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INTERGENERATIONAL ALTRUISM AND SOCIAL WELFARE: A CRITIQUE OF THE DYNASTIC MODEL

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ABSTRACT

In this paper, I show that, under relatively weak conditions, dynastic equilibria are never welfare optima. If a social planner sets policy to maximize a social welfare function, then, except in extreme cases where the planner cares only about a single generation, successive generations will never be linked through altruistically motivated transfers. This suggests that the dynastic model is unsuitable for normative analysis, and, to the extent governments actually behave in this manner, the model is also inappropriate for positive analysis. In addition, I show that, except in a few special cases, the planner's preferences are dynamically inconsistent. If the planner can successfully resolve this inconsistency, then the central result is somewhat modified.

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1. Introduction

A substantial body of recent research has established that a variety of important issues concerning national savings policy hinge critically upon the nature of economic relationships within families. If, in particular, parents and children are linked by altruistically motivated resource transfers, then each family may behave as though it is a single, infinite lived, "dynastic" unit. This observation has a variety of implications, including the well-known Ricardian equivalence theorem (see Barro [1974]).

Although the dynastic framework has become a standard analytical tool for applied theorists, particularly in the areas of public finance and macroeconomics, it has also been criticized on a variety of grounds. Many economists have challenged the empirical relevance of this framework (see e.g. Feldstein [1976] and Buiter and Tobin [1981]), while others have noted certain logical difficulties (see e.g. Bernheim and Bagwell [1987] and Abel and Bernheim [1986]).

In this paper, I discuss a new criticism of the dynastic model: under relatively weak conditions, I show that dynastic equilibria are never welfare optima. If the social planner sets policy to maximize a social welfare function, then, except in extreme cases where the planner cares only about a single generation, successive generations will never be linked through altruistically motivated transfers. To the extent one believes that the government does not behave in this manner, the dynastic model remains a legitimate tool for <u>positive</u> analysis. It is, however, unsuitable for <u>normative</u> analysis. Accordingly, studies which contemplate optimal government policies while maitaining dynastic assumptions (see e.g. Judd [1985] and Chamley [1985]) may lack internal coherence.

The intuition for this result is quite simple. Suppose that the social planner places some weight on the well-being of two successive generations. In a dynastic equilibrium, the older generation is indifferent about marginal transfers to the younger generation. But the planner will attach some importance to the younger generation's strict preference for receiving such transfers. Welfare optimal intergenerational resource distribution can therefore be attained only if the planner first drives the older generation to a corner, and then undertakes additional transfers.1/

In the abstract, this result is not new: similar arguments have been used to draw related conclusions in other contexts (see Roberts [1984] and Bernheim, Shleifer, and Summers [1985]). Yet certain peculiar aspects of the dynastic problem warrant further probing. Specifically, except in a few special cases, the planner's preferences are dynamically inconsistent. Resolution of this inconsistency requires that, at each point in time, the planner behaves as though he continues to place weight on the well-being of past (deceased) generations. This fundamentally alters the nature of his problem.

I assume that the planner discounts the stream of utilities from successive generations at the rate ρ, while individuals discount the stream of felicities at the rate δ. Letting the number of prior generations go infinity, I find that the welfare optimum converges to the dynastic equilibrium if and only if $\delta \geqq \rho$. Accordingly, the dynastic model turns out to be an appropriate normative tool as long as (i) the planner never disregards the preferences of deceased generations, and (ii) individuals do not discount felicity more rapidly than the government discounts utility. If, on the other hand, $\rho > \delta$, dynastic equilibria will always involve excessive levels of current consumption relative to the limiting welfare optima. Indeed, as long as $\delta \leq \rho$, changes in the level of private altruism have absolutely no effect on the planner's preferences.

The paper is organized as follows. I describe the model is section 2. Section 3 contains the basic criticism of the dynastic framework, along with a discussion of dynamic consistency. In section 4, I discuss the relationship between dynastic equilibria and welfare optima in an economy where the planner successfully resolves dynamic inconsistency.

2. The Model

The economy considered here corresponds quite closely to that analyzed by Barro [1974]. There is an infinite sequence of periods, labelled $t = 0,1,2,\ldots$, as well as an infinite sequence of consumers, which I also label $t = 0,1,2,\ldots$ in order to emphasized the fact that generation t is born in period t. Each generation lives for two periods. Accordingly, two generations are alive in each period, with the exception of period 0, during which generation 0 is alone. This special treatment of period 0 is for analytical convenience only (it allows me to avoid carrying around special terms for the "first" generation); one could easily add an "old" generation in period 0.

