explores the implications of tax rate uncertainty, identifying circumstances in which revenue-neutral tax rate variability increases profitability, economic activity, and the efficiency of resource allocation. Furthermore, with heterogeneous taxpayers, tax rate variability is shown to perform an efficiency-enhancing screening function, imposing heavier expected tax burdens on less responsive taxpayers. And while efficient tax uncertainty enables governments to reduce average costs of taxation, it necessarily increases the marginal cost of taxation over some ranges of expected revenue, so may reduce efficient levels of government spending.

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1. Introduction

Thoughtful observers have long criticized uncertainty in tax matters. One of Adam Smith's (1802, p. 256) canons of taxation is that "the tax which each individual is bound to pay ought to be certain," while for Alexander Hamilton (1782), "The genius of liberty...exacts that every man, by a definite and general rule, shall know what proportion of his property the State demands." Their concern was largely the risk of arbitrariness in the exercise of taxing powers; but the notion that tax uncertainty in itself has inherently damaging consequences has recently come to the fore of practical policy discussions. Perceptions of an increasingly uncertain tax environment¹ prompted the G20 (2016), for example, to stress "the benefits of tax certainty to promote investment and trade," and to put the IMF and OECD (2017) to work on how to enhance it.² This has become a concern for the European Union too.³ Against the background of this long-lasting and now high profile concern—and wider interest in the impact of policy uncertainty⁴—the purpose of this paper is to explore whether and when tax uncertainty reduces economic activity and economic welfare.

Any efficiency case for tax certainty must rest on the properties of behavioral responses to taxation. Tax-induced price uncertainty poses challenges to consumers and firms but need not be harmful to their interests, since while fluctuating conditions can be costly they also create opportunities. Indeed, one of the fundamental propositions of price theory is that a competitive firm's profits are convex in input and output prices (Mas-Colell, Whinston and Green, 1995, pp. 138, 141), implying that its expected profits increase with mean-preserving variability in these prices—and hence too with corresponding randomization of input or output tax rates. The reason is straightforward: input price variation permits firms to economize on expensive input purchases while prices are high, substituting with other inputs and reducing output; and they can expand the use of inexpensive inputs when prices are low. By adjusting its purchases and sales,

¹ Largely, though not only, in relation to implementation of the outcomes of the G20-OECD project on Base Erosion and Profit Shifting (BEPS), intended to curb tax avoidance by multinationals.

² This work is updated in IMF and OECD (2018).

³ See Zangari, Caiumi and Hemmelgarn (2017).

⁴ The index of economic policy uncertainty developed in the influential work of by Baker, Boom and Davis (2016), for example, includes an element intended to capture tax uncertainty.

a firm can effectively use price variability to reduce the average unit cost of its inputs, thereby improving after-tax profits.

Remarkable though these results are, however, they are seriously incomplete as guides to policy, since the firm responses just described mean that randomizing a tax rate generally reduces expected tax revenue, requiring the government to compensate with a higher average rate. This consideration suggests that the effect of tax uncertainty is appropriately gauged by holding constant expected tax revenue rather than expected tax rates. The point is of considerable practical importance. In the context of current policy concerns, it is important—but rarely attempted—to disentangle business worries over tax uncertainty from those related to expected tax liabilities, since with constant expected tax revenue businesses facing fluctuating tax rates will also face higher average tax rates. While such empirical evidence as is currently available suggests that tax uncertainty may discourage investment, it generally does little to distinguish the effects of uncertainty as such from the effects of expected levels of taxation.⁵

A further limitation of the standard price theory result is that it speaks only to the effect of tax uncertainty on after-tax profits, whereas the effect of tax uncertainty on input use—as with the G20’s concern for investment—and economic output are also important for policy assessment. There are of course some results on this, notably those presented by Hartman (1972) and Abel (1983), who show that a mean preserving spread in an output tax rate leads a competitive firm to increase its capital stock, so long as the marginal value product of capital is convex in that output price. Again, however, this does not condition on expected tax revenue—as is also true of more recent results on the effect of tax uncertainty with irreversible investment. Without considering the tax revenue implications of randomization, moreover, it is difficult to perform welfare analysis or draw implications for economic policy.

This paper takes up these and related issues, examining the effect of tax uncertainty on profitability, economic activity, and the efficiency cost of raising government revenue. The analysis takes tax uncertainty to consist of randomizing the tax rate on a productive input, subject to keeping expected tax revenue unchanged. Firms are assumed to make decisions after the tax

⁵ As reviewed in IMF and OECD (2017) and Zangari, Caiumi and Hemmelgarn (2017). Edmiston (2004), for example, uses the deviation of tax rates from trend as indicator of tax uncertainty, without reference to the expected level of revenue. Baker, Bloom and Davis (2016) use as their indicator of tax uncertainty the dollar value of tax provisions set to expire, which holds expected revenue constant only under limited circumstances.

rate is revealed, so they do not incur losses due to sunk costs. This is in contrast to much of the recent literature on tax uncertainty, which posits that taxpayers make irreversible decisions prior to knowing the future tax regime. Both of these specifications capture important features of reality that can significantly influence decisions,⁶ with accompanying implications for deadweight loss.⁷ But many aspects of government policies are understood in advance, and the analysis here focuses on their effects.

There appears to be a widely held presumption, shared by the G20, that tax uncertainty inefficiently reduces economic activity and depresses profits; some see this as simply ‘common sense.’⁸ Higher tax rates discourage taxed activity, and the resulting economic losses are commonly presumed not only to rise with the amount of tax revenue collected, but to do so at an increasing rate, making it ever more expensive—in an efficiency sense—to collect additional tax revenue. The familiar Harberger triangle approximation, that deadweight loss is proportional to the square of the tax rate (Harberger, 1964, 1971; Auerbach, 1985; Hines, 1998), lends support to this intuition—when the approximation is valid. It implies that tax rate variability increases the deadweight loss associated with raising a given level of expected tax revenue; the cost of this deadweight loss is borne by consumers in the form of higher prices and business in the form of lower profits, and these discourage economic activity.

This intuition, however, can be seriously misleading, and the common sense view misplaced. They rely on the notion that tax rate variability reduces the efficiency of the tax system, but there are two reasons why this may not be the case. The first is that the discouraging effects of higher increments to tax rates can become more moderated at high rates, causing the marginal deadweight loss of collecting additional tax revenue to decline with tax revenue. This possibility reflects an important inaccuracy in the intuitive approximation to the size of the deadweight loss triangle, and is the basis of earlier normative arguments (e.g., Stiglitz, 1982b)

⁶ On the effects of tax and other forms of uncertainty on firm investments, see for example Hartman (1972), Pindyck (1982), Abel (1983), Auerbach and Hines (1988), Alvarez, Kannianen, and Sodersten (1998), Hassett and Metcalf (1999), Niemann (2004, 2011), Bloom, Bond and van Reenen (2007) and Bloom (2014).

⁷ Weiss (1976), Stiglitz (1982a,b), Alm (1988), Skinner (1988), and Pestieau, Possen, and Slutsky (1998) consider the efficiency properties of tax systems with rates that are revealed only after actors make consequential economic choices.

⁸ As for instance do Zangari, Caiumi and Hemmelgarn (2017).

made in favor of random taxation. The analysis below identifies the circumstances—looking first at the impact on a single firm—in which, conditional on expected tax revenue, tax rate variability reduces expected profits, output and input use, as well as those in which, counter to ‘common sense,’ tax uncertainty actually increases all or some of these.

A second, previously unnoticed but commonly more powerful, reason why tax uncertainty can improve the efficiency of the tax system emerges from the analysis below. This is that tax uncertainty serves as a screening device, effectively imposing higher tax rates on firms and activities that are relatively unresponsive to taxation, and lower tax rates on firms and activities that are more responsive—which, by standard intuition, improves efficiency. If there is any type of taxpayer heterogeneity, then the fraction of taxed activity undertaken by taxpayers that are most responsive to taxation will decline as tax rates rise. Consequently, as tax rates rise the high tax rates increasingly bear on activities that are relatively unresponsive to taxation. The opposite happens when tax rates decline: the resulting low tax rates apply to a population that is relatively dominated by taxpayers and activities that are highly responsive to taxation. Tax uncertainty therefore implements a subtle version of the Ramsey rule, imposing higher tax rates on the activities of less-responsive taxpayers, and lower tax rates on the activities of more-responsive taxpayers – which has the effect of reducing the total cost of taxation. If the government cannot distinguish firms and activities any other way, it can do so indirectly with tax rate variability.

All of this carries implications for efficient tax policy that are also subtle and to some degree surprising. In the absence of tax uncertainty, and with a single tax instrument, tax policy is dictated by the government’s revenue needs. Tax uncertainty expands the range of possibilities, permitting the government to reduce the total economic cost of taxation by randomizing between disparate tax rates, each associated with relatively low deadweight loss per dollar of tax revenue collection. While over some ranges of expected tax revenue such policies will reduce the marginal cost of public funds, it is also the case that over equivalently-sized ranges they will increase the marginal cost of public funds. Consequently, there are many circumstances in which the adoption of efficiency-enhancing tax uncertainty will reduce the efficient level of government spending even though it reduces the average cost of taxation.

Section 2 sets out the basic framework of the analysis. Section 3 analyzes the effects of tax uncertainty on firm profits and economic activity in settings with identical taxpayers. Section 4 introduces taxpayer heterogeneity, showing it to have a marked effect on the efficiency properties of tax uncertainty by playing the screening function described above. Section 5 considers the efficient design of uncertain taxes and its implications for the marginal cost of public funds. Section 6 concludes by noting additional implications and generalizations.

2. *Preliminaries*

A perhaps surprising degree of theoretical richness is available from considering a simple setting in which a fixed number of identical price-taking competitive firms each purchases a taxed input in amount x , paying suppliers a unit price, and also paying a specific tax of τ per unit purchased;⁹ it is assumed that $\tau > 0$, but will be seen later that the results extend readily to cases in which τ is strictly negative. Each firm has a production function¹⁰ $q(x)$ that is increasing, continuous and continuously differentiable as needed in x , and exhibits decreasing returns, so that $q'(x) > 0$ and $q''(x) < 0$. There may be other untaxed inputs, in which case $q(x)$ represents output net of the cost of these optimally chosen additional inputs.¹¹ Firms sell their output at unit price (and untaxed) on the world market. The profits of the representative firm(π), arising from the presence of some fixed factor(s), are therefore given by

⁹ Specific and ad valorem taxes have equivalent effects in competitive markets in the absence of uncertainty in the tax-exclusive price of the taxed input, as noted by Suits and Musgrave (1953). The effects of (non-stochastic) specific and ad valorem taxes will, however, differ in competitive markets with price uncertainty (Goerke, 2011; Kotsogiannis and Serfes, 2014; Goerke, Herzberg, and Upmann, 2014) and in imperfectly competitive markets (Delipalla and Keen, 1992; Weyl and Fabinger, 2013). When, for either of these reasons, equivalence fails, the distinction between specific and ad valorem taxes will be material for the impact of tax uncertainty; this paper's analysis, however, abstracts from these issues.

¹⁰ It is an important feature of the model that production functions are exogenously given. If firms could choose among possible production functions then their choices, and therefore output, would be affected by the anticipated distribution of tax rates. Furthermore, the production functions considered here exhibit no adjustment costs or other forms of intertemporal dependence, so input choices and output are determined only by contemporaneous prices and taxes.

¹¹ Appendix A extends the analysis to allow for other variable inputs taxed at fixed rates.

$$(1) \quad \pi = q(x) - (1 + \tau)x.$$

The first order condition¹² for profit maximization is $q'(x) = (1 + \tau)$, which generates a derived demand for x as a function of τ , $x(\tau)$, that is continuous and continuously differentiable, with $x'(\tau) < 0$.

