NBER WORKING PAPER SERIES

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Working Paper 25239
http://www.nber.org/papers/w25239

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2018

We are grateful to our discussant, Gadi Barlevy, for his useful comments and suggestions. Lagos thanks support from the C.V. Starr Center for Applied Economics at NYU. Zhang thanks support from the Centre for Macroeconomics at LSE. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

At least one co-author has disclosed a financial relationship of potential relevance for this research. Further information is available online at http://www.nber.org/papers/w25239.ack

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JEL No. D83,E31,E52,G12

ABSTRACT

We develop a model of monetary exchange in bilateral over-the-counter markets to study the effects of monetary policy on asset prices and financial liquidity. The theory predicts asset prices carry a speculative premium that reflects the asset's marketability and depends on monetary policy and the market microstructure where it is traded. These liquidity considerations imply a positive correlation between the real yield on stocks and the nominal yield on Treasury bonds—an empirical observation long regarded anomalous.

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1 Introduction

We develop a monetary model of financial exchange in bilateral over-the-counter (OTC) markets and use it to show how the details of the market microstructure and the quantity of money (*i*) shape the performance of OTC markets (e.g., as gauged by standard measures of market liquidity), (*ii*) generate speculative premia, and (*iii*) explain the positive correlation between the nominal yield on Treasury bonds and the real yield on stocks (the basis for the so-called "Fed Model" of equity valuation popular among financial practitioners).

We consider a setting in which a financial asset that yields a dividend flow of consumption goods (e.g., an equity or a real bond) is traded by investors who have time-varying heterogeneous valuations for the dividend. In order to achieve the gains from trade that arise from their heterogeneous private valuations, investors participate in a bilateral market with random search. In the bilateral market, which has the stylized features of an OTC market structure, investors trade the financial asset using fiat money as a medium of exchange. Periodically, investors are able to rebalance their portfolios in a frictionless (Walrasian) market.

First, we use the theory to study the role that the quantity of money plays in shaping asset prices and the performance of OTC markets more generally. Since money serves as means of payment in financial transactions, the quantity of real balances affects the equilibrium allocation of the asset. Anticipated inflation reduces real balances and distorts the asset allocation by causing too many assets to remain in the hands of investors with relatively low valuations.

Second, we show that in a monetary equilibrium the asset price is larger than the expected present discounted value that any agent assigns to the dividend stream. This difference is a "speculative premium" that investors are willing to pay because they anticipate capital gains from reselling the asset to investors with higher valuations in the future. We show that the speculative premium and the asset price depend on the market structure where the asset is traded, e.g., both the premium and the asset price are decreasing functions of the expected execution delay. This theoretical result is broadly consistent with the behavior of illiquidity premia in response to variations in measures of trading activity documented in the recent empirical literature.

Third, we show how monetary policy affects speculative motives and the resulting speculative premium. An increase in anticipated inflation reduces the real money balances used to finance asset trading, which limits the ability of high-valuation traders to purchase the asset from low-valuation traders. As a result, the speculative premium and the real asset price are decreasing in the expected rate of inflation. This mechanism rationalizes the positive correlation between the real yield on stocks and the nominal yield on Treasury bonds—an empirical observation long regarded anomalous and that, for lack of an alternative theory, has been attributed to money illusion since the 1970s. We also use the model to study the effects of monetary policy on measures of financial liquidity of OTC markets, such as trade volume and price dispersion.

The remaining sections are organized as follows. Section 2 presents the model. Section 3 describes the efficient allocation. Equilibrium is characterized in Section 4. Section 5 analyzes the effects of monetary policy and OTC frictions on asset prices. Section 6 shows how monetary policy and OTC frictions influence measures of market liquidity, such as trade volume and price dispersion. In Section 7 we show asset prices typically exceed the expected present discounted value that any agent assigns to the dividend stream by an amount that reflects a speculative premium. In that section we also show the size of this speculative premium declines with inflation and the degree of OTC frictions. In Section 8 we show our theory offers a novel theoretical foundation for the "Fed Model"—a construct popular among practitioners and policymakers that is based on the documented positive correlation between nominal bond yields and real equity yields. Section 9 concludes. The appendix contains all proofs.

1.1 Related literature

This paper is related to three areas of research: search-theoretic models of money, search-theoretic models of financial trade in OTC markets, and resale-option theories of asset pricing.

From a methodological standpoint, this paper bridges the search-theoretic monetary literature that has largely focused on macro issues, and the search-theoretic financial OTC literature that focuses on micro considerations in the market-microstructure tradition. Specifically, we embed an OTC financial trading arrangement similar to Duffie et al. (2005) into a Lagos and Wright (2005) economy. Here, money serves as a medium of exchange for financial assets, whereas in the standard Lagos-Wright framework it is used as a medium of exchange for consumption goods. This makes the financial OTC model in Duffie et al. (2005) amenable to general equilibrium analysis, and delivers a natural transmission mechanism through which monetary policy influences asset prices and the standard measures of financial liquidity that are the main focus of the micro strand of the OTC literature.

Geromichalos et al. (2007), Jacquet and Tan (2010), Lagos and Rocheteau (2008), Lagos

(2010a, 2010b, 2011), Lester et al. (2012), and Nosal and Rocheteau (2013), introduce a real asset that can (at least to some degree) be used along with money as a medium of exchange for consumption goods in variants of Lagos and Wright (2005). These papers identify the liquidity value of the asset with its usefulness in exchange, and find that when the asset is valuable as a medium of exchange, this manifests itself as a "liquidity premium" that makes the real asset price higher than the expected present discounted value of its financial dividend. High anticipated inflation reduces real money balances; this tightens bilateral trading constraints, which in turn increases the liquidity value and the real price of the asset. In contrast, here we find that real asset prices are decreasing in the anticipated rate of inflation.[1]

This paper shares with three recent papers, Geromichalos and Herrenbrueck (2016), Lagos and Zhang (2018), and Trejos and Wright (2016), the general interest in bringing models of OTC trade in financial markets into the realm of modern monetary general equilibrium theory. Trejos and Wright (2016) offer an in-depth analysis of a model that nests Duffie et al. (2005) and the prototypical "second generation" monetary search model with divisible goods, indivisible money and unit upper bound on individual money holdings (e.g., Shi, 1995 or Trejos and Wright, 1995). Trejos and Wright emphasize the different nature of the gains from trade in both classes of models: In monetary models agents value consumption goods differently and use assets to buy goods, while in Duffie et al. (2005) agents trade because they value assets differently, and goods which are valued the same by all investors are used to pay for asset purchases. In the formulation we study here, there are gains from trading assets, as in Duffie et al. (2005), but agents pay with money, as in standard monetary models. Another difference with Trejos and Wright (2016) is that rather than assuming indivisible assets and unit upper bound on individual asset holdings as in Shi (1995), Trejos and Wright (1995) and Duffie et al. (2005), we work with divisible assets and unrestricted portfolios, as in Lagos and Wright (2005) and Lagos and Rocheteau (2009).

[1]There are some models that also build on Lagos and Wright (2005) where agents can use a real asset as collateral to borrow money that they subsequently use to purchase consumption goods. In those models, anticipated inflation reduces the demand for real balances which can in turn reduce the real price of the collateral asset needed to borrow money (see, e.g., He et al., 2012, and Li and Li, 2012). The difference is that in our setup, inflation reduces the real asset price by constraining the reallocation of the financial asset from investors with low valuations to investors with relatively high valuations. In the model that we study here, money is the only asset used as means of payment. It would be straightforward, however, to enrich the asset structure so that investors may choose to carry other real assets that can be used as means of payment in the OTC market, e.g., along the lines of Lagos and Rocheteau (2008) or Lagos (2010a, 2010b, 2011). As long as money is valued in equilibrium, we anticipate that the main results emphasized here would continue to hold.

Geromichalos and Herrenbrueck (2016) extend Lagos and Wright (2005) by incorporating a real asset that cannot be used to purchase goods in the decentralized market. The twist is that at the beginning of every period, agents learn whether they will want to buy or sell consumption goods in the subsequent decentralized market, and at that point they have access to a bilateral search market where they can retrade money and assets. This market allows agents to rebalance their positions depending on their need for money, e.g., those who will be buyers seek to buy money and sell assets. So although assets cannot be directly used to purchase consumption goods, agents can use assets to buy goods indirectly, i.e., by exchanging assets for cash in the additional bilateral trading round at the beginning of the period. Geromichalos and Herrenbrueck use the model to revisit the link between asset prices and inflation. Their core results are similar to those obtained in models where the asset can be used directly as a medium of exchange for consumption goods, i.e., the asset carries a liquidity premium and higher inflation increases the real asset price in the centralized market. There are relevant differences between our work and Geromichalos and Herrenbrueck (2016). In the setup we present here, money allows agents to exploit gains from trading assets (as in Duffie et al.) rather than consumption goods (as in the money literature), which is why we instead find that inflation reduces asset prices.

In Lagos and Zhang (2018) we develop a related theory for OTC markets intermediated by broker-dealers. That theory allows us to study the effect of monetary policy on spreads, trade volume, and dealers' incentives to supply liquidity services—the dimensions of financial liquidity that search-based theories of OTC markets seek to explain. The theory we lay out here is instead tailored to markets where trade is purely bilateral. This allows us to study the effect of monetary policy on measures of financial liquidity that are characteristic of pure-bilateral OTC markets, such as price dispersion.

The fact that the equilibrium asset price is larger than the expected present discounted value that any agent assigns to the dividend stream is reminiscent of the literature on speculative trading that can be traced back to Harrison and Kreps (1978). As in Harrison and Kreps and more recent work, e.g., Scheinkman and Xiong (2003a, 2003b) and Scheinkman (2013), speculation in our model arises because traders have heterogeneous asset valuations that change over time. Our model offers a new angle on the speculative premium embedded in the asset price, by showing how it depends on the underlying financial market structure and the prevailing monetary policy that jointly determine the likelihood and profitability of future resale opportunities.

Through this channel our theory can generate a positive correlation between trade volume and the size of speculative premia, a stylized fact emphasized by Scheinkman and Xiong (2003b).

2 The model

Time is represented by a sequence of periods indexed by $t = 0, 1, ...$ Each time-period is divided into two subperiods where different activities take place. There is a continuum of infinitely lived investors, each identified with a point in the set $\mathcal{I} = [0, 1]$. They discount payoffs across periods with factor, $\beta \in (0, 1)$. In every period there is a continuum of productive units (or *trees*) with measure $A^s \in \mathbb{R}_{++}$. Every productive tree yields an exogenous *dividend* $y_t \in \mathbb{R}_+$ of a perishable consumption good at the end of the first subperiod of period t. (Each tree yields the same dividend as every other tree, so y_t is also the aggregate dividend.) At the beginning of every period t, every tree is subject to an independent idiosyncratic shock that renders it permanently unproductive with probability $1 - \pi \in [0, 1)$ (unproductive trees physically disappear). If a tree remains productive, its dividend in period $t + 1$ is $y_{t+1} = \gamma_{t+1} y_t$ where γ_{t+1} is a nonnegative random variable with cumulative distribution function Γ, i.e., $\Pr(\gamma_{t+1} \leq \gamma) = \Gamma(\gamma)$, and mean $\bar{\gamma} \in (0, (\beta\pi)^{-1})$. The time-$t$ dividend becomes known to all investors at the beginning of period t, and at that time each tree that failed is replaced by a new tree that yields dividend y_t in the initial period and follows the same stochastic process as other productive trees thereafter (the dividend of the initial set of trees, $y_0 \in \mathbb{R}_{++}$, is given at $t = 0$). In the second subperiod of every period, every investor has access to a linear production technology that transforms a unit of the investor's effort into a unit of a perishable homogeneous consumption good.

Each productive tree has outstanding one durable and perfectly divisible equity share that represents the bearer's ownership of the tree and confers him the right to collect the dividends. At the beginning of every period $t \geq 1$, each investor receives an endowment of $(1 - \pi) A^s$ equity shares corresponding to the new trees created in that period. When a tree fails, its equity share disappears with the tree. There is a second financial instrument, money, which is intrinsically useless (it is not an argument of any utility or production function, and unlike equity, ownership of money does not constitute a right to collect any resources). The stock of money at time t is denoted A_t^m. The initial stock of money, $A_0^m \in \mathbb{R}_{++}$, is given, and $A_{t+1}^m = \mu A_t^m$, with $\mu \in \mathbb{R}_{++}$. A monetary authority injects or withdraws money via lump-sum transfers or taxes in the second subperiod of every period. At the beginning of period $t = 0$, each investor is endowed with a portfolio of equity shares and money. All financial instruments are perfectly recognizable, cannot be forged, and can be traded among investors in every subperiod.

In the second subperiod of every period, all investors can trade the consumption good produced in that subperiod, equity shares, and money, in a spot Walrasian market. In the first subperiod of every period, trading is organized as follows: Investors can trade equity shares and money in a random bilateral *OTC market*. We use $\alpha \in [0,1]$ to denote the probability that an individual investor is able to contact another investor in the OTC market. Once the two investors have contacted each other, the pair negotiates a trade involving equity shares and money. We assume that, with probability $\eta \in [0,1]$, the terms of the trade are chosen by the investor who values the equity dividend the most, and by the other investor with complementary probability.[2] After the transaction has been completed, the investors part ways. The timing assumption is that the round of OTC trade between investors takes place in the first subperiod of a typical period t, and ends before trees yield dividends. Hence equity is traded *cum dividend* in the OTC market of the first subperiod, but *ex dividend* in the Walrasian market of the second subperiod. We assume that investors cannot make binding commitments, that there is no enforcement, and that histories of actions are private in a way that precludes any borrowing and lending, so any trade must be *quid pro quo.* This assumption and the structure of preferences described below create the need for a medium of exchange.[3]

An individual investor's preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\varepsilon_{ti} y_{ti} + c_{ti} - h_{ti} \right)$$

where y_{ti} is the quantity of the dividend good that investor i consumes at the end of the first subperiod of period t, c_{ti} is his consumption of the homogeneous good that is produced, traded and consumed in the second subperiod of period t, and h_{ti} is the utility cost from exerting h_{ti} units of effort to produce this good. The variable ε_{ti} denotes the realization of a preference shock that is distributed independently over time and across investors, with a differentiable cumulative distribution function G on the support $[\varepsilon_L, \varepsilon_H] \subseteq [0, \infty]$, and $\bar{\varepsilon} = \int \varepsilon dG(\varepsilon)$. Investor i learns his realization ε_{ti} at the beginning of period t, before the OTC trading round. The expectation

[2]In the event that both investors value the dividend the same, each gets selected to make a take-it-or-leave it offer with equal probability.

[3]Notice that under these conditions there cannot exist a futures market for fruit, so an investor who wishes to consume the fruit dividend must be holding the equity share at the time the dividend is paid. A similar assumption is typically made in search models of financial OTC trade, e.g., see Duffie et al. (2005) and Lagos and Rocheteau (2009).

operator $\mathbb{E}_0$ is with respect to the probability measure induced by the dividend process, the investor's preference shock and the random trading process in the OTC market.

