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CAPITAL SHARE DYNAMICS WHEN FIRMS INSURE WORKERS

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ABSTRACT

Although the aggregate capital share of U.S. firms has increased, the firm-level capital share of a typical U.S. firm has decreased. This divergence is due to mega-firms that now produce a larger output share without a proportionate increase in labor compensation. We develop a model in which firms insure workers against firm-specific shocks, where more productive firms allocate more rents to shareholders, while less productive firms endogenously exit. Increasing firm-level risk delays exit and increases the measure of mega-firms, which raises the aggregate capital share while lowering the average firm's capital share. An increase in the level of rents quantitatively magnifies this effect. We present evidence supporting this mechanism.

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Over the last several decades, publicly traded U.S. firms have experienced large increases in the firm-specific volatility of both firm-level cash flow and returns (see, e.g., Campbell, Lettau, Malkiel, and Xu, 2001; Comin and Philippon, 2005; Xiaolan, 2014; Bloom, 2014; Herskovic, Kelly, Lustig, and Van Nieuwerburgh, 2015). At the same time, the share of total value added that accrues to the owners of these firms (i.e., the aggregate capital share) has also increased (see Karabarbounis and Neiman, 2014; Piketty and Zucman, 2014). We find that the aggregate factor shares are largely determined by the firm-level factor shares of the largest U.S. firms in the right tail of the size distribution. These mega-firms have experienced substantial increases in their capital share even though the capital share of the average U.S. firm has decreased.

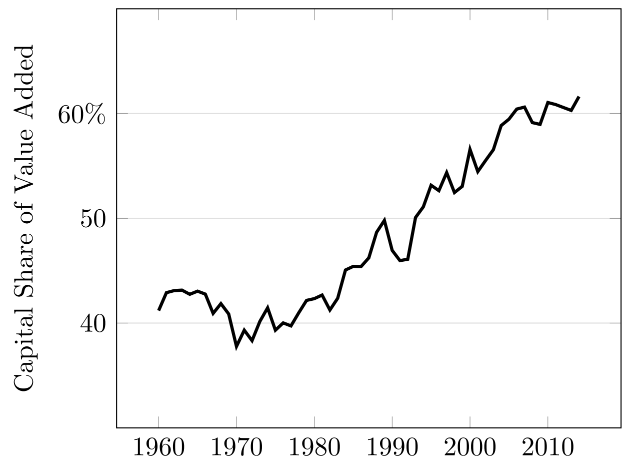
The U.S. economy's aggregate factor share dynamics are well understood, but its firm-level factor share dynamics are not. Between 1960 and 2010, the U.S. labor share of total output in the non-farm business sector of the U.S. economy shrank by 15%, and this phenomenon does not appear to be limited to the U.S. (see, e.g., Karabarbounis and Neiman, 2014). Figure 1 plots the capital share over time for the universe of U.S. publicly traded firms, demonstrating an increase from 40% to 60% since 1980. In this plot, capital share is measured as the ratio of capital income to valued added, where capital income is defined as operating income before depreciation (OIBDP) and value added is defined as operating income plus labor expenses.[1] Our key empirical contribution is to show that the increase in the capital share is concentrated among the largest publicly traded firms in the U.S. Figure 2 shows the relationship between firm size and the ratio of capital income to sales, which is a measure of the capital share of profits. In 1970, there was essentially no relation between firm size and the capital-income-to-sales ratio, but by 2010, this ratio had strongly increased in size. This shift caused the average and aggregate capital shares to diverge, and the equal-weighted average capital share of publicly traded companies has declined since the 1980s. The U.K. and continental Europe have experienced similar divergences between the aggregate and average factor shares over this sample, and increases in idiosyncratic risk. Japan, in contrast, has experienced neither.

Recently, scholars have documented similar findings in a different universe of U.S. firms. Kehrig and Vincent (2017) find similar evidence using establishment-level data in the manufacturing industry. Using census data, in work preceded by ours, Autor, Dorn, Katz, Patterson, and Reenen (2017) attribute the decrease in labor share to the low labor share of the largest firms, which is consistent with our earlier findings, but they abstract from the equally important changes in factor shares in the left tail of the size distribution.

We develop an equilibrium model that links our observations regarding volatility and factor shares and provides novel implications for national income accounting. When shareholders insure workers against idiosyncratic risk, capital shares vary substantially across firms, with the largest and most productive firms having the highest capital share. In our model, changes in the size distribution of firms triggered by changes in firm-level risk have first-order implications for the aggregate and average capital shares in two ways. First, in our model, an increase in idiosyncratic

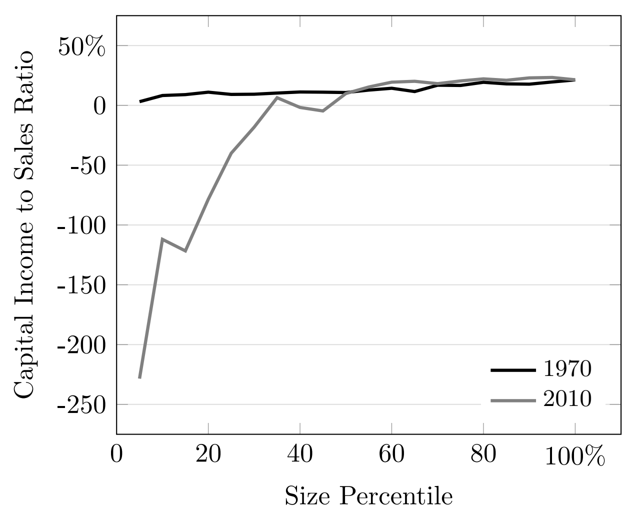
[1] We impute labor expenses for the Compustat sample using the method developed by Donangelo (2016).

Figure 1: Aggregate Capital Share of Total Value Added for Public Firms.



The figure presents the aggregate capital share for all of the firms in the Compustat public firms database. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014). The aggregate capital share is defined as $\sum_i$ capital income$_i$ divided by $\sum_i$ value added$_i$ for each year. Capital income is defined as operating income before depreciation (OIBDP). Value added is defined as operating income plus labor expenses. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

Figure 2: Firm-Level Capital Income to Sales Ratio by Size.



This figure presents the relation between the ratio of capital income (OIBDP) to sales and firm size for all of the firms in the Compustat public firms database. Firm size is measured as total assets. Each point represents the within-bin average of the ratio after grouping firms into 20 size bins. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

volatility coupled with an increase in economic rents can quantitatively explain both the shift in the aggregate and the average capital share. Second, consistent with our model, U.S. industries that saw larger increases in idiosyncratic volatility experienced larger drops in the average firm-level capital share. We document similar findings for the U.K., Europe, and, to a lesser extent, Japan.

Shareholders of publicly traded firms can decrease idiosyncratic firm-specific risk through diversification, while risk-averse workers cannot; therefore, it is efficient to provide workers with insurance against firm-specific risk. We analyze a simple optimal contracting model that embeds this intuition in an equilibrium model of industry dynamics (see, e.g., Hopenhayn, 1992). The optimal contract pays workers a fixed wage while allocating the remainder of the profits to shareholders. The level of compensation is set in equilibrium to capture the value of ex-ante identical firms. Ex post, these firms are subject to permanent idiosyncratic shocks that lead some firms to increase in size and productivity while others decrease and potentially exit. We use this model as a laboratory to analyze the impact of changes in firm-level risk on the distribution of rents.

