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COLLECTIVE CHOICE IN DYNAMIC PUBLIC GOOD PROVISION

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ABSTRACT

Two heterogeneous agents contribute over time to a joint project, and collectively decide its scope. A larger scope requires greater cumulative effort and delivers higher benefits upon completion. We show that the efficient agent prefers a smaller scope, and preferences are time-inconsistent: as the project progresses, the efficient (inefficient) agent's preferred scope shrinks (expands). We characterize the equilibrium outcomes under dictatorship and unanimity, with and without commitment. We find that an agent's degree of efficiency is a key determinant of control over project scopes. From a welfare perspective, it may be desirable to allocate decision rights to the inefficient agent.

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1 Introduction

In many economic settings, agents must collectively decide the goal or *scope* of a joint project. A greater scope reflects a more ambitious project, which requires more cumulative effort from the agents but yields a larger reward upon completion. Such collective decisions are common among countries who collaborate on a project. For example, the International Space Station (ISS) was a collaboration among the United States, Russia, the European Union, Japan, and Canada that cost approximately $150 billon. The Asian Highway Network, running about eighty-seven thousand miles and costing over $25 billion, is a collaboration among thirty-two Asian countries, the United Nations, and other entities to facilitate greater trade throughout the region. In both examples, the projects took several decades to implement, and an agreement that determined the project scope was signed by all stakeholders.[1]

If the agents' preferences over the project scope are aligned, it is natural for the agents to agree on the ideal scope, and there will be little debate. Yet, it is common to find disagreement about when and at what stage to complete a project. For example, the process of identifying roads to be included in the Asian Highway Network began in the late 1950s, and it was not until the 1990s that the majority of the work began, owing to the endorsement of the United Nations (see Yamamoto et al. 2003). The World Trade Organization's (WTO's) Doha Round began in 2001 and has (infamously) yet to be concluded fifteen years later. The delay is due, in part, to differences between member countries over which industries the agreement should cover and to what extent (see Bhagwati & Sutherland 2011). Central to many of these conflicts is the asymmetry between participants—often large contributors versus small contributors. In this paper, we investigate how the agents' cost of effort and their stake in the project affect their incentives to contribute and their preferences over the project scope, and, ultimately, how these parameters impact their influence on the project scope under different decision protocols.

We focus on projects with three key features. First, progress on the project is gradual, and hence the problem is dynamic in nature. Second, the project generates a payoff only upon completion, and each agent receives a fixed fraction of that payoff. Third, the scope of the project is endogenous, and the project's payoff increases with its scope. Thus, the scope of the project is a crucial determinant of not only the magnitude of the payoffs and effort, but also their timing.

Several other examples fit our framework. For instance, many new business ventures require costly effort before payoffs can be realized. Academics working on a joint research project exert effort over time, and the reward is largely realized after submission and publication of the findings. In both cases, agents must agree at some point in time on the

[1] Other notable multi-country collaborations include the International Thermonuclear Experimental Reactor (ITER) under construction in France, and the Joint European Torus (JET) in the United Kingdom.

scope of the project. Does the venture seek a blockbuster product or something that may have a quicker (if smaller) payoff? Do the coauthors target highly regarded general interest journals or work towards a more specific field journal? Indeed, there is often dissent on when a joint project is ready to be monetized through the launch of the product, sale of the company, or submission of the article.

A key ingredient of our framework is the agents' ability to decide on the project scope. The decision is collective and can be made at any time, with or without the ability to commit. As an example, it is common for the scope of a public infrastructure project to change throughout its development, a phenomenon often referred to as "scope creep." In such cases, the parties cannot commit to not renegotiate. In other cases, such as with an entrepreneurial venture, legally binding contracts can be enforced. An agreement can then be made at any time during the project, and parties can commit to it, preventing subsequent renegotiation. The ability to commit is considered a part of the economic environment and is not a choice of the agents. The actual choice of the project scope is determined by a protocol we refer to as *collective choice institution*. In the examples of the ISS and the Asian Highway Network, each country must sign a formal agreement for the project to enter into force (see Yakovenko 1999, Yamamoto et al. 2003). In these examples, the scope of the project cannot be decided without the consent of all parties: the collective choice institution is unanimity. An agreement may also designate a single party with the right to complete the project, such as when one party has a controlling share of an entrepreneurial joint venture. In this paper, we focus on these two common instances of collective choice institution: dictatorship and unanimity.

Our modeling approach is based on the dynamic public good provision framework of Marx & Matthews (2000). It is well established in this setting that free-riding occurs when agents make voluntary contributions to a public good, and basic comparative statics (e.g., the effect of changes in effort costs, discount rates, scope, etc.) are well understood when agents are symmetric. However, little is known about this problem when agents are heterogeneous. We begin by studying a simple two-agent model. The agent with the lower effort cost per unit of benefit is referred to as the *efficient* agent, and the agent with the higher effort cost per unit of benefit is referred to as the *inefficient* agent. The solution concept we use is Markov perfect equilibrium (hereafter MPE), as is standard in this literature. When multiple equilibria exist, we refine the set of equilibria to the Pareto-dominant ones.

To lay the foundations for the collective choice analysis, in Section 3, we consider the setting in which the project scope is exogenously fixed. We show that the efficient agent exerts more effort than the inefficient agent at every stage of the project and, moreover, obtains a lower discounted payoff (normalized by his project stake). It is known in this setting that the agents' efforts increase as the project nears completion (see Cvitanić &

Georgiadis (2016)), and we further show that the efficient agent's effort level increases at a faster rate than that of the inefficient agent. We use these results to derive the agents' preferences over the project scope. A lower normalized payoff for the efficient agent means that at every stage of the project, he prefers a smaller project scope than does the inefficient agent. Furthermore, we show that the scope of the project that the efficient agent prefers *decreases* as the project progresses, while the opposite is true for the inefficient agent. This is because the efficient agent's share of the remaining project cost is not only higher than the inefficient agent's, but also *increases* as the project progresses. The agents' preferences over the project scope are thus time-inconsistent and divergent.

In Section 4, we endogenize the project scope and analyze the equilibrium outcomes under dictatorship and unanimity, with and without commitment.[2] With commitment, we show that the project scope is decided at the start of the project in equilibrium under any institution. When either agent is dictator, he chooses his ex-ante ideal project scope, whereas under unanimity, the project scope lies between the agents' ex-ante ideal scopes. Without commitment, if the efficient agent is the dictator, then he completes the project at his ideal scope. However, if the inefficient agent is dictator, then there exists a continuum of equilibria on the Pareto frontier, which lie between the agents' ideal scopes. That is because the project scope that is implemented in equilibrium depends on when inefficient agent expects the other agent to stop working (at which moment the former optimally completes the project immediately). Lastly, because the inefficient agent always prefers a larger project scope than the efficient agent, without commitment, the set of equilibria under unanimity are the same as when the inefficient agent is dictator.

While institutions can enforce authority of an agent, the scope that is implemented remains an equilibrium outcome. That is, even if an agent has dictatorship rights, he has to account for the other agent's actions when deciding the project scope. Akin to Aghion & Tirole (1997), we say that an agent has real authority if his preferences are implemented in equilibrium. With commitment, whichever agent has formal authority (i.e., the dictator), also has real authority. However, this is not the case without commitment. In particular, if the efficient agent is dictator, then he also has real authority. In contrast, if the inefficient agent is dictator, then he has real authority while the project is in progress, but it eventually "runs out", and at the completion state, it is the efficient agent who has real authority. Therefore, real authority changes hands as the project progresses.

From a social welfare perspective, with or without commitment, when the efficient agent is the dictator, the equilibrium project scope is too small relative to the social planner's. The reason is that he retains full control of the scope and his ideal project scope does not

[2]The former refers to the case in which the agents can commit to a decision about the project scope at any time. The latter refers to the case in which the agents cannot commit to an ex-ante decision. Therefore, at every moment, they can either decide to complete the project immediately, or continue working.

internalize the inefficient agent's higher dynamic payoff. Therefore, it may be desirable to confer some formal authority to the inefficient agent (via dictatorship or unanimity) as a means to counter the real authority that the efficient agent obtains in equilibrium.[3]

To test the robustness of our results, we consider two extensions of the model in Section 5. If transfers are allowed, then the social planner's project scope can be implemented in equilibrium under all institutions. When the agents can choose the stakes (or shares) of the project ex-ante, simulations show that the efficient agent is always allocated a higher share than the inefficient agent. With the efficient agent as dictator, the share awarded to him is naturally the largest. We also consider the case in which the project progresses stochastically, and using simulations, we illustrate that the main results continue hold.[4] Finally, we discuss the case in which the group comprises of more than two agents in Section 6.

Related Literature

Our model draws from the literature on the dynamic provision of public goods, including classic contributions by Levhari & Mirman (1980) and Fershtman & Nitzan (1991). Similarly to our approach, Admati & Perry (1991), Marx & Matthews (2000), Compte & Jehiel (2004), Yildirim (2006), and Georgiadis (2016) consider the case of public good provision when the benefit is received predominantly upon completion. Bonatti & Rantakari (2015) consider collective choice in a public good game, where each agent exerts effort on an independent project, and the collective choice is made to adopt one of the projects at completion. Battaglini et al. (2014) study a public good provision game without a terminal date, in which each agent receives a flow benefit that depends on the stock of the public good, in contrast to our setting. We contribute to this literature by endogenizing the provision point of the public good, and studying how different collective choice institutions influence the project scope that is implemented in equilibrium.

This paper joins a large political economy literature studying collective decision-making when the agents' preferences are heterogeneous, including the seminal work of Romer & Rosenthal (1979). More recently, this literature has turned its attention to the dynamics of collective decision making, including papers by Baron (1996), Dixit et al. (2000), Battaglini & Coate (2008), Strulovici (2010), Diermeier & Fong (2011), Besley & Persson (2011) and Bowen et al. (2014). Other papers, for example, Lizzeri & Persico (2001), have looked at alternative collective choice institutions. To the best of our knowledge, this is the first paper

[3]This resonates with Galbraith (1952), who argues that when one party is strong and the other weak, it is preferable to give formal authority to the latter.

[4]The models with uncertainty and endogenous choice of project shares in the voluntary contribution game with heterogeneous agents that we study is analytically intractable, so we examine them numerically. All other results are obtained analytically.

to study collective decision-making in the context of a group of agents collaborating to complete a project.

The application to public projects without the ability to commit relates to a large number of articles studying international agreements. Several of these study environmental agreements (for example, Nordhaus 2015, Battaglini & Harstad forthcoming) and trade agreements (see Maggi 2014).[5] To our knowledge, this literature has not examined the dynamic selection of project scope (or goals) in these agreements with asymmetric agents or identified the source of authority. Our theory sheds light on the dominance of large countries in many trade and environmental agreements in spite of unanimity being the formal institution.

Finally, our interest in real and formal authority relates to a literature studying the source of authority and power, including the influential work of Aghion & Tirole (1997) and more recent contributions by Callander (2008), Levy (2014), Callander & Harstad (2015), Hirsch & Shotts (2015), and Akerlof (2015). Unlike this paper, these authors focus on the role of information in determining real authority. Bester & Krähmer (2008) and Georgiadis et al. (2014) consider a principal-agent setting in which the principal has formal authority to choose which project to implement, but that choice is restricted by the agent's effort incentives; or she can delegate the project choice decision to the agent. Acemoglu & Robinson (2008) consider the distinction of *de jure* and *de facto* political power, which are the analogs of formal and real authority, but the source of the latter is attributed to various forces outside the model. In contrast, we are able to endogenously attribute the source of real authority under different collective choice institutions to the agents' effort costs and stake in our simpler setting of a public project.

2 Model

We present a stylized model of two heterogeneous agents $i \in \{1, 2\}$ deciding the scope of a public project $Q \geq 0$.[6] Time is continuous and indexed by $t \in [0, \infty)$. A project of scope Q requires voluntary effort from the agents over time to be completed. Let $a_{it} \geq 0$ be agent i's instantaneous effort level at time t, which induces flow cost $c_i(a_{it}) = \gamma_i a_{it}^2/2$ for some $\gamma_i > 0$. Agents are risk-neutral and discount time at common rate $r > 0$.

We denote the cumulative effort (or progress on the project) up to time t by q_t, which we call the project state. The project starts at initial state $q_0 = 0$ and progresses according to

$$dq_t = (a_{1t} + a_{2t})\, dt \,.$$

[5] Bagwell & Staiger (2002) discuss the economics of trade agreements in depth. Others look at various aspects of specific trade agreements, such as flexibility or forbearance in a non-binding agreement, (see, for example, Beshkar & Bond 2010, Bowen 2013).

[6] We discuss the case of $n \geq 3$ agents in Section 6.

It is completed at the moment that the state reaches the chosen scope Q.[7] The project yields no payoff while it is in progress, but upon completion, it yields a payoff $\alpha_i Q$ to agent i, where $\alpha_i \in \mathbb{R}_+$ is agent i's stake in the project.[8] Agent i's project stake therefore captures all of the expected benefit from the project.[9]All information is common knowledge.

Therefore, given an arbitrary set of effort paths $\{a_{1s}, a_{2s}\}_{s \geq t}$ and project scope Q, agent i's discounted payoff at time t satisfies

$$J_{it} = e^{-r(\tau - t)}\alpha_i Q - \int_t^\tau e^{-r(s-t)}\frac{\gamma_i}{2}a_{is}^2 ds\,,$$

where τ denotes the equilibrium completion time of the project (and $\tau = \infty$ if the project is never completed).

By convention, we assume that the agents are ordered such that $\frac{\gamma_1}{\alpha_1} \leq \frac{\gamma_2}{\alpha_2}$. Intuitively, this means that agent 1 is *relatively more efficient* than agent 2, in that his marginal cost of effort relative to his stake in the project is smaller than that of agent 2. In sequel, we say that agent 1 is *efficient* and agent 2 is *inefficient*.

The project scope Q is decided by collective choice at any time $t \geq 0$, i.e., at the start of the project, or after some progress has been made. The set of decisions available to each agent depends on the collective choice institution, which is either dictatorship or unanimity. To lay the foundations for the collective choice analysis, we shall assume that the project scope Q is fixed in the next section. When we consider the collective choice problem in Section 4, we will enrich the model by introducing additional notation as necessary.

3 Analysis with fixed project scope Q

In this section, we lay the foundations for the collective choice analysis. We begin by considering the case in which the project scope Q is specified exogenously at the outset of the game and characterize the (essentially unique) stationary Markov Perfect equilibrium (MPE) of this game.[10] We then derive each agent's preferences over the project scope Q given the MPE payoffs induced by a choice of Q. Finally, we characterize the social planner's benchmark. In Section 4, we consider the case in which the agents decide the project scope via collective choice.

[7]We make the simplifying assumption that the project state progresses deterministically. See Section 5.2 for an extension in which the state progresses stochastically.

[8]Without loss of generality, one can assume that upon completion, the project yields a stochastic payoff to agent i that has expected value $\alpha_i Q$.

[9]The sum $\alpha_1 + \alpha_2$ reflects the *publicness* of the project, and if $\alpha_1 + \alpha_2 = 1$, then the project stake can be interpreted as the project share. We assume that these stakes are exogenously fixed. In Section 5.1, we extend our model to allow the agents to use transfers to re-allocate shares.

[10]As is standard in this literature, we focus on MPE. These equilibria require minimal coordination between the agents, and in this sense they are simple. The simplicity of MPE make them naturally focal in the collective choice setting.

3.1 Markov perfect equilibrium with exogenous project scope

In a MPE, at every moment, each agent chooses his effort level as a function of the current project state q to maximize his discounted payoff while anticipating the other agents' effort choices. Let us denote each agent i's discounted continuation payoff and effort level when the project state is q by $J_i(q)$ and $a_i(q)$, respectively. Using standard arguments (for example, Kamien & Schwartz 2012) and assuming that $\{J_1(\cdot), J_2(\cdot)\}$ are continuously differentiable, it follows that they satisfy the Hamilton-Jacobi-Bellman (hereafter HJB) equation

$$rJ_i(q) = \max_{\widehat{a}_i \geq 0} \left\{ -\frac{\gamma_i}{2}\widehat{a}_i^2 + (\widehat{a}_i + a_j(q))\, J_i'(q) \right\}, \tag{1}$$

subject to the boundary condition

$$J_i(Q) = \alpha_i Q, \tag{2}$$

where $a_j(\cdot)$ is agent i's conjecture for the effort chosen by agent $j \neq i$. We refer to such MPEs as *well-behaved.*

The right side of (1) is maximized when $\widehat{a}_i = \max\{0, J_i'(q)/\gamma_i\}$. Intuitively, at every moment, each agent either does not put in any effort, or he chooses his effort level such that the marginal cost of effort is equal to the marginal benefit associated with bringing the project closer to completion. In any equilibrium we have $J_i'(q) \geq 0$ for all i and q, that is, each agent is better off the closer the project is to completion.[11] By substituting each agent's first-order condition into (1), it follows that each agent i's discounted payoff function satisfies

$$rJ_i(q) = \frac{[J_i'(q)]^2}{2\gamma_i} + \frac{1}{\gamma_j} J_i'(q)\, J_j'(q), \tag{3}$$

subject to the boundary condition (2), where j denotes the agent other than i. By noting that each agent's problem is concave, and thus the first-order condition is necessary and sufficient for a maximum, it follows that every well-defined MPE is characterized by the system of ordinary differential equations (ODEs) defined by (3) subject to (2).[12] The following Proposition, which builds upon Cvitanić & Georgiadis (2016) characterizes the MPE.

Proposition 1. *For any project scope Q, there exists a unique well-behaved MPE. Moreover for any project scope Q, exactly one of two cases can occur.*

1. *The MPE is project-completing: both agents exert effort at all states and the project is completed. Then, $J_i(q) > 0$, $J_i'(q) > 0$, and $a_i'(q) > 0$ for all i and $q \geq 0$.*

[11] See the proof of Proposition 1.

[12] This system of ODEs can be normalized by letting $\widetilde{J}_i(q) = \frac{J_i(q)}{\gamma_i}$. This becomes strategically equivalent to a game in which $\gamma_1 = \gamma_2 = 1$, and agent i receives $\frac{\alpha_i}{\gamma_i}Q$ upon completion of the project.

2. *The MPE is not project-completing: agents do not ever exert any effort, and the project is not completed.*

If Q is sufficiently small, then case (1) applies, while otherwise, case (2) applies.

All proofs are provided in Appendix A.