3 Efficiency

Consider a social planner who wishes to maximize the sum of investors' expected discounted utilities, subject to the same meeting frictions that investors face in the decentralized formulation. Specifically, in the first subperiod of every period, the planner can only reallocate assets within the pairs of the measure α of investors who have contacted each other directly. Let $\mathcal{B}_t \subseteq \mathcal{I}$ denote the subset of investors who get a bilateral trading opportunity with another investor in the OTC market of period t. For any $i \in \mathcal{B}_t$, let $b(i) \in \mathcal{B}_t$ denote investor i's partner in the bilateral meeting. Notice that $\int_{\mathcal{B}_t} di = \alpha$ is the measure of investors who have an OTC meeting with another investor, and $\int_{\mathcal{B}_t} \mathbb{I}_{\{i \leq b(i)\}} di = \alpha/2$ is the total number of direct bilateral transactions between investors in the OTC market. We restrict attention to symmetric allocations (identical agents receive equal treatment). Let $c_t(\varepsilon)$ and $h_t(\varepsilon)$ denote consumption and production of the homogeneous consumption good in the second subperiod of period t by an investor with idiosyncratic preference type ε. Let $\tilde{a}_t$ denote the beginning-of-period-t (before depreciation and endowment) asset holding of an investor. Finally, let $\underline{a}_{tij}(\varepsilon_i, \varepsilon_j)$ denote the post-trade equity holding of an investor i with preference type ε_i who has a direct bilateral trade opportunity with an investor j with preference type ε_j. The planner's problem consists of choosing a nonnegative allocation,

$$\left\{\tilde{a}_t, \left[\left(\underline{a}_{tib(i)}(\varepsilon_i, \varepsilon_{b(i)})\right)_{i \in \mathcal{B}_t}, c_t(\varepsilon_i), h_t(\varepsilon_i)\right]_{\varepsilon_i, \varepsilon_{b(i)} \in [\varepsilon_L, \varepsilon_H]}\right\}_{t=0}^{\infty}, \tag{1}$$

to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg[\int_{\varepsilon_L}^{\varepsilon_H} [(1-\alpha)\varepsilon y_t a_t + c_t(\varepsilon) - h_t(\varepsilon)] \, dG(\varepsilon)$$
$$+ \int_{\mathcal{B}_t} \int \int \mathbb{I}_{\{i \leq b(i)\}} \left[\varepsilon_i \underline{a}_{tib(i)}(\varepsilon_i, \varepsilon_{b(i)}) + \varepsilon_{b(i)} \underline{a}_{tb(i)i}(\varepsilon_{b(i)}, \varepsilon_i)\right] y_t dG(\varepsilon_i) \, dG(\varepsilon_{b(i)}) di \Bigg]$$

(the expectation operator $\mathbb{E}_0$ is with respect to the probability measure induced by the dividend process) subject to

$$\underline{a}_{tib(i)}\left(\varepsilon_i, \varepsilon_{b(i)}\right) + \underline{a}_{tb(i)i}\left(\varepsilon_{b(i)}, \varepsilon_i\right) \leq 2a_t \tag{2}$$

$$\tilde{a}_t \leq A^s \tag{3}$$

$$\int_{\varepsilon_L}^{\varepsilon_H} c_t(\varepsilon)\, dG(\varepsilon) \leq \int_{\varepsilon_L}^{\varepsilon_H} h_t(\varepsilon)\, dG(\varepsilon) \tag{4}$$

$$a_t = \pi \tilde{a}_t + (1-\pi) A^s. \tag{5}$$

Proposition 1 *The efficient allocation has* $\tilde{a}_t = A^s$ *and* $\underline{a}_{tib(i)}\left(\varepsilon_i, \varepsilon_{b(i)}\right) = \mathbb{I}_{\{\varepsilon_{b(i)}<\varepsilon_i\}} 2A^s + \mathbb{I}_{\{\varepsilon_{b(i)}=\varepsilon_i\}} a^o$ *for all* $i \in \mathcal{B}_t$*, where* $a^o \in [0, 2A^s]$*.*

According to Proposition 1, the efficient allocation is achieved if in every bilateral trade, all the equity shares are allocated to the investor with the highest valuation.

4 Equilibrium

Let $V_t(\boldsymbol{a}_{ti}, \varepsilon)$ denote the maximum expected discounted payoff of an investor who has preference type ε and is holding portfolio $\boldsymbol{a}_{ti} \equiv (a_{ti}^m, a_{ti}^s)$ at the beginning of the OTC round of period t. Let $W_t(\boldsymbol{a}_t)$ denote the maximum expected discounted payoff of an investor who is holding portfolio $\boldsymbol{a}_t$ at the beginning of the second subperiod of period t (after the trees have borne dividends). Then,

$$W_t(\boldsymbol{a}_t) = \max_{c_t, h_t, \tilde{\boldsymbol{a}}_{t+1}} \left[c_t - h_t + \beta \mathbb{E}_t \int V_{t+1}\left(\boldsymbol{a}_{t+1}, \varepsilon'\right) dG(\varepsilon') \right] \tag{6}$$

$$\text{s.t. } c_t + \boldsymbol{\phi}_t \tilde{\boldsymbol{a}}_{t+1} \leq h_t + \boldsymbol{\phi}_t \boldsymbol{a}_t + T_t$$

$$c_t, h_t \in \mathbb{R}_+, \; \tilde{\boldsymbol{a}}_{t+1} \in \mathbb{R}_+^2$$

$$\boldsymbol{a}_{t+1} = (\tilde{a}_{t+1}^m, \pi \tilde{a}_{t+1}^s + (1-\pi) A^s),$$

where T_t is the real value of the time-t lump-sum monetary transfer (tax, if negative). Since ε is i.i.d. over time, $W_t(\boldsymbol{a}_t)$ is independent of ε and the portfolio that each investor chooses to carry into period $t+1$ is independent of ε. Consequently, in what follows we can write $dH_t(\boldsymbol{a}_{ti}, \varepsilon) = dF_t(\boldsymbol{a}_{ti})\, dG(\varepsilon)$, where F_t is the joint cumulative distribution function of investors' money and equity holdings at the beginning of the OTC round of period-t.

Consider a bilateral meeting in the OTC trading round of period t, between investor i with portfolio $\boldsymbol{a}_{ti}$ and preference type ε_i, and investor j with portfolio $\boldsymbol{a}_{tj}$ and preference type ε_j. Let

$\tilde{\eta}(\varepsilon_i, \varepsilon_j) \equiv \eta \mathbb{I}_{\{\varepsilon_j < \varepsilon_i\}} + (1-\eta)\mathbb{I}_{\{\varepsilon_i < \varepsilon_j\}} + (1/2)\mathbb{I}_{\{\varepsilon_i = \varepsilon_j\}}$ denote the probability that the investor with preference type ε_i has the power to make a take-it-or-leave-it offer in a bilateral trade with an investor with preference type ε_j. When investor i makes the take-it-or-leave-it offer to investor j, the resulting post-trade portfolios of investors i and j are denoted

$$[\underline{a}_{i^*}^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t), \underline{a}_{i^*}^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t)]$$
$$[\underline{a}_{j}^m(\boldsymbol{a}_{tj}, \boldsymbol{a}_{ti}, \varepsilon_j, \varepsilon_i; \boldsymbol{\psi}_t), \underline{a}_{j}^s(\boldsymbol{a}_{tj}, \boldsymbol{a}_{ti}, \varepsilon_j, \varepsilon_i; \boldsymbol{\psi}_t)],$$

respectively. With probability $1-\tilde{\eta}(\varepsilon_i, \varepsilon_j)$ the terms of trade are determined by a take-it-or-leave-it offer by investor j, and the resulting post-trade portfolios of investors i and j are

$$[\underline{a}_{i}^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t), \underline{a}_{i}^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t)]$$
$$[\underline{a}_{j^*}^m(\boldsymbol{a}_{tj}, \boldsymbol{a}_{ti}, \varepsilon_j, \varepsilon_i; \boldsymbol{\psi}_t), \underline{a}_{j^*}^s(\boldsymbol{a}_{tj}, \boldsymbol{a}_{ti}, \varepsilon_j, \varepsilon_i; \boldsymbol{\psi}_t)],$$

respectively.[4] We can now write the value function of an investor who enters the OTC round of period t with portfolio $\boldsymbol{a}_{it}$ and preference type ε_i,

$$\begin{aligned} V_t(\boldsymbol{a}_{ti}, \varepsilon_i) = \alpha \int & \tilde{\eta}(\varepsilon_i, \varepsilon_j) \{\varepsilon_i y_t \underline{a}_{i^*}^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t) + \\ & W_t[\underline{a}_{i^*}^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t), \underline{a}_{i^*}^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t)]\} dH_t(\boldsymbol{a}_{tj}, \varepsilon_j) \\ & + \alpha \int [1-\tilde{\eta}(\varepsilon_i, \varepsilon_j)] \{\varepsilon_i y_t \underline{a}_{i}^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t) + \\ & W_t[\underline{a}_{i}^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t), \underline{a}_{i}^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t)]\} dH_t(\boldsymbol{a}_{tj}, \varepsilon_j) \\ & + (1-\alpha)[\varepsilon_i y_t a_{ti}^s + W_t(\boldsymbol{a}_{ti})]. \end{aligned} \tag{7}$$

Consider a bilateral meeting in the OTC trading round of period t, between an investor i with portfolio a_{ti} and preference type ε_i, and an investor j with portfolio a_{tj} and preference type ε_j. With probability $\tilde{\eta}(\varepsilon_i, \varepsilon_j)$, investor i has the power to make a take-it-or-leave-it offer to investor j, and in that event investor i chooses an offer of post-trade portfolios for himself, $(\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s)$, and for investor j, $(\underline{a}_{tj}^m, \underline{a}_{tj}^s)$, by solving

$$\max_{\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s, \underline{a}_{tj}^m, \underline{a}_{tj}^s} [\varepsilon_i y_t \underline{a}_{ti^*}^s + W_t(\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s)]$$

[4] In what follows, we sometimes denote $\underline{a}_{i^*}^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t)$ with $\underline{a}_{ti^*}^m$, and $\underline{a}_{j}^s(\boldsymbol{a}_{tj}, \boldsymbol{a}_{ti}, \varepsilon_j, \varepsilon_i; \boldsymbol{\psi}_t)$ with $\underline{a}_{tj}^s$.

$$
\begin{aligned}
\text{s.t. } \underline{a}_{ti^*}^m + \underline{a}_{tj}^m &\le a_{ti}^m + a_{tj}^m \\
\underline{a}_{ti^*}^s + \underline{a}_{tj}^s &\le a_{ti}^s + a_{tj}^s \\
\varepsilon_j y_t \underline{a}_{tj}^s + W_t(\underline{a}_{tj}^m, \underline{a}_{tj}^s) &\ge \varepsilon_j y_t a_{tj}^s + W_t(a_{tj}^m, a_{tj}^s) \\
\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s, \underline{a}_{tj}^m, \underline{a}_{tj}^s &\in \mathbb{R}_+.
\end{aligned}
$$

The first two constraints imply that in a bilateral meeting the two investors can (only) reallocate money and assets between themselves. The third constraint ensures it is individually rational for investor j to accept i's offer. The following result characterizes the bargaining outcome.

Lemma 1 *Consider the bargaining problem between investor i with portfolio (a_{ti}^m, a_{ti}^s) and preference type ε_i, and investor j with portfolio (a_{tj}^m, a_{tj}^s) and preference type ε_j in the OTC market of period t. Suppose that investor i has the power to choose the terms of trade, then his post-trade portfolio is $\underline{\boldsymbol{a}}_{ti^*} = (\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s)$ with*

$$
\underline{a}_{ti^*}^s \begin{cases} = a_{ti}^s + \min\left[\frac{a_{ti}^m}{p_t(\varepsilon_j)}, a_{tj}^s\right] & \text{if } \varepsilon_j < \varepsilon_i \\ \in \left[a_{ti}^s - \min\left[\frac{a_{tj}^m}{p_t(\varepsilon_j)}, a_{ti}^s\right], a_{ti}^s + \min\left[\frac{a_{ti}^m}{p_t(\varepsilon_j)}, a_{tj}^s\right]\right] & \text{if } \varepsilon_j = \varepsilon_i \\ = a_{ti}^s - \min\left[\frac{a_{tj}^m}{p_t(\varepsilon_j)}, a_{ti}^s\right] & \text{if } \varepsilon_i < \varepsilon_j \end{cases}
$$

$$
\underline{a}_{ti^*}^m = \begin{cases} a_{ti}^m - \min\left[p_t(\varepsilon_j) a_{tj}^s, a_{ti}^m\right] & \text{if } \varepsilon_j < \varepsilon_i \\ a_{ti}^m + p_t(\varepsilon_j)\left(a_{ti}^s - \underline{a}_{ti^*}^s\right) & \text{if } \varepsilon_j = \varepsilon_i \\ a_{ti}^m + \min\left[p_t(\varepsilon_j) a_{ti}^s, a_{tj}^m\right] & \text{if } \varepsilon_i < \varepsilon_j, \end{cases}
$$

where

$$
p_t(\varepsilon) \equiv \frac{\varepsilon y_t + \phi_t^s}{\phi_t^m},
$$

and investor j's post-trade portfolio is $\underline{\boldsymbol{a}}_{tj} = (\underline{a}_{tj}^m, \underline{a}_{tj}^s)$, with $\underline{a}_{tj}^s = a_{tj}^s + a_{ti}^s - \underline{a}_{ti^*}^s$ and $\underline{a}_{tj}^m = a_{tj}^m + a_{ti}^m - \underline{a}_{ti^*}^m$.

In Lemma 1, if $\varepsilon_j < \varepsilon_i$, then investor i wishes to purchase all of investor j's equity. Since he has all the bargaining power, investor i sets the terms of trade at $p_t(\varepsilon_j)$ dollars per equity share, i.e., the dollar price of equity that makes investor j just indifferent between selling equity for dollars or not. The quantity of equity that investor i is able to purchase will depend on his money holdings, a_{ti}^m. If $a_{ti}^m \ge p_t(\varepsilon_j) a_{tj}^s$, then he buys all of investor j's equity holdings, a_{tj}^s, in exchange for $p_t(\varepsilon_j) a_{tj}^s$ dollars. If $a_{ti}^m < p_t(\varepsilon_j) a_{tj}^s$, then i gives investor j all his money holdings, a_{ti}^m, in exchange for $\frac{a_{ti}^m}{p_t(\varepsilon_j)}$ equity shares. Conversely, if $\varepsilon_i < \varepsilon_j$, then investor i wishes to sell all of his equity to investor j. Similarly, the quantity of equity that investor i will sell to investor j depends on j's money holdings, a_{tj}^{m}. If $a_{tj}^{m} \geq p_t(\varepsilon_j)a_{ti}^{s}$, then j buys all of investor i's equity holdings, a_{ti}^{s}, in exchange for $p_t(\varepsilon_j)a_{ti}^{s}$ dollars. If $a_{tj}^{m} < p_t(\varepsilon_j)a_{ti}^{s}$, then j gives investor i all his money holdings, a_{tj}^{m}, in exchange for $\frac{a_{tj}^{m}}{p_t(\varepsilon_j)}$ equity shares. The bargaining outcomes can be substituted in the value function (7) to obtain the following result.