Generation t is endowed with wealth, w_t, in period t. In addition, it receives a bequest, b_{t-1}, from its immediate predecessor (without loss of generality, I adopt the convention that $b_{-1} \equiv 0$). In period t, it divides its resources between consumption, c_t^y, and investment. I assume for simplicity that production is constant-returns-to-scale for capital investments, so that saving yields a fixed gross return, $\beta > 1$. [2/] In period $t + 1$, generation t divides the procedes from its investments between consumption, c_t^o, and a bequest to its successor, b_t. Thus, generation t faces the following intertemporal budget constraint:

$$b_{t-1} + w_t = c_t^y + \beta^{-1}(c_t^o + b_t) \tag{1}$$

In addition, generation t must also respect non-negativity constraints:

$$c_t^y,\ c_t^o,\ b_t \geqq 0 \tag{2}$$

I assume that each generation cares about the infinite stream of consumption enjoyed by itself and its descendents:

$$u_t = u(c_t^y, c_t^o, c_{t+1}^y, c_{t+1}^o, \ldots)$$

I take these functions to be non-paternalistic, and additively separable. Formally, I assume that there exists a function $v\colon \mathbb{R}_+^2 \to \mathbb{R}$ such that

$$u(c_t^y, c_t^o, \ldots) = v(c_t^y, c_t^o) + \delta u(c_{t+1}^y, c_{t+1}^o, \ldots) \tag{3}$$

Barro's analysis of the Ricardian equivalence theorem introduced a notion of equilibrium for such environments. In subsequent work, this notion has been termed a "dynastic" equilibrium. Formally, $\langle \hat{c}_t^y, \hat{c}_t^o, \hat{b}_t \rangle_{t=0}^{\infty}$ is a dynastic equilibrium if it maximizes

$$\sum_{t=0}^{\infty} \delta^t v(c_t^y, c_t^o)$$

subject to (1) and (2).

Note that in a dynastic equilibrium, $(\hat{c}_t^y, \hat{c}_t^o)$ must solve, for each t,

$$\max_{c_t^y, c_t^o} v(c_t^y, c_t^o)$$

$$\text{subject to} \quad c_t^y + \beta^{-1} c_t^o = \hat{c}_t^y + \beta^{-1} \hat{c}_t^o \quad .$$

We can therefore simplify the description of an equilibrium as follows. Let

$$V(C) = \max_{C \geq c \geq 0} v(c, \beta(C - c)) \quad ,$$

and consider the solution to

$$\max_{\langle C_t, b_t \rangle_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t V(C_t) \tag{4}$$

subject to

$$C_t + \beta^{-1} b_t = w_t + b_{t-1} \tag{5}$$

and

$$C_t, b_t \geqq 0 \quad . \tag{6}$$

This solution, $\langle \hat{C}_t, \hat{b}_t \rangle_{t=0}^{\infty}$, corresponds exactly to a dynastic equilibrium, in the sense that $\hat{C}_t = \hat{c}_t^y + \beta^{-1}\hat{c}_t^o$. Henceforth, we will work with this "reduced form" notion of an equilibrium.

This definition becomes still more familiar if an equilibrium has the property that $\hat{b}_t > 0$ for all t. In that case, $\langle \hat{C}_t \rangle_{t=0}^{\infty}$ solves

$$\max_{\langle C_t \rangle_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t V(C_t) \tag{7}$$

subject to

$$\sum_{t=0}^{\infty} \beta^{-t} C_t = W \equiv \sum_{t=0}^{\infty} \beta^{-t} w_t \quad . \tag{8}$$

and

$$C_t \geqq 0 \quad . \tag{9}$$

That is, whenever transfers are positive for all generations, $\langle \hat{C}_t \rangle_{t=0}^{\infty}$ is simply the solution to a standard programming problem. At this point, it is useful to define

$$S \equiv \left\{ \langle C_t \rangle_{t=0}^{\infty} \mid (8) \text{ and } (9) \text{ are satisfied} \right\}$$

I note for future reference that S is compact in the product topology. I will use C to denote an element of S.

Throughout, I make the following additional assumptions on preferences.

Assumption 1: V is differentiable, monotonically increasing, strictly concave, and satisfies $\lim_{C\to 0} V'(C) = +\infty$, $\lim_{C\to\infty} V'(C) = 0$, and $V(0) > -\infty$.

All aspects of assumption 1 are standard, except the final restriction, which places a lower bound on the dynastic optimization problem. One can dispense with this lower bound at the cost of some additional, tedious analysis in section 4, below. Note that the more traditional restrictions on V guarantee that consumption is strictly positive in all periods.