Proposition 1 characterizes the unique MPE for any given project scope Q. In any project-completing MPE, payoffs and efforts are strictly positive, and each agent increases his effort as the project progresses towards completion, i.e., $a_i'(q) > 0$ for all i and q. Because the agents discount time and they are rewarded only upon completion, their incentives are stronger the closer the project is to completion.

If the agents are symmetric (i.e., if $\frac{\gamma_1}{\alpha_1} = \frac{\gamma_2}{\alpha_2}$), then in the unique project-completing MPE, each agent i's discounted payoff and effort function can be characterized analytically as follows:

$$J_i(q) = \frac{r\gamma_i (q-C)^2}{6} \quad \text{and} \quad a_i(q) = \frac{r(q-C)}{3}, \tag{4}$$

where $C = Q - \sqrt{\frac{6\alpha_i Q}{r\gamma_i}}$ (see Georgiadis et al. (2014)). A project-completing MPE exists if $C < 0$. While the solution to the system of ODEs given by (3) subject to (2) can be found with relative ease in the case of symmetric agents, no closed-form solution can be obtained for the case of asymmetric agents. Nonetheless, we are able to derive important properties of the solution, which will be useful for understanding the intuition behind the results in Section 3.2. The following proposition compares the equilibrium effort levels and payoffs of the two agents.

Proposition 2. *Suppose that $\frac{\gamma_1}{\alpha_1} < \frac{\gamma_2}{\alpha_2}$. In any project-completing MPE:*

1. *Agent 1 exerts higher effort than agent 2 in every state, and agent 1's effort increases at a greater rate than agent 2's. That is, $a_1(q) \geq a_2(q)$ and $a_1'(q) \geq a_2'(q)$ for all $q \geq 0$.*

2. *Agent 1 obtains a lower discounted payoff normalized by project stake than agent 2. That is, $\frac{J_1(q)}{\alpha_1} \leq \frac{J_2(q)}{\alpha_2}$ for all $q \geq 0$.*

Suppose instead that $\frac{\gamma_1}{\alpha_1} = \frac{\gamma_2}{\alpha_2}$. In any project-completing MPE, $a_1(q) = a_2(q)$ and $\frac{J_1(q)}{\alpha_1} = \frac{J_2(q)}{\alpha_2}$ for all $q \geq 0$.

It is intuitive that the more efficient agent always exerts higher effort than the less efficient agent, as well as that the more efficient agent raises his effort at a faster rate than the less efficient agent. Notice that each agent i's effort incentives are proportional to his normalized gross payoff $e^{-r(\tau-t)}\frac{\alpha_i}{\gamma_i}Q$. Therefore, as the project progresses (i.e., as $\tau - t$

decreases), the incentives of the efficient agent grow at a faster rate than the incentives of the inefficient agent.

What is perhaps surprising is that the more efficient agent obtains a lower discounted payoff (normalized by his stake) than the other agent. This is because the more efficient agent not only works harder than the other agent, but he also incurs a higher total discounted cost of effort (normalized by his stake). To examine the robustness of this result, in Appendix B.1, we consider a larger class of effort cost functions, and we show that this result holds as long as each agent's effort cost is weakly log-concave in the effort level; i.e., $\{c_1(\cdot), c_2(\cdot)\}$ are not "super-convex".

3.2 Preferences over project scope

In this section, we characterize each agent's optimal project scope without institutional restrictions. That is, we determine the Q that maximizes each agent's discounted payoff given the current state q and assuming that both agents follow the MPE characterized in Proposition 1 for the project scope Q. Notice that the agents will choose a project scope such that the project is completed in equilibrium.

Agents working jointly

To make the dependence on the project scope explicit, we let $J_i(q;Q)$ denote agent i's payoff at project state q when the project scope is Q. Let $Q_i(q)$ denote agent i's ideal project scope when the state of the project is q. That is,

$$Q_i\left(q\right) = \arg\max_{Q \geq q} \left\{ J_i\left(q;Q\right) \right\}.$$

For each agent i there exists a unique state q, denoted by $\overline{Q}_i$ such that he is indifferent between terminating the project immediately or an instant later, and $\overline{Q}_2 \geq \overline{Q}_1$.[13] Throughout the remainder of this paper, we shall assume that the parameters of the problem are such that $Q \mapsto J_i(q;Q)$ is strictly concave on $[q, \overline{Q}_2]$.[14] Observe that the strict concavity assumption implies that $J_i(0,Q) > 0$ for all i and $Q \in (0, \overline{Q}_2)$, so the corresponding MPE is project-completing.

The following proposition establishes properties of each agent's ideal project scope.

Proposition 3. *Consider agent i's optimal project scope Q when both agents choose their effort strategies based on Q.*

[13] The value of $\overline{Q}_i$ is provided in Lemma 7 in the proof of Proposition 3.

[14] This condition is satisfied in the symmetric case $\frac{\gamma_1}{\alpha_1} = \frac{\gamma_2}{\alpha_2}$ (see Georgiadis et al. (2014) for details) and, by a continuity argument, it is also satisfied for neighboring, asymmetric parameter values. While we do not make a formal claim regarding the set of parameters values for which the condition is satisfied, numerical simulations suggest that this condition holds generically.

1. *If the agents are symmetric (i.e., $\frac{\gamma_1}{\alpha_1} = \frac{\gamma_2}{\alpha_2}$), then for all states q, their ideal project scopes are equal and given by $Q_1(q) = Q_2(q) = \frac{3\alpha_i}{2\gamma_i r}$.*

2. *If the agents are asymmetric (i.e., $\frac{\gamma_1}{\alpha_1} < \frac{\gamma_2}{\alpha_2}$), then:*

   (a) *The efficient agent prefers a strictly smaller project scope than the inefficient agent at all states up to $\overline{Q}_2$, i.e., $Q_1(q) < Q_2(q)$ for all $\overline{Q}_2$.*

   (b) *The efficient agent's ideal scope is strictly decreasing in the project state up to $\overline{Q}_1$, while the inefficient agent's scope is strictly increasing for all q, i.e., $Q_1'(q) < 0$ for all $q < \overline{Q}_1$ and $Q_2'(q) > 0$ for all q.*

   (c) *Agent i's ideal is to complete the project immediately at all states greater than $\overline{Q}_i$, i.e., $Q_i(q) = q$ for all $q \geq \overline{Q}_i$.*

Part 1 asserts that when the agents are symmetric, they have identical preferences over project scope, and these preferences are time-consistent.

Part 2 characterizes each agent's ideal project scope when the agents are asymmetric, and is illustrated in Figure 1. Part 2 (a) asserts that the more efficient agent always prefers a strictly smaller project scope than the less efficient agent for $q < \overline{Q}_2$.[15] Note that each agent trades off the bigger gross payoff from a project with a larger scope and the cost associated with having to exert more effort and wait longer until the project is completed. Moreover, agent 1 not only always works harder than agent 2, but at every moment, his discounted total cost remaining to complete the project normalized by his stake (along the equilibrium path) is larger than that of agent 2.[16] Therefore, it is intuitive that agent 1 prefers a smaller project scope than agent 2.

Part 2 (b) shows that both agents are time-inconsistent with respect to their preferred project scope: as the project progresses, agent 1's optimal project scope becomes smaller, whereas agent 2 would like to choose an ever larger project scope. To see the intuition behind this result, recall that $a_1'(q) \geq a_2'(q) > 0$ for all q; that is, both agents increase their effort with progress, but the rate of increase is greater for agent 1 than it is for agent 2. This implies that for a given project scope, the closer the project is to completion, the larger is the share of the remaining effort carried out by agent 1. Therefore, agent 1's optimal project scope decreases. The converse holds for agent 2, and as a result, his preferred project scope becomes larger as the project progresses.

Recall that $\overline{Q}_i$ is the project scope such that agent i is indifferent between stopping immediately (when $q = \overline{Q}_i$) and continuing one instant longer. This is the value of the state

[15] The agents' ideal project scopes are equal for $q \geq \overline{Q}_2$ by Proposition 3.2 part (c).

[16] Formally and as implied by Proposition 2.2, for every $t \in [0, \tau)$, we have $\frac{\gamma_1}{\alpha_1} \int_t^\tau e^{-rt} \frac{a_1^2(q_t;Q)}{2} dt > \frac{\gamma_2}{\alpha_2} \int_t^\tau e^{-rt} \frac{a_2^2(q_t;Q)}{2} dt$ along the equilibrium path of the project.

at which $Q_i(q)$ hits the 45° line. Part 2 (c) shows that at every state $q \geq \overline{Q}_i$, agent i prefers to stop immediately.

Agents working independently

This section characterizes each agent's optimal project scope when he works alone on the project. We use this to characterize the equilibrium with endogenous project scope in Section 4. Let $\widehat{J}_i(q, Q)$ denote agent i's discounted payoff function when he works alone on the project, the project scope is Q, and he receives $\alpha_i Q$ upon completion.[17] We define agent i's optimal project scope as

$$\widehat{Q}_i(q) = \arg\max_{Q \geq q} \left\{ \widehat{J}_i(q; Q) \right\}.$$

The following lemma characterizes $\widehat{Q}_i(q)$.

Lemma 1. *Suppose that agent i works alone and he receives $\alpha_i Q$ upon completion of a project with scope Q. Then his optimal project scope satisfies*

$$\widehat{Q}_i(q) = \frac{\alpha_i}{2r\,\gamma_i},$$

for all $q \leq \frac{\alpha_i}{2r\,\gamma_i}$, and otherwise, $\widehat{Q}_i(q) = q$. Moreover, for all q,

$$\widehat{Q}_2(q) \leq \widehat{Q}_1(q) \leq Q_1(q) \leq Q_2(q).$$

Lemma 1 asserts that if an agent works solo, then his preferences over the scope are time-consistent (as long as he does not want to stop immediately). As such, we will abuse notation and write $\widehat{Q}_i = \frac{\alpha_i}{2r\,\gamma_i}$.

Intuitively, when the agent works alone, he bears the entire cost to complete the project, in contrast to the case in which the two agents work jointly. The second part of this lemma rank-orders the agents' ideal project scopes. If an agent works in isolation, then he cannot rely on the other to carry out any part of the project, and therefore the less efficient agent prefers a smaller project scope than the more efficient one. Last, it is intuitive that the more efficient agent's ideal project scope is larger when he works with the other agent relative to when he works alone.

3.3 Social Optimum

To conclude this section, we consider a social planner choosing the project scope that maximizes the sum of the agents' discounted payoffs, conditional on the agents choosing effort strategically. For this analysis, we assume that the social planner cannot coerce the

[17] The value of $\widehat{J}_i(q; Q)$ is given in the proof of Lemma 1 in the Appendix.

agents to exert effort, but she can dictate the state at which the project is completed.[18] Let

$$Q^*(q) = \arg\max_{Q \geq q} \{J_1(q;Q) + J_2(q;Q)\}$$

denote the project scope that maximizes the agents' total discounted payoff.

Lemma 2. *The project scope that maximizes the agents' total discounted payoff satisfies* $Q^*(q) \in (Q_1(q), Q_2(q))$.

Lemma 2 shows that the social planner's optimal project scope $Q^*(q)$ lies between the agents' optimal project scopes for every state of the project. This is intuitive, since she maximizes the sum of the agents' payoffs. Note that in general, $Q^*(q)$ is dependent on q; i.e., the social planner's optimal project scope is also time-inconsistent. We illustrate Proposition 3, and Lemmas 1 and 2 in Figure 1 below.

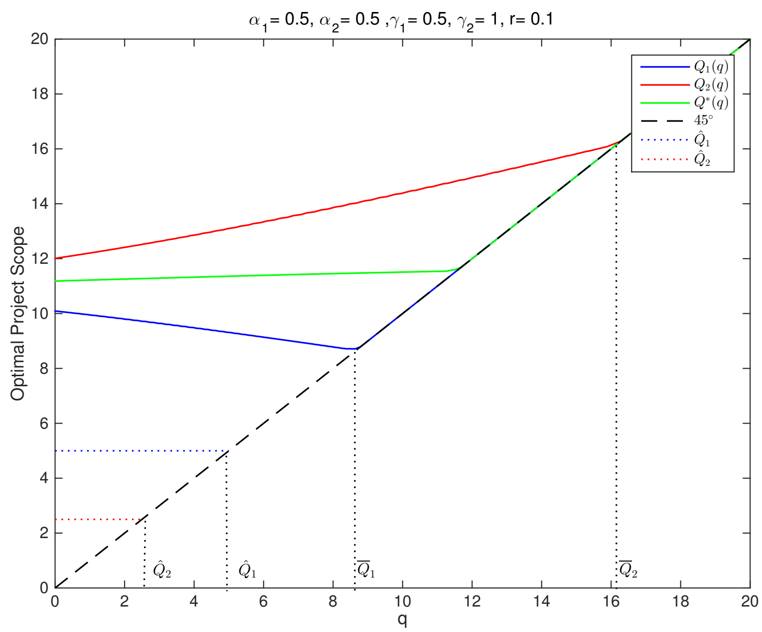


Figure 1: Agents' and social planner's ideal project scope

4 Endogenous Project Scope

We now allow agents to choose the project scope via a collective choice institution. The project scope in this section is thus *endogenous*, in contrast to the analysis in Section 3. In

[18]This implies that the social planner is unable to completely overcome the free-rider problem. We consider the benchmark in which the social planner chooses both the agents' effort levels, and the project scope in Appendix B.2. However, as it is unlikely that a social planner can coerce agents to exert a specific amount of effort, we use the result in the following lemma as the appropriate benchmark.

Section 4.1 and 4.2, we characterize the MPE under dictatorship and unanimity, respectively, while in Section 4.3 we discuss the implications for real authority and welfare. Whenever multiple equilibria exist, we shall focus on the Pareto-efficient ones (i.e., equilibrium outcomes such that in no other equilibrium outcome can a party get a strictly larger ex-ante payoff without a reduction of the other party's payoff).

4.1 Dictatorship

In this section, one of the two agents, denoted agent i, has dictatorship rights. The other agent, agent j, can contribute to the project, but has no formal authority to end it. We consider that the dictator can either commit to the project scope or not.

We enrich the baseline model of Section 2 by defining a strategy for agent i (the dictator) to be a pair of maps $\{a_i(q,Q),\theta_i(q)\}$, where $q \in \mathbb{R}_+$, $Q \in \mathbb{R}_+ \cup \{-1\}$, and $Q=-1$ denotes the case in which the project scope has not yet been decided yet.[19] The function $a_i(q,Q)$ gives the dictator's effort level in state q when project scope Q has been decided, where $Q=-1$ represents the case in which a decision about the project scope is yet to be made. The value $\theta_i(q)$ gives the dictator's choice of project scope in state q, which applies under the assumption that no project scope has been committed to before state q. We set by convention $\theta_i(q)=-1$ if the dictator does not yet wish to commit to a project scope at state q, and $\theta_i(q)\geq q$ otherwise. Similarly, a strategy for agent $j\neq i$ is a map $a_j(q,Q)$ associated with his effort level in state q and the project scope decided by the dictator Q (or $Q=-1$ if a decision has not yet been made). Notice that each agent's strategy conditions only on the payoff-relevant variables q and Q, and hence they are Markov in the sense of Maskin & Tirole (2001).

Dictatorship with Commitment

We first consider dictatorship with commitment. In this institution, the dictator can at any time announce a particular project scope, and, following this announcement, the project scope is set once and for all. Therefore, at every state q before some project scope Q has been committed to, the dictator chooses $\theta_i(q) \in \{-1\} \cup [q,\infty)$. After a project scope has been set, it is definitive, so $\theta_i(\cdot)$ becomes obsolete.

After a project scope Q has been committed to, it is completed and each agent obtains his reward as soon as the cumulative contributions reach Q. If the agents do not make sufficient contributions, then the project is never completed: both agents incur the cost of their effort, but neither gets any reward. The project cannot be completed before the dictator announces a project scope.

[19] Before the project scope has been decided, in equilibrium, the agents correctly anticipate the project scope that will be implemented, and choose their effort levels optimally.

The following proposition characterizes the equilibrium. Under commitment, each agent finds it optimal to impose his ideal project scope. The time inconsistency of the dictator's preferences implies that the scope is always chosen at the beginning of the project.

Proposition 4. *Under dictatorship with commitment, there exists a unique MPE in which agent i commits to his ex-ante ideal project scope $Q_i(0)$, and the project is completed.*

Dictatorship without Commitment

We now consider dictatorship without commitment. In this case, the dictator does not have the ability to credibly commit to a particular project scope, so at every instant, he must decide whether to complete the project immediately or continue one more instant. Formally, at every state q while the project is in progress, the dictator chooses $\theta_i(q) \in \{-1, q\}$.[20] Note that in contrast to the commitment case, the strategies no longer condition on any agreed upon project scope Q, as no agreement on the project scope is reached before the project is completed. As soon as the project is completed, both agents collect their payoffs.The following Proposition characterizes the MPE.

Proposition 5. *Under dictatorship without commitment, if agent* 1 *(i.e., the efficient agent) is the dictator, then there exists a unique MPE, and scope $Q = \overline{Q}_1$ is implemented. If agent* 2 *is the dictator, then any $Q \in [Q_1(0), Q_2(0)]$ can be part of an MPE.*

We provide a heuristic proof, which is useful for understanding the intuition for this result. First, recall from Lemma 1 that $\widehat{Q}_2 < \widehat{Q}_1 < \overline{Q}_1 < \overline{Q}_2$. Assume that agent i is dictator, fix some $Q \in (\widehat{Q}_i, \overline{Q}_i)$, and conjecture the following strategies. Agent i stops the project immediately when $q \geq Q$; i.e., he chooses $\theta_i(q) = q$ for all $q \geq Q$, and $\theta_i(q) = -1$ otherwise. For all $q < Q$, both agents exert effort according to the MPE with fixed project scope Q characterized in Proposition 1, and exert no effort thereafter. We shall argue that neither agent has an incentive to deviate, and hence these strategies constitute an MPE. Notice that the agents' efforts constitute an MPE for a fixed project scope Q, so they have no incentive to exert more or less effort at any $q < Q$. Because $Q \leq \overline{Q}_i$, agent i has no incentive to stop the project at any $q < Q$. Moreover, anticipating that he will contribute alone to the project at any $q \geq Q$, and noting that $Q \geq \hat{Q}_i$, agent i cannot benefit by completing the project at any state greater than Q.

Finally, observe that both agents' (ex-ante) payoffs increase (decrease) in the project scope for all $Q < Q_1(0)$ ($Q > Q_2(0)$). Therefore, if agent 1 is the dictator, then there

[20] Any announcement of project scope other than the current state cannot be committed to. Thus any announcement by agent i other than the current state is ignored by agent j in equilibrium. Thus, agent i's strategy collapses to an announcement to complete the project immediately, or keep working.

exists a unique Pareto-efficient MPE in which $Q = \overline{Q}_1$. If agent 2 is the dictator, then any $Q \in [Q_1(0), Q_2(0)]$ can be part of a Pareto-efficient MPE.