Lemma 2 *The value function of an investor who enters the OTC round of period t with portfolio $\boldsymbol{a}_{ti} = (a_{ti}^{m}, a_{ti}^{s})$ and preference type ε_i is given by*

$$\begin{aligned} V_t\left(a_{ti}^{m}, a_{ti}^{s}, \varepsilon_i\right) &= \phi_t^{m} a_{ti}^{m} + \left(\varepsilon_i y_t + \phi_t^{s}\right) a_{ti}^{s} + W_t\left(\mathbf{0}\right) \\ &+ \alpha \int \mathbb{I}_{\{\varepsilon_j \leq \varepsilon_i\}} \eta \frac{\left(\varepsilon_i - \varepsilon_j\right) y_t}{\varepsilon_j y_t + \phi_t^{s}} \min\left[\phi_t^{m} a_{ti}^{m}, \left(\varepsilon_j y_t + \phi_t^{s}\right) A^{s}\right] dG\left(\varepsilon_j\right) \\ &+ \alpha \int \mathbb{I}_{\{\varepsilon_i < \varepsilon_j\}} \left(1 - \eta\right) \frac{\left(\varepsilon_j - \varepsilon_i\right) y_t}{\varepsilon_j y_t + \phi_t^{s}} \min\left[\phi_t^{m} A_t^{m}, \left(\varepsilon_j y_t + \phi_t^{s}\right) a_{ti}^{s}\right] dG\left(\varepsilon_j\right) \qquad (8) \end{aligned}$$

where $\mathbb{I}_{\{\varepsilon_j \leq \varepsilon_i\}}$ is an indicator function that takes the value 1 if $\varepsilon_j \leq \varepsilon_i$, and 0 otherwise.

To interpret (8), notice that the first line represents the value to the investor of holding the portfolio of money and equity until the end of the period. The remaining two terms represent the expected net gains from trading with another investor in the OTC market. Consider the penultimate term: with probability α the investor contacts another investor in the OTC market, if the other investor's preference type, ε_j, is smaller than ε_i, then the investor with preference type ε_i has the bargaining power with probability η and he spends $\min\left[a_{ti}^{m}, p_t\left(\varepsilon_j\right) A^{s}\right]$ dollars purchasing $\min\left[a_{ti}^{m}/p_t\left(\varepsilon_j\right), A^{s}\right]$ equity shares from the other investor, for a net gain from trade equal to

$$\left(\varepsilon_i y_t + \phi_t^{s}\right) \min\left[a_{ti}^{m}/p_t\left(\varepsilon_j\right), A^{s}\right] - \phi_t^{m} \min\left[a_{ti}^{m}, p_t\left(\varepsilon_j\right) A^{s}\right] = \frac{\left(\varepsilon_i - \varepsilon_j\right) y_t}{\varepsilon_j y_t + \phi_t^{s}} \min\left[\phi_t^{m} a_{ti}^{m}, (\varepsilon_j y_t + \phi_t^{s}) A^{s}\right].$$

Similarly, with probability α the investor contacts another investor in the OTC market, and if the other investor's preference type, ε_j, is larger than ε_i, then the investor with preference type ε_i has the bargaining power with probability $1 - \eta$ and he sells $\min\left[A_t^{m}/p_t\left(\varepsilon_j\right), a_{ti}^{s}\right]$ equity shares in exchange for $\min\left[A_t^{m}, p_t\left(\varepsilon_j\right) a_{ti}^{s}\right]$ dollars, for a net gain from trade equal to

$$\phi_t^{m} \min\left[A_t^{m}, p_t\left(\varepsilon_j\right) a_{ti}^{s}\right] - \left(\varepsilon_i y_t + \phi_t^{s}\right) \min\left[A_t^{m}/p_t\left(\varepsilon_j\right), a_{ti}^{s}\right] = \frac{\left(\varepsilon_j - \varepsilon_i\right) y_t}{\varepsilon_j y_t + \phi_t^{s}} \min\left[\phi_t^{m} A_t^{m}, \left(\varepsilon_j y_t + \phi_t^{s}\right) a_{ti}^{s}\right].$$

The following result uses Lemma 2 to characterize the solutions to the portfolio problems that a typical investor solves in the second subperiod of period t.

Lemma 3 *Let* $\left(\tilde{a}^m_{t+1i}, \tilde{a}^s_{t+1i}\right)$ *denote the portfolios chosen by an investor in the second subperiod of period* t*. The first-order necessary and sufficient conditions for optimization that these portfolios must satisfy are*

$$\phi^m_t \geq \beta \mathbb{E}_t \left[1 + \alpha\eta \int_{\left[\frac{\phi^m_{t+1} a^m_{t+1i}}{A^s} - \phi^s_{t+1}\right]\frac{1}{y_{t+1}}}^{\varepsilon_H} \int_{\varepsilon_j}^{\varepsilon_H} \frac{(\varepsilon_i - \varepsilon_j)\, y_{t+1}}{\varepsilon_j y_{t+1} + \phi^s_{t+1}} dG(\varepsilon_i)\, dG(\varepsilon_j) \right] \phi^m_{t+1} \tag{9}$$

$$\phi^s_t \geq \beta\pi \mathbb{E}_t \left[\phi^s_{t+1} + \left(\bar{\varepsilon} + \alpha(1-\eta) \int_{\varepsilon_L}^{\left[\frac{\phi^m_{t+1} A^m_{t+1}}{a^s_{t+1i}} - \phi^s_{t+1}\right]\frac{1}{y_{t+1}}} \int_{\varepsilon_L}^{\varepsilon_j} (\varepsilon_j - \varepsilon_i)\, dG(\varepsilon_i)\, dG(\varepsilon_j) \right) y_{t+1} \right] \tag{10}$$

where (9) holds with "=" if $\tilde{a}^m_{t+1i} > 0$*, and (10) holds with "=" if* $\tilde{a}^s_{t+1i} > 0$.

Condition (9) is the investor's Euler equation for money. The left side is the real cost of purchasing a dollar in the second subperiod of period t. The right side is the discounted expected benefit from carrying this additional dollar into the following period, which consists of three components: (i) the expected benefit from holding the dollar until the second subperiod of period $t+1$ (i.e., if the investor does not spend the dollar in the OTC market) and (ii) the expected gain from using the dollar to purchase equity from another investor in the OTC market of period $t+1$. To interpret (9), it is useful to rewrite it as

$$\phi^m_t \geq \beta \mathbb{E}_t \left\{ \phi^m_{t+1} + \alpha \omega^m_{t+1}(a^m_{t+1i}, A^s)\eta \mathbb{E}\left[\frac{\varepsilon_i y_{t+1} + \phi^s_{t+1}}{p_{t+1}(\varepsilon_j)} - \phi^m_{t+1} \middle| (\varepsilon_i, \varepsilon_j) \in \Omega^m_{t+1}(a^m_{t+1i}, A^s) \right] \right\}$$

where $\mathbb{E}_t$ denotes the conditional expectation of y_{t+1}, $\mathbb{E}[\cdot|\cdot]$ denotes the conditional expectation of $(\varepsilon_i, \varepsilon_j)$, and for any $(a^m_{t+1i}, a^s_{t+1j}) \in \mathbb{R}^2_+$,

$$\Omega^m_{t+1}(a^m_{t+1i}, a^s_{t+1j}) = \left\{ (\varepsilon_i, \varepsilon_j) \in [\varepsilon_L, \varepsilon_H]^2 : \varepsilon_j < \varepsilon_i \text{ and } a^m_{t+1i} < p_{t+1}(\varepsilon_j)\, a^s_{t+1j} \right\}$$

and $\omega^m_{t+1}(a^m_{t+1i}, a^s_{t+1j}) \equiv \int\int \mathbb{I}_{\{(\varepsilon_i,\varepsilon_j)\in\Omega^m_{t+1}(a^m_{t+1i}, a^s_{t+1j})\}} dG(\varepsilon_i)\, dG(\varepsilon_j)$. With probability α, investor, call him i, contacts another investor, e.g., investor j. Then $\omega^m_{t+1}(a^m_{t+1i}, A^s)\eta$ denotes the joint probability that i's preference type is higher than j's (so i acts as a buyer of equity), and i has bargaining power (which happens with conditional probability η), and the bilateral gains from trade are constrained by i's money holdings (which given i's money holdings, a^m_{t+1i}, and j's equity holdings, A^s, at the time of the trade, occurs if the bilateral dollar price of equity is large enough, i.e., if j's individual valuation of equity, ε_j, is large enough). In this event, carrying an additional dollar into period $t+1$ helps investor i reap gains from trade in the bilateral trade with the other investor, and i's expected gain from trading the marginal dollar is the (conditional expected) value of the additional equity he purchases, i.e., $\frac{1}{p_{t+1}(\varepsilon_j)}$ equity shares each worth $\varepsilon_i y_{t+1} + \phi^s_{t+1}$, minus the value of the dollar, ϕ^m_{t+1}.

Condition (10) is the investor's Euler equation for equity. To interpret this condition it is useful to rewrite it as

$$\phi^s_t \geq \beta\pi\mathbb{E}_t \left\{ \bar{\varepsilon}y_{t+1} + \phi^s_{t+1} \right. \\ \left. + \alpha\omega^s_{t+1}(a^s_{t+1i}, A^m_{t+1})(1-\eta)\,\mathbb{E}\left[p_{t+1}(\varepsilon_j)\,\phi^m_{t+1} - \left(\varepsilon_i y_{t+1} + \phi^s_{t+1}\right) | (\varepsilon_i, \varepsilon_j) \in \Omega^s_{t+1}(a^s_{t+1i}, A^m_{t+1})\right] \right\}$$

where for any $(a^s_{t+1i}, a^m_{t+1j}) \in \mathbb{R}^2_+$,

$$\Omega^s_{t+1}(a^s_{t+1i}, a^m_{t+1j}) = \left\{ (\varepsilon_i, \varepsilon_j) \in [\varepsilon_L, \varepsilon_H]^2 : \varepsilon_i < \varepsilon_j \text{ and } p_{t+1}(\varepsilon_j)\, a^s_{t+1i} < a^m_{t+1j} \right\}$$

and $\omega^s_{t+1}(a^s_{t+1i}, a^m_{t+1j}) \equiv \int\int \mathbb{I}_{\{(\varepsilon_i,\varepsilon_j)\in\Omega^s_{t+1}(a^s_{t+1i},a^m_{t+1j})\}} dG(\varepsilon_i)\, dG(\varepsilon_j)$. The left side is the real cost of purchasing an additional equity share in the second subperiod of t. The right side is the discounted expected benefit from carrying an additional equity share into the following period, which consists of two terms. First, $\bar{\varepsilon}y_{t+1} + \phi^s_{t+1}$, the expected benefit of holding the equity share until the end of period $t+1$ (i.e., if the investor does not sell the equity in the OTC market). Second, with probability α investor i contacts another investor j in the OTC market. Then $\omega^s_{t+1}(a^s_{t+1i}, A^m_{t+1})(1-\eta)$ denotes the joint probability that i's preference type is lower than j's (so i acts as a seller of equity), and i has bargaining power (which happens with conditional probability $1-\eta$), and the bilateral gains from trade are constrained by i's equity holdings (which given i's equity holdings, a^s_{t+1i}, and j's money holdings, A^m_{t+1}, at the time of the trade, occurs if the bilateral dollar price of equity is low enough, i.e., if j's individual valuation of equity, ε_j, is low enough). In this event, an additional equity share helps investor i reap gains from trade in the bilateral trade with the other investor, and i's expected gain from trading the marginal share is the (conditional expected) value of the real balances he receives, i.e., $p_{t+1}(\varepsilon_j)\,\phi^m_{t+1}$, minus the (conditional expected) value of the equity share he sells, i.e., $\varepsilon_i y_{t+1} + \phi^s_{t+1}$.

Let $\tilde{A}^m_{t+1} \equiv \int_{\mathcal{I}} \tilde{a}^m_{t+1i} di$ and $\tilde{A}^s_{t+1} \equiv \int_{\mathcal{I}} \tilde{a}^s_{t+1i} di$ denote the aggregate quantities of money and shares held by investors at the end of period t. We are now ready to define equilibrium.

Definition 1 *An equilibrium is a sequence of terms of trade in the OTC market,* $\{\langle \underline{\boldsymbol{a}}_{ti^*}, \underline{\boldsymbol{a}}_{ti} \rangle_{i\in\mathcal{I}}\}^\infty_{t=0}$*, as given in Lemma 1, together with a sequence of asset holdings,* $\{\langle \boldsymbol{a}_{t+1i}, \tilde{\boldsymbol{a}}_{t+1i} \rangle_{i\in\mathcal{I}}\}^\infty_{t=0}$*, and* *prices, $\{\boldsymbol{\psi}_t\}_{t=0}^{\infty} \equiv \{\phi_t^m, \phi_t^s\}_{t=0}^{\infty}$, such that for all t, (i) the asset allocation solves the investor's optimization problem (6) taking prices as given, and (ii) prices are such that all Walrasian markets clear, i.e., $\tilde{A}_{t+1}^s = A^s$ (the end-of-period-t Walrasian market for equity), $\tilde{A}_{t+1}^m = A_{t+1}^m$ (the end-of-period-t Walrasian market for money). An equilibrium is "monetary" if $\phi_t^m > 0$ for all t, and "nonmonetary" otherwise.*

In what follows, we specialize the analysis to stationary equilibria in which real asset prices are time-invariant functions of the aggregate dividend, i.e., $\phi_t^s = \phi^s y_t$ and $\phi_t^m A_t^m = Z y_t$. Hence, in a stationary equilibrium, $\phi_{t+1}^s/\phi_t^s = \gamma_{t+1}$ and $\phi_t^m/\phi_{t+1}^m = \mu/\gamma_{t+1}$. Throughout the analysis we let $\bar{\beta} \equiv \beta\bar{\gamma}$ and maintain the assumption $\mu > \bar{\beta}$, but the following proposition considers the limiting case $\mu \to \bar{\beta}$.

Proposition 2 *The allocation implemented by the stationary monetary equilibrium converges to the symmetric efficient allocation as $\mu \to \bar{\beta}$.*

Let $q_{t,k}^B$ denote the nominal price in the second subperiod of period t of an N-period risk-free pure-discount nominal bond that matures in period $t+k$, for $k = 0, 1, 2, ..., N$ (so k is the number of periods until the bond matures). Assume that the bond is illiquid in the sense that it cannot be traded in the OTC market. Then in a stationary monetary equilibrium, $q_{t,k}^B = (\bar{\beta}/\mu)^k$, and

$$\iota = \frac{\mu - \bar{\beta}}{\bar{\beta}} \tag{11}$$

is the time-t nominal yield to maturity of the bond with k periods until maturity. Thus, the optimal monetary policy described in Proposition 2 in which the money supply grows at rate $\bar{\beta}$ can be interpreted as a policy that implements the *Friedman rule*, i.e., $\iota = 0$ for all contingencies at all dates.