Standard national income accounting, which is applied in this model, yields a new perspective on capital share dynamics. The worker's compensation is set such that the net present value of starting a new firm, computed by integrating over all paths using the density for a new firm and discounting, is zero. In contrast, national income accounts integrate only over firms that are currently active using the stationary size distribution, without discounting. As a result, the aggregate capital share calculation puts more probability mass on the right tail than the NPV calculation. As firm-level risk increases and the right tail of the firm size distribution grows, workers capture a smaller fraction of aggregate rents ex post even though they capture all of the ex-ante rents. This effect is partially offset by a larger mass of unprofitable firms in the left tail of the stationary size distribution.

In our model, an increase in firm-level risk invariably increases the aggregate capital share because we use time discounting when computing firm values but not when aggregating output. In the optimal contract, the level of compensation is proportional to the average value of newly established firms, and the average value of new firms is in turn equal to the present value of all future paths of productivity. When volatility increases, a greater fraction of future paths lead to high levels of productivity; however, these paths are discounted when calculating firm value. In contrast, aggregate productivity is simply the average of current firm level productivity with respect to the stationary distribution. When idiosyncratic volatility increases, the right tail of the stationary distribution increases in mass. This increase in mass is fully reflected in aggregate output without discounting. Thus, an increase in idiosyncratic volatility increases aggregate output by more than the aggregate compensation, and the aggregate capital share increases as a result.

At the heart of this mechanism is the selection effect that arises from measuring the distribution of rents while excluding firms that have endogenously exited[2]: the capital share computed

[2]Jovanovic (1982) is the first study of selection in an equilibrium model of industry dynamics. Selection has also been found to be quantitatively important. Luttmer (2007) attributes about 50% of output growth to selection using a model with firm-specific productivity improvements, selection of successful firms, and imitation by entrants. This effect is closely related to Hopenhayn (2002)'s observation that selection biases use average Tobin's Q estimates for industries above one.

in national income accounts produces a biased estimate of the ex-ante profitability of new firms. Moreover, an increase in selection increases the size of this bias. This effect explains the measured divergence between aggregate compensation and profits: compensation is tied to ex-ante profitability, but not to ex-post realized profits. This result also has a natural insurance interpretation. When idiosyncratic risk increases, workers effectively pay a larger idiosyncratic insurance premium ex post to shareholders. The increase in this ex-post premium leads to an increase in the aggregate capital share even though the shareholders are risk-neutral and receive zero rents ex ante.

This selection effect is quantitatively important for capital share dynamics. In a calibrated version of our model, we find that doubling the size of economic rents (see, e.g., Furman and Orszag, 2015; Barkai, 2016; De Loecker and Eeckhout, 2017) together with an increase in volatility from 20% to 40% replicates the increase in the aggregate capital share and the decrease in the average capital share. The joint increase in volatility and rents is essential to the quantitative performance of our model. The increases in volatility change how rents are shared between a firm's owners and its workers, while the increase in the level of rents amplifies the effect of changes in rent sharing on the capital share. We note that our calibration exercises compare outcomes in stationary equilibria. Thus, although the increase in idiosyncratic volatility in the data occurs mostly in the earlier half our sample, we should not expect an increase in the capital share in the data to immediately reflect the calibrated increase in the stationary equilibrium.

To provide further evidence to support our model, we conduct a variety of empirical tests. We show by plotting the capital-income-to-sales ratio within firm size quantiles that the aggregate capital share increase is driven by the largest firms, affirming Gabaix (2011)'s observation that we need to study the behavior of large firms to understand macroeconomic aggregates. In the smallest quantile, the average capital-income-to-sales ratio decreased from around 10% to less than −100% from 1960 to 2010. In contrast, this same ratio in the largest quantile remained almost unchanged at around 10%. Thus, the capital share has become more dispersed across the size distribution. We also show that this effect is most pronounced in the health products and technology industry, which also experienced a large increase in volatility. To directly assess the link between industry-level idiosyncratic volatility and dispersion in the capital share, we regress the industry-level capital-income-to-sales ratio on idiosyncratic volatility. We find that a within-industry increase in idiosyncratic volatility is associated with a decrease in the average capital-income-to-sales ratio.

Our paper contributes to the growing literature on the decline in the labor share of output. Karabarbounis and Neiman (2014) argue that this decline is due to a decrease in equipment prices that leads firms to substitute capital for labor, but this mechanism does not predict a divergence between the average labor share and the aggregate labor share that we document in the data.[3]

More recent evidence from manufacturing establishments and the U.S. Economic Census data on superstar firms (Kehrig and Vincent, 2017; Autor et al., 2017) confirm the validity of our earlier findings in the Compustat sample of publicly traded firms. Their findings are consistent with our

[3] An exception is the introduction of heterogeneous size-dependent technology choices in the last two decades, but not before that. Elsby, Hobijn, and Şahin (2013) show that labor share decreases most among industries exposed to import shocks, which indicates that the decline may be due to the offshoring of labor.

mechanism in that we also predict that relatively large and productive firms will have high capital shares. However, our paper differs from Autor et al. (2017) in several key respects. First, although we both identify the effect that large firms have on the capital share, we provide a model-based explanation for the relationship between firm size and the capital share that relies on firm-level risk and selection. In contrast, Autor et al. assume that the labor share of larger firms is smaller due to each firm's need for a fixed amount of overhead labor. They attribute the shift in the size distribution of firms to an exogenous increase in concentration, while we attribute it to an endogenous shift in the size distribution triggered by an increase in firm-level risk. Second, our mechanism also accounts for the increase in the mass of low capital share firms in the left tail, a key feature of the data, while their paper is silent on this. Third, we show evidence from the cross-section of industries that the effect of idiosyncratic volatility on industry-level factor shares is robust to the inclusion of industry concentration measures; we found similar effects in Europe and the U.K. Finally, our equilibrium framework allows us to conduct a quantitative analysis of our model's ability to match the data. We show that increases in both firm-level risk and rents are required to quantitatively match the data.

The rest of this paper is organized as follows: Section 1 describes the new stylized facts. Section 2 describes the setup of our model. Section 3 describes the equilibrium of our model and its implications for the aggregate capital share. Section 4 uses a calibrated version of our economy as a laboratory to explore the quantitative effect of changes in volatility on factor shares. Finally, Section 5 presents new empirical evidence on U.S. capital share dynamics, and we conclude by showing that compensation inequality has not kept pace with size inequality.

1 New Stylized Facts: U.S. Factor Share Dynamics, Volatility, and Firm Size Distribution

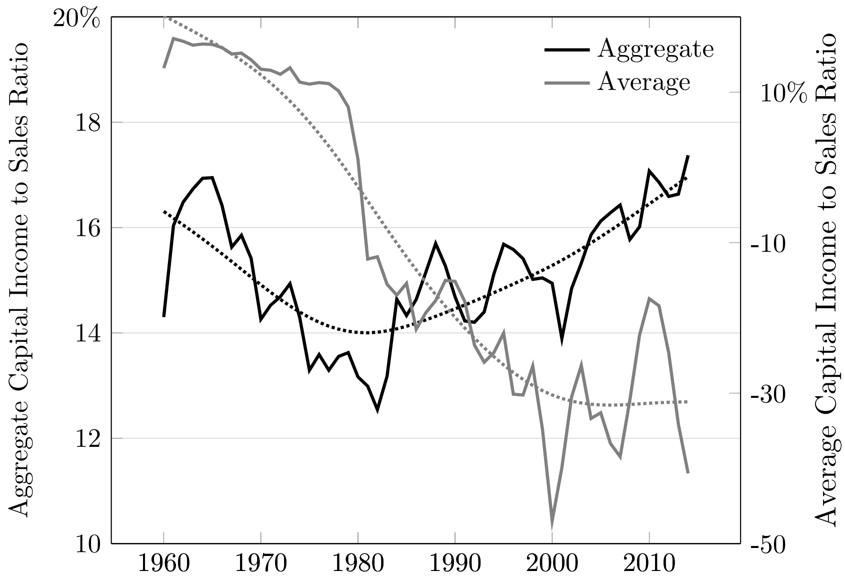
1.1 Factor Share Dynamics

To measure capital share at the firm level, we use widely available accounting data from the Compustat/CRSP Merged Fundamentals Annual. The sample ranges from 1960 to 2014. We exclude financial firms that have SIC codes in the interval 6000-6799, and we exclude firms whose sales, employee numbers and total asset values are negative. We measure the aggregate capital share of output as the ratio of aggregate capital income to aggregate value added. Capital income is measured as operating income before depreciation (OIBDP), which equals sales minus operating expenses including the cost of goods sold, labor costs, and other administrative expenses. Value added is computed as the sum of OIBDP and XLR, which records staff expenses. We provide further details in the data appendix (see section A of the Appendix).

