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A SUFFICIENT STATISTICS APPROACH FOR AGGREGATING FIRM-LEVEL
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A Sufficient Statistics Approach for Aggregating Firm-Level Experiments

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ABSTRACT

We consider a dynamic economy populated by heterogeneous firms subject to generic capital frictions: adjustment costs, taxes and financing constraints. A random subset of firms in this economy receives an empirical "treatment", which modifies the parameters governing these frictions. An econometrician observes the firm-level response to this treatment, and wishes to calculate how macroeconomic outcomes would change if all firms in the economy were treated. Our paper proposes a simple methodology to estimate this aggregate counterfactual using firm-level evidence only. Our approach takes general equilibrium effects into account, requires neither a structural estimation nor a precise knowledge on the exact nature of the experiment and can be implemented using simple moments of the distribution of revenue-to-capital ratios. We provide a set of sufficient conditions under which these formulas are valid and investigate the robustness of our approach to multiple variations in the aggregation framework.

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1 Introduction

Governments around the world have a wide range of policies to facilitate business investment and growth. A burgeoning empirical literature seeks to evaluate the effectiveness of these policies using firm-level data and well-identified empirical settings. Some papers look at financial market liberalizations (see for instance [Aghion et al. \(2007\)](#), [Bertrand et al. \(2007\)](#), [Larrain and Stumpner \(2017\)](#)). Others analyze firm response to the availability of subsidized credit (e.g. [Lelarge et al. \(2010\)](#), [Banerjee and Duflo \(2014\)](#), [Brown and Earle \(2017\)](#)), or changes in bank lending behavior ([Fraisie et al. \(2017\)](#), [Blattner et al. \(2017\)](#)). Another set of papers study the effect of capital taxes or subsidies on firm investment and hiring ([Yagan \(2015\)](#), [Zwick and Mahon \(2015\)](#), [Rauh \(2006\)](#), [Rotemberg \(2017\)](#)). By comparing treated firms to a plausibly exogenous control group, these papers quantify the relative effect of these policy interventions on *treated* firms. However, they remain silent on how these firm-level effects would aggregate, were the intervention generalized to a broader set of firms. In this paper, we offer simple formulas to estimate such an aggregate counterfactual using firm-level evidence. This approach does not require the estimation of a structural model of firm behavior and, in particular, does not require that the empiricist precisely knows how the intervention affects firm-level distortions.

Aggregating firm-level responses to these policies is a non-trivial exercise because it requires an explicit modeling of how firms and workers interact with one another. First, standard general equilibrium (GE) effects will typically dampen firm-level responses: for instance, if a growth-enhancing policy is extended to a larger set of firms, labor demand increases, which in turn raises the equilibrium wage and mitigates the initial direct effect. Second, extending the policy to a larger scale reallocates inputs across firms: as distortions are reduced, capital and labor flow from firms with low marginal productivity to firms with high marginal productivity, which leads to an increase in aggregate productivity.

To account for such equilibrium effects and map firm-level estimates into aggregate outcomes, we proceed in three steps. First, we set up a general equilibrium (GE) model with heterogeneous firms who face stochastic productivity shocks and are subject to several forms of distortions: adjustment costs, taxes and financing frictions. We relate aggregate output and total factor productivity (TFP) to the economy-wide distribution of revenue to capital ratios. Under some assumptions, the distribution of the revenue to capital ratio¹

¹The revenue to capital ratio is commonly called “marginal revenue product of capital” (MRPK) or equivalently “capital wedge” in the misallocation literature ([Restuccia and Rogerson \(2008\)](#), [Hsieh and Klenow \(2009\)](#)).

captures the extent of distortions in the economy: A firm with a relatively high revenue to capital ratio is a firm that invests too little, because of adjustment costs, financing constraints or taxes. The distribution of revenue to capital ratios is not a deep structural parameter, in the sense that it depends on firms' histories and choices. But the effect of a given policy on this distribution can easily be estimated using standard datasets and a well-designed experimental setting.

In a second step, we introduce such an empirical setting into our model. We assume that a policy intervention targets a random subset of firms in the economy. This treatment affects parameters governing firm-level frictions, but we do not need to specify which ones exactly. Using firm-level data, we assume that an econometrician estimates the effect of such a treatment on the distribution of revenue to capital ratios (e.g., its mean and variance, or its covariance with firm-level productivity). A natural solution to the aggregation exercise we consider – computing the effect of a generalization of the policy to all firms in the economy – is then to simply plug in the estimated treatment effect of the policy on the distribution of revenue to capital ratios into the aggregation formulas described above. But this solution, adopted by a few recent papers ([Larrain and Stumpner \(2017\)](#), [Rotemberg \(2017\)](#) and [Blattner et al. \(2017\)](#)) and which follows the insight of [Restuccia and Rogerson \(2008\)](#) and [Hsieh and Klenow \(2009\)](#), takes the distribution of revenue to capital ratios as independent of the market equilibrium.

However, in principle, the distribution of these capital wedges can depend on the market equilibrium. For instance, with a higher wage in the labor market, firms are less profitable and hence more financially constrained, which may result in a different distribution of revenue to capital ratios. This scale-dependence is a significant threat to the methodology outlined above. To see this, consider a policy that relaxes financing constraints for a small subset of firms in the economy. The econometrician estimates a significant reduction in the mean and variance of revenue to capital ratio for treated firms, implying that the policy boosts firm-level efficiency and improves the allocation of capital within treated firms. These treatment effects are the firm-level estimates that one would like to plug in the aggregation formulas. However, as this policy is generalized to more firms in the economy, labor demand goes up, which leads to an increase in the equilibrium wage in the labor market. Since firms are on average more constrained, the firm-level statistics estimated in the economy where only a small number of firms are treated could not be valid anymore.² The bottom-line is that the revenue to capital ratios measured in a particular experiment

²for instance, reducing firm-level frictions may lead to a greater reduction in the average revenue to capital ratio when the equilibrium wage is higher

are only observed *conditional* on current equilibrium conditions, and may not be used in alternative counterfactuals.

A key contribution in this paper is to provide conditions under which the revenue to capital ratio is *independent* of general equilibrium conditions. This property allows us to safely use the estimated effect of the policy into the formulas for GE outcome. It relies on two key assumptions. First, the sources of distortions (financing frictions, tax schedules, adjustment costs) are assumed to be homogeneous of degree 1. The intuition for this is that homogeneity guarantees that frictions remain on average constant on a size-adjusted basis. Hence, a change in general equilibrium, which affects firm size, will not affect distortions. This assumption is only mildly restrictive and consistent with most models of firm dynamics used in macro-finance. The second assumption is the firm-level Cobb-Douglas production function, which is another common assumption in the literature.

The formulas we obtain for aggregate output and TFP combine parameters of the macroeconomic model (labor share, goods substitutability, labor supply elasticity) and three sufficient statistics for the joint distribution of log productivity and log revenue to capital. The first statistic is the effect of the treatment on average log wedge. It captures the extent to which the treatment affects the aggregate amount of savings available to firms. The second statistic is the treatment effect on the variance of log wedges. It measures how the treatment distorts the allocation of capital across firms. The final statistic is the treatment effect on the covariance of log wedges and log productivities. Intuitively, if the treatment reduces this covariance, it will make the productive firms relatively less distorted which is good for aggregate output.

We then consider a series of relevant extensions to the basic setup and show how our aggregation formulas extend to these different settings. First, our formulas can easily accommodate the situation where the aggregation exercise is *partial*, in the sense that in the counterfactual, only a larger subset of firms – instead of all firms – receive the treatment. This is a relevant extension since in many settings (e.g., small firms subsidies), the policy focuses by design on a subset of firms, so that the relevant aggregation should be done within this subset. Second, we allow for a more realistic market structure, where firms imperfectly compete within industries, which are allowed to be heterogeneous in terms of their labor shares, price elasticity of demand, and total output shares. Third, we allow for decreasing returns to scale in production. Finally, we provide formulas that make no parametric assumptions about the distribution of revenue to capital ratios.

Our paper is first and foremost of interest for the growing literature that empirically

analyzes firm-level distortions using experimental-like settings. Many of the papers cited above estimate the *firm-level* effect of policies promoting business investment but do not speak to how these policies would affect macroeconomic outcome were the policy extended to all firms in the economy. Our paper provides a simple framework to answer this question using similar identification strategies but focusing on a set of sufficient statistics typically not computed in these studies (mean log revenue to capital ratio, its variance, and its covariance with log productivity). Recent exceptions are [Blattner et al. \(2017\)](#), [Rotemberg \(2017\)](#) and [Larrain and Stumpner \(2017\)](#), who consider an aggregation framework somewhat similar to ours, but in which the distribution of revenue to capital ratios are assumed to be exogenous, and in particular independent of aggregate conditions. One of our contributions is to provide sufficient conditions under which endogenous capital wedges are in fact independent of the market equilibrium. Our paper is also related to the rising literature that seeks to bridge reduced form analysis and structural approach by isolating simple “sufficient statistics” that help measure aggregate outcomes out of simple firm or household-level statistics. For instance, [Davila \(2016\)](#) writes down a model of household borrowing with costs of bankruptcy. He derives the optimal bankruptcy exemption as a function of sufficient statistics that can be observed, in particular, the reduction of consumption measured among bankrupt households and the probability of defaulting. More closely related is recent work by [Baqae and Farhi \(2017\)](#), who derive sufficient statistics to aggregate micro-level estimates. Their approach diverges from ours in that they use a very general aggregation framework, but, in the spirit of [Restuccia and Rogerson \(2008\)](#) and [Hsieh and Klenow \(2009\)](#) assume distortions can be represented by exogenous wedges over the cost of capital. We use a simpler aggregation framework but allow wedges to vary as they do in a large class of models. We also focus on sufficient statistics generated by policy experiments.

Section 2 lays out the economic model. Section 3 develops our methodology. Section 4 shows that the assumptions of Section 2, which are necessary to this result, are consistent with most of the literature on firm dynamics. Section 5 investigates the robustness of our formulas to various extensions to the basic set-up. The last Section concludes.

2 The Economic Model

2.1 Set-up

The economy is dynamic ($t = 0, 1, \dots, \infty$), but there is no aggregate uncertainty and the economy is assumed to be in steady state. We first consider a simple market structure and extend the analysis to include heterogeneous industries in Section 5. At each date t , a continuum of monopolists produce imperfectly substitutable intermediate goods in quantity y_{it} at a price p_{it} (Dixit and Stiglitz (1977)). There is a perfectly competitive final good market, which aggregates intermediate output according to a CES technology:

$$Y = \left(\int_i y_{it}^\theta di \right)^{\frac{1}{\theta}}, \quad (1)$$

where we omit the t subscript for aggregate output because the economy is in steady state. We use the final good as the numeraire. Profit maximization in the final good market implies that the demand for product i is given by: $p_{it} = \left(\frac{Y}{y_{it}} \right)^{1-\theta}$ and $-\frac{1}{1-\theta}$ is the price elasticity of demand.