Notice that because the agents have conflicting preferences on $[\overline{Q}_1, \overline{Q}_2]$ (i.e., agent 1 prefers to complete the project, whereas agent 2 prefers to continue), the Pareto-efficient MPE are also renegotiation-proof.

4.2 Unanimity

In this section, we consider the case in which both agents must agree on the project scope. One of the agents, whom we denote by i, is (exogenously) chosen to be the agenda setter, and he has the right to make proposals for the project scope. The other agent (agent j) must respond to the agenda setter's proposals by either accepting or rejecting each proposal.[21] If a proposal is rejected, then no decision is made about the project scope at that time. The project cannot be completed until a project scope has been agreed to.

A strategy for agent i (the agenda setter) is a pair of maps $\{a_i(q, Q), \theta_i(q)\}$ defined for $q \in \mathbb{R}_+$ and $Q \in \mathbb{R}_+ \cup \{-1\}$. Here, $a_i(q, Q)$ denotes the effort level of the agenda setter when the project state is q and the project scope agreed upon is Q; by convention, $a_i(q, -1)$ denotes his effort level when no agreement has been reached yet. The value of $\theta_i(q)$ is the project scope proposed by the agenda setter in project state q; by convention, $\theta_i(q) = -1$ if the agent does not make a proposal at state q. Similarly, the map $a_j(q, Q)$ denotes the effort level in state q when project scope Q has been agreed upon; by convention, $Q = -1$ if no agreement has been reached yet. The map $Y_j(q, Q)$ is the acceptance strategy of agent j if agent i proposes project scope Q at state q, where $Y_j(q, Q) = 1$ if agent j accepts, and $Y_j(q, Q) = 0$ if he rejects.

Unanimity with Commitment

We first consider the case in which the agents can commit to a decision about the project scope. At any instant, the agenda setter can propose a project scope. Upon proposal, the other agent must decide to either accept or reject the offer. If he accepts, then the project scope agreed upon is set once and for all, and cannot be changed. From that instant onwards, the agenda setter stops making proposals, so $\{\theta_i(\cdot), Y_j(\cdot)\}$ become obsolete. The agents may continue to work on the project, and the project is completed and the agents collect their payoffs if and only if the state reaches the agreed upon project scope. If agent j rejects the proposal, then no project scope is decided upon, and the agenda setter may continue to make further proposals.

[21] The set of equilibrium project scopes is independent of who is the agenda-setter.

The following Proposition characterizes the set of MPE for the game in which both agents must agree to a particular project scope, and they can commit ex-ante.

Proposition 6. *Under unanimity with commitment, the project scope is agreed to at the beginning of the project, and any* $Q \in [Q_1(0), Q_2(0)]$ *can be part of an MPE.*

In other words, the project scope is decided at the outset, and it lies between the agents' ideal project scopes.

Unanimity without Commitment

Now suppose that the agenda setter cannot commit to a future project scope. Given the current state q, the agenda setter either proposes to complete the project immediately, or he does not make any proposal; i.e., $\theta_i(q) \in \{-1, q\}$. The following Proposition shows that without commitment, unanimity generates the same set of equilibria as the game when the inefficient agent is the dictator.

Proposition 7. *Without commitment, under unanimity, the set of MPE outcomes are the same as when agent* 2 *(i.e., the inefficient agent) is the dictator. That is, any* $Q \in [Q_1(0), Q_2(0)]$ *can be part of an MPE.*

Recall from Proposition 3 that agent 2 always prefers a larger project scope than agent 1 (i.e., $Q_2(q) \geq Q_1(q)$ for all q). Therefore, at any state q such that agent 2 would like to complete the project immediately, agent 1 wants to do so as well, but the opposite is not true. Because both agents must agree to complete the project, effectively, it is agent 2 who has the decision rights over the project scope.

Note that there is another institution wherein at every moment, the agents must both agree to continue the project. By a symmetric argument, the set of MPE outcomes are the same as when agent 1 is the dictator; i.e., there exists a unique MPE in which $Q = \overline{Q}_1$ is implemented. However, to remain consistent with the previous cases analyzed, we focus on the institution in which both agents must agree to stop the project.

4.3 Implications

In this section, we elaborate on two implications of our results. First, we seek to understand how closely the equilibrium project scope is aligned with each agent's preferences, or equivalently, which agent has "real authority" over the scope of the project. Second, we examine the welfare implications associated with each collective choice institution.

Real Authority

Naturally, institutions can enforce an agent's authority. However, the scope that is eventually implemented remains an equilibrium outcome, and thus the agent with formal decision power (e.g., the dictator) also has to account for his anticipation of the other agent's actions. In that sense, the scope implemented in equilibrium may be better aligned with the preferences of the agent who does not exercise formal decision power.

We model formal authority as the ability to determine the state at which the project ends and rewards are collected. We consider formal authority to be enforceable by courts: an agent has formal authority if he has the right to "sign the documents" or "pull the plug." It is determined by the collective choice institution. In the dictatorship setting, formal authority goes to the agent chosen as the dictator. By convention, in the unanimity setting, we say that the agents share formal authority. In contrast, the agent who has effective control over the project scope (i.e., he whose preferences are implemented in equilibrium), is said to have real authority. We define real authority as follows.

Definition 1. *Suppose the state is q, and a project scope has not been decided at any $\tilde{q} < q$. Agent i has real authority if either:*

1. *The project scope Q is decided at q and $Q = Q_i(q)$; or,*
2. *The project scope Q is not decided at q and $Q_i(q) > q$.*

Note that this definition applies only until a project scope is committed to. After the project scope has been decided, the game becomes one of dynamic contributions with a fixed, exogenous scope, and the concept of real authority is no longer relevant.

Our notions of real and formal authority are much like those described in Aghion & Tirole (1997). As pointed out in Aghion & Tirole (1997), the agent endowed with formal authority is not necessarily able to control the project. For example, consider a developed country assisting a developing country to construct a large infrastructure project. The project, being carried out on the developing country's soil, is subject to its laws and jurisdiction. The developing country thus has formal authority over the project and can specify the termination state, but it is not clear that the developing country does so at a state that is its ideal scope, due to the incentives of the donor developed country.

With commitment, the project scope is decided at the beginning of the project, and whichever agent has formal authority (i.e., dictatorship rights), also has real authority. Under unanimity, recall that any $Q \in [Q_1(0), Q_2(0)]$ can be part of an equilibrium, so depending on which scope is implemented, either agent can have real authority, or neither.

Without commitment, because the agents' preferences over project scope are time-inconsistent, real authority has a temporal component, and therefore richer implications.

The following remark elaborates.

Remark 1. *Consider the case without commitment. For all $q < \overline{Q}_1$, the agents share real authority. For $q \geq \overline{Q}_1$:*

1. *If agent 1 is dictator, then he has real authority at the completion state $q = \overline{Q}_1$.*

2. *If agent 2 is dictator (or under unanimity) and $Q \in [Q_1(0), Q_2(0)]$ is implemented, then he has real authority for all $q \in \left[\overline{Q}_1, Q\right)$. However, agent 1 has real authority at the completion state Q.*

First note that for all $q < \overline{Q}_1$, both agents prefer to continue the project. Thus, the domain in which the agents have conflicting preferences is $\left[\overline{Q}_1, \overline{Q}_2\right]$. The main takeaway from this remark is that if the efficient agent is dictator, then he completes the project at his ideal project scope, so he has real authority at the completion state $\overline{Q}_1$. In contrast, when the inefficient agent is the dictator (or under unanimity), the inefficient agent has real authority while the project is ongoing (since he prefers to continue, whereas the efficient agent would like to complete the project immediately), but his real authority eventually "runs out", and upon completion, it is the efficient agent who has real authority.

This may also help explain why it is often the case that agreements formally governed by unanimity still appear to be heavily influenced by large contributors. These large donors are the more efficient agents, who contribute more to the public project and hence have the incentive to stop the project before the inefficient agent.

Welfare

Finally, we discuss the welfare implications associated with each collective choice institution. In particular, we are interested in the question — which institutions can maximize total welfare. The following remark summarizes.

Remark 2. *With commitment, the social planner's ex-ante ideal project scope can be implemented only with unanimity. Without commitment, the social planner's project scope can be implemented if the inefficient agent is dictator or with unanimity.*

First, notice that these are possibility results. Because in some cases, multiple MPE exist, this is the best one can hope for. The main takeaway is that from a welfare perspective, it may be desirable to give the weaker party (i.e., the inefficient agent) formal authority, because the stronger party obtains real authority in equilibrium. If instead the efficient agent is conferred formal authority, then because he does not internalize the positive externality associated with a larger project scope, total welfare will be lower.

5 Extensions

In this section, we extend our model in two directions. First, we allow the agents to use monetary transfers in exchange for (a) implementing a particular project scope, or (b) reallocating the shares $\{\alpha_1, \alpha_2\}$. Second, we consider the case in which the project progresses stochastically.

5.1 Transfers

So far we have assumed that each agent's project stake α_i is exogenous, and transfers are not permitted. These are reasonable assumptions if agents are liquidity constrained. However, if transfers are available, then there are various ways to mitigate the inefficiencies associated with the collective choice problem. Our objective in this section is to shed light on how transfers can be useful for improving the efficiency properties of the collective choice institutions. We consider that agents choose effort levels strategically, so free-riding still occurs. We look at two types of transfers. First, we discuss the possibility that the agents can make lump-sum transfers at the beginning of the game to directly influence the project scope that is implemented. Second, we consider the case in which the agents can bargain over the allocation of shares in the project in exchange for transfers. In both cases, we assume that the agents commit to the project scope, transfers, and reallocation of shares at the outset of the game.

Transfers contingent on project scope

Let us consider the case in which one of the agents is dictator, and he can commit to a particular project scope.[22] Assume that agent 1 is dictator and makes a take-it-or-leave-it offer to agent 2, which specifies a transfer in exchange for committing to some project scope Q. Then agent 1 solves the following problem:

$$\begin{aligned} \max_{Q \geq 0,\, T \in \mathbb{R}} \quad & J_1(0; Q) - T \\ \text{s.t.} \quad & J_2(0; Q) + T \geq J_2(0; Q_1(0)) \,. \end{aligned}$$

Put in words, agent 1 chooses the project scope and the corresponding transfer to maximize his ex-ante discounted payoff, subject to agent 2 obtaining a payoff that is at least as great as his payoff if he were to reject agent 1's offer, in which case agent 1 would commit to the status quo project scope $Q_1(0)$, and no transfer would be made. Because transfers are unlimited, the constraint binds in the optimal solution, and the problem reduces to

$$\max_{Q \geq 0} \{J_1(0; Q) + J_2(0; Q) - J_2(0; Q_1(0))\} \,.$$

[22] The analysis for the other cases is similar, and yields the same insights.

Note that the optimal choice of Q maximizes total surplus. This is intuitive: because the agents have complete and symmetric information, bargaining is efficient. Moreover, it is straightforward to verify that the same result holds under any one-shot bargaining protocol irrespective of which agent has dictatorship rights, and for any initial status quo.[23]

Transfers contingent on reallocation of shares

We now consider $\alpha_1 + \alpha_2 = 1$, so the project stakes can be interpreted as project shares. We consider an extension of the model in which, at the outset, the agents start with an exogenous allocation of shares and then engage in a bargaining game in which shares can be reallocated in exchange for a transfer. After the re-allocation of shares, the collective choice institution determines the choice of scope as given in Section 4. Note that the allocation of shares influences the agents' incentives and consequently the equilibrium project scope. Because this is a game with complete information, the agents reallocate the shares so as to maximize the ex-ante total discounted surplus, taking the collective choice institution as given. For the cases in which the Pareto-efficient MPE is not unique, we further refine the MPE to the one in which total surplus is maximal.[24]

Based on the analysis of Section 4, there are three cases to consider:

1. Agent i is dictator, for $i \in \{1, 2\}$, and he has the ability to commit. As such, he commits to $Q = Q_i(0)$ at the outset, by Proposition 4.

2. Agent 1 is dictator, but he is unable to commit. In this case, the project is completed at state $\overline{Q}_1$, by Proposition 5.

3. Agent 2 is dictator, but he is unable to commit, or decisions must be made unanimously, with or without commitment. In these cases, the equilibrium project scope is $Q^*(0)$ by Propositions 5, 6, and 7, and the refinement to the total surplus-maximizing MPE.

We focus the analysis on the case in which agent 1 is dictator and can commit to a particular project scope at the outset; the other cases lead to similar insights. To begin, let $Q_1(0; \alpha)$ denote the (unique) equilibrium project scope when agent 1 is dictator and has the ability to commit, conditional on the shares $\{\alpha_1, 1 - \alpha_1\}$. Assume that agent 1 makes a take-it-or-leave-it offer to agent 2, which specifies a transfer in exchange for reallocating the parties' shares from the status quo shares $\{\overline{\alpha}_1, 1 - \overline{\alpha}_1\}$ to $\{\alpha_1, 1 - \alpha_1\}$. Let $J_i(q; Q, \alpha)$ denote the

[23] One might also consider the case in which commitment is not possible. Because $Q_1(q) \leq Q_2(q)$ for all q, to influence the project scope at some state, agent 1 might offer a lump-sum transfer to agent 2 in exchange for completing the project immediately, whereas agent 2 might offer flow transfers to agent 1 to extend the scope of the project. This model is intractable, so we do not pursue it in the current paper.

[24] This is the case under dictatorship without commitment, and unanimity with or without commitment. Simulations indicate that the findings are robust to the equilibrium selection rule.

continuation value for agent i when the state is q, the chosen project scope is Q and the chosen share to agent 1 is α. Then agent 1 solves the following problem:

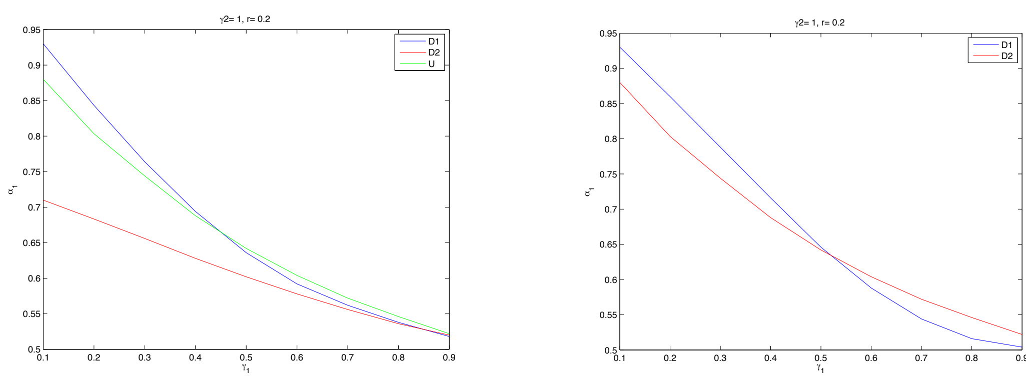
$$\begin{aligned} \max_{\alpha_1 \in [0,1],\, T \in \mathbb{R}} \quad & J_1\left(0;\, Q_1\left(0;\alpha_1\right), \alpha_1\right) - T \\ \text{s.t.} \quad & J_2\left(0;\, Q_1\left(0;\alpha_1\right), \alpha_1\right) + T \geq J_2\left(0;\, Q_1\left(0;\overline{\alpha}_1\right), \overline{\alpha}_1\right). \end{aligned}$$

Because transfers are unlimited and each agent's discounted payoff increases in his share, the incentive compatibility constraint binds in the optimal solution, and so the problem reduces to

$$\max_{\alpha_1 \in [0,1]} \left\{ J_1\left(0;\, Q_1\left(0;\alpha_1\right), \alpha_1\right) + J_2\left(0;\, Q_1\left(0;\alpha_1\right), \alpha_1\right) - J_2\left(0;\, Q_1\left(0;\overline{\alpha}_1\right), \overline{\alpha}_1\right) \right\} .$$

The optimal choice of α_1 maximizes total surplus, conditional on the scope subsequently selected by the collective choice institution. In all other cases, and under any one-shot bargaining protocol, the agents will agree to re-allocate their shares to maximize total surplus.

The problem of optimally reallocating shares is analytically intractable, therefore we find the solution numerically. Figure 2 below illustrates the share allocated to agent 1, as a function of his effort cost. Note that without commitment, both the case of unanimity and the case in which agent 2 is dictator deliver the same result, and hence the result for unanimity is omitted.



(a) Commitment (b) No commitment

Figure 2: Agent 1's optimal project share

In all cases, it is optimal for agent 1, who is more productive (i.e., $\gamma_1 < \gamma_2$), to possess the majority of the shares. Moreover, his optimal allocation decreases as his effort costs increase, i.e., as he becomes less productive. In other words, if one agent is substantially more productive than the other, then the former should possess the vast majority of the shares. Indeed, it is efficient to provide the stronger incentives to the more productive agent, and the smaller the disparity in productivity between the agents, the smaller should be the difference in the shares that they possess.

5.2 Collective Choice under Uncertainty

To examine the robustness of our results, in this section, we consider the case in which the project progresses stochastically according to

$$dq_t = (a_{1t} + a_{2t})\, dt + \sigma dZ_t,$$

where Z_t is a standard Brownian motion, and $\sigma > 0$ captures the degree of uncertainty associated with the evolution of the project. We discuss the results for collective choice under this form of uncertainty.

As in the deterministic case studied in Section 3, we begin by establishing the existence of an MPE with an exogenous project scope Q. In an MPE, each agent's discounted payoff function satisfies

$$rJ_i(q) = \frac{[J_i'(q)]^2}{2\gamma_i} + \frac{1}{\gamma_j} J_i'(q)\, J_j'(q) + \frac{\sigma^2}{2} J_i''(q)$$

subject to the boundary conditions $\lim_{q \to -\infty} J_i(q) = 0$ and $J_i(Q) = \alpha_i Q$ for each i. It follows from Georgiadis (2015) that for any project scope Q, an MPE exists and satisfies $J_i(q) > 0$, $J_i'(q) > 0$, $a_i(q) > 0$ and $a_i'(q) > 0$ for all i and q. This is the analog of Proposition 1 in the case of uncertainty.

We next establish the key properties of the MPE with exogenous project scope for asymmetric agents.

Proposition 8. *Consider the model with uncertainty, and suppose that* $\frac{\gamma_1}{\alpha_1} < \frac{\gamma_2}{\alpha_2}$.