Let

$$\tilde{\mu} \equiv \bar{\beta}\left[1 + \alpha\eta \int_{\varepsilon_L}^{\varepsilon_H} \int_{\varepsilon_j}^{\varepsilon_H} \frac{\varepsilon_i - \varepsilon_j}{\varepsilon_j + \frac{\beta\pi}{1-\beta\pi}\bar{\varepsilon}} dG(\varepsilon_i)\, dG(\varepsilon_j)\right], \tag{12}$$

and define the function $\varphi : [\varepsilon_L, \varepsilon_H] \to \mathbb{R}$ by

$$\varphi(\varepsilon) \equiv \int_{\varepsilon_L}^{\varepsilon} \int_{\varepsilon_L}^{\varepsilon_j} (\varepsilon_j - \varepsilon_i)\, dG(\varepsilon_i)\, dG(\varepsilon_j).$$

Intuitively, $\varphi(\varepsilon)$ is the expected surplus for investor i to sell a dividend good to investor j, conditional on investor i's marginal value over the dividend good being less than ε.

Proposition 3 *(i) There is no stationary monetary equilibrium if* $\mu \geq \tilde{\mu}$. *(ii) In the nonmonetary equilibrium there is no trade in the OTC market, and the equity price in the Walrasian market is*

$$\phi^s = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\bar{\varepsilon}y.$$

(iii) If $\mu \in (\bar{\beta}, \tilde{\mu})$*, then there is one stationary monetary equilibrium and asset prices are*

$$\phi_t^s = \phi^s y_t, \text{ with } \phi^s = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\bar{\varepsilon} + \alpha(1-\eta)\varphi(\varepsilon^c)\right] \tag{13}$$

$$\phi_t^m = Z\frac{y_t}{A_t^m}$$

where

$$Z = (\varepsilon^c + \phi^s) A^s, \tag{14}$$

and for any $\mu \in \left(\bar{\beta}, \tilde{\mu}\right)$, $\varepsilon^c \in (\varepsilon_L, \varepsilon_H)$ *is the unique solution to*

$$\int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H} \frac{(1-\bar{\beta}\pi)(\varepsilon_i - \varepsilon_j)}{(1-\bar{\beta}\pi)\varepsilon_j + \bar{\beta}\pi\left[\bar{\varepsilon} + \alpha(1-\eta)\varphi(\varepsilon^c)\right]} dG(\varepsilon_i)\, dG(\varepsilon_j) - \frac{\mu - \bar{\beta}}{\bar{\beta}\alpha\eta} = 0. \tag{15}$$

(iv) (a) As $\mu \to \tilde{\mu}$, $\varepsilon^c \to \varepsilon_L$ *and* $\phi_t^s \to \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\bar{\varepsilon}y_t$. *(b) As* $\mu \to \bar{\beta}$, $\varepsilon^c \to \varepsilon_H$ *and* $\phi_t^s \to \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\bar{\varepsilon} + \alpha(1-\eta)\varphi(\varepsilon_H)\right] y_t$.

A stationary monetary equilibrium does not exist if the inflation rate is too high, i.e., if $\mu \geq \tilde{\mu}$, and in this case there is no equity trade and the equity price is equal to the expected discounted present value of the dividend. If the inflation rate is low enough, i.e., $\mu \in (\bar{\beta}, \tilde{\mu})$, then a unique stationary equilibrium exists. In this case an investor i starts every period t with a portfolio of money and equity, and he is randomly matched with another investor j during the OTC round of trade. If i's preference shock is larger than j's, i.e., if $\varepsilon_j < \varepsilon_i$, then i will want to purchase all of j's equity holdings. Whether he is able to do so depends the quantity of assets that j holds, which in equilibrium equals A^s, and the dollar price that i has to pay for the equity. In turn, the dollar price will depend on whether investor i or investor j has the bargaining power. If i has the power (this happens with probability η), then the dollar price he pays j for each unit of equity is $p_t(\varepsilon_j) = \frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m}$, and since i holds A_t^m dollars in equilibrium, he can afford to buy all of j's equity only if $\frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m} A^s \leq A_t^m$, or equivalently, if $(\varepsilon_j + \phi^s) A^s \leq Z$ which using (14) can be rewritten as $\varepsilon_j \leq \varepsilon^c$. Thus as μ falls, real balances and ε^c increase, and the investor who wishes to buy equity and has the bargaining power, is cash constrained in a smaller fraction of the bilateral meetings. The value of money depends on the gains from trade of the relatively high valuation investors who buy equity. The equity price (13) on the other hand, reflects the gains from trade of the relatively low valuation investors who sell equity in bilateral transactions ($\alpha(1-\eta)\varphi(\varepsilon^c)$ captures the expected gain from selling equity to another investor in the OTC round).

5 Asset prices

In this section we study the properties of the equilibrium asset prices characterized in Proposition 3. We focus on how they depend on monetary policy and the degree of OTC frictions.

5.1 Inflation

The real price of equity in a monetary equilibrium is in part determined by the option available to low-valuation investors to resell the equity to high-valuation investors. A higher inflation rate causes real money balances to decline. This reduction in real balances enlarges the set of joint realizations of preference types in bilateral meetings in which the cash constraint binds for high-valuation buyers. In turn, this reduces the value of the marginal preference type, ε^c, of the buyer who is just able to purchase all of the equity from a seller in a bilateral meeting in which the seller has the bargaining power. The result is that ex ante, the period before the OTC round, investors anticipate that the expected gains from selling equity in the OTC market are smaller, and this manifests itself as a smaller equity price in the centralized round of trade. The following proposition formalizes this intuition.

Proposition 4 *In the stationary monetary equilibrium: (i)* $\partial\phi^s/\partial\mu < 0$*, (ii)* $\partial Z/\partial\mu < 0$ *and* $\partial\phi_t^m/\partial\mu < 0$*.*

5.2 OTC frictions: trading delays and market power

The value of holding equity increases with the bilateral meeting probability, as this increases the probability that the investor may find an opportunity to sell the asset to another investor with higher valuation. Similarly, the value of money increases with α as this increases the probability the investor may be able to use money to buy equity if he were to meet a counterparty with lower valuation.

Proposition 5 *In the stationary monetary equilibrium: (i) $\partial\phi^s/\partial\alpha > 0$, (ii) $\partial Z/\partial\alpha > 0$ and $\partial\phi_t^m/\partial\alpha > 0$.*

The effect of the bargaining power of asset buyers, η, on the asset price is non-monotonic. When η is 0, investors receive no trade surplus when they buy assets. So, they have no incentive to hold money and there does not exist monetary equilibrium, nor trade. The value of the option to resell the equity is then zero. When η is equal to 1, investors receive no trade surplus when they sell assets. The value of the option to resell the equity is also zero. The value of the resale option may be positive when η is between 0 and 1.

6 Financial liquidity

In this section we use the theory to study the determinants of two standard measures of market liquidity: trade volume and price dispersion.

6.1 Volume

According to Lemma 1, the quantity traded in a meeting between two investors depends on whether the buyer or the seller of equity has the bargaining power. Suppose that investor i has preference type ε_i and investor j has preference type $\varepsilon_j < \varepsilon_i$. If investor i (in this case the buyer) makes the offer, then he purchases $\min\{A_t^m/p_t(\varepsilon_j), A^s\} = \mathbb{I}_{\{\varepsilon^c<\varepsilon_j\}}\frac{Z}{\varepsilon_j+\phi^s} + \mathbb{I}_{\{\varepsilon_j\leq\varepsilon^c\}}A^s$ equity shares. Conversely, if investor j has the bargaining power, then investor i purchases $\min\{A_t^m/p_t(\varepsilon_i), A^s\} = \mathbb{I}_{\{\varepsilon^c<\varepsilon_i\}}\frac{Z}{\varepsilon_i+\phi^s} + \mathbb{I}_{\{\varepsilon_i\leq\varepsilon^c\}}A^s$ equity shares. Hence the total quantity of equity shares traded in the OTC market is

$$\mathcal{V} = \alpha\left\{\eta\left[\int_{\varepsilon_L}^{\varepsilon^c}[1-G(\varepsilon_j)]\,dG(\varepsilon_j) + \int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H}\frac{\varepsilon^c+\phi^s}{\varepsilon_j+\phi^s}dG(\varepsilon_i)\,dG(\varepsilon_j)\right]\right.$$
$$\left. + (1-\eta)\left[\int_{\varepsilon_L}^{\varepsilon^c}G(\varepsilon_i)\,dG(\varepsilon_i) + \int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_L}^{\varepsilon_i}\frac{\varepsilon^c+\phi^s}{\varepsilon_i+\phi^s}dG(\varepsilon_j)\,dG(\varepsilon_i)\right]\right\}A^s.$$

In trades where the investor with no bargaining power has preference type $\varepsilon < \varepsilon^c$, all the equity holdings of the seller, A^s, are traded. In meetings where the investor with no bargaining power has preference type $\varepsilon > \varepsilon^c$, the cash constraint of the buyer binds, and only $\frac{Z}{\varepsilon_j+\phi^s} = \frac{(\varepsilon^c+\phi^s)A^s}{\varepsilon_j+\phi^s}$ equity shares are traded. Notice that inflation only affects $\mathcal{V}$ indirectly, through its effect on ε^c (or equivalently, real balances, Z). Higher inflation reduces the value of real balances and this implies that the cash constraint will bind in more trades, causing trade volume to decline along the intensive margin (i.e., by reducing the quantity of equity traded in trades in which the agent with no bargaining power has relatively high valuation for the dividend). An increase in the contact probability α increases $\mathcal{V}$ along the extensive margin (more meetings among investors naturally result in larger trade volume), but an increase in α also increases real balances and therefore induces an increase in trade volume along the intensive margin. This intuition is formalized in the following proposition.

Proposition 6 *In the stationary monetary equilibrium: (i) $\partial \mathcal{V}/\partial \mu < 0$, and (ii) $\partial \mathcal{V}/\partial \alpha > 0$.*

6.2 Price dispersion

In empirical work, measures of price dispersion are often used as proxies for market illiquidity.[5] In our theory, the coefficient of variation,

$$\mathcal{D}_t^s \equiv \frac{SD\left[p_t(\varepsilon)\right]}{\mathbb{E}\left[p_t(\varepsilon)\right]} = \frac{SD(\varepsilon)}{\mathbb{E}(\varepsilon)+\phi^s}, \tag{16}$$

with $SD(x) \equiv \sqrt{\mathbb{E}[x-\mathbb{E}(x)]^2}$, is a natural measure of dispersion in the transaction price in the OTC market, $p_t(\varepsilon)$. Since $\mathcal{D}_t^s$ is decreasing in the asset price, ϕ^s, the following result is a corollary of Propositions 4 and 5.

Proposition 7 *In the stationary monetary equilibrium: (i) $\partial \mathcal{D}_t^s/\partial \mu > 0$, and (ii) $\partial \mathcal{D}_t^s/\partial \alpha < 0$.*

So far we have interpreted the asset in the model as an equity share.[6] To think of the asset as a bond, we can interpret y_t as the coupon payment in period t, and $\delta \equiv (1-\pi)^{-1}$ as the (expected) time until the bond matures.[7] Consider a set of N bonds, each indexed by a different maturity, $\delta_s \equiv (1-\pi_s)^{-1}$, with $\pi_s \in [0,1]$ for $s \in \{1,\ldots,N\}$. Then let $p_t^s(\varepsilon)$ denote the cum-coupon nominal price in a bilateral transaction in the first subperiod of period t of a bond with expected duration δ_s. For a bond of type $s \in \{1,\ldots,N\}$, the bond yield for an investor with valuation ε is

$$r_t^s(\varepsilon) \equiv \frac{p_t^s(\varepsilon)}{\phi_t^s/\phi_t^m} - 1 = \frac{\varepsilon}{\phi^s}$$

[5] See, e.g., Jankowitsch et al. (2011).

[6] While most equities in the US are traded in organized exchanges, one fifth of stocks are traded in OTC markets. See Ang et al. (2013) for an empirical study on the liquidity and pricing of OTC-traded stocks. Most fixed income securities, including Treasuries, are traded in OTC markets.

[7] This bond yields a stream of coupons $\{y_t\}$ until maturity, with no principal to be rapaid upon maturity.

and the average bond yield is

$$\bar{r}_t^s \equiv \mathbb{E}[r_t^s(\varepsilon)] = \frac{\mathbb{E}(\varepsilon)}{\phi^s}.$$

Hu et al. (2013) propose the following "noise" statistic as an empirical measure of the illiquidity of bonds

$$\text{Noise}_t = \sqrt{\frac{1}{N}\sum_{s=1}^{N}(\tilde{r}_t^s - \hat{r}_t^s)^2}, \tag{17}$$

where $\tilde{r}_t^s$ denotes an observation of the yield of bond s on day t, and $\hat{r}_t^s$ denotes the estimate of the average yield of bond s on day t that results from a standard yield-curve fitting exercise. In our theory,

$$\mathcal{N}_t = \sqrt{\frac{1}{N}\sum_{s=1}^{N}\{SD[r_t^s(\varepsilon)]\}^2}, \tag{18}$$

with $SD[r_t^s(\varepsilon)] = \sqrt{\mathbb{E}[r_t^s(\varepsilon) - \bar{r}_t^s]^2}$, would be an analogous marketwide average notion of price dispersion, or pricing "noise." The noise measure (18) is closely related to the price dispersion measure (16). To see this, suppose the coupon is small, so that $\mathbb{E}(\varepsilon^s)/\phi^s \approx 0$ and

$$\mathcal{D}_t^s \approx \frac{SD(\varepsilon)}{\phi^s}. \tag{19}$$

In this case, $\mathcal{D}_t^s \approx SD[r_t^s(\varepsilon)]$, so

$$\mathcal{N}_t \approx \sqrt{\frac{1}{N}\sum_{s=1}^{N}(\mathcal{D}_t^s)^2}. \tag{20}$$

The theoretical measure $\mathcal{N}_t$ is based on the yields implied by the whole cross section of transaction prices on day t for each bond s. Suppose these detailed cross-sectional data were not available, and that instead one is only able to obtain an imperfect proxy for $SD[r_t^s(\varepsilon)]$, for instance, $\tilde{r}_t^s - \bar{r}_t^s$, where $\tilde{r}_t^s \equiv \tilde{\varepsilon}_t^s/\phi^s$ denotes an observation of the yield of bond s on day t. Then,

$$\tilde{\mathcal{N}}_t = \sqrt{\frac{1}{N}\sum_{s=1}^{N}(\tilde{r}_t^s - \bar{r}_t^s)^2},$$

would be the corresponding proxy for $\mathcal{N}_t$. Notice that $\tilde{\mathcal{N}}_t$ coincides with the noise measure (17) proposed by Hu et al. (2013) as long as their yield-curve estimation delivers estimates $\hat{r}_t^s$ that are close to the true average yields $\bar{r}_t^s$.

Variable	Regression (21)
$\Delta\hat{i}_t$	.6054**
	(.018)
constant	.01957
	(0.571)
Number of Obs.	137

Table 1: Response of noise measure for Treasury bonds to moneteray policy shocks.