Assumption 2: $\delta\beta < 1$

This condition is also standard, and guarantees that the dynastic optimization problem is bounded above. Under assumptions 1 and 2, one can demonstrate that the objective function in (7) is continuous over S in the product topology. Accordingly, the solution to (7) is well-defined.

Before proceding, it is worthwhile to reflect beiefly on the justification for using this notion of an equilibrium. It is easy to demonstrate that a dynastic equilibrium is subgame perfect (see Selten [1965, 1975]). However, Gale [1985] has shown that it is not the only subgame perfect equilibrium for this model. Indeed, there is a continuum of subgame perfect outcomes, and one cannot discard these alternative solutions on the basis of any natural criterion, such as dominance. Why then focus on dynastic equilibria?

There are at least three reasons for doing so. First, the strategic structure of a dynastic equilibrium is particularly simple--each agent simply expects that everyone will act to maximize the dynastic welfare function, and assumes that this is common knowledge. Other equilibria involve more complex history-dependent punishments. Since all generations can never meet to discuss strategies, it seems unlikely that such complex strategic behavior will arise. Second, the dynastic equilibrium is analytically convenient, since it corresponds to the solution of a simple programming problem. Third, dynastic equilibria have certain well-known properties, such as Ricardian equivalence. Other refinements of the equilibrium set may or may not have these properties, depending upon whether punishments remain feasible after policy changes (see Bernheim and Bagwell [1987] and O'Connell and Zeldes [1986]). While the second and third points do not validate the dynastic concept, they no doubt account for part of its popularity as an analytic device (see e.g. Judd [1985]).

I now turn from consumers to the social planner. The planner has a social welfare function, ψ, which I assume to be additively separable:3/

(10) $$\psi(u_0, u_1, \ldots) = \sum_{t=0}^{\infty} \rho_t u_t \quad ,$$

where

(11) $$\sum_{t=0}^{\infty} \rho_t = 1 \quad ,$$

and where u_t is given by (3). The planner maximizes (10) subject to (3) and

(12) $$c_t^y, \; c_t^o \geqq 0$$

(note that he is <u>not</u> bound by the non-negativity constraint on transfers).

So far, there is no guarantee that the sum in equation (10) converges. I therefore impose

<u>Assumption 3</u>: There exists T and η with $\eta\beta < 1$ such that

$$\sum_{t=0}^{\tau} \rho_t \delta^{\tau-t} \leqq \eta^{\tau}$$

for all $\tau > T$.

Again, standard arguments suffice to establish that, under assumption 3, ψ is continuous on S in the product topology, and therefore achieves a maximum (the reasons for this will perhaps become clearer at the beginning of section 3).

Our objective in subsequent sections is to determine the relationship between dynastic equilibria and welfare optima.

3. Difficulties with the Dynastic Concept

In many instances, dynastic models have been used to study optimal public policy. The main result of this section suggests that, quite generally, the preceding sentence is logically inconsistent, since the optimum necessarily entails the use of policies which lead to a violation of the basic dynastic assumptions.

I begin by deriving through recursive substitution a direct expression for the utility of generation t:

$$u_t = \sum_{\tau=t}^{\infty} \delta^{\tau-t} V(C_\tau) \tag{13}$$

By assumption, this sum converges on S. Next, I write out the planner's objective function explicitly in terms of consumption. Since welfare optimal plans must always satisfy

$$v_1(c_t^y, c_t^o) = \beta v_2(c_t^y, c_t^o)$$

(where subscripts denote partial derivatives) for all t, I can, as before, simplify by expressing welfare as a function of $\langle C_t \rangle_{t=0}^{\infty}$:

$$\begin{aligned} \Psi(\langle C_t \rangle_{t=0}^{\infty}) &= \sum_{t=0}^{\infty} \rho_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} V(C_\tau) \\ &= \sum_{\tau=0}^{\infty} \left(\sum_{t=0}^{\tau} \rho_t \delta^{\tau-t} \right) V(C_\tau) \end{aligned} \tag{14}$$

(one can now see the basis for my claim that assumption 3 bounds achievable social welfare on S).

