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RESOURCES, CONFLICT, AND ECONOMIC DEVELOPMENT IN AFRICA

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ABSTRACT

Natural resources have driven both growth and conflict in modern Africa. We model the interaction of parties engaged in potential conflict over such resources. The likelihood of conflict depends on both the absolute and relative resource endowments of the parties. Resources fuel conflict by raising the gains from expropriation and by increasing fighting strength. Economic prosperity, as a result, is a function of equilibrium conflict prevalence determined not just by a region's own resources but also by the resources of its neighbors. Using high-resolution spatial data on resources, conflicts, and nighttime illumination across the whole of sub-Saharan Africa, we find evidence confirming each of the model's predictions. Structural estimates of the model reveal that conflict equilibria are more prevalent where institutional quality (measured by, e.g., risk of expropriation, property rights, voice and accountability) is worse.

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1 Introduction

No understanding of the development of modern Africa would be complete without an appreciation of the importance of natural resources. Rents from natural resources can drive economic growth; yet the countries with the highest resource endowments tend to have the slowest rates of growth (Gylfason, 2001; Sachs and Warner, 2001). One reason for this is that as the gains from expropriating resources rise, conflict becomes more likely (Buonanno et al., 2015; Caselli et al., 2015; De La Sierra, 2015; Dube and Vargas, 2013; Fearon, 2005). Similarly, resources can empower the state and its rivals, and can be used to fuel repressive and destructive activities (Acemoglu and Robinson, 2001; Caselli and Tesei, 2016; Mitra and Ray, 2014; Nunn and Qian, 2014). Where these motives are salient, and where conflict is destructive enough, resource windfalls may indeed hamper economic development (Bannon and Collier, 2003).

Critical in these arguments is the idea of strategic interaction between rival factions in the face of economic incentives. Conflict and growth are outcomes of a game in which both a party's own resources as well as the resources of rivals determine equilibrium choices (Esteban et al., 2012; Esteban and Ray, 2011a,b; Grossman, 1991; Mitra and Ray, 2014). How does the joint distribution of resources across potential rivals work to foment or prevent conflict, and what economic outcomes ensue in equilibrium? These questions are the focus of our paper.

As evidence on the importance of resources across potential rivals in the African context, consider the following correlations in the spatial distributions of conflict prevalence, economic development, and resources. In Figure 1, we plot the log density of nighttime illumination (a proxy for prosperity) against a resource index for 0.5° by 0.5° grid cells across the entirety of sub-Saharan Africa, grouped into percentile bins.¹ Importantly, to capture the idea that resource endowments of potential rivals matter, we split all neighboring grid-cells within a 500km radius into being either above or below the median value of this resource index.

We see in Figure 1 a remarkable difference in the relationship between natural resources and prosperity by neighbors' resources. With resource-rich neighbors, there is a clear inverse U-shaped relationship between natural resources and the density of nighttime illumination – after a certain point, when neighbors are rich, one's own resources become bad for economic development.² In contrast, with resource-poor neighbors, there is an increasing and fairly linear relationship between natural resources and nighttime illumination. That is, when neighbors are relatively poor, resources continue to be good for development at all levels.

Figure 2 helps to explain part of these divergent relationships. Here, we graph conflict incidence

¹The resource index is comprised of the first principal component of (i) annual rainfall averaged over a ten year period (1998-2008), (ii) oil or gas reserves, (iii) lootable diamonds, (iv) gold, (v) zinc, and (vi) cobalt in the 0.5×0.5 degree grid cell.

²This is similar to, for example, Figure 1 in Sachs and Warner (2001).

against the resource index for sub-Saharan African countries in the same time period, again splitting the sample by neighbors’ resources. The resulting relationship is substantially more positive for resource-rich neighbors – that is, natural resources fuel conflict to a greater extent when potential rivals also have greater resources. If the impacts of conflict on development are destructive enough, then the increased likelihood of conflict could dominate the positive effects of natural resources to produce the inverse U-shaped relationship we see in Figure 1.

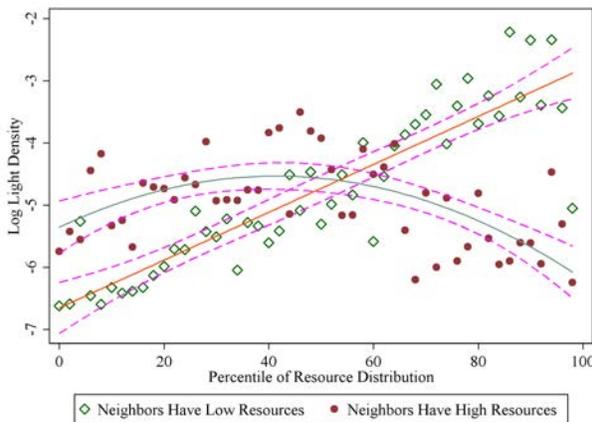


Figure 1: Log Light Density vs. Resources

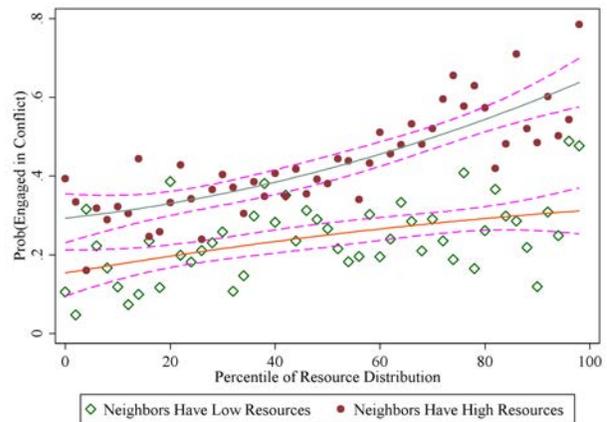


Figure 2: Prob(Conflict) vs. Resources

Observations for 0.5° by 0.5° grid cells across Sub Saharan Africa, as binned averages. Graphs plot quadratic fits with confidence intervals of the relationship between a resource index against (1) Log (Light Density) in 2008 and (2) Whether the region was engaged in conflict between 1998 and 2008. The Resource Index consists of the first principal component of (i) annual rainfall averaged over a ten year period (1998-2008), (ii) oil or gas reserves, (iii) lootable diamonds, (iv) gold, (v) zinc, and (vi) cobalt in the 0.5×0.5 degree grid cell. Confidence intervals are estimated over the underlying data rather than binned averages. The slopes for each sub-group are statistically different in each graph. Neighbors are defined as grid cells in a 500km radius.

In this paper, we formalize these linkages and shed light on the underlying mechanisms through the lens of strategic interaction. We present a simple game-theoretic model in which two groups decide whether or not to engage in conflict. Rather than incorporating mechanisms separately as much of the literature does, we simultaneously integrate several mechanisms into a single model that can be tested empirically. We model both supply-side channels that determine the ease of engaging in conflict, and demand-side channels that capture the benefits of acquiring territory.

In our model, offensive and defensive capabilities for each group increase with resource endowments. This endowment effect of resources on conflict is a feature of early models of conflict (e.g., Grossman and Kim (1995); Hirshleifer (1989)) though it has received limited attention in more recent empirical work, with the exception of a handful of important recent studies (Caselli et al., 2015; Dube and Vargas, 2013; Mitra and Ray, 2014). In addition to this endowment effect, we explicitly model a “rapacity effect,” by stipulating that each group’s return to fighting is increasing

in the neighboring group’s resources.³ In our baseline model, there is a fixed cost of participating in conflict.⁴ Conflict arises as an outcome if either group chooses to fight. If both fight, the probability of success is determined by the relative strength of each group, which itself depends on resource endowments.

Nash equilibria in this model are determined by the resource endowments of each group (along with other fundamentals such as the cost of fighting and the fraction expropriated when winning). When both groups have low levels of resources, peace results. This is because neither group has much strength, and the gains from fighting are also not high for either group, since contestable resources are few. When one group has more resources (loosely speaking) than the other, a one-sided conflict equilibrium results – what one might call an “uncontested attack.” When both groups have abundant resources, both are impelled to conflict. In line with the mechanisms outlined in [Acemoglu and Johnson \(2005\)](#); [Acemoglu et al. \(2001, 2005\)](#), we model the quality of institutions as shifting the cutoffs for conflict onset, by either changing the costs of war or the fraction of appropriable resources, or both.

We test the model’s predictions using disaggregated spatial data on resource endowments, conflict, and satellite data on nighttime lights. Rather than focusing on countries, we partition sub-Saharan Africa into a 0.5 x 0.5 degree grid. At each point, we match the likelihood of conflict events and the intensity of nighttime lights to a “natural resource” indicator (which equals 1 if any natural resource from the following is present: oil and natural gas reserves, deposits of “lootable” diamonds, gold, zinc, and cobalt) at that point. We also use historical rainfall patterns as an alternative agricultural measure of long-term wealth accumulation and resource abundance. We then match these points i to every neighbor j within a given radius. We use two sources of data on conflict, from (a) the Peace Research Institute Oslo (PRIO), which allows us to measure the spatial intensity of conflicts, and (b) the Armed Conflict Location & Event Data Project (ACLED), which allows us to focus specifically on territorial conflicts. Using these data, we ascertain whether two regions were involved in joint conflict over the past 10 years.

The model’s main predictions amount to a partitioning of the ij “resource space” into Nash equilibria regions defined by resource thresholds. Both a region’s own resources and also the resources in surrounding areas jointly determine the likelihood of conflict. In the empirical analysis, we begin by drawing a heat map of the raw data on the involvement of shared conflict for points i and j over the resource index for these points. In this simple plot, we find striking confirmation of the model’s implications regarding equilibria regions over the ij resource space. Groups represented by our disaggregated points i and j behave in a manner markedly consistent with the predictions of our simple model.

³We extend the model by incorporating a sharing rule, and show within this augmented framework that societies who share more are less likely to choose conflict.

⁴We explore other functional forms in an extension of the model that incorporates the idea that the opportunity cost of engaging in conflict depends on the resource endowment, as in, e.g., [Brückner and Ciccone \(2010\)](#); [Hsiang et al. \(2013\)](#); [Jia \(2014\)](#); [Miguel and Satyanath \(2011\)](#); [Miguel et al. \(2004\)](#).

In a regression framework, we test these implications by estimating the relationships between i - and j -specific natural resource endowments with whether or not regions i and j were involved in the same conflict. In keeping with recent work that finds time-invariant characteristics are often better at predicting conflict than shocks (Bazzi et al., 2017), we focus on the longer-run “endowment” of resources in the cross-section. To address concerns regarding identification that arise from the use of the cross-sectional dimension of the data rather than time-varying shocks, we show that our results are robust to controlling for local geographic, agricultural, and climatological characteristics, as well as spatial fixed effects of varying size. Standard errors are clustered using conservatively-defined geographic levels to account for potential spatial correlation in the error term. When testing the model using rainfall data, we find a (two-dimensional) structural break in the relationship between region i and j ’s historical rainfall patterns on the one hand and conflict on the other.⁵

The results of this analysis are in line with the heat map evidence and in strong support of the model’s predictions. Own and neighbor resource endowments are both statistically and economically significant determinants of the spatial distribution of conflict in sub-Saharan Africa. These patterns are consistent when analyzing many different resources and both sources of conflict data, further alleviating concerns of omitted confounders driving the results. We complement this evidence with procedures relying on optimal bandwidth regression discontinuity (RD) methods (Calonico et al., 2014) to measure the rise in the likelihood of conflict when crossing the resource threshold from a peace to a conflict equilibrium region.

We also investigate contexts in which resources *do not* lead to conflict. In keeping with the importance of institutions as mediators of conflict and development (Acemoglu and Johnson, 2005; Acemoglu et al., 2001, 2005; Barro, 1996; Caselli and Tesei, 2016; Mehlum et al., 2006), we show that for ij pairs where baseline institutional quality (measured in alternative specifications by property rights, risk of expropriation, political stability, and voice and accountability) is lower the estimated resource value at which conflict ensues is lower (i.e., conflict is more likely to break out for smaller resource endowments) and the explanatory power of our model is substantially higher. These results suggest that good institutions at baseline potentially raise the costs of conflict and/or lower the gains from expropriation, and weaken the link between resources and conflict overall. In particular, the roles played by rapacity and relative strength in conflict may become less important in the presence of stronger baseline property rights and lower risks of expropriation.

Finally, we estimate analogous regression equations for satellite data on nighttime lights (Henderson et al., 2012; Michalopoulos and Papaioannou, 2013; Pinkovskiy and Sala-i Martin, 2015) to highlight how the resource-conflict dependence results in a non-monotonic reduced form relationship, seen in Figure 1, between resource abundance and development (Sala-i Martin and Subramanian, 2013). Additional evidence using regression discontinuity methods and two-stage

⁵Our empirical approach is an extension of structural break methods used by Card et al. (2008) and Gonzalo and Wolf (2005), in which we use two-thirds of the sample to find the optimal cutoff and the remaining one-third to perform regression analysis using the estimated cutoff value.

least squares analyses supports this story, showing that as we move across regions on either side of the optimally determined threshold the rise in conflict correspondingly leads to a sharp drop in light density.

Our main contribution is to the literature on natural resources and conflict. The closest papers to ours in this large literature are [Besley and Persson \(2010\)](#); [Caselli et al. \(2015\)](#); [Harari and La Ferrara \(2013\)](#), and [Berman et al. \(2017\)](#). We build on these studies in several ways. First, we highlight the importance of strategic interaction in determining conflict prevalence and levels of development across space, simultaneously incorporating several mechanisms from this literature into a single theoretical model with clear testable predictions. Second, like [Harari and La Ferrara \(2013\)](#) and [Berman et al. \(2017\)](#), our analysis is on a granular (grid point) level and spans the entirety of sub-Saharan Africa. What is different is that we analyze point pairs in potential joint conflict, drawing focus on strategic interaction effects in the determination of equilibrium conflict. We also show consistent results for a broad set of natural resources, allowing for greater generalizability. Third, of the studies mentioned, only [Besley and Persson \(2010\)](#) explicitly model and examine equilibrium impacts on economic development; we are able to study this outcome at a highly granular level using satellite data on nighttime illumination.

We also contribute to the literature on the determinants of growth ([Barro, 1991](#); [Jones, 2016](#)). While both natural resources and conflict have long been regarded as playing key roles in the economic growth of developing nations ([Rodrik, 1999](#); [Sachs and Warner, 2001](#)), few papers have disciplined the interrelationships between resources, conflict, and growth through a model of strategic interaction, and tested its predictions across a large set of countries incorporating data on resource endowments of many types. We are also able to directly incorporate the role of property-related institutions in the model to study the potential for institutional quality to modulate the aforementioned relationships ([Mehlum et al., 2006](#)).

The remainder of the paper is structured as follows. Section 2 sets up our model and delivers its main predictions through a set of lemmas and propositions. Section 3 describes our data. Section 4 details our empirical strategy, and section 5 presents and interprets our results. Finally, section 6 is a concluding discussion.

2 Model

We model the interaction of two parties, i and j , who play a symmetric, simultaneous game that determines peace or conflict between them. Our static model generates testable empirical predictions, and extensions to the basic model, including heterogeneity in institutional structures, provide additional refinements to our predictions.⁶ Importantly, we integrate various mechanisms by which

⁶We do not show any dynamics. A repeated game generates more equilibria including the Nash equilibria from the static model we highlight here. We stick to the static version given the ambiguity in equilibria in the dynamic

resources may affect conflict into one model. We add additional mechanisms to the baseline setup and show that our primary predictions stay the same.

The parties choose strategies s from the set $\{F, N\}$, where F denotes the decision to fight (i.e., engage in conflict), and N denotes the decision not to fight. We denote a strategy profile by (s_i, s_j) for $s_i, s_j \in \{F, N\}$.

Each party is endowed with resource wealth, denoted $r_i, r_j \in (0, \infty)$ for resources in i and j , respectively. If neither party fights (N, N) , each keeps its own wealth. If a party fights, it expends fixed cost c in conflict, which we assume for simplicity is the same for i and j . If a party wins, it seizes a fraction δ of the opposing party's wealth. Both c and δ vary with institutional quality, like the risk of expropriation and property rights.

If one party fights and the other chooses not to fight $((F, N)$ or $(N, F))$, the fighting party succeeds with probability 1. If, on the other hand, both parties choose to fight (F, F) , then with probability $p \equiv \frac{r_i}{r_i + r_j}$ party i wins.⁷ If i wins in this scenario, it seizes a proportion δ of j 's remaining assets, (i.e. $\delta(r_j - c)$).

The game is summarized in Figure 3. Note that in (F, F) , we evaluate the expected payoff to each party given probability of success p defined above.

Figure 3: The payoff-matrix for the game between i and j .

		j	
		F	N
i	F	$p(r_i - c + \delta(r_j - c)) + (1 - p)(1 - \delta)(r_i - c),$ $(1 - p)(r_j - c + \delta(r_i - c)) + p(1 - \delta)(r_j - c)$	$r_i - c + \delta r_j,$ $(1 - \delta)r_j$
	N	$(1 - \delta)r_i,$ $r_j - c + \delta r_i$	$r_i,$ r_j

Notes: p is the probability of victory for party i , r_k are the level of resources for parties $k = \{i, j\}$, c is the cost of engaging in conflict, and δ is the fraction of resources that the victorious party expropriates.

2.1 Best Responses

The best responses of each party to the other's actions depend on the model parameters, and in particular the realizations of wealth r_i and r_j . The following lemma determines the best response functions (denoted $BR_k(s_{-k})$ for $k \in \{i, j\}$) for i and j with wealth $(r_i, r_j) \in \mathbb{R}_+^2$.

Proposition 2.1 *The following are best response functions for agent k :*

model, the strong adherence of the data with the simple static model, and our empirical set-up of cross-sectional relationships.