According to Proposition 7, the noise measure $\mathcal{N}_t$ should increase when monetary policy tightens. This theoretical prediction can be tested by estimating the following regression:

$$\Delta \text{Noise}_t = \text{constant} + b \times \Delta i_t + \epsilon_t, \tag{21}$$

where $\Delta\text{Noise}_t = \text{Noise}_{t+1} - \text{Noise}_t$, i_t is a proxy for the market expectation of the policy rate, $\Delta i_t = i_t - i_{t-1}$, and ϵ_t is an error term.[8] To deal with endogeneity of policy rate changes to market liquidity, we use a standard instrument, the changes in Fed funds futures rate around a narrow window before and after the announcements of FOMC meetings.[9]

Table 1 reports the estimation results. Both, the change in policy rate and the noise measure are expressed in basis points. The estimate of b is statistically significant at the 5% level (p-value of 1.8%). The estimate implies that a 10-basis-point unexptected increase in the policy rate increases the noise measure by about 6 basis points. Given (19) and (20), this result lends empirical support to Proposition 7.

7 Speculation

According to Proposition 3, in a monetary equilibrium the equity price, ϕ^s, is larger than the expected present discounted value that any agent assigns to the dividend stream, i.e., $\hat{\phi}_t^s \equiv \left[\bar{\beta}\pi/(1-\bar{\beta}\pi)\right]\bar{\varepsilon}y_t$. We follow Harrison and Kreps (1978) and call the value of the asset in excess of the expected present discounted value of the dividend under no trade, i.e., $\phi_t^s - \hat{\phi}_t^s$,

[8]We use the change in the illiquidity measure from date t to $t+1$ because FOMC announcements are typically made at around 2pm of a trading day. Since the number of transactions in the OTC market is small, the illiquidity measure constructed from transaction data may respond with delay. Although monetary policy in the model we presented above is deterministic, one could extend the theory to allow for random monetary policy shocks as in Lagos and Zhang (2018). The key result that unexpected tightening of monetary policy lowers the asset price and increases market illiquidity also holds in the stochastic formulation.

[9]The data for the instrument and policy rate changes is from the online appendix of Gorodnichenko and Weber (2016), which include 137 FOMC meeting announcments from February 4, 1994 to December 16, 2009. The daily noise measure is from the online appendix of Hu et al. (2013).

the *speculative premium* which investors are willing to pay in anticipation of the capital gains they will reap when reselling the asset to investors with higher valuations in the future.[10]

According to Proposition 3, the speculative premium is $\mathcal{P}_t = \mathcal{P}y_t$, where

$$\mathcal{P} = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\alpha\left(1-\eta\right)\varphi\left(\varepsilon^c\right).$$

Again, $\mathcal{P}_t \geq 0$, with "=" only if $\mu = \bar{\mu}$. Higher inflation reduces real balances and therefore ε^c, which reduces the expected resale value of equity in the OTC market, so $\mathcal{P}$ decreases with inflation. An increase in the trade probability α has a positive direct effect on $\mathcal{P}$ (increase in the meeting probability) and also an indirect positive effect (α increases ε^c which in turn increases $\mathcal{P}$). These effects are summarized below.

Proposition 8 *In the stationary monetary equilibrium: (i) $\partial\mathcal{P}/\partial\mu < 0$, and (ii) $\partial\mathcal{P}/\partial\alpha > 0$.*

Notice that together, Proposition 5 and Proposition 6 imply that changes in the trading probability will generate a positive correlation between trade volume and the size of the speculative premium.[11] The positive correlation between trade volume and the size of speculative premia is a feature of historical episodes that are usually regarded as bubbles—a point emphasized by Scheinkman and Xiong (2003a, 2003b) and Scheinkman (2013).

8 The Fed Model and the Modigliani-Cohn hypothesis

The high inflation of the 1970s stimulated researchers to ask whether stocks are a good hedge against inflation. In a well-known paper, Fama and Schwert (1977) found that, contrary to long-held beliefs, common stocks were rather perverse as hedges against inflation. They found that common stock returns were negatively related to the expected inflation rate during the

[10] As in Harrison and Kreps (1978), investors exhibit *speculative behavior* if the right to resell a stock makes them willing to pay more for it than they would pay if obliged to hold it forever. Investors exhibit speculative behavior in the sense that they buy with the expectation to resell, and naturally the asset price incorporates the value of this option to resell: investors are willing to pay more for the asset than they would pay if obliged to hold it forever. This notion of *speculative premium* corresponds to the notion of *speculative bubble* that is used in the modern literature on bubbles. See, e.g., Barlevy (2007), Brunnermeier (2003), Scheinkman and Xiong (2003a,b), Scheinkman (2013), and Xiong (2013), who discuss Harrison and Kreps (1978) in the context of what is generally known as the *resale-option theory of bubbles*. The notion of a "bubble," however, requires one to take a stand on a notion of "fundamental value," which may open up several possibilities in economies where investors have heterogeneous valuations. See Barlevy (2015) for a more in-depth discussion.

[11] From a normative standpoint, our model implies that a large speculative premium is a sign of a better allocation of the asset.

1953-71 period, and that they also seemed to be negatively related to the unexpected inflation rate.[12] In line with these observations, in the late 1970s Modigliani and Cohn (1979) pointed out that the ratio of market value to profits of firms had declined consistently since the late 1960s. They observed that this fact was consistent with investors who capitalize equity earnings using a nominal interest rate instead of a real one, and settled on this kind of money illusion as the most reasonable explanation. More recently, Sharpe (2002), Asness (2000), Lansing (2004) and many others have documented that yields on stocks (e.g., as measured by the dividend-price ratio) are highly correlated with nominal bond yields.[13] Since stocks are claims to cash flows from real capital and inflation is the main driver of nominal interest rates, this correlation has proven difficult to rationalize with conventional asset pricing theory.[14]

Theory aside, this correlation has led financial practitioners to adopt the so-called *Fed Model* of equity valuation to calculate the "correct" price of stocks.[15] In its simplest form, the Fed Model says that, because stocks and nominal Treasury bonds compete for space in investors' portfolios, their yields should be positively correlated. That is, if the yield on bonds rises, then the yield on stocks must also rise to maintain the competitiveness of stocks vis a vis bonds. Practitioners use this reasoning to argue that the yield on nominal bonds (plus a risk premium to account for the relative riskiness of stocks) defines a "normal" yield on stocks: if the measured stock yield is below this normal yield, then stocks are considered overpriced; if the measured stock yield is above this normal yield, then stocks are considered underpriced.[16]

[12] At the time of Fama and Schwert's writing, Lintner (1975), Jaffe and Mandelker (1976), Bodie (1976), and Nelson (1976) had offered similar empirical evidence. Cagan (1974) is an early effort to study these issues using some historical records.

[13] Along similar lines, Bordo and Wheelock (2007) review the histories of major 20th century stock market booms in the United States and nine other countries. They find that booms usually arose when inflation was below its long-run average, and that booms typically ended when inflation began to rise and/or monetary authorities tightened policy in response to rising or a threatened rise in inflation. Ritter and Warr (2002) argue that the decline in inflation was a major factor leading to the bull market of 1982-1999.

[14] See the discussions in Ritter and Warr (2002), Asness (2003), and Campbell and Vuolteenaho (2004).

[15] Sharpe (2002), Asness (2003), and Feinman (2005) discuss the popularity of the Fed Model among Wall Street analysts and strategists.

[16] The term *Fed Model* appears to have been first used by securities strategist Ed Yardeni in 1997 following the publication of the Federal Reserve Humphrey-Hawkins Report for July 1997. In Section 2 ("Economic and Financial Developments in 1997"), a chart plotted the time series for the earnings-price ratio of the S&P 500 against the 10-year constant-maturity nominal treasury yield and reported: "The run-up in stock prices in the spring was bolstered by unexpectedly strong corporate profits for the first quarter. Still, the ratio of prices in the S&P 500 to consensus estimates of earnings over the coming twelve months has risen further from levels that were already unusually high. Changes in this ratio have often been inversely related to changes in long-term Treasury yields, but this year's stock price gains were not matched by a significant net decline in interest rates. As a result, the yield on ten-year Treasury notes now exceeds the ratio of twelve-month-ahead earnings to prices by the largest amount since 1991, when earnings were depressed by the economic slowdown."

The relationship between equity prices and monetary policy also appears to have been clear in the minds of policymakers. Alan Greenspan, for example, famously held the view that stock market booms are more likely to occur when inflation is low. He saw a dilemma in the use of monetary policy to defuse stock market booms:

> "We have very great difficulty in monetary policy when we confront stock market bubbles. That is because, to the extent that we are successful in keeping product price inflation down, history tells us that price-earnings ratios under those conditions go through the roof. What is really needed to keep stock market bubbles from occurring is a lot of product price inflation, which historically has tended to undercut stock markets almost everywhere. There is a clear trade-off. If monetary policy succeeds in one, it fails in the other. Now, unless we have the capability of playing in between and managing to know exactly when to push a little here and to pull a little there, it is not obvious to me that there is a simple set of monetary policy solutions that deflate the bubble." (Alan Greenspan, FOMC transcript, September 24, 1996, pp. 30-31.)

The Fed model does not always hold in practice. For example, after the great recession, the policy rate hit the zero lower bound. Gourio and Ngo (2016), among others, makes the point that the correlation between inflation and the asset price may be quite different in this case. Our theory and the empirical work in Lagos and Zhang (2018) focus on a particular transmission channel through the liquidity of the financial market. While our findings support the Fed model, the correlation between inflation and the asset price could depend on the relative relevance of various transmission channels of monetary policy.

To fix ideas, consider a standard Lucas (1978) economy in which a risk-neutral investor with discount rate β prices a tree that is subject to a shock that renders it permanently unproductive with probability $1-\pi$. Conditional on remaining productive, the tree yields real dividend D_t, with $D_{t+1}=\gamma_{t+1}D_t$, where γ_{t+1} is a nonnegative random variable with mean $\bar{\gamma}\in(0,(\beta\pi)^{-1})$. The real price of an equity share of the tree is

$$P_t^s=\frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}D_t.$$

If we set $D_t=\bar{\varepsilon}y_t$, this is just the equity price in the nonmonetary equilibrium of Proposition 3. We can use this expression to obtain

$$\frac{\bar{D}_{t+1}}{P_t^s}=(1+r)-\bar{\gamma}\pi, \tag{22}$$

where $\bar{D}_{t+1} \equiv \bar{\gamma}\pi D_t$ denotes the expected dividend (conditional only on the tree having survived period t) and $1 + r = 1/\beta$ is the real risk-free rate. Condition (22) is known as the "Gordon growth model" (e.g., Gordon (1962), Williams (1938)). The left side is the *dividend (or stock) yield*, which is equal to the real risk-free rate, $1 + r$, minus the expected growth rate of the real dividend, $\bar{D}_{t+1}/D_t = \bar{\gamma}\pi$. All the variables in (22) are real. In particular, since a rational investor's Euler equation equates the expected real equity return to the risk-free real interest rate, i.e., $\mathbb{E}_t R^s_{t+1} = 1 + r$, where $R^s_{t+1} \equiv \pi \frac{P^s_{t+1} + y_{t+1}}{P^s_t}$, the investor uses the real interest rate r to discount future dividends.[17] According to the narrative behind the Fed Model, however, investors allocate their portfolio between stocks and nominal long-term bonds by comparing the expected real return on equity to the expected *nominal* bond yield, i.e., they use the wrong Euler equation, $\mathbb{E}_t R^s_{t+1} = 1 + \iota$. This leads to

$$\frac{\bar{D}_{t+1}}{P^s_t} = (1 + \iota) - \bar{\gamma}\pi,$$

which "explains" the positive relation between the nominal bond yield $\iota = (\mu - \bar{\beta})/\bar{\beta}$ and the stock yield $\bar{D}_{t+1}/P^s_t$ by saying that investors suffer from money illusion in the sense that they discount future real dividends using the nominal rate, ι, rather than the real rate, r. This is the Modigliani-Cohn hypothesis. Financial analysts are often ambivalent toward the Fed Model. On the one hand, "it works" empirically. On the other hand, they are reluctant to recommend it to clients because the conventional logic behind it is fundamentally flawed; it is inconsistent with investor rationality.[18]

[17] As before, $\mathbb{E}_t$ is the expectation over y_{t+1}, conditional on the information available in period t, as well as on the tree surviving period $t + 1$.

[18] For example, Feinman (2005) (Chief Economist at Deutsche Management Americas in New York) writes: "If interest rates are to be brought into the calculus at all, they should be real rates, not nominal. This is not to deny that equity prices seem to be set as if investors are comparing equity yields with nominal interest rates. But this just demonstrates the error of money illusion. It should not be construed as recommending that error." Similarly, Asness (2003) writes: "Historically S&P 500 earnings-price ratio and 10-Year Treasury Rate have been strongly related... The correlation of these two series over this period [1965-2001] is an impressive +0.81. I am far from unique in presenting a graph like figure 1. It's a rare Wall Street strategist that in the course of justifying the Fed Model (or similar analytic) does not pull out a version of this figure... If you are trying to explain why price-earning ratios are where they are, based on investors behaving in a similar manner in the past (errors and all), then feel free to use the Fed Model (hopefully modified for volatility as in this paper), but do not confuse that with a tool for making long-term recommendations to investors." Siegel (2002): "It is true that bonds are the major asset class that competes with stocks in an investor's portfolio, so one might expect that low interest rates would be favorable to stocks. But since in the long run low interest rates are caused by low inflation, the rate of growth of nominal earnings, which depends in large part on the rate of inflation, will be lower also. Over long periods of time, changes in the inflation rate cause changes in earnings growth of the same magnitude and do not change the valuation of stocks." See also Ritter and Warr (2002). Modigliani and Cohn

Despite much skepticism, however, the Modigliani-Cohn hypothesis remains the leading explanation for the positive correlation between stock yields and nominal bond yields, and for the empirical success of the Fed Model. Campbell and Vuolteenaho (2004) empirically decompose the S&P500 stock yield into three components: (*i*) a rational forecast of long-run expected dividend growth, (*ii*) a subjective risk premium (identified from a cross-sectional regression), and (*iii*) a residual "mispricing term." They evaluate three hypotheses for why low stock prices coincide with high inflation: (1) High inflation coincides with low expected dividend growth. (2) High inflation coincides with a high (subjective) risk premium. (3) Investors suffer from money illusion. Campbell and Vuolteenaho (2004) assess these hypotheses by regressing the three components of the dividend yield (the expected dividend growth, risk premium, and the residual mispricing term) on an exponentially smoothed moving average of inflation. The regression coefficient of expected dividend growth on inflation is positive and large, so the raw correlation between inflation and expected dividend growth is not negative as required by the first hypothesis. The regression coefficient of the risk premium on inflation is negative but small, indicating that the risk premium is not increasing with inflation as required by the second hypothesis. Thus, Campbell and Vuolteenaho (2004) reject the two conventional rational hypotheses for the positive correlation between the dividend yield and inflation. The regression coefficient of the residual mispricing term on inflation is positive, large, and statistically significant. Moreover, the R^2 on this regression is 77.90, indicating that inflation accounts for about 80% of the variability in the mispricing term. Based on this evidence, Campbell and Vuolteenaho (2004) conclude that the positive correlation of the dividend yield with inflation is mostly due to the mispricing term, i.e., stocks appear to be undervalued by conventional measures when inflation is high.[19] In the remainder of this section

(1979) themselves expressed some skepticism about their money illusion hypothesis: "...we readily admit that our conclusion is indeed hard to swallow—and especially hard for those of us who have been preaching the gospel of efficient markets. It is hard to accept the hypothesis of a long-lasting, systematic mistake in a well-organized market manned by a large force of alert and knowledgeable people."