Suppose now that we have some dynastic equilibrium, $\langle \hat{C}_t, \hat{b}_t \rangle_{t=0}^{\infty}$. First order conditions imply that

$$V'(\hat{C}_t) = \delta\beta V'(\hat{C}_{t+1}) \tag{15}$$

wherever $\hat{b}_t > 0$. Suppose further that $\rho_t > 0$ for some $t^* > 0$. Consider a transfer of consumption, α, from generation $t^* - 1$ to t^*:

$$\frac{dC_t}{d\alpha} = \begin{cases} -1 & \text{if} \quad t = t^* - 1 \\ \beta & \text{if} \quad t = t^* \\ 0 & \text{otherwise} \end{cases}$$

What effect does this have on welfare? Differentiating,

$$\left.\frac{d\Psi}{d\alpha}\right|_{\hat{C}} = -\left(\sum_{t=0}^{t^*-1} \rho_t \delta^{t^*-1-t}\right) V'(\hat{C}_{t^*-1})$$

$$+ \beta\left(\sum_{t=0}^{t^*} \rho_t \delta^{t^*-t}\right) V'(\hat{C}_{t^*})$$

$$= \beta\left(\sum_{t=0}^{t^*} \rho_t \delta^{t^*-t}\right) V'(\hat{C}_{t^*})$$

$$-\beta\left(\sum_{t=0}^{t^*-1} \rho_t \delta^{t^*-t}\right) V'(\hat{C}_{t^*})$$

$$= \rho_{t^*} \beta V'(\hat{C}_{t^*}) > 0$$

(where the second equality follows from substituting equation (15)). Accordingly, $\langle \hat{C}_t \rangle_{t=0}^{\infty}$ could not be a welfare optimum. We have therefore proven:

<u>Theorem 1:</u> <u>Suppose that</u> $\rho_\tau > 0$ <u>for some</u> $\tau > 0$. <u>Let</u> $\langle \hat{C}_t, \hat{b}_t \rangle_{t=0}^{\infty}$ <u>be a dynastic equilibrium.</u> <u>If</u> $\hat{b}_{\tau-1} > 0$, <u>then this equilibrium is not a welfare optimum.</u>

The intuition for this result is extremely simple. When $\hat{b}_{\tau-1} > 0$, generation $\tau - 1$ must be indifferent between the consumption of $\tau - 1$ and τ, as must all prior generations. However, τ strictly prefers to receive and consume a larger bequest. Future generations are simply indifferent. Since the planner attaches positive weight to τ's well-being, he must also prefer such a redistribution.

Theorem 1 has an immediate and important corollary:

Corollary 1: Suppose that $\rho_t > 0$ for all t. Let $\langle \hat{C}_t, \hat{b}_t \rangle_{t=0}^{\infty}$ be a dynastic equilibrium. If $\langle \hat{C}_t \rangle_{t=0}^{\infty}$ is a welfare optimun, then $\hat{b}_t = 0$ for all t.

Thus, in the standard case where the planner cares about every generation directly, successive generations would never be linked through operative transfers.

One might, of course, take the view that the planner is an elected government, and that this government should only represent the preferences of its current constituency. One might then argue that it is appropriate to take $\rho_0 = 1$ and $\rho_t = 0$ for $t > 0$ in period 0. $\rho_1 = 1$ and $\rho_t = 0$ for $t > 1$ in period 1, and so forth. This would indeed validate the dynastic framework. However, this argument is faulty. In general, several generations coexist in each period, and the government should be somewhat responsive to each of these. This is precisely why I formulated the problem in the context of an overlapping generations model: one would, at a minimum, expect to have ρ_0 and

ρ_1 strictly positive in period 0 (similarly in subsequent periods). As long as the concurrent government is responsive to both living generations in every period, one should never observe positive private transfers from old to young.

In the following section, I provide a partial resolution to the problem described above. I motivate this resolution somewhat indirectly by, perhaps paradoxically, introducing a second problem. This concerns dynamic consistency. Normative policy analyses which employ the dynamic framework typically proceed by assuming that the social planner acts to maximize welfare through an optimal sequence of choices. If the planner's objectives are dynamically inconsistent, then this prescription is inappropriate, since the planner should rationally anticipate future deviations from the plan which appears optimal at each instant.

I will consider two different notions of dynamic consistency. The appropriateness of each notion depends upon whether one takes the social planner to be a representative government (as in the immediately preceding discussion), or a "social architect." I elaborate on this distinction below.