Tax revenue is $R \equiv \tau x(\tau)$. The analysis confines attention to ranges of tax rates over which tax revenue monotonically increases with the tax rate, which requires that

$$(2) \quad x(\tau) + \tau x'(\tau) > 0.$$

Of course, (2) is not always satisfied, particularly at high tax rates. It is nonetheless constructive to restrict attention to ranges of tax rates over which (2) is satisfied, since there are predictably strange consequences of randomizing tax rates into ranges in which higher tax rates are associated with reduced tax collections. Thus, it is assumed throughout that (2) holds, or, equivalently, that

$$(3) \quad 1 + \varepsilon(\tau) > 0,$$

where

$$(4) \quad \varepsilon(\tau) \equiv \tau \frac{x'(\tau)}{x(\tau)} < 0$$

denotes the elasticity of input demand with respect to the tax rate. This elasticity (strictly negative at any positive tax rate), and its properties, plays a central role in the analysis that follows.

From the implied one-to-one relationship between the tax rate and tax revenue, it is possible to express the tax rate as an implicit function of tax revenue, $\tau(R)$, with

¹² The assumption that $q''(x) < 0$ ensures that the firm's second order condition is satisfied.

$$(5) \quad \tau'(R) = \frac{1}{x(\tau) + \tau x'(\tau)} > 0.$$

Input demand is then also an implicit function of tax revenue, $x(\tau(R))$, and so too are both output $q[x(\tau(R))]$ and profits $\pi(R)$. The device of characterizing firm behavior and profits as a function of tax revenue, used repeatedly in the next section, makes it possible to infer the effect of revenue-neutral tax rate randomization from their convexity or concavity in revenue. For example, a local randomization of tax rates that leaves expected revenue unchanged¹³ increases the expected value of profits if and only if $\pi(R)$ is convex in R .

3. *The impact of tax uncertainty with identical firms*

To identify clearly the considerations that influence the effect of tax uncertainty on expected profits, input use and output, it is helpful to start by considering uncertainty that applies to a single firm or to a set of identical firms.

3.1 *Expected profits*

Consider first the impact of a revenue neutral tax rate randomization on the firm's expected profits. Shephard's lemma implies, from (1), that $\pi'(R) = -x(\tau)\tau'(R)$ and so, using (5), that:

$$(6) \quad \pi'(R) = \frac{-1}{1 + \varepsilon(\tau)}.$$

Recalling (3), it follows that at any positive tax rate $\pi'(R) < -1$: profits decline by more than any increment to tax revenue, reflecting that the marginal deadweight loss associated with raising an additional dollar is positive—a point returned to in Section 5. It also follows from inspection of (6) that if $\varepsilon'(\tau) < 0$, so that the elasticity of input demand has a larger negative magnitude as

¹³ For brevity this is denoted a revenue neutral reform.

the tax rate increases, then increments of tax revenue will be associated with ever larger reductions in firm profits, making the profit function concave in revenue. More formally, differentiating (6) produces:

$$(7) \quad \pi''(R) = \frac{\varepsilon'(\tau)}{[1 + \varepsilon(\tau)]^2} \tau'(R).$$

Hence given (5), profits are strictly concave in tax revenue if $\varepsilon'(\tau) < 0$ and strictly convex in tax revenue if $\varepsilon'(\tau) > 0$. Therefore:¹⁴

PROPOSITION 1: *A revenue neutral local tax rate randomization reduces expected profits if and only if $\varepsilon'(\tau) < 0$, and increases expected profits if and only if $\varepsilon'(\tau) > 0$.*

The impact of tax rate randomization thus turns on whether the elasticity of derived input demand increases or decreases with the tax rate.

Figure 1 provides some intuition for this. It displays the relationship between tax rates and firm profits, with the solid locus in the figure plotting firm profit as a decreasing convex function of the tax rate. This captures the notion that a mean-preserving price spread gives firms more options than they would have under price stability; formally, convexity is guaranteed by the combination of Shephard's Lemma, which implies that the slope of the profit function is $-x(\tau)$, and the input demand implication that $x'(\tau) < 0$. The figure describes a setting with tax rate uncertainty: the tax rate takes the high value τ_H with probability 0.5, and the low value τ_L with probability 0.5.

Consider then a revenue-neutral reform that increases the scope of tax rate uncertainty by increasing τ_H by an amount that would raise an additional \$1 in tax revenue while lowering τ_L by an amount that would lose \$1 in tax revenue. Denoting input demand at τ_H by $x_H \equiv x(\tau_H)$, and analogously the input demand elasticity at τ_H by $\varepsilon_H \equiv \varepsilon(\tau_H)$, it follows

¹⁴ Equation (7) readily generalizes to cases in which there are multiple taxed inputs, as elaborated in Appendix A.

from (5) that the new high value of the tax rate is $\tau_H + \tau'(R_H) = \tau_H + \left[\frac{1}{x_H(1 + \varepsilon_H)} \right]$. Since the slope of the profit function at τ_H is given by $-x_H$, the change (a reduction) in profit if this small tax increase were to arise is the product of these two terms, $\frac{-x_H}{x_H(1 + \varepsilon_H)} = \frac{-1}{(1 + \varepsilon_H)}$. By a similar calculation, the increase in profit if the tax were reduced is $\frac{1}{(1 + \varepsilon_L)}$. Consequently, with equal probabilities of high and low tax rates, the net increase in expected profits from widening the spread of tax rates in this expected revenue-neutral way is given by $0.5 \left[\frac{1}{(1 + \varepsilon_L)} - \frac{1}{(1 + \varepsilon_H)} \right]$. Expected profits therefore fall if and only if $\varepsilon_H < \varepsilon_L$, which is the case if and only if the tax elasticity of input demand declines (is more negative) at higher tax rates—which is precisely the condition that $\varepsilon'(\tau) < 0$, just as in Proposition 1.¹⁵

Much thus turns on the sign of the derivative $\varepsilon'(\tau)$. This, as a general matter, is ambiguous. To see the forces at work, denote by $\eta(\tau) \equiv (1 + \tau) \frac{x'(\tau)}{x(\tau)} < 0$ the price elasticity of demand for the taxed input, so that $\varepsilon(\tau) = \left(\frac{\tau}{1 + \tau} \right) \eta(\tau)$ and hence

$$\varepsilon'(\tau) = \frac{1}{(1 + \tau)^2} \eta(\tau) + \frac{\tau}{(1 + \tau)} \eta'(\tau).$$

Since the first term on the right points toward $\varepsilon'(\tau) < 0$, it is sufficient, but not necessary, for $\varepsilon'(\tau) < 0$ that the price elasticity of the input demand be decreasing or constant in the input price—which will be the case, for instance, if input demand is linear or (a case explored further below) Cobb-Douglas. Conversely, it can be the case that $\varepsilon'(\tau) > 0$ – so that revenue-neutral randomization of tax rates raises expected profits – only if the price elasticity $\eta(\tau)$ not only increases with the input price but does so sufficiently rapidly to

¹⁵ While this exercise starts from a situation with tax uncertainty, its implications also apply to introducing a small amount of tax uncertainty into a setting with tax certainty.

offset the mechanical effect of a higher tax rate. That this is a real possibility is illustrated by considering firms with production functions

$$(8) \quad q(x) = q(\bar{x}) + b^{\frac{1}{c}} \int_{\bar{x}}^x \left[\ln \left(\frac{z}{a} \right) \right]^{\frac{-1}{c}} dz,$$

with a , b and c all strictly positive constants. The production function (8) generates input demands

$$(9) \quad x = ae^{b(1+\tau)^{-c}},$$

which imply that $\varepsilon(\tau) = \frac{-bc\tau}{(1+\tau)^{1+c}}$, and for b sufficiently small, $0 > \varepsilon(\tau) > -1$. It follows that

$$(10) \quad \varepsilon'(\tau) = \frac{-bc(1-\tau c)}{(1+\tau)^{2+c}}.$$

Equation (10) then implies that that $\varepsilon'(\tau) > 0$ whenever $\tau > 1/c$.¹⁶

While theory thus leaves the sign of $\varepsilon'(\tau)$ open, and clearly there are instances in which $\varepsilon'(\tau) > 0$, there is perhaps a cautious presumption that in many cases $\varepsilon'(\tau) < 0$, at least in the sense that this can be shown to be case for the specific functional forms often found convenient to work with and used to guide intuition. The implication is a similarly guarded presumption that, with identical firms, revenue neutral tax rate randomization will commonly lower expected profits. This, of course, stands in sharp contrast to the established result that randomization of tax rates holding the expected tax rate constant increases expected profits: controlling for the expected impact on revenue makes a great deal of difference to the likely consequences of tax uncertainty for firm profitability.

3.2 Expected input use

¹⁶ More generally it is readily shown that the necessary and sufficient condition for $\varepsilon'(\tau) < 0$ is that the price elasticity of the price elasticity of input demand be greater than $-1/\tau$.

The impact of revenue neutral tax rate randomization on input use turns on the convexity or concavity of demand for input x expressed as a function of R . With $x'(R) = x'(\tau)\tau'(R)$, imposing (5) gives:

$$(11) \quad x'(R) = \frac{\varepsilon(\tau)}{\tau[1 + \varepsilon(\tau)]}.$$

Equation (11) then implies:

PROPOSITION 2: *Expected input use is reduced by revenue neutral local tax rate*

randomization if and only if $\varepsilon'(\tau) < \frac{\varepsilon(\tau)[1 + \varepsilon(\tau)]}{\tau} < 0$. Conversely, expected input use is

increased by revenue neutral local tax rate randomization if and only if $\varepsilon'(\tau) > \frac{\varepsilon(\tau)[1 + \varepsilon(\tau)]}{\tau}$.

Proof: Differentiating both sides of (11) with respect to R produces:

$$(12) \quad x''(R) = \frac{1}{\tau[1 + \varepsilon(\tau)]^2} \tau'(R) \left\{ \varepsilon'(\tau) - \frac{\varepsilon(\tau)[1 + \varepsilon(\tau)]}{\tau} \right\}.$$

Since the first two terms on the right side of (12) are positive, the sign of $x''(R)$ depends on the sign of the term in braces, from which the proposition follows. \square

Proposition 2 implies that $\varepsilon'(\tau) > 0$ is a sufficient condition for expected input use to increase with tax uncertainty. So, from Proposition 1, if tax uncertainty increases expected profits then it will also increase expected input use. The central implication of Proposition 2, however, is that the converse is not true: if $\varepsilon'(\tau) < 0$, so that tax uncertainty reduces expected profits, it may nonetheless increase expected input use if $\varepsilon'(\tau)$ is sufficiently small in magnitude. When $\varepsilon'(\tau)$ is negative but vanishingly small, for instance, a revenue neutral tax rate randomization reduces expected profits but increases expected input use.

3.3 Expected output

The effect of tax uncertainty on expected output can be analyzed in the same way. Expressing output as a function of tax revenue, $q(R)$, the output effect of greater tax revenue is $q'(R) = q'(x)x'(\tau)\tau'(R)$. Imposing the firm's first order condition that $q'(x) = (1 + \tau)$, together with (5), gives:

$$(13) \quad q'(R) = \frac{(1 + \tau)\varepsilon(\tau)}{\tau[1 + \varepsilon(\tau)]}.$$

Differentiating equation (13) then gives:

PROPOSITION 3: *Expected output declines with revenue neutral local tax rate randomization if and only if $\varepsilon'(\tau) < \frac{\varepsilon(\tau)[1 + \varepsilon(\tau)]}{\tau(1 + \tau)}$. Conversely, expected output increases with revenue neutral local tax rate randomization if and only if $\varepsilon'(\tau) > \frac{\varepsilon(\tau)[1 + \varepsilon(\tau)]}{\tau(1 + \tau)}$.*

Proof: Differentiating both sides of (13) with respect to R produces:

$$(14) \quad q''(R) = \frac{(1 + \tau)}{\tau[1 + \varepsilon(\tau)]^2} \tau'(R) \left\{ \varepsilon'(\tau) - \frac{\varepsilon(\tau)[1 + \varepsilon(\tau)]}{\tau(1 + \tau)} \right\}.$$

Since the first two terms on the right side of (14) are positive, the sign of $q''(R)$ depends on the sign of the term in braces, from which the proposition follows. \square

The results in Propositions 2 and 3 can be visualized in a fashion similar to that of Proposition 1, though with the important difference that, while profits, input demand and output are all decreasing in the tax rate, only profit is guaranteed to be convex in the tax rate. Appendix B offers an illustration of Proposition 3.