⁷We choose this functional form for p for its parsimony and because intuitively p should be increasing in r_i and decreasing in r_j . Recent work on conflict uses similar functions or simple transformations of this function (Andersen et al., 2018).

$$1. BR_k(s_{-k} = N) = \begin{cases} F, & \text{if } r_{-k} > \frac{c}{\delta} \\ N, & \text{else} \end{cases}$$

$$2. \text{ Let } \psi(r_k) := \frac{-\delta r_k^2 + c(1+\delta)r_k}{\delta r_k - c(1-\delta)}.$$

$BR_k(s_{-k} = F) = F$, for all (r_k, r_{-k}) such that

$$\{(r_k, r_{-k}) : r_k \in (c\frac{1-\delta}{\delta}, \infty), r_{-k} > \psi(r_k)\} \quad (1)$$

And $BR_k(s_{-k} = F) = N$, for all (r_k, r_{-k}) such that

$$\{(r_k, r_{-k}) : r_k \in (0, c\frac{1-\delta}{\delta})\} \cup \{(r_k, r_{-k}) : r_k \in (c\frac{1-\delta}{\delta}, \infty), r_{-k} < \psi(r_k)\} \quad (2)$$

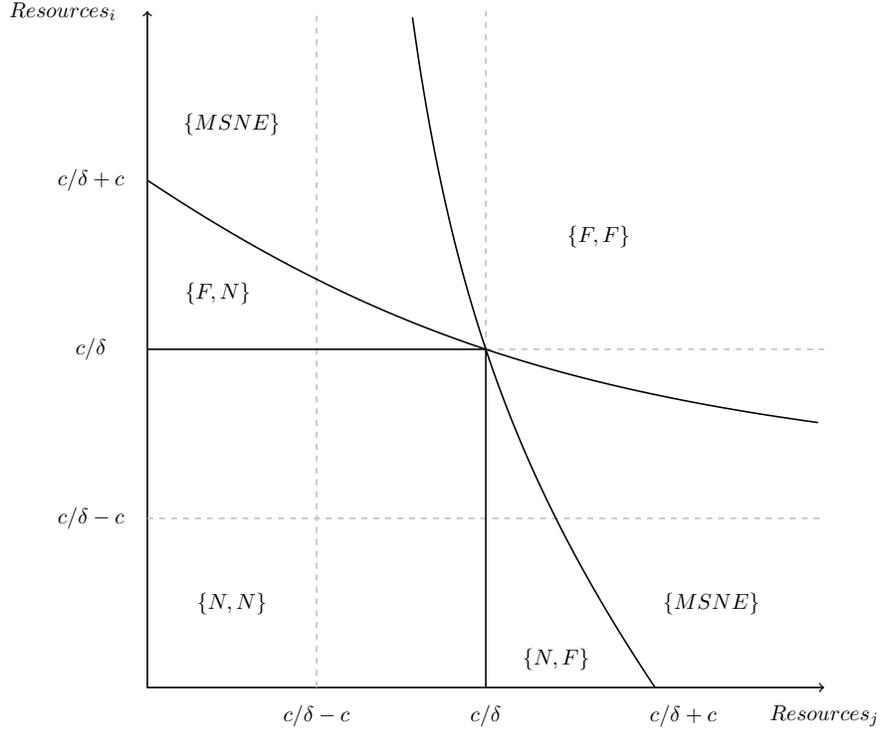
2.2 Equilibria

These best response functions help characterize the set of pure strategy Nash Equilibria in the (r_i, r_j) space. Figure 4 divides the (r_i, r_j) space into the Nash Equilibrium regions. The space can then be described by the following lemmas which are proved in Appendix A.1.:

1. For $r_i, r_j \in (0, \frac{c}{\delta})$, (N, N) is the unique pure-strategy Nash Equilibrium.
2. (F, F) is the unique pure strategy Nash Equilibrium in the region $\{(r_i, r_j) : r_i \in (c\frac{1-\delta}{\delta}, \infty), r_j > \psi(r_i)\} \cap \{(r_i, r_j) : r_j \in (c\frac{1-\delta}{\delta}, \infty), r_i > \psi(r_j)\}$
3. (N, F) is the unique pure strategy Nash Equilibrium in the region $\{(r_i, r_j) : r_i \in (\frac{c}{\delta}, \infty), r_j < \psi(r_i)\}$
4. (F, N) is the unique pure strategy Nash Equilibrium in the region $\{(r_i, r_j) : r_j \in (\frac{c}{\delta}, \infty), r_i < \psi(r_j)\}$
5. \exists a unique mixed-strategies Nash Equilibrium (MSNE) in the region $\{(r_i, r_j) : r_j \in (\frac{c}{\delta}, \psi(r_i)), r_i > \psi(r_j)\} \cup \{(r_i, r_j) : r_i \in (\frac{c}{\delta}, \psi(r_j)), r_j > \psi(r_i)\}$

Intuitively, these lemmas organize the (r_i, r_j) plane into several regions summarized in Figure 4. In the convex hull comprised of large realizations of wealth for both parties, each party's dominant strategy is F . This is brought on by two motives. First, when i and j both have high levels of resource wealth, but i has relatively more, it is prone to fight because the probability of success in capturing some of j 's wealth is relatively high. On the other hand, when j has relatively more, i prefers fighting because if it does win, it captures some of j 's considerable wealth. The intuition behind the proposition that i wishes to fight j when j has higher wealth comes from the 'rapacity

Figure 4: Nash Equilibria in the (r_i, r_j) space



The figure plots the Nash equilibrium regions for any given draw of resources for parties i and j . c is the cost of engaging in conflict, and δ is the fraction of resources that the victorious party expropriates.

effect’, where i wishes to capture a fraction of j ’s larger resource pie. The intuition behind the finding that i wishes to fight when i has higher wealth comes from the ‘relative strength’ mechanism where i has more resources to build a stronger army and therefore a higher probability of victory against j .

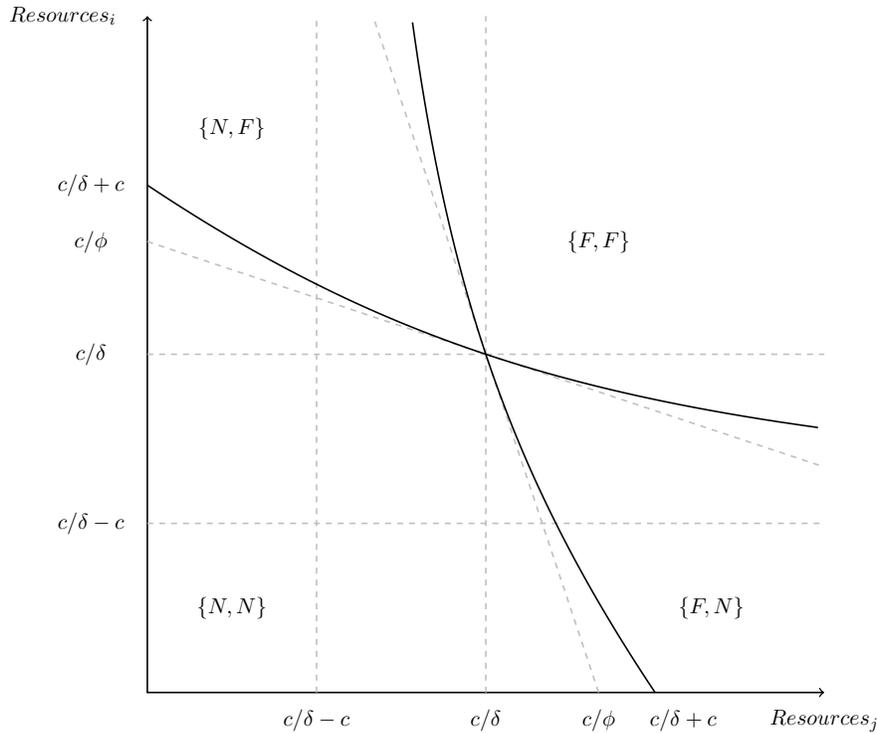
Notice that the cutoffs depend on fundamentals like c and δ . Better institutions may raise the cost of war c or lower the amount that can be appropriated δ , raising the cutoffs and enlarging the (N, N) region of the figure. We explicitly test for how institutional quality affects the structural parameters of the model, with the prediction that stronger institutions raise the cutoff.

2.3 A Sharing Rule

The possibility that conflict can be mitigated by the sharing of resources can be captured by a sharing rule, whereby each party shares a proportion ϕ of their wealth with the other party if and only if neither party chooses to fight. This changes the payoffs in the (N, N) portion of the game to be $(1 - \phi)r_i + \phi r_j$ and $(1 - \phi)r_j + \phi r_i$. That is, in the absence of anybody fighting, party i receives $(1 - \phi)$ of its own resources, and a ϕ portion of party j ’s resources. The modified game is presented

in Appendix Figure A1. This sharing rule expands the region of the (N, N) Nash Equilibrium as can be seen in Figure 5.⁸

Figure 5: Pure-strategy Nash Equilibria in the (r_i, r_j) space with the Sharing-Rule



The figure plots the Nash equilibrium regions for any given draw of resources for parties i and j , in the presence of a sharing rule. c is the cost of engaging in conflict, and δ is the fraction of resources that the victorious party expropriates. ϕ is the fraction of resources shared.

The best response functions, game matrices, proofs of propositions, and a description of the Nash Equilibrium regions under the sharing rule can be found in Appendix A.2. Intuitively, the easier it is to trade and share the fruits of higher resources with your neighbors, the lower is the likelihood of conflict. Importantly, the model's predictions of the likelihood of conflict in different regions remain similar to the baseline setup.⁹

2.4 The Opportunity Cost of Fighting

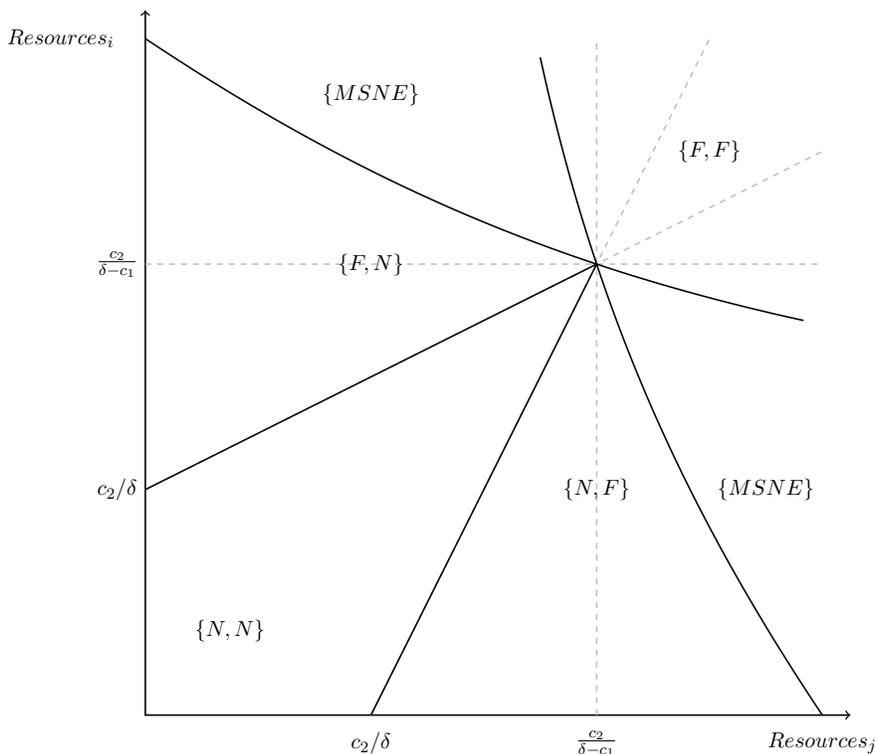
In many instances we may expect engaging in conflict to have a cost that varies with the amount of resources that can be lost to violence. For instance, the opportunity cost of war in terms of foregone earnings in the labor market will be higher for richer economies. Since the strength of the labor

⁸The figure restricts ϕ to values of $\delta > \phi > \frac{\delta}{1+\delta}$ for clarity.

⁹In addition, this setup also captures spillovers in resources across regions, showing that such spillovers are consistent with our qualitative predictions.

market depends on the strength of the overall economy, we may expect that this opportunity cost is larger in places that have more resources. In order to incorporate this aspect into our baseline model, we disaggregate the cost of war into two different types of costs – c_1 is a fixed proportion of the resources, whereas c_2 is the same fixed cost we had in the baseline model. The variable cost $c_1 \times r_k$ captures additional channels like the opportunity cost of war. The payoff matrix that includes this term is shown in Appendix Figure A2. These variable costs expand the (N, N) Nash Equilibrium regions, as shown in Figure 6.

Figure 6: Nash Equilibria in the (r_i, r_j) space



The figure plots the Nash equilibrium regions for any given draw of resources for parties i and j . c_1 is the variable cost of engaging in conflict, c_2 is the fixed cost of engaging in conflict, and δ is the fraction of resources that the victorious party expropriates.

The best response functions, game matrices, proofs of propositions, and a description of the Nash Equilibrium regions under the opportunity cost extension can be found in Appendix A.3. The $\{N, N\}$ region is now larger, since even as resources increase, the opportunity cost motive dampens the likelihood of conflict.

In general, across the various model specifications it is clear that the rapacity effect and the relative-strength mechanism divide the resource space into a few areas with different probabilities of conflict. The sharing rule and opportunity cost extension change the shape of these areas, but maintain the overall predicted patterns that we test empirically.

We empirically explore the patterns highlighted in this model section. Regions i and j that both have a high number of resources will engage in conflict with a higher likelihood than if one cell has resources and another doesn't. But the lowest likelihood of conflict arises when both cells have few resources. For continuous measures of resources, like agricultural wealth (proxied by rainfall), it will be important to also estimate the cutoffs that divide the space into the different Nash regions using structural break methods. The advantage of the model is that we can estimate the parameters c and δ that determine where these regions lie. Finally, better institutions will enlarge the $\{N, N\}$ by either raising c or reducing δ . How these structural parameters and the quality of model-fit change with the quality of institutions illustrates the mitigating effects of good institutions.

3 Data

To test the implications of the model, we combine spatial data on rainfall, oil and gas reserves, diamond deposits, gold mines, zinc deposits, cobalt mines, conflicts, and nighttime lights. We begin with a data set at the 0.5 degree by 0.5 degree latitude/longitude grid level covering the whole of Sub Saharan Africa.¹⁰ Each observation is a grid-cell pair: whether or not those two cells were involved in a conflict, and the resources and the other characteristics of each of the cells. The same cell will show up once as cell i , and may appear multiple times as cell j .¹¹ We construct the pairs between any two grid-cells within a specific distance radius. In our main specifications we use a 500km radius, but we do robustness checks to show that our results are not sensitive to any specific radius, and, as we show, are even more powerful at smaller distances, such as at a 150km radius.

The resolution of these cells is chosen to match the data on rainfall that are available in the well-known series from [Matsuura and Willmott \(2009\)](#). This is the lowest-resolution spatial data that we use in our baseline analysis. Hosted by the University of Delaware, these provide monthly temperature and rainfall for each 0.5 degree by 0.5 degree latitude/longitude grid between 1900 and 2010.¹² These data are commonly used as measures of localized economic development (e.g. [Dell et al. \(2012\)](#)). We collapse all our inter-temporal data into a single cross-section, allowing us to study the spatial patterns of conflict and development, rather than “shocks” to resources as much of the literature does. We use mean annual rainfall experienced in each grid cell over the period

¹⁰In keeping with the literature, we exclude the northern African countries of Algeria, Morocco, Egypt, Libya and Tunisia. We perform a robustness check on all our tables by also including these countries and each of our results hold for the entire African continent.

¹¹While our model considered ‘parties’, in the empirical analysis we use ‘region-cells.’ It is possible to do a similar analysis at the ethnic-group or political-boundary level, as some of the literature has done. However, as we are not studying ethnic-war or geopolitical battles, but rather a model of territorial control of resources, the grid-level data allows us to focus on this issue at the greatest extent possible. Furthermore, one may be concerned that ethnic-group and political boundaries are endogenously determined.

¹²See <http://climate.geog.udel.edu/~climate/>. The data comes from weather stations and interpolates between areas. The interpolation may introduce some measurement error that may attenuate results. Alternatively, we explore satellite-sources of precipitation from the Global Precipitation Climatology Project and the ERA-40 project, but satellite data is too coarse for our purposes as the data is either 2.5×2.5 degrees or 1.25×1.25 degrees.

1998 to 2008. The final year of our sample is constrained by the availability of the geographic coordinates of one of our conflict datasets, whereas the first year of our data is chosen so as study conflict over a ten year period.¹³

We then combine this rainfall data with data on oil and gas reserves and lootable diamonds, available from the International Peace Research Institute, Oslo (PRIO). The diamonds dataset, first created by Gilmore et al. (2005), lists all known diamond deposits in the world, coded with precise geographic coordinates. The oil and gas reserves dataset, developed by Lujala et al. (2007), depicts polygons for each deposit. As the data on diamonds and mines is in point format, we take the centroid of the polygon and merge this data with data on mines from the United States Geological Survey (USGS).¹⁴ The USGS has geolocations for mines across the world, and we pick the minerals that are most prevalent in Africa for our analysis.¹⁵ We merge this resources data, now at the point-level, with data on conflict at the center of the 0.5 degree by 0.5 degree latitude/longitude grid. We restrict ourselves to resources already discussed by the conflict literature: rainfall (Harari and La Ferrara, 2013), oil (Caselli et al., 2015), diamonds (Balestri and Maggioni, 2014) and other mines (Berman et al., 2017).

We use two main sources of conflict data and show our results for both. The first is the Uppsala Conflict Data Program (UCDP) / International Peace Research Institute, Oslo (PRIO) Armed Conflict Dataset, Version 4 - 2011. This is a widely used data set (e.g. Miguel and Satyanath (2011)) that lists conflicts and the years during which they occur. Initially coded by Gleditsch et al. (2002), these data report conflicts occurring between 1946 and 2010. To assign geographic coordinates to these conflicts, we add additional data, taken from Raleigh et al. (2006). For conflicts in the base PRIO data up to 2008, these report a latitude/longitude coordinate as well as a radius in kilometers. The circle defined by these numbers is taken as the area affected by the conflict, and we consider any 0.5×0.5 grid cell point lying within this circle, in the year of conflict, as being a part of the same conflict. Additional information on these data are provided by Raleigh et al. (2006) and Hallberg (2012). In particular, the latitude and longitude coordinate for a conflict is defined as the mid-point of all known locations of battles. The radius is constructed in multiples of 50 km and encompasses all of these battle locations, except for sporadic violence far from the remaining events.¹⁶

Our second source of conflict data is The Armed Conflict Location & Event Data Project (ACLED) database records conflicts at each latitude and longitude, the parties involved on each opposing side, and the type of conflict. We study territorial conflicts and exclude observations that

¹³We do robustness exercises extending this window to be 20 or 30 years long.

¹⁴See <https://data.usgs.gov/>

¹⁵We pick the minerals that are most prevalent in terms of number of mines across sub-Saharan Africa, with a minimum of at least thirty occurrences. Our results are robust to relaxing this cutoff and including the next set of prevalent minerals (phosphate and barite which occur in few mines).