The program which maximizes (10) subject to (3) and (12) satisfies <u>stationary dynamic consistency</u> if, for all t, the continuation program maximizes

$$\sum_{\tau=t}^{\infty} \rho_{\tau-t} u_\tau$$

This corresponds to the notion of dynamic consistency proposed by Strotz [1956] and Pollak [1968]. Alternatively, the program which maximizes

(10) subject to (3) and (12) satisfies <u>non-stationary dynamic consistency</u> if, for all t, the continuation program maximizes

$$\sum_{\tau=t}^{\infty} \rho_\tau u_\tau$$

It is useful to think first about these notions of dynamic consistency in the case where $\delta = 0$ (i.e., all generations are selfish). Strotz's well known result establishes that the maximizing program satisfies stationary dynamic consistency if and only if the welfare weights are geometric, i.e.

$$\rho_t = \rho^t$$

for some $\rho \in [0,1)$, and for all t. In contrast, the maximizing program <u>always</u> satisfies non-stationary dynamic consistency when $\delta = 0$.

When are these two notions appropriate? If we think of the planner as a government which responds to current political pressures, then, unless the relative political strengths of old and young change systematically over time, stationary dynamic consistency is relevant. If the maximizing program does not satisfy stationary dynamic consistency, then a current government cannot count on its successor to persue the same objectives by continuing the same policies. Successive governments will then be forced to treat policy formation as a strategic problem.

If, on the other hand, we think of the planner as a social architect, such as an infinitely lived dictator (i.e. a true dynasty, in the original sense of the world), then non-stationary dynamic consistency is relevant. The planner simply attaches weights to specific generations, and these relative weights do not change as time passes. However, if the planner's objective function does not satisfy non-stationary dynamic consistency, he may not be able to stick to an optimal plan chosen in period 0. In particular, he would have an incentive to deviate from this plan in subsequent periods on the theory that what previous (deceased) generations don't know can't hurt them. Anticipating such thinking at time 0, he would then act strategically *vis a vis* himself, in general choosing an action which is inconsistent with unconstrained maximization of (10).

Unfortunately, in the presence of altruism $(\delta > 0)$, the planner's problem is almost always dynamically inconsistent, in both senses. I establish this through two theorems.

Theorem 2: The maximizing program satisfies stationary dynamic consistency if and only if for some $\gamma > \delta$

$$\rho_o = \frac{1 - \gamma}{1 - \delta}$$

and

$$\rho_t = \gamma^t(\gamma - \delta)\rho_o$$

for $t \geqq 1$.

Proof: Noting from equation (14) that ψ is additively separable, we apply Strotz' [1956] well-known theorem to conclude that there must exist k and γ such that

$$\text{(16)} \qquad k\gamma^t = \sum_{\tau=0}^{t} \rho_\tau \delta^{t-\tau}$$

for all t. Thus,

$$k(\frac{\gamma}{\delta})^{t+1} - k(\frac{\gamma}{\delta})^t = \rho_{t+1}\delta^{-(t+1)}$$

Accordingly,

$$\rho_{t+1} = k\gamma^{t+1} - k\gamma^t\delta = k\gamma^t(\gamma - \delta)$$

for all $t \geqq 0$. $\rho_t > 0$ implies $\gamma > \delta$. Substituting $t = 0$ into (16) yields $\rho_o = k$. The constraint that $\sum_{t=0}^{\infty} p_t = 1$ then implies that $k = (1 - \gamma)/(1 - \delta)$. This establishes necessity. Sufficiency is trivial. Q.E.D.

Note that, for stationary dynamic consistency to hold, the planner's welfare weights must decline geometrically after generation 0, but that generation 0 must receive extra weight. I should also emphasize that the planner's problem is never dynamically consistent in the stationary sense for the class of welfare functions satisfying

$$\rho_0 + \rho_1 = 1 \quad ,$$

and $\rho_t = 0$ for $t > 1$, and that, within an overlapping generations framework, this is the most natural class of welfare functions for a representative government.

I now turn to the second notion of dynamic consistency.

<u>Theorem 3: The maximizing program satisfies non-stationary dynamic consistency if and only if there exists some</u> t^* <u>such that</u>

$$\rho_{t^*} + \rho_{t^*+1} = 1 \quad ,$$

<u>and</u> $\rho_t = 0$ <u>for</u> $t < t^*$ <u>and</u> $t > t^* + 1$.

<u>Proof</u>: Note that

$$\sum_{t=s}^{\infty} \rho_t u_t = \sum_{\tau=s}^{\infty} \Big(\sum_{t=s}^{\tau} \rho_t \delta^{\tau-t} \Big) V(C_\tau) \quad . \tag{17}$$

Dynamic consistency requires that for each s, the weights on $V(C_\tau)$, $\tau \geqq s$ in (17) are proportional to the weights in (14). That is, for each s there exists k_s such that for all $\tau \geqq s$,

$$k_s \sum_{t=0}^{\tau} \rho_t \delta^{\tau-t} = \sum_{t=s}^{\tau} \rho_t \delta^{\tau-t} \tag{18}$$

There are three cases to consider.