Tax uncertainty thus exerts many of the same effects on expected output as it does on expected input use. In particular, if tax uncertainty increases expected profits (because $\varepsilon'(\tau) > 0$) then it also increases expected output. Again, however, the converse is not

true: tax uncertainty reduces expected profits but increases expected output if

$$0 > \varepsilon'(\tau) > \frac{\varepsilon(\tau)[1 + \varepsilon(\tau)]}{\tau(1 + \tau)}.$$

Comparing the critical values in Propositions 2 and 3, the

additional $(1 + \tau)$ in the denominator of the latter means that while a reduction in expected input use is a sufficient condition for expected output also to fall, it is not necessary: there is a range of values of $\varepsilon'(\tau)$ over which tax uncertainty is associated with greater expected input use but reduced expected output.

3.4 Interpretation

Figure 2 summarizes the relationships between the value of $\varepsilon'(\tau)$ and the signs of the effects of tax uncertainty on expected profits, input use, and output implied by Propositions 1, 2 and 3. At sufficiently low (negative) values of $\varepsilon'(\tau)$, for example, revenue neutral tax rate randomization reduces the expected levels of profits, input use, and output. At the other extreme, if (and only if) $\varepsilon'(\tau)$ is strictly positive does a revenue neutral tax rate randomization increases all three. In between, there is a ready intuition for the hierarchy of results shown in Figure 2. Concavity of the production function implies that for any expected level of input use, the expected level of output declines with tax-induced variation in input use. This means, loosely speaking, that output is more likely to be a concave function of revenue than is input use—and so uncertainty reduces output more readily than it reduces inputs. And since profits are concave in output they are in turn more likely to be reduced by uncertainty than are outputs.

To illustrate these results, consider the effects of taxation when firms have identical Cobb-Douglas production functions given by:

$$(15) \quad q(x) = kx^\alpha,$$

with $k > 0$ and $1 > \alpha > 0$. The first order condition for profit maximization implies

$x^{1-\alpha} = \frac{k\alpha}{(1+\tau)}$; differentiating this with respect to τ yields:

$$(16) \quad \varepsilon(\tau) = \frac{-\tau}{(1+\tau)(1-\alpha)}.$$

Differentiating again produces:

$$(17) \quad \varepsilon'(\tau) = \frac{-1}{(1+\tau)^2(1-\alpha)}.$$

It is clear from (17) that, as claimed earlier, in this case $\varepsilon'(\tau) < 0$, so that, by Proposition 1, tax uncertainty reduces firm profits. This reflects the fact that in the Cobb-Douglas case the tax elasticity of input demand is zero in the absence of taxation, steadily rising in magnitude with higher tax rates and ultimately converging to $\frac{-1}{(1-\alpha)}$. Furthermore, (16) and (17) together imply:

$$(18) \quad \frac{\varepsilon'(\tau)}{\varepsilon(\tau)[1+\varepsilon(\tau)]} = \frac{1}{\tau} \frac{(1-\alpha)}{(1-\alpha-\alpha\tau)} > \frac{1}{\tau},$$

where the inequality in (18) follows from the restriction that $\tau'(R) > 0$, which implies $\varepsilon(\tau) > -1$ and therefore, from (16), that $\alpha(1+\tau) < 1$. Applying Propositions 2 and 3, equation (18) implies that with Cobb-Douglas production functions tax uncertainty also reduces both expected input use and expected output.

A numerical example for this Cobb-Douglas case, to be exploited further below, is presented in Table 1. The initial tax rate is $\tau = 2$ and the production function has $\alpha = 0.2$, and k chosen so that $x = 10$ initially.¹⁷ With these input demand parameters, initial tax revenue is 20, output is 150, and profits are 120. Tax uncertainty takes the form of reducing the tax rate to 1.6 with probability 0.5, and raising it to 2.86 with probability 0.5, an asymmetric change that leaves

¹⁷ This entails $k = 94.64$.

expected tax revenue unchanged at 20. As reflected in the table, this tax rate fluctuation reduces expected profits to 118.5, and similarly reduces expected input use to 9.65 and expected output to 148.15. The rising tax sensitivity of input demand requires a significantly higher tax rate in the state of the world with high taxes to compensate for a lower tax rate in the state of the world with low taxes, distorting production and reducing expected input use and profits.

4. *Heterogeneous firms*

The focus so far has been on economies with representative firms, or equivalently, economies in which all firms are identical—and, moreover, are taxed identically *ex post*. Of course this is not an accurate representation of the world. This section considers the consequences of taxpayer heterogeneity, finding that differences among taxpayers significantly increase the likelihood that tax uncertainty will lead to higher profits, greater input use, and greater output.

4.1 *Heterogeneity with a fixed number of firms*

Consider first a setting with a fixed number of heterogeneous firms, indexed by i . Letting $x_i(\tau)$ denote firm i 's demand for productive input x , $\varepsilon_i(\tau)$ its corresponding tax elasticity of input demand, and $\pi_i(\tau)$ its profits, the requirement that aggregate tax revenue increase with the tax rate now implies that $\sum x_i(\tau)[1 + \varepsilon_i(\tau)] > 0$. Taking this requirement to be met, the tax rate can again be written as a function of aggregate revenue, $\tau(R)$, with

$$(19) \quad \tau'(R) = \frac{1}{[1 + \varepsilon_A(\tau)] \sum x_i(\tau)},$$

in which $\varepsilon_A(\tau)$ is the elasticity of aggregate input demand, which in turn is simply a weighted average of the individual demand elasticities:

$$(20) \quad \varepsilon_A(\tau) \equiv \frac{\tau \sum x_i'(\tau)}{\sum x_i(\tau)} = \sum w_i(\tau) \varepsilon_i(\tau),$$

with the weight $w_i(\tau) \equiv \frac{x_i(\tau)}{\sum x_i(\tau)}$ being firm i 's share of aggregate input demand.

Consider then the impact of revenue neutral local tax rate randomization on aggregate profits $\Pi(R) \equiv \sum \pi_i(\tau(R))$. From Shephard's lemma, $\Pi'(R) = -\tau'(R) \sum x_i(R)$; hence from

(19), it follows that $\Pi'(R) = \frac{-1}{1 + \varepsilon_A(\tau)}$. Thus (7) also applies to aggregate profits in the

heterogeneous firm case, with $\varepsilon(\tau)$ replaced by $\varepsilon_A(\tau)$, and $\varepsilon'(\tau)$ replaced by $\varepsilon'_A(\tau)$.

Differentiating (20), noting that

$$(21) \quad w'_i(\tau) = \frac{1}{\tau} w_i(\tau) [\varepsilon_i(\tau) - \varepsilon_A(\tau)],$$

and using $\sum w'_i(\tau) = 0$ to replace $\sum w'_i(\tau) \varepsilon_i(\tau)$ with $\sum w'_i(\tau) [\varepsilon_i(\tau) - \varepsilon_A(\tau)]$, gives

$$(22) \quad \varepsilon'_A(\tau) = \sum w_i(\tau) \varepsilon'_i(\tau) + \frac{1}{\tau} \sum w_i(\tau) [\varepsilon_i(\tau) - \varepsilon_A(\tau)]^2.$$

Consequently:

PROPOSITION 4: *With an unchanging set of active firms a revenue neutral local tax rate randomization reduces expected aggregate profits if and only if $\varepsilon'_A(\tau) < 0$, where $\varepsilon'_A(\tau)$ is given by (22). Conversely, a revenue neutral local tax rate randomization increases expected aggregate profits if and only if $\varepsilon'_A(\tau) > 0$.*

Proposition 4 is a straightforward generalization of Proposition 1. The structure of the tax derivative of aggregate demand for the taxed input, however, points to potentially quite different outcomes. For while the first term on the right side of (22) is negative if all of the $\varepsilon'_i(\tau)$ s are negative, the second term, which is the variance of the elasticities (weighted by input shares), is unambiguously positive under the very mild condition that the elasticities $\varepsilon_i(\tau)$ differ—and that points to $\varepsilon'_A(\tau) > 0$. In this sense, taxpayer heterogeneity increases the

likelihood that revenue neutral tax rate randomization increases aggregate expected profits; and this is so even if $\varepsilon'_i(\tau) > 0$ for each individual firm.¹⁸

If, for example, all taxpayers have constant input demand elasticities, but those elasticities differ, then (22) implies that the aggregate demand elasticity is unambiguously increasing in the tax rate ($\varepsilon'_A(\tau) > 0$). This follows simply from the fact that the variance term in (22) is positive. Intuitively, what happens in this case is that, as the tax rate increases, input demands by firms with relatively inelastic demands decline proportionately less than input demands by firms with more elastic demands, so aggregate demand $\varepsilon_A(\tau)$ becomes less elastic. Conversely, as the tax rate decreases input demands by firms with relatively elastic demands increase disproportionately, so aggregate demand becomes more elastic. The resulting induced positive correlation between the tax rate and the aggregate demand elasticity gives the aggregate demand elasticity a positive derivative. Profits increase because higher tax rates apply disproportionately to firms with inelastic demands: tax uncertainty serves, in effect, as a screening device for focusing taxation on firms with less elastic input demands.¹⁹

Proceeding similarly for aggregate output and aggregate input, analogs to Propositions 2 and 3 follow easily, the only difference being that terms in $\varepsilon(\tau)$ are replaced by terms in $\varepsilon_A(\tau)$, and $\varepsilon'(\tau)$ is replaced by $\varepsilon'_A(\tau)$. But this difference evidently matters; and even if there is insufficient taxpayer heterogeneity to make the tax derivative of the aggregate demand elasticity positive, heterogeneity may so elevate the derivative of the aggregate demand elasticity that tax uncertainty will stimulate greater expected input use or output.

To illustrate these results, it is instructive to introduce firm heterogeneity into the example of section 3.4. Suppose that half of input demand comes from firms with identical Cobb-Douglas production functions given by (15), while the other half comes from firms with production functions that give them (over the relevant tax range) perfectly inelastic input

¹⁸ It is perhaps counterintuitive that there are situations in which, from the standpoint of any individual firm, a revenue neutral tax rate randomization would reduce its profits, yet an aggregate revenue neutral tax rate randomization for all firms subject to the same tax rates would increase aggregate profits. The explanation is simply that with heterogeneous firms, a tax rate randomization that keeps aggregate tax payments constant generally will not keep constant the expected tax payments of each firm.

¹⁹ Appendix B illustrates Proposition 4 by revisiting the analysis in Figure 1 above.

demands. Since half of input demand arises from firms with zero demand elasticities, it follows that the aggregate demand elasticity is half its value in (16):

$$(23) \quad \varepsilon_A(\tau) = \frac{-\tau}{2(1+\tau)(1-\alpha)}.$$

Furthermore, (19) and (23) together imply that, for the half of the firm population with Cobb-Douglas production functions, $\varepsilon'(\tau) = 2\varepsilon_A(\tau)/[\tau(1+\tau)]$. Consequently, (22) implies:

$$(24) \quad \varepsilon'_A(\tau) = \frac{\varepsilon_A(\tau)}{\tau(1+\tau)} + \frac{\varepsilon_A(\tau)^2}{\tau}.$$

And applying (23), (24) produces:

$$(25) \quad \varepsilon'_A(\tau) = \frac{\tau - 2(1-\alpha)}{4(1-\alpha)^2(1+\tau)^2}.$$

It is clear from (25) that there exist values of τ and α for which $\varepsilon'_A(\tau) > 0$, and for which therefore expected inputs, outputs, and profits all increase with tax uncertainty; and this is so despite the fact that neither type of individual firm exhibits $\varepsilon'(\tau) > 0$.