¹⁶UCDP also publishes a companion dataset that covers the same time period– the GED polygon dataset, that uses polygons instead of radii and includes other events like one-sided violence among non-state actors.

correspond with riots, protests or non-violent events. Since ACLED has information on parties involved which allows us to determine whether the same parties were on opposing sides of a conflict in two different regions. Specifically, if two parties are on opposing sides of a conflict in two neighboring regions we code them as a conflict pair. This measure ensures that we are not looking at parties that are on the same side of the conflict. While restricting our analysis to territorial conflicts lowers the number of conflicts we study by almost half, the advantage is that we are then focusing on the types of interactions most closely related to the model setup.

Despite the possibility of measurement error in either conflict dataset, we are unable to think of any reasons why the effect of the spatial distribution of resources may be biased by measurement in our left-hand side variable. Since we construct grid-cell conflict pairs, each conflict will have more than one observation, one for each party involved. A grid-cell pair will have $conflict_{ij} = 1$ if they are both in conflict with each other during that 10 year period, and $conflict_{ij} = 0$ if there was no conflict between cells i and j .

We also consider the development implications of resources and conflict. We follow past researchers such as Henderson et al. (2012), Michalopoulos and Papaioannou (2013) and Pinkovskiy and Sala-i Martin (2016) in using night-time lights as a proxy for economic activity. Luminosity data are taken from the Defense Meteorological Satellite Program’s Operational Linescan System. Major advantages of these data include their arbitrary divisibility, their consistency across multiple political jurisdictions, their high spatial resolution, and their availability given the weaknesses of official data on African economic activity (Jerven, 2013; Pinkovskiy and Sala-i Martin, 2015). Henderson et al. (2012) provide additional information on these data. These data are constructed as an annual average of satellite images of the earth taken daily between 20:30 and 22:00 local time. The raw data are at a 30 second resolution, which implies that each pixel in the raw data is roughly one square kilometer. We average over pixels within a 0.5×0.5 grid cell point. The raw luminosity data for each pixel is reported as a six-bit integer ranging from 0 to 63. We average these pixels within grid-cell; doing this ensures that almost no grid-cell has the maximum value.¹⁷

In most specifications we only consider the variation within agro-ecological zones (AEZs). These AEZs are determined by the Food and Agricultural Organization (FAO) and depend on long-term climatic conditions, soil resources, terrain, land cover and other measures of crop suitability. For instance, the elevation component is divided into lowland, mid-elevation and high, the humidity component is divided into dry, semi-arid, sub-humid, humid and moist, and the soil component is divided into desert, savanna and forest. A combination of these features ensures that each AEZ is specific in its characteristics. As noted elsewhere, our results are robust to including grid-cell level fixed effects.¹⁸

¹⁷As only about one-fourth of the sample has non-zero values of light-density, we show our results using various transformations used in the literature (like inverse hyperbolic sine functions, or re-normalizing the zeros to number less than the minimum non-zero value).

¹⁸And subsequently country-level fixed effects which are subsumed by the grid fixed effects.

4 Estimation Strategy

The theoretical model allows us to divide the conflict-resources space into four distinct Nash Equilibrium regions. When there are low resources for both parties, there is a lower probability of conflict as neither party has resources to build an army and there is little wealth to expropriate from one’s neighbor. On the other hand, having a large amount of resources for either party leads to more conflict, and this is especially true when both parties have high levels of resources.

In order to capture this pattern produced by the Nash regions in Figure 4, we use the following regression specification at the $i - j$ grid-cell pair level:¹⁹

$$conflict_{ij} = \gamma_0 + \gamma_1 r_i + \gamma_2 r_j + \gamma_3 (r_i \times r_j) + \gamma_{\mathbf{x}} \mathbf{X} + \nu_a + \epsilon_{ij}, \quad (3)$$

where $conflict_{ij} = 1$ if the grid-cell i was ever engaged in the same conflict as grid cell j between 1998 and 2008. Our first resource measure, the presence of oil or gas, diamonds, gold, zinc or cobalt, is a discrete measure. Therefore, we can define $r_k = 1$ for $k = \{i, j\}$ if the region k contains any of these resources. The γ_0 captures the probability of conflict in the (No, No) region in the south-west section of Figure 4 where neither party fights. Similarly, $\gamma_0 + \gamma_1$ captures the north-western quadrant of the graph, which is represented by a $(Fight, No)$ region and a MSNE region. $\gamma_0 + \gamma_2$ corresponds to the $(No, Fight)$ and the MSNE region in the south-east section of the graph, and $\gamma_0 + \gamma_1 + \gamma_2 + \gamma_3$ captures the $(Fight, Fight)$ quadrant in the north-eastern portion of the figure. Given the model’s predictions, we should therefore expect $\gamma_1 \geq 0$, $\gamma_2 \geq 0$ and sum of coefficients, $\gamma_1 + \gamma_2 + \gamma_3 > 0$. There is no prediction on γ_3 alone as it may either be positive or negative. As our empirical setup tests for how a neighboring region’s resources affects the likelihood of conflict, we are explicitly testing for spillovers, but in a manner driven by our model’s predictions.

When using a continuous variable such as rainfall, our model predicts that conflict will be higher above certain rainfall cutoffs r^c :

$$conflict_{ij} = \beta_0 + \beta_1 \mathbb{1}_{r_i > r^c} + \beta_2 \mathbb{1}_{r_j > r^c} + \beta_3 \mathbb{1}_{r_i > r^c} \times \mathbb{1}_{r_j > r^c} + \beta_{\mathbf{x}} \mathbf{X} + \nu_a + \epsilon_{ij} \quad (4)$$

In this formulation, r^c represents the cutoffs in Figure 4 separating the Nash regions. As a region’s own resources r_i cross the cutoff r^c , we enter a different Nash region. Like before, β_0 captures the (No, No) region in the south-west section of Figure 4, $\beta_0 + \beta_1$ captures the north-western quadrant of the graph, and $\beta_0 + \beta_2$ corresponds to the south-eastern quadrant. Finally,

¹⁹Notice that this setup differs meaningfully from what other studies have done in the past, as our empirical framework follows from our model. For instance, [Caselli et al. \(2015\)](#) look at how far a given oil field is from a national border to evaluate whether parties fight over that oil field, whereas we study whether or not each party have any resources within their own territory.

$\beta_0 + \beta_1 + \beta_2 + \beta_3$ captures the (*Fight, Fight*) quadrant in the north-east portion of the figure. Given the model’s predictions, we should expect $\beta_1 \geq 0$, $\beta_2 \geq 0$ and sum of coefficients, $\beta_1 + \beta_2 + \beta_3 > 0$.

Notice, given our model’s results, there is no prediction on the sign of β_3 alone. Indeed, it is entirely plausible that $\beta_3 < 0$ even though the north-east quadrant displays a high probability of conflict $\beta_1 + \beta_2 + \beta_3 > 0$, and perhaps even higher than the south-eastern or north-western quadrants $\beta_1 + \beta_2 + \beta_3 > \beta_2$ and $\beta_1 + \beta_2 + \beta_3 > \beta_1$.

While the amount and geographic location of resources is exogenous, in the sense that it is taken as given by the actors involved, it is important to control for other factors, \mathbf{X} , that otherwise influence the likelihood of conflict in Africa and that may be correlated with resource endowments. Our analysis exploits the cross-section rather than the panel dimension as we focus on long-run growth.²⁰ We, therefore, need to establish that our relationships are robust to including an extensive set of fixed effects and controls. These controls include (for both points i and j) latitude and longitude, measures of land quality, malaria prevalence, humidity, population density, ruggedness and a quadratic in the distance between the two points. Furthermore, we also restrict attention to the variation within continuous regions by including fixed effects, ν_a , for Agro-Ecological Zones (AEZ) and, alternately, latitude-longitude grids of various sizes. Importantly, our results are robust to omitting controls or fixed effects suggesting that our estimated resource cutoffs are not systematically correlated with other observable variables.

4.1 Cutoffs

In order to estimate Equation 4 for a continuous variable like rainfall, it is necessary to identify the cutoff r^c . One simple approach would be to use the median level of resources. While all our results are consistent with using the median as a cutoff, there is no reason to believe that the median is the correct threshold. The literature on structural breaks has made progress in identifying such unknown cutoffs often in macroeconomics (Bai, 1997a,b, 2010; Bai and Perron, 1998; Gonzalo and Pitarakis, 2002; Gonzalo and Wolf, 2005; Hansen, 2000). These papers propose that the cutoff can be estimated by using a search algorithm that identifies the threshold that minimizes the residual sum of squares of the model, or alternatively maximizes the partial R-squared for the the variable of interest. Under a correctly specified model, this process leads to a consistent estimate of the cutoff and the parameters of interest.

While most of this literature focuses on structural breaks in time-series data, there are applications using cross-sectional micro-data (Card et al., 2008). In these cases, likelihood ratio (LR) tests under the null of no structural breaks do not allow for conventional hypothesis testing, and instead alternative methods that do not suffer from the drawbacks of such LR tests are used (Gonzalo and

²⁰Recent work shows that cross-sectional variation is a better predictor of conflict than time-varying shocks (Bazzi et al., 2017).

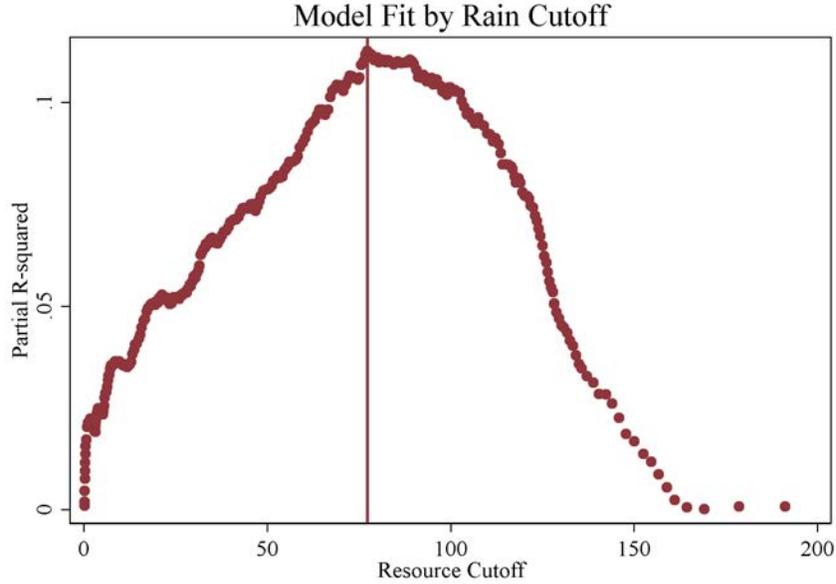


Figure 7: Model Fit and Optimal Rain Cutoff

The figure plots the partial R-square of the regression model in Equation 4 for each rainfall cutoff. Rainfall measure is annual rainfall, in mm, averaged over 1998 to 2008. The optimal cutoff is at 77.68mm of rainfall.

Wolf, 2005). An advantage of having a large sample is that we can use a split-sample approach – while one portion of the sample is used to identify the cutoff, the rest is utilized in running the regression of interest taking the cutoff as given (Angrist et al., 1999; Angrist and Krueger, 1995). Due to the independence of the sub-samples, the cutoff has a standard distribution under the null. One application of this can be found in Card et al. (2008), who use two-thirds of the sample to identify a threshold and the remaining one-third to identify the coefficients of interest. Similarly, Gonzalo and Wolf (2005) use the intuition behind Politis and Romano (1994) to propose using many randomly selected sub-samples to describe the distribution of cutoffs and coefficients of interest.

In keeping with the literature, therefore, the following empirical strategy is used. Two-thirds of the data are randomly selected, upon which the search algorithm is performed to identify the cutoff that minimizes the residual sum of squares. This can be seen in Figure 7, which identifies the value of the resources cutoff for which the partial R-squared is maximized in Equation 4. The remaining one-third is then used to identify the coefficients in the equation. This process is repeated with various randomly selected sub-samples to describe the distribution of the cutoffs and the parameters estimated. In our case, however, the process of repeatedly picking different sub-samples did not affect the estimates, largely because there was little to no change in the optimal cutoff across iterations.²¹

²¹Since our estimated resource cutoff parameter has little to no variation, we do not report a standard error on the cutoff value.

While identifying the cutoff is necessary for estimating β_1, β_2 and β_3 , it is also an informative parameter in itself since it represents the threshold amount of resources that pushes parties into conflict. As theory suggests, this threshold may be lower for regions that either have a lower cost of conflict or higher potential returns to conflict. Facilitation of trade or any other sharing-rules may, alternatively, raise the threshold necessary for the outbreak of conflict. Indeed, we test for one dimension of heterogeneity in the optimal cutoff and model fit below. We investigate whether the optimal cutoff is lower, and corresponding model fit improved, where institutions are weaker.

We then estimate a similar regression model to study how light density changes at these resource cutoffs. After identifying the cutoffs based on structural breaks in the likelihood of conflict, we regress log light density on these cutoffs.²² Since rainfall directly affects light density, we also control for continuous measures of own rainfall, neighbor’s rainfall and the interaction between the two.

4.2 Standard Errors

One additional issue is that of the estimation of standard errors. A standard result in the structural break literature is that the sampling error in the break can be ignored when estimating the size of the break (Bai, 1997b; Card et al., 2008). Given the iterative nature of the split-sample approach, it is possible to obtain a distribution of the coefficients of interest. However, in our context, this produces extremely tight standard errors due to a very precisely estimated cutoff value, and a more conservative approach may be warranted. Given the possibility of spatial correlation in the errors, the approach we use is to cluster the standard errors at various geographic levels. The data consists of points of a size spanned by 0.5×0.5 degrees in latitude and longitude, matched to each of its “neighbors,” i.e. all points within a 500km radius. Standard errors can therefore be clustered at the point level, or two-way clustered errors can be calculated for the point and each of its neighbors (Cameron et al., 2011). Estimates that allow for a greater degree of spatial correlation can be obtained by calculating errors at latitude-longitude grids of larger sizes, ranging from a 1×1 degree grid, to a more conservative 2×2 degree grid which consists of sixteen adjacent points and spans approximately 50 thousand square kilometers at the equator. Our results are robust to clustering at larger grid-sizes and at the country-level as well.

4.3 Discontinuities

The cutoffs not only allow for the estimation of Equation 4, but also the estimation of the size of the discontinuity at each boundary. In doing so, we can rely on the Regression Discontinuity (RD) literature to identify how the probability of conflict changes at each threshold. We do this using

²²We also do a robustness check where we transform the light density variable to be $\log(\text{light density} + 0.001)$ to account for the 0 values, as Michalopoulos and Papaioannou (2013) do in the context of Africa.

the latest methods developed by [Calonico et al. \(2014\)](#), who calculate the optimal bandwidths, and provide a robust bias-corrected estimate of the coefficients and standard errors. The methods developed by the RD literature can be used for two different results.²³ The first is just to see what happens to conflict when the region’s own rainfall crosses the threshold, whereas the second looks at the effect of the neighboring region’s rainfall at the cutoff. We should expect the likelihood of conflict to discontinuously rise at the cutoff, and in turn we should expect light density to jump downwards in response to this increase in the likelihood of conflict. Given this response, we perform a two-stage least squares exercise, again using the RD methods, where in the first stage we estimate the increase in the likelihood of conflict in crossing the resource cutoff, and in the second stage we estimate the corresponding fall in light density. The assumption underlying this 2SLS exercise is that, other than conflict, there are no alternative underlying features of the data that produce discontinuous jumps to light-density at the cutoff. With the help of this assumption we measure the impact of conflict on light-density and regional development. For the exercise we translate light density into GDP using the elasticities for low-income countries and Africa discussed in the literature ([Henderson et al., 2012](#); [Michalopoulos and Papaioannou, 2013](#)).

5 Results

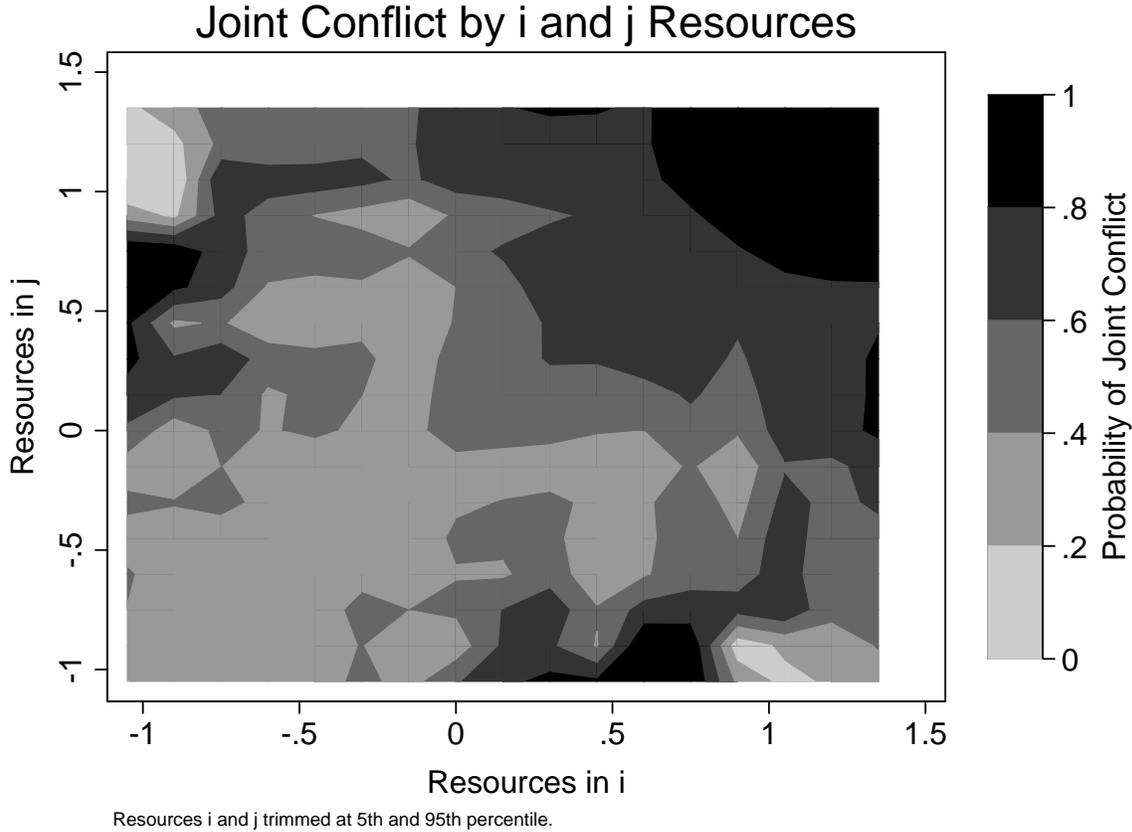
In this section, we present and discuss empirical evidence in support of the model developed in section 2. The analysis is carried out in multiple stages as discussed in section 4 above.