(i) $k_s = 1$. Then, by (18), $\rho_t = 0$ for all $t < s$.

(ii) $k_s = 0$. Then, by (18), $\rho_t = 0$ for all $t \geqq s$.

(iii) $k_s \neq 0, 1$. Using (18), we see that for all $\tau \geqq s$,

$$k_s \sum_{t=0}^{\tau+1} \rho_t \delta^{\tau+1-t} = \sum_{t=s}^{\tau+1} \rho_t \delta^{\tau+1-t} \quad ,$$

or, dividing by δ,

$$k_s \sum_{t=0}^{\tau+1} \rho_t \delta^{\tau-t} = \sum_{t=s}^{\tau+1} \rho_t \delta^{\tau-t} \quad . \tag{19}$$

Subtracting (18) from (19) yields

$$k_s \rho_{\tau+1} = \rho_{\tau+1}$$

Thus, $\rho_t = 0$ for all $t > s$.

Let t^* be the smallest t for which ρ_t is positive (note: t^* may be 0). Then $k_s \neq 1$ for all $s > t^*$. Thus, $\rho_t = 0$ for all s and t satisfying $t > s > t^*$, or, more simply, for all $t > t^* + 1$.

Q.E.D.

This result is extremely ironic. The class of social welfare functions satisfying non-stationary dynamic consistency include those functions described above as "most natural" for a representative government within an overlapping generations framework. Yet non-stationary dynamic consistency is not a relevant concept if one conceives of the planner as a representative government. This concept is appropriate when the planner is a social architect, but in that case, the class of social welfare functions described in Theorem 3 are extremely unnatural.

I conclude that, in the presence of intergenerational altruism, social dynamic inconsistency is a serious problem regardless of how one conceives of the planner.

4. A Partial Resolution

Let us reflect for a moment on the plight of the social architect. In his case, dynamic inconsistency arises not from the fact that relative welfare weights change over time, but rather from an inability to commit in advance to respect the preferences of each generation after its demise. In practice, societies may well develop techniques based upon both laws and customs which allow the planner at each point in time to restrict his future choices (i.e. through the adoption of a constitution), and which may therefore alleviate problems stemming from dynamic inconsistency. I shall not investigate here the nature or efficacy of such techniques. For the purposes of this study, I simply assume that considerations of this sort allow the planner to resolve intertemporal conflicts without loss of efficiency. Accordingly, I look at first best welfare optima, and compare them with dynastic equilibria.

Yet this discussion suggests that the comparison of equilibria and optima should be conducted a bit differently than in the preceding section. Specifically, if the planner continues to respect the preferences of each generation after its demise, then, in period 0, it should also recall and respect the preferences of prior generations. As we shall see, allowing for memory of this sort partially resolves the problem raised by Theorem 1 (i.e. the divergence of equilibria and optima).

Throughout this discussion, we will assume that welfare weights are geometric:

$$\rho_t = \rho^t$$

for some $\rho \in (0,1)$, and for all t. Further, we will assume that the planner honors the preferences of r past generations, so that the social welfare function is given by

$$\Psi^r = \sum_{t=-r}^{\infty} \rho^t u_t \quad ,$$

where

$$u_t = \sum_{\tau=\max(t,0)}^{\infty} \delta^{\tau-t} V(C_\tau)$$

(note that we are only concerned with the consumptions of generations $t = 0,1,\ldots,$ even though the preferences of prior generations come into play). As in section 3, we express social welfare directly as a function of consumptions:

$$\begin{aligned}
\Psi^r(C) &= \sum_{t=-r}^{\infty} \rho^t \sum_{\tau=\max(t,0)}^{\infty} \delta^{\tau-t} V(C_t) \\
&= \sum_{\tau=0}^{\infty} \sum_{t=-r}^{\tau} \rho^t \delta^{\tau-t} V(C_t) \qquad (20) \\
&= (\delta/\rho)^r \sum_{\tau=0}^{\infty} \sum_{t=0}^{\tau+r} \rho^t \delta^{\tau-t} V(C_t)
\end{aligned}$$

I will assume that we are interested in an economy that has been operating for quite a long time, so that r is large. Formally, I will let r go to infinity, and investigate the limiting properties of Ψ^r. In some cases, the welfare function may blow up in the limit. Since multiplication by a scalar does not change the ordinal properties of Ψ^r, it is appropriate in such cases to select a sequence of scalars, $\langle\lambda_r\rangle_{r=0}^{\infty}$, such that $\lambda_r \Psi^r(C)$ converges on S.

