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THE MARGINAL VALUE OF SOCIAL SECURITY

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ABSTRACT

If annuities such as Social Security are not choosen freely, the consumption path typically cannot be determined independently of the path of annuities. This constraint reduces the value of the annuity from the point of view of the annuitant. I measure the value of the annuity by the marginal rate of substitution (MRS), the amount of bequeathable wealth that will substitute for a dollar of annuity wealth. In the analytical section of the paper, I show that the MRS increases as bequeathable wealth increases; in that sense the wealthy benefit more from Social Security than the poor. In the empirical section, I estimate the MRS for a sample of retired single elderly. The MRS varies considerably from individual to individual because of differences in the mix of bequeathable wealth and annuities. For the parameter values that best fit the data, a substantial fraction of the sample has more Social Security than it would like in that it would be willing to trade, at the margin, a claim to Social Security for an increase in bequeathable wealth.

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1. Introduction

In a simple model in which lifetime utility only depends on the flow of consumption, and in which the only source of uncertainty is the date of death, a well-functioning annuity market can increase substantially the expected utility of a retired person (Yaari, 1965). Essentially the market increases the interest rate on the wealth held to finance retirement. This model, however, envisions that the consumption path can be chosen independently from the annuity path, or, equivalently, the annuity path can be chosen to match a desired consumption path. In the United States, and surely in many other countries, this assumption is not even approximately true because the great fraction of annuities are not privately purchased annuities but pensions and Social Security. In fact, for most of the elderly in the United States, Social Security is the by far the most important annuity. Because an annuitant cannot borrow against future Social Security benefits, in some cases the consumption path cannot be chosen independently from the annuity path. For example, someone who desires a consumption path that declines with age but who has little bequeathable wealth will be constrained to a consumption path the follows the annuity path. The utility from this consumption path will be smaller than the utility that could have been achieved from an annuity with the same expected present value, but in which the consumption path could be chosen independently from the annuity path. This example is by no means academic: a high fraction of the single retired elderly have small amounts of bequeathable wealth compared to annuity wealth (Hurd and Shoven, 1983), and eventually almost all must follow constrained consumption paths.

Another limitation of the simple annuity model is that in the model bequests do not give utility; that is, there is no bequest motive for saving. Although the empirical evidence is mixed on the importance of the bequest motive, a number of authors have called for models that do include a bequest motive (Kotlikoff and Summers (1981), Menchik and David (1983), Kurz (1984), and Modigliani (1986)). This is important for the valuation of Social Security and other annuities because they cannot be inherited. A strong bequest motive will, therefore, decrease the value of Social Security relative to bequeathable wealth.

The value to the annuitant of exogenously given annuities is of considerable policy interest because it bears on the issue of the optimal size of the Social Security program. If, at the margin, Social Security beneficiaries value an increase in the benefit stream less than the cost to the Social Security system of the incremental stream, an increase based on the annuity value of Social Security cannot be justified. An estimate of of the value can be based on what an annuitant would be willing to pay for an increase in Social Security: if this is less than the cost of the increase, the beneficiary would not want an increase in which each beneficiary paid for the increase through taxes.

Bernheim (1987) found analytically the compensating variation associated with a small change in exogenously given annuities; that is, he found the change in bequeathable wealth that would keep lifetime utility constant following a change in the annuity stream. This is not quite the same thing as comparing the value to the individual of the change in Social Security benefits with the cost of the change to the Social Security system: one would want the compensating variation associated with a small change in the cost of the annuity stream, not in the level of the annuity stream. But this is a minor difference involving renormalizing by the expected present value of the annuity stream. The model Bernheim analyzed, however, is based on a specific utility function which has no provision for a bequest motive, and on a simple representation of mortality rates. The analysis should be extended to a general utility function and to a realistic representation of mortality rates because the nature of the solution may change considerably.[1] Furthermore, Bernheim's illustrative examples of the compensating variation are not based on estimated parameters or on actual data, so one has little sense of whether or not they are reasonable.

In this paper I study both analytically and empirically the value at the margin of an exogenously given annuity path. To the extent that individuals choose their work effort independently from their desired level of Social Security retirement benefits, this model approximates the value individuals place on Social Security benefits. In the first part of the paper, I analyze how the marginal rate of substitution (MRS) of bequeathable wealth for annuity wealth varies with the level of annuities and bequeathable wealth. The main finding is that at the margin Social Security wealth becomes more valuable as bequeathable wealth increases. Someone with very little bequeathable wealth may hold an excess of annuities in the sense that he would be willing to give up some claim to Social Security benefits in exchange for bequeathable wealth at a rate of exchange that would be favorable to the Social Security system; that is, the marginal benefit of an increase in Social Security benefits costs the Treasury more than it is worth to the individual. At the other extreme a wealthy individual would like to increase his level of benefits: it costs the Treasury less to finance an increase in his benefits than the increase is worth to him.

In the second part of the paper, I use a model of consumption to estimate the MRS for a sample of single people in the United States. Because of the complexity, the model must be solved numerically. Even though the consumption model assigns the same utility function parameters to all individuals, the MRS varies across individuals due to variation in bequeathable wealth and annuites. Thus, the model must be solved for each individual.

The estimates verify that the MRS increases in bequeathable wealth. Most individuals have a MRS that is greater than one, which implies that they would like to purchase at an actuarially fair price higher Social Security benefits. A substantial fraction of the observations, however, have a MRS that is less than one: they would like to reduce at a fair rate of exchange their holdings of Social Security.