1. *Agent* 1 *exerts higher effort than agent* 2 *in every state, and agent* 1*'s effort increases at a greater rate than agent 2's. That is,* $a_1(q) \geq a_2(q)$ *and* $a_i'(q) \geq a_2'(q)$ *for all* q.

2. *Agent* 1 *obtains a lower discounted payoff normalized by project stake than agent* 2. *That is,* $\frac{J_1(q)}{\alpha_1} \leq \frac{J_2(q)}{\alpha_2}$ *for all* q.

If instead $\frac{\gamma_1}{\alpha_1} = \frac{\gamma_2}{\alpha_2}$, *then* $a_1(q) = a_2(q)$ *and* $\frac{J_1(q)}{\alpha_1} = \frac{J_2(q)}{\alpha_2}$ *for all* q.

Proposition 8 is the analog of Proposition 2 in the case of uncertainty. It asserts that, under uncertainty, if agents are asymmetric, then the efficient agent exerts higher effort at every state of the project, and the efficient agent's effort increases at a higher rate than that of the inefficient agent. Furthermore, the efficient agent achieves a lower discounted payoff (normalized by the stake α_i) at every state of the project.

As for the agents' preferences over project scopes, while we are unable to prove the counterpart of the results in Section 3.2, numerical computations suggest that they continue to hold. This is not surprising given the result in Proposition 8 and because the intuition for the ordering and divergence of preferences is identical to that for the case without uncertainty. An example is illustrated in Figure 3.

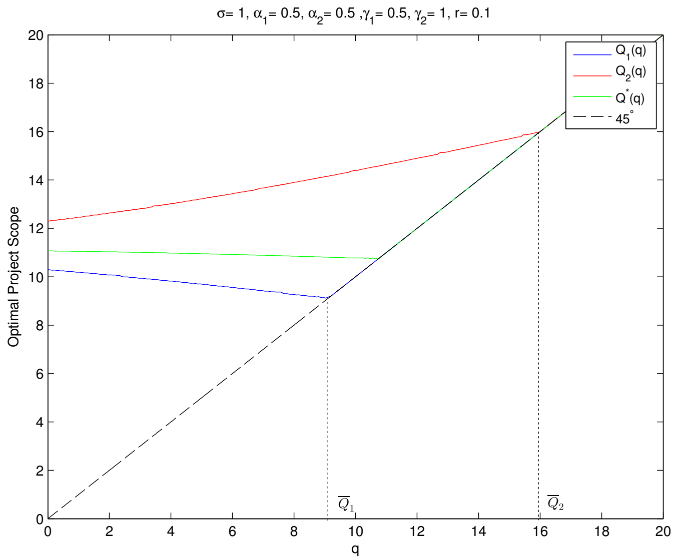


Figure 3: Agent i's ideal project scope $Q_i(q)$ with uncertainty

As Figure 3 illustrates, the inefficient agent prefers a larger scope than the efficient agent at every state, and furthermore, his ideal project scope increases over the course of the project, whereas the efficient agent's ideal project scope decreases. Moreover, for each agent, there exists a threshold such that he prefers to complete the project immediately at every state larger than that threshold.

Notice that the results of Section 4 rely on the key properties of the preferences illustrated in Figure 3. Conditional on these preferences, all results of Propositions 4–7 will hold.

6 Discussion

We study a dynamic game in which two heterogeneous agents make costly contributions towards the completion of a public project. The scope (i.e., the size) of the project is endogenous, and it can be decided by a predetermined collective choice institution at any time.

Three main takeaways arise from our analysis. First, due to free-riding incentives, the agents' preferences with respect to their ideal project scope are time-inconsistent, and the more efficient agent prefers to implement a smaller project relative to the less efficient agent. Second, absent the ability to commit to a decision about the project scope, if the efficient agent has dictatorship rights, then he also has real authority of the project scope that is implemented. In contrast, if the inefficient agent is the dictator or under unanimity, then real authority has a temporal component: for a duration of time, the dictator has real authority, but it eventually runs out, and upon completion of the project, it is the efficient agent who has real authority. Third, from a welfare perspective, it may be desirable to give some formal authority to the inefficient agent (via dictatorship rights or unanimity).

Finally, we suggest several directions for future research. A natural next step is to extend some of our results to a model with an arbitrary number of players, and understand, first, how incentives for effort interact, and second, how different collective choice institutions influence the project scope that is implemented in equilibrium. Notice that with even three players, other collective choice institutions can be considered, such as majority voting. As an example, Figure 4 illustrates each agent's ideal project scope, as well as the socially optimal project scope for a group of 4 agents. Similar to the 2-player case, the agents'

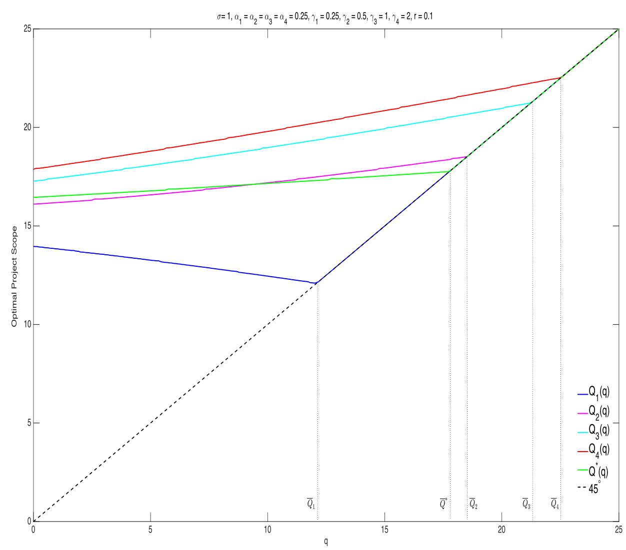


Figure 4: Agents' and social planner's ideal project scopes with 4 agents.

preferences over project scope are time-inconsistent, and rank-ordered from most to least efficient. Second, a richer contracting space may be considered, where each agent's payoff is conditioned both the project scope, and the completion time. Lastly, our model assumes complete information. This aids tractability, but likely misses important effects related to learning about the project's benefit and the agents' effort costs over time. For example, if an agent's cost of effort is private information, then the efficient agent may have an incentive to mimic the inefficient agent, thus contributing a smaller amount of effort. This may lead to a greater ideal project scope for the efficient agent, which will be welfare enhancing if the efficient agent is the dictator, but the welfare implications are not immediate because the distribution of work will likely be further away from that of the social planner.

A Proofs

A.1 Proof of Proposition 1

We first establish two Lemmas that will be used throughout the proof of this Proposition, as well as in the proof of Proposition 3. We consider the benchmark game of Section 3 with exogenous project scope Q.

Lemma 3. *Let (J_1, J_2) be a pair of well-behaved value functions associated with an MPE. Then $J_i(q) \in [0, \alpha_i Q]$ and $J_i'(q) \geq 0$ for all i and q.*

Proof of Lemma 3. Because each agent i can guarantee himself a payoff of zero by not exerting any effort, in any equilibrium, it must be the case that $J_i(q) \geq 0$ for all q. Moreover, because he receives reward $\alpha_i Q$ upon completion of the project, he discounts time, and the cost of effort is nonnegative, his payoff satisfies $J_i(q) \leq \alpha_i Q$ for all q. Next, suppose that $J_i'(q^*) < 0$ for some i and q^*. Then agent i exerts zero effort at q^*, and it must be the case that agent $j \neq i$ also exerts zero effort, because otherwise it implies $J_i(q^*) < 0$, which cannot occur in equilibrium. Since both agents exert zero effort at q^*, the project is never completed, and so $J_1(q^*) = J_2(q^*) = 0$. Therefore, for sufficiently small $\epsilon > 0$, we have $J_i(q^* + \epsilon) < 0$, which is a contradiction, implying $J_i'(q) \geq 0$ for all i and q. □

Observe that dividing both sides of equation (3) by γ_i the system of ODEs defined by (3) subject to (2) can be rewritten as

$$r\widetilde{J}_i(q) = \frac{1}{2}\left[\widetilde{J}_i'(q)\right]^2 + \widetilde{J}_i'(q)\,\widetilde{J}_j'(q) \tag{5}$$

subject to $\widetilde{J}_i(Q) = \frac{\alpha_i}{\gamma_i}Q$ for all $i \in \{1, 2\}$ and $j \neq i$. The following lemma derives an explicit system of ODEs that is equivalent to the implicit form given in (5) of Section 3.

Lemma 4. *Let (J_1, J_2) be a pair of well-behaved value functions associated with an MPE, and let $\widetilde{J}_i(q) = \frac{J_i(q)}{\gamma_i}$. Then if, at state q, the project is completing, the following explicit ODEs are satisfied on the range (q, Q):*[25]

$$\widetilde{J}_1' = \sqrt{\frac{r}{6}}\sqrt{2\sqrt{\widetilde{J}_1^2 + \widetilde{J}_2^2 - \widetilde{J}_1\widetilde{J}_2} + \widetilde{J}_1 + \widetilde{J}_2} + \sqrt{\frac{r}{2}}\sqrt{2\sqrt{\widetilde{J}_1^2 + \widetilde{J}_2^2 - \widetilde{J}_1\widetilde{J}_2} - \widetilde{J}_1 + \widetilde{J}_2},$$

$$\widetilde{J}_2' = \sqrt{\frac{r}{6}}\sqrt{2\sqrt{\widetilde{J}_1^2 + \widetilde{J}_2^2 - \widetilde{J}_1\widetilde{J}_2} + \widetilde{J}_1 + \widetilde{J}_2} - \sqrt{\frac{r}{2}}\sqrt{2\sqrt{\widetilde{J}_1^2 + \widetilde{J}_2^2 - \widetilde{J}_1\widetilde{J}_2} - \widetilde{J}_1 + \widetilde{J}_2}.$$

Proof of Lemma 4. In an MPE in which the project is completing at state q, $\widetilde{J}_1' + \widetilde{J}_2' > 0$ on $[q, Q)$ as otherwise both agents put zero effort at some intermediary state and the project is not completed.

[25] We say that the *project is completing at state q* to indicate that if the state is q, then the project will be completed. In contrast, we say that the *project is completed at state Q* to indicate that state Q is the termination state.

Using (5), subtracting $\widetilde{J}_2$ from $\widetilde{J}_1$ and adding $\widetilde{J}_2$ to $\widetilde{J}_1$ yields

$$r(\widetilde{J}_1 - \widetilde{J}_2) - \frac{1}{2}(\widetilde{J}_1' + \widetilde{J}_2')(\widetilde{J}_1' - \widetilde{J}_2') = 0 \text{ , and}$$
$$r(\widetilde{J}_1 + \widetilde{J}_2) - \frac{1}{2}(\widetilde{J}_1' + \widetilde{J}_2')^2 = \widetilde{J}_1'\widetilde{J}_2',$$

respectively, where for notational simplicity we drop the argument q. Letting $G = \widetilde{J}_1 + \widetilde{J}_2$ and $F = \widetilde{J}_1 - \widetilde{J}_2$, these equations can be rewritten as

$$rF - \frac{1}{2}F'G' = 0$$
$$rG - \frac{1}{2}(G')^2 = \frac{1}{4}(G')^2 - \frac{1}{4}(F')^2.$$

From the first equation we have $F' = \frac{2rF}{G'}$ (and recall that we have assumed $G' > 0$), while the second equation, after plugging in the value of F', becomes

$$rG - \frac{1}{2}(G')^2 = \frac{1}{4}(G')^2 - r^2\frac{F^2}{(G')^2},$$

This equation is quadratic in $(G')^2$, and noting by Lemma 3 that in any project-completing MPE we have $G' > 0$ on $[0, Q]$, the unique strictly positive root is

$$(G')^2 = \frac{2r}{3}\left(\sqrt{G^2 + 3F^2} + G\right) \Longrightarrow G' = \sqrt{\frac{2r}{3}}\sqrt{\sqrt{G^2 + 3F^2} + G}\,.$$

Since $G' > 0$ on the interval of interest, we have

$$F' = \frac{2rF}{G'} = \frac{\sqrt{6r}F}{\sqrt{\sqrt{G^2 + 3F^2} + G}} \Longrightarrow F' = \sqrt{2r}\sqrt{\sqrt{G^2 + 3F^2} - G}\,.$$

By using that $\widetilde{J}_1 = \frac{1}{2}(G + F)$ and $\widetilde{J}_2 = \frac{1}{2}(G - F)$, we obtain the desired expressions. □

Existence. Fix some $Q > 0$, and let $\widetilde{J}_i(q) = \frac{J_i(q)}{\gamma_i}$. As in Lemma 4, we note that the system of ODEs of Section 3 defined by (3) subject to (2) can be rewritten as

$$r\widetilde{J}_i(q) = \frac{1}{2}\left[\widetilde{J}_i'(q)\right]^2 + \widetilde{J}_i'(q)\,\widetilde{J}_j'(q) \tag{6}$$

subject to $\widetilde{J}_i(Q) = \frac{\alpha_i}{\gamma_i}Q$ for all $i \in \{1, 2\}$ and $j \neq i$. If a solution to this system of ODEs exists and $\widetilde{J}_i'(q) \geq 0$ for all i and q, then it constitutes an MPE, and each agent i's effort level satisfies $a_i(q) = \widetilde{J}_i'(q)$.

Lemma 5. *For every $\epsilon \in \left(0, \min_i\left\{\frac{\alpha_i}{\gamma_i}Q\right\}\right)$, there exists some $q_\epsilon < Q$ such that there exists a unique solution $\left(\widetilde{J}_1, \widetilde{J}_2\right)$ to the system of ODEs on $[q_\epsilon, Q]$ that satisfies $\widetilde{J}_i \geq \epsilon$ on that interval for all i.*

Proof of Lemma 5. This proof follows the proof of Lemma 4 in Cvitanić & Georgiadis (2016) closely. It follows from Lemma 4 above that we can write (3) as

$$\widetilde{J}_i'(q) = H_i\left(\widetilde{J}_1(q), \widetilde{J}_2(q)\right), \tag{7}$$

with

$$H_1(x,y) = \sqrt{\frac{r}{6}}\sqrt{2\sqrt{x^2+y^2-xy}+x+y} + \sqrt{\frac{r}{2}}\sqrt{2\sqrt{x^2+y^2-xy}-x+y},$$
$$H_2(x,y) = \sqrt{\frac{r}{6}}\sqrt{2\sqrt{x^2+y^2-xy}+x+y} - \sqrt{\frac{r}{2}}\sqrt{2\sqrt{x^2+y^2-xy}-x+y}.$$

For given $\epsilon > 0$, let

$$M_H = \max_i \max_{\epsilon \le x_i \le \frac{\alpha_i}{\gamma_i}Q} H_i(x_1, x_2).$$

Let us choose $q_\epsilon < Q$ sufficiently large such that, for all i,

$$\frac{\alpha_i}{\gamma_i}Q - (Q - q_\epsilon)M_H \ge \epsilon.$$

Then, define $\Delta q = \frac{Q-q_\epsilon}{N}$ and functions $\widetilde{J}_i^N$ by Euler iterations (see, for example, Atkinson et al. (2011)). Going backwards from Q,

$$\widetilde{J}_i^N(Q) = \frac{\alpha_i}{\gamma_i}Q$$

$$\widetilde{J}_i^N(Q-\Delta q) = \frac{\alpha_i}{\gamma_i}Q - \Delta q H_i\left(\frac{\alpha_1}{\gamma_1}Q, \frac{\alpha_2}{\gamma_2}Q\right)$$

$$\widetilde{J}_i^N(Q-2\Delta q) = J_i^N(Q-\Delta q) - \Delta q H_i\left(J_1^N(Q-\Delta q), \dots, J_n^N(Q-\Delta q)\right)$$
$$= \frac{\alpha_i}{\gamma_i}Q - \Delta q H_i\left(\frac{\alpha_1}{\gamma_1}Q, \frac{\alpha_2}{\gamma_2}Q\right) - \Delta q H_i\left(J_1^N(Q-\Delta q), \dots, J_n^N(Q-\Delta q)\right),$$

and so on, until $\widetilde{J}_i^N(Q - N\Delta q) = \widetilde{J}_i(q_\epsilon)$. We then complete the definition of function $\widetilde{J}_i^N$ by making it piecewise linear between the points $Q - k\Delta q$, $k = 1, \dots, N$. Note from the assumption on $Q - q_\epsilon$ that $\widetilde{J}_i^N(Q - k\Delta q) \ge \epsilon$, for all $k = 1, \dots, N$. Since the H_i's are continuously differentiable, they are Lipschitz continuous on the 2−dimensional bounded domain $\left[\epsilon, \frac{\alpha_1}{\gamma_1}Q\right] \times \left[\epsilon, \frac{\alpha_2}{\gamma_2}Q\right]$. Therefore, following standard arguments, the sequence $\left\{\widetilde{J}_i^n\right\}_{n=1}^N$ converges to a unique solution $\widetilde{J}_i$ of the system of ODEs, and we have $\widetilde{J}_i(q) > \epsilon$ for all $q \in [q_\epsilon, Q]$. □

Let

$$\underline{q} = \inf_{\epsilon > 0} q_\epsilon. \tag{8}$$

Lemma 5 shows that the system of ODEs has a unique solution on $[q_\epsilon, Q]$ for every $\epsilon > 0$. Thus, there exists a unique solution on $\left(\underline{q}, Q\right]$. Then, by standard optimal control arguments, it follows that $\widetilde{J}_i(q)$ is the value function of agent i for every initial project value $q > \underline{q}$.

To establish convexity, we differentiate (5) with respect to q to obtain

$$r\widetilde{J}_i'(q) = \left[\widetilde{J}_1'(q) + \widetilde{J}_2'(q)\right]\widetilde{J}_i''(q) + \widetilde{J}_i'(q)\,\widetilde{J}_j''(q)\,,$$

or equivalently, in matrix form,

$$r\begin{bmatrix}\widetilde{J}_1'\\ \widetilde{J}_2'\end{bmatrix} = \begin{bmatrix}\widetilde{J}_1'+\widetilde{J}_2' & \widetilde{J}_1'\\ \widetilde{J}_2' & \widetilde{J}_1'+\widetilde{J}_2'\end{bmatrix}\begin{bmatrix}\widetilde{J}_1''\\ \widetilde{J}_2''\end{bmatrix} \Longrightarrow \begin{bmatrix}\widetilde{J}_1''\\ \widetilde{J}_2''\end{bmatrix} = \frac{r}{\left(\widetilde{J}_1'\right)^2+\left(\widetilde{J}_2'\right)^2+\widetilde{J}_1'\widetilde{J}_2'}\begin{bmatrix}\left(\widetilde{J}_1'\right)^2\\ \left(\widetilde{J}_2'\right)^2\end{bmatrix}. \quad (9)$$

Note that $a_i'(q) = \widetilde{J}_i''(q) > 0$ if and only if $\widetilde{J}_i'(q) > 0$ for all i, or equivalently, if and only if $q > \underline{q}$.