[19]To the extent that these types of studies do not fully control for risk, the results may confound the impact of risk attitudes and attribute them to some anomaly such as money illusion. Cohen et al. (2005) revisit the robustness of the results of Campbell and Vuolteenaho (2004) by further controlling for changes in the risk premium. They exploit the fact that if the equity premium is high for risk-related reasons, then there is a cross-sectional implication, namely that high-beta stocks should outperform low-beta stocks in such periods. The Modigliani-Cohn hypothesis, on the other hand, implies that inflation-driven mispricing will apply to all stocks equally, causing all stocks to be equally underpriced when inflation is high. Cohen et al. (2005) show the latter is the case and interpret this as further confirmation of the Modigliani-Cohn hypothesis. Bekaert and Engstrom (2010) find the bulk of the contribution to the covariance between equity and bond yields comes from the positive comovements between expected inflation and a residual term, just like Campbell and Vuolteenaho (2004).

we show our theory offers a novel explanation for this finding: the effects that inflation has on asset prices through the liquidity (or resalability) channel. The new explanation we propose will be transparent because our theory does not assume irrational investors that suffer from money illusion and it abstracts from the other channels through which high inflation may depress real asset prices, such as the possibility that it may adversely affect firms' profitability or riskiness.[20]

According to our theory, the equilibrium equity price is

$$\phi_t^s = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\epsilon\left(\iota\right)y_t,$$

where $\epsilon\left(\iota\right) \equiv \bar{\varepsilon} + \alpha\left(1-\eta\right)\varphi\left(\varepsilon^c\right)$. Since ε^c is decreasing in the growth rate of the money supply, μ, and the nominal bond yield, ι, is increasing in μ, we have $\epsilon'\left(\iota\right) < 0$. Let $\bar{y}_{t+1} \equiv \bar{\gamma}\pi y_t$ denote the expected dividend (conditional only on the tree having survived period t). The log dividend yield is

$$\log \bar{y}_{t+1} - \log \phi_t^s = \log\left[\left(1+r\right) - \bar{\gamma}\pi\right] - \log \epsilon\left(\iota\right), \tag{23}$$

and it is increasing in the nominal yield, ι. Thus, (23) rationalizes the Fed Model, despite the fact that agents do not suffer from money illusion (they discount payoffs using the risk-free real rate $1+r$), risk premia do not change (since agents are risk-neutral here), and the expected growth rate of the dividend, $\bar{\gamma}\pi$, is unaffected by monetary considerations.[21]

However, Bekaert and Engstrom (2010) claim that this is due to a correlation between expected inflation and two plausible proxies for rational time-varying risk premia: a measure of economic uncertainty (the uncertainty among professional forecasters regarding real GDP growth) and a consumption-based measure of risk aversion. Thus, they offer a rational channel that could potentially explain why the Fed model "works:" their explanation is that high expected inflation coincides with periods of high risk aversion and/or economic uncertainty, which conflicts with Campbell and Vuolteenaho (2004) and Cohen et al. (2005). The papers differ on the specifics of how they measure equity risk premia and on the sample period (Bekaert and Engstrom focus on the post-war subsample while Campbell and Vuolteenaho and Cohen et al. go back to 1930s).

[20]Basak and Yan (2010) explore the asset pricing implications of the money illusion hypothesis and Campbell et al. (2014) focus on the monetary policy drivers of the riskiness of bonds and equities.

[21]There is a literature that focuses on the effects of inflation on real prices of assets that trade in markets were trading delays and intermediation costs are substantial, such as housing. For example, Brunnermeier and Julliard (2008) decompose the housing price-rent ratio in the UK into three components: expected future returns on housing investment, rent growth rates, and a mispricing component. They find that inflation and nominal interest rates explain a large share of the time series variation in the mispricing term, and that through this term, a reduction in inflation can generate substantial increases in housing prices. Like the literature discussed in this section, they attribute this mispricing term to money illusion. Motivated by similar observations, Piazzesi and Schneider (2008) develop a housing model with investors who suffer from money illusion (they believe that changes in nominal interest rates reflect changes in real interest rates) and investors who understand the Fisher equation (i.e., that bond returns are given by the nominal rate minus expected inflation). Since borrowing is assumed to be backed by real estate, disagreement about real rates among investors increases house prices. Through this mechanism, inflation shocks can cause house-price booms.

9 Conclusion

We have developed a model in which money is used as a medium of exchange in financial transactions that take place in over-the-counter markets. In any monetary equilibrium the real asset price contains a speculative premium that is positively related to the quantity of real money balances and therefore negatively correlated with anticipated inflation and the long-term nominal interest rate. As a result, the asset price generically exceeds the expected present discounted value that any agent assigns to the dividend stream. We have shown that this simple mechanism rationalizes the positive correlation between the real yield on stocks and the nominal yield on Treasury bonds—an empirical observation long regarded anomalous. We have also used the model to study how monetary considerations and the microstructure of the market where the asset is traded jointly determine the standard measures of financial liquidity of OTC markets, such as trade volume and price dispersion.

The model could be useful to interpret the behavior of asset prices in OTC markets. Recently Ang et al. (2013) have analyzed a large cross section of OTC-traded common stocks over time and find that equity returns are increasing in the proportion of non-trading days (i.e., days in which the stock was not traded) and decreasing in the trade volume of the stock. They interpret these findings through the lens of asset pricing theories that emphasize differences in investors' individual valuations (e.g., due to differences in opinions about the fundamentals) and limits on short sales. Our theory also has heterogeneous valuations, but in addition, it is explicit about the search and bargaining frictions that are defining characteristics of OTC markets. It is also consistent with the behavior of the illiquidity premia in response to variations in the measures of liquidity documented by Ang et al. (2013); e.g., in the stationary monetary equilibrium the expected financial return on the equity, $(\phi^s_{t+1} + y_{t+1})/\phi^s_t$, is decreasing with α.

The theory we have developed also has sharp implications about how the effect of monetary policy on asset prices depends on the microstructure of the market. For instance, it predicts that the speculative premium, and therefore the typical residual mispricing term, should be larger and more responsive to inflation in markets that are more liquid from an investor standpoint, i.e., markets where investors are able to trade fast and face narrow spreads. In Lagos and Zhang (2018), we provide related empirical evidence based on the differential effects of high-frequency monetary shocks on the returns of stocks that differ in terms of their turnover rates.

A Proofs

Proof of Proposition 1. Clearly, (3) and (4) must bind for every t at an optimum, so $\tilde{a}_t = a_t = A^s$, and the planner's problem is equivalent to

$$\max_{\left\{\left[\underline{a}_{tib(i)}\right]_{i\in\mathcal{B}_t}\right\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{(1-\alpha)\bar{\varepsilon}A^s + \int_{\mathcal{B}_t}\int\int \mathbb{I}_{\{i\leq b(i)\}}\right.$$
$$\left.\left[\varepsilon_i \underline{a}_{tib(i)}\left(\varepsilon_i,\varepsilon_{b(i)}\right) + \varepsilon_{b(i)}\underline{a}_{tb(i)i}\left(\varepsilon_{b(i)},\varepsilon_i\right)\right] dG\left(\varepsilon_i\right) dG\left(\varepsilon_{b(i)}\right) di\right\} y_t,$$

subject to (2). Let W^* denote the value of this problem. Then $W^* \leq \bar{W}^*$, where

$$\begin{aligned}\bar{W}^* &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\alpha)\bar{\varepsilon}A^s + \int_{\mathcal{B}_t}\int\int \mathbb{I}_{\{i\leq b(i)\}} \max\left(\varepsilon_i,\varepsilon_{b(i)}\right) 2A^s dG\left(\varepsilon_i\right) dG\left(\varepsilon_{b(i)}\right) di\right] y_t \\ &= \left[(1-\alpha)\bar{\varepsilon} + \alpha\varepsilon_B\right]\left(\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t y_t\right) A^s,\end{aligned}$$

with

$$\varepsilon_B \equiv \int\int \max\left(\varepsilon,\varepsilon'\right) dG\left(\varepsilon\right) dG\left(\varepsilon'\right).$$

The allocation $\underline{a}_{tib(i)}\left(\varepsilon_i,\varepsilon_{b(i)}\right) = \mathbb{I}_{\{\varepsilon_{b(i)}<\varepsilon_i\}} 2A^s + \mathbb{I}_{\{\varepsilon_{b(i)}=\varepsilon_i\}} a^o$, where $a^o \in [0, 2A^s]$ achieves $\bar{W}^*$ and therefore solves the planner's problem. ■

Proof of Lemma 1. Notice that (6) can be written as

$$W_t\left(\boldsymbol{a}_t\right) = \boldsymbol{\phi}_t \boldsymbol{a}_t + W_t\left(\boldsymbol{0}\right) \tag{24}$$

where

$$\begin{aligned}&W_t\left(\boldsymbol{0}\right) = T_t + \max_{\tilde{\boldsymbol{a}}_{t+1}\in\mathbb{R}_+^2}\left[\beta\mathbb{E}_t \int V_{t+1}\left(\boldsymbol{a}_{t+1},\varepsilon\right) dG\left(\varepsilon\right) - \boldsymbol{\phi}_t \tilde{\boldsymbol{a}}_{t+1}\right] \\ &\text{s.t. } \boldsymbol{a}_{t+1} = \left(\tilde{a}_{t+1}^m, \pi\tilde{a}_{t+1}^s + (1-\pi)A^s\right).\end{aligned}$$

With (24) investor i's problem when choosing his take-it-or-leave it offer to investor j reduces to

$$\max_{\underline{a}_{ti*}^m,\underline{a}_{ti*}^s,\underline{a}_{tj}^m,\underline{a}_{tj}^s}\left[\left(\varepsilon_i y_t + \phi_t^s\right)\underline{a}_{ti*}^s + \phi_t^m \underline{a}_{ti*}^m\right]$$

$$
\begin{aligned}
\text{s.t. } \underline{a}_{ti^*}^m + \underline{a}_{tj}^m &\leq a_{ti}^m + a_{tj}^m \\
\underline{a}_{ti^*}^s + \underline{a}_{tj}^s &\leq a_{ti}^s + a_{tj}^s \\
\varepsilon_j y_t \underline{a}_{tj}^s + \phi_t^m \underline{a}_{tj}^m + \phi_t^s \underline{a}_{tj}^s &\geq \varepsilon_j y_t a_{tj}^s + \phi_t^m a_{tj}^m + \phi_t^s a_{tj}^s \\
\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s, \underline{a}_{tj}^m, \underline{a}_{tj}^s &\in \mathbb{R}_+.
\end{aligned}
$$

If $\phi_t^m = 0$, then $\underline{a}_{ti^*}^s = a_{ti}^s$ and $\underline{a}_{tj}^s = a_{tj}^s$ (the bargaining outcome is no trade between investors i and j) so suppose $\phi_t^m > 0$ for the rest of the proof. The Lagrangian corresponding to investor i's problem is

$$
\begin{aligned}
\mathcal{L} = &\left(\phi_t^m + \varsigma_i^m - \xi^m\right) \underline{a}_{ti^*}^m + \left(\varepsilon_i y_t + \phi_t^s + \varsigma_i^s - \xi^s\right) \underline{a}_{ti^*}^s \\
&+ \left(\rho \phi_t^m + \varsigma_j^m - \xi^m\right) \underline{a}_{tj}^m + \left[\rho \left(\varepsilon_j y_t + \phi_t^s\right) + \varsigma_j^s - \xi^s\right] \underline{a}_{tj}^s + K'',
\end{aligned}
$$

where $K'' \equiv \xi^m(a_{ti}^m + a_{tj}^m) + \xi^s(a_{ti}^s + a_{tj}^s) - \rho(\varepsilon_j y_t a_{tj}^s + \phi_t^m a_{tj}^m + \phi_t^s a_{tj}^s)$, $\xi^m \in \mathbb{R}_+$ is the multiplier associated with the bilateral constraint on money holdings, $\xi^s \in \mathbb{R}_+$ is the multiplier associated with the bilateral constraint on equity holdings, $\rho \in \mathbb{R}_+$ is the multiplier on investor j's individual rationality constraint, and $\varsigma_i^m, \varsigma_i^s, \varsigma_j^m, \varsigma_j^s \in \mathbb{R}_+$ are the multipliers for the nonnegativity constraints on $\underline{a}_{ti^*}^m$, $\underline{a}_{ti^*}^s$, $\underline{a}_{tj}^m$ and $\underline{a}_{tj}^s$, respectively. The first-order necessary and sufficient conditions are

$$\phi_t^m + \varsigma_i^m - \xi^m = 0 \tag{25}$$

$$\varepsilon_i y_t + \phi_t^s + \varsigma_i^s - \xi^s = 0 \tag{26}$$

$$\rho \phi_t^m + \varsigma_j^m - \xi^m = 0 \tag{27}$$

$$\rho\left(\varepsilon_j y_t + \phi_t^s\right) + \varsigma_j^s - \xi^s = 0 \tag{28}$$

and the complementary slackness conditions

$$\xi^m(a_{ti}^m + a_{tj}^m - \underline{a}_{ti^*}^m - \underline{a}_{tj}^m) = 0 \tag{29}$$

$$\xi^s(a_{ti}^s + a_{tj}^s - \underline{a}_{ti^*}^s - \underline{a}_{tj}^s) = 0 \tag{30}$$

$$\rho(\varepsilon_j y_t \underline{a}_{tj}^s + \phi_t^m \underline{a}_{tj}^m + \phi_t^s \underline{a}_{tj}^s - \varepsilon_j y_t a_{tj}^s - \phi_t^m a_{tj}^m - \phi_t^s a_{tj}^s) = 0 \tag{31}$$

$$\varsigma_i^m \underline{a}_{ti^*}^m = 0 \tag{32}$$

$$\varsigma_i^s \underline{a}_{ti^*}^s = 0 \tag{33}$$

$$\varsigma_j^m \underline{a}_{tj}^m = 0 \tag{34}$$

$$\varsigma_j^s \underline{a}_{tj}^s = 0. \tag{35}$$

If $\xi^m = 0$, (25) implies $0 < \phi_t^m = -\varsigma_i^m \leq 0$, a contradiction. If $\xi^s = 0$, (26) implies $0 < \varepsilon_i y_t + \phi_t^s = -\varsigma_i^s \leq 0$, another contradiction. Hence $\xi^m > 0$ and $\xi^s > 0$, so (29) and (30) imply

$$\underline{a}_{ti^*}^m + \underline{a}_{tj}^m = a_{ti}^m + a_{tj}^m \tag{36}$$

$$\underline{a}_{ti^*}^s + \underline{a}_{tj}^s = a_{ti}^s + a_{tj}^s. \tag{37}$$

If $\rho = 0$, (27) and (28) imply $\varsigma_j^m = \xi^m > 0$ and $\varsigma_j^s = \xi^s > 0$, and (34) and (35) imply $\underline{a}_{tj}^m = \underline{a}_{tj}^s = 0$. From investor's j individual rationality constraint, this can only be a solution if $a_{tj}^m = a_{tj}^s = 0$, and if this is the case (29) and (30) imply $(\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s) = (a_{ti}^m, a_{ti}^s)$. Hereafter suppose $\rho > 0$ which using (31) implies

$$\phi_t^m \underline{a}_{tj}^m + (\varepsilon_j y_t + \phi_t^s)\underline{a}_{tj}^s = \phi_t^m a_{tj}^m + (\varepsilon_j y_t + \phi_t^s) a_{tj}^s. \tag{38}$$