2. The Effect of Annuities on Utility.

In a typical model of lifetime utility under uncertainty about the date of death, the introduction of actuarially fair annuities will increase utility (Yaari, 1965). The reasoning is as follows. In the absence of annuities the budget constraint is

$$(1) \qquad \int c_t e^{-rt} dt = w$$

in which r is the constant rate of interest, c_t is the consumption stream, and w is initial wealth. An actuarially fair annuity is priced such that

$$\int A_t e^{-rt} a_t dt = w,$$

where A_t is the stream of annuity payments, and a_t, the life rate, is the probability of being alive at t. $a_t < 1$ for $t > 0$. The left hand side of this equation is just the expected present value of the annuity stream. If there are good capital markets and actuarially fair life insurance is available, the consumption path can be chosen independently from the annuity path. For example, suppose the annuity path is flat and the desired consumption path declines with age. Then a consumer can borrow against the future annuity stream and simultaneously buy life insurance to protect the lender against the borrower's premature death. If the consumption trajectory can be chosen independently from the annuity path, then the budget constraint on consumption is

$$(2) \qquad \int c_t e^{-rt} a_t dt = w.$$

A comparison of (1) and (2) shows that any consumption path that is feasible under (1) will be feasable under (2) but not the reverse. Therefore, utility can never be lower when there are actuarially fair annuities, and in general it will be higher. An example is when $a_t = e^{-\lambda t}$, which implies that the conditional mortality rate (the hazard rate) is λ. The budget constraint becomes

$$\int c_t e^{-t(r+\lambda)} dt = w,$$

so that the mortality risk with actuarially fair annuities acts exactly like an increase in the interest rate. In that the annuitants are lenders, an increase in the interest rate must increase utility.

This model of annuities is not well-suited to United States data. First, neither privately purchased annuities nor life insurance are actuarially fair: typically they have a load factor of about 35% (Friedman and Warshawsky, 1985). This means that it is costly to choose the consumption path independently from the annuity path. Second, almost all annuities are job related, either private or government pensions or Social Security. Furthermore, for most people, Social Security is the largest part of job-related pensions. The benefit stream from Social Security cannot be used as collateral for a loan. Whether the benefit stream from a private pension could be used as collateral would depend on the details of the particular pension program. In the United States the path of Social Security benefits is fixed in real terms; therefore, if someone wanted a declining consumption path he would need bequeathable wealth. Third, a substantial fraction of the elderly have low levels of bequeathable wealth. This means their consumption paths will eventually have to follow the annuity path. Fourth, in the model I have outlined utility does not depend on bequests: a bequest motive will lower the utility-value of an annuity.

I now turn to models of utility maximization that are better suited to U.S. data. The first model, which does not depend on any specific utility function, accounts for annuities, but it has no bequest motive. I show analytically that the MRS increases as bequeathable wealth increases. The second model is based on a specific utility function including a bequest motive. It is too complicated to be studied analytically, so, using data on retired individuals, I solve numerically for the MRS, and show how the MRS varies with bequeathable wealth and annuities.

2.1. A Utility Model with Annuities.

I assume that individuals maximize in the consumption path $\{c_t\}$ lifetime utility

(3) $$\int u(c_t)a_t e^{-\rho t}dt$$

in which

$$a_t = 1-\int_0^t m_s ds$$

is the probability that the individual is alive at t; m_t is the instantaneous mortality rate. I have not given an upper bound to the integral in (3), but I have in mind a finite-time problem. That is, eventually, a_t becomes zero. ρ is the subjective time rate of discount; r is the real interest rate which is taken to be known and fixed. $u(c_t)$ is the instantaneous utility from consuming c_t; $u' > 0$; $u'' < 0$. The resources available are bequeathable wealth, w_t, and annuities, including pensions, Social Security and Medicare/Medicaid. Annuities are distinguished from bequeathable wealth in that they cannot be borrowed against. The conditions on the utility maximization are that initial bequeathable wealth, w, is given, and that

(4) $$w_t = we^{rt} + \int_0^t (A_s-c_s)e^{(t-s)r}ds \geqq 0 \text{ for all } t,$$

A_s is the flow of annuities at time s. This formulation differs from the usual intertemporal utility maximization problem in that the annuity stream cannot be summarized by its expected present value. Because many of the elderly have large annuities relative to their bequeathable wealth, corner solutions are important.

The Pontryagin necessary conditions associated with this problem are

(5) $$c_t = A_t \text{ if } w_t = 0$$

and

(6) $$u_t = u_\tau(a_\tau/a_t)e^{(r-\rho)(\tau-t)}$$

over an interval (t,τ) in which $w_t > 0$. u_t and u_τ are the marginal utilities of consumption and t and τ.

If

$$(a_\tau/a_t)e^{(r-\rho)(\tau-t)} < 1,$$

the marginal utility trajectory will slope upward. As $\tau-t \to 0$, equation (6) becomes

$$(1/u_t)du_t/dt = \rho + h_t - r$$

in which $h_t = m_t/a_t$ is the conditional mortality rate or the hazard rate. Therefore, marginal utility will increase with t if $(h_t + \rho) > r$. In a finite horizon problem and in actual mortality data h_t increases in t; in fact h_t is well approximated by the function $e^{\xi t}$, $\xi > 0$, for ages over, say, 60. I simply assume that h_t increases in t. Therefore, for large t, $(\rho + h_t) > r$ and marginal utility will increase with age. This means consumption will eventually fall with age.

The complete solution will depend on both the consumption and annuity paths. Suppose that annuities are constant in real terms: $A_t = A$. Two possible consumption paths, each based on different parameter values, are shown in Figure 1. Initially consumption may increase as in cons_1, but eventually it must decrease. The reasoning is based on the continuity of the consumption path. If dc_t/dt were positive as t approached T, c_t would be less than A because $c_T = A$. But this implies that dw_t/dt would be positive; hence, w_T would be positive which violates a condition of the solution.

At t^* in Figure 1, the parameters generating $cons_1$ are such that $r = (\rho + h_{t^*})$. The parameters generating $cons_2$ are such that $r < (\rho + h_0)$.

With constant annuities, the consumption path follows equation (6) until bequeathable wealth is exhausted at T. The present value of the area under the consumption path and above the annuity path equals initial bequeathable wealth. The solution is implicitly defined by (6) and

(7.1) $c_T = A$

(7.2) $w_T = w_0 e^{rT} + \int_o^T (A - c_s) e^{(T-s)r} ds$

(7.3) $w_T = 0$

(7.4) $c_t = A, \ t > T.$

2.2. The Marginal Rate of Substitution.

In this section I analyze how the MRS varies with initial bequeathable wealth and the level of annuities. For this analytical section, I assume the annuity stream is constant at A; for many elderly in the U.S. this is roughly accurate because the only pension of many is Social Security, which, after retirement, is fixed in real terms.

Under these assumptions the optimal consumption path is given implicitly by (6) and (7). Maximum lifetime utility is given by

$$U = \int_o^T u(c_t) a_t e^{-\rho t} dt$$

In a change of notation, c_t is now the optimal consumption path.