To produce, firms combine labor and capital according to a Cobb-Douglas production function: $y_{it} = e^{z_{it}} k_{it}^\alpha l_{it}^{1-\alpha}$, where k_{it} is firm i 's capital stock in period t , l_{it} is the labor input in period t , α is the capital share and z_{it} is firm i 's idiosyncratic productivity shock in period t . With monopolistic competition and the demand system in Equation (1), firm i revenue in period t is $p_{it}y_{it} = Y^{1-\theta} y_{it}^\theta$. We assume that there is no adjustment costs to labor so that labor is a static input. If w is the steady state wage, static labor optimization implies that firm i 's profit becomes:

$$\pi_{it} = p_{it}y_{it} - wl_t = \kappa_0 \left(\frac{Y}{w^{\frac{1-\alpha}{\alpha}} \frac{\phi}{1-\phi}} \right)^{1-\phi} e^{\frac{\phi}{\alpha} z_{it}} k_{it}^\phi,$$

where $\phi = \frac{\alpha\theta}{1-(1-\alpha)\theta} < 1$. κ_0 is a function of α and ϕ . Productivity shocks (z_{it}) are Markovian and $H(z_{it+1}|z_{it})$ is the c.d.f of z_{it+1} conditional on z_{it} .

The capital good consists of final good – so that its price is also 1 – and it depreciates at a rate δ . Capital investment in period t is subject to a one period time-to-build. Firms can finance investment using the profits they realize from operations or through external financing. The first source of outside financing is debt. b_{it+1} is the total real payment due to creditors in period $t + 1$. To simplify notations, we define $\mathbf{x}_{it} = (k_{it}, k_{it+1}, b_{it}, b_{it+1})$.

We introduce Θ_i , a vector containing all the model's parameters for firm i . $r_{it}=r(z_{it}, \mathbf{x}_{it}; \Theta_i, w, Y)$ is the interest rate on the loan granted at date t , so that $\frac{b_{it+1}}{1+r_{it}}$ is the proceed from debt financing received in period t . As will become clear, this flexible function is designed to allow for risky debt and loss given default. We allow the firm's investment and debt financing at date t to be subject to adjustment costs $\Gamma(z_{it}, \mathbf{x}_{it}; \Theta_i, w, Y)$. We also assume that firms pay taxes and receive subsidies: $\mathcal{T}(z_{it}, \mathbf{x}_{it}; \Theta_i, w, Y)$ corresponds to the net tax paid by the firm.

The second source of outside funding is equity. The firm can raise funds from shareholders in the equity market, or distribute excess funds to shareholders: e_{it} is the equity issuance (if negative) or distribution (if positive) made by firm i in period t : it corresponds to the financing gap left after all other sources of financing have been used:

$$\begin{aligned} e_{it} &= \pi_{it} - (k_{t+1} - (1 - \delta)k_t) - \Gamma(z_{it}, \mathbf{x}_{it}; \Theta_i, w, Y) \\ &+ \left(\frac{b_{it+1}}{1 + r(z_{it}, \mathbf{x}_{it}; \Theta_i, w, Y)} - b_{it} \right) - \mathcal{T}(z_{it}, \mathbf{x}_{it}; \Theta_i, w, Y) \\ &= e(z_{it}, \mathbf{x}_{it}; \Theta_i, w, Y) \end{aligned}$$

We consider generic financing frictions. First, equity issuance may be costly, and we note $\mathcal{C}(z_{it}, \mathbf{x}_{it}; \Theta_i, w, Y)$ these equity issuance costs. Second, the amount of outside financing may be constrained, a friction that we capture through a vector of constraint: $M(z_{it}, \mathbf{x}_{it}; \Theta_i, w, Y) \leq 0$.

The timing is standard in models of firm dynamics. At the beginning of period t , productivity z_{it} is realized. The firm then combines capital in place k_{it} with labor l_{it} to produce and receive the corresponding profits. It then selects the next period stock of capital k_{it+1} , pays the corresponding adjustment costs, reimburse its existing debt b_{it} and receive the proceed from debt issuance $\frac{b_{it+1}}{1+r_{it}}$

Omitting the it index and denoting with prime next-period variables, we can represent firms optimization problem through the following Bellman equation:

$$V(z, k, b; \Theta, w, Y) = \begin{cases} \max_{k', b'} e(z, \mathbf{x}; \Theta, w, Y) - \mathcal{C}(z, \mathbf{x}; \Theta, w, Y) + \beta^F \mathbb{E}_{z'} [V(z', k', b'; \Theta, w, Y) | z], \\ M(z, \mathbf{x}; \Theta, w, Y) \leq 0 \end{cases} \quad (2)$$

where β^F is the firm's discount rate. In what follows, we assume the standard conditions on the cost functions and the constraints so that the contraction mapping theorem applies to this Bellman equation and there is a unique value function.

The household side of the economy is stripped down to its essentials. A representative household has GHH preferences (Greenwood et al. (1988)) over consumption and leisure: $u(c_t, l_t) = \frac{1}{1-\gamma} \left(c_t - \frac{l_t^{1+\frac{1}{\epsilon}} \bar{w}}{1+\frac{1}{\epsilon} \bar{L}^{\frac{1}{\epsilon}}} \right)^{1-\gamma}$, where c_t is period t consumption, l_t is period t labor supply, ϵ is the Frisch elasticity and (\bar{w}, \bar{L}) are constant. The representative household owns all firms in the economy. β^H is the representative household's discount rate. In the absence of aggregate uncertainty, optimal consumption and labor supply decisions imply that $L_t^s = \bar{L} \left(\frac{w}{\bar{w}} \right)^\epsilon$ and $\beta^H = \frac{1}{1+r}$, where r is the exogenous risk-free rate. Note that since households portfolios are well diversified across firms, we also have $\beta^F = \frac{1}{1+r}$ even though the model potentially allows for firms' default.

2.2 Introducing capital wedges

Instead of solving the model explicitly, we will characterize its equilibrium as a function of the distribution of objects defined as capital wedges τ which vary over time and across firms. These wedges are defined as the ratio of a firm's marginal revenue product of capital to some frictionless user cost of capital R for firm i in period t . R is arbitrary but fixed throughout the analysis. Wedges measure how much firms' capital stock deviates from frictionless optimization. In our model, firms potentially deviate from frictionless optimum for three reasons: financing frictions, adjustment costs, and taxes.

Definition 1 (Definition of wedges).

$$\begin{aligned} 1 + \tau(z, \mathbf{x}; \Theta, w, Y) &= \frac{1}{R} \frac{\partial py}{\partial k}(z, \mathbf{x}; \Theta, w, Y) \\ &= \frac{\alpha \theta}{R} \frac{py(z, \mathbf{x}; \Theta, w, Y)}{k} \end{aligned}$$

As previously noted in the literature, in this Cobb-Douglas framework, wedges are easy to measure since they are proportional to the ratio of revenue to capital. Both revenue and capital can be approximated using standard firm-level datasets containing financial statements. A priori, wedges are complicated functions of productivity, firm-level policies variables (debt and capital), model parameters Θ and aggregate conditions (w, Y) . In the following, we will summarize the information on wedges through the joint distribution of log wedges $\log(1 + \tau)$ and log productivity z , whose c.d.f. we note $F(z, \tau; \Theta, w, Y)$. This distribution reflects the fact that otherwise similar firms may have different wedges because of

different histories embedded in their state variables. For instance, firms who experienced a long sequence of adverse productivity shocks have little capital and therefore little ability to borrow, even if the current productivity goes up. Their investment decision will exhibit a larger wedge. The c.d.f. $F(z, \tau; \Theta, w, Y)$ also reminds us that the wedge distribution is a priori conditional on the aggregate state of the economy (e.g., an increase in aggregate output Y may lead to firms being less constrained on average) and the parameters governing the optimization problem they face (Θ).

2.3 Equilibrium

We now solve the competitive equilibrium of this economy in the steady state as a function of the distribution of the capital wedges defined in Definition 1. In this simple model, the steady state equilibrium corresponds to an equilibrium wage w that clears the labor market and aggregate output Y that clears the final good market.

Given the definition of capital wedges τ , the static FOC in labor, we can write down the market clearing conditions in labor and final good. We note $F(z, \tau; \Theta, w, Y)$ the c.d.f. of log wedges $\log(1 + \tau)$ and log productivity z .

To simplify exposition, we assume that all firms face the same parameters so Θ is the same for all firms and therefore does not need to be indexed by i . This assumption is not necessary but makes equations clearer. This leads to the following formulas describing aggregate production and TFP, as in Hsieh and Klenow (2009) and Midrigan and Xu (2014):

$$Y \propto \left(\int_{z, \tau} \left(\frac{e^z}{(1 + \tau)^\alpha} \right)^{\frac{\theta}{1-\theta}} dF(z, \tau; \Theta, w, Y) \right)^{(1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta}} \quad (3)$$

$$TFP = \frac{\left(\int_{z, \tau} \left(\frac{e^z}{(1+\tau)^\alpha} \right)^{\frac{\theta}{1-\theta}} dF(z, \tau; \Theta, w, Y) \right)^{\frac{1-(1-\alpha)\theta}{\theta}}}{\left(\int_{z, \tau} \frac{e^{\frac{\theta}{1-\theta}z}}{(1+\tau)^{\frac{1-(1-\alpha)\theta}{1-\theta}}} dF(z, \tau; \Theta, w, Y) \right)^\alpha} \quad (4)$$

Clearly, the above equations do not solve for (w, Y) , since analytically solving the model is in general not feasible. These two equations offer relationships between macro outcomes (output and TFP) and the distribution of wedges, which can be observed. To make these equation easier to apprehend, we now introduce a simplification.

2.4 Small Perturbation Approximation

Equations 3 and 4 show how aggregate TFP and output depend on the joint distribution of log productivity z and wedges τ . We simplify these expressions by focusing on the following case:

Assumption 1. *Variations in z and $\log(1 + \tau)$ are small relative to their respective means.*

In this multiplicative set-up, Assumption 1 is equivalent to assuming that $\log(1 + \tau)$ and z are jointly normally distributed (which is the assumption made for instance in Hsieh and Klenow (2009)). Since $\log\left(\frac{py}{k}\right) = \ln(1 + \tau) + \text{cst}$, Assumption 1 implies that the log revenue to capital ratio is also normally distributed. We test the relevance of this assumption using data from BvD AMADEUS Financials for the year 2014. As in Gopinath et al. (2015), we measure py_{it} as the value added of the firm, i.e. the difference between gross output (operating revenue) and materials. We measure the capital stock, k_{it} , with the book value of fixed tangible and intangible. For 9 countries in our sample (France, Italy, Spain, UK, Portugal, Croatia, Sweden, Bulgaria and Romania), we report in Figure 1 normal probability plots, i.e. plots of the empirical c.d.f. of the standardized log revenue to capital ratios against the c.d.f. of a normal distribution. Figure 1 shows that the log-normality assumption is a reasonable one.