If $\varsigma_i^m > 0$ and $\varsigma_j^m > 0$, (32) and (34) imply $\underline{a}_{ti^*}^m = \underline{a}_{tj}^m = 0$ which by (36), is only possible if $a_{ti}^m = a_{tj}^m = 0$. But then (38) implies $\underline{a}_{tj}^s = a_{tj}^s$, and (37) implies $\underline{a}_{ti^*}^s = a_{ti}^s$. Similarly, if $\varsigma_i^s > 0$ and $\varsigma_j^s > 0$, (33) and (35) imply $\underline{a}_{ti^*}^s = \underline{a}_{tj}^s = 0$ which by (37), is only possible if $a_{ti}^s = a_{tj}^s = 0$. But then (38) implies $\underline{a}_{tj}^m = a_{tj}^m$, and (36) implies $\underline{a}_{ti^*}^m = a_{ti}^m$. If $\varsigma_i^m > 0$ and $\varsigma_i^s > 0$, then (32) and (33) imply $\underline{a}_{ti^*}^m = \underline{a}_{ti^*}^s = 0$, and according to (36), (37) and (38), this is only possible if $a_{ti}^m = a_{ti}^s = 0$. Conditions (36) and (37) in turn imply $(\underline{a}_{tj}^m, \underline{a}_{tj}^s) = (a_{tj}^m, a_{tj}^s)$. Similarly, if $\varsigma_j^m > 0$ and $\varsigma_j^s > 0$, then (34) and (35) imply $\underline{a}_{tj}^m = \underline{a}_{tj}^s = 0$, and according to (38) this is only possible if $a_{tj}^m = a_{tj}^s = 0$. Conditions (36) and (37) in turn imply $(\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s) = (a_{ti}^m, a_{ti}^s)$. So far we have simply verified that there is no trade between investors i and j, i.e., $(\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s) = (a_{ti}^m, a_{ti}^s)$ and $(\underline{a}_{tj}^m, \underline{a}_{tj}^s) = (a_{tj}^m, a_{tj}^s)$, if $a_{ti}^m = a_{tj}^m = 0$, or $a_{ti}^s = a_{tj}^s = 0$, or $a_{ti}^m = a_{ti}^s = 0$, or $a_{tj}^m = a_{tj}^s = 0$. Thus there are seven binding patterns for $(\varsigma_i^m, \varsigma_i^s, \varsigma_j^m, \varsigma_j^s)$ that remain to be considered.

(i) $\varsigma_i^m = \varsigma_i^s = \varsigma_j^m = \varsigma_j^s = 0$. Conditions (25)-(28) imply that this case is only possible if $\varepsilon_i = \varepsilon_j$, and conditions (36), (37) and (38), imply that the solution consists of any pair of post trade portfolios $(\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s)$ and $(\underline{a}_{tj}^m, \underline{a}_{tj}^s)$ that satisfy

$$\begin{aligned}
\underline{a}_{tj}^m &= a_{tj}^m - \frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m} \left(a_{ti}^s - \underline{a}_{ti^*}^s\right) \\
\underline{a}_{ti^*}^m &= a_{ti}^m + \frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m} \left(a_{ti}^s - \underline{a}_{ti^*}^s\right) \\
\underline{a}_{tj}^s &= a_{ti}^s + a_{tj}^s - \underline{a}_{ti^*}^s \\
\underline{a}_{ti^*}^s &\in \left[a_{ti}^s - \min\left(\frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s} a_{tj}^m, a_{ti}^s\right), a_{ti}^s + \min\left(\frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s} a_{ti}^m, a_{tj}^s\right)\right].
\end{aligned}$$

(ii) $\varsigma_i^s = \varsigma_j^m = \varsigma_j^s = 0 < \varsigma_i^m$. Condition (32) implies $\underline{a}_{ti^*}^m = 0$, and from (36) we obtain $\underline{a}_{tj}^m = a_{ti}^m + a_{tj}^m$. Then condition (38) yields

$$\underline{a}_{tj}^s = a_{tj}^s - \frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s} a_{ti}^m$$

and condition (37) implies

$$\underline{a}_{ti^*}^s = a_{ti}^s + \frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s} a_{ti}^m.$$

Notice that $\varsigma_j^s = 0$ requires $\underline{a}_{tj}^s \geq 0$ which is equivalent to

$$\phi_t^m a_{ti}^m \leq (\varepsilon_j y_t + \phi_t^s)\, a_{tj}^s.$$

Conditions (25)-(28) imply $\varsigma_i^m = (\varepsilon_i - \varepsilon_j)\, y_t \frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s}$, so $\varsigma_i^m > 0$ requires $\varepsilon_j < \varepsilon_i$.

(iii) $\varsigma_i^m = \varsigma_j^m = \varsigma_j^s = 0 < \varsigma_i^s$. Condition (33) implies $\underline{a}_{ti^*}^s = 0$, and from (37) we obtain $\underline{a}_{tj}^s = a_{ti}^s + a_{tj}^s$. Then condition (38) yields

$$\underline{a}_{tj}^m = a_{tj}^m - \frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m} a_{ti}^s$$

and condition (36) implies

$$\underline{a}_{ti^*}^m = a_{ti}^m + \frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m} a_{ti}^s.$$

Notice that $\varsigma_j^m = 0$ requires $\underline{a}_{tj}^m \geq 0$ which is equivalent to

$$(\varepsilon_j y_t + \phi_t^s)\, a_{ti}^s \leq \phi_t^m a_{tj}^m.$$

Conditions (25)-(28) imply $\varsigma_i^s = (\varepsilon_j - \varepsilon_i)\, y_t$, so $\varsigma_i^s > 0$ requires $\varepsilon_i < \varepsilon_j$.

(iv) $\varsigma_i^m = \varsigma_i^s = \varsigma_j^s = 0 < \varsigma_j^m$. Condition (34) implies $\underline{a}_{tj}^m = 0$, and from (36) we obtain $\underline{a}_{ti^*}^m = a_{ti}^m + a_{tj}^m$. Then (37) and (38) imply

$$\underline{a}_{tj}^s = a_{tj}^s + \frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s} a_{tj}^m$$

$$\underline{a}_{ti^*}^s = a_{ti}^s - \frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s} a_{tj}^m.$$

Notice that $\varsigma_i^s = 0$ requires $\underline{a}_{ti^*}^s \geq 0$ which is equivalent to

$$\phi_t^m a_{tj}^m \leq (\varepsilon_j y_t + \phi_t^s)\, a_{ti}^s.$$

Conditions (25)-(28) imply $\varsigma_j^m = (\varepsilon_j - \varepsilon_i)\, y_t \frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s}$, so $\varsigma_j^m > 0$ requires $\varepsilon_i < \varepsilon_j$.

(v) $\varsigma_i^m = \varsigma_i^s = \varsigma_j^m = 0 < \varsigma_j^s$. Condition (35) implies $\underline{a}_{tj}^s = 0$, and from (37) we obtain $\underline{a}_{ti^*}^s = a_{ti}^s + a_{tj}^s$. Then (36) and (38) imply

$$\underline{a}_{tj}^m = a_{tj}^m + \frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m} a_{tj}^s$$
$$\underline{a}_{ti^*}^m = a_{ti}^m - \frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m} a_{tj}^s.$$

Notice that $\varsigma_i^m = 0$ requires $\underline{a}_{ti^*}^m \geq 0$ which is equivalent to

$$(\varepsilon_j y_t + \phi_t^s)\, a_{tj}^s \leq \phi_t^m a_{ti}^m.$$

Conditions (25)-(28) imply $\varsigma_j^s = (\varepsilon_i - \varepsilon_j)\, y_t$, so $\varsigma_j^s > 0$ requires $\varepsilon_j < \varepsilon_i$.

(vi) $\varsigma_i^m, \varsigma_j^s \in \mathbb{R}_{++}$ and $\varsigma_i^s = \varsigma_j^m = 0$. In this case, conditions (32) and (35) give $\underline{a}_{ti^*}^m = \underline{a}_{tj}^s = 0$, and (36) and (37) imply $\underline{a}_{tj}^m = a_{ti}^m + a_{tj}^m$ and $\underline{a}_{ti^*}^s = a_{ti}^s + a_{tj}^s$. Condition (38) implies the following restriction must be satisfied

$$\phi_t^m a_{ti}^m = (\varepsilon_j y_t + \phi_t^s) a_{tj}^s.$$

Conditions (25)-(28) imply $\varsigma_i^m = (\rho - 1)\, \phi_t^m$ and $\varsigma_j^s = (\varepsilon_i - \varepsilon_j)\, y_t - (\rho - 1)\, (\varepsilon_j y_t + \phi_t^s)$, so $\varsigma_i^m > 0$ requires $\rho > 1$, and ς_j^s requires $\varepsilon_j < \varepsilon_i$.

(vii) $\varsigma_i^m = \varsigma_j^s = 0$ and $\varsigma_i^s, \varsigma_j^m \in \mathbb{R}_{++}$. In this case, conditions (33) and (34) give $\underline{a}_{ti^*}^s = \underline{a}_{tj}^m = 0$, and (36) and (37) imply $\underline{a}_{ti^*}^m = a_{ti}^m + a_{tj}^m$ and $\underline{a}_{tj}^s = a_{ti}^s + a_{tj}^s$. Condition (38) implies the following restriction must be satisfied

$$\phi_t^m a_{tj}^m = (\varepsilon_j y_t + \phi_t^s) a_{ti}^s.$$

Conditions (25)-(28) imply $\varsigma_j^m = (1 - \rho)\, \phi_t^m$ and $\varsigma_i^s = (\varepsilon_j - \varepsilon_i)\, y_t - (1 - \rho)\, (\varepsilon_j y_t + \phi_t^s)$, so $\varsigma_j^m > 0$ requires $\rho \in (0, 1)$, and $\varsigma_i^s > 0$ requires $\varepsilon_i < \varepsilon_j$. ■

Proof of Lemma 2. With (24) and the notation introduced in Lemma 1, (7) becomes

$$\begin{aligned} V_t\left(a_{ti}^m, a_{ti}^s, \varepsilon_i\right) = \alpha \int &\tilde{\eta}\left(\varepsilon_i, \varepsilon_j\right) \left[\phi_t^m \left(\underline{a}_{ti^*}^m - a_{ti}^m\right) + \left(\varepsilon_i y_t + \phi_t^s\right) \left(\underline{a}_{ti^*}^s - a_{ti}^s\right)\right] dH_t\left(\boldsymbol{a}_{tj}, \varepsilon_j\right) \\ &+ \alpha \int \left[1 - \tilde{\eta}\left(\varepsilon_i, \varepsilon_j\right)\right] \left[\phi_t^m \left(\underline{a}_{ti}^m - a_{ti}^m\right) + \left(\varepsilon_i y_t + \phi_t^s\right) \left(\underline{a}_{ti}^s - a_{ti}^s\right)\right] dH_t\left(\boldsymbol{a}_{tj}, \varepsilon_j\right) \\ &+ \phi_t^m a_{ti}^m + \left(\varepsilon_i y_t + \phi_t^s\right) a_{ti}^s + W_t\left(\mathbf{0}\right). \end{aligned}$$

Use $\tilde{\eta}(\varepsilon_i, \varepsilon_j) \equiv \eta \mathbb{I}_{\{\varepsilon_j < \varepsilon_i\}} + (1-\eta)\mathbb{I}_{\{\varepsilon_i < \varepsilon_j\}} + (1/2)\mathbb{I}_{\{\varepsilon_i = \varepsilon_j\}}$ and substitute the bargaining outcomes reported in Lemma 1 to obtain

$$\begin{aligned} V_t\left(a_{ti}^m, a_{ti}^s, \varepsilon_i\right) = \alpha\eta \int\int \mathbb{I}_{\{\varepsilon_j \leq \varepsilon_i\}} \Big[& -\phi_t^m \min\left\{p_t(\varepsilon_j) a_{tj}^s, a_{ti}^m\right\} \\ & + \left(\varepsilon_i y_t + \phi_t^s\right) \min\left\{\tfrac{a_{ti}^m}{p_t(\varepsilon_j)}, a_{tj}^s\right\}\Big] dF_t\left(\boldsymbol{a}_{tj}\right) dG\left(\varepsilon_j\right) \\ & + \alpha(1-\eta) \int\int \mathbb{I}_{\{\varepsilon_i < \varepsilon_j\}} \Big[\phi_t^m \min\left\{p_t(\varepsilon_j) a_{ti}^s, a_{tj}^m\right\} \\ & - \left(\varepsilon_i y_t + \phi_t^s\right) \min\left\{\tfrac{a_{tj}^m}{p_t(\varepsilon_j)}, a_{ti}^s\right\}\Big] dF_t\left(\boldsymbol{a}_{tj}\right) dG\left(\varepsilon_j\right) \\ & + \phi_t^m a_{ti}^m + \left(\varepsilon_i y_t + \phi_t^s\right) a_{ti}^s + W_t(\mathbf{0}). \end{aligned} \tag{39}$$

From (6), we anticipate that as in Lagos and Wright (2005), the beginning-of-period distribution of assets across investors will be degenerate, i.e., $(a_{t+1j}^m, a_{t+1j}^s) = (A_{t+1}^m, A^s)$ for all $j \in \mathcal{I}$, so (39) can be written as (8). ■