The MRS should give the change in bequeathable wealth required to keep utility constant in response to a unit change in annuity wealth. That is, it should not be defined in terms of the annuity stream, A, but in terms of the cost to the government of the stream A. Let that cost be S; then,

$$S = \int A e^{-rt} a_t dt$$

This would also be the cost to an individual if the annuity were purchased in a fair market. Hence, if the annuity were freely chosen, S would be the value to him at the margin.[2]

Let MRS be the absolute value of $\frac{dw}{dS}$ holding utility constant. Then

$$MRS = \frac{\partial U/\partial S}{\partial U/\partial w} = \frac{(\partial U/\partial A)(\partial A/\partial S)}{\partial U/\partial w}.$$

$\partial A/\partial S$ is just $(\int e^{-rt} dt)^{-1}$.

To find the MRS, we need the marginal utility of initial wealth and of annuities.

$$(8) \qquad \partial U/\partial w = \int u_t (\partial c_t/\partial w) a_t e^{-\rho t} dt$$

$$= \int_0^T u_t (\partial c_t/\partial w) a_t e^{-\rho t} dt$$

in that $c_t = A$ for $t \geq T$. From the budget constraint

$$e^{rT} = \int_0^T (\partial c_t/\partial w) e^{r(T-t)} dt$$

or

$$1 = \int_0^T (\partial c_t/\partial w) e^{-rt} dt.$$

Using (6) at $\tau = T$ and (8) we see that

$$\partial U/\partial w = \int_0^T u_T (a_T/a_t) e^{(r-\rho)(T-t)} (\partial c_t/\partial w) a_t e^{-\rho t} dt$$

$$= u_T a_T e^{(r-\rho)T} \int_0^T (\partial c_t/\partial w) e^{-rt} dt$$

$$= u_T a_T e^{(r-\rho)T}$$

This can be rewritten as

(9) $$\partial U/\partial w = u_t a_t e^{(r-\rho)t}, \quad t \leqq T.$$

The RHS is the expected discounted marginal utility of consumption at t. All along the consumption path this is put equal to the marginal utility of initial wealth. Because $u_t a_t e^{(r-\rho)t}$ is also the expected (at $t = 0$) discounted marginal utility of wealth at t, (9) has the interpretation of an Euler equation: the consumption path is chosen is that the marginal utility of wealth at $t = 0$ equals the expected marginal utility of wealth at all t.

The marginal utilility of annuities is

$$\partial U/\partial A = \int_0 u_t (\partial c_t/\partial A) a_t e^{-\rho t} dt$$

$$= \int_0^T u_t (\partial c_t/\partial A) a_t e^{-\rho t} dt + \int_T u_t a_t e^{-\rho t} dt$$

where the last term follows from $\partial c_t/\partial A = 1$, $t \geqq T$. From the budget constraint

$$\int_0^T (\partial c_t/\partial A - 1) e^{-rt} dt = 0.$$

Then, by using (6),

$$\partial U/\partial A = \int_0^T u_T (a_T/a_t) e^{(r-\rho)(T-t)} (\partial c_t/\partial A) a_t e^{-\rho t} dt$$

$$+ \int_T u_t a_t e^{-\rho t} dt$$

$$= e^{(r-\rho)T} u_T a_T \int_0^T e^{-rt} (\partial c_t/\partial A) dt + u_T \int_T a_t e^{-\rho t} dt,$$

using $u_t = u_T$, $t \geq T$.

$$\partial U/\partial A = e^{(r-\rho)T} u_T a_T \int_0^T e^{-rt} + u_T \int_T a_t e^{-\rho t} dt$$

$$= \frac{1}{r} u_T a_T e^{(r-\rho)T}(1-e^{-rT}) + u_T \int_T a_t e^{-\rho t} dt$$

This gives the marginal utility of A. Using this result, the calculation of $\partial U/\partial w$ and $(\partial U/\partial S)\int e^{-rt} = \partial U/\partial A$, one can write

$$\text{(10)} \qquad MRS = \frac{\partial U/\partial S}{\partial U/\partial w} = \frac{\frac{1}{r} u_T a_T e^{(r-\rho)T}(1-e^{-rT}) + u_T \int_T a_t e^{-\rho t} dt}{u_T a_T e^{(r-\rho)T} \int e^{-rt} dt}.$$

If initial bequeathable wealth is zero, T is zero, $a_T = 1$, and

$$MRS_0 = \frac{\int a_t e^{-\rho t} dt}{\int a_t e^{-rt} dt}$$

If ρ, the subjective time rate of discount, is greater than the interest rate, MRS_0 will be less than one. This happens because the discounting in the utility function of the flat consumption trajectory is at the effective rate of $(h_t + \rho)$ whereas the discounting of the trajectory from the point of view of the government is at the rate $(h_t + r)$. Thus the annuity costs the government more at the margin than it is worth to the individual. Said differently, the individual would like to cash in part of his annuity at the actuarially fair rate. In that sense he is overannuitized. As far as I know, the only estimates of ρ in the context of mortality risk and annuities are in my paper, "Mortality Risk and Bequests." As I report there and discuss below, the simplest estimator produced an estimate of ρ of 0.05 when r was assumed to be 0.03. If the hazard rate were constant at 0.03, which is approximately the conditional mortality rate of a 65 year old male, $a_t = e^{-0.03t}$; then, the MRS for someone with these parameter values and initial wealth of zero would be 0.06/0.08 = 0.75. Such an individual would, at the margin, be willing to give up a dollar in expected discounted Social Security benefits in exchange for 75 cents in initial wealth.

If initial wealth is large, T becomes large and

$$\text{MRS} \approx \frac{1}{r\int e^{-rt}a_t dt} > \frac{1}{r\int e^{-rt}dt}$$

$$= 1.$$

This result follows almost directly from the statement of the budget constraint when the consumption path can be chosen independently from the annuity path: when no annuities are available $\frac{1}{r}$ is the wealth required to produce a unit annuity flow; when annuities are available only $\int a_t e^{-rt}dt$ in wealth will produce the unit flow. A constant hazard of 0.03 will yield a MRS of (0.03 + 0.03)/0.3 = 2 in the high wealth case.