Neither of these assumptions (small deviations or log normality) is necessary to our approach and we provide in our robustness section formulas that do not rest on it. But assumption 1 proves useful to clarify the logic of our approach, as it summarizes the distribution of wedges in a handful of moments.

The second order Taylor expansion of equations (3-4) in $\log(1 + \tau)$ and z around their means leads to simple formulas summarized in the following proposition:

Proposition 1. *Under Assumption 1, at equilibrium, aggregate output and TFP can be written as simple functions of three moments of the joint distribution of log wedges $\ln(1 + \tau)$ and log productivity z .*

$$\log Y = \frac{\alpha(1 + \epsilon)}{1 - \alpha} \left(-\mu_\tau(\Theta, w, Y) + \frac{1}{2} \frac{\theta}{1 - \theta} (\alpha\sigma_\tau^2(\Theta, w, Y) - 2\sigma_{\tau z}(\Theta, w, Y)) \right) + \text{cst} \quad (5)$$

$$\log(TFP) = -\frac{\alpha}{2} \left(1 + \frac{\alpha\theta}{1 - \theta} \right) \sigma_\tau^2(\Theta, w, Y) \quad (6)$$

where $\mu_\tau(\Theta, w, Y)$ and $\sigma_\tau^2(\Theta, w, Y)$ are the steady-state mean and variance of the ergodic distribution of log capital wedges $\log(1 + \tau)$ for an economy of firms facing optimization

problem 2 with parameters Θ and aggregate conditions (w, Y) . Similarly, $\sigma_{\tau z}(\Theta, w, Y)$ is the steady-state covariance between log productivity and log wedges.

Proof. See Appendix B.1. □

These formulas illustrate forces already discussed in the literature. Dispersion of wedges impairs aggregate efficiency because it creates capital misallocation (Hsieh and Klenow (2009)). A positive correlation between productivity and wedges also hurts aggregate production: output is lower when the most productive firms experience the largest distortions (Hopenhayn (2014)). However, in our setting, such a correlation does not affect aggregate TFP. This result emanates from the small deviation assumption (or alternatively, from log normality). For instance, it does not hold in Restuccia and Rogerson (2008), who use a binary distribution for the distribution of distortions.

The above formulas suggest a very simple methodology to aggregate firm-level evidence:

1. Measure the treatment effect of a policy experiment on the three moments introduced in Proposition 1 (mean and variance of log wedges, and covariance of log wedges with log productivity). These three moments are easy to compute using firm-level data since log wedges are equal to log revenue-to-capital ratios up to a constant.
2. Plug these treatment effects into formulas (5-6). This would lead to the aggregate effect (in terms of log-changes in aggregate output and TFP) of generalizing the experiment to all firms in the economy. This approach is originally the one of Hsieh and Klenow (2009) who use the variance of log revenue to capital ratios of the US to investigate TFP losses among Indian firms due to misallocation. It has since been taken on in a few recent papers based on quasi-experimental frameworks (Blattner et al. (2017), Larrain and Stumpner (2017), Rotemberg (2017)).

However, this methodology faces a challenge, well illustrated by the above formulas. The distribution of wedges is only observed *conditional* on the economy's equilibrium (w, Y) . The inference on the joint distribution of $(z, \log(1 + \tau))$ in step 1 may thus depend on the size of the experiment, as we expect aggregate outcomes to change as the experiment scales up. In particular, when all firms receive the treatment, the equilibrium distribution of $(z, \log(1 + \tau))$ may differ from the distribution observed in an experimental setting where only a fraction of firms receive the treatment. The next section details this issue and shows a set of sufficient conditions for the model presented in this section such that this issue does not arise and the above methodology is valid.

3 Inference and Aggregation of Policy Experiments

In this section, we show how to perform aggregate counterfactuals using firm-level estimates. To simplify the exposition, we assume in this section that the empirical exercise consists of a simple binary treatment, where a random subset of firms are treated. Our approach can easily be generalized to continuous treatments.

3.1 Definition of the Empirical Treatment

An econometrician observes data on an infinite number of firms, of which a subset is subject to an empirical treatment. The treatment is binary: firm i is either treated ($T_i = 1$) or untreated ($T_i = 0$). This treatment is a policy that affects the parameters Θ governing financing constraints, adjustment costs or taxes. Θ_0 (resp. Θ_1) correspond to the parameters of non-treated (resp. treated) firms. The econometrician does not necessarily know how the treatment affects these parameters. However, we assume that she knows that the treatment leaves the following three parameters unchanged: the capital share in production α , the price elasticity of demand θ , and the labor supply elasticity ϵ .

To be consistent with our model, we assume that the econometrician observes the ergodic distribution of firms in the economy. As a result of this assumption, we do not need to worry about the exogeneity of the treatment: at the steady state, the model converges to its new ergodic distribution, which is independent of initial conditions. This arises because, in our simple setting, heterogeneity only stems from temporary productivity shocks. Our results extend directly in the presence of persistent productivity shocks, although they require the additional assumption that the treatment is orthogonal to long-term productivity.

3.2 Aggregating the Treatment

Our goal is to aggregate the firm-level evidence gathered from the experiment. This aggregation exercise consists of measuring the effect of generalizing the policy from the subset of treated firms to *all* firms in the economy. An alternative version of our aggregation exercise consists of extending the policy to just a larger fraction of firms in the economy. We focus on full aggregation here, and explore partial aggregation in Section 5. We summarize the total aggregation exercise in the paragraph below.

Objective 1 (Aggregation of the Treatment). *The aggregation of the treatment consists of computing the log-change in output and TFP when moving from an economy where no firms are non-treated ($\Theta = \Theta_0$ for all firms) to an economy where all firms are treated ($\Theta = \Theta_1$).*

Note (w_0, Y_0, TFP_0) (resp. (w_1, Y_1, TFP_1)) the equilibrium quantities in the economy where no firms (resp. all firms) are treated. Then:

$$\Delta \log Y = \frac{\log(Y_1) - \log(Y_0)}{\log(Y_0)} = \frac{\alpha(1 + \epsilon)}{1 - \alpha} \left(-\Delta\mu_\tau + \frac{1}{2} \frac{\theta}{1 - \theta} (\alpha\Delta\sigma_\tau^2 - 2\Delta\sigma_{\tau z}) \right) \quad (7)$$

$$\Delta \log(TFP) = \frac{\log(TFP_1) - \log(TFP_0)}{\log(TFP_0)} = -\frac{\alpha}{2} \left(1 + \frac{\alpha\theta}{1 - \theta} \right) \Delta\sigma_\tau^2 \quad (8)$$

where:

$$\begin{aligned} \Delta\mu_\tau &= \mu_\tau(\Theta_1, w_1, Y_1) - \mu_\tau(\Theta_0, w_0, Y_0) \\ \Delta\sigma_\tau^2 &= \sigma_\tau^2(\Theta_1, w_1, Y_1) - \sigma_\tau^2(\Theta_0, w_0, Y_0) \\ \Delta\sigma_{z\tau} &= \sigma_{z\tau}(\Theta_1, w_1, Y_1) - \sigma_{z\tau}(\Theta_0, w_0, Y_0) \end{aligned}$$

The equations above make clear the challenge faced by the econometrician. The econometrician does not directly observe the distribution of wedges and productivities *conditional* on the new equilibrium (w_1, Y_1), since this new equilibrium is by definition not observed. Therefore, empirically, the econometrician cannot directly estimate $\Delta\mu_\tau$ using the available evidence, but instead can only estimate the following statistic $\widehat{\Delta\mu_\tau}$, which is a priori not equal to $\Delta\mu_\tau$:

$$\begin{aligned} \widehat{\Delta\mu_\tau} &= \mathbb{E} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_i = 1 \right) - \mathbb{E} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_i = 0 \right) \\ &= \mu_\tau(\Theta_1, w_0, Y_0) - \mu_\tau(\Theta_0, w_0, Y_0) \\ &\neq \Delta\mu_\tau = \mu_\tau(\Theta_1, \mathbf{w}_1, \mathbf{Y}_1) - \mu_\tau(\Theta_0, w_0, Y_0) \quad \text{a priori} \end{aligned}$$

To perform the aggregation exercise stated in Objective 1, however, the econometrician needs to estimate $\Delta\mu_\tau$ and not $\widehat{\Delta\mu_\tau}$. Of course, this problem is not specific to the mean but is relevant for all moments of the joint distribution of $(z, \log(1 + \tau))$. The next Section presents sufficient conditions under which these statistics are in fact similar.

3.3 Scale Invariance of the Wedge Distribution

This section presents one of our main results: we provide sufficient conditions under which the joint distribution of wedges and productivity does not depend on the equilibrium quantities w and Y . As we detail below, the assumptions necessary to obtain this result are satisfied in a large class of models of firm dynamics, commonly used in macro-finance.

Proposition 2 (Distribution of wedges).

Let $S = \frac{Y}{w \frac{(1-\alpha)\phi}{(1-\phi)\alpha}}$ be the steady state “scale” of the economy. Assume that:

1. (1) the adjustment cost $\Gamma()$, (2) taxes $\mathcal{T}()$, (3) the vector of funding constraint $M()$ and, (4) the equity issuance cost function $\mathcal{C}()$ all satisfies the following property:

$$\forall (z, \mathbf{x}; \Theta, w, Y), \quad Q(z, \mathbf{x}; \Theta, w, Y) = S \times Q\left(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1\right) \quad (9)$$

2. The interest rate function $r()$ satisfies the following property:

$$\forall (z, \mathbf{x}; \Theta, w, Y), \quad r(z, \mathbf{x}; \Theta, w, Y) = r\left(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1\right)$$

Then, the joint-distribution of z and τ does not depend on (w, Y) :

$$F(z, \tau; \Theta, w, Y) \equiv F(z, \tau; \Theta)$$

Proof. See Appendix [B.2](#) □

This proposition shows that, given parameters Θ , the ergodic distribution of capital wedges does not depend on the scale of the economy. It is the key result of the paper. It implies that under the assumptions highlighted in Proposition 2, the wedge distribution observed for a subset of firms operating under parameters Θ does not depend on the behavior of other firms, in particular on the parameters $\tilde{\Theta}$ faced by other firms in the economy. This result is necessary to be able to estimate the aggregate counterfactual presented in Objective 1 using firm-level evidence. This result rests on two key assumptions. The first assumption is that firm-level production is Cobb-Douglas, since the multiplicative property of Cobb-Douglas technology implies that firm-level (revenue-)productivity scales proportionally to S . The second assumption, explicitly in Proposition 2 is that frictions have to satisfy properties that resemble homogeneity in firm policies k and b . Our assumption is a bit more general as it only requires homogeneity with respect to S , an assumption that

firm's operating profit satisfy – although they may not be strictly homogeneous. As it turns out, these assumptions are valid in a large class of existing models of firm dynamics. We discuss the validity of these homogeneity assumptions in greater detail in Section 4.