So far, we have shown that for any given Q, there exists some $\underline{q} < Q$ (which depends on the choice of Q) such that the system of ODEs defined by (3) subject to (2) has a project-completing solution on $\left(\underline{q}, Q\right]$. In this solution, $J_i(q) > 0$, $J_i'(q) > 0$, and $a_i'(q) > 0$ for all i and $q > \underline{q}$. On the other hand, Lemma 6 implies that $J_i(q) = J_i'(q) = 0$ for all $q \le \underline{q}$. Therefore, the game starting at $q_0 = 0$ has a project-completing MPE if and only if $\underline{q} < 0$.

As shown in Lemma 1 regarding the single agent case, for small enough Q, each agent would be exerting effort and completing the project by himself even if the other agent were to exert no effort. A fortiori, the project will complete in an equilibrium where both agents can exert effort. Hence, for Q small enough, the MPE is project-completing.

As is shown in Section 3.2 regarding the socially optimal effort levels, for large enough Q, agents are better off not starting the project. A fortiori, for such project scopes, the project will not complete in an equilibrium where both agents can exert effort. Hence, for Q large enough, the MPE is not project-completing. Instead, neither agent puts any effort on the project and the project is never started.

Uniqueness. We show that if (J_1^a, J_2^a) and (J_1^b, J_2^b) are two well-behaved solutions to (3) subject to the boundary constraint (2) and subject to the constraint that each of the four functions is nondecreasing, then $(J_1^a, J_2^a) = (J_1^b, J_2^b)$ on the entire range $[0, Q]$. If the value functions associated with some MPE are well-behaved, then they must satisfy (3) subject to (2), and by Lemma 3 they must be nondecreasing. As the value functions uniquely pin down the equilibrium actions, it implies that for any project scope Q there exists a unique MPE with well-behaved solutions to the HJB equations.

The following Lemma shows that at every state q, $J_1(q) > 0$ if and only if $J_2(q) > 0$.

Lemma 6. *Let (J_1, J_2) be a pair of well-behaved value functions associated with an MPE. Then for every state q, $J_1(q) > 0$ if and only if $J_2(q) > 0$. Furthermore, if the project is completing at state q, then both J_1' and J_2' are strictly positive on (q, Q).*

Proof of Lemma 6. Fix agent i and let j denote the other agent. If $J_i(q) > 0$, then the project is completing at state q. By Lemma 4, $\widetilde{J}_1'$ is bounded strictly above 0 on (q, Q), thus J_1' is also bounded strictly above zero on that range, and as an agent's action is proportional to the slope of the value function, agent 1's effort is also bounded strictly above 0 on the range (q, Q). This implies that, if agent 2 chooses to exert no effort on (q, Q), potentially deviating from his equilibrium strategy, the project is still completed by agent 1—and thus agent 2 makes a strictly positive discounted payoff at state q without exerting any effort from state q onwards. Agent 2's equilibrium strategy provides at least as much payoff as in the case of agent 2 exerting no effort past state q, thus agent 2's equilibrium discounted payoff at state q, $J_2(q)$ should be strictly positive. To summarize, $J_1(q) > 0$ and $J_2(q) > 0$. Thus, if the project is completing at state q, then $J_1(q)$ and $J_2(q)$ are both strictly positive. By Lemma 3, $J_1'(q) \geq 0$ and $J_2'(q) \geq 0$ and therefore J_1 and J_2 are strictly positive on (q, Q). Equation (5) then implies that J_1' and J_2' are strictly positive on (q, Q). Hence, if in some MPE the project is completing at state q, both agents exert strictly positive effort at all states beyond q (and up to completion of the project). □

First, consider the case $J_1^a(0) > 0$. Then $J_2^a(0) > 0$ by Lemma 6. As J_1^a and J_2^a are nondecreasing, it follows from Lemma 5 that $(J_1^a, J_2^a) = (J_1^b, J_2^b)$ on the entire range $[0, Q]$. If instead $J_1^b(0) > 0$, the symmetric argument applies.

Next consider the case $J_1^a(0) = J_1^b(0) = 0$, and let $q^a = \sup\{q \geq 0 \mid J_1^a(q) = 0\}$. As $J_1^a(0) = 0$ we have $q^a \geq 0$. The boundary condition (2) and the continuity of J_1 implies that $q^a < Q$. Moreover, on the non-empty interval $(q^a, Q]$ we have $J_1^a > 0$, and thus by Lemma 6, $J_1^b > 0$ on that same interval. Lemma 5 then implies that $(J_1^a, J_2^a) = (J_1^b, J_2^b)$ on every $[q^a + \epsilon, Q]$ for $\epsilon > 0$, and thus that $(J_1^a, J_2^a) = (J_1^b, J_2^b)$ on $(q^a, Q]$. Now let us consider the range $[0, q^a]$. By continuity of J_1^a we have $J_1^a(q^a) = 0$. As J_1^a is nondecreasing and nonnegative, then $J_1^a(q^a) = 0$ implies that $J_1^a = 0$ on the interval $[0, q^a]$. As $J_1^a(q) = 0$ if and only if $J_2^a(q) = 0$, we get that $J_2^a = 0$ on the interval $[0, q_0]$. Thus, $(J_1^a, J_2^a) = 0$ on $[0, q^a]$.

Similarly let $q^b = \sup\{q \mid J_1^b(q) = 0\}$. We have $q^b \in [0, Q)$, and by a symmetric argument $(J_1^b, J_2^b) = 0$ on $[0, q^b]$. If $q^b < q^a$, then we get by Lemma 5 that $(J_1^a, J_2^a) = (J_1^b, J_2^b) > 0$ on $(q^b, Q]$, which contradicts $(J_1^a, J_2^a) = 0$ on $[0, q^a]$. If instead $q^b > q^a$, then we get that $(J_1^a, J_2^a) = (J_1^b, J_2^b) > 0$ on $(q^a, Q]$, which contradicts that $(J_1^b, J_2^b) = 0$ on $[0, q^b]$. Hence $q^a = q^b$.

Altogether this implies that on the interval $[0, q^a]$, $(J_1^a, J_2^a) = (J_1^b, J_2^b) = 0$, and on the interval $(q^a, Q]$, $(J_1^a, J_2^a) = (J_1^b, J_2^b) > 0$. Hence the HJB equations define a unique value function and thus a unique MPE. ■

A.2 Proof of Proposition 2

First, we fix some $Q > 0$, and we use the normalization $\widetilde{J}_i(q) = \frac{J_i(q)}{\gamma_i}$ as in the proof of Proposition 1.

To prove part 1, assume that $\frac{\gamma_1}{\alpha_1} < \frac{\gamma_2}{\alpha_2}$, let $\widetilde{D}(q) = \widetilde{J}_1(q) - \widetilde{J}_2(q)$, and note that $\widetilde{D}(\cdot)$ is smooth, $\widetilde{D}(q) = 0$ for $q \le \underline{q}$ and $\widetilde{D}(Q) = \left(\frac{\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2}\right) Q > 0$, where $\underline{q}$ is given by (8), in the proof of Proposition 1. Observe that either $\widetilde{D}'(q) > 0$ for all $q \ge 0$, or there exists some $\overline{q} \in [0, Q]$ such that $\widetilde{D}'(\overline{q}) = 0$. Suppose that the latter is the case. Then it follows from (5) that $\widetilde{D}(\overline{q}) = 0$, which implies that $\widetilde{D}(q) \ge 0$ for all q, and $\widetilde{D}'(q) > (=) 0$ if and only if $\widetilde{D}(q) > (=) 0$. Therefore, $\widetilde{D}'(q) \ge 0$, which implies that $a_1(q) \ge a_2(q)$ for all $q \ge 0$. Observe from equation (9) in the proof of Proposition 1, that $J_i''(q) = \beta \cdot (J_i'(q))^2$, where $\beta = r/[(\widetilde{J}_1')^2 + (\widetilde{J}_2')^2 + \widetilde{J}_1'\widetilde{J}_2']$, and note that $a_i(q) = \widetilde{J}_i'(q)$. Moreover, we know from part 1 of Proposition 2 that $a_1(q) \ge a_2(q)$, which implies that $J_1''(q) \ge J_2''(q)$, or equivalently, $a_1'(q) \ge a_2'(q)$ for all $q \ge 0$.

To prove part 2, note first the result for actions follows from the previous paragraph with all weak inequalities replaced with strict inequalities. Let $D(q) = \frac{J_1(q)}{\alpha_1} - \frac{J_2(q)}{\alpha_2}$, and note that $D(\cdot)$ is smooth, $D(q) = 0$ for q sufficiently small, and $D(Q) = 0$. Therefore, either $D(q) = 0$ for all q, or $D(\cdot)$ has an interior extreme point. Suppose that the former is true. Then for all q, we have $D(q) = D'(q) = 0$, which using (3) implies that

$$rD(q) = \frac{[J_1'(q)]^2}{2\alpha_1^2}\left(\frac{\alpha_2}{\gamma_2} - \frac{\alpha_1}{\gamma_1}\right) = 0 \Longrightarrow J_1'(q) = 0\,.$$

However, this is a contradiction, and so the latter must be true. Then there exists some $\overline{q}$ such that $D'(\overline{q}) = 0$. Using (3) and the fact that $J_i'(q) \ge 0$ for all q and $J_i'(q) > 0$ for some q, this implies that $D(\overline{q}) \le 0$. Therefore, $D(q) \le 0$ for all q, which completes the proof.

Finally, if $\frac{\alpha_1}{\gamma_1} = \frac{\alpha_2}{\gamma_2}$, then it follows from the analysis above that $\widetilde{D}'(q) = 0$ and $D(q) = 0$, which implies that $a_1(q) = a_2(q)$ and $\frac{J_1(q)}{\alpha_1} = \frac{J_2(q)}{\alpha_2}$ for all $q \ge 0$. ■

A.3 Proof of Proposition 3

To prove part 1, first suppose that $\frac{\gamma_1}{\alpha_1} = \frac{\gamma_2}{\alpha_2}$. In this case, we know from equation (4) that each agent's discounted payoff function satisfies

$$J_i(q; Q) = \frac{r\,\gamma_i}{6}\left[q - Q + \sqrt{\frac{6\alpha_i Q}{r\gamma_i}}\right],$$

and by maximizing $J_i(q; Q)$ with respect to Q, we obtain that $Q_1(q) = Q_2(q) = \frac{3\alpha_i}{2r\gamma_i}$ for all q.

To prove part 2, consider the case in which $\frac{\gamma_1}{\alpha_1} < \frac{\gamma_2}{\alpha_2}$. This part of the proof comprises 3 steps. To begin, in the following lemma, we characterize the values $\overline{Q}_i$ for $i = 1, 2$ that are defined to be the project state that makes each agent i indifferent between terminating the project at this state, and continuing the project one more instant.

Lemma 7. *Assume the agents are asymmetric, i.e.,* $\frac{\gamma_1}{\alpha_1} < \frac{\gamma_2}{\alpha_2}$. *The values of* $\overline{Q}_1$ *and* $\overline{Q}_2$ *are unique and given by*

$$\sqrt{\overline{Q}_1} = \frac{\sqrt{2/3}\sqrt{\mu}\alpha_1/\gamma_1}{\sqrt{r}\alpha_1/\gamma_1 + \frac{\sqrt{r}}{12}\left[\sqrt{\mu}+\sqrt{3\nu}\right]^2}$$

and

$$\sqrt{\overline{Q}_2} = \frac{\sqrt{2/3}\sqrt{\mu}\alpha_2/\gamma_2}{\sqrt{r}\alpha_2/\gamma_2 + \frac{\sqrt{r}}{12}\left[\sqrt{\mu}-\sqrt{3\nu}\right]^2}$$

where

$$\mu = 2\sqrt{\left(\frac{\alpha_1}{\gamma_1}\right)^2 + \left(\frac{\alpha_2}{\gamma_2}\right)^2 - \frac{\alpha_1}{\gamma_1}\frac{\alpha_2}{\gamma_2}} + \frac{\alpha_1}{\gamma_1} + \frac{\alpha_2}{\gamma_2}$$

and

$$\nu = 2\sqrt{\left(\frac{\alpha_1}{\gamma_1}\right)^2 + \left(\frac{\alpha_2}{\gamma_2}\right)^2 - \frac{\alpha_1}{\gamma_1}\frac{\alpha_2}{\gamma_2}} - \frac{\alpha_1}{\gamma_1} + \frac{\alpha_2}{\gamma_2}.$$

Furthermore, $\overline{Q}_1 < \overline{Q}_2$.

Proof of Lemma 7. Throughout this proof, we consider a project of a given scope Q. Let $\overline{a}_i(Q)$ denote the equilibrium effort agent i exerts at the very end of the project, when the terminal state is Q. Recall that, in equilibrium, the action of agent i at state q is given by

$$a_i(q) = J_i'(q)/\gamma_i,$$

and thus $\overline{a}_i(Q) = J_i'(Q)/\gamma_i = \widetilde{J}_i'(Q)$. From Lemma 4 and noting that $\widetilde{J}_i(Q) = (\alpha_i/\gamma_i)Q$, we get

$$\overline{a}_1(Q) = \sqrt{\frac{rQ}{6}}\left(\sqrt{\mu}+\sqrt{3\nu}\right) \tag{10}$$

$$\overline{a}_2(Q) = \sqrt{\frac{rQ}{6}}\left(\sqrt{\mu}-\sqrt{3\nu}\right), \tag{11}$$

with μ and ν defined as in the statement of the current lemma.

For a project of scope Q, agent i gets value $\alpha_i Q$ at the completion of the project, when $q = Q$. If the project is instead of scope $Q + \Delta Q$ (for small enough ΔQ), and if the current state is $q = Q$, there is a delay ϵ before the project is completed. To the first order in ϵ, the relationship $\Delta Q = (\overline{a}_1(Q) + \overline{a}_2(Q))\epsilon$ holds. Thus, to the first order in ϵ, the net discounted value of the project to agent i at state $q = Q$ is

$$\alpha_i\left[Q + (\overline{a}_1(Q) + \overline{a}_2(Q))\epsilon\right]e^{-r\epsilon} - \frac{\gamma_i}{2}(\overline{a}_i(Q))^2\epsilon.$$

At project scope $Q = \overline{Q}_i$, the agent is indifferent between stopping the project now (corresponding to a project scope $\overline{Q}_i$) and waiting an instant later (corresponding to a project scope $\overline{Q}_i + \Delta Q$ for an infinitesimal ΔQ). So to the first order,

$$\alpha_i \overline{Q}_i = \alpha_i(\overline{Q}_i + (\overline{a}_1(\overline{Q}_i) + \overline{a}_2(\overline{Q}_i))\epsilon)e^{-r\epsilon} - \frac{\gamma_i}{2}(\overline{a}_i(\overline{Q}_i))^2\epsilon.$$

So:

$$\alpha_i(\overline{a}_1(\overline{Q}_i) + \overline{a}_2(\overline{Q}_i)) - r\alpha_i\overline{Q}_i - \frac{\gamma_i}{2}(\overline{a}_i(\overline{Q}_i))^2 = 0.$$

Solving this equation for $i = 1, 2$ yields

$$\sqrt{\overline{Q}_1} = \frac{\sqrt{2/3}\sqrt{\mu}\alpha_1/\gamma_1}{\sqrt{r}\alpha_1/\gamma_1 + \frac{\sqrt{r}}{12}\left[\sqrt{\mu} + \sqrt{3\nu}\right]^2} \quad \text{and} \quad \sqrt{\overline{Q}_2} = \frac{\sqrt{2/3}\sqrt{\mu}\alpha_2/\gamma_2}{\sqrt{r}\alpha_2/\gamma_2 + \frac{\sqrt{r}}{12}\left[\sqrt{\mu} - \sqrt{3\nu}\right]^2}.$$

Note that

$$\frac{\sqrt{\overline{Q}_1}}{\sqrt{\overline{Q}_2}} = \frac{12 + \left(\frac{\alpha_2}{\gamma_2}\right)^{-1}\left[\sqrt{\mu} - \sqrt{3\nu}\right]^2}{12 + \left(\frac{\alpha_1}{\gamma_1}\right)^{-1}\left[\sqrt{\mu} + \sqrt{3\nu}\right]^2}.$$

In particular, $\overline{Q}_1 < \overline{Q}_2$ if and only if the inequality

$$\left(\frac{\alpha_2}{\gamma_2}\right)^{-1/2}\left[\sqrt{\mu} + \sqrt{3\nu}\right] - \left(\frac{\alpha_1}{\gamma_1}\right)^{1/2}\left(\frac{\alpha_2}{\gamma_2}\right)^{-1/2}\left(\frac{\alpha_2}{\gamma_2}\right)^{-1/2}\left[\sqrt{\mu} - \sqrt{3\nu}\right] > 0 \qquad (12)$$

holds. Let

$$f(x) = \sqrt{1 + x + 2\sqrt{1 + x^2 - x}} \quad \text{and} \quad g(x) = \sqrt{1 - x + 2\sqrt{1 + x^2 - x}}.$$

Note that

$$\left(\frac{\alpha_2}{\gamma_2}\right)^{-1/2}\left[\sqrt{\mu} + \sqrt{3\nu}\right] = f\left(\left(\frac{\alpha_1}{\gamma_1}\right)\left(\frac{\alpha_2}{\gamma_2}\right)^{-1}\right) + \sqrt{3}g\left(\left(\frac{\alpha_1}{\gamma_1}\right)\left(\frac{\alpha_2}{\gamma_2}\right)^{-1}\right)$$

and

$$\left(\frac{\alpha_2}{\gamma_2}\right)^{-1/2}\left[\sqrt{\mu} - \sqrt{3\nu}\right] = f\left(\left(\frac{\alpha_1}{\gamma_1}\right)\left(\frac{\alpha_2}{\gamma_2}\right)^{-1}\right) - \sqrt{3}g\left(\left(\frac{\alpha_1}{\gamma_1}\right)\left(\frac{\alpha_2}{\gamma_2}\right)^{-1}\right).$$

Since, by assumption, $\alpha_1/\gamma_1 < \alpha_2/\gamma_2$, (12) is satisfied if $[f(x)+\sqrt{3}g(x)]-x[f(x)-\sqrt{3}g(x)] > 0$ for every $x \in (0, 1)$. Note that, as $f, g > 0$ on $(0, 1)$, so

$$\begin{aligned}
[f(x) + \sqrt{3}g(x)] - x[f(x) - \sqrt{3}g(x)] &\geq x[f(x) + \sqrt{3}g(x)] - x[f(x) - \sqrt{3}g(x)] \\
&\geq x[f(x) + g(x)] - x[f(x) - g(x)] \\
&= 2xg(x) > 0.
\end{aligned}$$

This establishes the inequality (12), and thus $\overline{Q}_1 < \overline{Q}_2$. □

Equations (10) and (11) show that the agent's action at time of termination is strictly increasing with the project scope.