These examples give typical values of MRS at the extremes of initial bequeathable wealth. I next show how MRS varies with w and with A.

Examination of (10) shows that MRS only depends on w and A through T. That is,

$$\partial\text{MRS}/\partial w = (\partial\text{MRS}/\partial T)(\partial T/\partial w)$$

and similarly for ∂MRS/∂A. I first show that ∂MRS/∂T is positive.

$$\partial\text{MRS}/\partial T = -(E/D)[a_T(r-\rho) + da_T/dT]/(a_T^2 e^{(r-\rho)T})$$

where $E = \int a_t e^{-\rho t}dt$ and $D = \int a_t e^{-rt}dt$. But $da_T/dT = -m_T$, so that $\partial\text{MRS}/\partial T > 0$ if $(r - \rho - m_T/a_T) < 0$. This condition will always hold at T because a condition for the optimum is that consumption is falling at T, which requires that $(r - \rho - m_T/a_T) < 0$.

$\partial T/\partial w$ is easily seen to be positive: the consumption paths associated with different initial wealth levels cannot cross which implies that the consumption path associated with a particular initial wealth will lie above the consumption path associated with any lower initial wealth. Therefore, the higher wealth will lead to a greater T.

In that both $\partial MRS/\partial T$ and $\partial T/\partial w$ are positive, I conclude that $\partial MRS/\partial w > 0$.

The sign of $\partial T/\partial A$ depends on the particular form of the instantaneous utility function; one, therefore, cannot in general give a sign to $\partial MRS/\partial A$. I demonstrate this result by giving two examples: the first is a simple utility function in which I show graphically that $\partial T/\partial A$ is positive; the second is a widely used utility function in which I show analytically that $\partial T/\partial A$ is negative.

First, consider the utility function u^*:

$$\begin{aligned} u^*(c) &= c, \quad c \leq \alpha, \\ &= \alpha, \quad c > \alpha. \end{aligned}$$

u^* and $\tilde{u}$, another utility function to be discussed below, are illustrated in Figure 2. Let $A < \alpha$, $(h_t + \rho) = \lambda$, a constant, and $r = 0$. The utility maximizing consumption path is

$$\begin{aligned} c_t &= \alpha, \quad t < T \\ &= A, \quad t \geq T. \end{aligned}$$

The path is illustrated in Figure 3. T is given by $(\alpha - A)T = w$. $\partial T/\partial w = 1/(\alpha - A) > 0$. $\partial T/\partial A = w/(\alpha - A)^2 > 0$. Therefore, both $\partial MRS/\partial w$ and $\partial MRS/\partial A$ are positive.

One can also directly verify from

$$U = \int u^*(c_t)e^{-\lambda t}dt$$

that MRS = $\lambda T + 1$. Then, $\partial MRS/\partial w = \lambda \partial T/\partial w$ and $\partial MRS/\partial A = \lambda \partial T/\partial A$ both of which are positive.

Of course, this utility function does not satisfy the strict concavity assumption. But, as illustrated in Figure 2, a slightly modified utility function, $\tilde{u}$, would, and it would lead to the same qualitative result on T.

In the second example I take the utility function to be

$$u(c) = c^{1-\gamma}/(1-\gamma).$$

This utility function has been the subject of considerable analysis; the empirical work to be reported later in this paper is based on it.

The optimal consumption trajectory is given by equations (7) and

$$c_t^{-\gamma} a_t = A^{-\gamma} a_T e^{(r-\rho)(T-t)}. \tag{11}$$

This equation implies that

$$\partial c_t/\partial A = c_t(1/A + \theta \partial T/\partial A)$$

in which $\theta = (\rho + h_T - r)/\gamma$. From the budget constraint

$$\begin{aligned}\int_0^T e^{-rt}dt &= \int_0^T (\partial c_t/\partial A)e^{-rt}dt \\ &= (1/A + \theta\partial T/\partial A)\int_0^T c_t e^{-rt}dt. \\ &= (1/A + \theta\partial T/\partial A)(w + A\int_0^T e^{-rt}).\end{aligned}$$

Thus,

$$\delta/(w+A\delta) = 1/A + \theta\partial T/\partial A$$

where $\delta = \int_0^T e^{-rt}dt$. Then

$$\theta \partial T/\partial A = -w/(wA+A^2\delta).$$

Because θ and δ are positive, $\partial T/\partial A$ is negative. Therefore, with this utility function $\partial MRS/\partial A$ is negative.[3]

3. Estimation.

Although the analytical model may give a good approximation to the MRS, it has several shortcomings. First, people may desire to leave a bequest: because annuity wealth is not bequeathable, this desire will lower the MRS. Second, most private pensions are not indexed; yet, the analysis assumed that the annuity flow was fixed in real terms. Although the assumption is correct for many people, some have a mixture of real and nominal annuities. Finally it is desirable to have magnitudes for the MRS rather than just the ranges given by the analysis. Expanding the model to include bequests and nominal annuities requires that the model be solved numerically, which, in turn, means definite utility functions for consumption and for bequests must be specified. I assume that individuals maximize in the consumption path $\{c_t\}$ lifetime utility

$$(12) \qquad \int u(c_t)e^{-\rho t}a_t dt + \int V(w_t)e^{-\rho t}m_t dt$$

in which

$$u(c_t) = c_t^{1-\gamma}/(1-\gamma).$$

$V(.)$ is the utility from bequests. This formulation of utility maximization with bequests is due to Yaari (1965).

I parameterize the bequest function by assuming that the marginal utility of bequests is constant. This assumption may be defended in several ways. First, from a practical point of view, without such an assumption the model cannot be solved; yet, the estimation requires a model solution. Second, in other work I found that the strength of the bequest motive did not seem to depend on the wealth level.[4] Third, variations in the level of wealth cause only small variations in the level of the wealth of the heirs; therefore, the marginal utility of wealth of the heirs will roughly be constant over variations in wealth of the older generation, and one would expect the marginal utility of bequests to be constant.

The Pontryagin necessary conditions associated with this problem are

(13) $\quad c_t = A_t$, if $w_t = 0$,

and

(14) $$c_t^{-\gamma} a_t = c_{t+h}^{-\gamma} a_{t+h} e^{h(r-\rho)} + \alpha \int_t^{t+h} e^{(s-t)(r-\rho)} m_s ds$$

over an interval $(t, t+h)$ in which $w_t > 0$. α is the constant marginal utility of bequests.