5.1 Joint Conflict

We start by showing a heat map of joint conflict for points i and j as a function of the relative resource endowments between the points in the pair. The heat map, shown in Figure 8, bears remarkable resemblance to the graph depicting the model predictions in Figure 4. That is, the region adjacent to the origin shows little to no likelihood of joint conflict; while the upper-right quadrant of the heat map corresponds to the highest likelihood of joint conflict, as predicted by the model. The no-conflict region at the origin extends along both the x and y axes until about the 0 point of the resource index for both i and j , beyond which the likelihood of conflict appears to increase starkly. Specifically, a high degree of inequality between resources in i and j leads to a higher probability of joint conflict, as one region has the resources to attack its neighbor, and the other wishes to expropriate its neighbor’s resources. This probability of conflict diminishes as the inequality diminishes (i.e., resources in j approach the high level of resources in i), but then jumps up again as we approach equally high resource levels for both i and j (i.e., as we approach the upper-right quadrant). Note further that the corner portions of the lower-right and upper-left

²³Our exercise is not strictly an RD since we are using an estimation procedure to first identify the discontinuity. Our exercise using RD methods is to provide an estimate of the corresponding size of this discontinuity.

Figure 8: Heat Map of the Probability of Conflict by Rainfall



Annual rainfall averaged over 1998 and 2008. Probability of joint conflict is the likelihood of cells i and j ever being involved in the same conflict over a ten year period (1998 to 2008).

quadrants representing extreme inequality actually show a reduction in conflict corresponding to the mixed strategy equilibrium regions in Figure 4. This non-monotonicity in the conflict-resource relationship lends preliminary empirical support to the predictions of the model.

Next, we discuss results from a more formal regression analysis of the model’s predictions. We split up our main results into two tables – Table 1 uses PRIO conflict data and covers all conflicts, whereas Table 2 focuses in on territorial conflicts based on the ACLED data.

For both sets of tables, in our first two columns we look at the relationship between the presence of any resource – oil, gas, diamonds or mines – with the probability of region i and j being in conflict, whereas in our last two columns we do a similar exercise for rainfall being above the cutoff. Using the procedure discussed in section 4, we estimate the cutoff level of rainfall above which the probability of joint conflict is higher for a randomly selected two-thirds of the sample. Using this cutoff value and the remaining one-third of the sample, we then regress the probability of a joint conflict between cell-points i and j on whether rainfall in i being above the cutoff, and

Table 1: Resources and Conflict (PRIO)

Dependent Variable: Probability(Region i&j in same conflict)				
Resource variable:	Oil, diamonds, mines		Rainfall	
Resource i	0.0639	0.0305	0.0728	0.0598
SE cluster: Point i	(0.0180)***	(0.0105)***	(0.0140)***	(0.00921)***
2 by 2 grid	(0.0315)**	(0.0164)*	(0.0351)**	(0.0198)***
2-way: Point i&j	(0.0216)***	(0.0238)	(0.0241)***	(0.0312)*
Resource j	0.0680	0.0129	0.135	0.0602
SE cluster: Point i	(0.00387)***	(0.00277)***	(0.00857)***	(0.00451)***
2 by 2 grid	(0.0124)***	(0.00878)	(0.0250)***	(0.0114)***
2-way: Point i&j	(0.0210)***	(0.0236)	(0.0177)***	(0.0260)**
Resource i&j	-0.0170	0.00610	0.0835	0.0641
SE cluster: Point i	(0.0115)	(0.00871)	(0.0118)***	(0.00720)***
2 by 2 grid	(0.0224)	(0.0152)	(0.0345)**	(0.0199)***
2-way: Point i&j	(0.0195)	(0.0187)	(0.0198)***	(0.0210)***
Sum of Coefficients	0.115	0.050	0.292	0.184
SE cluster: Point i	(0.021)***	(0.013)***	(0.014)***	(0.010)***
2 by 2 grid	(0.041)***	(0.023)	(0.038)***	(0.026)***
2-way: Point i&j	(0.036)***	(0.035)	(0.033)***	(0.047)***
R-squared	0.334	0.631	0.354	0.639
Controls (i & j)	All	All	All	All
Fixed Effects	AEZ	Grid 7 by 7	AEZ	Grid 7 by 7
Observations	1,901,074	1,901,074	633,821	633,821
Mean dependent var	0.34	0.34	0.34	0.34

Conflict data: PRIO. Oil, diamonds and mines: PRIO and USGS. Rainfall: University of Delaware. Regressions of ever being involved in the same conflict over a ten year period (1998 to 2008) on resources for the region and neighboring region.

In the first two columns, our resource measure is binary to indicate whether or not the region has any oil, gas, diamonds, gold, zinc or cobalt deposits.

In the last two columns our resource measure is binary to indicate whether rainfall is above the optimal cutoff. Search algorithm for the rainfall cutoff is described in the empirical section. One-third of the sample were randomly selected for the search procedure. Rainfall data is averaged over a ten year period between 1998 and 2008. The optimal cutoff for all conflicts is 77.68mm.

Observations including region $i&j$ pairs where region j is within 500 kilometers of region i .

Controls include Agro-Ecological Zone Fixed Effects or Grid Fixed Effects, and measures of (for both points i and j) latitude, longitude, ruggedness index, land quality index, humidity, malaria, population density, and a quadratic of the distance between two points in kms. Robustness to specifications without controls shown in other tables.

Standard errors clustered at latitude-longitude degree grids - For instance, a 2 by 2 grid consists of sixteen adjacent points.

Table 2: Resources and Conflict (ACLED)

Dependent Variable: Probability(Region i&j in same conflict)				
Resource variable:	Oil, diamonds, mines		Rainfall	
Resource i	0.0205	0.0133	0.0225	0.00509
SE cluster: Point i	(0.00505)***	(0.00449)***	(0.00270)***	(0.00219)**
2 by 2 grid	(0.00712)***	(0.00557)**	(0.00577)***	(0.00386)
2-way: Point i&j	(0.00540)***	(0.00572)**	(0.00388)***	(0.00432)
Resource j	0.0198	0.0137	0.0201	0.00777
SE cluster: Point i	(0.00166)***	(0.00146)***	(0.00190)***	(0.00135)***
2 by 2 grid	(0.00362)***	(0.00292)***	(0.00375)***	(0.00219)***
2-way: Point i&j	(0.00529)***	(0.00553)**	(0.00310)***	(0.00371)**
Resource i&j	0.0464	0.0460	0.0104	0.0126
SE cluster: Point i	(0.00781)***	(0.00758)***	(0.00229)***	(0.00204)***
2 by 2 grid	(0.0141)***	(0.0135)***	(0.00435)**	(0.00397)***
2-way: Point i&j	(0.0105)***	(0.0106)***	(0.00317)***	(0.00334)***
Sum of Coefficients	0.087	0.073	0.053	0.026
SE cluster: Point i	(0.010)***	(0.010)***	(0.004)***	(0.003)***
2 by 2 grid	(0.018)***	(0.016)***	(0.009)***	(0.006)***
2-way: Point i&j	(0.014)***	(0.015)***	(0.007)***	(0.007)***
R-squared	0.045	0.079	0.045	0.078
Controls (i & j)	All	All	All	All
Fixed Effects	AEZ	Grid 7 by 7	AEZ	Grid 7 by 7
Observations	1,901,074	1,901,074	633,821	633,821
Mean dependent var	0.028	0.028	0.028	0.028

Conflict data: ACLED, sub-sample of territorial conflicts only. Oil, diamonds and mines: PRIO and USGS. Rainfall: University of Delaware.

Regressions of ever being involved in the same conflict over a ten year period (1998 to 2008) on resources for the region and neighboring region.

In the first two columns, our resource measure is binary to indicate whether or not the region has any oil, gas, diamonds, gold, zinc or cobalt deposits. In the last two columns our resource measure is binary to indicate whether rainfall is above the optimal cutoff. Search algorithm for the rainfall cutoff is described in the empirical section. Two-thirds of the sample were randomly selected for the search procedure, and the final regression was run on the remaining one-third of the sample. The optimal cutoff for territorial conflicts is 20.67mm. Rain is averaged over a ten year period 1998-2008.

Observations including region i & j pairs where region j is within 500 kilometers of region i .

Controls include Agro-Ecological Zone Fixed Effects or Grid Fixed Effects, and measures of (for both points i and j) latitude, longitude, ruggedness index, land quality index, humidity, malaria, population density, and a quadratic of the distance between two points in kms. Robustness to specifications without controls shown in other tables.

Standard errors clustered at latitude-longitude degree grids - For instance, a 2 by 2 grid consists of sixteen adjacent points.

similarly for point j , and both i and j combined. As we use only the randomly selected one-third of the sample for the rainfall regressions, the number of observations in the final regression are smaller.²⁴

As discussed in section 4, we estimate two different specifications with different fixed effects. The results of these regressions are presented in Table 1 and 2 with the controls and fixed effects denoted for each column in the rows below the estimated coefficients and standard errors. These are our most conservative specifications, and the specifications without controls or fixed effects is shown in Table 4 and discussed in the next subsection.

In Tables 1 and 2 it is evident that the presence of a resource raises the likelihood of conflict. More resources in i increase the probability of a joint conflict, as do more resources in j . Furthermore, as predicted by the model, the probability of a joint conflict increases further when both i and j resources are high. Together, the estimates show that the likelihood of conflict is higher when moving from south-west to the north-east portion of the resource distribution represented in Figure 4. These results verify that the patterns depicted in the heat map in Figure 8 (i.e., low probability of joint conflict in the lower-left quadrant, a higher probability of conflict in the upper-left and lower-right quadrants and the highest probability of conflict in the upper-right quadrant) are indeed statistically significant.²⁵

For the PRIO data in Table 1, the increase in the probability of conflict relative to the baseline is also economically significant – the presence of oil, diamonds or mines raises the likelihood of conflict by at least 15% relative to the baseline, when moving from the lower-left no-conflict region to the upper-right high conflict region. This increase is even larger, an increase of at least 54%, if rainfall crosses the optimal cutoff into the upper-right high-conflict region. We find the optimal rainfall cutoff to be 77.7 mm (reported in the last row of Table 1). The distribution around this estimate of the cutoff for the full sample of data is shown in Figure 7.

The ACLED results display a similar pattern in Table 2 for the sub-sample of territorial conflicts. While the baseline level of territorial conflicts is small, the percentage increase in conflict in going from the lower-left low conflict quadrant to the upper-right high conflict region is high: around 92% for rainfall being above the cutoffs.

The results are robust across the different specifications including controls for both points i and j , along with fixed effects for agro-ecological zones (AEZ). The statistical significance of the results is unaffected when clustering standard errors at larger squares of the point i geospatial grid as well as two way clustering by squares in both the i and j geospatial grids.

²⁴As the random selection is done at the cell i level and not all cells have an equal number of neighbors, given coastlines, the number of observations are not precisely one-third.

²⁵As a reminder, the model’s predictions are that the coefficients on $Resource_i > 0$ and $Resource_j > 0$, and the sum of the coefficients on $Resource_i + Resource_j + Resource(i&j) > 0$. There is no prediction on the coefficient on the interaction term $Resource(i&j)$ alone.

5.2 Different Samples and Specifications

We next demonstrate the robustness of these main results in three important ways – changing the sample of analysis, using different model specifications, and focusing on specific resources. We then extend the analysis using RD methods to estimate the size of the discontinuity at these resource thresholds.

In Table 3 we replicate our main results for different cuts of the data. We test our model’s predictions that the coefficients on $Resource_i$, $Resource_j$ and the sum of coefficients are positive.²⁶ First, we use the entire African continent rather than sub-Saharan, and show that across both data sets (ACLED and PRIO) and across different resources (rainfall and the combination of oil, diamonds or metal mines), our results are both economically and statistically significant. Second, we restrict the sample only to i and j pairs that are of different ethnicities as defined by the [Murdock \(1959\)](#) Atlas of Africa. Since ethnic wars are a major focus of the large part of this literature, it is not surprising that our results are strong when considering such pairs of cells as well. Thirdly, we narrow the radius of i - j pairs to be only for pairs within 150km of each other. This restricts the sample to only 9% of our total observations. Once again, across data sets and resources, our results tell the same story.

In Table 4 we are able to study how sensitive our results are to different modeling specifications. In the top half of the table we show specifications without controls and without fixed effects. These results are similar to our baseline results. Our empirical analysis depends crucially on the validity of the comparison between i and j pairs. Most importantly, estimates should ideally be obtained from a comparison of pairs with sufficient variation in relative resources but otherwise common unobservables. That is, we want to be careful to not confuse differences in unobservables across disparate regions of the continent with the marginal deviations in relative resources underlying the intuition of the model’s predictions. While the specification including AEZ fixed effects presented in Tables 1 and 2 start to address this concern, we show further robustness of the results to restricting identifying variation within smaller contiguous areas as captured by the 5 degrees by 5 degrees grid fixed effects in the bottom half of Table 4.

We then proceed to focus on specific resources in Table 5. Given that the literature on conflict in Africa has often discussed the presence of oil and gas, and the presence of diamond deposits as drivers of conflict, we study whether the presence of these resources in neighboring regions individually predict joint conflict as our model suggests. Indeed, we find that a region having these resources predicts conflict with a neighboring region, and the same is true if the neighboring region possesses the resource as well.

²⁶As there is no prediction on the interaction term $Resource(i\&j)$ alone, we omit it from this table.

Table 3: Alternative Samples

Type	Data	Variable	Resource i	Resource j	Sum of Coeffs
Full Africa	PRIO	Resources	0.058 (0.007)***	0.046 (0.00309)***	0.121 (0.010)***
Full Africa	PRIO	Rain	0.050 (0.008)***	0.055 (0.00475)***	0.145 (0.010)***
Full Africa	ACLED	Resources	0.009 (0.002)***	0.009 (0.000987)***	0.010 (0.003)***
Full Africa	ACLED	Rain	0.005 (0.002)**	0.008 (0.00135)***	0.026 (0.003)***
Diff Ethnic	PRIO	Resources	0.063 (0.018)***	0.066 (0.00394)***	0.111 (0.022)***
Diff Ethnic	PRIO	Rain	0.084 (0.014)***	0.141 (0.00834)***	0.303 (0.014)***
Diff Ethnic	ACLED	Resources	0.017 (0.004)***	0.016 (0.00161)***	0.073 (0.010)***
Diff Ethnic	ACLED	Rain	0.023 (0.002)***	0.020 (0.00190)***	0.053 (0.004)***
Radius 150km	PRIO	Resources	0.102 (0.0197)***	0.115 (0.00980)***	0.153 (0.024)***
Radius 150km	PRIO	Rain	0.0489 (0.0210)**	0.0467 (0.0172)***	0.194 (0.017)***
Radius 150km	ACLED	Resources	0.0348 (0.00964)***	0.0361 (0.00668)***	0.154 (0.018)***
Radius 150km	ACLED	Rain	0.0526 (0.0105)***	0.0524 (0.00786)***	0.074 (0.010)***

Each row is a separate regression of $conflict_{ij}$ on resources and rainfall. The regressions with ‘Resources’ have 1901074 observations, whereas the regressions with ‘Rainfall’ have 633821 observations as the rest of the sample was used in the search procedure for the rainfall cutoff.

‘Full Africa’ replicates main results for the entire continent. ‘Diff Ethnic’ restricts the sample to be only for i and j pairs of different ethnicities. ‘Radius 150km’ restricts the sample to be only for i and j pairs within 150km of each other.

Conflict: ACLED or PRIO. Oil, diamonds and mines: PRIO & USGS. Rainfall: University of Delaware. Regressions of ever being involved in the same conflict over a ten year period (1998 to 2008) on resources for the region and neighboring region.

Resources: measure is binary to indicate whether or not the region has any oil, gas, diamonds, gold, zinc or cobalt deposits.

Rainfall: whether rainfall is above the optimal cutoff. Search algorithm for the rainfall cutoff is described in the empirical section. Two-thirds of the sample were randomly selected for the search procedure, and the final regression was run on the remaining one-third of the sample. Rainfall data is averaged over a ten year period between 1998 and 2008.

Observations including region i & j pairs where region j is within 500 kilometers of region i .

Controls include Agro-Ecological Zone Fixed Effects, and measures of (for both points i and j) latitude, longitude, ruggedness index, land quality index, humidity, malaria, population density, and a quadratic of the distance between two points in kms. Standard errors clustered at latitude-longitude degree grid level.

Table 4: Alternative Specifications

Type	Data	Variable	Resource i	Resource j	Sum of Coeffs
No controls	PRIO	Resources	0.058 (0.021)***	0.052 (0.00527)***	0.104 (0.022)***
No controls	PRIO	Rain	0.119 (0.013)***	0.119 (0.0109)***	0.367 (0.009)***
No controls	ACLED	Resources	0.025 (0.005)***	0.024 (0.00189)***	0.107 (0.011)***
No controls	ACLED	Rain	0.005 (0.001)***	0.004 (0.00133)***	0.038 (0.001)***
No Fixed Effects	PRIO	Resources	0.085 (0.018)***	0.080 (0.00454)***	0.125 (0.022)***
No Fixed Effects	PRIO	Rain	0.173 (0.013)***	0.176 (0.00917)***	0.456 (0.011)***
No Fixed Effects	ACLED	Resources	0.020 (0.005)***	0.019 (0.00175)***	0.084 (0.010)***
No Fixed Effects	ACLED	Rain	0.022 (0.002)***	0.021 (0.00198)***	0.050 (0.004)***
Grid 5x5 FE	PRIO	Resources	0.022 (0.008)**	0.004 (0.0027)	0.040 (0.013)***
Grid 5x5 FE	PRIO	Rain	0.025 (0.008)***	0.045 (0.00439)***	0.129 (0.009)***
Grid 5x5 FE	ACLED	Resources	0.007 (0.004)	0.011 (0.00133)***	0.065 (0.009)***
Grid 5x5 FE	ACLED	Rain	0.003 (0.003)	0.005 (0.00126)***	0.022 (0.003)***

Each row is a separate regression of $conflict_{ij}$ on resources and rainfall. The regressions with ‘Resources’ have 1901074 observations, whereas the regressions with ‘Rainfall’ have 633821 observations as the rest of the sample was used in the search procedure for the rainfall cutoff.

‘Grid 5x5 FE’ looks at the variation only within small grid-cells of 5 degrees latitude by 5 degrees longitude. ‘No Fixed Effects’ is the specification with all controls but no fixed effects. ‘No controls’ is the specification with no controls nor fixed effects.

Conflict: ACLED or PRIO. Oil, diamonds and mines: PRIO & USGS. Rainfall: University of Delaware. Regressions of ever being involved in the same conflict over a ten year period (1998 to 2008) on resources for the region and neighboring region.