Table 2 revisits the tax uncertainty scenario examined in Table 1, now adding an equal number of firms with inelastic input demands.²⁰ Given the assumed parameter values, and as above, at a tax rate $\tau = 2$ firms of both types demand 10 units of x , produce output of 150, have profits of 120, and generate tax revenue of 20. A tax rate of 1.6 is associated with greater input use and output by the type 1 firms with Cobb-Douglas production functions, and unchanged input use and output by the type 2 firms; profits rise for both firm types, and tax revenues decline. It is now the case that a high tax rate of only 2.43 is necessary to accompany the low tax rate of 1.6 for the government to collect (in expectation) the same revenue as with a certain tax rate of 2. This high tax rate is of course associated with reduced input demand and output by firms of type 1, but over the two possible tax rates firms of type 1 have expected input demand of

²⁰ These firms have production functions $q = 0, \forall x < 10$ and $q = 150, \forall x \geq 10$.

10.25, exceeding their input demand with a certain tax of 2, and an expected output of 150.3, also exceeding output with tax certainty. Since firms of type 2 do not change their inputs and outputs, aggregate expected input demand and output increase with tax uncertainty. Similarly, tax uncertainty increases firm 1's expected profits from 120 to 120.25, and since firm 2's expected profits decline from 120 to 119.85, aggregate expected profits rise.

4.2 *Endogenous entry and exit*

In practice, tax and subsidy policies can affect numbers of active firms—indeed that is sometimes an objective, especially of subsidy policy. This makes it important also to consider settings with endogenous entry, particularly since firm heterogeneity means that different firms will enter and exit at different tax rates.

Note first that while a small tax change may induce firm entry or exit, these entries and exits will not directly affect aggregate profits, since entering and exiting firms have zero profits. Consequently, even with endogenous entry the effect of a small tax change on aggregate profits is given by $-\sum x_i(\tau)$. Similarly, if entering and exiting firms have zero input demands and zero outputs at their points of entry, then endogenous entry does not alter the effect of a small tax change on levels of aggregate inputs, outputs, and tax revenue. Proposition 5 identifies a sufficient condition for this to be the case.

PROPOSITION 5: *With endogenous entry and exit, if firm output is nonnegative in the absence of inputs, then Propositions 1, 2, 3 and 4 continue to hold.*

Proof: Denoting firm i 's production function by $q_i(x_i)$, Proposition 5 posits that $q_i(0) \geq 0$. If $q_i(0) > 0$ then there is no value of τ at which the firm exits; hence entry and exit occur only for firms with $q_i(0) = 0$, so attention can be restricted to this case. Consequently:

$$(26) \quad q_i(\tilde{x}_i) = \int_0^{\tilde{x}_i} q'_i(z) dz .$$

The firm's first order condition implies that if \tilde{x}_i is a profit maximizing input choice then

$q'_i(\tilde{x}_i) = (1 + \tau)$. Strict concavity of the production function $q''_i(x_i) < 0$ implies that

$q'_i(z) > (1 + \tau) \forall z < \tilde{x}_i$, so applying (26) yields, for any $x_i > 0$:

$$(27) \quad q_i(\tilde{x}_i) > (1 + \tau)\tilde{x}_i.$$

It follows from (27) and the definition of firm profits (1) that firms are profitable at any positive input levels that satisfy their first order conditions for profit maximization. Hence entry and exit, if they occur at all, occur only when a firm's first order condition is satisfied at zero inputs, and then only when output and profits are also zero. Consequently, endogenous entry and exit in these circumstances does not change the effect of small tax changes on aggregate input use, output, tax revenue, or profits. \square

Proposition 5 requires that firm output not be negative at zero input levels, which corresponds to there being zero net fixed costs—or, in settings with additional productive inputs, that the firm be profitable even without purchasing any of input x . If instead some firms have fixed costs that make them unprofitable in the absence of input x then entry occurs once the tax rate becomes low enough that a firm can generate sufficient quasirents from use of x to compensate for the fixed costs it must incur. The resulting discontinuity of input demand and output at the point of firm entry or exit would then invalidate the differential analysis used in Propositions 1, 2, 3, and 4; but in the absence of such fixed costs these propositions remain valid.

5. *Efficient tax uncertainty and the marginal cost of public funds*

Any kind of taxation is apt to affect the efficiency of resource allocation. While the analysis in sections 3 and 4 largely concerns the effects of tax uncertainty on incentives and outcomes for firms, it also carries implications for aggregate economic efficiency. This section turns to implications for deadweight loss and efficient tax design, beginning by characterizing the ranges over which tax randomization is efficient and then considering the effect of efficient randomization on the marginal cost of public funds.

5.1 Deadweight loss

Proposition 1 can be interpreted in terms of the deadweight loss associated with input taxation, denoted $DL(R)$ and defined as the difference between the tax-induced loss of profits and the amount of tax revenue collected:

$$(28) \quad DL(R) \equiv \pi(0) - \pi(R) - R \geq 0.$$

Rearranging gives:

$$(29) \quad \pi(R) = \pi(0) - R - DL(R),$$

from which it follows directly that profit is concave (resp. convex) in tax revenue if and only if deadweight loss is convex (concave) in tax revenue. The effects of an increase in tax revenue on profits and on deadweight loss are thus mirror images: a revenue neutral tax rate randomization reduces expected profit, for instance, if and only if deadweight loss is convex in tax revenue, so that the randomization also increases expected deadweight loss. And convexity of deadweight loss in tax revenue, as stressed at the outset, is not implied by conditions that make deadweight loss convex in the tax rate. Specifically, it follows from (28) and (7) that

$$(30) \quad DL''(R) = -\pi''(R) = \frac{-\varepsilon'(\tau)}{[1 + \varepsilon(\tau)]^2} \tau'(R),$$

so that, consistent with Proposition 1, it is the tax elasticity of derived demand that shapes the convexity/concavity of deadweight loss in revenue and so determines the effect of tax rate randomization. Equation (30) implies that:

PROPOSITION 6: *A revenue neutral local tax rate randomization increases expected deadweight loss if and only if $\varepsilon'(\tau) < 0$. Conversely, a revenue neutral local tax rate randomization reduces expected deadweight loss if and only if $\varepsilon'(\tau) > 0$.*

5.2 Efficient uncertain tax ranges

In order to understand the effect of tax uncertainty on the marginal cost of public funds it is necessary to broaden the analysis to incorporate more than local tax rate randomizations. In circumstances when it is efficient to impose uncertain taxes, the resulting tax rates over which the government randomizes will differ substantially, producing a wide range of rates that the government will never find it efficient to impose with certainty. The top panel of Figure 3 depicts the possible scope for and consequences of tax rate randomization that is efficient in the sense of minimizing expected deadweight loss subject to raising some given amount of expected tax revenue. The solid locus plots the deadweight losses produced by tax rates implied by different tax revenue levels; it is possible to reduce expected deadweight loss by randomizing tax collections between any two points on this solid locus for which a line connecting them lies below it. While there are many such possibilities between the revenue levels R_1 and R_2 at which marginal deadweight loss is equal, intuition suggests (and it will shortly be proved) that the greatest efficiency gains are to be had by randomizing between these two levels.

It is clear that the region $R_1 - R_2$ includes points over which deadweight loss is concave in tax revenue; indeed, the purpose of randomization is to exploit these ranges of tax revenue to minimize the expected cost of tax collections. But it is also apparent from Figure 3 that the region $R_1 - R_2$ contains points over which deadweight loss is convex in tax revenue, including the endpoints R_1 and R_2 themselves. It may seem paradoxical to randomize tax collections between points at which deadweight loss is convex in tax revenue, but this follows simply from the nature of the exercise being performed. An efficient program cannot include positive probability of imposing taxes at points where deadweight loss is concave in tax revenue, since randomizing away from these points reduces expected deadweight loss while maintaining expected revenue; hence deadweight loss must be convex in tax revenue at R_1 and R_2 . Indeed in order for the deadweight loss minimizing choices of R_1 and R_2 fully to exploit the opportunity created by the concavity of deadweight loss over a region of revenue levels between them, that interval must also contain ranges over which deadweight loss is convex in revenue. More precisely, the following formalization of these intuitions shows that the interval $R_1 - R_2$ must contain equally sized (with appropriate weighting) regions over which deadweight loss is concave and convex; consequently, a greater degree of local concavity of the deadweight loss function widens the $R_1 - R_2$ range.

PROPOSITION 7: *For any range $R_1 - R_2$ of tax revenue over which tax rates are efficiently randomized: (i) Marginal deadweight loss is equal at R_1 and R_2 , and so marginal deadweight loss under efficient randomization is perfectly certain; (ii) There are equal-sized (weighted) regions over which deadweight loss is concave and convex in tax revenue; and (iii) Deadweight loss is convex in tax revenue at the revenue levels R_1 and R_2 .*

Proof: If the government efficiently randomizes tax rates over some range $R_1 - R_2$, then there exists an expected revenue level \bar{R} for which R_1 , R_2 and γ minimize expected deadweight loss, given by

$$(31) \quad \gamma DL(R_1) + (1 - \gamma) DL(R_2),$$

where γ is the probability of the tax rate corresponding to revenue level R_1 , subject to the expected revenue requirement

$$(32) \quad \gamma R_1 + (1 - \gamma) R_2 \geq \bar{R},$$

and the restriction that $1 \geq \gamma \geq 0$. This minimization problem's first order conditions over the choices of R_1 , R_2 and γ are:

$$(33) \quad DL'(R_1) = DL'(R_2) = \lambda$$

$$(34) \quad DL(R_1) - DL(R_2) = \lambda (R_1 - R_2),$$

in which λ is the multiplier corresponding to the constraint (32). Since the analysis considers circumstances in which randomization is efficient, the restrictions on γ are assumed not to bind, so that both revenue levels occur with strictly positive probability.

The first part of Proposition 7 follows from (33), which also implies that $DL'(R_2) - DL'(R_1) = 0$. Applying the fundamental theorem of the calculus to this equation,

$$(35) \quad \int_{R_1}^{R_2} DL''(R) dR = 0.$$

Defining A_1 to consist of tax revenue levels in $R_1 - R_2$ over which $DL''(R) > 0$, and A_2 to consist of tax revenue levels over which $DL''(R) < 0$, (35) implies

$$(36) \quad \int_{A_1} DL''(R)dR = -\int_{A_2} DL''(R)dR.$$

Then taking the absolute magnitude of the degree of curvature of deadweight loss with respect to tax revenue, $|DL''(R)|$, to be weights, the second part of the proposition follows.

The third part of Proposition 7 comes from the second-order conditions corresponding to the cost-minimizing choices of R_1 and R_2 . These conditions are

$$(37a) \quad DL''(R_1) > 0$$

$$(37b) \quad DL''(R_2) > 0,$$

from which this part of the proposition follows directly. \square

5.3 *The marginal cost of public funds*

The marginal cost of public funds is the cost that government imposes on society in the course of raising an additional dollar of tax revenue. This includes not only the dollar of resources extracted from the population but also any cost of economic distortions that accompany the use of the tax system. In the setting depicted in Figure 3 the marginal cost of public funds at revenue level R_1 , for example, is $[1 + DL'(R_1)]$, reflecting the additional deadweight loss associated with an incremental dollar of tax revenue. Recall too that efficient randomization implies that marginal deadweight loss is perfectly certain, and equal to its common value at R_1 and R_2 .