The solution depends on the parameters, initial wealth and the annuity path. Unless initial wealth is very large or annuities very small, bequeathable is eventually consumed. Then the solution is given by

(15.1) $\quad c_T = A_T$

(15.2) $\quad c_0^{-\gamma} = c_t^{-\gamma} a_t e^{t(r-\rho)} + \alpha \int_0^t e^{(r-\rho)s} m_s ds$

(15.3) $\quad w_T = w_0 e^{rT} + \int_0^T (A_s - c_s) e^{(T-s)r} ds$

(15.4) $\quad w_T = 0$

If initial wealth is very large, wealth will never go to zero, and the nature of the solution is different. Although these cases are taken care of in the estimation to be reported below, I will not discuss them here because empirically they are not important.

In previous work I have estimated the parameters of this model. Given the parameters, the model can be solved for the optimal consumption trajectory and for the maximum utility. The solution will depend on initial bequeathable wealth, the real annuity stream, the nominal annuity stream, actual mortality data, and the marginal utility of bequests. A second solution at a slighly higher level of bequeathable wealth can be used to calculate a numerical approximation to $\partial U/\partial w$. A third solution at the original level of bequeathable wealth but at a higher level of Social Security benefits will lead to a numerical approximation to $\partial U/\partial S$. The MRS is estimated by taking ratios. These simulations are done for each single person in the sample.

In that the estimated MRS depends critically on the parameter estimates of the model in equations (15) I outline the data and estimation methods on which they are based.

3.1. Estimation of a Consumption Model.[5]

The data are from the Longitudinal Retirement History Survey, which was commissioned by the United States Social Security Administration. About 11,000 households whose heads were born in 1906-1911 were interviewed every two years from 1969 through 1979. Detailed questions were asked about all assets (except a meaningful question on life insurance), and the data were linked with offical Social Security records so that one can calculate exactly Social Security benefits. There are some data on consumption, but they are not complete, so I estimated the parameters of the model over wealth data. Bequeathable wealth includes stocks and bonds, property, businesses and savings accounts, all less debts. As suggested by King and Dicks-Mireaux (1982), I excluded housing wealth because the costs of adjusting housing consumption are substantial; therefore, people may not follow their desired housing consumption path. As long as the consumption of other goods follows its desired path, the parameters may be estimated over bequeathable wealth excluding housing wealth. Annuities include pensions, Social Security benefits, an estimated income value from Medicare/Medicaid, privately purchased annuities (which are very small), welfare transfers, and transfers from relatives. See Hurd and Shoven (1985) for a detailed description of the data.

The estimation method is to use equations (15) to solve for the consumption path as a function of an initial choice of the parameter values. This requires numerical integreation and a search for T. The solution will depend on initial wealth. Then, wealth in the next survey, w_2, is predicted from equation (4). That is, the necessary conditions and the boundary conditions, equations (15), implicitly define

$$w_2 = f(w, \{A\}, \boldsymbol{\theta}),$$

in which w is initial wealth, {A} is the annuity stream, and $\boldsymbol{\theta}$ is the parameter vector (γ ρ α). The parameter space is searched to minimize a function of (w_2 - f).

Although α is, in principle, identified through nonlinearities in the functional form, the identification is very weak. Therefore, I specify that α is zero if a household has no living children.[6] The interpretation of α is the increase in the marginal utility of bequests across households according to whether they have living children or not.

The first set of parameter estimates comes from solving

$$\min_{\boldsymbol{\theta}} \Sigma(w_2 - f(w_0, \{A\}, \boldsymbol{\theta}))^2$$

The estimated parameter values, which I refer to later as the nonlinear least squares (NLLS) estimates, are

γ	ρ	α
.729	.0501	$5.0\text{x}10^{-7}$
(.091)	(.004)	$(1\text{x}10^{-4})$

Number of observations = 5452

An analysis of the residuals was consistent with the hypothesis that wealth is observed with error. Therefore, I estimated the parameters by nonlinear two-stage least squares (NL2SLS), in which the parameter estimates come from solving

$$\min_{\theta} [\mathbf{w}_2-\mathbf{f}(\theta)]'\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'[\mathbf{w}_2-\mathbf{f}(\theta)]$$

$\mathbf{X}$ is an nx15 matrix of observations on income from wealth; these data are not derived from the wealth data but come from separate questions in the RHS. Thus they should not be correlated with the observation errors in w.

The results from the NL2SLS are

$\underline{\gamma}$	$\underline{\rho}$	$\underline{\alpha}$
1.12	-0.011	$6.0\text{x}10^{-7}$
(.074)	(.002)	$(32\text{x}10^{-7})$

Number of observations = 5452

The major difference between the two sets of results is in $r-\rho$, which, if the mortality rate were zero, would control the slope of the consumption trajectory. r is taken to be 0.03 so in the NLLS $r-\rho$ is approximately -0.02; even with a bequest motive, the consumption path will slope downward. In the NL2SLS estimates $r-\rho$ is about 0.04. Even without a bequest motive, the consumption slope will have a positive slope until the conditional mortality rate, m_t/a_t, exceeds 0.04. The NL2SLS consumption trajectories will look like cons_1 in Figure 1, and the NLLS like cons_2. Both sets of estimates produce an estimate of γ that is much smaller that what has typically been assumed in the literature. For example, Kotlikoff, Shoven and Spivak (1983, 1984) and Kotlikoff and Spivak (1981) use a value of 4 in their simulations. Hubbard (1984) uses values of 0.75, 2 and 4. Davies (1981) "best guess" for his simulations is 4. Large values of γ mean that the slope of the consumption trajectory is not sensitive to variations in mortality rates; my estimates imply that the consumption paths of the elderly will have substantial variation with mortality rates.

The marginal utility of bequests, α, is estimated to be very small, which is consistent with other estimates I have made in a model that is almost free of functional form restrictions.[7] The small estimate of α is caused by the fact that in the data there is little difference between the saving rates of households with children and households without children.