Resources: measure is binary to indicate whether or not the region has any oil, gas, diamonds, gold, zinc or cobalt deposits.

Rainfall: whether rainfall is above the optimal cutoff. Search algorithm for the rainfall cutoff is described in the empirical section. Two-thirds of the sample were randomly selected for the search procedure, and the final regression was run on the remaining one-third of the sample. Rainfall data is averaged over a ten year period between 1998 and 2008.

Observations including region i & j pairs where region j is within 500 kilometers of region i .

Controls include Agro-Ecological Zone Fixed Effects, and measures of (for both points i and j) latitude, longitude, ruggedness index, land quality index, humidity, malaria, population density, and a quadratic of the distance between two points in kms. Standard errors clustered at latitude-longitude degree grid level.

Table 5: Oil and Diamonds

Type	Data	Resource i	Resource j	Sum of Coeffs
Diamonds	PRIO	0.102 (0.026)***	0.139 (0.00535)***	0.290 (0.022)***
Oil and Gas	PRIO	0.093 (0.033)***	0.020 (0.00795)***	0.009 (0.035)
Diamonds	ACLED	0.014 (0.008)*	0.015 (0.00279)***	0.129 (0.017)***
Oil and Gas	ACLED	0.065 (0.013)***	0.062 (0.00525)***	0.174 (0.024)***

Each row is a separate regression of $conflict_{ij}$ on a specific resource: oil or gas, and diamond deposits. Each regression has 1901074 observations.

Conflict: ACLED or PRIO. Oil, diamonds: PRIO & USGS

Regressions of ever being involved in the same conflict over a ten year period (1998 to 2008) on resources for the region and neighboring region.

Observations including region i & j pairs where region j is within 500 kilometers of region i .

Controls include Agro-Ecological Zone Fixed Effects, and measures of (for both points i and j) latitude, longitude, ruggedness index, land quality index, humidity, malaria, population density, and a quadratic of the distance between two points in kms. Standard errors clustered at latitude-longitude degree grid level.

5.2.1 The Size of the Discontinuity

Next, having verified the differences in the probability of joint conflict across the four quadrants depicted in Figure 8, we test whether the relationships between the probability of joint conflict and resources in points i and j are in fact discontinuous at the cutoffs estimated. These cutoffs correspond to the optimal model fit shown in Figure 7 and used in the tables for the rainfall results. We present graphical depictions of discontinuities in joint conflict as a function of i and j resources, in turn, in Figures 9 to 12. We test statistically for these discontinuities more formally using the latest semi-parametric methods developed in Calónico et al. (2014). The method determines an optimal data-driven bandwidth h for both the primary estimation and a bias-correction exercise with a larger bandwidth and different polynomial order p . The semi-parametric RD estimate is: $\tau(h, p) = \beta_+(h, p) - \beta_-(h, p)$, where β_+ is the estimate above the cutoff, and β_- is the estimate below the cutoff.

Table 6 reports results from the estimation of regression discontinuity specifications analogous to the exercises depicted in Figures 9 to 12 for both the PRIO and ACLED data. The results show that the discontinuities in the probability of joint conflict at the cutoff values in resources are indeed statistically significant. At the cutoff, there is an increase of about 0.1 percentage points in the likelihood of conflict for the PRIO conflicts and a 0.013 percentage point increase in the likelihood of territorial conflicts as determined by the ACLED data. These increases are meaningful as they are approximately one-third the size of the baseline levels of conflict.

Figure 9: PRIO: Conflict on own rainfall

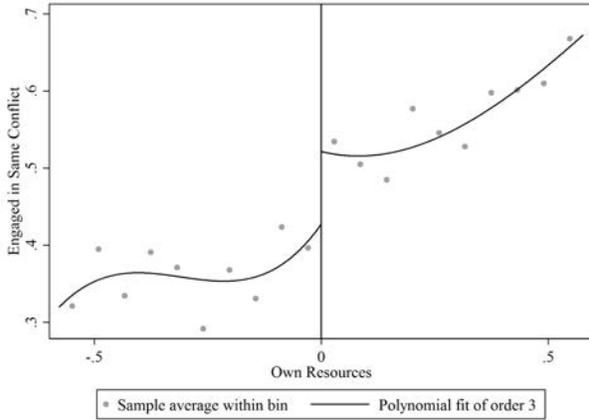


Figure 11: ACLED: Conflict on own rainfall

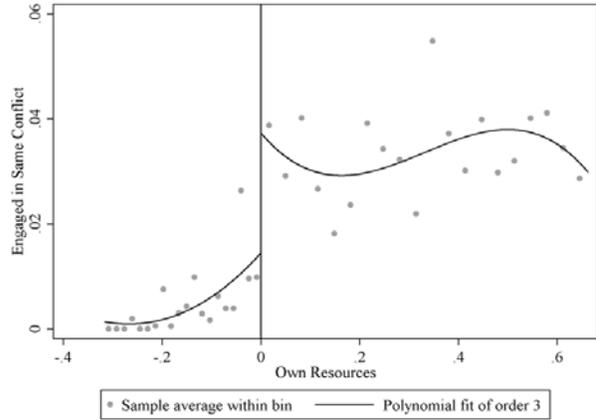


Figure 10: PRIO: Conflict on neighbor's rain

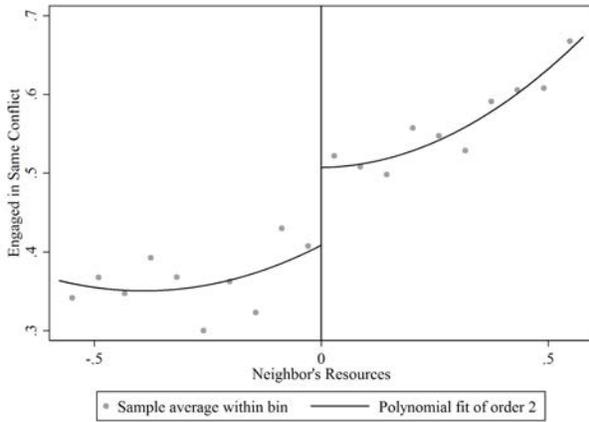
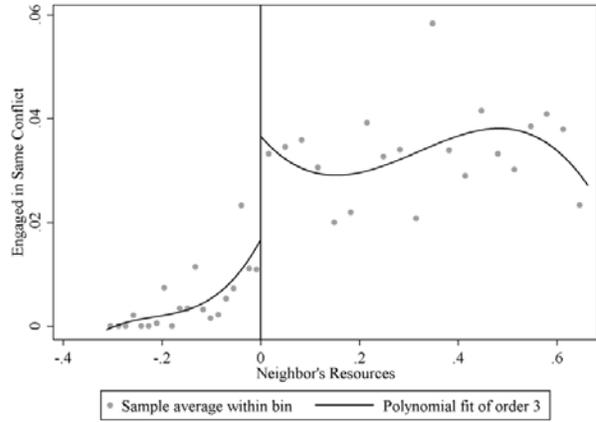


Figure 12: ACLED: On neighbor's rain



RD graphs of being engaged in the same conflict on rainfall. Rainfall centered around estimated cutoff. Graphs were produced using the [Calonico et al. \(2014\)](#) procedure of identifying the optimal bin sizes.

We interpret these large and significant discontinuities in the probability of joint conflict as further evidence in support of the model. While the patterns depicted in Figure 8 and verified statistically in Table 1 validate the non-monotonic relationship between resources and conflict that the model predicts should arise from the strategic interaction, the regression discontinuity results demonstrate strong support of the specific functional form of this relationship. In addition, the strength of these results validates the structural break estimation methodology we employ to identify the resource cutoff.

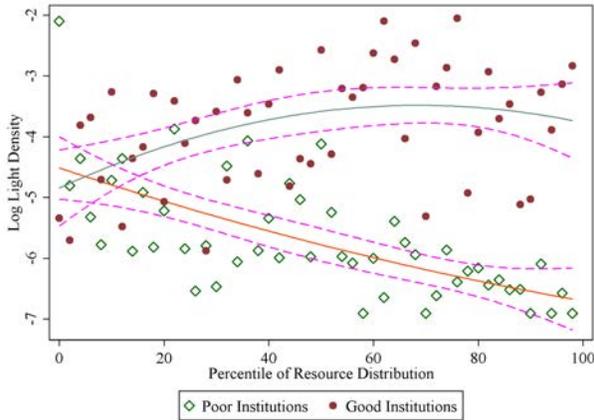


Figure 13: Log Light Density vs. Resources

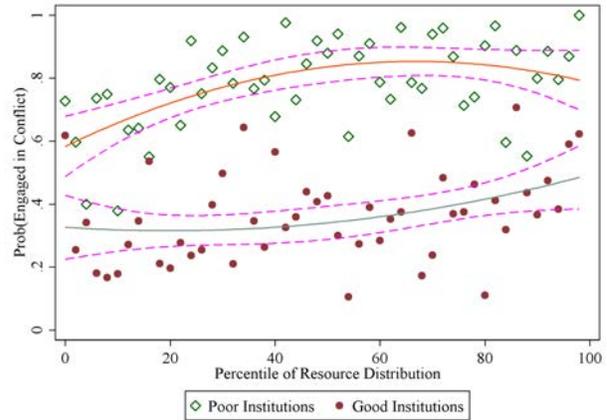


Figure 14: Prob(Conflict) vs. Resources

Observations for 0.5° by 0.5° grid cells across Sub Saharan Africa, as binned averages. Graphs plot quadratic fits with confidence intervals of the relationship between a resource index against (1) Log (Light Density) in 2008 and (2) Whether the region was engaged in conflict between 1998 and 2008. The Resource Index consists of the first principal component of (i) annual rainfall averaged over a ten year period (1998-2008), (ii) oil or gas reserves, (iii) lootable diamonds, (iv) gold, (v) zinc, and (vi) cobalt in the 0.5×0.5 degree grid cell. Good institutions are defined as having below the median value in the risk of expropriation index, whereas poor institutions are defined as having above the median value. Confidence intervals are estimated over the underlying data rather than binned averages.

5.3 Institutions and Mitigating Conflict

We next investigate the degree to which the strength of the conflict-resource relationship predicted by the model is moderated by local institutions. In particular, we investigate whether stronger institutions weakens the impulse for strategic conflict. If stronger institutions reduce the return to fighting (e.g., by introducing some probability of legal retribution) or increases the cost of conflict, the model predicts that the resource cutoff above which conflict becomes the optimal strategy should rise. In addition, we might suspect that stronger institutions might dampen the ability of the model to predict the drivers of conflict overall as the model focuses on rapacity and relative strength in conflict which should become less important as institutions such as those protecting legal rights become stronger.

We begin by plotting the relationships between natural resources, nighttime illumination, and conflict, akin to the plots in Figures 1 and 2 in the introduction, but here splitting the data by places with ‘good’ v. ‘poor’ institutions, defined as having either more or less than the median value of the ‘risk of expropriation’ index (Acemoglu et al., 2001). Figure 13 plots resources against log light density, finding starkly different relationships across places with good and bad institutions. In particular, nighttime illumination is increasing in resource endowments for places with good institutions, but clearly decreasing in resources when institutions are weak. Figure 14 provides

Table 6: Discontinuity Methods: Probability of Conflict at the Cutoff

Dependent Variable: Probability(Region i&j in same conflict)		
PRIO Data	Own Cutoff	Neighbor's Cutoff
RD Estimate	0.101 (0.0160)***	0.127 (0.00648)***
Robust Confidence Intervals	[0.065, 0.140]	[0.118, 0.144]
Mean dependent variable	0.340	0.340

Dependent Variable: Probability(Region i&j in same conflict)		
ACLED Data	Own Cutoff	Neighbor's Cutoff
RD Estimate	0.0158 (0.00772)**	0.0132 (0.00270)***
Robust Confidence Intervals	[-0.001, 0.008]	[0.034, 0.02]
Mean dependent variable	0.028	0.028

Regression discontinuity estimates of the change in the probability of conflict at the estimated conflict cutoff. Top panel uses PRIO data, whereas bottom panel uses ACLED data. Search algorithm for rainfall cutoffs described in the empirical section, where cutoffs predict largest changes in probability of conflict. Running variable is annual rainfall averaged over 1998 and 2008. Optimal bandwidth selection procedure, as described in [Calonico et al. \(2014\)](#). The robust, bias corrected confidence intervals reported using the [Calonico et al. \(2014\)](#) where the standard errors are clustered at the point i level using 150 nearest neighbors. Conventional standard errors reported in parentheses, centered around the point estimate.

empirical support for the idea that part of this difference is via conflict incidence: there is a lower probability of conflict at every point of the resource distribution for places with stronger institutions and the slope of the relationship is flatter. This preliminary evidence is exactly in line with what is found by [Mehlum et al. \(2006\)](#).

To formalize the intuition laid out in the figures discussed above, we use various measures of institutions including variables from the Aggregate Governance Indicators and the International Country Risk Guide published by the Political Risk Services Group. These measures are commonly employed in related studies (see, e.g., [Acemoglu et al. \(2001\)](#); [Michalopoulos and Papaioannou \(2015\)](#)). We use values from 1996, pre-dating the first year of our data. To first study how institutions affect the optimal cutoff, we repeat the exercise presented in Figure 7 for sub-samples of points with above and below the median values of our institutions measures. Then, to study the explanatory power of the model, we estimate the partial R-squared at the optimal cutoff.

Figure 15: PRIO: by Property Rights

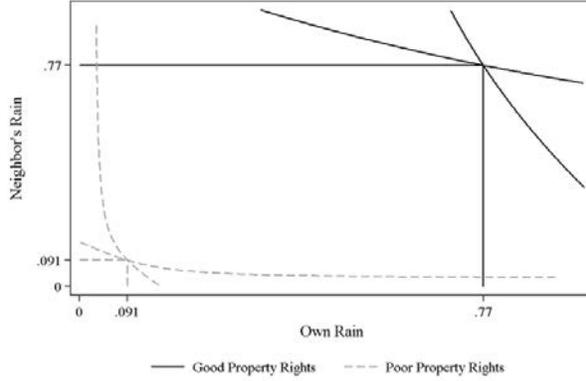
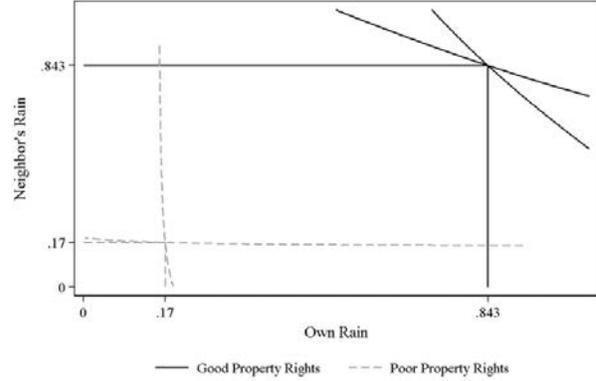


Figure 16: ACLED: by Property Rights



Structural estimates for cutoffs, costs of war c and fraction appropriated δ were estimated by simultaneously finding the optimal values that maximize the explanatory power of the model. For ‘Good Property Rights’, the sample is restricted to countries that have an above median measure of property rights. For ‘Poor Property Rights’ the sample is restricted to countries that have a below median measure of property rights.

In Figures 15 and 16 we present an example of the full structural estimation for one of our measures of institutions – the quality of property rights.²⁷ To estimate the full model, we estimate the optimal cutoff c/δ and the corresponding cost of war c , that maximizes the explanatory power of the model. We do this for sub-samples above and below the median measure of property rights and create figures analogous to Figure 4, with estimated structural parameters. We see that, indeed, better property rights produce a higher cutoff as the model would predict – indicating that in the presence of good institutions, the rapacity effect and endowment effect would have to be stronger to be able to produce conflict.

In Table 7 we summarize this exercise for four different measures of baseline institutional quality. As is evident from the table, for every measure and type of data, poor institutions at baseline correspond to lower rain-cutoffs.²⁸ A lower rain-cutoff indicates that even at low levels of resources, an increase in resources can lead to conflict. Good institutions mitigate this, and better institutions have higher rain cutoffs. Furthermore, the difference in the partial R-squared shows that the overall ability of the model to explain the relationship between conflict and resources in i and j is diminished when institutions are good. Our model is better at explaining conflict patterns in regions that have poor institutional quality.

²⁷As is evident from Table 7, the figures using other measures of institutions will look qualitatively similar. We focus on property rights given the importance in the literature (Acemoglu and Johnson, 2005).

²⁸Note that the ACLED territorial conflicts and PRIO all conflicts sample have different cutoffs as the two data sets have different definitions of conflicts and coding rules.

Table 7: Institutions as a Mitigating Factor

		PRIO		ACLED	
		Rain Cutoff	R^2 ratio	Rain Cutoff	R^2 ratio
Baseline		77.68	1.00	20.67	1.00
Property Rights	Poor	63.84	1.42	20.51	1.08
Property Rights	Good	76.67	0.35	84.25	0.93
Risk of Expropriation	Poor	72.58	1.45	21.31	1.37
Risk of Expropriation	Good	76.33	0.90	81.81	0.80
Political Stability	Poor	2.32	1.11	21.59	1.38
Political Stability	Good	79.77	0.85	96.17	0.40
Voice & Accountability	Poor	10.24	2.32	20.28	1.43
Voice & Accountability	Good	88.56	0.61	24.55	0.62

Institutional measures from Aggregate Governance Indicators (1996), and the International Country Risk Guide (Political Risk Services Group).

Rain cutoff in millimeters of rain.

For each exercise, the sample is divided at the median measure of institutions, after which the optimal rainfall cutoff is estimated using the method described in Section 4 and Figure 7.

The R^2 ratio is the ratio of the R^2 from the estimated model to the baseline model.

5.4 Development: Night Time Illumination

Finally, we present evidence of the relationship between development in point i as proxied by a measure of night time illumination (lights) and resources in points i and j , net of any intervening conflict. Even as African growth has recently picked up (Pinkovskiy and Sala-i Martin, 2014), indicators of development are still poor. Here we examine how big a role conflict plays in hindering economic activity in Africa. Figure 17 repeats the exercise in Figure 8 for $\log(\text{lights})$ as a function of resources in both points i and j . We see a pattern similar to the one depicted in Figure 1: at first luminosity increases with rainfall, only to fall at high levels of rainfall.²⁹ We see that the regions of joint conflict prevalence (e.g., upper-right) in Figure 8 correspond to low levels of lights or development in Figure 17; while the region of little to no conflict (i.e., lower-left) corresponds to low to moderate development as resources are low, but conflict is also low. The highest levels of development appear in the center of the heat map where resources are moderately high and conflict is avoided. These results are broadly consistent with the predictions of the model.