We start by examining the time series dynamics of the capital share of U.S. publicly traded firms as measured in the Compustat/CRSP Merged Fundamentals dataset. As we document in Figure 1, the aggregate capital share for these firms increased from 41% in 1970 to 62% in 2010.

However, this trend is not operative for a typical U.S. firm. To demonstrate this, we analyze

Figure 3: Average and Aggregate Capital Income to Sales Ratios



The aggregate capital income to sales ratio is $\sum_i$ Operating Income$_i$ divided by $\sum_i$ Sales$_i$ for each year. The average capital-income-to-sales ratio is the simple average of the firm-level capital-income-to-sales ratio for each year. The dotted lines are the HP-filtered trends. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

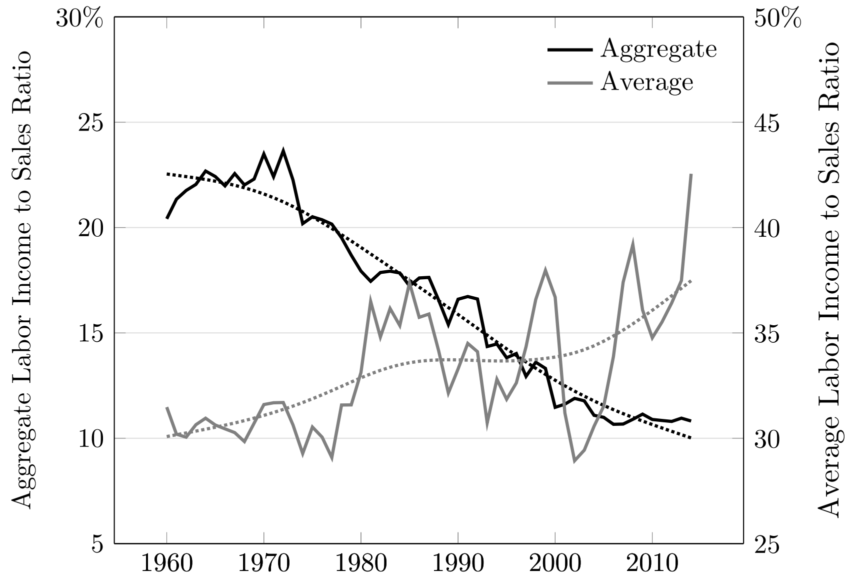
firm-level data. Our measure of *firm-level* value added can be negative. Because capital income can also be negative, the ratio of capital income to value added at the firm level is not an informative measure of the firm's capital share that can be readily compared across firms.[4] Instead, we use the ratio of capital income to sales as a proxy for *firm-level* capital shares in our firm-level analysis to obtain a well-ordered estimate. Our key empirical contribution is to show that the increase in the capital share is concentrated among the largest publicly traded firms in the U.S. Figure 2 shows the relationship between firm size and the ratio of capital income to sales (which is a measure of the capital share of profits). In 1970, there was essentially no relation between firm size and the capital-income-to-sales ratio, but by 2010, this ratio was strongly increasing. This shift caused the average and aggregate capital shares to diverge: the equal-weighted average capital share of publicly traded companies has declined since the 1980s.[5]

This is illustrated in Figure 3, which plots the average and aggregate capital shares as fractions of sales for the sample of publicly traded firms. The average ratio of capital income to sales is the cross-sectional mean of the capital-income-to-sales ratio for a given year. The aggregate ratio equals the sum of capital income (OIBDP) across all of the firms divided by aggregate sales. The large declines in the average capital-income-to-sales ratio are driven mostly by small firms that

[4]Over the past decade, 15% of public firms had negative value added. Dropping negative value-added firms arbitrarily truncates the left tail of the firm size distribution.

[5]In section A of the Separate Online Appendix, we show that the results are not exclusively driven by the accession of NASDAQ to the Compustat database. Even in the universe of NYSE firms, we document a similar, though smaller, divergence between the average and aggregate capital shares.

Figure 4: Aggregate and Average Labor-Income-to-Sales Ratios



The aggregate-labor-income-to-sales ratio is $\sum_i$ Extended XLR_i divided by $\sum_i \text{Sales}_i$ for each year, and the average labor-income-to-sales ratio is the simple average of the within-firm-labor-income-to-sales ratio for each year. The dashed lines are the HP-filtered trends. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

have negative operating margins. The average ratio dropped from 13% in 1960 to -40% in 2014, while the aggregate ratio increased, albeit less dramatically, from 14% in 1960 to 17% in 2014. [6] Importantly, the decline in the average capital-income-to-sales ratio is mostly a within-industry phenomenon, but the increase in the aggregate ratio is largely driven by cross-industry effects: some industries with high aggregate capital-income-to-sales ratios have grown much more than others, and they thus account for most of the increase in the aggregate capital-income-to-sales ratio, as documented in section A of the Separate Online Appendix.

We also compute measures of the labor income share. One drawback of the Compustat data is the lack of comprehensive labor expense data: XLR in Compustat is sparse, with the sample being only about 13% firm-year observations. To address this weakness, we adopt Donangelo (2016)'s imputation procedure to construct the extended labor cost (extended XLR) for firms that failed to report staff expenses. To implement this measure, we group firms into 17 industries and then sort them into 20 size groups within each industry based on their total assets, thus obtaining a total of 340 industry-size cells. We estimate the average labor cost per employee (XLR/EMP) within each industry/size cell for each year using the available XLR observations and then use this estimate to impute labor costs to firms that have missing XLR data as the number of employees times the

[6] In section F.1 of the Separate Online Appendix, we report additional time series evidence using an alternative measure of capital income: OIBDP + R&D expenses because R&D expenses can arguably be considered as an investment instead of expenses. The initial decline in the aggregate capital-income-to-sales ratio becomes less dramatic when we add back the expensing of R&D to operating income. Since R&D expenses include compensation to R&D employees, we do not use this adjustment as our main measure of capital income.

average labor cost per employee of the same industry/size cell during that year.[7]

The aggregate-labor-income-to-sales ratio in the non-farm business sector declined by 15%. However, the average labor share of output did not decline in our sample of publicly traded firms. Figure 4 shows the time series of both the average and aggregate labor-income-to-sales ratios in our sample using the estimated labor cost. The average labor share rose from 32% in 1960 to 40% in 2014, while the aggregate labor share (labor income to sales ratio) dropped from 22% to 11% during the same period. Our alternative measure of aggregate labor share, the ratio of labor income to value added, declined over the same sample period, from 59% to 40% (1 minus the capital share from Figure 1). Our measure of the ratio of labor income to value added is lower than the BEA's because our sample includes only publicly traded firms. As we show in section B of the appendix, the aggregate labor share in the non-publicly-traded sector actually increased over the last several decades. As a consequence, the drop in the aggregate labor share that we record in the publicly traded sector exceeds that for the U.S. economy as a whole.