A key reason for the centrality of concept of the marginal cost of public funds is in shaping the efficient level of public spending. Efficient government policy equates the marginal value of additional public spending to the marginal cost of public funds, so a higher marginal

cost of public funds usually entails a lower efficient level of public spending. This may not be the case, however, with efficient randomization of tax rates.

To illustrate this, the bottom panel of Figure 3 depicts the marginal cost of public funds at different revenue levels; this is simply one plus the slope of the corresponding solid locus in the top panel of Figure 3. As stressed above, the marginal cost of public funds at revenue level R_1 must equal that at R_2 . Between R_1 and R_2 the marginal cost of public funds first rises, then falls, and then rises again, exceeding its initial level over roughly half of the $R_1 - R_2$ interval, and lying below it over the other half. Since with efficient tax rate randomization the marginal cost of public funds is constant at its R_1 and R_2 levels over the entire $R_1 - R_2$ range, it follows that the effect of tax rate randomization (relative to fully certain levels of tax revenue within this range) is to reduce the marginal cost of public funds at some expected revenue levels and increase it at others. And indeed, equation (36) implies that area A in Figure 3 is equal in size to area B. Proposition 8 brings out a key implication of these observations:

PROPOSITION 8: *Relative to fully certain tax policy, efficient tax rate randomization reduces the marginal cost of public funds at some expected revenue levels and increases it at others.*

Proof: Equations (34) and (33), together with the fundamental theorem of the calculus, collectively imply

$$(38) \quad DL(R_2) - DL(R_1) = \int_{R_1}^{R_2} DL'(R) dR = (R_2 - R_1) DL'(R_1).$$

It follows from (38) that:

$$(39) \quad \int_{R_1}^{R_2} \{ [1 + DL'(R)] - [1 + DL'(R_1)] \} dR = 0.$$

Equation (33) indicates that when tax rates are efficiently randomized the additional deadweight loss associated with raising a marginal unit of public funds is given by $\lambda = DL'(R_1)$, so the second bracketed term in (39), $[1 + DL'(R_1)]$, is the marginal cost of funds with efficient tax rate randomization. The first bracketed term is the marginal cost of public funds at each revenue level without randomization. Since from (37a), $[1 + DL'(R)]$ will exceed $[1 + DL'(R_1)]$ at

revenue levels just exceeding R_1 , and from (37b), $[1 + DL'(R)]$ will be less than $[1 + DL'(R_1)]$ at revenue levels just below R_2 , it follows that $[1 + DL'(R)]$ must differ from $[1 + DL'(R_1)]$ at various points in the $R_1 - R_2$ range. From equation (39) the integral of these differences is zero, implying that since randomization reduces the marginal cost of public funds at some revenue levels it must increase it at others. \square

The intuition for Proposition 8 stems from recognizing that randomization does not reduce total deadweight loss at revenue level R_2 . Consequently, to the extent that randomization reduces marginal deadweight loss at revenue levels above R_1 , it must increase marginal deadweight loss at some revenue levels in the interval $R_1 - R_2$. Indeed, the property that tax rate randomization increases the marginal cost of public funds at some levels of expected tax revenue and reduces it at others does not depend on the randomization being chosen to minimize expected deadweight loss.

The importance of Proposition 8 lies in pointing to the existence of circumstances in which a spending rule equating the marginal value of public expenditure to the marginal cost of public funds will entail reduced public expenditures upon the adoption of efficiency-enhancing tax rate randomization, notwithstanding that this tax rate randomization reduces the deadweight cost of collecting any given level of revenue within the range of the randomization. This spending implication follows from the mixed effect of tax rate randomization on the marginal cost of public funds; Figure 4 offers an illustration. This figure reproduces the bottom panel of Figure 3, superimposing schedules D_1 and D_2 representing two alternative specifications of marginal values of public expenditures (which for illustration are taken to be declining in spending levels and unaffected by tax policy choices). If public expenditures are valued according to D_1 , then equating marginal valuation to the marginal cost of public funds, and balancing the government's budget only in expectation, entails a spending level of S_1 . In this case, adoption of efficient tax rate randomization reduces the marginal cost of public funds and so supports a higher efficient spending level $S_2 > S_1$. If, however, public expenditures are valued according to D_2 , then the adoption of efficient tax rate randomization increases the marginal cost of public funds and so lowers the efficient spending level from S_4 to S_3 . In either case, however, the randomization improves efficiency in taxing and spending.

One further implication is of note. Efficient tax uncertainty consists of tax rates that exhibit discrete jumps between settings or time periods. Gradually rising government revenue needs will not change the rates at which activities are taxed, but will slowly increase the likelihood that the government imposes a higher tax rate, and correspondingly reduce the likelihood that the government imposes a lower tax rate. Consequently, one sign that a government has adopted an efficiency-enhancing uncertain tax policy is that its tax rates do not change gradually in response to changing revenue needs.

6. *Evaluating tax uncertainty*

The proposition that price uncertainty enhances firm profitability is due originally to Oi (1961), which considered output prices and attracted immediate qualifications that the proposition depends on firms' abilities costlessly to adjust production levels (Tisdell, 1963; Oi, 1963) and that uncertainty take the form of mean-preserving price variability (Zucker, 1965). As it happens, an earlier²¹ contribution of Waugh (1944), together with comments (Howell, 1945; Lovasy, 1945) and reply (Waugh, 1945), considered the symmetric problem from the consumer standpoint, analogously concluding that price instability increases consumer welfare. Samuelson (1972) subjected these lines of inquiry to scorching critiques, noting their inattention to budget constraints and the patent invalidity of their apparent joint implication that random price variations unconnected to fundamentals somehow benefit both firms and consumers. As Samuelson (1972) notes, the contemplated random price variations are infeasible,²² in that the gains to consumers come from reducing the expected returns of producers, and vice versa, so the only way for both consumers and producers to gain would be with the injection of external

²¹ And apparently overlooked, at least until the appearance of Waugh (1966), by those analyzing the effect of price uncertainty on firm profits. Samuelson (1972, p. 476) notes that the first version of Samuelson's 1972 critique of Waugh (1944) was accepted for publication by the *Quarterly Journal of Economics* in 1944/45, but that "when the manuscript was lost in the editorial process, the exigencies of war did not seem to warrant preparing a new copy."

²² "That is, unless you have a Santa Claus; but then if you do have a Santa Claus available, who needs the Waugh-Oi theorems?" (Samuelson, 1972, p. 488).

resources. Thus “*Waugh’s result can never be applied so as to permit a society to lift its welfare by its own bootstraps through manufactured instability.*”²³

This brutal conclusion does not apply, however, to tax uncertainty: its properties differ from those of the price uncertainty considered by this earlier literature, as taxation creates inefficiencies whether or not tax rates are uncertain.²⁴ The general lesson of the literature on marginal deadweight loss is that prescriptions are situation-specific, implying that the simple intuition that marginal deadweight loss rises with tax revenue, making total deadweight loss convex in tax revenue, need not describe reality, as there can be ranges, such as those depicted in Figure 3, over which marginal deadweight loss declines.²⁵ There is a common presumption, drawn from implicitly linearizing behavioral response functions, that tax uncertainty will increase deadweight loss due to the convexity of deadweight loss in tax revenue. While this presumption is intuitively appealing for a world of identical economic agents, even then it is valid in an unknown portion of cases; and is considerably less likely to hold in settings characterized by extensive taxpayer heterogeneity.

Tax stability entails choosing just one point on the deadweight loss-revenue locus in Figure 3, with the choice dictated by government revenue needs. Tax uncertainty can afford the government the opportunity to extend its taxation into ranges of tax rates over which marginal deadweight loss declines, making it possible to reduce total deadweight loss. The ability to exploit tax ranges over which there are low values of marginal deadweight loss is part of the basis of Stiglitz’s (1982b) normative argument favoring random taxation.²⁶ Interpretation of the Stiglitz model is complicated by its randomization across individuals with differing social welfare weights and marginal propensities to consume taxed goods, but its case for

²³ Samuelson (1972, p.476).

²⁴ This is the basis of the welfare consequences of random taxation analyzed by Balcer and Sadka (1982, 1986) and those of insurance policy differentiation considered by Arnott and Stiglitz (1988).

²⁵ See, for example, Atkinson and Stern (1974), Ballard (1990), Mayshar (1990), Triest (1990), Fullerton (1991), Kaplow (1996), Snow and Warren (1996), Auerbach and Hines (2002), and Gahvari (2006); Dahlby (2008) offers an interpretive survey of this literature.

²⁶ This is implied by conditions (13), (14) and (14') of Stiglitz (1982b); analogous terms appear in Chang and Wildasin (1986) and Brito et al. (1995), though in all cases without the elasticity formulation of Proposition 1.

randomization relies at least in part on the ability of governments that randomize to raise tax revenue over ranges in which marginal deadweight loss is particularly low.

In the spirit of Stiglitz's argument, the model of sections 2-4 might be applied across individual firms, with some finding themselves facing higher tax rates and others lower tax rates, subject to an aggregate revenue requirement. Firm-specific application would not change the model's implications, since input and output prices are exogenously determined; though the possibility that tax uncertainty increases expected profits would presumably offer cold comfort to those firms randomly subject to high rates of taxation. Governments with fixed budgets might additionally consider whether input subsidies more effectively stimulate aggregate output if spread equally across identical firms or if concentrated among a smaller number; Proposition 3 identifies the relevant condition. Evaluating the welfare impact of tax rate randomization taken to the level of individual firms would require a framework that incorporates the resulting distributional consequences, though it remains possible to evaluate the effect of randomization on aggregate profits and firm operations.

One notable implication of the possibility that tax rate uncertainty can increase expected profits is that firms themselves may seek to create such uncertainty, even at no expected revenue cost to the government. Taxpayers might, for instance, seek Advance Pricing Agreements (governing acceptable transfer pricing practices) as ways of testing the authorities' reaction to aggressive tax planning that holds some prospect of being rebutted by the authorities, and even penalized, but which nonetheless have some chance of success. Or exporters may claim favorable classifications of certain items for tariff purposes, rather than simply accept some moderate rate, in the knowledge that this risks being challenged and triggering penalties large enough to equal the possible reduction in duties. Firms might even lobby for preferential tax treatment to be given to some but not all firms. Taking risky tax positions can be good corporate strategy even if not expected to reduce the amount of tax paid.

It is also worth noting that the paper's analysis of taxation applies with equal force to subsidies. With $\tau < 0$, the elasticity $\varepsilon(\tau)$ as defined in (4) becomes positive, but it is readily verified that all the results above—as summarized for Propositions 1-4 in Figure 2, for

example²⁷—continue to hold as stated.²⁸ For example, a government eager to stimulate business activity with a limited budget will find that a revenue neutral local subsidy rate randomization increases expected input use if and only if the condition identified in Proposition 2 holds.

Economists have long noted the profit opportunities created by price uncertainty even while extolling the benefits of tax stability in limiting deadweight loss and maintaining incentives for forward-looking economic activities. Since in the framework of this paper firms bear the cost of inefficiencies introduced by the tax system,²⁹ the effect of tax uncertainty on profitability is really the effect of tax uncertainty on efficiency. The second-best nature of resource allocation in the presence of taxation creates the possibility that tax uncertainty can reduce deadweight loss and encourage output by imposing taxes in ranges over which aggregate behavior is relatively unresponsive to taxation. The possibility, counter to ‘common sense,’ that tax uncertainty encourages firm operations and enhances profitability is just one aspect of the reality that tax policy adjustments may have very different effects on economies significantly distorted by prior taxation than they do on economies starting from efficient production points in the absence of taxation.

²⁷ Note that $\varepsilon(\tau)/\tau$ remains negative.