I now turn to the results.

<u>Theorem 4</u>: <u>Suppose</u> $\delta = \rho$. <u>Then</u> $r^{-1}\Psi^r(C)$ <u>converges uniformly on</u> S <u>to</u> $\sum_{t=0}^{\infty} \delta^t V_t(C)$.

<u>Proof</u>: Using (20), we see that

$$r^{-1}\Psi^r(C) = r^{-1} \sum_{\tau=0}^{\infty} (r + \tau + 1)\delta^\tau V(C_\tau)$$

$$= \sum_{\tau=0}^{\infty} \delta^\tau V(C_\tau) + r^{-1} \sum_{\tau=0}^{\infty} (\tau + 1)\delta^\tau V(C_\tau)$$

Under assumptions 1 and 2, there exists a finite B such that for all $C \varepsilon S$,

$$-B < \sum_{\tau=0}^{\infty} (\tau + 1)\, \delta^\tau V(C_\tau) < B$$

Choose some $\varepsilon > 0$. Let $R = B/\varepsilon$. For all $r > R$, we have

$$|r^{-1}\Psi^r(C) - \sum_{\tau=0}^{\infty} \delta^\tau V(C_\tau)| < \varepsilon$$

for all $C \varepsilon S$. Uniform convergence is thereby established. Q.E.D.

The interpretation of Theorem 4 is as follows. If private individuals discount <u>felicity</u> at the rate δ, and if the planner discounts <u>utility</u> at the rate δ, then, as r becomes large, the planner discounts <u>felicity</u> at the rate δ. This result is somewhat surprising since, for $r = 0$, the planner discounts felicity at a very different rate. Intuitively, it holds for the following reason. In the limit, $\Psi^r(C)$ fails to converge, so asymptotically $r^{-1}\Psi^r(C)$ places all of the weight on the preferences of deceased generations. Yet each generation $r < 0$ has the same preferences, $\sum_{t=0}^{\infty} \delta^t V_t(C_t)$, regarding $\langle C_t \rangle_{t=0}^{\infty}$.

Theorem 5: Suppose $\rho > \delta$. *then* $(1 - \delta/\rho)\Psi^r(C)$ *converges uniformly on* S *to* $\sum_{t=0}^{\infty} \rho^t V_t(C_t)$.

Proof: Using (20), we see that

$$(1 - \delta/\rho)\Psi^r(C) = \sum_{\tau=0}^{\infty} (1 - \delta/\rho)\Big[\sum_{t=0}^{\tau+r} (\delta/\rho)^{\tau+r-t}\Big]\rho^\tau V(C_\tau)$$

$$= \sum_{\tau=0}^{\infty} (1 - \delta/\rho)\Big[\sum_{t=0}^{\tau+r} (\delta/\rho)^{t}\Big]\rho^\tau V(C_\tau)$$

$$= \sum_{\tau=0}^{\infty} \big[1 - (\delta/\rho)^{\tau+r}\big]\rho^\tau V(C_\tau)$$

$$= \sum_{\tau=0}^{\infty} \rho^\tau V(C_\tau) + (\delta/\rho)^r \sum_{\tau=0}^{\infty} \delta^\tau V(C_\tau)$$

Under assumptions 1 and 2, there exists a finite B such that

$$-B < \sum_{\tau=0}^{\infty} \delta^\tau V(C_\tau) < B \quad .$$

Choose some $\varepsilon > 0$. Let $R = \ell n(B/\varepsilon)/\ell n(\delta/\rho)$. For all $r > R$, we have

$$\Big|(1 - \delta/\rho)\Psi^r - \sum_{t=0}^{\infty} \rho^t V(C_t)\Big| < \varepsilon \quad .$$

Uniform convergence is thereby established. Q.E.D.

Theorem 5 tells us that if individuals discount *felicity* at the rate δ, the planner discounts *utility* at the rate ρ, and $\rho > \delta$, then, as r becomes large, the planner discounts *felicity* at the rate ρ. This result is surprising for two reasons. First, ρ is defined as a discount factor for utilities, and not felicities. Second, in the limit the private discount factor δ completely disappears from the planner's objective function. Thus, the presence, absence, or degree of private altruism is completely irrelevant to the planner, as long as altruism is not too strong (i.e. $\delta > \rho$). Since Ψ^r does converge in this case, the intuition provided following Theorem 4 does not apply.