3.2. Estimation of the MRS.

In that the utility model does not apply to couples, I estimated the MRS of each single person who was observed for two consecutive interviews. For the ith individual the estimate of the MRS is

$$\widehat{MRS}_i = (\Delta U/\Delta S)/(\Delta U/\Delta w)_i,$$

where ΔU is the change in utility associated with a change of ΔS in Social Security wealth, or a change of Δw in bequeathable wealth. The estimated parameter values from either the NLLS or the NL2SLS and the individual's actual data are used to solve for the optimal consumption trajectories and utility levels. Thus, the distribution of the MRS will depend on the distributions of initial bequeathable wealth and real annuities. The distribution will also depend on the distributions of children, nominal annuities, and mortality rates. These last variables were not considered in the analytical section; but they are taken into account in the estimates. Children enter through the parameter α, the marginal utility of bequests: this parameter only affects the consumption path of individuals with children. Nominal annuities are almost all job-related pensions which are fixed in nominal terms. They, therefore, decline in real terms as the owner ages. Mortality rates vary from individual to individual because of differences in initial age, initial year (each of the five initial years has a different mortality table), sex and race.

Initial bequeathable wealth (excluding housing wealth) is \$15791 on average and \$4720 at the median. The initial mean annual flow of real annuities is \$2984. This represents about \$36000 of annuity wealth. The sample, which is mostly widows, is certainly not very wealthy, and it has the majority of its wealth in real annuities.

According to the NLLS parameter estimates, bequeathable wealth declines rapidly: T, the mean time to exhaustion of bequeathable wealth, is 6.7 years. One would expect the average MRS to be quite low, and that, indeed, is the case: the mean MRS is just 1.06. Of course, there is substantial variation in the MRS, as the following table shows.

Distribution of MRS: NLLS estimates

Percentile Point	100	99	95	90	75	50	25	10	5	1	0
MRS	2.34	1.60	1.40	1.30	1.17	1.05	0.96	0.84	0.75	0.57	0.31

The median MRS is only slightly above one, which according to these estimates, implies that a substantial fraction of the sample of singles is over-annuitized. The variation in the MRS means that different individuals would be willing to pay different amounts for variations in Social Security: for example, someone at the 95th percentile with a MRS of 1.40 would be willing to pay almost twice as much as someone at the 5th percentile with a MRS of 0.75.

Although the bequest motive for saving could, in principle, cause the MRS to vary by a great deal, in fact it makes very little difference. This is, of course, a consequence of the small estimate of α. The estimated value of α changes the consumption path by a negligible amount, and the addition to utility in equation (3) that arises from holding wealth is very small. In fact, the average MRS over individuals with children is 1.09; over individuals without children it is 1.05. This is the opposite of what one would expect *cet. par.*, but, of course, these averages do not hold constant other, more powerful, determinents of the MRS such as bequeathable wealth and annuities.

In Table 1, I give the average MRS for the NLLS parameter estimates.

As the last column shows, the MRS increases in wealth. The difference between the highest and lowest intervals is rather substantial when the MRS are viewed as prices: had the annuity levels been freely chosen the price at the margin of annuities for someone in the upper wealth interval would have been 1/1.28 = 0.78. For someone in the lowest wealth interval the price would have been 1/0.91 = 1.10, which is 41% higher.

The last row has variation in MRS by initial real annuity level. Although the theoretical analysis of the simple model indicated that the MRS should decrease with increasing real annuities, the row shows little, if any, variation. Of course, other determinents of the MRS vary across the annuity intervals.

The first column of the table shows how MRS varies with wealth catagory holding annuities roughly constant. The variation is large, especially at the lowest annuity level: there is about a 74% difference in the MRS between the lowest and highest wealth intervals. At higher annuity levels the variation with initial bequeathable wealth is smaller because at low wealth levels there is an increase in MRS with annuities. This is counter to the analytical results; it is caused by variation in the other determinents of the MRS. It is not clear how to hold these other determinents constant in that they cannot be summarized as single numbers: for example, both the mortality rates and the nominal annuities are vectors of length 55.

The NL2SLS estimates of the parameters imply much flatter consumption paths than the NLLS parameter estimates: the consumption path slopes upward until the conditional mortality rate exceeds $r-\rho$ which is about 0.04. Under the NL2SLS parameter values the mean time to exhaustion of bequeathable wealth is 15.5 years. Because the desired consumption path is closer to the annuity path, one would expect the MRS to be higher. The mean MRS is 1.41, and the distribution is

Distribution of MRS: NL2SLS Estimates

Percentile Point	100	99	95	90	75	50	25	10	5	1	0
MRS	2.41	1.86	1.71	1.62	1.50	1.41	1.35	1.27	1.06	0.71	0.30

Less that 5% of the sample has an estimated MRS less than one. This is a much smaller fraction of the sample than under the NLLS parameter estimates.

Table 2 has the averages of MRS by initial bequeathable wealth and by initial real annuity interval. The pattern is about the same as before although there is somewhat less variation by wealth interval. As before, the MRS increases with bequeathable wealth, and it has little variation by annuity level. Holding annuity level constant, the MRS increases in wealth, especially at the lowest annuity level. As before, the variation with annuity level depends on the wealth interval: the MRS increases at low wealth levels and decreases at high wealth levels.

4. Conclusion.

When annuities are given exogenously, and actuarially fair life insurance is not available, the desired consumption path may differ from the actual consumption path. In a simplified model the analytical results showed that the greater the initial bequeathable wealth the more valuable, at the margin, the annuity stream. This result was verified in the empirical part of the paper. The MRS varied substantially from individual to individual even though everyone was assumed to have the same utility function parameters. The variation was entirely due to variations in economic resources, age and demographic variables.

Under the NLLS parameter estimates, 90% of the retired single individuals had a MRS less than 1.3. We are used to valuing an annuity stream by its expected present value; but these results suggest that from the point of the view of the individual, such a valuation is very inaccurate. In this sample a fair annuity would pay a return of about 2.5 times that of bequeathable wealth with some variation due to differences in initial age. One would normally think, therefore, that the elderly would be willing to exchange 2.5 dollars of bequeathable wealth for one dollar of annuity wealth. But these results suggest that for most individuals that rate of exchange is too large by a factor of about two.