²⁸ Restricting τ to negative values may, however, change the likelihood of various possibilities; for instance, $\varepsilon'(\tau)$ becomes unambiguously negative for the production function in (8).

²⁹ If either inputs or outputs were less than perfectly elastically supplied or demanded then their prices would be affected by tax rates, and firms would share some of the tax burden with other economic actors. Even with firms bearing the full burden of the tax plus deadweight loss, imposition of the tax may well affect the real incomes of economic actors outside the model, since in general equilibrium their returns are functions of supply and demand, notwithstanding the indiscernible effect of the tax on input and output prices (Bradford, 1979; Kotlikoff and Summers, 1987).

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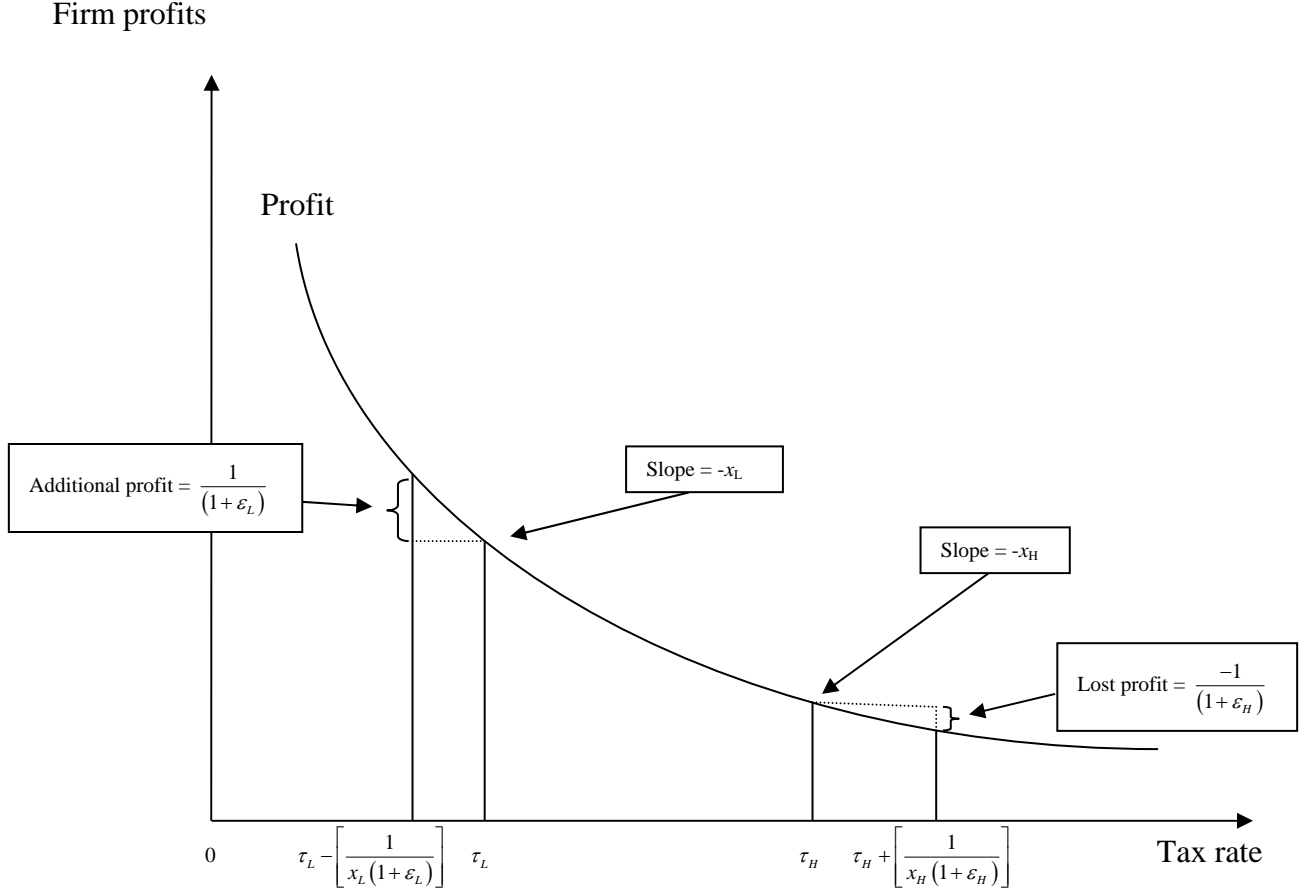
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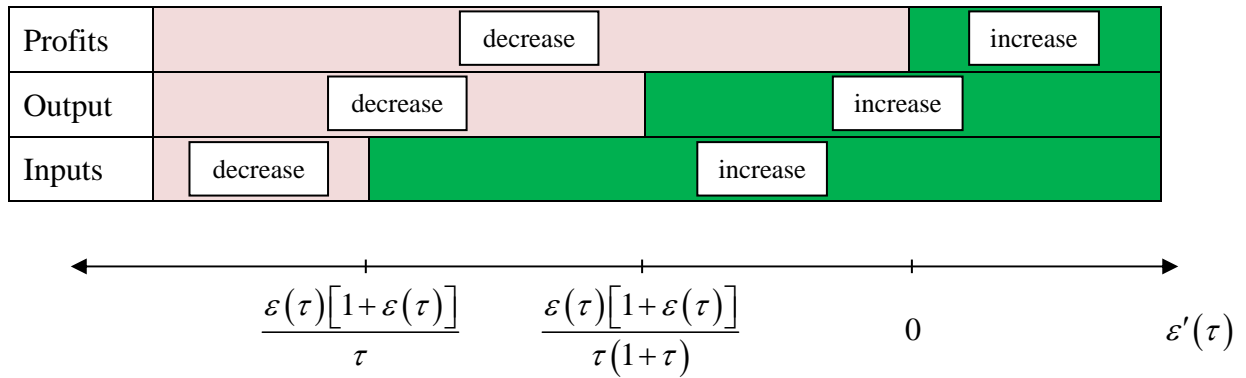
Figure 1

Effect of Tax Uncertainty on Firm Profitability



Note: Figure 1 depicts the effect on firm profits of broadening the scope of tax uncertainty. The solid locus is firm profits as a function of the tax rate. Initially there is tax uncertainty: the tax rate is τ_H with probability 0.5 and τ_L with probability 0.5. The figure considers the effect of increasing τ_H by an amount that raises \$1 of additional tax revenue while lowering τ_L by an amount that reduces tax revenue by \$1. This variation increases τ_H by $\frac{1}{[X_H(1+\varepsilon_H)]}$, which reduces profits by $\frac{1}{(1+\varepsilon_H)}$, and reduces τ_L by $\frac{1}{[X_L(1+\varepsilon_L)]}$, which reduces profits by $\frac{1}{(1+\varepsilon_L)}$. Consequently, the net effect on profits depends on the extent to which $\frac{1}{[1+\varepsilon(\tau)]}$ increases or decreases with τ .

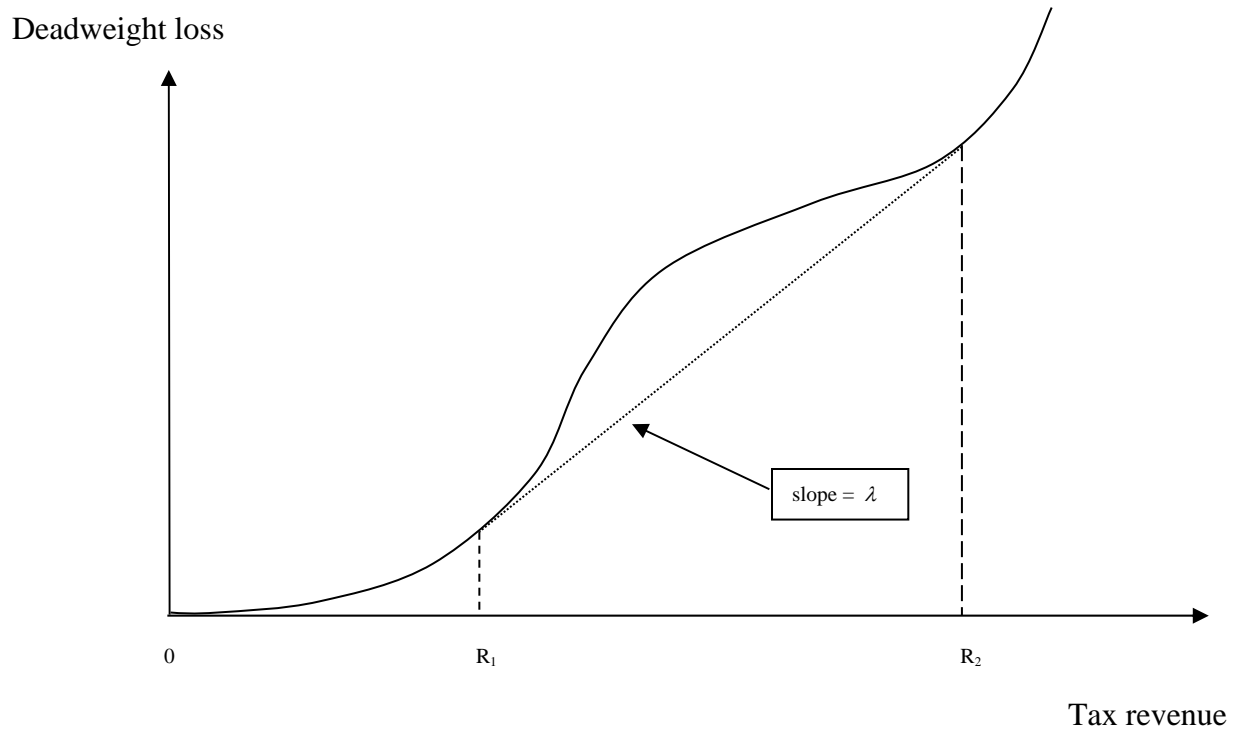
Figure 2
Effects of Tax Uncertainty on Inputs, Output, and Profit



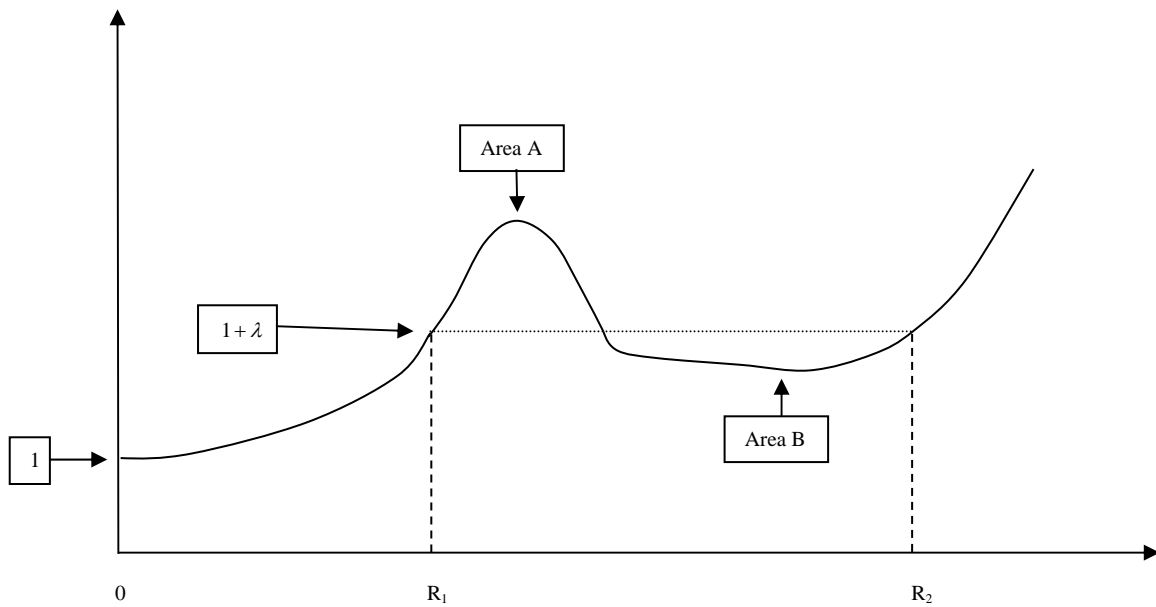
Note: Figure 2 depicts the signs of the effects of tax uncertainty on expected inputs, expected output, and expected profits for differing values of $\varepsilon'(\tau)$. If $\varepsilon'(\tau) > 0$ then tax uncertainty increases expected input use, output, and profits. Tax uncertainty is associated with greater expected output even for some negative values of $\varepsilon'(\tau)$, and is associated with greater expected input use for an even wider range of negative values of $\varepsilon'(\tau)$.