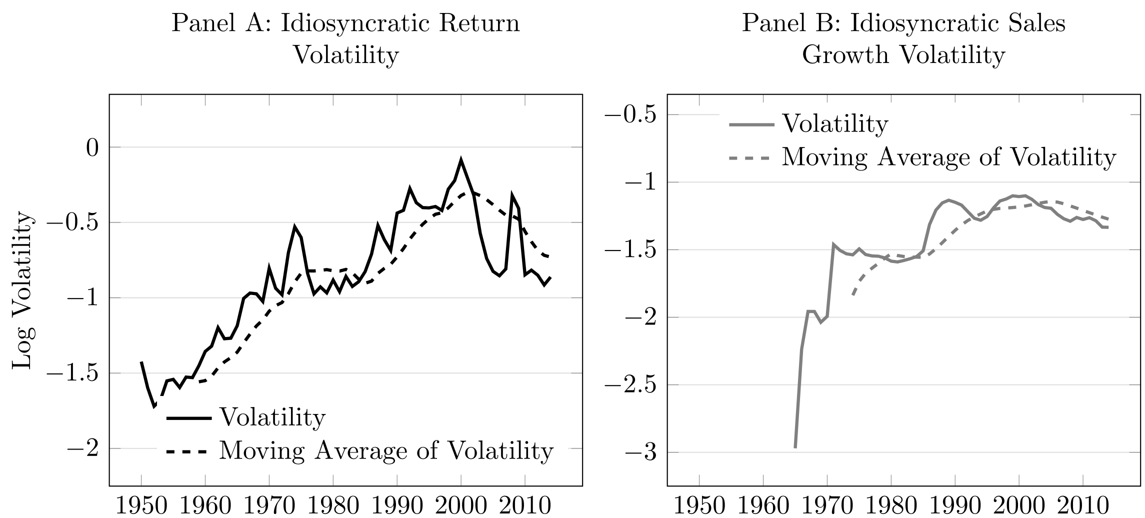
1.2 Firm-level Volatility and the Firm Size Distribution

Over the same period, U.S. firms experienced a large increase in volatility. Figure 5 plots the log of firm-level volatility, which is computed as the equal-weighted average of the log volatility of the idiosyncratic component of stock returns or cash flows for all publicly traded U.S. firms. These volatility measures more than doubled over the period 1950-2000 (see, e.g., Campbell et al., 2001; Comin and Philippon, 2005; Xiaolan, 2014; Bloom, 2014; Herskovic et al., 2015). As volatility increases, the right tail of the firm size distribution increases in mass. However, it is important to note that while Figure 5 plots annual changes in volatility, these changes will gradually be impounded into the size distribution. As such, we also plot the log of the 10-year moving average of idiosyncratic volatility for both returns and sales growth, both of which have substantially increased over our sample period. These increases are borne out in a considerable shift in the size distribution of firms. Figure 6 indicates that from 1960 to 2014, the power law exponent γ, which is given by the solution to the equation $P(\text{Size} > X) = kX^{-\gamma}$ for some constant k, decreased substantially. The average of our estimate for the power law exponent across the years 1960-1970 is 1.48 when measuring either total assets or sales. For the years 1990-2014, this average decreases to 1.13 when measuring size by total assets and to 1.11 when measuring size by sales.

We note that our estimates for the power law exponent of the firm size distribution differ in an important way from estimates in the literature, such as in Axtell (2001), **?** Luttmer (2007), Gabaix and Landier (2008a), and Ai, Kiku, and Li (2013). Research has focused on the distribution of firm size in the later part of our sample: for example, Axtell (2001) begins his analysis in 1988, while Gabaix and Landier (2008a) begin their analysis in 1980. We show that the firm size distribution in the early part of our sample had a substantially higher power law exponent.

[7] We follow Donangelo (2016) and use the Fama-French 17 industry classifications. To check that our results are not an artifact of this imputation procedure, we also report capital income as a fraction of sales. This measure of the capital share does not rely on the imputation.

Figure 5: Firm-Level Volatility of U.S. Public Firms



The black line indicates idiosyncratic firm-level stock return volatility in logs. The Idiosyncratic return for firm i is the residual from the following Fama and French (1993) 3-factor model:

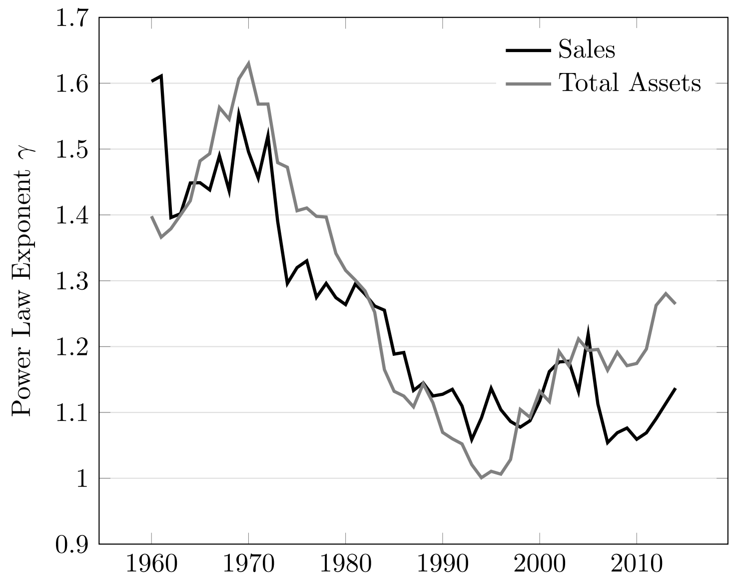
$$r_{it} = \gamma_{0,i} + \gamma_{1,i}\mathrm{MKT}_t + \gamma_{2,i}\mathrm{HML}_t + \gamma_{3,i}\mathrm{SMB}_t + \varepsilon_{i,t}$$

where t is a daily observation within year T. Idiosyncratic return volatility for firm i in year T is the standard deviation of $\varepsilon_{i,t}$. A time series of idiosyncratic volatility is then obtained by averaging across firms at each year. The dashed black line is the 10-year moving average of idiosyncratic return volatility. The gray line indicates the firm-level idiosyncratic cash flow volatility in logs. The idiosyncratic sales growth for firm i is the residual from the following factor model:

$$g_{it} = \gamma_{0,i} + \boldsymbol{\gamma}_i \mathbf{F}_t + \varepsilon_{i,t}$$

where t is a quarter in the prior 20 quarters up to calendar year T and $\mathbf{F}$ is the first five principal components of the cross section of sales growth for the 20 quarters prior to year T. Idiosyncratic return sales growth volatility for firm i in year T is the standard deviation of $\varepsilon_{i,t}$ at the fourth quarter of year T. A time series of idiosyncratic volatility is obtained by averaging across firms at each year. The dashed gray line is the 10-year moving average of idiosyncratic sales growth volatility. Source: CRSP 1960-2014 and Compustat/CRSP Merged Fundamentals Annual 1950-2014.

Figure 6: Power Law Coefficient of Firm Size: 1960-2014



The variable γ is the power law exponent given by $P(\text{Size} > X) = kX^{-\gamma}$, where size is either Total Assets or Sales. We estimate γ_t for each year t using the following regression:

$$\log(\text{Rank}_{i,t}) = \alpha_t + \gamma_t \log(\text{Size}_i) + \varepsilon_{i,t}$$

using the top n largest firms, where n is defined by the 95th percentile of firm size in the year. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014). Note: Variables are not winsorized.

To summarize, we document a divergence in the moments of the firm-level capital and labor share distributions that is broadly consistent with the mechanism that we now highlight in our model. Specifically, the trends that we observe in the data are consistent with changes in firm-level volatility, and they cause a shift in the firm size distribution that favors the owners of capital. As firm-level volatility increases, the aggregate and average capital shares in the model will diverge.