These results have some bearing on why so few elderly purchase annuities: with the normal loading of annuities, anyone with a MRS less than about 1.35 will find the annuities too costly. According to the NLLS estimates, this covers about 90-95% of the sample. In addition, of course, privately purchased annuities are risky because they have no inflation protection. Finally, many elderly surely have a precautionary motive for saving: they want to protect themselves against bad health outcomes or against variations in rates of return on their assets. Annuities do not satisfy the precautionary motive.

A substantial fraction of the sample had a MRS less than one, which implies they would like to reduce their holdings of annuities at an actuarially fair rate of exchange. The results also imply that they would not want an expansion of the Social Security system unless there were a favorable transfer compenent in Social Security.

A comparison of the MRS based on the NLLS parameter estimates with those based on the NL2SLS estimates shows that the level of the MRS is quite sensitive to the parameters. But, even though the median and average MRS are higher according to the NL2SLS parameter estimates, the bulk of the distribution is much below 2.5, so the same general conclusions hold. In particular, under either set of parameter estimates the wealthy can make better use of an increase in Social Security benefits than the poor. Of course, there are some parameter values that would so flatten the consumption trajectories that bequeathable wealth would remain positive for the lifetimes of almost everyone in the sample; then the MRS would be about 2.5 for everyone. Such parameter values would be very different from the values behind these estimates because the principle reason the MRS are low is that most of the sample has much less bequeathable wealth than annuity wealth.

Footnotes

1. The specific utility function he uses implies that the compensating variation decreases as Social Security benefits increase; but this result does not generally hold. See footnote 3 and the associated discussion.

2. Bernheim calculates the compensating variation to be $(\partial U/\partial A)/(\partial U/\partial w)$ in a model with a constant hazard. He takes the instantaneous utility function to be the constant relative risk aversion utility function which has marginal utility $c^{-\gamma}$. For this special case, his results differ from mine by $\partial A/\partial S$.

3. Bernheim bases a test of the Life Cycle Hypothesis on the variation in $d(\ln w_t)/dt$ with A/w. His analysis of the constant risk aversion utility function calls for a negative relationship. He takes his finding of a positive relationship to be evidence against the LCH. The two examples I give here indicate that because the sign of $\partial T/\partial A$ is indeterminate $d(\ln w_t)/dt$ may either increase or decrease in A. Thus his test depends on the special nature of the utility function he chose, and, in general, is not valid.

4. See my "Savings of the Elderly and Desired Bequests."

5. This discussion of the estimation is a summary of material in my "Mortality Risk and Bequests."

6. Although the RHS does not have information about the ages of the children, because of the ages of the RHS population the median age of the children would be about 30 in the first year of the survey. Thus, almost all the children will have their own households.

7. "Savings of the Elderly and Desired Bequests."