Figure 3

Deadweight Loss and the Marginal Cost of Public Funds



Marginal cost of public funds

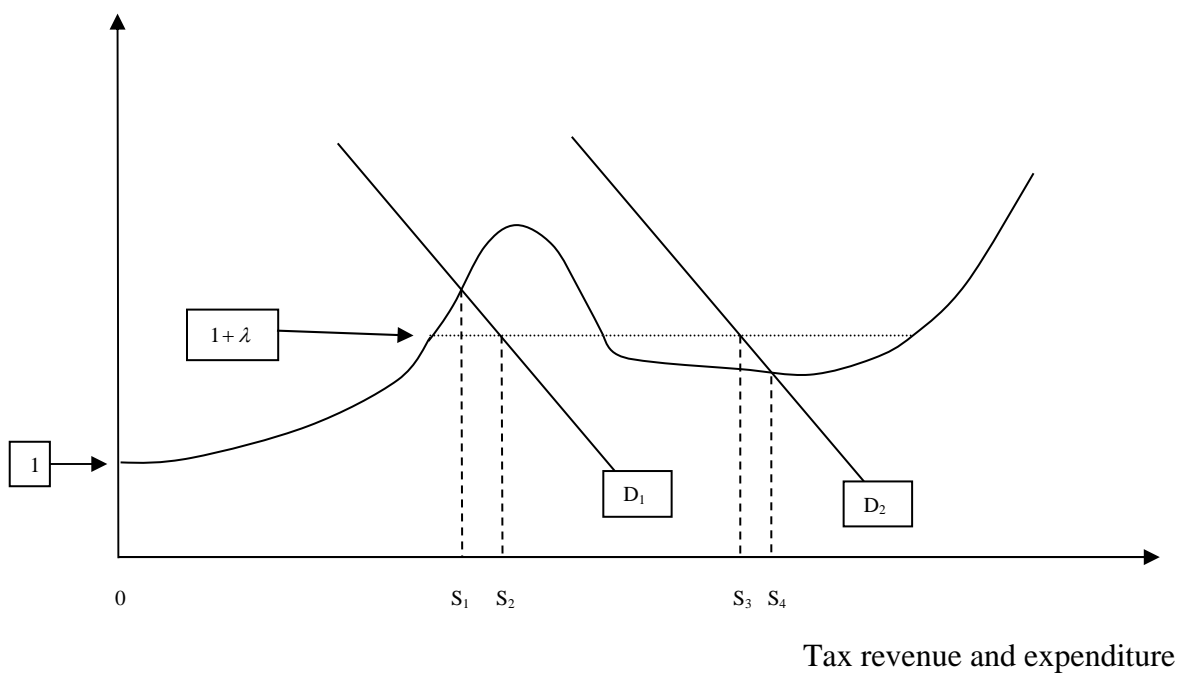


Note to Figure 3: The solid locus in the top panel depicts deadweight loss as a function of tax revenue. Deadweight loss is increasing in tax revenue, with regions of convexity and concavity. A deadweight-loss-minimizing program does not impose taxes that collect revenues between R_1 and R_2 , but instead randomizes tax collections between R_1 and R_2 in order to raise needed revenue in that range. The solid locus in the bottom panel of Figure 3 depicts the marginal cost of public funds, which is one plus marginal deadweight loss (given by the slope of the locus in the top panel). Randomizing tax rates between R_1 and R_2 produces a marginal cost of public funds equal to $1 + \lambda$, where λ is the slope of the dotted line in the top panel. Area A is equal in size to Area B.

Figure 4

Efficient Government Spending with and without Tax Uncertainty

Marginal cost of public funds and
marginal valuation of public services



Note: Figure 4 presents the marginal cost of public funds schedules that appear in the bottom panel of Figure 3, superimposing two functions representing marginal valuations of public expenditure. If the marginal valuation of public expenditure is given by D_1 , then in the absence of tax rate randomization the efficient spending level is S_1 , whereas with efficient tax rate randomization the marginal cost of public funds is $1 + \lambda$, and the implied efficient spending level $S_2 > S_1$. If instead the marginal valuation of public expenditure is given by D_2 , then in the absence of tax rate randomization the efficient spending level is S_4 , whereas with efficient tax rate randomization the implied efficient spending level $S_3 < S_4$.

Table 1 Tax Uncertainty with Cobb-Douglas Production Functions					
Tax Rate		Inputs	Output	Profits	Tax Revenue
2.0		10	150	120	20
1.6		12	155.5	124.4	19.1
2.86		7.3	140.8	112.6	20.9
<p>Note: the table presents inputs, output, profits and tax revenue produced by firms with production functions given by $q = (94.64)x^{0.2}$. These firms seek to maximize profits given by $q - (1 + \tau)x$, in which x is their input demand; and they produce tax revenue of τx.</p>					

<p>Table 2</p> <p>Tax Uncertainty with Heterogeneous Firms</p>						
Tax Rate		Firm	Inputs	Output	Profits	Tax Revenue
2.0		1	10	150	120	20
		2	10	150	120	20
1.6		1	12	155.5	124.4	19.1
		2	10	150	124	16
2.43		1	8.5	145.1	116.1	20.6
		2	10	150	115.7	24.3
<p>Note: the table presents inputs, output, profits and tax revenue generated by two types of firms. Firms of type 1 have production functions given by $q = (94.64)x^{0.2}$; firms of type 2 have production functions given by $q = 0, \forall x < 10$ and $q = 150, \forall x \geq 10$. Both types of firms seek to maximize profits of $q - (1 + \tau)x$, in which x is their input demand; and each type generates tax revenue of τx.</p>						

Appendix A: Multiple Taxed Inputs

The model in Sections 2 and 3 easily generalizes to incorporate scenarios in which firms have multiple taxed inputs. Denoting each of n inputs by x^j , the effect on total tax revenue of a change in the tax rate on input i is $x^i + \sum_{j=1}^n \tau_j \frac{\partial x^j}{\partial \tau_i}$, in which τ_j is the tax rate on input j . As before, the analysis is restricted to ranges over which higher tax rates yield greater tax revenue. From Shephard's lemma, the effect of the tax rate change on firm profits remains $-x^i$. Consequently, if the government is constrained to raise any additional revenue by adjusting only the tax on input i , then:

$$(A1) \quad \pi'(R) \equiv \frac{-x^i}{x^i + \sum_{j=1}^n \tau_j \frac{\partial x^j}{\partial \tau_i}}.$$

It is possible to simplify (A1) by using symmetry of the input demand matrix to replace $\frac{\partial x^j}{\partial \tau_i}$ with $\frac{\partial x^i}{\partial \tau_j}$, and by denoting the cross elasticity of demand for input i with respect to the tax on input j as $\varepsilon_{ij} \equiv \frac{\tau_j}{x^i} \frac{\partial x^i}{\partial \tau_j}$. Doing so produces:

$$(A2) \quad \pi'(R) \equiv \frac{-1}{1 + \sum_{j=1}^n \varepsilon_{ij}}.$$

Then differentiating both sides of (A2) with respect to R yields:

$$(A3) \quad \pi''(R) \equiv \frac{\sum_{j=1}^n \frac{\partial \varepsilon_{ij}}{\partial \tau_i}}{\left[1 + \sum_{j=1}^n \varepsilon_{ij}\right]^2} \tau_i'(R).$$

The expression (A3) clearly incorporates (7) as a special case with a single taxed input, and has a similar interpretation, in that the sign of $\pi''(R)$, and therefore the concavity or convexity of the profit function, depends on the effects of tax changes on the sum of cross-price tax elasticities of demand for good i .³⁰ A similar exercise for input demands and output in settings with multiple taxed inputs produces expressions that are analogous to (12) and (14).

³⁰ For an alternative interpretation, it can be shown that $\sum_j \varepsilon_{ij}$ is the elasticity of demand for x_i with respect to a uniform proportionate increase in all input taxes; denoting this elasticity by $\hat{\varepsilon}_i$, it plays a role analogous to that of $\varepsilon(\tau)$ in the main text, in the sense that the sign of $\pi''(R)$ is the same as that of partial derivative $\partial \hat{\varepsilon}_i / \partial \tau_i$.

Appendix B: Propositions 3 and 4 Illustrated.

This appendix offers diagrammatic expositions of Propositions 3 and 4.

Proposition 3

Figure B1 illustrates the effect of greater tax uncertainty on expected output, as posited in Proposition 3. Consider a revenue neutral tax rate randomization, one that increases τ_H to

$\tau_H + \left[\frac{1}{x_H(1 + \varepsilon_H)} \right]$ and reduces τ_L to $\tau_L - \left[\frac{1}{x_L(1 + \varepsilon_L)} \right]$. The solid locus depicts firm output as a

function of the tax rate. The change in output induced by tax rate change is given by $q'(\tau)x'(\tau)$,

and that is the slope of the locus in Figure B1. The firm's first-order condition is

that $q'(x) = (1 + \tau)$; and from the definition of the tax elasticity of input demand,

$x'(\tau) = \frac{\varepsilon(\tau)x(\tau)}{\tau}$. Hence the slope of the locus in Figure B1 is given by $\frac{\varepsilon(\tau)x(\tau)(1 + \tau)}{\tau}$.

Consequently, the lost output from increasing τ_H is the product of this slope and the tax change,

which is $\frac{\varepsilon_H(1 + \tau_H)}{\tau_H(1 + \varepsilon_H)}$. By a similar calculation, reducing τ_L to $\tau_L - \left[\frac{1}{x_L(1 + \varepsilon_L)} \right]$ increases output

by $\frac{-\varepsilon_L(1 + \tau_L)}{\tau_L(1 + \varepsilon_L)}$. The net output effect of additional tax uncertainty is thus to reduce output if and

only if $\frac{\varepsilon(\tau)(1 + \tau)}{\tau[1 + \varepsilon(\tau)]}$ decreases with the tax rate; which is the condition in Proposition 3.