Proof of Lemma 3. From (8),

$$\begin{aligned} \int V_{t+1}\left(a_{t+1}^m, a_{t+1}^s, \varepsilon_i\right) dG(\varepsilon_i) = {} & \phi_{t+1}^m a_{t+1}^m + \int \left(\varepsilon_i y_{t+1} + \phi_{t+1}^s\right) a_{t+1}^s dG(\varepsilon_i) + W_{t+1}(\mathbf{0}) \\ & + \alpha\eta \int_{\left[\frac{\phi_{t+1}^m a_{t+1}^m}{A^s} - \phi_{t+1}^s\right]\frac{1}{y_{t+1}}} \int_{\varepsilon_j} \frac{(\varepsilon_i - \varepsilon_j) y_{t+1}}{\varepsilon_j y_{t+1} + \phi_{t+1}^s} \phi_{t+1}^m a_{t+1}^m dG(\varepsilon_i) dG(\varepsilon_j) \\ & + \alpha\eta \int^{\left[\frac{\phi_{t+1}^m a_{t+1}^m}{A^s} - \phi_{t+1}^s\right]\frac{1}{y_{t+1}}} \int_{\varepsilon_j} (\varepsilon_i - \varepsilon_j) y_{t+1} A^s dG(\varepsilon_i) dG(\varepsilon_j) \\ & + \alpha(1-\eta) \int_{\left[\frac{\phi_{t+1}^m A_{t+1}^m}{a_{t+1}^s} - \phi_{t+1}^s\right]\frac{1}{y_{t+1}}} \int^{\varepsilon_j} \frac{(\varepsilon_j - \varepsilon_i) y_{t+1}}{\varepsilon_j y_{t+1} + \phi_{t+1}^s} \phi_{t+1}^m A_{t+1}^m dG(\varepsilon_i) dG(\varepsilon_j) \\ & + \alpha(1-\eta) \int^{\left[\frac{\phi_{t+1}^m A_{t+1}^m}{a_{t+1}^s} - \phi_{t+1}^s\right]\frac{1}{y_{t+1}}} \int^{\varepsilon_j} (\varepsilon_j - \varepsilon_i) y_{t+1} a_{t+1}^s dG(\varepsilon_i) dG(\varepsilon_j) \end{aligned}$$

so the investor's problem (6) can be written as in (24), with

$$
\begin{aligned}
W_t(\mathbf{0}) = {} & \max_{\tilde{a}^m_{t+1}\in\mathbb{R}_+} \Bigg\{ -\phi^m_t \tilde{a}^m_{t+1} \\
& + \beta\mathbb{E}_t\Bigg[\Bigg(1+\alpha\eta\int_{\left[\frac{\phi^m_{t+1}a^m_{t+1}}{A^s}-\phi^s_{t+1}\right]\frac{1}{y_{t+1}}}\int_{\varepsilon_j}\frac{(\varepsilon_i-\varepsilon_j)\,y_{t+1}}{\varepsilon_j y_{t+1}+\phi^s_{t+1}}dG(\varepsilon_i)\,dG(\varepsilon_j)\Bigg)\phi^m_{t+1}\tilde{a}^m_{t+1} \\
& + \alpha\eta\int^{\left[\frac{\phi^m_{t+1}\tilde{a}^m_{t+1}}{A^s}-\phi^s_{t+1}\right]\frac{1}{y_{t+1}}}\int_{\varepsilon_j}(\varepsilon_i-\varepsilon_j)\,y_{t+1}dG(\varepsilon_i)\,dG(\varepsilon_j)\,A^s\Bigg]\Bigg\} \\
& + \max_{\tilde{a}^s_{t+1}\in\mathbb{R}_+}\Bigg\{-\phi^s_t\tilde{a}^s_{t+1}+\beta\mathbb{E}_t\Bigg[\Bigg(\int\left(\varepsilon_i y_{t+1}+\phi^s_{t+1}\right)dG(\varepsilon_i) \\
& + \alpha(1-\eta)\int^{\left[\frac{\phi^m_{t+1}A^m_{t+1}}{a^s_{t+1}}-\phi^s_{t+1}\right]\frac{1}{y_{t+1}}}\int^{\varepsilon_j}(\varepsilon_j-\varepsilon_i)\,y_{t+1}dG(\varepsilon_i)\,dG(\varepsilon_j)\Bigg)a^s_{t+1} \\
& + \alpha(1-\eta)\int_{\left[\frac{\phi^m_{t+1}A^m_{t+1}}{a^s_{t+1}}-\phi^s_{t+1}\right]\frac{1}{y_{t+1}}}\int^{\varepsilon_j}\frac{(\varepsilon_j-\varepsilon_i)\,y_{t+1}}{\varepsilon_j y_{t+1}+\phi^s_{t+1}}dG(\varepsilon_i)\,dG(\varepsilon_j)\,\phi^m_{t+1}A^m_{t+1}\Bigg]\Bigg\} \\
& + T_t+\beta\mathbb{E}_t W_{t+1}(\mathbf{0}),
\end{aligned}
\tag{40}
$$

where $a^s_{t+1}=\pi\tilde{a}^s_{t+1}+(1-\pi)A^s$. The first-order necessary and sufficient conditions for optimization of (40) are (9) and (10). ■

Proof of Proposition 2. In a stationary monetary equilibrium the investor's Euler equations in Lemma 3 become

$$
\mu=\bar{\beta}\left[1+\alpha\eta\int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H}\frac{\varepsilon_i-\varepsilon_j}{\varepsilon_j+\phi^s}dG(\varepsilon_i)\,dG(\varepsilon_j)\right] \tag{41}
$$

$$
\phi^s=\frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\bar{\varepsilon}+\alpha(1-\eta)\,\varphi(\varepsilon^c)\right], \tag{42}
$$

where $\varepsilon^c\equiv Z/A^s-\phi^s$ and $\varphi(\varepsilon)\equiv\int_{\varepsilon_L}^{\varepsilon}\int_{\varepsilon_L}^{\varepsilon_j}(\varepsilon_j-\varepsilon_i)\,dG(\varepsilon_i)\,dG(\varepsilon_j)$. As $\mu\to\bar{\beta}$, (41) implies

$$
\alpha\eta\int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H}\frac{\varepsilon_i-\varepsilon_j}{\varepsilon_j+\phi^s}dG(\varepsilon_i)\,dG(\varepsilon_j)\to 0,
$$

a condition that can only hold if $\varepsilon^c\to\varepsilon_H$. The fact that $\varepsilon^c\to\varepsilon_H$ implies the investor with the higher valuation purchases all his counterparty's equity holdings (the investor who wishes to buy is never constrained by his real money balances as $\mu\to\bar{\beta}$). ■

Proof of Proposition 3. In an equilibrium with no money (or no valued money), there is no trade in the OTC market. The first-order condition for an investor i in the time-t Walrasian market is

$$\phi_t^s \geq \beta\pi\mathbb{E}_t\left(\bar{\varepsilon}y_{t+1}+\phi_{t+1}^s\right), \text{ `` = '' if } \tilde{a}_{t+1i}^s > 0.$$

In a stationary equilibrium the Walrasian market for equity can only clear if $\phi_t^s = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\bar{\varepsilon}y_t$. This establishes parts (ii) in the statement of the proposition. In a stationary monetary equilibrium, the Euler equations for an investor obtained in Lemma 3 reduce to

$$\mu = \bar{\beta}\left[1+\alpha\eta\int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H}\frac{\varepsilon_i-\varepsilon_j}{\varepsilon_j+\phi^s}dG(\varepsilon_i)\,dG(\varepsilon_j)\right] \tag{43}$$

$$\phi^s = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\bar{\varepsilon}+\alpha(1-\eta)\int_{\varepsilon_L}^{\varepsilon^c}\int_{\varepsilon_L}^{\varepsilon_j}(\varepsilon_j-\varepsilon_i)\,dG(\varepsilon_i)\,dG(\varepsilon_j)\right] \tag{44}$$

where

$$\varepsilon^c \equiv \frac{Z}{A^s}-\phi^s. \tag{45}$$

Condition (43) can be substituted into (44) to obtain a single equation in the unknown ε^c, namely $\bar{T}(\varepsilon^c)=0$, where $\bar{T}:[\varepsilon_L,\varepsilon_H]\to\mathbb{R}$ is defined by

$$\bar{T}(\varepsilon^c) \equiv \bar{\beta}\alpha\eta\int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H}\frac{\varepsilon_i-\varepsilon_j}{\varepsilon_j+\frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\bar{\varepsilon}+\alpha(1-\eta)\int_{\varepsilon_L}^{\varepsilon^c}\int_{\varepsilon_L}^{\varepsilon_j}(\varepsilon_j-\varepsilon_i)dG(\varepsilon_i)dG(\varepsilon_j)\right]}dG(\varepsilon_i)\,dG(\varepsilon_j)+\bar{\beta}-\mu.$$

Notice that $\bar{T}(\varepsilon_H)=\bar{\beta}-\mu<0$ and

$$\bar{T}(\varepsilon_L) = \bar{\beta}\alpha\eta\int_{\varepsilon_L}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H}\frac{\varepsilon_i-\varepsilon_j}{\varepsilon_j+\frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\bar{\varepsilon}}dG(\varepsilon_i)\,dG(\varepsilon_j)+\bar{\beta}-\mu,$$

so since $\bar{T}$ is continuous, a stationary monetary equilibrium exists if $\mu<\tilde{\mu}$ with $\tilde{\mu}$ defined as in (12). In addition,

$$\bar{T}'(\varepsilon^c) = -\left[\bar{\beta}\alpha\eta\int_{\varepsilon^c}^{\varepsilon_H}\frac{\varepsilon_i-\varepsilon^c}{\varepsilon^c+\phi^s}dG(\varepsilon_i)\,G'(\varepsilon^c)\right.$$
$$\left.+\frac{(\bar{\beta}\alpha)^2\pi\eta(1-\eta)}{1-\bar{\beta}\pi}\int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H}\frac{(\varepsilon_i-\varepsilon_j)\int_{\varepsilon_L}^{\varepsilon^c}(\varepsilon^c-\varepsilon)dG(\varepsilon)G'(\varepsilon^c)}{(\varepsilon_j+\phi^s)^2}dG(\varepsilon_i)\,dG(\varepsilon_j)\right]$$

is negative, so a stationary monetary equilibrium exists if and only if $\mu<\tilde{\mu}$, and there cannot be more than one stationary monetary equilibrium. Condition (13) is just (44), condition (15) is $\bar{T}(\varepsilon^c)=0$, and (14) follows from (45). This establishes parts (i) and (iii). Part (iv) is immediate from (15). ■

Proof of Proposition 4. First, notice that $\partial\varepsilon^c/\partial\mu = 1/\bar{T}'(\varepsilon^c) < 0$, where $\bar{T}(\cdot)$ is the mapping defined in the proof of Proposition 3. (i) Differentiate (13) to obtain

$$\frac{\partial\phi^s}{\partial\mu} = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\alpha(1-\eta)G'(\varepsilon^c)\int_{\varepsilon_L}^{\varepsilon^c}(\varepsilon^c-\varepsilon_i)\,dG(\varepsilon_i)\frac{\partial\varepsilon^c}{\partial\mu} < 0.$$

(ii) From (14), $\partial Z/\partial\mu = (\partial\varepsilon^c/\partial\mu + \partial\phi^s/\partial\mu)A^s < 0$, and since $Z = \phi_t^m A_t^m/y_t$, $\partial\phi_t^m/\partial\mu = (\partial Z/\partial\mu)(y_t/A_t^m) < 0$. ■

Proof of Proposition 5. Implicit differentiation of $\bar{T}(\varepsilon^c) = 0$ implies

$$\frac{\partial\varepsilon^c}{\partial\alpha} = \frac{\int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H}\frac{\eta(1-\bar{\beta}\pi)(\varepsilon_i-\varepsilon_j)\left[(1-\bar{\beta}\pi)\varepsilon_j+\bar{\beta}\pi\bar{\varepsilon}\right]}{\left\{(1-\bar{\beta}\pi)\varepsilon_j+\bar{\beta}\pi[\bar{\varepsilon}+\alpha(1-\eta)\varphi(\varepsilon^c)]\right\}^2}dG(\varepsilon_i)dG(\varepsilon_j)}{\int_{\varepsilon^c}^{\varepsilon_H}\frac{\alpha\eta(1-\bar{\beta}\pi)(\varepsilon_i-\varepsilon_j)}{(1-\bar{\beta}\pi)\varepsilon^c+\bar{\beta}\pi[\bar{\varepsilon}+\alpha(1-\eta)\varphi(\varepsilon^c)]}dG(\varepsilon_i)G'(\varepsilon^c)+\int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H}\frac{\bar{\beta}\pi\alpha^2\eta(1-\eta)(1-\bar{\beta}\pi)(\varepsilon_i-\varepsilon_j)\varphi'(\varepsilon^c)}{\left\{(1-\bar{\beta}\pi)\varepsilon_j+\bar{\beta}\pi[\bar{\varepsilon}+\alpha(1-\eta)\varphi(\varepsilon^c)]\right\}^2}dG(\varepsilon_i)dG(\varepsilon_j)} > 0.$$

(i) Differentiate (13) to arrive at

$$\frac{\partial\phi^s}{\partial\alpha} = \frac{\bar{\beta}\pi(1-\eta)}{1-\bar{\beta}\pi}\left[\varphi(\varepsilon^c)+\alpha\int_{\varepsilon_L}^{\varepsilon^c}(\varepsilon^c-\varepsilon_i)\,dG(\varepsilon_i)\,dG(\varepsilon^c)\frac{\partial\varepsilon^c}{\partial\alpha}\right] > 0.$$

(ii) From (14),

$$\frac{\partial Z}{\partial\alpha} = \left(\frac{\partial\varepsilon^c}{\partial\alpha}+\frac{\partial\phi^s}{\partial\alpha}\right)A^s > 0,$$

and since $Z = \phi_t^m A_t^m/y_t$, it follows that $\partial\phi_t^m/\partial\alpha > 0$. ■

Proof of Proposition 6. Rewrite $\mathcal{V}$ as

$$\mathcal{V} = \alpha A^s\int_{\varepsilon_L}^{\varepsilon^c}\{\eta[1-G(\varepsilon_i)]+(1-\eta)G(\varepsilon_i)\}\,dG(\varepsilon_i)$$
$$+\alpha A^s\int_{\varepsilon^c}^{\varepsilon_H}\{\eta[1-G(\varepsilon_i)]+(1-\eta)G(\varepsilon_i)\}\frac{\varepsilon^c+\phi^s}{\varepsilon_i+\phi^s}dG(\varepsilon_i).$$

Differentiate to obtain

$$\frac{\partial\mathcal{V}}{\partial\varepsilon^c} = \alpha A^s\int_{\varepsilon^c}^{\varepsilon_H}\{\eta[1-G(\varepsilon_i)]+(1-\eta)G(\varepsilon_i)\}\frac{\partial}{\partial\varepsilon^c}\left[\frac{\varepsilon^c+\phi^s}{\varepsilon_i+\phi^s}\right]dG(\varepsilon_i),$$

where

$$\frac{\partial}{\partial\varepsilon^c}\left[\frac{\varepsilon^c+\phi^s}{\varepsilon_i+\phi^s}\right] = \frac{\varepsilon_i+\phi^s+(\varepsilon_i-\varepsilon^c)\frac{\partial\phi^s}{\partial\varepsilon^c}}{(\varepsilon_i+\phi^s)^2}A^s > 0 \text{ for } \varepsilon_i > \varepsilon^c.$$

Hence, $\partial\mathcal{V}/\partial\varepsilon^c > 0$. Thus $\partial\mathcal{V}/\partial\mu = (\partial\mathcal{V}/\partial\varepsilon^c)(\partial\varepsilon^c/\partial\mu) < 0$, since $\partial\varepsilon^c/\partial\mu < 0$ (see proof of Proposition 4), which establishes (*i*). For part (*ii*), simply notice that $\partial\mathcal{V}/\partial\alpha = \mathcal{V}/\alpha + (\partial\mathcal{V}/\partial\varepsilon^c)(\partial\varepsilon^c/\partial\alpha) > 0$. ■

Proof of Proposition 8. (*i*) $\partial\mathcal{P}/\partial\mu = \left[\bar{\beta}\pi/(1-\bar{\beta}\pi)\right]\alpha(1-\eta)\varphi'(\varepsilon^c)(\partial\varepsilon^c/\partial\mu) < 0$. (*ii*) $\partial\mathcal{P}/\partial\alpha = \left[\bar{\beta}\pi/(1-\bar{\beta}\pi)\right](1-\eta)\{\alpha\varphi'(\varepsilon^c)(\partial\varepsilon^c/\partial\alpha) + \varphi(\varepsilon^c)\} > 0$. ■

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