In Section B of the Separate Online Appendix, we show that the U.K. and, to a lesser extent, Europe, have experienced similar divergences between the aggregate and average factor shares over this sample and increases in firm-level risk. Japan, in contrast, has experienced neither.

2 A Dynamic Model of Industry Equilibrium with Entry and Exit

In this section, we present a model to rationalize the facts presented in Figures 1 and 2 and Section 1. In our model, firms produce cash flows according to a simple production function. Importantly, the shareholders of a given firm hold an option to cease operations when productivity falls: this is the classic abandonment option. As is standard in the real options literature, increasing the volatility of the firms' cash flows increases the value of the option to wait to abandon, which lowers the threshold of productivity at which the firm ceases operations.

We embed an optimal risk-sharing contract between the skilled workers and the firm in our model. Beginning with the seminal work of Harris and Holmström (1982), a large body of literature has developed on the study of the settings under which it is optimal for firms to shield workers from risk. In Harris and Holmström (1982), the optimal labor contract protects a risk-averse worker from negative shocks to her perceived ability. However, positive shocks to worker ability increase her outside option and must be reflected in higher wages. Thus, the optimal contract features downward rigidity as a form of partial insurance. Berk, Stanton, and Zechner (2010) build on Harris and Holmström (1982) and show that in the presence of leverage, the optimal wage contract features temporary wage cuts when the firm is in financial distress and bankruptcy. As a result, the optimal capital structure will take into account employees' risk aversion. Our setting differs from Harris and Holmström (1982) and Berk et al. (2010) in that risk in our model is entirely embodied in the firm. This means that the optimal wage contract in our model features full insurance for the skilled worker.[8]

There is strong evidence that firms insure workers. Guiso, Pistaferri, and Schivardi (2005), the first to study insurance within the firm using U.S. microdata, find that firms fully insure workers against transitory shocks but not against permanent shocks (see also Rute Cardoso and Portela, 2009; Fuss and Wintr, 2009; Lagakos and Ordoñez, 2011; Friedrich, Laun, Meghir, and Pistaferri, 2014; Fagereng, Guiso, and Pistaferri, 2017, for foreign evidence). Xiaolan (2014) finds direct evidence of increased cash flow volatility for firms that provide better insurance to workers.

[8] Other studies of the optimal wage contracts and insurance include Thomas and Worrall (1988); Holmstrom and Milgrom (1991); Kocherlakota (1996); Krueger and Uhlig (2006); Lustig, Syverson, and Nieuwerburgh (2011); Lagakos and Ordoñez (2011); Berk and Walden (2013); Eisfeldt and Papanikolaou (2013); Xiaolan (2014); Ai and Li (2015); **?**.

Lagakos and Ordoñez (2011) find that the wages of low-skill workers are more responsive to shocks than those of high-skill workers. In our model, unskilled labor does not benefit from insurance. In a model with systematic shocks, Eisfeldt and Papanikolaou (2013) show that the outside options of skilled workers increase with positive systematic shocks, which increases skilled workers' share of firm profits and the riskiness of shareholder equity.

When we introduce moral hazard and other frictions that hamper risk sharing, our mechanism is mitigated. However, we show in section D of the Appendix that when we allow workers some exposure to firm performance, our primary results remain unchanged. The selection mechanism still applies as long as a firm's owners provide some insurance to its workers and as long as the firm can exit when productivity declines.[9]

Finally, we characterize the stationary distribution of firms given the solution to the optimal abandonment problem. Increasing (idiosyncratic) cash flow volatility leads more firms to delay abandonment and to survive long enough to become highly productive. Thus, the average of the capital share of profits across firms increases in volatility. Here, we use insights from recent work on firm size distribution.[10]

2.1 Technology and Preferences

The economy is populated by a measure of ex-ante identical firms, and each firm operates a standard production technology. A given firm i with productivity X_{it} has a single skilled worker, rents physical capital K_{it}, and employs unskilled labor L_{it}. The total output produced by this firm is given by

$$Y_{it} = X_{it}^{\nu} F(K_{it}, L_{it})^{1-\nu},$$

where F is homogeneous of degree one and $0 < \nu < 1$. The parameter ν governs the decreasing returns to scale at the firm level. Lucas refers to ν as the span of the control parameter of the firm's manager. Atkeson and Kehoe (2005) show that a decrease in competition in a model with imperfect competition is equivalent to an increase in ν in our model, and thus we interpret ν as a measure of the level of economic rents in the economy. The aggregate supplies of physical capital and unskilled labor are denoted by k and l, respectively.

Firm productivity evolves according to

$$dX_{it} = \mu X_{it} dt + \sigma X_{it} dZ_t^i - X_{it} dN_{it}; \quad \text{for } X_{it} > X_{\min}, \tag{1}$$

[9] Gabaix and Landier (2008b); Edmans, Gabaix, and Landier (2009) find that equilibrium CEO compensation in a competitive market for CEO talent is composed of a cash component and an equity component. For our key results, we analyze the implications of this class of contracts.

[10] In a series of papers, Luttmer (2007, 2012) characterizes the stationary size distribution of firms when firm-specific productivity is subject to permanent shocks, and firms incur a fixed cost of operating. The selection effect of exit at the bottom of the distribution informs the shape of the stationary size distribution, which is a Pareto distribution with an endogenous tail index. Our work explores the impact of changes in the stationary size distribution on the distribution of rents in our laboratory economy. Other work on characterizing the firm size distribution includes Miao (2005); Gourio and Roys (2014); Moll (2016). Perla, Tonetti, Benhabib, et al. (2014) examine the endogenous productivity distribution in an environment in which firms choose to innovate, adopt new technology, or keep producing with old technology.

where Z_{it} is a standard Brownian motion that is independent across firms, N_{it} is a Poisson process with intensity λ, and $X_{\min} > 0$ is some minimum level of productivity. If $dN_{it} = 1$ or if X_{it} reaches $X_{\min}$, X_i jumps to zero and the firm exits. The process N_{it} gives rise to what is often referred to as an exogenous death rate of firms, and it is necessary to guarantee the existence of a stationary distribution of firms for all parametrizations of the model. Because all firms are identical up to their current level of productivity, we omit the subscript i for the remainder of the discussion.

Each firm is owned by an investor and requires one skilled worker to operate. We assume that investors are risk-neutral and discount cash flows at the risk-free rate of $r > \mu$, while skilled workers value a stream of payment $\{c_t\}_{t\geq 0}$ according to the following utility function:

$$U(\{c_t\}_{t\geq 0}) = E\left[\int_0^{\infty} e^{-rt} u(c_t) dt\right],$$

where $u'(c) \geq 0$ and $u''(c) < 0$. We normalize the measure of skilled workers in the economy to one.

Firms can enter and exit the economy at the discretion of their owners. When a firm exits, its owner receives the liquidation value of the firm, which we normalize to zero, and its skilled worker immediately re-enters the skilled labor market. There is a competitive fringe of shareholders waiting to create new firms. When a shareholder creates a new firm, she matches with a skilled worker, then pays a cost P for the technology blueprint to begin production. After creating a new firm, the firm's initial productivity is drawn from a Pareto distribution with a density of

$$f(X) = \frac{\rho}{X^{1+\rho}}; \quad X \in [1, \infty).$$

This distribution implies that the log-productivity of an entering firm is exponentially distributed with parameter $\rho > 1$, and it simplifies the characterization of the equilibrium that follows. We denote the rate at which new firms are created by ψ_t. Note that this implies that the entry rate at a given point X is $\psi_t f(X)$.