3.4 Taking Stock: Aggregation Formulas

In this Section, we summarize our aggregation methodology. Under the assumptions of Proposition 2, the econometrician can proceed in two steps. First, she uses the firm-level evidence to measure the treatment effect on the three moments introduced in Proposition 1. For instance, focusing on the average log wedge, the econometrician can estimate:

$$\begin{aligned}
\widehat{\Delta\mu_\tau} &= \mathbb{E} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_i = 1 \right) - \mathbb{E} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_i = 0 \right) \\
&= \mathbb{E} (\log (1 + \tau) | T_i = 1) - \mathbb{E} (\log (1 + \tau) | T_i = 0) \\
&= \mu_\tau(\Theta_1, w_0, Y_0) - \mu_\tau(\Theta_0, w_0, Y_0) \\
&= \mu_\tau(\Theta_1, w_1, Y_1) - \mu_\tau(\Theta_0, w_0, Y_0) \\
&= \Delta\mu_\tau
\end{aligned}$$

To go from the third to the fourth line in the previous equation is valid under the assumptions of Proposition 2, as the distribution of τ is then independent of w and Y . As a result, the empirical estimate $\widehat{\Delta\mu_\tau}$ now provides us with the relevant statistics to be used in the aggregation exercise. The same reasoning applies to the two other moments of Proposition 1, which can be estimated through:

$$\begin{aligned}
\widehat{\Delta\sigma_\tau^2} &= \text{Var} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_i = 1 \right) - \text{Var} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_i = 0 \right) = \Delta\sigma_\tau^2 \\
\widehat{\Delta\sigma_{z\tau}} &= \text{Cov} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right), z_{it} | T_i = 1 \right) - \text{Cov} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right), z_{it} | T_i = 0 \right) = \Delta\sigma_{z\tau}
\end{aligned}$$

In a second step, the econometrician then plugs these three statistics into the aggregation formulas (7-8). They require prior knowledge of only three parameters: the capital share α , the extent of competition θ and the labor supply elasticity ϵ . With these 3 parameters only, the econometrician can estimate an aggregate counterfactual which does not

require a full structural estimation of the firm-level problem 2 nor a precise mapping from the treatment to the parameters Θ affected by the experiment.

A standard calibration of these parameters in the aggregation formula (7-8) is $\alpha = .33$ (Bartelsman et al. (2013)), $\theta = .8$ (Broda and Weinstein (2006)) and $\epsilon = .5$ (Chetty (2012)). With this calibration, the aggregation formula imply that a policy that increases investment by about 2% at the firm-level and leaves the variance of wedges as well as the correlation of wedges with productivity unchanged would increase output by .63%.³ A policy that would leave the average wedge and the correlation of wedge and TFP unchanged, but would reduce the dispersion of wedges by 1.29 percentage points would similarly lead to an increase in aggregate output of .63%.

4 Relation with Standard Models of Firm Dynamics

Proposition 2 requires that the frictions faced by firms be homogeneous of degree 1. In this section, we show that standard models of firm dynamics with frictions satisfy these assumptions. We discuss in turn real frictions (adjustment cost), financial frictions and taxes.

4.1 Adjustment Costs

Consider first the case of adjustment costs. Quadratic adjustment costs to capital, linear adjustment costs, fixed costs that scale either with production, output and capital or discount for capital resale all satisfy the assumptions in Proposition 2. For instance, if $\Gamma(\cdot)$ is given by:

$$\Gamma(z, \mathbf{x}; \Theta, w, Y) = \gamma_1 \frac{(k' - (1 - \delta)k)^2}{k} + \gamma_2 k + \mathbb{1}_{\{k' - (1 - \delta)k \neq 0\}} (\gamma_3 y + \gamma_4 p y + \gamma_5 k) + \gamma_6 k \mathbb{1}_{\{k' - (1 - \delta)k < 0\}},$$

then, since $y(z, k; \Theta, w, Y) = S \times y(z, \frac{k}{S}; \Theta, 1, 1)$ and $p y(z, k; \Theta, w, Y) = S \times p y(z, \frac{k}{S}; \Theta, 1, 1)$, it is trivial to show that $\Gamma(z, \mathbf{x}; \Theta, w, Y) = S \times \Gamma(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1)$

³The revenue to capital ratio is given by: $\Delta \log \frac{p y}{k} = -\frac{1 - \theta}{1 - \theta(1 - \alpha)} \Delta \log k$, so that a 2% increase in investment corresponds to a .85% decline in the sales-to-capital ratio ($\Delta \mu_\tau = -.85\%$), which in turn leads to an increase in aggregate output of $\frac{\alpha(1 + \epsilon)}{1 - \alpha} \times .85\% = .63\%$.

4.2 Financing Frictions

Second, consider the financing side of the model. Our formulation encompass standard models of financing constraints and investment.

Let us start with the interest rate function. For instance, in [Michaels et al. \(2016\)](#) or [Gilchrist et al. \(2014\)](#), debt is risky and in the event that the firm is unable to repay, the lender can seize a fraction $1 - \zeta$ of the firm's fixed assets k . The firm's future market value is not collateralizable, so that a firm's access to credit is mediated by a net worth covenant, which restrains the firm's ability to sell new debt based on its current physical assets and liabilities. Concretely, default is triggered when net worth reaches 0, which defines a threshold value for productivity \hat{z} such that:

$$0 = \kappa_0 S^{1-\phi} e^{\frac{\phi}{\alpha} \hat{z}} k^\phi - b + c^k (1 - \delta) k, \quad (10)$$

where c_k is the second-hand price of capital, which we treat as a technological parameter. As in [Michaels et al. \(2016\)](#), the right side of the previous equation represents the resources that the firm could raise in order to repay its debt just prior to bankruptcy, which is why its capital is valued at the second-hand price c_k . The wage bill is absent from the previous equation because labor is paid in full, even if the firm subsequently defaults. Finally, the face value of debt discounted at the interest rate $r(z, \mathbf{x}; \Theta, w, Y)$ must equal the debt holder's expected payoff discounted at the risk-free rate:

$$\frac{1}{1+r} \left[\int_0^{\hat{z}} \left(\kappa_0 S^{1-\phi} e^{\frac{\phi}{\alpha} z'} k'^\phi + (1-\zeta)(1-\delta)k' \right) dH(z'|z) + (1-H(\hat{z}|z))b' \right] = \frac{b'}{1+r(z, \mathbf{x}; \Theta, w, Y)} \quad (11)$$

Equations (10) and (11) provides the joint definition for the interest rate function $r(z, \mathbf{x}; \Theta, w, Y)$, which satisfies the assumption in [Proposition 2](#). Note first that equation (10) can be rewritten as: $0 = \kappa_0 e^{\frac{\phi}{\alpha} \hat{z}} \frac{k^\phi}{S} - \frac{b}{S} + c^k (1 - \delta) \frac{k}{S}$. As a result, it is clear that $\hat{z}(k, b; \Theta, w, Y) = \hat{z}(\frac{k}{S}, \frac{b}{S}; \Theta, 1, 1)$. Also, we can rewrite Equation (11) as:

$$\frac{1}{1+r} \left[\int_0^{z(\frac{k'}{S}, \frac{b'}{S}; \Theta, 1, 1)} \left(\kappa_0 e^{\frac{\phi}{\alpha} z'} \frac{k'^\phi}{S} + (1-\zeta)(1-\delta) \frac{k'}{S} \right) dH(z'|z) + (1-H(\hat{z}|z)) \frac{b'}{S} \right] = \frac{\frac{b'}{S}}{1+r(z, \mathbf{x}; \Theta, w, Y)},$$

so that $r(z, \mathbf{x}; \Theta, w, Y) = r(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1)$.

Similarly, the specification of debt renegotiation in [Hennessy and Whited \(2007\)](#) would

also satisfy these assumptions. More generally, these models make the probability of default independent of the scale of the economy S , and the loss given default proportional to S . These properties ensure our assumption about $r(\cdot)$ in Proposition 2 is satisfied. Obviously, models of risk-free debt, such as [Midrigan and Xu \(2014\)](#), also satisfy our assumption.

Our assumption on the cost of equity is also verified in [Michaels et al. \(2016\)](#) and [Gilchrist et al. \(2014\)](#), who posit that equity issuances are subject to an underwriting fees such that there is a positive marginal cost to issue equity:

$$\mathcal{C}(z, \mathbf{x}; \Theta, w, Y) = \lambda |e(z, \mathbf{x}; \Theta, w, Y)| \mathbb{1}_{\{e(z, \mathbf{x}; \Theta, w, Y) < 0\}}$$

Given that $e(z, \mathbf{x}; \Theta, w, Y) = S e(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1)$, it is obvious that $\mathcal{C}(z, \mathbf{x}; \Theta, w, Y) = S \times \mathcal{C}(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1)$. Thus, the financing frictions specified in [Gilchrist et al. \(2014\)](#) and [Michaels et al. \(2016\)](#) satisfy the assumptions of Proposition 2. Additionally, it is obvious to see that fixed or quadratic issuance costs would satisfy our assumptions as long as they are appropriately scaled with the size of the firm. For instance, $\psi \frac{e_{it}^2}{k_{it}} \mathbb{1}_{e_{it} < 0}$, or $\iota k_{it} \mathbb{1}_{e_{it} < 0}$ would fall in this category.

Finally, our formulation of financing frictions also encompasses debt constraints as for instance in [Midrigan and Xu \(2014\)](#) or [Catherine et al. \(2017\)](#). In [Midrigan and Xu \(2014\)](#), debt is assumed to be risk-free through full collateralization: $b' \leq \xi k'$ so that $r(z, \mathbf{x}; \Theta, w, Y) = r_f$ and producers can only issue claims to a fraction χ of their future profits: $e(z, \mathbf{x}; \Theta, w, Y) \geq -\chi V(z, \mathbf{x}; \Theta, w, Y)$. In this case, the vector $M(z, \mathbf{x}; \Theta, w, Y)$ consists of the last two inequalities, and it is direct to see that both M and $r(\cdot)$ satisfy the assumptions of Proposition 2. Of course, any combination of the constraints in [Midrigan and Xu \(2014\)](#) and [Hennessy and Whited \(2007\)](#) would also satisfy these assumptions. Note also that our model also encompasses debt constraints where debt financing is limited by existing or future cash flows ($b \leq \iota e(z, \mathbf{x}; \Theta, w, Y)$).

4.3 Taxes

Standard specifications for the corporate income tax satisfy the assumption of Proposition 2: $\mathcal{T}(z, \mathbf{x}; \Theta, w, Y) = \tau \max(0, \pi(z, \mathbf{x}; \Theta, w, Y) - \delta k - b)$. However, a progressive tax system would violate our assumptions.