Lemma 8. *The value $J_i'(Q; Q)$ is strictly increasing in Q. Furthermore $\overline{Q}_i$ is the unique solution to the equation in Q, $J_i'(Q_i(Q); Q_i(Q)) = \alpha_i$.*

Proof of Lemma 8. Consider agent i's optimization problem given state q. We seek to find the unique q such that $q = \arg\max_{Q \geq q} \{J_i(q;Q)\}$. For such q, we have $\frac{\partial}{\partial Q} J_i(q;Q)\Big|_{q=Q} = 0$. Note that $J_i(Q;Q) = \alpha_i Q$, and totally differentiating this with respect to Q yields

$$\frac{dJ_i(Q;Q)}{dQ} = J_i'(Q;Q) + \frac{\partial J_i(q;Q)}{\partial Q}\Bigg|_{q=Q}$$

thus

$$J_i'(Q;Q) = \alpha_i \,. \tag{13}$$

By our assumption that $J_i(q;Q)$ is strictly concave in Q for all $q \leq Q \leq \overline{Q}_2$, it follows that (13) is necessary and sufficient for a maximum.

Noting that the explicit form of the HJB equations of Lemma 4 implies that $J_i'(Q;Q) = J_i'(1;1)\sqrt{Q}$, it follows that $J_i'(Q;Q)$ is strictly increasing in Q. Therefore, the solution to (13) is unique. $\square$

Step 1: *We show that $Q_2'(q) \geq 0$ for all $q \geq \overline{Q}_1$.*

To begin, we differentiate $\widetilde{J}_i(q;Q)$ in (5) with respect to Q to obtain

$$r\partial_Q \widetilde{J}_1(q;Q) = \partial_Q a_1(q;Q)\left[a_1(q;Q) + a_2(q;Q)\right] + a_1(q;Q)\,\partial_Q a_2(q;Q)$$
$$r\partial_Q \widetilde{J}_2(q;Q) = \partial_Q a_2(q;Q)\left[a_1(q;Q) + a_2(q;Q)\right] + a_2(q;Q)\,\partial_Q a_1(q;Q)$$

where we note $\partial_Q \widetilde{J}_i(q;Q) = \frac{\partial}{\partial Q}\widetilde{J}_i(q;Q)$, and where $\partial_Q a_i(q;Q) = \partial_Q \widetilde{J}_i'(q;Q) = \frac{\partial^2}{\partial Q\,\partial q}\widetilde{J}_i(q;Q)$, and $a_i(q;Q) = \widetilde{J}_i'(q;Q) = \frac{\partial}{\partial q}\widetilde{J}_i(q;Q)$.[26] Rearranging terms yields

$$\frac{(a_1+a_2)^2 - a_1 a_2}{r}(\partial_Q a_1) = (a_1+a_2)\left(\partial_Q \widetilde{J}_1\right) - a_1\left(\partial_Q \widetilde{J}_2\right) \tag{14}$$

$$\frac{(a_1-a_2)^2 + a_1 a_2}{r}(\partial_Q a_2) = (a_1+a_2)\left(\partial_Q \widetilde{J}_2\right) - a_2\left(\partial_Q \widetilde{J}_1\right)\,, \tag{15}$$

where we drop the arguments q and Q for notational simplicity. Because $a_i, a_j > 0$, note that $(a_1+a_2)^2 - a_1a_2 > 0$ and $(a_1-a_2)^2 + a_1a_2 > 0$. Recall $Q_i(q)$ is agent i's ideal project scope given the current state q. Then for all $q < Q_i(q)$ and for the smallest q such that $q = Q_i(q)$, we have $\frac{\partial}{\partial Q}\widetilde{J}_i(q;Q_i(q)) = 0$. Differentiating this with respect to q yields

$$\frac{\partial^2}{\partial Q\,\partial q}\widetilde{J}_i(q;Q_i(q)) + \frac{\partial^2}{\partial Q^2}\widetilde{J}_i(q;Q_i(q))\,Q_i'(q) = 0 \Longrightarrow Q_i'(q) = -\frac{\partial_Q a_i(q;Q_i(q))}{\partial_Q^2 \widetilde{J}_i(q;Q_i(q))}\,.$$

Since $\partial_Q^2 \widetilde{J}_i(q;Q) < 0$ (by our strict concavity assumption), it follows that $Q_i'(q) \leq 0$ if and only if $\partial_Q a_i(q;Q) \geq 0$.

Next, fix some $\widehat{q} \in \left(\overline{Q}_1, \overline{Q}_2\right)$. By the strict concavity of $\widetilde{J}_i(q;Q)$ in Q, it follows that $\partial_Q \widetilde{J}_1(\widehat{q}, Q_2(\widehat{q})) < 0$ and $\partial_Q \widetilde{J}_2(\widehat{q}, Q_2(\widehat{q})) = 0$; i.e., agent 1 would prefer to have completed

[26] Note $a_i(q;Q)$ is distinct from agent strategies in the case of commitment $a_i(q,Q)$. Here $a_i(q;Q)$ denotes agents' actions in the MPE with exogenous project scope Q.

the project at a smaller project scope than $Q_2(\widehat{q})$, whereas agent 2 finds it optimal to complete the project at $Q_2(\widehat{q})$ (the latter statement being true by definition of $Q_2(\widehat{q})$). Using (15) it follows that $\partial_Q a_2(\widehat{q}, Q_2(\widehat{q})) > 0$, which implies that $Q_2'(\widehat{q}) > 0$. Therefore, $Q_2'(q) > 0$ for all $q \in (\overline{Q}_1, \overline{Q}_2)$ and $Q_2(\overline{Q}_1) > \overline{Q}_1$, where the last inequality follows from the facts that by assumption $\widetilde{J}_2(q; Q)$ is strictly concave in Q for $q \leq Q \leq \overline{Q}_2$ and so it admits a unique maximum, and that $\widetilde{J}_2'(\overline{Q}_1; \overline{Q}_1) < \frac{\alpha_2}{\gamma_2}$, which implies that he prefers to continue work on the project rather than complete it at $\overline{Q}_1$.

Step 2: *We show that $Q_1'(q) \leq 0 \leq Q_2'(q)$ for all $q \leq \overline{Q}_1$. Moreover, $Q_1'(q) < 0 < Q_2'(q)$ for all q such that $Q_1(q) < Q_2(q)$.*

Because $Q_2(\overline{Q}_1) > \overline{Q}_1$ and $Q_i(\cdot)$ is smooth, there exists some $\overline{q} \geq 0$ such that $Q_2(q) > Q_1(q)$ for all $q \in (\overline{q}, \overline{Q}_1)$. Pick some q in this interval, and note that $\partial_Q \widetilde{J}_1(q, Q_2(q)) < 0$ and $\partial_Q \widetilde{J}_2(q, Q_2(q)) = 0$, which together with (15) implies that $\partial_Q a_2(q, Q_2(q)) > 0$. Similarly, we have $\partial_Q \widetilde{J}_1(q, Q_1(q)) = 0$ and $\partial_Q \widetilde{J}_2(q, Q_1(q)) > 0$, which together with (14) implies that $\partial_Q a_1(q, Q_1(q)) < 0$. Therefore, $Q_1'(q) < 0 < Q_2'(q)$ for all $q \in (\overline{q}, \overline{Q}_1)$.

Next, by way of contradiction, assume that there exists some q such that $Q_1(q) > Q_2(q)$ for some $q < \overline{q}$. Because $Q_i(q)$ is smooth, by the intermediate value theorem, there exists some $\widetilde{q}$ such that $Q_1(\widetilde{q}) > Q_2(\widetilde{q})$ and at least one of the following statements is true: $Q_1'(\widetilde{q}) < 0$ or $Q_2'(\widetilde{q}) > 0$. This implies that for such $\widetilde{q}$, we must have $\partial_Q \widetilde{J}_1(\widetilde{q}, Q_2(\widetilde{q})) > 0$, $\partial_Q \widetilde{J}_2(\widetilde{q}, Q_2(\widetilde{q})) = 0$, $\partial_Q \widetilde{J}_1(\widetilde{q}, Q_1(\widetilde{q})) = 0$ and $\partial_Q \widetilde{J}_2(\widetilde{q}, Q_1(\widetilde{q})) < 0$. Then it follows from (14) and (15) that $\partial_Q a_1(\widetilde{q}, Q_2(\widetilde{q})) > 0$ and $\partial_Q a_2(\widetilde{q}, Q_1(\widetilde{q})) < 0$. This in turn implies that $Q_1'(\widetilde{q}) > 0 > Q_2'(\widetilde{q})$, which is a contradiction. Therefore, it must be the case that $Q_2(q) \geq Q_1(q)$ for all q, and therefore $Q_1'(q) \leq 0$ *for all* $q \leq \overline{Q}_1$ *and* $Q_2'(q) \geq 0$ *for all* $q \leq \overline{Q}_2$.

Step 3: *We show that there does not exist any q such that $Q_1(q) = Q_2(q)$.*

First, we show that if there exists some $\overline{q}$ such that $Q_1(\overline{q}) = Q_2(\overline{q})$, then it must be the case that $Q_1(q) = Q_2(q)$ for all $q \leq \overline{q}$. Suppose that the converse is true. Then by the intermediate value theorem, there exists some $\widetilde{q}$ such that $Q_1(\widetilde{q}) < Q_2(\widetilde{q})$ and at least one of the following statements is true: either $Q_1'(\widetilde{q}) > 0$ or $Q_2'(\widetilde{q}) < 0$. This implies that for such $\widetilde{q}$, we must have $\partial_Q \widetilde{J}_1(\widetilde{q}, Q_2(\widetilde{q})) < 0$, $\partial_Q \widetilde{J}_2(\widetilde{q}, Q_2(\widetilde{q})) = 0$, $\partial_Q \widetilde{J}_1(\widetilde{q}, Q_1(\widetilde{q})) = 0$ and $\partial_Q \widetilde{J}_2(\widetilde{q}, Q_1(\widetilde{q})) > 0$. Then it follows from (14) and (15) that $\partial_Q a_1(\widetilde{q}, Q_2(\widetilde{q})) < 0$ and $\partial_Q a_2(\widetilde{q}, Q_1(\widetilde{q})) > 0$. This in turn implies that $Q_1'(\widetilde{q}) < 0 < Q_2'(\widetilde{q})$, which is a contradiction. Therefore, if there exists some $\overline{q}$ such that $Q_1(\overline{q}) = Q_2(\overline{q})$, then $Q_1(q) = Q_2(q)$ and $\partial_Q a_1(q; Q) = \partial_Q a_2(q; Q) = 0$ for all $q \leq \overline{q}$ and $Q = Q_1(q)$.

Next, note that each agent's normalized discounted payoff function can be written in integral form as

$$\widetilde{J}_i\left(q_t;Q\right)=e^{-r[\tau(Q)-t]}\frac{\alpha_i}{\gamma_i}Q-\int_t^{\tau(Q)}e^{-r(s-t)}\frac{\left(a_i\left(q_s;Q\right)\right)^2}{2}ds\,.$$

Differentiating this with respect to Q yields the first-order condition

$$e^{-r[\tau(Q)-t]}\frac{\alpha_i}{\gamma_i}\left[1-rQ\tau'\left(Q\right)\right]-e^{-r[\tau(Q)-t]}\tau'\left(Q\right)\frac{\left(a_i\left(Q;Q\right)\right)^2}{2}-\int_t^{\tau(Q)}e^{-r(s-t)}a_i\left(q_s;Q\right)\partial_Q a_i\left(q_s;Q\right)ds=0. \tag{16}$$

Now, by way of contradiction, suppose there exists some $\overline{q}$ such that $Q_1\left(\overline{q}\right)=Q_2\left(\overline{q}\right)=Q^*$. Then we have $Q_1\left(q\right)=Q_2\left(q\right)$ and $\partial_Q a_1\left(q;Q^*\right)=\partial_Q a_2\left(q;Q^*\right)=0$ for all $q\leq\overline{q}$. Therefore, fixing some $q\leq\overline{q}$ and $Q^*=Q_1\left(\overline{q}\right)$, it follows from (16) that

$$2\left[1-rQ^*\tau'\left(Q^*\right)\right]=\tau'\left(Q^*\right)\frac{\gamma_1}{\alpha_1}\left(a_1\left(Q^*;Q^*\right)\right)^2=\tau'\left(Q^*\right)\frac{\gamma_2}{\alpha_2}\left(a_2\left(Q^*;Q^*\right)\right)^2.$$

Observe that $\partial_Q a_1\left(q;Q^*\right)=\partial_Q a_2\left(q;Q^*\right)=0$, which implies that $\partial_Q\left[a_1\left(q;Q^*\right)+a_2\left(q;Q^*\right)\right]=0$, and hence $\tau'\left(Q^*\right)>0$. By assumption, $\frac{\gamma_1}{\alpha_1}<\frac{\gamma_2}{\alpha_2}$, and we shall now show that $\frac{\gamma_1}{\alpha_1}\left(a_1\left(Q^*;Q^*\right)\right)^2>\frac{\gamma_2}{\alpha_2}\left(a_2\left(Q^*;Q^*\right)\right)^2$. Let $D\left(q;Q^*\right)=\sqrt{\frac{\gamma_1}{\alpha_1}}\widetilde{J}_1\left(q;Q^*\right)-\sqrt{\frac{\gamma_2}{\alpha_2}}\widetilde{J}_2\left(q;Q^*\right)$, and note that $D\left(q;Q^*\right)=0$ for q sufficiently small, $D\left(Q^*;Q^*\right)=\left(\sqrt{\frac{\alpha_1}{\gamma_1}}-\sqrt{\frac{\alpha_2}{\gamma_2}}\right)Q^*>0$, and $D\left(\cdot;Q^*\right)$ is smooth. Therefore, either $D'\left(q;Q^*\right)>0$ for all q, or there exists some extreme point z such that $D'\left(z;Q^*\right)=0$. If the former is true, then $D'\left(Q^*;Q^*\right)>0$, and we obtain the desired result. Now suppose that the latter is true. It follows from (5) that

$$rD\left(z;Q^*\right)=\frac{\left[\widetilde{J}_1'\left(z;Q^*\right)\right]^2}{2}\left(\sqrt{\frac{\gamma_1}{\alpha_1}\frac{\alpha_2}{\gamma_2}}-1\right)<0\,,$$

which implies that any extreme point z must satisfy $D\left(z;Q^*\right)<0<D\left(Q^*;Q^*\right)$, and hence $D'\left(Q^*;Q^*\right)>0$. Therefore, $\frac{\gamma_1}{\alpha_1}\left(a_1\left(Q^*;Q^*\right)\right)^2>\frac{\gamma_2}{\alpha_2}\left(a_2\left(Q^*;Q^*\right)\right)^2$, which contradicts the assumption that there exists some q such that $Q_1\left(q\right)=Q_2\left(q\right)$.

We complete the proof of Proposition 3. From Lemma 7, we know that $\overline{Q}_1<\overline{Q}_2$. Steps 1 and 2 show that $Q_1'\left(q\right)\leq 0$ for all $q\leq\overline{Q}_1$ and $Q_2'\left(q\right)\geq 0$ for all $q\leq\overline{Q}_2$, respectively, while step 3 shows that there exists no $q<\overline{Q}_2$ such that $Q_1\left(q\right)=Q_2\left(q\right)$. This proves part 2(a). To see part 2(b), Step 3 shows that $Q_2(q)>Q_1(q)$ for all q (i.e. $\overline{q}=0$), which together with Step 2, implies that $Q_2'(q)>0>Q_1'(q)$ for all $q>0$. Finally, it follows from the strict concavity of $J_i\left(q;Q\right)$ in Q that $Q_i\left(q\right)=q$ for all $q\geq\overline{Q}_i$, which completes the proof of part 2(c). ■

A.4 Proof of Lemma 1

First, we characterize each agent i's effort and payoff function when he works alone on the project (and receives $\alpha_i Q$ upon completion).

Let $\widehat{J}_i(q;Q)$ be agent i's discounted payoff at state q for a project of scope Q. By standard arguments, under regularity conditions, the function $\widehat{J}_i(\cdot;Q)$ satisfies the HJB equation

$$r\widehat{J}_i(q;Q) = \max_{\breve{a}_i}\left\{-\frac{\gamma}{2}\breve{a}_i^2 + \breve{a}_i\widehat{J}_i'(q;Q)\right\} \tag{17}$$

subject to the boundary condition

$$\widehat{J}_i(Q;Q) = \alpha_i Q. \tag{18}$$

The game defined by (17) subject to the boundary condition (18) has a unique solution on $\left(\underline{q},Q\right]$ in which the project is completed, where $\underline{q} = Q - \sqrt{\frac{2\alpha_i Q}{r\gamma_i}}$. Then agent i's effort strategy and discounted payoff satisfies

$$\begin{aligned} \widehat{a}_i\left(q;Q\right) &= r\left(q - Q + \sqrt{\frac{2\alpha_i Q}{r\gamma_i}}\right) \\ \text{and} \quad \widehat{J}_i\left(q;Q\right) &= \frac{r\gamma_i}{2}\left(q - Q + \sqrt{\frac{2\alpha_i Q}{r\gamma_i}}\right)^2, \end{aligned}$$

respectively. Define

$$\widehat{Q}_i\left(q\right) = \arg\max_{Q\geq q}\left\{\widehat{J}_i\left(q;Q\right)\right\}.$$

It is straightforward to verify that $\widehat{Q}_i\left(q\right) = \frac{\alpha_i}{2r\gamma_i}$. The inequality $\widehat{Q}_2\left(q\right) < \widehat{Q}_1\left(q\right)$ follows from the fact that by assumption $\frac{\gamma_1}{\alpha_1} < \frac{\gamma_2}{\alpha_2}$.