Upon matching with a skilled worker, an investor in a new firm offers a long-term contract to the skilled worker before the realization of the firm's productivity and the firm's payment of the cost P. The skilled worker can reject the contract, at which point she is instantaneously matched with a new firm. Formally, this contract can be denoted by a process $\{c_t\}_{t\geq 0}$, which determines a payment to the skilled worker of c_t at time t. We assume that the investor cannot commit to continue operations or to pay the skilled worker after the firm has ceased operations. We also assume that the skilled worker can choose to exit the contract and match with a new firm at any time, and we assume that she does not have access to a savings technology. This contracting environment features a two-sided limited commitment problem similar to Ai et al. (2013) and Ai and Li (2015). Importantly, the outside option of the skilled worker depends on the value of starting a new firm, which is endogenously determined in equilibrium. Eisfeldt and Papanikolaou (2013) consider a similar mechanism to explore the implications of the division of the surplus between shareholders and skilled workers for the cross-section of returns.

2.2 The Investor's Problem

We denote the utility that the skilled worker receives upon entering this market by U_0, which is also the skilled worker's reservation utility. At the inception of the contract, the investor and the skilled worker take U_0 as exogenously given, although it will be determined in equilibrium by the market for skilled workers. The investor will continue operations as long as doing so yields a positive present value. This means that the investor's value for operating the firm is the solution to a standard *abandonment option*. Specifically, the investor operates the firm until a stopping time denoted by τ. The investor's problem is thus

$$\max_{K,L,\tau,c} E\left[\int_0^\tau e^{-rt}(Y_t - c_t - \kappa K_t - wL_t)dt\right], \tag{2}$$

such that

$$U_0 \leq E\left[\int_t^\tau e^{-r(s-t)}u(c_s)ds + e^{-r(\tau-t)}U_0\right] \text{ for all } t > 0. \tag{3}$$

The problem contained in Equations (2) and (3) is an optimal risk-sharing problem. Given that the investor is risk-neutral and the skilled worker is risk-averse, the investor chooses a compensation rule to minimize the skilled worker's exposure to risk. Intuitively, the skilled worker's limited commitment constraint given in Equation (3) must be binding, as promising her a greater continuation value at any point in time can only reduce the investor's value for the firm. As a result, the skilled worker's value for the contract is constant over time, and it is without loss of generality that we restrict attention to contracts that offer the skilled worker a fixed wage of c until the firm exits, at which point the skilled worker re-enters the market and receives her outside option.

2.3 Equilibrium

We focus our analysis on equilibria in which the measure of firms at any given level of productivity is stationary. We denote the stationary distribution of log-productivity by $\phi(x)$, where $x = \log(X)$ throughout.

Definition 1. A stationary equilibrium consists of a rental rate κ for physical capital, a demand for physical capital as a function of productivity $K(X)$, a wage rate w for unskilled labor, a demand for unskilled labor $L(X)$ as a function of X, a compensation c^* for the skilled workers, an entry rate of new firms ψ^*, an exit policy for the shareholder $\bar{X}$, and a stationary distribution $\phi(x)$, such that

1. The exit policy $\bar{X}$ solves the investor's problem given by (2) and (3).

2. The stationary distribution $\phi(x)$ is consistent with the entry rate of new firms of ψ and with the exit policy $\bar{X}$.

3. The markets for physical capital, unskilled labor, and skilled workers clear

$$\int_0^\infty K(x)\phi(x)dx = k, \quad \int_0^\infty L(x)\phi(x)dx = l, \quad \text{and} \int_0^\infty \phi(x)dx = 1.$$

4. Creating a new firm leaves the investor with zero expected NPV:

$$\int_1^\infty V(X;c)f(X)dX = P.$$

Conditions 1-3 are standard equilibrium conditions. Condition 4 derives from the existence of the competitive fringe of investors waiting to create new firms. If an investor in a new firm offers a contract that leaves her with positive ex-ante expected NPV, then the skilled worker will reject it because she can simply re-enter the market and instantaneously match with a new firm. Thus, Condition 4 is equivalent to allocating all of the ex-ante bargaining power to the skilled worker. This in turn determines the level of skilled worker compensation. An alternative definition for Condition 4 would be to allocate some bargaining power to the investor; however, doing so will change the division of ex-ante rents but not how that division qualitatively depends on the key parameters of the model.

3 Equilibrium Analysis

3.1 Equilibrium Allocation of Physical Capital and Unskilled Labor

To characterize equilibrium, we begin by considering the allocation of physical capital and unskilled labor across active firms. Given spot rates for physical capital and unskilled labor and some current level of productivity, a given firm chooses capital and labor to maximize profits, which are the net of the rental payments to physical capital and wages to unskilled labor:

$$(K_t, L_t) = \arg\max_{K,L} \left\{ X_t^\nu F(K,L)^{1-\nu} - wL - \kappa K \right\}.$$

The homogeneity of the production function F implies that the solution (K_t, L_t) of the maximization above is linear in X_t. Market clearing then implies that physical capital and unskilled labor are allocated across firms according to the following linear allocation rule:

$$K_t = \frac{k}{\widehat{X}} X_t, \tag{4}$$

$$L_t = \frac{l}{\widehat{X}} X_t, \tag{5}$$

where

$$\widehat{X} = \int_{x_{\min}}^\infty e^x \phi(x)dx$$

is the average productivity in the economy given the stationary distribution of log productivity $\phi(x)$. This allocation rule implies that the output of any given firm is a linear function of aggregate output:

$$Y_t = \frac{y}{\widehat{X}} X_t,$$

where $y = \widehat{X}^{\nu} F(k,l)^{1-\nu}$ is aggregate output. As a result, a firm's gross earnings (operating profits) are proportional to X_t:

$$Y_t - wL_t - \kappa K_t = \frac{\nu y}{\widehat{X}} X_t.$$

For convenience, we let $\widehat{F} = \frac{\nu y}{\widehat{X}}$. We refer to $\widehat{F}$ as the equilibrium rents normalized by (average) productivity $\hat{X}$.

3.2 Solving the investor's problem

Having determined the allocation of physical capital and unskilled labor, we can now analyze the investor's problem given in Equation (2). Taking equilibrium rents $\hat{F}$ as given and using the allocation rules given in Equations (4) and(5) allows us to simplify the investor's problem to

$$V(X; c, \hat{F}) = \max_{\tau} E\left[\int_0^{\tau} e^{-rt}\left(\widehat{F}X_t - c\right) dt | X_0 = X\right], \tag{6}$$

where $V(X; c, \hat{F})$ is the value of operating a firm with current productivity X given a skilled worker contract c and rents $\widehat{F}$. The payment c to the skilled worker then acts as a fixed cost or as operating leverage. As such, the optimal risk-sharing problem given in Equations (2) and (3) reduces to an optimal abandonment considered in the real options literature as in Brennan and Schwartz (1985) or to an optimal default problem as in Leland (1994). Thus, the investor in a given firm will choose to exit if productivity X is low enough. Without a loss of generality, we can restrict attention to firm exit times that are given by threshold rules of the form

$$\tau = \inf\{t | X_t \le \bar{X}\}$$

for some $\bar{X} \ge 0$.

An application of Ito's formula and the dynamic programming principle implies that $V(X; c)$ must satisfy the following ordinary differential equation:

$$(r+\lambda)V(X; c, \widehat{F}) = \widehat{F}X - c + \mu X \frac{\partial}{\partial X} V(X; c, \widehat{F}) + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2}{\partial X^2} V(X; c, \widehat{F}), \tag{7}$$

with the boundary conditions

$$V(\bar{X}(c,\widehat{F}));c,\widehat{F}))=0, \tag{8}$$

$$\frac{\partial}{\partial X}V(\bar{X}(c,\widehat{F}));c,\widehat{F}))=0, \tag{9}$$

$$\lim_{X\to\infty}\left|V(X;c,\widehat{F})-\left(\frac{\widehat{F}X}{r+\lambda-\mu}-\frac{c}{r+\lambda}\right)\right|=0, \tag{10}$$

where $\bar{X}(c,\widehat{F})$ is the abandonment threshold given c and $\widehat{F}$. Conditions (8) and (9) are the standard value-matching and smooth-pasting conditions that pin down the optimal exit threshold $\bar{X}$. Condition (10) arises because as X_t tends toward infinity, abandonment occurs with zero probability and firm value is just the present value of all future profits.