5 Robustness and Extensions

This section proposes extensions to the methodology developed in Section 3 and the model presented in Section 2.

5.1 Partial Aggregation

In some settings, the full aggregation exercise discussed in Section 3 may not be relevant, for instance in the presence of government budget constraint. The following proposition adapts our framework to the case of a partial aggregate counterfactual where the treatment is extended to a fraction $\lambda < 1$ of the firms in the economy. As before, the empirical setting consists of a random 0-measure set of firms that receive a treatment $T_i = 1$ that consists of a change in the parameters governing frictions (i.e Θ going from Θ_0 to Θ_1).

Proposition 3 (Partial Aggregation Formulas). *Assume that the assumptions in Proposition 2 hold. In addition, assume that the effect of the treatment is small in the sense that $\Delta\mu_\tau$, $\Delta\sigma_\tau^2$ and $\Delta\sigma_{z\tau} \ll 1$.*

The procedure to compute the partial aggregation counterfactual follows three steps:

1. *Estimate the following treatment effects in the data:*

$$\begin{aligned}\widehat{\Delta\mu_\tau} &= \mathbb{E} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_i = 1 \right) - \mathbb{E} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_i = 0 \right) \\ \widehat{\Delta\sigma_\tau^2} &= \text{Var} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_i = 1 \right) - \text{Var} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_i = 0 \right) \\ \widehat{\Delta\sigma_{z\tau}} &= \text{Cov} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right), z_{it} | T_i = 1 \right) - \text{Cov} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right), z_{it} | T_i = 0 \right)\end{aligned}$$

2. *Compute the modified treatment effects that reflect the partial nature of this aggregation:*

$$\begin{aligned}\widetilde{\Delta\mu_\tau} &= \lambda \widehat{\Delta\mu_\tau} \\ \widetilde{\Delta\sigma_\tau^2} &= \lambda \widehat{\Delta\sigma_\tau^2} + \lambda(1 - \lambda)(\widehat{\Delta\mu_\tau})^2 \\ \widetilde{\Delta\sigma_{z\tau}} &= \lambda \widehat{\Delta\sigma_{z\tau}}\end{aligned}$$

3. *Plug these modified treatment effects into the aggregation formulas (7-8).*

Proof. See Appendix B.3. □

The above proposition says that the formulas developed in Section 3 still apply under partial aggregation, but the data moments have to be adapted to take into account the fact that not all firms are treated. Quite intuitively, the effect of the differential mean wedge has to be multiplied by the fraction actually treated in the aggregation counterfactual (λ). The same intuition directly applies to the covariance between log productivity and log wedges. The variance of log wedges is, however, affected in a subtler way as the result of two effects. The first effect comes from the fact that the experiment itself increases the variance of wedges by $\Delta\sigma_\tau^2$, so the overall variance increases by λ times that amount. The second effect is specific to partial aggregation: treating only a fraction λ of firms will generate additional misallocation between treated and non-treated firms. \times

5.2 Heterogeneous Industries

In this section, we consider a more general market structure than the one presented in Section 2: the economy features industries that are heterogeneous in (1) their output share in total output (2) their labor share (3) the degree of competition between firms within the industry (4) the parameters that govern the firm-level dynamics of investment and hiring and (5) potentially the treatment they receive.

More precisely, let S be the number of industries and M_s the set of firms operating in industry s . Firms in each industry produce in monopolistic competition as in Section 2, and the price-elasticity of demand $-\frac{1}{1-\theta_s}$ can be industry specific: $Y_s = \left(\int_i y_{is}^{\theta_s}\right)^{\frac{1}{\theta_s}}$. The final good market produces by combining each industry output according to the following Cobb-Douglas production function:

$$\ln(Y) = \sum_{k=1}^S \phi_s \ln(Y_s) \quad \text{and} \quad \sum_{s=1}^S \phi_s = 1. \quad (12)$$

Within industry s , the production function is given by: $y_{it} = e^{z_{it}} k_{it}^{\alpha_s} l_{it}^{1-\alpha_s}$, i.e. the labor share is assumed to be industry-specific, while we assume that the distribution of idiosyncratic productivity shocks is common across industries. Beyond ϕ_s , α_s and θ_s , an industry is also characterized by the vector of parameters that govern the frictions in the firm-level optimization problem 2, Θ_s , which is now industry-specific. Finally, we also allow the binary treatment $T_i \in \{0, 1\}$ to be industry-specific: in industry s , treated (resp. control) firms operate under $\Theta = \Theta_s^1$ (resp. Θ_s^0). Beyond these modifications, the setting is similar to that of Section 3.

In this augmented setting, the aggregate counterfactual we consider is one where, in each industry s , the industry-specific treatment is extended to all firms in the industry (i.e., in each industry, Θ goes from Θ_0^s to Θ_s^1). This counterfactual can be estimated in the following way:

Proposition 4 (Aggregating Heterogeneous Industries). *Under the homogeneity assumptions of Proposition 2, the joint distribution of z and τ in industry s does not depend on w and $(Y_{s'})_{s' \in [1, S]}$.*

Define the following industry-level treatment effects:

$$\begin{aligned}\widehat{\Delta\mu_\tau}(s) &= \mathbb{E} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_i = 1, s_i = s \right) - \mathbb{E} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_i = 0, s_i = s \right) \\ \widehat{\Delta\sigma_\tau^2}(s) &= \text{Var} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_i = 1, s_i = s \right) - \text{Var} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_i = 0, s_i = s \right) \\ \widehat{\Delta\sigma_{z\tau}}(s) &= \text{Cov} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right), z_{it} | T_i = 1, s_i = s \right) - \text{Cov} \left(\log \left(\frac{p_{it}y_{it}}{k_{it}} \right), z_{it} | T_i = 0, s_i = s \right)\end{aligned}$$

Extending industry-level treatments to all firms in the industry lead to a change in aggregate output of:

$$\Delta \ln Y = \sum_s \left(\frac{\alpha_s \theta_s \phi_s (1 + \epsilon)}{\sum_{s'} (1 - \alpha_{s'}) \theta_{s'} \phi_{s'}} \right) \left(-\widehat{\Delta\mu_\tau}(s) + \frac{1}{2} \frac{\theta_s}{1 - \theta_s} \left(\alpha_s \widehat{\Delta\sigma_\tau^2}(s) - 2\widehat{\Delta\sigma_{z\tau}}(s) \right) \right)$$

Denote $\kappa_s = \frac{K_s^0}{K^0}$ the share of total capital employed by industry s . With homogeneous capital shares ($\alpha_s = \alpha$) and price elasticity of demand ($\theta_s = \theta$), and assuming small industry shares ($\phi_s \ll 1$), the aggregate counterfactual can be further simplified:

$$\begin{aligned}\Delta \ln Y &= \frac{\alpha(1 + \epsilon)}{1 - \alpha} \sum_s \phi_s \left(-\widehat{\Delta\mu_\tau}(s) + \frac{1}{2} \frac{\theta}{1 - \theta} \left(\alpha \widehat{\Delta\sigma_\tau^2}(s) - 2\widehat{\Delta\sigma_{z\tau}}(s) \right) \right) \\ \Delta \log TFP &= -\frac{\alpha}{2} \left(1 + \frac{\alpha\theta}{1 - \theta} \right) \widehat{\Delta\sigma_\tau^2} - \alpha(1 + \alpha\theta) \sum_s (\kappa_s - \phi_s) \left(-\widehat{\Delta\mu_\tau}(s) + \frac{1}{2} \frac{\theta}{1 - \theta} \left(\alpha \widehat{\Delta\sigma_\tau^2}(s) - 2\widehat{\Delta\sigma_{z\tau}}(s) \right) \right)\end{aligned}$$

In particular, if the treatment is homogeneous across sectors, the aggregation formulas (7-8) are unchanged.

Proof. See Appendix B.4. □

Three remarks about Proposition 4 are in order. First, this proposition shows that, provided the treatment is homogeneous across sectors, and capital shares and mark-ups are identical across sectors, our main formulas remain unchanged even though industry shares are heterogeneous. Second, if the treatment is industry-specific, the aggregation formula for output remain close to Equation 7 derived above: the industry-specific treatment effects is simply weighted by industry shares ϕ_s . The aggregation formula for TFP, however, exhibit an additional first-order term relative to the one-industry case in Equation 8, which arises because across industry distortions may be present in the baseline economy. Intuitively, some industries may be “too large” in terms of capital employed relative to the undistorted optimum ($\kappa_s > \phi_s$): in this case, if distortions decrease relatively more in these industries as a result of the treatment, aggregate TFP increase. Finally, if capital shares and mark-ups are industry-specific, the aggregation formula for output is a simple weighted-average of the one-industry aggregation formula in Equation (7).

5.3 Decreasing returns to scale

In this section, we extend our baseline model to allow for decreasing technological returns to scale (span of control): $y_{it} = e^{z_{it}}(k_{it}^\alpha l_{it}^{1-\alpha})^\nu$ where $\nu < 1$. The aggregation formulas are then similar to those in our baseline case, with minor modifications:

Proposition 5. *With decreasing technological returns to scale ν and under the homogeneity assumptions of Proposition 2, the joint-distribution of z and τ does not depend on (w, Y) .*

With decreasing returns to scale, the aggregation formulas become:

$$\begin{aligned}\Delta \log(Y) &= \frac{\alpha\nu(1+\epsilon)}{(1-\alpha)\nu + (1+\epsilon)(1-\nu)} \left(-\widehat{\Delta\mu_\tau} + \frac{1}{2} \frac{\nu\theta}{1-\nu\theta} \left(\alpha\widehat{\Delta\sigma_\tau^2} - 2\widehat{\Delta\sigma_{z\tau}} \right) \right) \\ \Delta \log(TFP) &= -\frac{\alpha\nu}{2} \left(1 + \frac{\alpha\nu\theta}{1-\nu\theta} \right) \widehat{\Delta\sigma_\tau^2},\end{aligned}$$

where $\widehat{\Delta\mu_\tau}$, $\widehat{\Delta\sigma_\tau^2}$, $\widehat{\Delta\sigma_{z\tau}}$ are the same treatment effects defined in Section 3.4.

Proof. See Appendix B.5. □

The modifications introduced by decreasing returns to scale are marginal. Proposition 5 makes clear that our approach also applies to models of perfect competition ($\theta = 1$) and decreasing returns to scale such as Hopenhayn (2014) or Midrigan and Xu (2014).

It also makes clear that the modifications induced by decreasing returns to scale $\nu < 1$ will quantitatively be small, since ν is typically close to 1.

5.4 Non-parametric Formulas

Finally, we explore here the effect of relaxing the assumption of small variations in distortions and productivity. It turns out that simple formulas, similar to (7-8) can be developed. These formulas rely more heavily on the Cobb-Douglas nature of production, but present the advantage that they do not require the estimation of firm-level TFP shocks z .