Next, we show that $\widehat{Q}_1\left(q\right) < \overline{Q}_1$. Define $\widehat{\Delta}\left(q\right) = J_1\left(q;\overline{Q}_1\right) - \widehat{J}_1\left(q;\overline{Q}_1\right)$. Note that $J_1'\left(\overline{Q}_1;\overline{Q}_1\right) = \alpha_1$, $\widehat{\Delta}\left(\overline{Q}_1\right) = 0$, $\widehat{\Delta}\left(q\right) = 0$ for sufficiently small q, and $\widehat{\Delta}\left(\cdot\right)$ is smooth. Therefore, either $\widehat{\Delta}\left(q\right) = 0$ for all q, or it has an interior local extreme point. In either case, there exists some z such that $\widehat{\Delta}'\left(z\right) = 0$. Using (3) and the fact that, from the single agent HJB equation, $r\widehat{J}_1(q;Q) = \left[\widehat{J}_1'(q;Q)\right]^2/(2\gamma_1)$, it follows that

$$r\widehat{\Delta}\left(z\right) = \frac{J_1'\left(z;\overline{Q}_1\right)J_2'\left(z;\overline{Q}_1\right)}{\gamma_2}.$$

Because $J_1'\left(q;\overline{Q}_1\right)J_2'\left(q;\overline{Q}_1\right) > 0$ for at least some q, it follows that it cannot be the case that $\widehat{\Delta}\left(q\right) = 0$ for all q. Because $J_1'\left(q;\overline{Q}_1\right)J_2'\left(q;\overline{Q}_1\right) \geq 0$, it follows that any extreme point z must satisfy $\widehat{\Delta}\left(z\right) \geq 0$, which together with the boundary conditions implies that $\widehat{\Delta}\left(q\right) \geq 0$ for all q. Therefore, $\widehat{\Delta}'\left(\overline{Q}_1\right) < 0$, which in turn implies that $\widehat{J}_1'\left(\overline{Q}_1;\overline{Q}_1\right) > J_1'\left(\overline{Q}_1;\overline{Q}_1\right) = \alpha_1$. By noting that $\widehat{J}_1'\left(\widehat{Q}_1\left(q\right);\widehat{Q}_1\left(q\right)\right) = \alpha_1$ and $\widehat{J}_1'\left(Q;Q\right)$ is strictly increasing in Q, it follows that $\widehat{Q}_1\left(q\right) < \overline{Q}_1$.

Since $Q_1'\left(q\right) < 0$ for all q, it follows that $\widehat{Q}_1\left(q\right) < Q_1\left(q\right)$ for all q, and we know from Proposition 3 that $Q_1\left(q\right) < Q_2\left(q\right)$ for all q. ■

A.5 Proof of Lemma 2

Let $S(q;Q) = J_1(q;Q) + J_2(q;Q)$. Because, by assumption, $J_i(q;Q)$ is strictly concave in Q for all i and $q \leq Q \leq \overline{Q}_2$, it follows that $S(q;Q)$ is also strictly concave in Q for all $q \leq Q \leq \overline{Q}_2$. Therefore, $Q^*(q)$ will satisfy $\frac{\partial}{\partial Q} S(q;Q) = 0$ at $Q = Q^*(q)$ and $\frac{\partial}{\partial Q} S(q;Q)$ is strictly decreasing in Q for all q. We know from Proposition 3 that $Q_1(q) < Q_2(q)$ for all $q \leq \overline{Q}_2$. Moreover, we know that (i) $\frac{\partial}{\partial Q} J_1(q;Q) \geq 0$ and $\frac{\partial}{\partial Q} J_2(q;Q) > 0$ and so $\frac{\partial}{\partial Q} S(q;Q) > 0$ for all $q \leq Q_1(q)$, and (ii) $\frac{\partial}{\partial Q} J_1(q;Q) < 0$ and $\frac{\partial}{\partial Q} J_2(q;Q) \leq 0$ and so $\frac{\partial}{\partial Q} S(q;Q) < 0$ for all $q \geq Q_2(q)$. Because $\frac{\partial}{\partial Q} S(q;Q)$ is strictly decreasing in Q, it follows that $\frac{\partial}{\partial Q} S(q;Q) = 0$ for some $Q \in (Q_1(q), Q_2(q))$. ■

A.6 Proof of Proposition 4

We first construct a project-completing MPE with project scope $Q_i(0)$, and then argue the uniqueness of the equilibrium project scope.

Consider the following strategy profile.

- *Effort levels:* let both agents exert no effort at all states before the project scope has been decided. Once a project scope Q has been decided, let both agents choose their respective effort level as in the benchmark setting of Section 3 for a project of exogenous scope Q at all states $q \leq Q$, and let them exert no effort for all states $q > Q$.

- *Dictator's decision:* at any state q where no scope has yet been decided, let the dictator set the project scope $Q_i(q)$.

We verify that such strategy profile is an MPE.

First, let us fix the strategy of the dictator. Then at any state q, if the dictator's decision is yet to be made, agent j anticipates the scope to be set immediately, and exerting no effort is a best response. At any state q, if a decision of scope Q has been made by the dictator, agent j's effort levels are, by definition, a best response to the dictator's effort strategy.

Second, let us fix the effort strategy of agent j. If, at state q, the project scope has not been decided, the dictator never profits by delaying the decision to commit because agent j exerts no effort before the project scope is decided. Therefore, it is a best response to commit at state q. Furthermore, if he commits to project scope $Q \neq Q_i(q)$, the dictator's discounted payoff is $J_i(q;Q) \leq J_i(q;Q_i(q))$. Hence committing at state q to project scope $Q_i(q)$ is a best response. The effort levels of the dictator are, by definition, a best response to agent j's strategy.

Finally, we note that in any MPE, the dictator commits at the beginning of the project. Suppose he were to commit after the project started, say when the project reaches state $\check{q} > 0$. Since $J_i(\check{q};Q)$ has a unique maximum in Q, he commits to $Q_i(\check{q})$ and obtains payoff

$J_i(\cdot; Q_i(\check{q}))$. Then at state $q = 0$ there is a profitable deviation to commit immediately to $Q_i(0)$ and obtain payoff $J_i(0; Q_i(0)) > J_i(0; Q_i(\check{q}))$. Hence there is no MPE in which the dictator delays the announcement of the project scope. ■

A.7 Proof of Proposition 5

We begin by showing that if a project-completing equilibrium exists with scope Q, and if agent i is dictator, then $Q \le \overline{Q}_i$. This helps identify the set of Pareto-efficient equilibrium outcomes.

In an equilibrium of project scope Q, both agents anticipate that the project will be completed at state Q. Therefore, they will both work as they would in the benchmark game of fixed project scope Q described in Section 3. In particular, at any state $q \in [0, Q]$, each agent $k \in \{1, 2\}$ gets continuation payoff $J_k(q; Q)$.

If $Q > \overline{Q}_i$, then at any state $q \in (\overline{Q}_i, Q)$, Proposition 3 implies that $J_i(q; q) > J_i(q; Q)$, i.e., the dictator is strictly better off stopping the project when at state q, instead of stopping at state Q. Thus, $Q \le \overline{Q}_i$ in equilibrium.

Next, we show that, if agent 1 is the dictator, then $Q = \overline{Q}_1$ can be sustained in an MPE, whereas if agent 2 is the dictator, then any $Q \in [Q_1(0), Q_2(0)]$ can be sustained in an MPE. Observe that these project scopes are the Pareto-efficient ones, subject to the constraint that $Q \le \overline{Q}_i$ when agent i is dictator.

Let $Q^\dagger = \overline{Q}_1$ if agent 1 is the dictator and let $Q^\dagger \in [Q_1(0), Q_2(0)]$ if agent 2 is the dictator. Recall that, as explained in Section 3, for any *fixed, exogenous* scope $Q \in [0, \overline{Q}_2)$, the resulting MPE is completing, owing to the assumed strict concavity of $Q \mapsto J_2(0, Q)$ over that range. We verify that there exists an MPE with project scope $Q^\dagger$.

Consider the following strategy profile:

- *Effort levels:* for any state $q \le Q^\dagger$, let both agents choose their effort optimally in a game of fixed project scope $Q^\dagger$, and for all $q > Q^\dagger$ let them exert no effort. Note that, because the unique MPE of a project of fixed scope $Q^\dagger$ is completing, both agents put positive effort at every state up to $Q^\dagger$.

- *Dictator's decision:* let the dictator stop the project immediately whenever $q \ge Q^\dagger$.

To show such strategy profile is an MPE, we must show that agents play a best response to each other at every state.

First, fix the dictator's strategy. Then agent j anticipates to be working on a project of scope $Q^\dagger$, and it follows directly from agent j's effort strategy that agent j plays a best response at every state $q \le Q^\dagger$. At any state $q > Q^\dagger$, agent j anticipates that the dictator completes the project immediately, and so putting no effort is a weakly best response.

Now, let us fix agent j's strategy. If the dictator completes the project at state $Q^\dagger$, then his effort level is optimal given j's effort level, by definition of agent i's effort strategy.

Let us check that terminating the project at every state $q \geq Q^\dagger$ is optimal for the dictator. Consider state $q \geq Q^\dagger$. As agent j exerts no effort for all states greater that $Q^\dagger$, and as $Q^\dagger \geq \overline{Q}_1 > \widehat{Q}_i$, the dictator has no incentive to continue the project by himself: he is always better off stopping the project immediately.

Now consider state $q < Q^\dagger$.

- If agent 1 is the dictator, then as $q < \overline{Q}_1 < Q_1(q)$, by our assumption that $Q \mapsto J_1(q; Q)$ is strictly concave on $[q, \overline{Q}_2)$ and is maximized for $Q = Q_1(q)$, it is also strictly increasing $[q, Q_1(q)]$. This implies that $J_1(q; Q_1(q)) > J_1(q; \overline{Q}_1) > J_1(q; q)$, and so the agent has no incentive to collect the termination payoff before reaching state $\overline{Q}_1$.

- If agent 2 is the dictator, then by Lemma 8 (see the proof of Proposition 3), $Q \mapsto J_2'(Q; Q)$ increases on $[\overline{Q}_1, \overline{Q}_2]$, and $J_2'(\overline{Q}_2; \overline{Q}_2) = J_2'(Q_2(\overline{Q}_2); Q_2(\overline{Q}_2)) = \alpha_2$. Besides, $J_2(Q; Q) = \alpha_2 Q$ and Proposition 1 shows that $J_2(q; Q)$ is strictly convex in q for $q \leq Q \leq \overline{Q}_2$. Hence $J_2'(q; Q) < \alpha_2$ for $q < Q < \overline{Q}_2$, which in turn implies that $J_2(q; Q) > \alpha_2 q$ for all $q < Q$ with $Q < \overline{Q}_2$. So, if $q < Q^\dagger$, then $J_2(q; q) = \alpha_2 q < J_2(q; Q^\dagger)$, and hence agent 2 has no incentive to complete the project before reaching state $Q^\dagger$.

In conclusion, the strategies defined above form a project-completing MPE with project scope $Q^\dagger$. ■

A.8 Proof of Proposition 6

Fix some $Q^\dagger \in [Q_1(0), Q_2(0)]$. We construct a project-completing MPE with project scope $Q^\dagger$. Observe that any project scope $Q' \notin [Q_1(0), Q_2(0)]$ is Pareto-dominated; that is , there exists some $Q^* \in [Q_1(0), Q_2(0)]$ such that $J_i(0; Q^*) \geq J_i(0; Q')$ for all i. Consider the following strategy profile.

- *Effort levels:* Before a project scope has been committed to, each agent i exerts effort $a_i(q; -1) = a_i(q; Q^\dagger)\mathbb{I}_{\{q<Q^\dagger\}}$. After a project scope Q has been committed to, each agent exerts effort $a_i(q; Q)\mathbb{I}_{\{q<Q\}}$, where $a_i(q; Q)$ is characterized in the benchmark setting of Section 3 for a project of exogenous scope Q.

- *Agenda setter proposals:* Let the agenda setter propose project scope $Q^\dagger$ at every state $q \leq Q^\dagger$, and propose to stop the project immediately at every state $q > Q^\dagger$.

- *Agent j's decisions:* In a project state $q > Q^\dagger$, agent j accepts the agenda setter's proposal to stop at Q for all Q with $J_j(q; Q) \geq J_j(q; q)$, and rejects the proposal otherwise. In a state $q \leq Q^\dagger$, let agent j accept the agenda setter's proposal to stop at Q whenever $J_j(q;Q) \geq J_j(q;Q^\dagger)$ and reject the proposal otherwise.

We now show that such strategy profile is an MPE. First, fix the agenda setter's strategy. It follows directly from agent j's strategy that agent j plays a best response at every state—both in terms of effort and response to proposals of the agenda setter.

Now take the strategy of agent j as given. If at state q a project scope Q has already been agreed upon, the agenda setter, who can no longer change the project scope, plays a best response (in terms of effort level) to the strategy of agent j. It remains to show that the agenda setter plays a best response at every q when no project state has been agreed on yet. If he anticipates the project scope to be $Q^\dagger$, then his effort levels are optimal in every state. Let us check that the proposal strategy is indeed optimal, and yield project scope $Q^\dagger$.

- If $q \geq Q^\dagger$, and agent 1 is the agenda setter, then agent 1 is better off if the project stops immediately: since $Q_1(q) = q$ as $Q^\dagger \geq \overline{Q}_1$, $J_1(q;q) > J_1(q;Q)$ for every $Q > q$. If agent 1 proposes to stop the project at state q, then agent 2 accepts, by definition of agent 2's strategy. Hence it is optimal for agent 1 to propose to stop the project at state q, and the conjectured equilibrium strategy of agent 1 is a best response to agent 2's strategy.

- If $q \geq Q^\dagger$, and agent 2 is the agenda setter, then agent 2 would prefer in some cases to pursue the project with agent 1, but never wants to pursue the project by himself, because $Q^\dagger > \widehat{Q}_2$. As agent 1 only accepts proposals to stop right away, and as he exerts no effort past state $Q^\dagger$ until a scope proposed is accepted, agent 2 is better off proposing to stop the project at the current state q—proposition accepted by agent 1. Hence the conjectured equilibrium strategy of agent 2 is a best response to agent 1's strategy.

- If $q < Q^\dagger$, and agent 1 is the agenda setter, then the agenda setter can guarantee himself a continuation payoff $J_i(q;Q^\dagger)$ by following the strategy defined in the above conjectured equilibrium profile. Assume by contradiction that there is an alternative strategy for the agenda setter that yields a strictly higher payoff. Such strategy must generate a different project scope, Q. In addition, that project scope must be less than $Q^\dagger$ for agent 1 to be better off, and so an agreement must be reached before state $Q^\dagger$. But then $J_2(q;Q) < J_2(q;Q^\dagger)$, and by definition of agent 2's strategy, agent 2 would not accept agent 1's proposal to set scope Q at any state $q < Q^\dagger$. Hence the conjectured equilibrium strategy of agent 1 is a best response to agent 2's strategy.

- If $q < Q^\dagger$, and agent 2 is the agenda setter, then as before the agenda setter can guarantee himself a continuation payoff $J_2(q;Q^\dagger)$ by following the strategy defined in the above conjectured equilibrium profile. Assume by contradiction that there is an alternative strategy for the agenda setter that yields strictly higher payoff with a different project scope Q. Then, as agent 2 is strictly better off, it must be that $Q > Q^\dagger$, as $J_2(q;Q)$ is strictly increasing in Q when $Q < Q^\dagger$. However, agent 1 would not accept such a proposal of project Q before reaching state $Q^\dagger$. He may accept such a proposal in state $q = Q$, however, between state $Q^\dagger$ and Q exerts no effort. As $Q^\dagger > \widehat{Q}_2$, agent 2 is never better off pursuing and completing the project by himself past state $Q^\dagger$, and thus a project scope $Q = Q^\dagger$ is optimal. Hence the conjectured equilibrium strategy of agent 2 is a best response to agent 1's strategy.

Therefore the conjectured strategy profile constitutes a project-completing MPE with project scope $Q^\dagger$. ■

A.9 Proof of Proposition 7

Fix some $Q^\dagger \in [Q_1(0), Q_2(0)]$. As in the proof of Proposition 6, we show that $Q^\dagger$ can be sustained in some MPE. Let us consider the following strategy profile.

1. *Effort levels:* let both agents choose an effort level optimal for a project of fixed scope $Q^\dagger$, and put zero effort for any state $q > Q^\dagger$.

2. *Agenda setter proposals:* let the agenda setter propose to stop the project for any state $q \geq Q^\dagger$, and continue to project for all $q < Q^\dagger$.

3. *Agent j's decisions:* let agent j accept the agenda setter's proposal to stop for all states $q \geq Q^\dagger$, and otherwise accept to stop whenever $J(q;q) \geq J(q;Q^\dagger)$.

Let us show that such strategy profile is an MPE.

Let us fix the strategy of the agenda setter and check that agent j's strategy is a best response at every state.

- First, suppose agent 1 is the agenda setter. If he proposes to stop the project at a state $q \geq Q^\dagger$, agent 2 should accept: agent 1 puts no effort past state $Q^\dagger$, and agent 2 would rather not work alone on the project as $\widehat{Q}_2 < Q^\dagger$. If agent 1 proposes to stop at a state $q < Q^\dagger$, then agent 2 should accept only if the payoff he makes from immediate project termination, $J_2(q;q)$ is no less than the payoff he makes by rejecting—which then pushes back the next anticipated proposal at state $Q^\dagger$, $J_2(q;Q^\dagger)$. Given the agenda setter's strategy, agent 2 expects to complete the project in state $Q^\dagger$, and by definition of agent 2's effort strategy, the effort levels of agent 2 are optimal at all states.

- Second, suppose agent 2 is the agenda setter. If agent 1 is offered to stop the project at $q \geq Q^\dagger$, then agent 1 finds it optimal to accept because $Q_1(q) = q$ for all $q \geq \overline{Q}_1$. If agent 1 is offered to stop the project at $q < Q^\dagger$, then he should accept only if the payoff from immediate project termination $J_1(q;q)$ is no less than the payoff he expects to make from rejecting, which as before is $J_1(q;Q^\dagger)$. Given the agenda setter's strategy, agent 1 expects to complete the project in state $Q^\dagger$, and by definition of agent 1's effort strategy, the effort levels of agent 1 are optimal at all states.

Next let us fix the strategy of agent j and check that the agenda setter's strategy is a best response at every state.