The solution to Equations (7)-(10) is given by

$$V(X;c,\widehat{F})=\frac{\widehat{F}X}{r+\lambda-\mu}\left(1-\left(\frac{X}{\bar{X}}\right)^{-(\eta+1)}\right)-\frac{c}{r+\lambda}\left(1-\left(\frac{X}{\bar{X}}\right)^{-\eta}\right), \tag{11}$$

where

$$\bar{X}=\left(\frac{\xi-1}{\xi}\right)\left(\frac{c}{\widehat{F}}\right), \tag{12}$$

$$\eta=\frac{\mu-\frac{1}{2}\sigma^2+\sqrt{(\mu-\frac{1}{2}\sigma^2)^2+2(r+\lambda)\sigma^2}}{\sigma^2},$$

and

$$\xi=\frac{-\left(\mu-\frac{1}{2}\sigma^2\right)+\sqrt{(\mu-\frac{1}{2}\sigma^2)^2+2(r+\lambda)\sigma^2}}{\sigma^2}$$

are the roots of the fundamental quadratic for Equation (7). Note that an increase in firm-level volatility σ invariably lowers the abandonment threshold (holding equilibrium rents $\hat{F}$ constant) simply because an increase in volatility raises the option value of keeping the firm alive. This feature of the abandonment option plays a key role in our analysis. Its importance becomes apparent when we discuss the stationary distribution of firm size. Specifically, an increase in firm-level volatility leads to an increase in the mass of firms that delay exit, thus increasing the mass of firms that have low productivity and the mass of firms that survive long enough to achieve high productivity.

Given the solution for firm value conditional on a skilled worker's wage c, and the Pareto entry distribution of new firms, it is straightforward to solve the investor's ex-ante zero profit condition for the equilibrium c. We have

$$c^*=\left(\frac{P(r+\lambda)(\rho-1)(\rho-\eta)}{\eta}\left(\frac{\xi-1}{\hat{F}\xi}\right)^{\rho}\right)^{-\frac{1}{\rho-1}}. \tag{13}$$

The derivation of c^* is given in Section C of the Appendix. Note that c^* depends on σ through η and ξ.

3.3 Stationary Size Distribution

We now consider the equilibrium distribution of productivity $\phi(x)$ given an exit threshold of $\bar{X}$ and rents $\widehat{F}$. For the $\phi(x)$ to be stationary, the expected change via inflow and outflow in the measure of firms at a given level of x must equal the measure of firms that exogenously die at rate λ, less the measure of firms that endogenously enter at rate $\psi g(x)$. This leads to the following Kolmogorov forward equation for the stationary distribution of log productivity $\phi(x)$:

$$\frac{1}{2}\sigma^2\phi''(x) - \left(\mu - \frac{1}{2}\sigma^2\right)\phi'(x) - \lambda\phi(x) + \psi g(x) = 0, \tag{14}$$

where $g(x) = \rho e^{-\rho x}$ is the density of initial log productivity x for entering firms. A similar argument gives a boundary condition for $\phi(x)$ at the exit barrier $\bar{x} = \log \bar{X}$

$$\phi(\bar{x}) = 0. \tag{15}$$

The final equation that determines the stationary distribution of firm size is given by the market-clearing condition for skilled workers:

$$\int_{\bar{x}}^{\infty} \phi(x)dx = 1. \tag{16}$$

The solution to equations (14)-(16) is given by

$$\phi(x) = \frac{\rho\gamma}{\rho - \gamma}\left(e^{-\gamma(x-\bar{x})} - e^{-\rho(x-\bar{x})}\right) \tag{17}$$

for $x \in [\bar{x}, \infty)$, where $\gamma = \frac{-(\mu - \frac{1}{2}\sigma^2) + \sqrt{(\mu-\frac{1}{2}\sigma^2)^2 + 2\sigma^2\lambda}}{\sigma^2}$. This solution also allows us to characterize the aggregate entry rate of new firms:

$$\psi = \frac{\gamma(\rho(\mu - \frac{1}{2}\sigma^2) + \frac{1}{2}\rho^2\sigma^2 - \lambda)}{\rho - \gamma} e^{\rho\bar{x}}. \tag{18}$$

We note that our assumption about the density of the productivity of entering firms allows for the simple closed-form solutions shown above. The general solution to the ODE given in Equation (14) is exponential. By assuming that $g(x)$ is also exponential, we are left with a solution to Equation (14) for which it is possible to solve the boundary condition given in Equation (15).

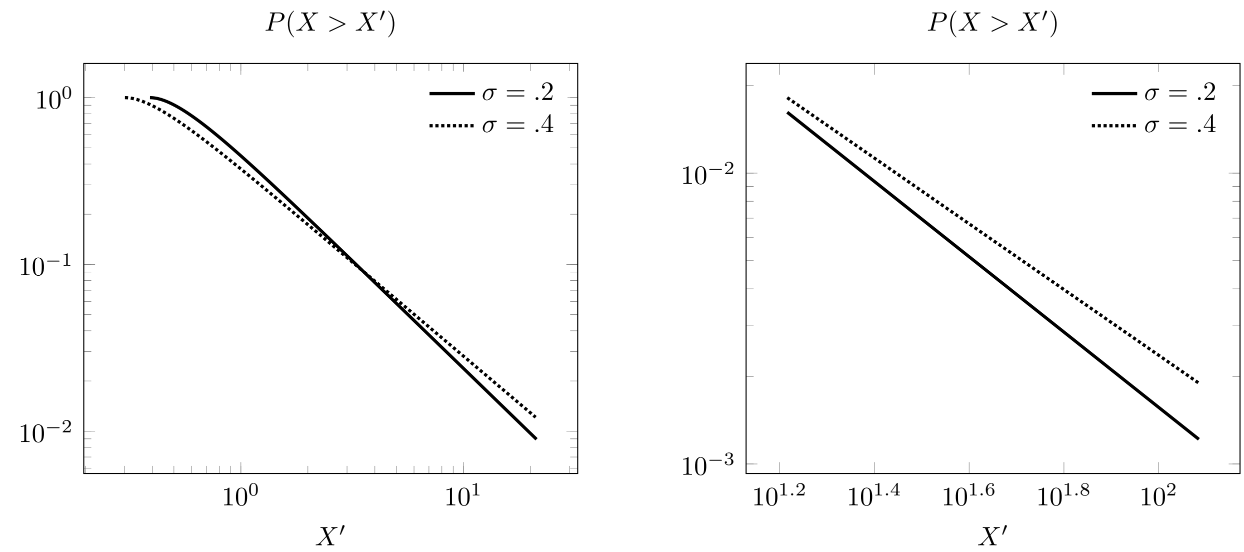
The equilibrium average productivity is then

$$\widehat{X}(\bar{X}) = \frac{\bar{X}\gamma\rho}{(\gamma - 1)(\rho - 1)}. \tag{19}$$

This implies that the equilibrium rents are

$$\widehat{F} = \nu\left(\frac{\bar{X}\gamma\rho}{(\gamma - 1)(\rho - 1)}\right)^{\nu - 1} F(k,l)^{1-\nu}. \tag{20}$$

Figure 7: The stationary distribution of productivity



Parameter values are given by $\sigma = .2, .4$, $r = 5\%, \mu = 2.5\%, \lambda = .04, \rho = 3.5$, $\alpha = .3$, $k/l = 1$, $\nu = .2$, and $P = 1$ and are taken from the calibration given in Section 4

An equilibrium is then characterized by a solution $(\bar{X}, \widehat{F})$ to Equations (12), (13), and (20). It can be shown that such a solution exists and is unique.