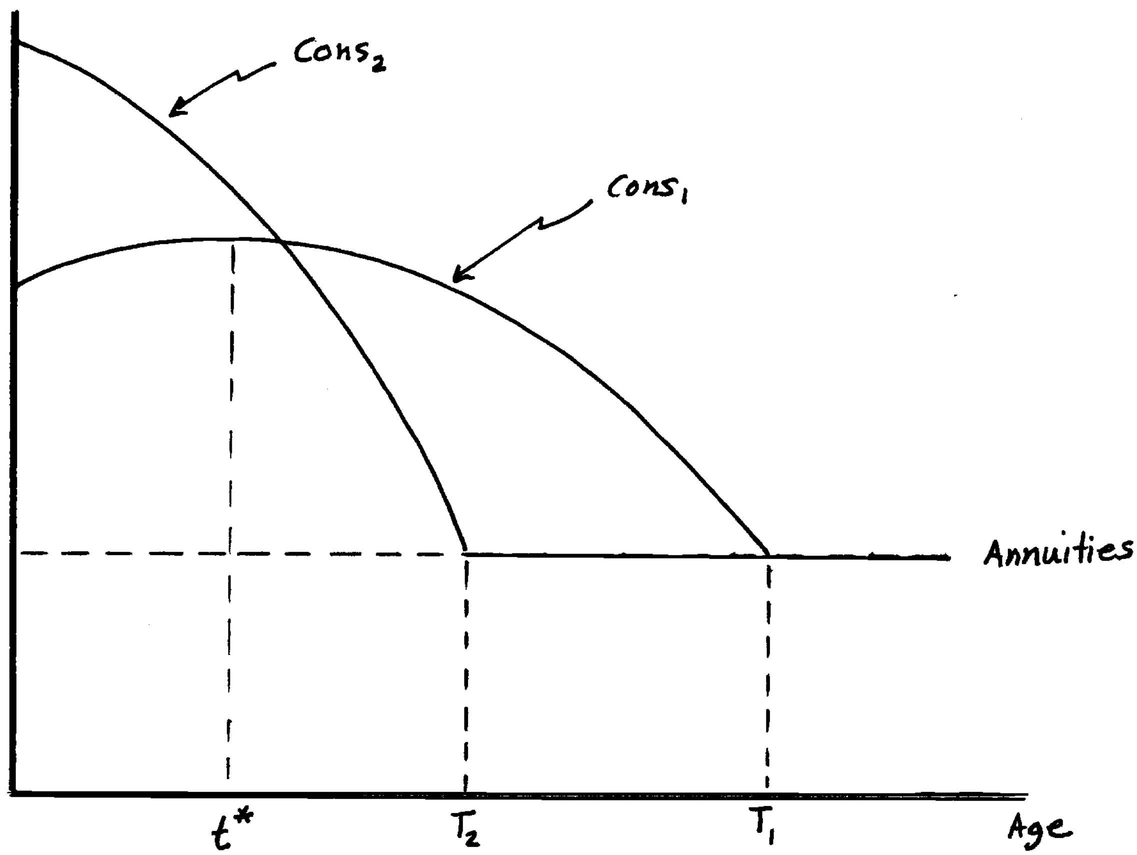

Figure 1

Two Consumption Paths

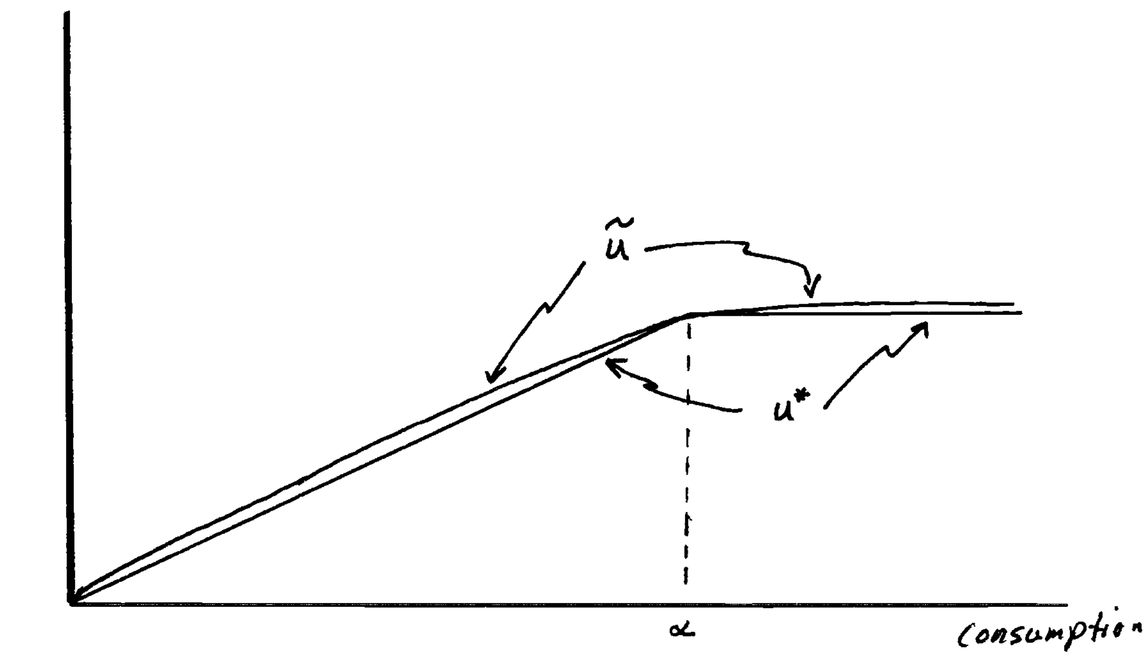

Figure 2

Two Utility Functions

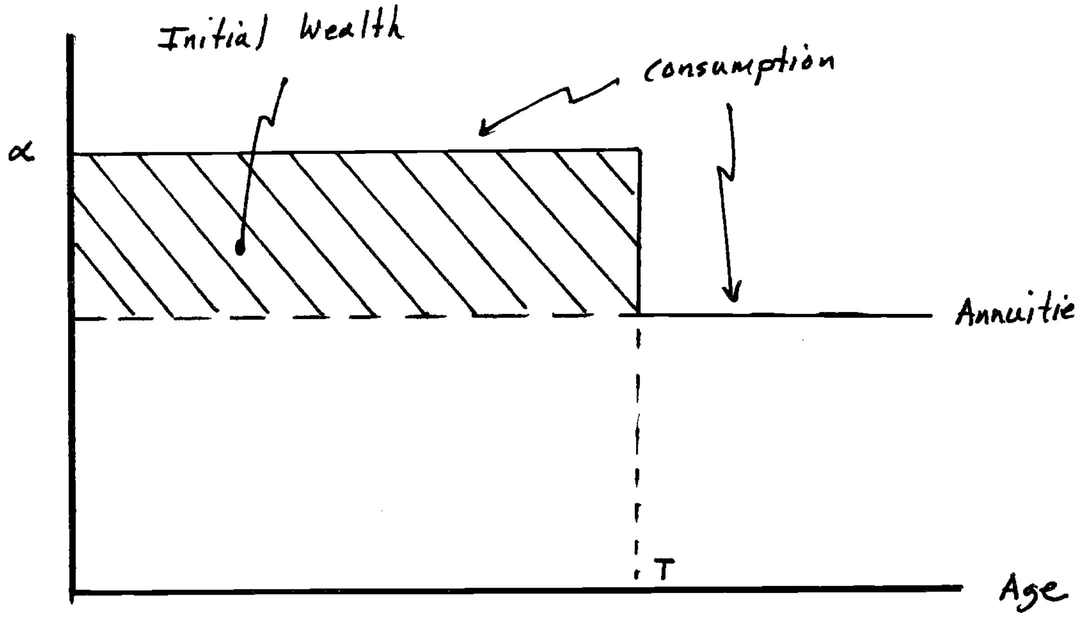

Figure 3

Consumption Path for u^*

Table 1

Average Marginal Rate of Substitution Based on the NLLS Parameter Estimates

Initial Wealth	Initial Annuity: Less Than $1,000	1,000 - 2,500	2,500 - 5,000	More Than 5,000	All
Less Than $1,000	0.85 (229)	0.87 (551)	0.98 (474)	1.06 (47)	0.91 (1301)
1,000 - 5,000	1.02 (224)	0.96 (551)	1.00 (619)	0.96 (86)	0.99 (1480)
5,000 - 20,000	1.23 (166)	1.10 (416)	1.10 (700)	1.04 (197)	1.11 (1479)
More Than 20,000	1.48 (127)	1.29 (259)	1.27 (522)	1.17 (249)	1.28 (1157)
All	1.09 (746)	1.02 (1777)	1.09 (2315)	1.08 (579)	1.06 (5417)

Note: Number in parenthesis is the number of observations.

Table 2

Average Marginal Rate of Substitution Based on the NL2SLS Parameter Estimates

	Initial Annuity				
Initial Wealth	Less Than $1,000	1,000 - 2,500	2,500 - 5,000	More Than 5,000	All
Less Than $1,000	1.13 (229)	1.26 (551)	1.33 (474)	1.32 (47)	1.27 (1301)
1,000 - 5,000	1.35 (224)	1.36 (551)	1.40 (619)	1.35 (86)	1.37 (1480)
5,000 - 20,000	1.52 (166)	1.45 (416)	1.45 (700)	1.40 (197)	1.45 (1479)
More Than 20,000	1.73 (127)	1.57 (259)	1.57 (522)	1.48 (249)	1.57 (1157)
All	1.39 (746)	1.38 (1777)	1.44 (2315)	1.42 (579)	1.41 (5417)

Note: Number in parenthesis is the number of observations.

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