In Table 8 we show how light-density falls as resources cross the estimated cutoffs. Using the cutoffs estimated for the conflict regression, we regress light-density on the cutoffs, controlling for continuous measures of own resources, neighbors' resources and the interaction between the two. As

²⁹Notice, that unlike previous pictures, here the outcome is light density in region i rather than a joint outcome between i and j . Resources in i will directly affect the outcome, negating the possibility of symmetry in the figure.

Table 8: Resources and Light Density

Dependent Variable:	Inverse			
	Log(Light Density)	Hyperbolic Sine	Log(Light+0.001)	Lights>0
Rain i>cutoff	-0.485	-0.171	-1.094	-0.171
SE cluster: Point i	(0.133)***	(0.0251)***	(0.128)***	(0.0222)***
2 by 2 grid	(0.183)***	(0.0486)***	(0.225)***	(0.0359)***
2-way: Point i&j	(0.145)***	(0.0252)***	(0.144)***	(0.0251)***
Rain j> cutoff	-0.136	-0.0649	-0.526	-0.0884
SE cluster: Point i	(0.0605)**	(0.0111)***	(0.0575)***	(0.00984)***
2 by 2 grid	(0.0911)	(0.0250)***	(0.120)***	(0.0186)***
2-way: Point i&j	(0.0787)*	(0.0134)***	(0.0800)***	(0.0134)***
Rain i&j> cutoff	-0.00677	0.0112	0.127	0.0203
SE cluster: Point i	(0.0903)	(0.0161)	(0.0805)	(0.0133)
2 by 2 grid	(0.107)	(0.0237)	(0.143)	(0.0238)
2-way: Point i&j	(0.113)	(0.0182)	(0.102)	(0.0169)
Sum of coefficients	-0.628	-0.225	-1.494	-0.239
SE cluster: Point i	(0.122)***	(0.024)***	(0.123)***	(0.021)***
2 by 2 grid	(0.194)***	(0.057)***	(0.250)***	(0.038)***
2-way: Point i&j	(0.143)***	(0.024)***	(0.152)***	(0.027)***
R-squared	0.092	0.071	0.098	0.087
Controls (i & j)	All	All	All	All
Fixed Effects	AEZ	AEZ	AEZ	AEZ
Observations	143,346	635,549	635,549	635,549
Mean dependent var	-1.299	-5.56	0.135	0.23

Regressions of Light Density on resources for the region and neighboring region being above an estimated cutoff. Light density measures averaged over 1998 to 2008.

Each column uses a different specification of light density. ‘Inverse Hyperbolic Sine’ is $\log(\text{lights} + \sqrt{(\text{lights}^2 + 1)})$. $\text{Lights} > 0$ is a binary indicator for whether or not there are any lights there or not.

Controls include continuous values of own resources and neighbor’s resources (both increase light density), and the interaction between the two continuous measures.

Rainfall measure is annual rainfall averaged over 1998 and 2008. Search algorithm for resource cutoffs described in the empirical section 4, where cutoffs predict largest changes in probability of conflict. Estimated cutoff is 77.68mm. Two-thirds of the sample were randomly selected for the search procedure, and the final regression was run on the remaining one-third of the sample.

Observations including region i & j pairs where region j is within 500 kilometers of region i - Data is averaged over a ten year period between 1998 and 2008. Controls include Agro-Ecological Zone Fixed Effects, and measures of (for both points i and j) latitude, longitude, and a quadratic of the distance between two points in kms. Standard errors clustered at latitude-longitude degree grids - For example, a 2 by 2 grid consisting of sixteen adjacent points.

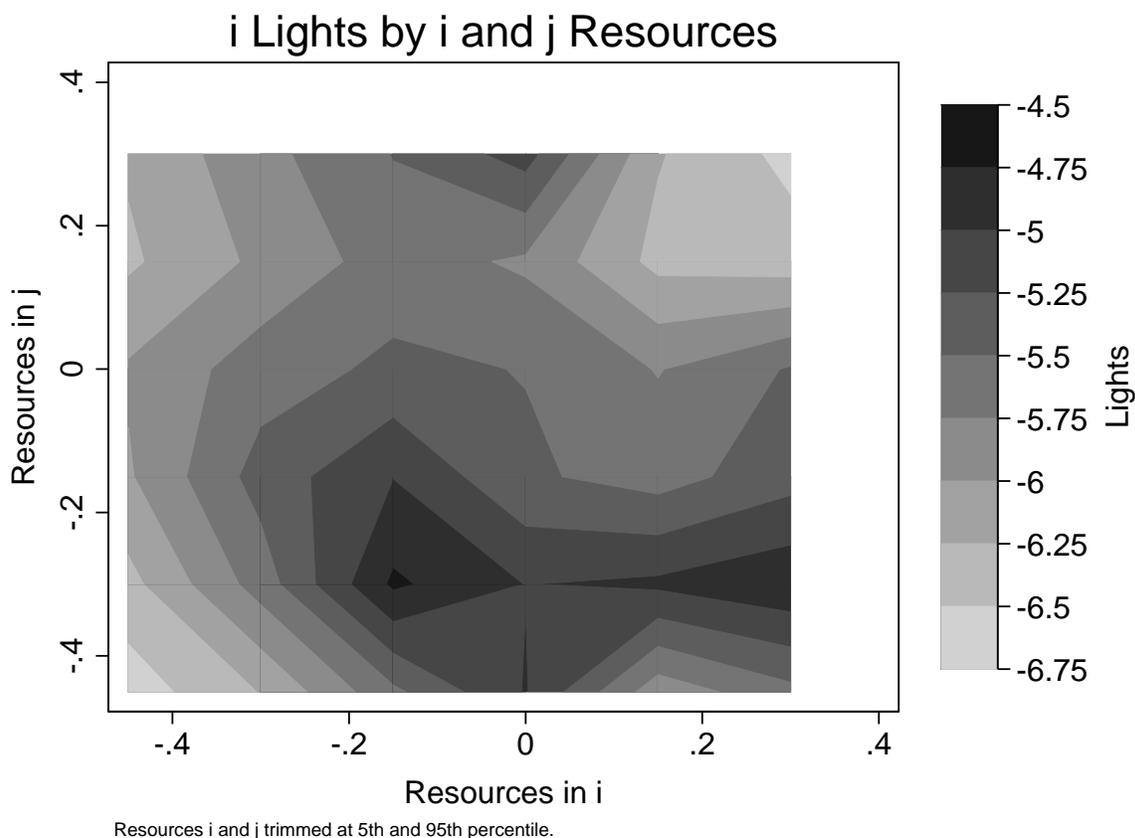


Figure 17: Heat Map of Light-Density by Rainfall

Log(light density) and annual rainfall averaged over 1998 and 2008.

only one-fourth of cells have non-zero light density, we show that our results are consistent across various measures of lights, including an inverse hyperbolic sine transformation and normalizing the zeros to 0.001 which is less than the smallest non-zero value of density. We see that light density falls at each of these cutoffs: own, neighbor's and the sum of all three. This shows how an increase in conflict may lead to a reduction in local economic activity.

In Table 9, we conduct various robustness checks for this exercise. First, we look at variation only within small grid-cells by including 5 by 5 degree grid fixed effects. We also restrict the sample to different ethnic groups, and lastly, instead of lights, we use the G-Econ measure of disaggregated GDP.³⁰ Our results are robust across these specifications.

We push this empirical test of the model one final step further by conducting the analogous analysis using the RD methods for light-density at the resource cutoffs estimated in the conflict results above. The results from these analyses are presented in the top panel of Table 10 and once

³⁰To learn more about this measure, see <http://gecon.yale.edu/>.

Table 9: Robustness: Resources and Light Density

Type	Resource i	Resource j	Sum of Coeffs
Grid 5 by 5 Fixed Effect	-0.262 (0.12)**	-0.0738 (0.04)*	-0.277 (0.12)**
Different Ethnicity	-0.374 (0.13)***	-0.152 (0.07)**	-0.364 (0.12)***
GDP (Dependent Variable)	-0.287 (0.0581)***	-0.16 (0.0378)***	-0.472 (0.07)***

Regression of Log(Light Density) or GDP on resources for the region and neighboring region being above an estimated cutoff. Light density measures averaged over 1998-2008.

Controls include continuous values of own resources and neighbor's resources (both increase light density), and the interaction between the two continuous measures.

Rainfall measure is annual rainfall averaged over 1998 and 2008. Search algorithm for resource cutoffs described in the empirical section 4, where cutoffs predict largest changes in probability of conflict. Two-thirds of the sample were randomly selected for the search procedure, and the final regression was run on the remaining one-third of the sample.

Observations including region $i&j$ pairs where region j is within 500 kilometers of region i - Data is averaged over a ten year period between 1998 and 2008. Controls include Agro-Ecological Zone Fixed Effects, and measures of (for both points i and j) latitude, longitude, and a quadratic of the distance between two points in kms. Standard errors clustered at latitude-longitude degree grids - For example, a 2 by 2 grid consisting of sixteen adjacent points.

again lend further support to the predictions of the model. Non-monotonicity in the relationship between resource endowments and development as proxied by lights is a particularly striking prediction, but one that helps to explain the relationship depicted in Figures 1 and 2. More striking still, is a sharp drop at the threshold, in an otherwise positive relationship between resources and development, and one not easily explained by any mechanism other than the intervention of conflict predicted by the model. We see in Table 10 that indeed there is a significant drop in lights at the resource cutoff at which the likelihood of conflict rises discontinuously.

Simple empirical correlations between development and conflict will suffer from endogeneity concerns. As a final empirical exercise, we exploit this common discontinuity in the resource-conflict and resource-lights relationships to obtain an estimate of how conflict affects overall economic activity. The exclusionary restriction underlying this exercise is that other than conflict, there are no underlying factors that cause light-density to change discontinuously at the threshold. In the bottom panel of Table 10, we report results from a two-stage least squares analysis of how log light density changes as the probability of conflict jumps at the resource cutoff. We then convert this to measures of GDP using the elasticities discussed in Henderson et al. (2012) and Michalopoulos and Papaioannou (2013).³¹ Together these results suggest that as we vary the probability of conflict

³¹For low income countries the elasticity of light density and GDP is about 0.3.

Table 10: Discontinuity Methods: Effect of Conflict on Light Density

Dependent Variable: Log Light Density		
	Own Cutoff	Neighbor's Cutoff
RD Estimate	-0.299 (0.109)***	-0.148 (0.0570)***
Robust Confidence Intervals	[-0.586, -0.085]	[-0.295, -0.068]
Dependent Variable: Log Light Density – 2SLS on Prob (Conflict)		
	Own Cutoff	Neighbor's Cutoff
RD Estimate	-2.249 (0.976)**	-0.969 (0.423)**
Robust Confidence Intervals	[-4.797, -0.329]	[-1.938, -0.262]

Regression discontinuity estimates display the change in log light density (top panel) and two-stage least squares fuzzy RD (bottom panel), where the dependent variable in the first stage is the probability of conflict on the cutoff, and the dependent variable in the second stage is the log light density. Search algorithm for resource cutoffs described in the empirical section, where cutoffs predict largest changes in probability of conflict. Resources include annual rainfall averaged over 1998 and 2008. Optimal bandwidth selection procedure, as described in [Calonico et al. \(2014\)](#). The robust, bias corrected confidence intervals reported using the [Calonico et al. \(2014\)](#) where the standard errors are clustered at the point i level using 150 nearest neighbors. Conventional standard errors reported in parentheses, centered around the point estimate.

over the interquartile range, GDP falls by 0.15 log points at the neighbor's resource cutoff and 0.33 log points at the own resources cutoff. This suggests that an increase from a 25% to a 75% likelihood of conflict can reduce GDP by as much as one-third. These tabulations suggest a significant effect of conflict on the economic prosperity of the region.

6 Conclusion

We present a model of resource-driven conflict and the corresponding impacts on development, and test its implications in sub-Saharan Africa. In our model, natural resources impel societies to conflict by increasing both the capacity for aggression as well as the gains from expropriation. We extend the model to account for 1) the idea that resources raise the opportunity costs of conflict, and 2) the possibility that neighboring societies share resources. Using a pairwise dataset of grid points in sub-Saharan Africa containing information on resource endowments, joint conflict, and

nighttime illumination (a proxy for local development), we test the predictions of this model. We find cutoffs for conflict onset in these data using a two-dimensional structural break estimator, and show that cutoffs are lower when institutions are poor. The results are a striking confirmation of our theory, and are robust to functional form assumptions and alternative data sources. We also demonstrate a non-monotonic relationship between resources and economic development with sharp reductions at conflict cutoffs.

The back-of-the-envelope exercise conducted in the previous section gives a sense of the importance of our results for understanding the development of sub-Saharan Africa in the context of natural resource-driven conflict. Moving from the bottom to the top end of the interquartile range of the probability of conflict, we estimate a GDP reduction of 33%, based on the results in Table 10. Though rough, these numbers convey the extent to which conflict stunts growth, and helps to explain the striking stylized fact that many of the most resource-rich countries grow no faster (indeed, often slower) than the most resource-poor countries.

References

- Acemoglu, D. and Johnson, S. (2005). Unbundling Institutions. *Journal of Political Economy*, 113(5):949–995.
- Acemoglu, D., Johnson, S., and Robinson, J. A. (2001). The Colonial Origins of Comparative Development: An Empirical Investigation. *The American Economic Review*, 91(5):1369–1401.
- Acemoglu, D., Johnson, S., and Robinson, J. A. (2005). Institutions as a fundamental cause of long-run growth. *Handbook of Economic Growth*, 1:385–472.
- Acemoglu, D. and Robinson, J. A. (2001). A theory of political transitions. *American Economic Review*, 91(4):938–963.
- Andersen, J. J., Nordvik, F. M., and Tesei, A. (2018). Oil and civil conflict: On and off (shore). *Working Paper*.
- Angrist, J. D., Imbens, G. W., and Krueger, A. B. (1999). Jackknife instrumental variables estimation. *Journal of Applied Econometrics*, 14(1):57–67.
- Angrist, J. D. and Krueger, A. B. (1995). Split-Sample Instrumental Variables Estimates of the Return to Schooling. *Journal of Business & Economic Statistics*, 13(2):225–235.
- Bai, J. (1997a). Estimating Multiple Breaks One at a Time. *Econometric Theory*, 13(3):315–352.
- Bai, J. (1997b). Estimation of a Change Point in Multiple Regression Models. *Review of Economics and Statistics*, 79(4):551–563.
- Bai, J. (2010). Common breaks in means and variances for panel data. *Journal of Econometrics*, 157(1):78–92.
- Bai, J. and Perron, P. (1998). Estimating and testing linear models with multiple structural changes. *Econometrica*, 66(1):47–78.
- Balestri, S. and Maggioni, M. A. (2014). Blood diamonds, dirty gold and spatial spill-overs measuring conflict dynamics in West Africa. *Peace Economics, Peace Science and Public Policy*, 20(4):551–564.
- Bannon, I. and Collier, P. (2003). *Natural Resources and Violent Conflict: Options and Actions*. World Bank Publications.
- Barro, R. (1996). Democracy and growth. *Journal of Economic Growth*, 1(1):1–27.
- Barro, R. J. (1991). Economic growth in a cross section of countries. *The Quarterly Journal of Economics*, 106(2):407–443.

- Bazzi, S., Blair, R., Blattman, C., Dube, O., Gudgeon, M., and Peck, R. (2017). The Limits of Conflict Prediction: Evidence From Colombia and Indonesia. *Working Paper*.
- Berman, N., Couttenier, M., Rohner, D., and Thoenig, M. (2017). This mine is mine! How minerals fuel conflicts in Africa. *American Economic Review*, 107(6).
- Besley, T. and Persson, T. (2010). State capacity, conflict, and development. *Econometrica*, 78(1):1–34.
- Brückner, M. and Ciccone, A. (2010). International Commodity Prices, Growth and the Outbreak of Civil War in Sub-Saharan Africa. *The Economic Journal*, 120(544):519–534.
- Buonanno, P., Durante, R., Prarolo, G., and Vanin, P. (2015). Poor institutions, rich mines: Resource curse in the origins of the Sicilian mafia. *The Economic Journal*, 125(586):F175–F202.
- Calonico, S., Cattaneo, M. D., and Titiunik, R. (2014). Robust Nonparametric Confidence Intervals for Regression-Discontinuity Designs. *Econometrica*, 82(6):2295–2326.
- Cameron, C., Gelbach, J., and Miller, D. (2011). Robust Inference with Multi-way Clustering. *Journal of Business and Economic Statistics*, 29(2):238–249.
- Card, D., Mas, A., and Rothstein, J. (2008). Tipping and the Dynamics of Segregation. *The Quarterly Journal of Economics*, 123(1):177–218.
- Caselli, F., Morelli, M., and Rohner, D. (2015). The Geography of Inter-State Resource Wars. *The Quarterly Journal of Economics*, 130(1):267–315.
- Caselli, F. and Tesei, A. (2016). Resource windfalls, political regimes, and political stability. *Review of Economics and Statistics*, 98(3):573–590.
- De La Sierra, R. S. (2015). On the Origin of States: Stationary Bandits and Taxation in Eastern Congo. *Mimeo*.
- Dell, M., Jones, B. F., and Olken, B. A. (2012). Temperature shocks and economic growth: Evidence from the last half century. *American Economic Journal: Macroeconomics*, 4(3):66–95.
- Dube, O. and Vargas, J. F. (2013). Commodity price shocks and civil conflict: Evidence from Colombia. *The Review of Economic Studies*, 80(4):1384–1421.
- Esteban, J., Mayoral, L., and Ray, D. (2012). Ethnicity and conflict: An empirical study. *The American Economic Review*, 102(4):1310–1342.
- Esteban, J. and Ray, D. (2011a). A model of ethnic conflict. *Journal of the European Economic Association*, 9(3):496–521.
- Esteban, J. and Ray, D. (2011b). Linking conflict to inequality and polarization. *The American Economic Review*, 101(4):1345–1374.