Proposition 6.

Consider the baseline framework of Section 2. Assume that the assumptions in Proposition 2 hold.

Define the following treatment effects for labor and capital:

$$\begin{cases} \widehat{\Delta l} = \ln(\mathbb{E}[l_{it}|T_i = 1]) - \ln(\mathbb{E}[l_{it}|T_i = 0]) \\ \widehat{\Delta k} = \ln(\mathbb{E}[k_{it}|T_i = 1]) - \ln(\mathbb{E}[k_{it}|T_i = 0]) \end{cases}$$

Then, the effect of generalizing the treatment to all firms in the economy on aggregate output and TFP is given by the following formulas:

$$\begin{cases} \Delta \log Y = \frac{(1 + \epsilon)(1 - \theta)}{(1 - \alpha)\theta} \times \widehat{\Delta l} \\ \Delta \log TFP = \left(\frac{1}{\theta} - (1 - \alpha) \right) \times \widehat{\Delta l} - \alpha \times \widehat{\Delta k} \end{cases}$$

Proof. See Appendix B.6. □

The formulas in Proposition 6 are intuitive and simply leverage the fact that reduction in distortions due to the treatment translates into changes in input use. The appeal of these formula is that they do not require assumptions about the joint-distribution of productivity and wedges, or equivalently, assumption about the size of variations of productivity and wedges. However, these formulas may be unpractical from an empirical standpoint: with log-normally distributed labor and capital, estimating treatment effects in levels is likely to be inconsistent.

6 Conclusion

This paper develops a simple sufficient statistics framework to aggregate well-identified firm-level evidence of policy experiments aiming to reduce frictions faced by firms. The methodology proceeds in two steps: (1) using firm-level data, the econometrician estimates the treatment effect of the policy on moments of the joint distribution of productivity and capital wedges (2) these treatment effects are applied to *all* firms in a general equilibrium model of firm dynamics with real frictions, financial frictions and taxes. Our approach yields simple aggregation formula, that can easily be estimated in (quasi-)experimental settings. These formula can easily be extended to more complex economies (e.g, allowing decreasing returns to scale or heterogeneous industries) or partial aggregation exercises where all firms do not receive the treatment.

While variants of this methodology have been used in recent applied work, our paper explicits a set of conditions under which such an approach is valid: (1) intermediate inputs are combined with (nests of) CES aggregators (2) production takes place according to a Cobb-Douglas technology combining labor and capital (3) capital adjustment costs, financing frictions and taxes satisfy a type of homogeneity condition. While these assumptions may appear restrictive, they are satisfied by a large class of models commonly used in the macro-finance literature.

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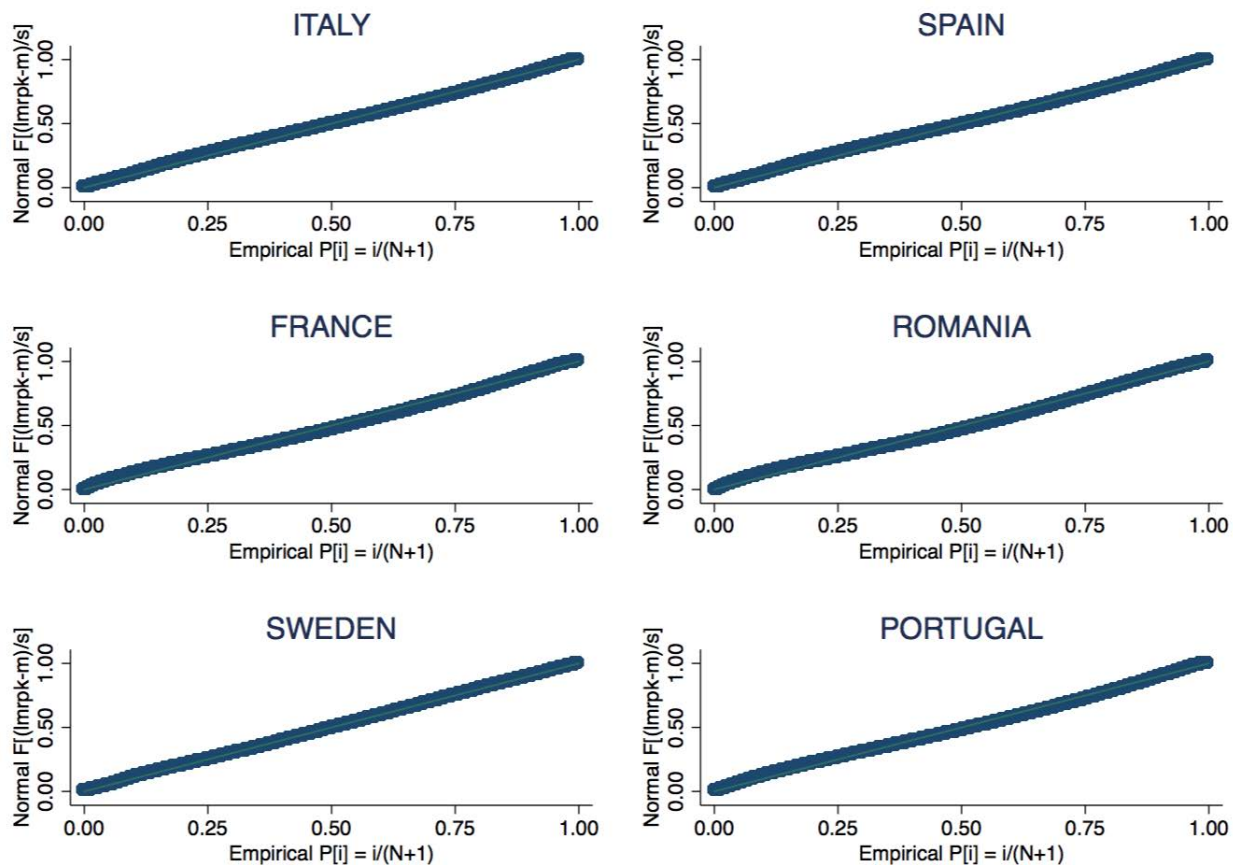
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A Figures and Tables

Figure 1: Normal probability plot of log-MRPK for firms in Amadeus



Source: BvD AMADEUS Financials, 2014. Note: This figure shows normal probability plots for 6 OECD countries (France, Spain, Italy, Portugal, Romania and Sweden) for the distribution of log-MRPK. Log-MRPK is computed as the ratio of value added (operating revenue minus materials) and total fixed assets.

B Proofs

B.1 Proof of Proposition 1

First, note $\mu_\tau = \mathbb{E}\tau$, $\sigma_\tau^2 = \text{Var}\tau$ and $\sigma_{z\tau} = \text{Cov}(z, \tau)$. Since the distribution of wedges is a function of Θ and the aggregate equilibrium (w, Y) , so are these moments, but we omit the dependence to ease notations.

We start with aggregate production (3):

$$\log Y = (1 + \epsilon) \frac{1 - \theta}{(1 - \alpha)\theta} \log \int_{z, \tau} \left(\frac{e^z}{(1 + \tau)^\alpha} \right)^{\frac{\theta}{1 - \theta}} dF(z, \tau; \Theta, w, Y)$$

Note $\delta_\tau = \log(1 + \tau) - \mu_\tau$, and $u = \frac{\theta}{1 - \theta} (z - \alpha\delta_\tau)$. Then:

$$\begin{aligned} \int_{z, \tau} \left(\frac{e^z}{(1 + \tau)^\alpha} \right)^{\frac{\theta}{1 - \theta}} dF(z, \tau; \Theta, w, Y) &= \mathbb{E} \left(e^{-\frac{\alpha\theta}{1 - \theta} \mu_\tau + u} \right) \\ &= e^{-\frac{\alpha\theta}{1 - \theta} \mu_\tau} \\ &\approx e^{-\frac{\alpha\theta}{1 - \theta} \mu_\tau} \mathbb{E} \left(1 + u + \frac{u^2}{2} \right) \\ &\approx e^{-\frac{\alpha\theta}{1 - \theta} \mu_\tau} \left(1 + \frac{\text{Var}u}{2} \right) \\ &\approx e^{-\frac{\alpha\theta}{1 - \theta} \mu_\tau} \left(1 + \frac{1}{2} \left(\frac{\theta}{1 - \theta} \right)^2 (\sigma_z^2 - 2\alpha\sigma_{z\tau} + \alpha^2\sigma_\tau^2) \right) \end{aligned}$$

so that:

$$\begin{aligned} \log \int_{z, \tau} \left(\frac{e^z}{(1 + \tau)^\alpha} \right)^{\frac{\theta}{1 - \theta}} dF(z, \tau; \Theta, w, Y) &\approx -\frac{\alpha\theta}{1 - \theta} \mu_\tau + \log \left(1 + \frac{1}{2} \left(\frac{\theta}{1 - \theta} \right)^2 (\sigma_z^2 - 2\alpha\sigma_{z\tau} + \alpha^2\sigma_\tau^2) \right) \\ &\approx -\frac{\alpha\theta}{1 - \theta} \mu_\tau + \frac{1}{2} \left(\frac{\theta}{1 - \theta} \right)^2 (\sigma_z^2 - 2\alpha\sigma_{z\tau} + \alpha^2\sigma_\tau^2) \end{aligned}$$

which leads to the result. Computation of the TFP formula follows the same logic.