Theorem 6: Suppose $\rho < \delta$. *Then* $(1 - \rho/\delta)(\rho/\delta)^r\Psi^r$ *converges uniformly on* S *to* $\sum_{t=0}^{\infty} \delta^t V_t(C_t)$.

Proof: Using (20), we see that

$$(1 - \rho/\delta)(\rho/\delta)^r\Psi^r = \sum_{r=0}^{\infty} (1 - \rho/\delta)[\sum_{t=0}^{\tau+r} (\rho/\delta)^t]\delta^\tau V(C_\tau)$$

$$= \sum_{\tau=0}^{\infty} [1 - (\rho/\delta)^{\tau+r}]\delta^\tau V(C_\tau)$$

$$= \sum_{\tau=0}^{\infty} \delta^\tau V(C_\tau) - (\rho/\delta)^r \sum_{r=0}^{\infty} \rho^\tau V(C_\tau)$$

Under assumptions 1 and 2, there exists a finite B such that

$$-B < \sum_{\tau=0}^{\infty} \rho^\tau V(C_\tau) < B$$

Choose some $\varepsilon > 0$. Let $R = \ln(B/\varepsilon)/\ln(\delta/\rho)$. For all $r > R$. we have

$$|(1 - \rho/\delta)(\rho/\delta)^r \Psi^r(C) - \sum_{t=0}^{\infty} \delta^t V(C_t)| < \varepsilon$$

for all $C \in S$. Uniform convergence is thereby established. Q.E.D.

Theorem 6 tells us that if individuals discount felicity at the rate δ, the planner discounts utility at the rate ρ, and $\delta > \rho$, then, as r becomes large, the planner discounts felicity at the rate δ. This result is surprising since, in the limit, the planner ignores his own discount factor (ρ) entirely. Indeed, his preferences coincide in the limit with those of generation 0 (the intuition is the same as for Theorem 4). This suggests that dynastic equilibria are indeed optimal when $\delta > \rho$.

To establish that this conclusion is in fact warranted, I argue as follows. Let $\hat{C}^\delta$ denote the solution to the "dynastic" optimization problem (i.e. equation (4)), and let $\hat{C}^\rho$ denote the solution to the same problem when ρ is substituted for δ. Finally, let $\hat{C}^r$ denote the solution to maximizing Ψ^r (given compactness of S, this is well-defined under assumptions 1 and 3). Noting that S is compact and that convergence of $\lambda_r \Psi^r$ is always uniform over S, we have

Corollary: If $\delta \geqq \rho$, then $\lim_{r\to\infty} \hat{C}^r = \hat{C}^\delta$. If $\rho \geqq \delta$, then $\lim_{r\to\infty} \hat{C}^r = \hat{C}^\rho$.

The striking aspect of this result is that, in the limit, either ρ or δ matters, but not both. In particular, supposing that $\delta \geqq \rho$ and taking r large implies that the dynastic equilibrium is arbitrarily close to a welfare optimum.

From the point of view of a social architect, the optimality of dynastic equilibria therefore depends upon a comparison of ρ and δ. It is important to acknowledge that there is no a priori reason for expecting that the inequality will go one way or the other. The case of $\rho = \delta$ has no particular significance, since ρ measures relative weights on utilities, while δ measures relative weights on felicities. One cannot, for example, appeal to the literature on the social rate of discount, arguing that, since the government ought to be more "forward looking" than the private sector, $\rho > \delta$. In fact, $\rho > 0$ is alone sufficient to guarantee that individuals are impatient relative to the planner.

However, this analysis does provide us with a useful framework for thinking about the economic importance of intergenerational altruism. As long as altruism is not too strong, it has absolutely no effect on the preferences of a social architect, and optimal policy will in general drive private transfers to zero. On the other hand, if altruism is sufficiently strong, the social architect's preferences coincide with those of the oldest generation in every period. A dynastic equilibrium with operative linkages then generates a first-best optimum.

Footnotes

1/ Technically, the planner could also achieve an optimum through Pigouvian subsidies, in which case he would not need to drive successive generations to coerners. In practice, it would be extremely difficult to implement such a policy, in that the planner would need to fine tune the subsidy separately for the degree of altruism in each family (indeed, it may not possess the necessary information). Note in addition that a policy of subsidization would also remove us from the traditional dynastic framework.

2/ One could easily allow β to depend upon the current capital stock, as in standard models. However, this would add an additional layer of obscurity to the analysis without changing anything of substance.

3/ Theorem 1 only requires that ψ be stirctly increasing in certain arguments.

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