- Fearon, J. D. (2005). Primary commodity exports and civil war. *Journal of Conflict Resolution*, 49(4):483–507.
- Gilmore, E., Gleditsch, N. P., Lujala, P., and Rod, J. K. (2005). Conflict Diamonds: A New Dataset. *Conflict Management and Peace Science*, 22(3):257–292.
- Gleditsch, N., Wallensteen, P., Eriksson, M., Sollenberg, M., and Strand, H. (2002). Armed conflict 1946-2001: A new dataset. *Journal of Peace Research*, 39(5):615–637.
- Gonzalo, J. and Pitarakis, J.-Y. (2002). Estimation and model selection based inference in single and multiple threshold models. *Journal of Econometrics*, 110(2):319–352.
- Gonzalo, J. and Wolf, M. (2005). Subsampling inference in threshold autoregressive models. *Journal of Econometrics*, 127(2):201–224.
- Grossman, H. I. (1991). A General Equilibrium Model of Insurrections. *The American Economic Review* 81(4): 912-21.
- Grossman, H. I. and Kim, M. (1995). Swords or plowshares? A theory of the security of claims to property. *Journal of Political Economy*, 103(6):1275–1288.
- Gylfason, T. (2001). Natural resources, education, and economic development. *European Economic Review*, 45(4):847–859.
- Hallberg, J. D. (2012). PRIO conflict site 1989–2008: A geo-referenced dataset on armed conflict. *Conflict Management and Peace Science*, 29(2):219–232.
- Hansen, B. E. (2000). Sample Splitting and Threshold Estimation. *Econometrica*, 68(3):575–603.
- Harari, M. and La Ferrara, E. (2013). Conflict, Climate and Cells: A Disaggregated Analysis. *CEPR Discussion Paper No. DP9277*.
- Henderson, J. V., Storeygard, A., and Weil, D. N. (2012). Measuring economic growth from outer space. *American Economic Review*, 102(2):994–1028.
- Hirshleifer, J. (1989). Conflict and rent-seeking functions: Ratio versus difference models of relative success. *Public Choice*, 63, 101-12.
- Hsiang, S. M., Burke, M., and Miguel, E. (2013). Quantifying the influence of climate on human conflict. *Science*, 341(6151):1235367.
- Jerven, M. (2013). *Poor numbers: How we are misled by African development statistics and what to do about it*. Cornell University Press.
- Jia, R. (2014). Weather shocks, sweet potatoes and peasant revolts in historical China. *The Economic Journal*, 124(575):92–118.

- Jones, C. I. (2016). The facts of economic growth. *Handbook of Macroeconomics*, 2:3–69.
- Lujala, P., Rod, J. K., and Thieme, N. (2007). Fighting over Oil: Introducing A New Dataset. *Conflict Management and Peace Science*, 24(3):239–256.
- Matsuura, K. and Willmott, C. J. (2009). Terrestrial precipitation: 1900–2008 gridded monthly time series. *Center for Climatic Research, University of Delaware*.
- Mehlum, H., Moene, K., and Torvik, R. (2006). Institutions and the Resource Curse. *The Economic Journal*, 116(508):1–20.
- Michalopoulos, S. and Papaioannou, E. (2013). Pre-Colonial Ethnic Institutions and Contemporary African Development. *Econometrica*, 81(1):113–152.
- Michalopoulos, S. and Papaioannou, E. (2015). On the ethnic origins of African development: Chiefs and precolonial political centralization. *The Academy of Management Perspectives*, 29(1):32–71.
- Miguel, E. and Satyanath, S. (2011). Re-examining Economic Shocks and Civil Conflict. *American Economic Journal: Applied Economics*, 3(4): 228–232.
- Miguel, E., Satyanath, S., and Sergenti, E. (2004). Economic shocks and civil conflict: An instrumental variables approach. *Journal of Political Economy*, 112(4):725–753.
- Mitra, A. and Ray, D. (2014). Implications of an economic theory of conflict: Hindu-Muslim violence in India. *Journal of Political Economy*, 122(4):719–765.
- Murdock, G. P. (1959). *Africa: Its peoples and their culture history*. McGraw-Hill New York.
- Nunn, N. and Qian, N. (2014). US food aid and civil conflict. *The American Economic Review*, 104(6):1630–1666.
- Pinkovskiy, M. and Sala-i Martin, X. (2014). Africa is on time. *Journal of Economic Growth*, 19:311–338.
- Pinkovskiy, M. and Sala-i Martin, X. (2015). Lights, camera, income! illuminating the national accounts-household surveys debate. *The Quarterly Journal of Economics*, 131(2):579–631.
- Pinkovskiy, M. and Sala-i Martin, X. (2016). Newer need not be better: Evaluating the penn world tables and the world development indicators using nighttime lights. *NBER Working Paper 22216*.
- Politis, D. N. and Romano, J. P. (1994). Large Sample Confidence Regions Based on Subsamples under Minimal Assumptions. *The Annals of Statistics*, 22(4):2031–2050.
- Raleigh, C., Cunningham, D., Wilhelmsen, L., and Gleditsch, N. P. (2006). Conflict Sites 1946–2008. *UCDP/PRIO Armed Conflicts Dataset*, 2.

- Rodrik, D. (1999). Where did all the growth go? external shocks, social conflict, and growth collapses. *Journal of Economic Growth*, 4(4):385–412.
- Sachs, J. D. and Warner, A. M. (2001). The curse of natural resources. *European Economic Review*, 45(4):827–838.
- Sala-i Martin, X. and Subramanian, A. (2013). Addressing the natural resource curse: An illustration from nigeria. *Journal of African Economies*, 22(4):570–615.

A Nash Equilibria Proofs

A.1 Baseline Model

Lemma A.1 For $r_i, r_j \in (0, \frac{c}{\delta})$, (N, N) is the unique pure-strategy Nash Equilibrium.

Proof. Consider i 's best response to $s_j = N$. i will choose N iff $r_i > r_i - c + \delta r_j$, which reduces to $\frac{c}{\delta} > r_j$. Similarly, j 's best response to $s_i = N$ is N iff $\frac{c}{\delta} > r_i$. Thus for $r_i, r_j \in (0, \frac{c}{\delta})$, N constitutes the best response to N for both i and j , and we have that (N, N) is a Nash equilibrium.

To show that (N, N) is the unique NE in this case, it must be that (F, F) is not a NE.³² We can show that (F, F) will not lie in this region. Take i 's best response to $s_j = F$. This best response is F iff

$$(1 - \delta)r_i < p(r_i - c + \delta(r_j - c)) + (1 - p)(1 - \delta)(r_i - c). \quad (5)$$

Given $p = \frac{r_i}{r_i + r_j}$, this inequality is equivalent to

$$r_j < \frac{-\delta r_k^2 + c(1 + \delta)r_k}{\delta r_k - c(1 - \delta)} \quad \text{for} \quad r_i \in (0, c\frac{1 - \delta}{\delta}) \quad (6)$$

$$r_j > \frac{-\delta r_k^2 + c(1 + \delta)r_k}{\delta r_k - c(1 - \delta)} \quad \text{for} \quad r_i \in (c\frac{1 - \delta}{\delta}, \frac{c}{\delta}) \quad (7)$$

Equation 6 would require that $r_j < 0$ and equation 7 that $r_j > \frac{c}{\delta}$. There is thus no $r_i, r_j \in (0, \frac{c}{\delta})$ for which the F is the best response for $s_j = F$

■

Lemma A.2 Let $\psi(r_k) := \frac{-\delta r_k^2 + c(1 + \delta)r_k}{\delta r_k - c(1 - \delta)}$. (F, F) is the unique pure strategy Nash Equilibrium in the region $\{(r_i, r_j) : r_i \in (c\frac{1 - \delta}{\delta}, \infty), r_j > \psi(r_i)\} \cap \{(r_i, r_j) : r_j \in (c\frac{1 - \delta}{\delta}, \infty), r_i > \psi(r_j)\}$

Proof. Consider i 's best response to $s_j = F$ in this region. i will choose F iff $p(r_i - c + \delta(r_j - c)) + (1 - p)(1 - \delta)(r_i - c) > (1 - \delta)r_i$, which amounts to the region $\{(r_i, r_j) : r_i \in (c\frac{1 - \delta}{\delta}, \infty), r_j > \psi(r_i)\}$. Similarly j 's best response to $s_i = F$ is F iff $(1 - p)(r_j - c + \delta(r_i - c)) + p(1 - \delta)(r_j - c) > (1 - \delta)r_j$, which is the region $\{(r_i, r_j) : r_j \in (c\frac{1 - \delta}{\delta}, \infty), r_i > \psi(r_j)\}$. Thus (F, F) is a Nash equilibrium in the intersection of these regions.

To show that (F, F) is the unique pure strategy Nash Equilibrium, it must be that (N, N) is

³²Note that for $r_i, r_j \in (0, \frac{c}{\delta})$, (F, N) and (N, F) cannot be NE, given that if one party plays N , the other's best response must be N for values of r_i and r_j in the specified range.

not an equilibrium.³³ We can show that (N, N) cannot be an equilibrium in this region. i 's best response to $s_j = N$ will be N iff $r_j < \frac{c}{\delta}$ and j 's best response to $s_i = N$ will be N iff $r_i < \frac{c}{\delta}$. However, the intersections of the regions $\{(r_i, r_j) : r_i \in (c\frac{1-\delta}{\delta}, \infty), r_j > \psi(r_i)\} \cap \{(r_i, r_j) : r_j \in (c\frac{1-\delta}{\delta}, \infty), r_i > \psi(r_j)\}$ and the region $r_i, r_j \in (0, \frac{c}{\delta})$ is a null set. In \mathbb{R}_+^2 , $\psi(r_i) = \psi(r_k)$ at the point $(\frac{c}{\delta}, \frac{c}{\delta})$. There is therefore no region in \mathbb{R}_+^2 , for which $r_i > \psi(r_j)$, $r_j > \psi(r_i)$ and $r_i, r_j \in (0, \frac{c}{\delta})$.

■

Lemma A.3 Let $\psi(r_k) := \frac{-\delta r_k^2 + c(1+\delta)r_k}{\delta r_k - c(1-\delta)}$. (N, F) is the unique pure strategy Nash Equilibrium in the region $\{(r_i, r_j) : r_i \in (\frac{c}{\delta}, \infty), r_j < \psi(r_i)\}$

Proof. Consider i 's best response to $s_j = F$ in this region. i will choose N iff $p(r_i - c + \delta(r_j - c)) + (1-p)(1-\delta)(r_i - c) < (1-\delta)r_i$, which amounts to the region $\{(r_i, r_j) : r_i \in (c\frac{1-\delta}{\delta}, \infty), r_j < \psi(r_i)\}$. Similarly j 's best response to $s_i = N$ is F iff $\frac{c}{\delta} < r_i$. Thus (N, F) is a Nash equilibrium if $r_i > \frac{c}{\delta}$ and $r_j < \psi(r_i)$.

To show that (N, F) is the unique pure strategy Nash Equilibrium, we must rule out the other equilibria. (N, N) can be ruled out because j 's best response to $s_i = N$ cannot be N if $\frac{c}{\delta} < r_i$. Similarly, (F, F) cannot be a Nash Equilibrium in this region because i 's best response to $s_j = F$ cannot be F if $\{(r_i, r_j) : r_i \in (\frac{c}{\delta}, \infty), r_j < \psi(r_i)\}$. Lastly, we must rule out (F, N) . i 's best response to $s_j = N$ is F iff $r_j > \frac{c}{\delta}$. However, $\{(r_i, r_j) : r_i \in (\frac{c}{\delta}, \infty), r_j < \psi(r_i)\} \cap \{(r_i, r_j) : r_j > \frac{c}{\delta}\} = \emptyset$, thus precluding an (F, N) equilibrium in this region.

■

Lemma A.4 Let $\psi(r_k) := \frac{-\delta r_k^2 + c(1+\delta)r_k}{\delta r_k - c(1-\delta)}$. (F, N) is the unique pure strategy Nash Equilibrium in the region $\{(r_i, r_j) : r_j \in (\frac{c}{\delta}, \infty), r_i < \psi(r_j)\}$

Proof. Consider j 's best response to $s_i = F$ in this region. j will choose N iff $\{(r_i, r_j) : r_j \in (c\frac{1-\delta}{\delta}, \infty), r_i < \psi(r_j)\}$. Similarly i 's best response to $s_j = N$ is F iff $\frac{c}{\delta} < r_j$. Thus (F, N) is a Nash equilibrium if $r_j > \frac{c}{\delta}$ and $r_i < \psi(r_j)$.

To show that (F, N) is the unique pure strategy Nash Equilibrium, we must rule out the other equilibria. (N, N) can be ruled out because i 's best response to $s_j = N$ cannot be N if $\frac{c}{\delta} < r_j$. Similarly, (F, F) cannot be a Nash Equilibrium in this region because j 's best response to $s_i = F$ cannot be F if $\{(r_i, r_j) : r_j \in (\frac{c}{\delta}, \infty), r_i < \psi(r_j)\}$. Lastly, we must rule out (N, F) . j 's best response to $s_i = N$ is F iff $r_i > \frac{c}{\delta}$. However, $\{(r_i, r_j) : r_j \in (\frac{c}{\delta}, \infty), r_i < \psi(r_j)\} \cap \{(r_i, r_j) : r_i > \frac{c}{\delta}\} = \emptyset$, thus preventing the possibility of an (N, F) equilibrium in this region.

³³Note that in this region, (F, N) and (N, F) cannot be NE, given that if one party plays F , the other's best response must be F for values of r_i and r_j in the specified range.

■

Lemma A.5 Let $\psi(r_k) := \frac{-\delta r_k^2 + c(1+\delta)r_k}{\delta r_k - c(1-\delta)}$. \exists a mixed-strategies Nash Equilibrium (MSNE) in the region $\{(r_i, r_j) : r_j \in (\frac{c}{\delta}, \psi(r_i)), r_i > \psi(r_j)\} \cup \{(r_i, r_j) : r_i \in (\frac{c}{\delta}, \psi(r_j)), r_j > \psi(r_i)\}$

Proof. In this region, each party k will play a mixed-strategy where they *Fight*, with probability q_k :

$$q_k = \frac{(\delta r_k - c)(r_k + r_{-k})}{\delta(r_{-k}^2 - r_k^2 - c(r_k + r_{-k} - 2))} \quad (8)$$

Given these probabilities that each party *Fights*, the expected payoff from fighting and not fighting are equalized.

■

A.2 Sharing Rule

Proposition A.6 For $\delta > \phi > \frac{\delta}{1+\delta}$ and $(r_i, r_j) \in \mathbb{R}_+^2$, the following are best response functions for agent k under a sharing-rule agreement:

$$1. BR_k(s_{-k} = N) = \begin{cases} F, & \text{if } r_{-k} > \frac{c - \phi r_k}{\delta - \phi} \\ N, & \text{else} \end{cases}$$

$$2. \text{ Let } \psi(r_k) := \frac{-\delta r_k^2 + c(1+\delta)r_k}{\delta r_k - c(1-\delta)}.$$

$BR_k(s_{-k} = F) = F$, for all (r_k, r_{-k}) such that

$$\{(r_k, r_{-k}) : r_k \in (c \frac{1-\delta}{\delta}, \infty), r_{-k} > \psi(r_k)\} \quad (9)$$

And $BR_k(s_{-k} = F) = N$, for all (r_k, r_{-k}) such that

$$\{(r_k, r_{-k}) : r_k \in (0, c \frac{1-\delta}{\delta})\} \cup \{(r_k, r_{-k}) : r_k \in (c \frac{1-\delta}{\delta}, \infty), r_{-k} < \psi(r_k)\} \quad (10)$$

The change in the best response functions, leads to an expansion of the (N, N) region of the Nash equilibrium.

Proposition A.7 The sharing-rule expands the Nash Equilibrium region of (No, No)

Figure A1: The game with a sharing-rule.

		j	
		F	N
i	F	$p(r_i - c + \delta(r_j - c)) + (1 - p)(1 - \delta)(r_i - c),$ $(1 - p)(r_j - c + \delta(r_i - c)) + p(1 - \delta)(r_j - c)$	$r_i - c + \delta r_j,$ $(1 - \delta)r_j$
	N	$(1 - \delta)r_i,$ $r_j - c + \delta r_i$	$(1 - \phi)r_i + \phi r_j,$ $(1 - \phi)r_j + \phi r_i$

p is the probability of victory for party i , r_k are the level of resources for parties $k = \{i, j\}$, c is the cost of engaging in conflict, δ is the fraction of resources that the victorious party expropriates, and ϕ is the fraction of resources shared.

Lemma A.8 Let $\chi(r_k) = \frac{c - \phi r_k}{\delta - \phi}$. (N, N) is the unique pure-strategy Nash Equilibrium for the region $\{(r_i, r_j) : r_j < \chi(r_i), r_i < \chi(r_j)\}$.

Proof. Consider i 's best response to $s_j = N$. i will choose N iff $(1 - \phi)r_i + \phi r_j > r_i - c + \delta r_j$, which reduces to $\chi(r_i) > r_j$. Similarly, j 's best response to $s_i = N$ is N iff $\chi(r_i) > r_i$. Thus for $\{(r_i, r_j) : r_j < \chi(r_i), r_i < \chi(r_j)\}$, N constitutes the best response to N for both i and j , and we have that (N, N) is a Nash equilibrium.

To show that (N, N) is the unique NE in this case, it must be that (F, F) is not an NE.³⁴ We can show that (F, F) will not lie in this region. Take i 's best response to $s_j = F$. This best response is F iff

$$(1 - \delta)r_i < p(r_i - c + \delta(r_j - c)) + (1 - p)(1 - \delta)(r_i - c). \quad (11)$$

Given $p = \frac{r_i}{r_i + r_j}$, this inequality is equivalent to

$$r_j < \frac{-\delta r_k^2 + c(1 + \delta)r_k}{\delta r_k - c(1 - \delta)} \quad \text{for} \quad r_i \in (0, c \frac{1 - \delta}{\delta}) \quad (12)$$

$$r_j > \frac{-\delta r_k^2 + c(1 + \delta)r_k}{\delta r_k - c(1 - \delta)} \quad \text{for} \quad r_i \in (c \frac{1 - \delta}{\delta}, \frac{c}{\phi}) \quad (13)$$

Equation 12 would require that $r_j < 0$ and equation 13 that $r_j > \chi(r_i)$ for $\delta > \phi > \frac{\delta}{1 + \delta}$. There is thus no $\{(r_i, r_j) : r_j < \chi(r_i), r_i < \chi(r_j)\}$ for which the F is the best response for $s_j = F$

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Lemma A.9 Let $\psi(r_k) := \frac{-\delta r_k^2 + c(1 + \delta)r_k}{\delta r_k - c(1 - \delta)}$. (F, F) is the unique pure strategy Nash Equilibrium in the region $\{(r_i, r_j) : r_i \in (c \frac{1 - \delta}{\delta}, \infty), r_j > \psi(r_i)\} \cap \{(r_i, r_j) : r_j \in (c \frac{1 - \delta}{\delta}, \infty), r_i > \psi(r_j)\}$.

³⁴Note that for $r_i, r_j \in (0, \frac{c}{\phi})$, (F, N) and (N, F) cannot be NE, given that if one party plays N , the other's best response must be N for values of r_i and r_j in the specified range.