B.2 Proof of Proposition 2

Remember that equity issuance / distributions are given by:

$$e_{it} = \frac{\alpha}{\alpha + (1 - \alpha)\phi} \left(\frac{(1 - \alpha)\phi}{\alpha + (1 - \alpha)\phi} \right)^{\frac{1-\alpha}{\alpha}\phi} S_t^{1-\phi} e^{\frac{\phi}{\alpha} z_{it}} k_{it}^{\phi} - (k_{t+1} - (1 - \delta)k_t) - \Gamma(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t) \\ + \left(\frac{b_{it+1}}{1 + r(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t)} - b_{it} \right) - \mathcal{T}(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t),$$

where $S_t = \frac{Y_t}{\frac{(1-\alpha)\phi}{\alpha(1-\phi)} w_t}$. By combining the different assumptions in Proposition 2, we get that:

$$e_{it} = S_t \left(\frac{\alpha}{\alpha + (1 - \alpha)\phi} \left(\frac{(1 - \alpha)\phi}{\alpha + (1 - \alpha)\phi} \right)^{\frac{1-\alpha}{\alpha}\phi} e^{\frac{\phi}{\alpha} z_{it}} \left(\frac{k_{it}}{S_t} \right)^{\phi} - \left(\frac{k_{t+1}}{S_t} - (1 - \delta) \frac{k_t}{S_t} \right) \right. \\ \left. - \Gamma \left(z_{it}, \frac{\mathbf{x}_{it}}{S_t}; \Theta, 1, 1 \right) + \left(\frac{b_{it+1}}{1 + r(z_{it}, \frac{\mathbf{x}_{it}}{S_t}; \Theta, 1, 1)} - \frac{b_{it}}{S_t} \right) - \mathcal{T} \left(z_{it}, \frac{\mathbf{x}_{it}}{S_t}; \Theta, 1, 1 \right) \right)$$

Therefore, $e(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t) = S_t e(z_{it}, \frac{\mathbf{x}_{it}}{S_t}; \Theta, 1, 1)$. Since the equity issuance cost $\mathcal{C}()$ also satisfies property 9, the flow variable in the Bellman equation 2 can be rewritten as:

$$e(z, \mathbf{x}; \Theta, w, Y) - \mathcal{C}(z, \mathbf{x}; \Theta, w, Y) = S \times \left(e(z_{it}, \frac{\mathbf{x}_{it}}{S_t}; \Theta, 1, 1) - \mathcal{C}(z_{it}, \frac{\mathbf{x}_{it}}{S_t}; \Theta, 1, 1) \right)$$

We now consider the steady-state of this economy: $w_t = w_{t+1} = w$ and $Y_t = Y_{t+1} = Y$. The Bellman equation 2 becomes:

$$V(z, k, b; \Theta, w, Y) = \begin{cases} \max_{k', b'} e(z, \mathbf{x}; \Theta, w, Y) - \mathcal{C}(z, \mathbf{x}; \Theta, w, Y) + \frac{\mathbb{E}_{z'}[V(z', k', b'; \Theta, w, Y)|z]}{1 + r_f} \\ M(z, \mathbf{x}; \Theta, w, Y) \leq 0 \end{cases}$$

Let \mathcal{B} be the Bellman operator:

$$\mathcal{B}f(z, k, b; \Theta, w, Y) = \begin{cases} \max_{k', b'} e(z, \mathbf{x}; \Theta, w, Y) - \mathcal{C}(z, \mathbf{x}; \Theta, w, Y) + \frac{\mathbb{E}_{z'}[f(z', k', b'; \Theta, w, Y)|z]}{1 + r_f} \\ M(z, \mathbf{x}; \Theta, w, Y) \leq 0 \end{cases}$$

Consider the set of functions \mathcal{F} such that for all $(z, k, b; \Theta, w, Y)$, $f(z, k, b; \Theta, w, Y) = S \times f(z, \frac{k}{S}, \frac{b}{S}; \Theta, 1, 1)$.

If $f \in \mathcal{F}$, then $\mathcal{B}f \in \mathcal{F}$:

$$\begin{aligned}
\mathcal{B}f(z, k, b; \Theta, w, Y) &= \begin{cases} \max_{k', b'} S \times \left(e(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1) - \mathcal{C}(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1) + \frac{\mathbb{E}_{z'}[f(z', \frac{k'}{S}, \frac{b'}{S}; \Theta, 1, 1)|z]}{1 + r_f} \right) \\ \mathcal{M}(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1) \leq 0 \end{cases} \\
&= S \times \begin{cases} \max_{k', b'} e(z, \frac{k}{S}, \frac{k'}{S}, \frac{b}{S}, \frac{b'}{S}; \Theta, 1, 1) - \mathcal{C}(z, \frac{k}{S}, \frac{k'}{S}, \frac{b}{S}, \frac{b'}{S}; \Theta, 1, 1) + \frac{\mathbb{E}_{z'}[f(z', \frac{k'}{S}, \frac{b'}{S}; \Theta, 1, 1)|z]}{1 + r_f} \\ \mathcal{M}(z, \frac{k}{S}, \frac{k'}{S}, \frac{b}{S}, \frac{b'}{S}; \Theta, 1, 1) \leq 0 \end{cases} \\
&= S \times \begin{cases} \max_{k', b'} e(z, \frac{k}{S}, k', \frac{b}{S}, b'; \Theta, 1, 1) - \mathcal{C}(z, \frac{k}{S}, k', \frac{b}{S}, b'; \Theta, 1, 1) + \frac{\mathbb{E}_{z'}[f(z', k', b'; \Theta, 1, 1)|z]}{1 + r_f} \\ \mathcal{M}(z, \frac{k}{S}, k', \frac{b}{S}, b'; \Theta, 1, 1) \leq 0 \end{cases} \\
&= S \times \mathcal{B}f(z, \frac{k}{S}, \frac{b}{S}; \Theta, 1, 1)
\end{aligned}$$

Since the contraction mapping theorem applies and \mathcal{F} is a compact space, this implies that the value function V also belongs to \mathcal{F} :

$$V(z, k, b; \Theta, w, Y) = S \times V(z, \frac{k}{S}, \frac{b}{S}; \Theta, 1, 1).$$

The previous equations also show that, in an economy with scale (w, Y) , if (k', b') are the optimal policies for a firm with state variable (z, k, b) , then $(\frac{k'}{S}, \frac{b'}{S})$ are the optimal policies for a firm with state variables $(z, \frac{k}{S}, \frac{b}{S})$ and in the economy with scale $(w = 1, Y = 1)$. As a result, the ergodic distribution of $\frac{k}{S}$ in the economy (w, Y) is equal to the ergodic distribution of k in the economy $(1, 1)$.

Remember that, by definition in the steady-state, capital wedges are equal to:

$$(1 + \tau_{it}) = \frac{\alpha \theta}{r_f + \delta} \frac{p_{it} y_{it}}{k_{it}} = \frac{\alpha \phi}{(\alpha + (1 - \alpha)\phi)(r_f + \delta)} e^{\frac{\phi}{\alpha} z_{it}} \left(\frac{k_{it}}{S} \right)^\phi$$

Since the ergodic distribution of $(\frac{k}{S})$ in the economy (w, Y) is the same as the ergodic distribution of k in the economy $(1, 1)$ and since the distribution of z is independent of (w, Y) , this implies that, in the steady state, the distribution of wedges τ_{it} does not depend on (w, Y) and can be written $G(\tau_i; \Theta)$.

B.3 Proof of Proposition 3

With heterogenous firm models, formulas (3-4) change a little bit. Let λ be the fraction of firms with Θ_1 and $1 - \lambda$ the fraction of firms with Θ_0 . Then, aggregate output writes:

$$\log Y = (1 + \epsilon) \frac{1 - \theta}{(1 - \alpha)\theta} \log(\lambda M(\Theta_1, w, Y) + (1 - \lambda)M(\Theta_0, w, Y))$$

where we note:

$$\begin{aligned}\log M(\Theta, w, Y) &= \int_{z, \tau} \left(\frac{e^z}{(1+\tau)^\alpha} \right)^{\frac{\theta}{1-\theta}} dF(z, \tau; \Theta, w, Y) \\ &= -\frac{\alpha\theta}{1-\theta}\mu_\tau + \frac{1}{2} \left(\frac{\theta}{1-\theta} \right)^2 (\sigma_z^2 - 2\alpha\sigma_{z\tau} + \alpha^2\sigma_\tau^2)\end{aligned}$$

Since:

$$\log M(\Theta_1, w, Y) = \log M(\Theta_0, w, Y) + \underbrace{\left[-\frac{\alpha\theta}{1-\theta}\Delta\mu_\tau + \frac{1}{2} \left(\frac{\theta}{1-\theta} \right)^2 (-2\alpha\Delta\sigma_{z\tau} + \alpha^2\Delta\sigma_\tau^2) \right]}_{=\Delta}$$

we have that:

$$\begin{aligned}\log Y &= (1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta} \log \left((1-\lambda)e^{\log M(\Theta_0, w, Y)} + \lambda e^{\log M(\Theta_1, w, Y)} \right) \\ &= \log Y_0 + (1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta} \log (1-\lambda + \lambda e^\Delta) \\ &\approx \log Y_0 + (1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta} \left(\lambda\Delta + \frac{\lambda(1-\lambda)}{2}\Delta^2 \right)\end{aligned}$$

since $\Delta\mu_\tau$, $\Delta\sigma_\tau^2$ and $\Delta\sigma_{z\tau} \ll 1$. We then substitute the expression of Δ , neglect terms beyond order 2, and obtain the formula for output in the proposition. The proof for aggregate TFP follows the same logic.

B.4 Proof of Proposition 4

First, notice that profit maximization of the final sector leads to:

$$p_s Y_s = \phi_s Y$$

and the FOC in labor nicely aggregates into:

$$wL = \left(\sum_s \theta_s \phi_s (1-\alpha_s) \right) Y$$

while industry-level aggregator leads to the following firm-level demand curve:

$$p_i = p_s Y_s^{1-\theta_s} y_i^{\theta_s-1}$$

We then start with the firm-level relations and omit the time subscripts:

$$l_i = \frac{\theta_s(1 - \alpha_s)p_i y_i}{w}$$

$$k_i = \frac{\theta_s \alpha_s p_i y_i}{R(1 + \tau_i)}$$

which we plug back into the definition of revenue to obtain:

$$p_i y_i \propto p_s^{\frac{1}{1-\theta_s}} Y_s \frac{e^{\frac{\theta_s}{1-\theta_s} z_i}}{w^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}} (1 + \tau_i)^{\frac{\alpha_s \theta_s}{1-\theta_s}}}$$

Aggregating at the industry level, we obtain that:

$$Y_s \propto Y J_s^{1-\theta_s} \frac{1}{w^{(1-\alpha_s)\theta_s}}$$

where:

$$\log J_s = \log \left(\int \frac{e^{\frac{z\theta_s}{(1-\theta_s)}}}{(1 + \tau_i)^{\frac{\alpha_s \theta_s}{1-\theta_s}}} dF(z, \tau) \right)$$

$$\approx \frac{\theta_s \alpha_s}{1 - \theta_s} \underbrace{\left(-\mu_\tau + \frac{1}{2} \frac{\theta_s}{1 - \theta_s} (\alpha \sigma_\tau^2 - 2\sigma_{z\tau}) \right)}_{A_s}$$

assuming small deviations of τ and z around their means.

We then reaggregate at the economy level using $\log Y = \sum_s \phi_s \log Y_s$ and the fact that $Y \propto wL$ so as to obtain:

$$\log \frac{Y}{L} = \text{cst} + \sum_s \frac{\phi_s \alpha_s \theta_s}{\sum_{s'} \phi_{s'} \theta_{s'} (1 - \alpha_{s'})} A_s$$

differentiating and taking into account the fact that $Y \propto w^{1+\epsilon}$ leads to the equation for Y .