Proposition 4

Figure B2 illustrates Proposition 4 by considering the profit functions of two firms, 1 and 2. The increment to τ_H necessary to generate an additional dollar of tax revenue is thus given

by $\left[\frac{1}{x_{1H}(1 + \varepsilon_{1H}) + x_{2H}(1 + \varepsilon_{2H})} \right]$, in which x_{1H} and ε_{1H} are respectively firm 1's input demand

and demand elasticity at τ_H , while x_{2H} and ε_{2H} are analogous values for firm 2. The slope of

firm 1's profit function at τ_H is $-x_{1H}$. Consequently the effect of such an increase in τ_H is to

reduce firm 1's profits by $\left[\frac{-x_{1H}}{x_{1H}(1 + \varepsilon_{1H}) + x_{2H}(1 + \varepsilon_{2H})} \right]$; this can be written as

$$\left[\frac{-w_{1H}}{1 + w_{1H}\varepsilon_{1H} + w_{2H}\varepsilon_{2H}} \right], \text{ in which } w_{1H} \text{ is firm 1's share of input demand at } \tau_H. \text{ By similar}$$

reasoning, and with analogous notation, the tax increase reduces firm 2's profits by

$$\left[\frac{-w_{2H}}{1 + w_{1H}\varepsilon_{1H} + w_{2H}\varepsilon_{2H}} \right]. \text{ Since } w_{1H} + w_{2H} = 1, \text{ the effect on total profits is a reduction of}$$

$$\left[\frac{-1}{1 + w_{1H}\varepsilon_{1H} + w_{2H}\varepsilon_{2H}} \right]. \text{ Similar calculations show that lowering } \tau_L \text{ by an amount required to}$$

reduce tax collections by a dollar will increase total profits by $\left[\frac{1}{1 + w_{1L}\varepsilon_{1L} + w_{2L}\varepsilon_{2L}} \right]$. Hence the

net aggregate profit effect of greater tax uncertainty depends on the extent to which

$$\left[\frac{1}{1 + w_1(\tau)\varepsilon_1(\tau) + w_2(\tau)\varepsilon_2(\tau)} \right] \text{ increases or decreases with the tax rate. This is a function not}$$

only of tax rate effects on $\varepsilon_1(\tau)$ and $\varepsilon_2(\tau)$, but also of the extent to which tax rates shift the

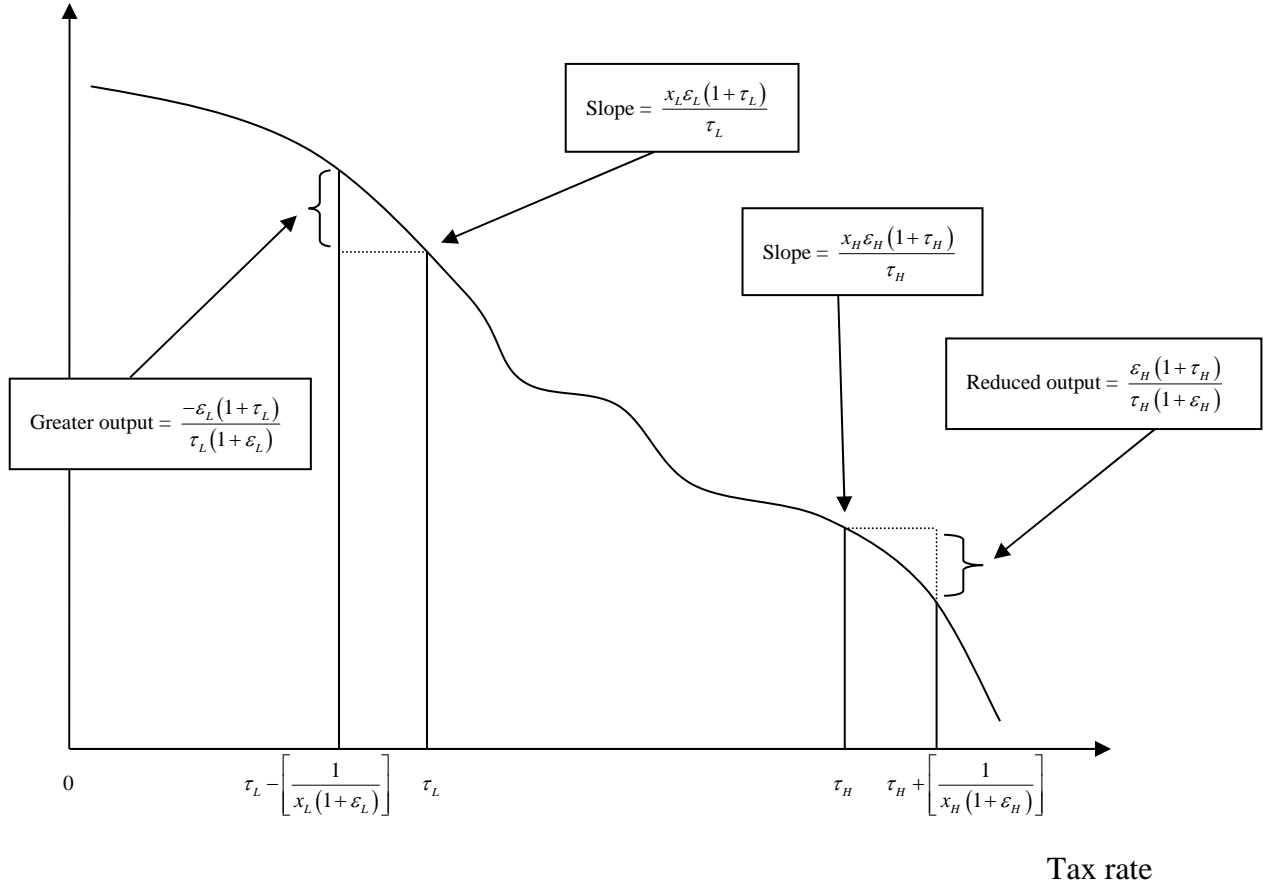
weights attached to the demand elasticities of different firms. Since $w'_1(\tau) > 0$ and $w'_2(\tau) < 0$ if

$\varepsilon_1(\tau) > \varepsilon_2(\tau)$, it follows that the endogeneity of firm weights with respect to the tax rate has the

effect of increasing the likelihood that tax uncertainty contributes to aggregate profitability.

Figure B1
Effect of Tax Uncertainty on Output

Output



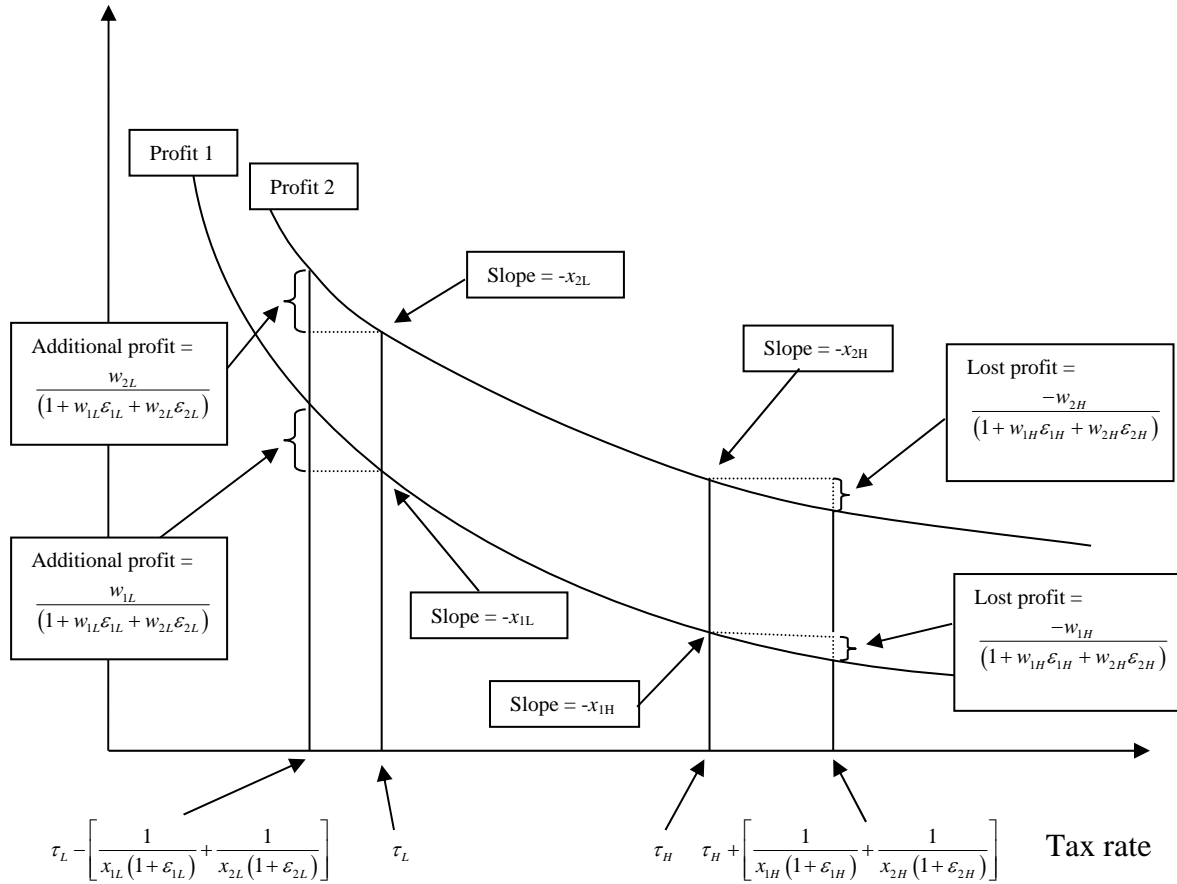
Note: Figure B1 depicts the effect on output of broadening the scope of tax uncertainty. The solid locus is firm output as a function of the tax rate. Initially there is tax uncertainty: the tax rate is τ_H with probability 0.5 and τ_L with probability 0.5. The figure considers the effect of increasing τ_H by an amount that raises \$1 of additional tax revenue while lowering τ_L by an amount that reduces tax revenue by \$1. This variation increases τ_H by $\left[\frac{1}{x_H(1+\varepsilon_H)} \right]$, which reduces output by $\frac{\varepsilon_H(1+\tau_H)}{\tau_H(1+\varepsilon_H)}$, and reduces τ_L by $\left[\frac{1}{x_L(1+\varepsilon_L)} \right]$, which increases output by

$\frac{-\varepsilon_L(1+\tau_L)}{\tau_L(1+\varepsilon_L)}$. Consequently, the net effect on profits depends on the extent to which
 $\frac{\varepsilon(\tau)(1+\tau)}{\tau[1+\varepsilon(\tau)]}$ increases or decreases with τ .

Figure B2

Effect of Tax Uncertainty on Profitability with Heterogeneous Firms

Firm profits



Note: Figure B2 depicts the effect on the profits of two firms, 1 and 2, of broadening the scope of tax uncertainty. The lower solid locus describes the profits of firm 1 as a function of the tax rate; the higher solid locus describes the profits of firm 2. Initially there is tax uncertainty: the tax rate is τ_H with probability 0.5 and τ_L with probability 0.5. The figure considers the effect of increasing τ_H by an amount that raises \$1 of additional tax revenue while lowering τ_L by an amount that reduces tax revenue by \$1. This variation increases τ_H by

$$\left[\frac{1}{x_{1H}(1+\varepsilon_{1H})} + \frac{1}{x_{2H}(1+\varepsilon_{2H})} \right], \text{ which reduces profits of firm 1 by } \frac{-w_{1H}}{(1+w_{1H}\varepsilon_{1H}+w_{2H}\varepsilon_{2H})} \text{ and}$$

reduces profits of firm 2 by $\frac{-w_{2H}}{(1 + w_{1H}\varepsilon_{1H} + w_{2H}\varepsilon_{2H})}$, thereby reducing total profits by $\frac{-1}{(1 + w_{1H}\varepsilon_{1H} + w_{2H}\varepsilon_{2H})}$. The variation also reduces τ_L by $\left[\frac{1}{x_{1L}(1 + \varepsilon_{1L})} + \frac{1}{x_{2L}(1 + \varepsilon_{2L})} \right]$, which increases profits of firm 1 by $\frac{w_{1L}}{(1 + w_{1L}\varepsilon_{1L} + w_{2L}\varepsilon_{2L})}$ and increases profits of firm 2 by $\frac{w_{2L}}{(1 + w_{1L}\varepsilon_{1L} + w_{2L}\varepsilon_{2L})}$, so increases total profits by $\frac{1}{(1 + w_{1L}\varepsilon_{1L} + w_{2L}\varepsilon_{2L})}$. Consequently, the net effect on profits depends on the extent to which $\frac{1}{[1 + w_1(\tau)\varepsilon_1(\tau) + w_2(\tau)\varepsilon_2(\tau)]}$ increases or decreases with τ .