Proof. Consider i 's best response to $s_j = F$ in this region. i will choose F iff $p(r_i - c + \delta(r_j - c)) + (1-p)(1-\delta)(r_i - c) > (1-\delta)r_i$, which amounts to the region $\{(r_i, r_j) : r_i \in (c\frac{1-\delta}{\delta}, \infty), r_j > \psi(r_i)\}$. Similarly j 's best response to $s_i = F$ is F iff $(1-p)(r_j - c + \delta(r_i - c)) + p(1-\delta)(r_j - c) > (1-\delta)r_j$, which is the region $\{(r_i, r_j) : r_j \in (c\frac{1-\delta}{\delta}, \infty), r_i > \psi(r_j)\}$. Thus (F, F) is a Nash equilibrium in the intersection of these regions.

To show that (F, F) is the unique pure strategy Nash Equilibrium, it must be that (N, N) is not an equilibrium.³⁵ We can show that (N, N) cannot be an equilibrium in this region. i 's best response to $s_j = N$ will be N iff $r_j < \frac{c}{\delta}$ and j 's best response to $s_i = N$ will be N iff $r_i < \frac{c}{\delta}$. However, the intersections of the regions $\{(r_i, r_j) : r_i \in (c\frac{1-\delta}{\delta}, \infty), r_j > \psi(r_i)\} \cap \{(r_i, r_j) : r_j \in (c\frac{1-\delta}{\delta}, \infty), r_i > \psi(r_j)\}$ and the region $r_i, r_j \in (0, \frac{c}{\delta})$ is a null set. In \mathbb{R}_+^2 , $\psi(r_i) = \psi(r_k)$ at the point $(\frac{c}{\delta}, \frac{c}{\delta})$. There is therefore no region in \mathbb{R}_+^2 , for which $r_i > \psi(r_j)$, $r_j > \psi(r_i)$ and $r_i, r_j \in (0, \frac{c}{\delta})$.

■

Lemma A.10 Let $\psi(r_k) := \frac{-\delta r_k^2 + c(1+\delta)r_k}{\delta r_k - c(1-\delta)}$, and let $\chi(r_k) = \frac{c - \phi r_k}{\delta - \phi}$. (N, F) is the unique pure strategy Nash Equilibrium in the region $\{(r_i, r_j) : r_i \in (0, \frac{c}{\delta} - c), \chi(r_j) < r_i\} \cup \{(r_i, r_j) : r_i \in (\frac{c}{\delta} - c, \infty), r_j < \psi(r_i)\}$

Proof. Consider i 's best response to $s_j = F$ in this region. i will choose N iff $p(r_i - c + \delta(r_j - c)) + (1-p)(1-\delta)(r_i - c) < (1-\delta)r_i$, which amounts to the region $\{(r_i, r_j) : r_i \in (c\frac{1-\delta}{\delta}, \infty), r_j < \psi(r_i)\}$. Similarly j 's best response to $s_i = N$ is F iff $\chi(r_j) < r_i$. Thus (N, F) is a Nash equilibrium in the region $\{(r_i, r_j) : r_i \in (0, \frac{c}{\delta} - c), \chi(r_j) < r_i\} \cup \{(r_i, r_j) : r_i \in (\frac{c}{\delta} - c, \infty), r_j < \psi(r_i)\}$.

To show that (N, F) is the unique pure strategy Nash Equilibrium, we must rule out the other equilibria. (N, N) can be ruled out because j 's best response $s_i = N$ cannot be N if $\chi(r_j) < r_i$. Similarly, (F, F) cannot be a Nash Equilibrium in this region because i 's best response to $s_j = F$ cannot be F if $\{(r_i, r_j) : r_i \in (\frac{c}{\delta}, \infty), r_j < \psi(r_i)\}$. Lastly, we must rule out (F, N) . i 's best response to $s_j = N$ is F iff $r_j > \chi(r_i)$. However, $\{(r_i, r_j) : r_i \in (\frac{c}{\delta}, \infty), r_j < \psi(r_i)\} \cap \{(r_i, r_j) : r_j > \chi(r_i)\} = \emptyset$, thus precluding an (F, N) equilibrium in this region.

■

Lemma A.11 Let $\psi(r_k) := \frac{-\delta r_k^2 + c(1+\delta)r_k}{\delta r_k - c(1-\delta)}$, and let $\chi(r_k) = \frac{c - \phi r_k}{\delta - \phi}$. (F, N) is the unique pure strategy Nash Equilibrium in the region $\{(r_i, r_j) : r_j \in (0, \frac{c}{\delta} - c), \chi(r_i) < r_j\} \cup \{(r_i, r_j) : r_j \in (\frac{c}{\delta} - c, \infty), r_i < \psi(r_j)\}$

Proof. Consider j 's best response to $s_i = F$ in this region. j will choose N iff $\{(r_i, r_j) : r_j \in (c\frac{1-\delta}{\delta}, \infty), r_i < \psi(r_j)\}$. Similarly i 's best response to $s_j = N$ is F iff $\chi(r_i) < r_j$. Thus (F, N) is a

³⁵Note that in this region, (F, N) and (N, F) cannot be NE, given that if one party plays F , the other's best response must be F for values of r_i and r_j in the specified range.

Nash equilibrium in the region $\{(r_i, r_j) : r_j \in (0, \frac{c}{\delta} - c), \chi(r_i) < r_j\} \cup \{(r_i, r_j) : r_j \in (\frac{c}{\delta} - c, \infty), r_i < \psi(r_j)\}$.

To show that (F, N) is the unique pure strategy Nash Equilibrium, we must rule out the other equilibria. (N, N) can be ruled out because i 's best response $s_j = N$ cannot be N if $\chi(r_i) < r_j$. Similarly, (F, F) cannot be a Nash Equilibrium in this region because j 's best response to $s_i = F$ cannot be F if $\{(r_i, r_j) : r_j \in (\frac{c}{\delta}, \infty), r_i < \psi(r_j)\}$. Lastly, we must rule out (N, F) . j 's best response to $s_i = N$ is F iff $r_i > \chi(r_j)$. However, $\{(r_i, r_j) : r_j \in (\frac{c}{\delta}, \infty), r_i < \psi(r_j)\} \cap \{(r_i, r_j) : r_i > \chi(r_j)\} = \emptyset$, thus precluding an (N, F) equilibrium in this region.

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A.3 The Opportunity Cost of Conflict

Figure A2: The payoff-matrix for the game between i and j .

		j	
		F	N
i	F	$p(r_i(1 - c_1) - c_2 + \delta(r_j(1 - c_1) - c_2)) + (1 - p)(1 - \delta)(r_i(1 - c_1) - c_2),$ $(1 - p)(r_j(1 - c_1) - c_2 + \delta(r_i(1 - c_1) - c_2)) + p(1 - \delta)(r_j(1 - c_1) - c_2)$	$r_i(1 - c_1) - c_2 + \delta r_j,$ $(1 - \delta)r_j$
	N	$(1 - \delta)r_i,$ $r_j(1 - c_1) - c_2 + \delta r_i$	$r_i,$ r_j

Notes: p is the probability of victory for party i , r_k are the level of resources for parties $k = \{i, j\}$, c_1 is the variable cost of engaging in conflict, c_2 is the fixed cost of conflict, and δ is the fraction of resources that the victorious party expropriates.

Proposition A.12 For $\delta^2 > c_1$ and $(r_i, r_j) \in \mathbb{R}_+^2$, the following are best response functions for agent k :

$$1. BR_k(s_{-k} = N) = \begin{cases} F, & \text{if } r_{-k} > \frac{c_2 + c_1 r_k}{\delta} \\ N, & \text{else} \end{cases}$$

$$2. \text{ Let } \psi(r_k) := \frac{-(\delta - c_1)r_k^2 + c_2(1 + \delta)r_k}{(\delta - c_1)r_k - c_2(1 - \delta)}.$$

$BR_k(s_{-k} = F) = F$, for all (r_k, r_{-k}) such that

$$\{(r_k, r_{-k}) : r_k \in \left(c_2 \frac{1 - \delta}{\delta - c_1}, \infty \right), r_{-k} > \psi(r_k)\} \quad (14)$$

And $BR_k(s_{-k} = F) = N$, for all (r_k, r_{-k}) such that

$$\{(r_k, r_{-k}) : r_k \in \left(0, c_2 \frac{1 - \delta}{\delta - c_1} \right)\} \cup \{(r_k, r_{-k}) : r_k \in \left(c_2 \frac{1 - \delta}{\delta - c_1}, \infty \right), r_{-k} < \psi(r_k)\} \quad (15)$$

Lemma A.13 Let $\chi(r_k) = \frac{c_2 - c_1 r_k}{\delta}$. (N, N) is the unique pure-strategy Nash Equilibrium for the region $\{(r_i, r_j) : r_j < \chi(r_i), r_i < \chi(r_j)\}$.

Proof. Consider i 's best response to $s_j = N$. i will choose N iff $r_i(1 - c_1) - c_2 + \delta r_j < r_i$. This inequality reduces to $r_j < \frac{c_2 + c_1 r_i}{\delta}$. Similarly, j 's best response to $s_i = N$ is N iff $r_i < \frac{c_2 + c_1 r_j}{\delta}$. Thus for $\{(r_i, r_j) : r_j < \chi(r_i), r_i < \chi(r_j)\}$, N constitutes the best response to N for both i and j , and we have that (N, N) is a Nash equilibrium.

To show that (N, N) is the unique PSNE in this case, it must be that (F, F) is *not* an NE.³⁶ We can show that (F, F) will not lie in this region. Take i 's best response to $s_j = F$. This best response is F iff

$$p(r_i(1 - c_1) - c_2 + \delta(r_j(1 - c_1) - c_2)) + (1 - p)(1 - \delta)(r_i(1 - c_1) - c_2) > (1 - \delta)r_j \quad (16)$$

Given $p = \frac{r_i}{r_i + r_j}$, this inequality is equivalent to

$$r_j < \frac{-(\delta - c_1)r_i^2 + c_2(1 + \delta)r_i}{(\delta - c_1)r_i - c_2(1 - \delta)} \quad \text{for} \quad r_i \in \left(0, c_2 \frac{1 - \delta}{\delta - c_1}\right) \quad (17)$$

$$r_j > \frac{-(\delta - c_1)r_i^2 + c_2(1 + \delta)r_i}{(\delta - c_1)r_i - c_2(1 - \delta)} \quad \text{for} \quad r_i \in \left(c_2 \frac{1 - \delta}{\delta - c_1}, \frac{c_2}{\delta - c_1}\right) \quad (18)$$

Equation 17 and 18 ensure that there is no $\{(r_i, r_j) : 0 < r_j < \chi(r_i), 0 < r_i < \chi(r_j)\}$ for which the F is the best response for $s_j = F$

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Lemma A.14 Let $\psi(r_k) := \frac{-(\delta - c_1)r_k^2 + c_2(1 + \delta)r_k}{(\delta - c_1)r_k - c_2(1 - \delta)}$. (F, F) is the unique pure strategy Nash Equilibrium in the region $\{(r_i, r_j) : r_i \in \left(c_2 \frac{1 - \delta}{\delta - c_1}, \infty\right), r_j > \psi(r_i)\} \cap \{(r_i, r_j) : r_j \in \left(c_2 \frac{1 - \delta}{\delta - c_1}, \infty\right), r_i > \psi(r_j)\}$.

Proof. Consider i 's best response to $s_j = F$ in this region. i will choose F iff $p(r_i(1 - c_1) - c_2 + \delta(r_j(1 - c_1) - c_2)) + (1 - p)(1 - \delta)(r_i(1 - c_1) - c_2) > (1 - \delta)r_i$, which amounts to the region $\{(r_i, r_j) : r_i \in (c_2 \frac{1 - \delta}{\delta - c_1}, \infty), r_j > \psi(r_i)\}$. Similarly j 's best response to $s_i = F$ is F iff $(1 - p)(r_j(1 - c_1) - c_2 + \delta(r_i(1 - c_1) - c_2)) + p(1 - \delta)(r_j(1 - c_1) - c_2) > (1 - \delta)r_j$, which is the region $\{(r_i, r_j) : r_j \in (c_2 \frac{1 - \delta}{\delta - c_1}, \infty), r_i > \psi(r_j)\}$. Thus (F, F) is a Nash equilibrium in the intersection of these regions.

To show that (F, F) is the unique pure strategy Nash Equilibrium, it must be that (N, N) is not an equilibrium.³⁷ We can show that (N, N) cannot be an equilibrium in this region. i 's best response to $s_j = N$ will be N iff $r_j < \frac{c_2 + c_1 r_i}{\delta}$ and j 's best response to $s_i = N$ will be N iff $r_i < \frac{c_2 + c_1 r_j}{\delta}$. However, the intersections of the regions $\{(r_i, r_j) : r_i \in (c_2 \frac{1 - \delta}{\delta - c_1}, \infty), r_j > \psi(r_i)\} \cap \{(r_i, r_j) : r_j \in (c_2 \frac{1 - \delta}{\delta - c_1}, \infty), r_i > \psi(r_j)\}$ and the region $r_i, r_j \in (0, \frac{c_2 + c_1 r_i}{\delta})$ is a null set. In \mathbb{R}_+^2 , $\psi(r_i) = \psi(r_k)$ at the point $(\frac{c_2}{\delta - c_1}, \frac{c_2}{\delta - c_1})$. There is therefore no region in \mathbb{R}_+^2 , for which $r_i > \psi(r_j)$, $r_j > \psi(r_i)$ and $r_i, r_j \in (0, c_2 \frac{1 - \delta}{\delta - c_1})$.

■

³⁶Note that for $r_i, r_j \in (0, \frac{c_2}{\delta})$, (F, N) and (N, F) cannot be NE, given that if one party plays N , the other's best response must be N for values of r_i and r_j in the specified range.

³⁷Note that in this region, (F, N) and (N, F) cannot be NE, given that if one party plays F , the other's best response must be F for values of r_i and r_j in the specified range.

Lemma A.15 Let $\psi(r_k) := \frac{-(\delta-c_1)r_k^2+c_2(1+\delta)r_k}{(\delta-c_1)r_k-c_2(1-\delta)}$, and let $\chi(r_k) = \frac{c_2+c_1r_k}{\delta}$. (N, F) is the unique pure strategy Nash Equilibrium in the region $\{(r_i, r_j) : r_i \in (\frac{c_2}{\delta}, \infty), \chi(r_j) < r_i\} \cap \{(r_i, r_j) : r_i \in (0, \frac{c_2(1+\delta)}{\delta-c_1}), r_j < \psi(r_i)\}$

Proof. Consider i 's best response to $s_j = F$ in this region. i will choose N iff $p(r_i(1-c_1) - c_2 + \delta(r_j(1-c_1) - c_2)) + (1-p)(1-\delta)(r_i(1-c_1) - c_2) < (1-\delta)r_i$, which amounts to the region $\{(r_i, r_j) : r_i \in (c_2\frac{1-\delta}{\delta-c_1}, \infty), r_j < \psi(r_i)\} \cup \{(r_i, r_j) : r_i \in (0, c_2\frac{1-\delta}{\delta-c_1})\}$. Similarly j 's best response to $s_i = N$ is F iff $\{(r_i, r_j) : r_i \in (\frac{c_2}{\delta}, \infty), \chi(r_j) < r_i\}$. Thus (N, F) is a Nash equilibrium in the region $\{(r_i, r_j) : r_i > \chi(r_j), r_j < \psi(r_i)\}$.

To show that (N, F) is the unique pure strategy Nash Equilibrium, we must rule out the other equilibria. (N, N) can be ruled out because j 's best response $s_i = N$ cannot be N if $\chi(r_j) < r_i$. Similarly, (F, F) cannot be a Nash Equilibrium in this region because i 's best response to $s_j = F$ cannot be F if $\{(r_i, r_j) : r_i \in (\frac{c_2}{\delta}, \infty), r_j < \psi(r_i)\}$. Lastly, we must rule out (F, N) . i 's best response to $s_j = N$ is F iff $r_j > \chi(r_i)$. However, $\{(r_i, r_j) : r_i \in (\frac{c_2}{\delta}, \infty), r_i > \chi(r_j), r_j < \psi(r_i)\} \cap \{(r_i, r_j) : r_j > \chi(r_i)\} = \emptyset$, thus precluding an (F, N) equilibrium in this region.

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Lemma A.16 Let $\psi(r_k) := \frac{-(\delta-c_1)r_k^2+c_2(1+\delta)r_k}{(\delta-c_1)r_k-c_2(1-\delta)}$, and let $\chi(r_k) = \frac{c_2+c_1r_k}{\delta}$. (F, N) is the unique pure strategy Nash Equilibrium in the region $\{(r_i, r_j) : r_j \in (\frac{c_2}{\delta}, \infty), \chi(r_i) < r_j\} \cap \{(r_i, r_j) : r_j \in (0, \frac{c_2(1+\delta)}{\delta-c_1}), r_i < \psi(r_j)\}$

Proof. Consider j 's best response to $s_i = F$ in this region. j will choose N iff $(1-p)(r_j(1-c_1) - c_2 + \delta(r_i(1-c_1) - c_2)) + p(1-\delta)(r_j(1-c_1) - c_2) < (1-\delta)r_j$, which amounts to the region $\{(r_i, r_j) : r_j \in (c_2\frac{1-\delta}{\delta-c_1}, \infty), r_i < \psi(r_j)\} \cup \{(r_i, r_j) : r_j \in (0, c_2\frac{1-\delta}{\delta-c_1})\}$. Similarly i 's best response to $s_j = N$ is F iff $\{(r_i, r_j) : r_j \in (\frac{c_2}{\delta}, \infty), \chi(r_i) < r_j\}$. Thus (F, N) is a Nash equilibrium in the region $\{(r_i, r_j) : r_j > \chi(r_i), r_i < \psi(r_j)\}$.

To show that (F, N) is the unique pure strategy Nash Equilibrium, we must rule out the other equilibria. (N, N) can be ruled out because i 's best response $s_j = N$ cannot be N if $\chi(r_i) < r_j$. Similarly, (F, F) cannot be a Nash Equilibrium in this region because j 's best response to $s_i = F$ cannot be F if $\{(r_i, r_j) : r_j \in (\frac{c_2}{\delta}, \infty), r_i < \psi(r_j)\}$. Lastly, we must rule out (N, F) . j 's best response to $s_i = N$ is F iff $r_i > \chi(r_j)$. However, $\{(r_i, r_j) : r_j \in (\frac{c_2}{\delta}, \infty), r_j > \chi(r_i), r_i < \psi(r_j)\} \cap \{(r_i, r_j) : r_i > \chi(r_j)\} = \emptyset$, thus precluding an (N, F) equilibrium in this region.

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