We now turn to aggregate TFP. We start from the following natural definition:

$$\log TFP = \left(\sum_s \phi_s (1 - \alpha_s) \right) \log \frac{Y}{L} + \left(\sum_s \phi_s \alpha_s \right) \log \frac{Y}{K}$$

We thus need to focus on $\log \frac{Y}{K}$. Start from the fact that:

$$k_i \propto \frac{p_i y_i}{1 + \tau_i} \propto p_s^{\frac{1}{1-\theta_s}} Y_s \frac{e^{\frac{\theta_s}{1-\theta_s} z_i}}{w^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}} (1 + \tau_i)^{1 + \frac{\alpha_s \theta_s}{1-\theta_s}}}$$

which we aggregate at the sector level into:

$$K_s \propto Y_s \frac{I_s}{J_s}$$

where:

$$\begin{aligned} \log I_s &= \log \left(\int \frac{e^{\frac{z\theta_s}{(1-\theta_s)}}}{(1+\tau_i)^{1+\frac{\alpha_s\theta_s}{1-\theta_s}}} dF(z, \tau) \right) \\ &\approx \left(1 + \frac{\theta_s\alpha_s}{1-\theta_s} \right) \left(-\mu_\tau + \frac{1}{2} \frac{\theta_s}{1-\theta_s} (\alpha\sigma_\tau^2 - 2\sigma_{z\tau}) + \frac{1}{2}\sigma_\tau^2 \right) \\ &\approx \left(1 + \frac{\alpha_s\theta_s}{1-\theta_s} \right) \left(A_s + \frac{\sigma_\tau^2}{2} \right) \end{aligned}$$

Hence:

$$\log \frac{K_s}{Y} = cst + \left(1 + \frac{\alpha_s\theta_s}{1-\theta_s} \right) \frac{\sigma_\tau^2}{2} + (1 + \alpha_s\theta_s) A_s - (1 - \alpha_s)\theta_s \sum_{s'} \frac{\phi_{s'}\alpha_{s'}\theta_{s'}}{\sum_{s''} \phi_{s''}\theta_{s''}(1 - \alpha_{s''})} A_{s'}$$

Differentiating and linearizing $\log \frac{K}{Y}$, we obtain:

$$\Delta \log \frac{K}{Y} \approx \sum_s \frac{K_s^0}{K^0} \left(\Delta \log \frac{K_s}{Y} + \frac{1}{2} \left(1 - \frac{K_s^0}{K^0} \right) \left(\Delta \log \frac{K_s}{Y} \right)^2 \right)$$

which lead to a large, ugly formula for TFP.

Assume that $\alpha_s = \alpha$, $\theta_s = \alpha$. Note $\kappa_s = \frac{K_s^0}{K^0}$ the capital share of each sector in the control economy where no firm is treated. In this case:

$$\Delta \log \frac{Y}{L} = \frac{\alpha}{1-\alpha} \sum_s \phi_s \Delta A_s$$

and:

$$\begin{aligned} \Delta \log \frac{K}{Y} &= -\frac{1}{2} \left(1 + \frac{\alpha_s\theta_s}{1-\theta_s} \right) \Delta\sigma_\tau^2 + \alpha(1 + \alpha\theta) \sum_s (\phi_s - \kappa_s) \Delta A_s \\ &\quad + \frac{1}{2} \sum_s \kappa_s(1 - \kappa_s) \left((1 + \alpha\theta) \Delta A_s - \alpha\theta \sum_{s'} \phi_{s'} \Delta A_{s'} \right)^2 \end{aligned}$$

We further assume that industries are small, which allows us to neglect the last term:

$$\Delta \log TFP = -\frac{\alpha}{2} \left(1 + \frac{\alpha\theta}{1-\theta} \right) \Delta\sigma_\tau^2 + \alpha(1 + \alpha\theta) \sum_s (\phi_s - \kappa_s) \Delta A_s$$

which leads to the formula in the proposition.

B.5 Proof of Proposition 5

We first show that Proposition 2 still holds with decreasing returns to scale.

With monopolistic competition and decreasing returns to scale, for a firm i with a stock of capital k_{it} , operating profits after optimizing labor demand are given by:

$$\begin{aligned} p_i y_i - w l_i &= (1 - (1 - \alpha)\nu\theta) \left(\frac{(1 - \alpha)\nu\theta}{w} \right)^{\frac{(1 - \alpha)\nu\theta}{1 - (1 - \alpha)\nu\theta}} Y^{\frac{1 - \theta}{1 - (1 - \alpha)\nu\theta}} e^{z_i \frac{\theta}{1 - (1 - \alpha)\nu\theta}} k_i^{\frac{\alpha\nu\theta}{1 - (1 - \alpha)\nu\theta}} \\ &= S \times (1 - (1 - \alpha)\nu\theta) \left(\frac{(1 - \alpha)\nu\theta}{w} \right)^{\frac{(1 - \alpha)\nu\theta}{1 - (1 - \alpha)\nu\theta}} e^{z_i \frac{\theta}{1 - (1 - \alpha)\nu\theta}} \left(\frac{k_i}{S} \right)^{\frac{\alpha\nu\theta}{1 - (1 - \alpha)\nu\theta}} \end{aligned}$$

where $S = \frac{Y^{\frac{1 - \theta(1 - \alpha\nu)}{1 - (1 - \alpha)\nu\theta}}}{w^{\frac{\nu\theta}{1 - (1 - \alpha)\nu\theta}}}$.

It follows directly from the proof of Proposition 2 that in this economy, and under the assumptions of Proposition 2, the ergodic joint distribution of capital wedges and productivity is independent of (w, Y) and depend only on the parameters Θ . Let $F(z, \tau; \Theta)$ denote this distribution as before.

With decreasing returns to scale ν , profit maximization for firm i in industry s as a function of a capital wedge τ_{is} leads to:

$$\begin{cases} k_i \propto \left(\frac{1}{w} \right)^{\frac{\nu(1 - \alpha)\theta}{1 - \nu\theta}} Y^{\frac{1 - \theta}{1 - \nu\theta}} e^{z_i \frac{\theta}{1 - \nu\theta}} \left(\frac{1}{(1 + \tau_i)} \right)^{\frac{1 - (1 - \alpha)\nu\theta}{1 - \nu\theta}} \\ l_i \propto \left(\frac{1}{w} \right)^{\frac{1 - \alpha\nu\theta}{1 - \theta}} Y^{\frac{1 - \theta}{1 - \nu\theta}} e^{z_i \frac{\theta}{1 - \nu\theta}} \left(\frac{1}{(1 + \tau_i)} \right)^{\frac{\alpha\nu\theta}{1 - \nu\theta}} \end{cases}$$

Firm i output at the optimum is given by:

$$p_i y_i \propto Y^{\frac{1 - \theta}{1 - \nu\theta}} \left(\frac{1}{w} \right)^{\frac{(1 - \alpha)\nu\theta}{1 - \nu\theta}} \left(\frac{1}{1 + \tau_i} \right)^{\frac{\alpha\nu\theta}{1 - \nu\theta}} e^{z_i \frac{\theta}{1 - \nu\theta}} \quad (13)$$

Omitting the i subscripts, equilibrium on the product market implies that:

$$w \propto Y^{-\frac{1 - \nu}{(1 - \alpha)\nu}} \left(\int_{z, \tau} \frac{e^{z \frac{\theta}{1 - \nu\theta}}}{(1 + \tau)^{\frac{\alpha\nu\theta}{1 - \nu\theta}}} dF(z, \tau; \Theta) \right)^{\frac{1 - \nu\theta}{(1 - \alpha)\nu\theta}} \quad (14)$$

Equilibrium on the labor market implies that $Y \propto w^{1 + \epsilon}$

Combining these two equations provides the following expression for aggregate output:

$$Y \propto \left(\int_{(z, \tau)} \frac{e^{z \frac{\theta}{1 - \nu\theta}}}{(1 + \tau)^{\frac{\alpha\nu\theta}{1 - \nu\theta}}} dF(z, \tau; \Theta) \right)^{\frac{(1 + \epsilon)(1 - \nu\theta)}{\theta((1 - \alpha)\nu + (1 + \epsilon)(1 - \nu))}}$$

which then leads to the expression in the proposition after Taylor expansion.

Finally, aggregate TFP admits a simple expression:

$$TFP = \frac{Y}{K^{\alpha\nu} L^{(1-\alpha)\nu}}$$

$$= \left(\int_{z,\tau} \frac{z^{\frac{\theta}{1-\nu\theta}}}{(1+\tau)^{\frac{\alpha\nu\theta}{1-\nu\theta}}} dF(z, \tau; \Theta) \right)^{\frac{1-(1-\alpha)\nu\theta}{\theta}} \left(\int_{z,\tau} \frac{z^{\frac{\theta}{1-\nu\theta}}}{(1+\tau)^{\frac{1-(1-\alpha)\nu\theta}{1-\nu\theta}}} dF(z, \tau; \Theta) \right)^{-\alpha\nu}$$

which then leads to the formula in the proposition after straightforward Taylor expansion.

B.6 Proof of Proposition 6

Optimal labor demand as a function of firm-level capital wedge is:

$$l_i \propto \left(\frac{1}{w} \right)^{\frac{1-\alpha\theta}{1-\theta}} Y e^{\frac{\theta}{1-\theta} z_i} \left(\frac{1}{(1+\tau_i)} \right)^{\frac{\alpha\theta}{1-\theta}}$$

Assume treated firms are a zero-measure set. Then, the following sufficient statistic can be computed for both the treatment and control groups:

$$\mathbb{E}[l_i | T_i = T] \propto \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{\frac{\alpha\theta}{1-\theta}}} dF(z, \tau, \Theta_T)$$

where $T \in \{0, 1\}$.

We now introduce the log difference in mean employment:

$$\begin{aligned} \widehat{\Delta l} &= \log(\mathbb{E}[l_i | T_i = 1]) - \log(\mathbb{E}[l_i | T_i = 0]) \\ &= \log \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{\frac{\alpha\theta}{1-\theta}}} dF(z, \tau, \Theta_1) - \log \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{\frac{\alpha\theta}{1-\theta}}} dF(z, \tau, \Theta_0) \end{aligned}$$

Given the output equation (3), it follows directly that

$$\Delta \log Y = \frac{(1+\epsilon)(1-\theta)}{(1-\alpha)\theta} \widehat{\Delta l}$$

We now compute TFP, which requires calculating the capital stock. Similarly, optimal capital demand implies that:

$$k_i \propto \left(\frac{1}{w} \right)^{\frac{(1-\alpha)\theta}{1-\theta}} Y e^{\frac{\theta}{1-\theta} z_i} \left(\frac{1}{1+\tau_i} \right)^{\frac{1-(1-\alpha)\theta}{1-\theta}}$$

Like for employment, we use this to compute the new capital sufficient statistic:

$$\begin{aligned} \widehat{\Delta k} &= \log(\mathbb{E}[k_i | T_i = 1]) - \log(\mathbb{E}[k_i | T_i = 0]) \\ &= \log \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{1+\frac{\alpha\theta}{1-\theta}}} dF(z, \tau, \Theta_1) - \log \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{1+\frac{\alpha\theta}{1-\theta}}} dF(z, \tau, \Theta_0) \end{aligned}$$

Given the TFP formula (4), $\widehat{\Delta}l$ and $\widehat{\Delta}k$ can be straightforwardly combined into the formula given in the proposition.