- First, suppose agent 1 is the agenda setter. Then agent 1 expects to make payoff $J_1(q;Q^\dagger)$ by following the conjectured equilibrium strategy. To make a better payoff, he would have to complete the project at a state $Q < Q^\dagger$. However such a proposal to stop the project early would not be accepted by agent 2, who is better off working towards a project of scope $Q^\dagger$, because $J_2(q;Q)$ is increasing in Q for all $Q \leq Q^\dagger \leq Q_2(q)$. Hence not proposing to stop before state $Q^\dagger$ is a (weak) best response. As agent 2 accepts to stop at all states $q \geq Q^\dagger$, agent 1 is better off proposing to stop at all states $q \geq Q^\dagger$, because $Q_1(q) = q$ for all $q \geq Q^\dagger \geq \overline{Q}_1$. Therefore, agent 1 anticipates the project scope to be $Q^\dagger$ and his effort levels are optimal for such a project scope.

- Second, suppose agent 2 is the agenda setter. Then agent 2 expects to make payoff $J_2(q;Q^\dagger)$ by following the conjectured equilibrium strategy, and to make a larger payoff would require completing the project at a state $Q > Q^\dagger$. Therefore it is never optimal for agent 2 to stop at any $Q < Q^\dagger$. However it is always optimal to stop at every $Q \geq Q^\dagger$, as agent 1 plans to put in no effort after Q, and agent 2 prefers not to work alone on the project since $\widehat{Q}_2 < Q^\dagger$.

Hence the conjectured strategy profile constitutes a project-completing MPE with project scope $Q^\dagger$. ■

A.10 Proof of Proposition 8

Fix some $Q > 0$. We use the normalization $\widetilde{J}_i(q) = \frac{J_i(q)}{\gamma_i}$ as in the proof of Proposition 1.

To prove part 1, assume that $\frac{\gamma_1}{\alpha_1} < \frac{\gamma_2}{\alpha_2}$, let $\widetilde{D}(q) = \widetilde{J}_1(q) - \widetilde{J}_2(q)$, and note that $\widetilde{D}(\cdot)$ is smooth, $\lim_{q\to-\infty}\widetilde{D}(q) = 0$ and $\widetilde{D}(Q) = \left(\frac{\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2}\right)Q > 0$. Suppose that $\widetilde{D}(\cdot)$ has an interior global extreme point, and denote such extreme point by $\overline{q}$. Because $\widetilde{D}(\cdot)$ is smooth, it must be the case that $\widetilde{D}'(\overline{q}) = 0$. Then it follows from (5) that $r\widetilde{D}(\overline{q}) = \frac{\sigma^2}{2}\widetilde{D}''(\overline{q})$. If $\overline{q}$ is a maximum, then $\widetilde{D}''(\overline{q}) \leq 0$, so $\widetilde{D}(\overline{q}) \leq 0$, which contradicts the fact that $\lim_{q\to-\infty}\widetilde{D}(q) = 0$ and the assumption that $\overline{q}$ is a maximum. On the other hand, if $\overline{q}$ is a minimum, then $\widetilde{D}''(\overline{q}) \geq 0$, so $\widetilde{D}(\overline{q}) \geq 0$, which contradicts the fact that $\lim_{q\to-\infty}\widetilde{D}(q) = 0$ and the assumption that $\overline{q}$ is a minimum. Therefore, $\widetilde{D}'(q) > 0$ for all q, which implies that $a_1(q) > a_2(q)$ for all q.

To prove part 2, let $D(q) = \frac{J_1(q)}{\alpha_1} - \frac{J_2(q)}{\alpha_2}$, and note that $D(\cdot)$ is smooth, $\lim_{q\to-\infty} D(q) = 0$, and $D(Q) = 0$. Therefore, either $D(q) = 0$ for all q, or $D(\cdot)$ has an interior global extreme point. Suppose that the former is true. Then for all q, we have $D(q) = D'(q) = D''(q) = 0$, which using (3) implies that

$$rD(q) = \frac{[J_1'(q)]^2}{2\alpha_1^2}\left(\frac{\alpha_2}{\gamma_2} - \frac{\alpha_1}{\gamma_1}\right) = 0 \Longrightarrow J_1'(q) = 0\,.$$

By Proposition 1, we have $J_i' > 0$ in any project-completing MPE so this is a contradiction. Thus the latter must be true. Then there exists some $\overline{q}$ such that $D'(\overline{q}) = 0$. Using (3), this implies that

$$rD(\overline{q}) = \frac{[J_1'(\overline{q})]^2}{2\alpha_1^2}\left(\frac{\alpha_2}{\gamma_2} - \frac{\alpha_1}{\gamma_1}\right) + \frac{\sigma^2}{2}D''(\overline{q})\ ,$$

and note that $J_1'(\overline{q}) > 0$. Suppose that $\overline{q}$ is a maximum. Then $D''(\overline{q}) \leq 0$, which together with the fact that $\frac{\alpha_2}{\gamma_2} < \frac{\alpha_1}{\gamma_1}$ implies that $D(\overline{q}) < 0$. Therefore, $D(q) \leq 0$ for all q, which completes the proof of part 2.

Finally, if $\frac{\alpha_1}{\gamma_1} = \frac{\alpha_2}{\gamma_2}$, then it follows from the analysis above that $\widetilde{D}(q) = \widetilde{D}'(q) = 0$ and $D(q) = 0$, which implies that $a_1(q) = a_2(q)$ and $\frac{J_1(q)}{\alpha_1} = \frac{J_2(q)}{\alpha_2}$ for all $q \geq 0$. ■

B Additional Results

B.1 Propositions 1 and 2 hold under broader assumptions

In this Appendix, we show that Propositions 1 and 2 hold under a broader class of effort cost functions. In particular, suppose that effort level a induces flow cost equal to $c_i(a) = \gamma_i c(a)$ to agent i, where $\gamma_i > 0$, and $c(\cdot)$ is some arbitrary function that satisfies $c', c'' > 0$, $c''' \geq 0$, $c(0) = 0$, and $\lim_{a\to\infty} c(a) = \infty$. Using similar arguments as in Section 3, it follows that for any fixed $Q > 0$, each agent i's payoff function satisfies the HJB equation

$$rJ_i(q) = \max_{\hat{a}_i}\left\{-\gamma_i c(\hat{a}_i) + (\hat{a}_i + a_j(q))\, J_i'(q)\right\}$$

subject to the boundary condition $J_i(Q) = \alpha_i Q$. By using the normalization $\tilde{J}_i(q) = \frac{J_i(q)}{\gamma_i}$, it follows that in a well-behaved MPE, each agent's discounted payoff satisfies the following system of ODE:

$$r\tilde{J}_i(q) = -c\left(f\left(\tilde{J}_i'(q)\right)\right) + \left[f\left(\tilde{J}_i'(q)\right) + f\left(\tilde{J}_j'(q)\right)\right]\tilde{J}_i'(q) \tag{19}$$

subject to $\tilde{J}_i(Q) = \frac{\alpha_i}{\gamma_i}Q$, where $f(\cdot) = c'^{-1}(\cdot)$, and each agent's effort level is given by $a_i(q) = f\left(\tilde{J}_i'(q)\right)$. Cvitanić & Georgiadis (2016) show that if a project-completing MPE exists, then Proposition 1, part 1 holds; i.e., $J_i(q) > 0$, $J_i'(q) > 0$, and $a_i'(q) > 0$ for all i and $q \geq 0$.

The following result establishes conditions such that Proposition 2 holds under a broader class of effort cost functions.

Proposition 9. *Suppose that $\frac{\gamma_1}{\alpha_1} < \frac{\gamma_2}{\alpha_2}$. In any project-completing MPE:*

1. *Agent 1 exerts higher effort than agent 2 in every state; i.e., $a_1(q) \geq a_2(q)$ for all $q \geq 0$.*

2. *Agent 1's effort increases at a greater rate than agent 2 (i.e., $a_1'(q) \geq a_2'(q)$ for all $q \geq 0$) if $c'(\cdot)$ is weakly log-concave; i.e., $\log c'(a)$ is weakly concave in a.*

3. *Agent 1 obtains a lower discounted payoff normalized by project state than agent 2 (i.e., $\frac{J_1(q)}{\alpha_1} \leq \frac{J_2(q)}{\alpha_2}$ for all $q \geq 0$) if $c(\cdot)$ is weakly log-concave.*

Proof of Proposition 9.

Statement 1. Define $\tilde{D}(\cdot) = \tilde{J}_1(\cdot) - \tilde{J}_2(\cdot)$, and note that $\tilde{D}(\cdot)$ is smooth, $\tilde{D}(q) = 0$ for q sufficiently small (possibly $q < 0$), and $\tilde{D}(Q) = \left(\frac{\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2}\right) Q > 0$. Therefore, either $\tilde{D}'(q) \geq 0$ for all q, or $\tilde{D}(\cdot)$ has at least one interior extreme point. Suppose that the latter is true. Then there exists some z such that $\tilde{D}'(z) = 0$ and substituting into (19) yields

$$r\tilde{D}(z) = 0$$

Because any interior extreme point z must satisfy $\tilde{D}(z) = 0$ and $\tilde{D}(\cdot)$ is continuous, it must be the case that $\tilde{D}(q) \geq 0$ and $\tilde{D}'(q) \geq 0$ for all q. Therefore, $\tilde{J}_1'(q) \geq \tilde{J}_2'(q)$ for all q, and because $f(\cdot)$ is monotone, it follows that $a_1(q) \geq a_2(q)$ for all q.

Statement 2. To prove the second part, we differentiate (19) with respect to q, which yields in matrix form

$$r \begin{bmatrix} \tilde{J}_1' \\ \tilde{J}_2' \end{bmatrix} = \begin{bmatrix} f\left(\tilde{J}_1'\right) + f\left(\tilde{J}_2'\right) & \tilde{J}_1' f'\left(\tilde{J}_2'\right) \\ \tilde{J}_2' f'\left(\tilde{J}_1'\right) & f\left(\tilde{J}_1'\right) + f\left(\tilde{J}_2'\right) \end{bmatrix} \begin{bmatrix} \tilde{J}_1'' \\ \tilde{J}_2'' \end{bmatrix},$$

where we used that $c'(f(x)) = x$, and we omitted the dependence of $\left\{\tilde{J}_1, \tilde{J}_2\right\}$ on q for notational convenience. If the determinant of the above matrix is positive; i.e., if

$$\det := \left[f\left(\tilde{J}_1'\right) + f\left(\tilde{J}_2'\right)\right]^2 - \tilde{J}_1'\tilde{J}_2' f'\left(\tilde{J}_1'\right) f'\left(\tilde{J}_2'\right) > 0,$$

then it is invertible. A sufficient condition for this to be true is that $c''' \geq 0$.[27] Then we have that

$$\begin{bmatrix} \tilde{J}_1'' \\ \tilde{J}_2'' \end{bmatrix} = \frac{r}{\det} \begin{bmatrix} f\left(\tilde{J}_1'\right) + f\left(\tilde{J}_2'\right) & -\tilde{J}_1' f'\left(\tilde{J}_2'\right) \\ -\tilde{J}_2' f'\left(\tilde{J}_1'\right) & f\left(\tilde{J}_1'\right) + f\left(\tilde{J}_2'\right) \end{bmatrix} \begin{bmatrix} \tilde{J}_1' \\ \tilde{J}_2' \end{bmatrix}.$$

Note that $a_1'(q) \geq a_2'(q)$ if and only if $\tilde{J}_1''(q) \geq \tilde{J}_2''(q)$, which is true if and only if

$$\begin{aligned} \left[f\left(\tilde{J}_1'\right) + f\left(\tilde{J}_2'\right)\right] \tilde{J}_1' - \tilde{J}_1' \tilde{J}_2' f'\left(\tilde{J}_2'\right) &\geq -\tilde{J}_1' \tilde{J}_2' f'\left(\tilde{J}_1'\right) + \left[f\left(\tilde{J}_1'\right) + f\left(\tilde{J}_2'\right)\right] \tilde{J}_2' \\ \Leftrightarrow \left[f\left(\tilde{J}_1'\right) + f\left(\tilde{J}_2'\right)\right]\left(\tilde{J}_1' - \tilde{J}_2'\right) + \tilde{J}_1' \tilde{J}_2' \left[f'\left(\tilde{J}_1'\right) - f'\left(\tilde{J}_2'\right)\right] &\geq 0 \quad (20) \end{aligned}$$

Recall that $\left[f\left(\tilde{J}_1'\right) + f\left(\tilde{J}_2'\right)\right]^2 > \tilde{J}_1' \tilde{J}_2' f'\left(\tilde{J}_1'\right) f'\left(\tilde{J}_2'\right)$ and $\tilde{J}_1' \geq \tilde{J}_2'$. Therefore, (20) is satisfied if

$$\begin{aligned} \tilde{J}_2' f'\left(\tilde{J}_2'\right)\left(\tilde{J}_1' - \tilde{J}_2'\right) + \tilde{J}_1' \tilde{J}_2' \left[f'\left(\tilde{J}_1'\right) - f'\left(\tilde{J}_2'\right)\right] &\geq 0 \\ \Leftrightarrow \tilde{J}_2' \left[\tilde{J}_1' f'\left(\tilde{J}_1'\right) - \tilde{J}_2' f'\left(\tilde{J}_2'\right)\right] &\geq 0 \end{aligned}$$

Noting that $f\left(\tilde{J}_i'\right) = a_i$, $\tilde{J}_i' = c'(a_i)$, $f = c'^{-1}$, and $f'\left(\tilde{J}_i'\right) = \frac{1}{c''(a_i)} > 0$, it follows that the above inequality holds if and only if $\frac{c'(a)}{c''(a)}$ is increasing in a. This is true if and only if

$$\left[c''(a)\right]^2 \geq c'(a)\, c'''(a) \quad \text{for all } a,$$

or equivalently if $c'(a)$ is weakly log-concave.

Statement 3. Recall that in any well-defined MPE, each agent's payoff satisfies the system of ODE

$$rJ_i(q) = -\gamma_i c\left(f\left(\frac{J_i'(q)}{\gamma_i}\right)\right) + \left[f\left(\frac{J_i'(q)}{\gamma_i}\right) + f\left(\frac{J_j'(q)}{\gamma_j}\right)\right] J_i'(q) \quad \text{s.t. } J_i(Q) = \alpha_i Q. \quad (21)$$

Define $D(\cdot) = \frac{J_1(\cdot)}{\alpha_1} - \frac{J_2(\cdot)}{\alpha_2}$, and note that $D(\cdot)$ is smooth, $D(q) = 0$ for q sufficiently small, and $D(Q) = 0$. Therefore, there must exist an interior point z such that $D'(z) = 0$, and substituting into *(21)* yields

$$\begin{aligned} rD(z) &= -\frac{\gamma_1}{\alpha_1} c\left(f\left(\frac{J_1'(z)}{\gamma_1}\right)\right) + \frac{\gamma_2}{\alpha_2} c\left(f\left(\frac{J_2'(z)}{\gamma_2}\right)\right) \\ \Rightarrow r\alpha_1 D(z) &= -\gamma_1 c\left(f\left(\frac{J_1'(z)}{\gamma_1}\right)\right) + \underbrace{\frac{\alpha_1\gamma_2}{\alpha_2}}_{>\gamma_1} c\left(f\left(\frac{\alpha_2}{\alpha_1}\frac{J_1'(z)}{\gamma_2}\right)\right) \end{aligned}$$

Notice that if $D(z) \leq 0$, then this will imply that $D(q) \leq 0$ for all q, which will complete the proof. To establish $D(z) \leq 0$, notice that it suffices to show that $\frac{c(f(\lambda x))}{\lambda}$ is increasing in λ for all $x > 0$ and $\lambda > 0$. That is because letting $x = J_1'(z)$, $\lambda_1 = \frac{1}{\gamma_1}$ and $\lambda_2 = \frac{\alpha_2}{\alpha_1\gamma_2}$, where

[27] For details, see footnote 20 on on p. 29 in Cvitanic and Georgiadis.

$\lambda_1 > \lambda_2$ we will have $-\frac{c(f(\lambda_1 x))}{\lambda_1} + \frac{c(f(\lambda_2 x))}{\lambda_2} \leq 0$.

Fix x, and let $g(\lambda) = \frac{c(f(\lambda x))}{\lambda}$. Then

$$\begin{aligned} g'(\lambda) &= \frac{x}{\lambda} c'(f(\lambda x)) f'(\lambda x) - \frac{c(f(\lambda x))}{\lambda^2} \\ &= x^2 \frac{1}{c''(f(\lambda x))} - \frac{c(f(\lambda x))}{\lambda^2} \geq 0 \\ &\Leftrightarrow (\lambda x)^2 \geq c(f(\lambda x))\, c''(f(\lambda x)) \end{aligned}$$

Letting $a = f(\lambda x) = c'^{-1}(\lambda x)$, observe that $\lambda x = c'(a)$, and substituting this into the above inequality yields

$$\left[c'(a)\right]^2 \geq c(a)\, c''(a) \ ,$$

which holds for all a if and only if $c(\cdot)$ is weakly log-concave.

□

B.2 Social planner's project scope and effort level

A classic benchmark of the literature is the cooperative environment in which agents follow the social planner's recommendations for effort. Here, we present, for completeness, the solution when the social planner chooses both the agents' level of effort and the project scope.

For a fixed project scope Q, the social planner's relevant HJB equation is

$$rS(q) = \max_{a_1, a_2} \left\{ -\tfrac{\gamma_1}{2} a_1^2 - \tfrac{\gamma_2}{2} a_2^2 + (a_1 + a_2) S'(q) \right\} ,$$

subject to $S(Q) = Q$. Each agent's first-order condition is $a_i = \frac{S'(q)}{\gamma_i}$, and substituting this into the HJB equation, we obtain the ordinary differential equation $rS(q) = \frac{\gamma_1 + \gamma_2}{2\gamma_1 \gamma_2} [S'(q)]^2$. This admits the closed form solution for the social planner's value function $S(q) = \frac{r\gamma_1\gamma_2}{2(\gamma_1+\gamma_2)} (q - C)^2$, where $C = Q - \sqrt{\frac{2Q(\gamma_1+\gamma_2)(\alpha_1+\alpha_2)}{r\gamma_1\gamma_2}}$. Agent i's effort level is thus $a_i(q) = \frac{r\gamma_{-i}}{\gamma_1+\gamma_2} (q - C)$. Note that $a_1(q) > a_2(q)$ for all q if and only if $\gamma_1 < \gamma_2$. That is, the social planner would have the efficient agent do the majority of the work, and incur the majority of the effort cost. It is straightforward to show that the social planner's discounted payoff function is maximized at

$$Q^{**} = \frac{(\gamma_1 + \gamma_2)(\alpha_1 + \alpha_2)}{2r\gamma_1\gamma_2}$$

at every state of the project, and thus, the planner's preferences are time-consistent. This is intuitive, as the time-inconsistency problem is due to the agents not internalizing the externality of their actions and choices.

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