Figure 7 plots the complementary cumulative distribution function for the stationary distribution of firm productivity X on the log axes for different levels of σ. The other parameters are calibrated at $r = 5\%, \mu = 2.5\%, \lambda = .05, \rho = 3.5$, and $P = 1$ and are taken from our calibration of the model given in Section 4. As X becomes large, the log of the mass of firms with productivity to the right of X depends linearly on $\log(X)$, indicating that the stationary distribution of X tends to a power law. The power law exponent of X is given by the term γ: as σ increases, the power law exponent γ decreases and the stationary distribution becomes more diffuse, with a fatter right tail. It also shifts to the left because as firm-level volatility increases, the value of the option to wait to exit also increases, and the optimal point at which the investor chooses to exit necessarily decreases.

3.4 National Income Accounting

Armed with the stationary equilibrium, we can conduct national income accounting within our model. Specifically, we can calculate the aggregate capital share of output as

$$\begin{aligned}\text{Capital Share of Output} = \Pi &= \frac{y - wl - c}{y} \\ &= 1 - \underbrace{(1-\nu)(1-\alpha(k,l))}_{\text{Unskilled Labor Share}} - \underbrace{\frac{c}{y}}_{\text{Skilled Labor Share}}\end{aligned}$$

where

$$1 - \alpha(k, l) = \frac{1}{F(k, l)^{-1}} \frac{\partial F(k, l)}{\partial l}$$

is the elasticity of the production function F with respect to unskilled labor. In other words, the capital share of output is one minus the total labor share, where the labor share aggregates the share of output that accrues to unskilled and skilled labor.

3.4.1 Idiosyncratic Volatility and the Capital Share

We now consider how the aggregate capital share of output depends on idiosyncratic volatility. Note that the share of output that accrues to unskilled labor is independent of σ, so it suffices to consider how the share that accrues to skilled workers $\frac{c}{y}$ depends on σ and then to abstract from the unskilled labor share by setting $\nu = 1$. In this special case, $y = \hat{X}$, so aggregate output is just equal to the average productivity in the economy.

To provide intuition for the effect of a comparative static change in idiosyncratic volatility on the capital share, it is useful to decompose the (skilled) labor share into its constituent parts. The denominator is the total output of all firms in the economy, and is given by

$$\int_{\bar{x}}^{\infty} e^x \phi(x) dx = \left(\frac{\gamma}{\gamma - 1}\right) \left(\frac{\rho}{\rho - 1}\right) \bar{X}.$$

The numerator is the total compensation paid to skilled workers c. We demonstrate that the optimality of the exit threshold $\bar{x}$ implies that c is proportional using a quantity that we call "discounted average productivity." Although we derived the optimal exit threshold using the smooth pasting condition in Equation (9), the following first-order condition for exit

$$\frac{\partial}{\partial \bar{x}} \left(\int_0^{\infty} E\left[\int_0^{\tau} e^{-rt}(e^{x_t} - c) dt | x_0 = x \right] g(x) dx \right) = 0 \tag{21}$$

is equivalent. This condition states that the optimal exit threshold $\bar{x}$ must maximize the ex-ante firm value prior to the realization of the initial log productivity x_0.

Using an argument similar to that used to derive the stationary distribution $\phi(x)$, it can be shown that

$$\int_0^{\infty} E\left[\int_0^{\tau} e^{-rt}(e^{x_t} - c) dt | x_0 = x \right] g(x) dx = A \left(\int_{\bar{x}}^{\infty} e^x \theta(x) dx - c \right) \tag{22}$$

where

$$A = \frac{e^{-\rho \bar{x}} \eta}{(r + \lambda)(\eta + \lambda)} sfgs \tag{23}$$

and

$$\theta(x) = \frac{\xi \rho}{\xi - \rho} (e^{-\xi(x - \bar{x})} - e^{-\rho(x - \bar{x})}) \tag{24}$$

where

$$\xi = \frac{-(\mu - \frac{1}{2}\sigma^2) + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2(r+\lambda)\sigma^2}}{\sigma^2}. \tag{25}$$

Note that A is the survival annuity factor of the firm because it is equal to the ex-ante present value of a claim to a cash flow of one dollar for as long as the firm survives.

Comparing $\theta(x)$ to $\phi(x)$, or more specifically ξ to γ, reveals that $\theta(x)$ is the stationary distribution of log productivity in an economy equivalent to the one we describe above, but with a rate of exogenous exit equal to $r + \lambda$ to account for the effect of discounting. Thus, it represents a transformed version of the stationary distribution that puts less weight on paths that are far from the center of mass of the entry distribution. We refer to

$$\tilde{X} = \int_{\bar{x}}^{\infty} e^x \theta(x) dx = \frac{\xi \rho \bar{X}}{(\xi - 1)(\rho - 1)} \tag{26}$$

as the discounted average productivity of the economy.

We can rewrite the first-order condition in equation (21) to obtain

$$c\frac{\partial A}{\partial \bar{x}} = \frac{\partial A}{\partial \bar{x}}\tilde{X} + A\frac{\partial \tilde{X}}{\partial \bar{x}}. \tag{27}$$

The right-hand side of Equation (27) represents the marginal cost (in ex-ante terms) of delaying exit, i.e., the present value of the additional wages that the firm will have to pay to the skilled worker. The left-hand side represents the marginal benefit (again in ex-ante terms) of delaying exit, i.e., the present value of the additional gross cash flows that will accrue to the firm. This marginal benefit is determined by the effect that such a delay has on both the discounted average productivity of the firm $\tilde{X}$ and the survival annuity factor A.

Note that $\frac{\partial A}{\partial \bar{x}} = -\rho A$ and $\frac{\partial \tilde{X}}{\partial \bar{x}} = \tilde{X}$. We can thus solve Equation (27) to obtain the following expression for compensation:

$$c = \left(\frac{\rho - 1}{\rho}\right)\tilde{X}. \tag{28}$$

The wage paid to the skilled worker is proportional to the ex-ante discounted average productivity of the firm, where the constant of proportionality adjusts for survivorship bias. Thus, if $\nu = 1$, the capital share is given by

$$\Pi = 1 - \left(\frac{\rho - 1}{\rho}\right)\frac{\tilde{X}}{\hat{X}} = 1 - \left(\frac{\rho - 1}{\rho}\right)\left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{\xi}{\xi - 1}\right). \tag{29}$$

Note that $\bar{X}$ appears in both the numerator and the denominator, and it thus cancels. Intuitively, as σ increases, the right tail of the stationary distribution $\phi(x)$ becomes fatter, i.e., the tail exponent γ decreases, and aggregate productivity $\bar{X}$ increases. The right tail of θ also becomes fatter, i.e., ξ decreases, because the chance of any one firm becoming large increases. As a result, the discounted average productivity $\tilde{X}$ also increases, but not by as much as $\hat{X}$ because the paths that lead to large x are now discounted in $\theta(x)$. As a result, the capital share increases. In the absence of discounting ($r = 0$), ξ and γ are identical, and the equilibrium capital share is governed only by the exogenous entry distribution parameter ρ: $\Pi = 1 - \left(\frac{\rho-1}{\rho}\right)$ and is therefore unaffected by volatility. The above argument assumed that $\nu = 1$, but this was not essential. We state this result formally in the following proposition.

Proposition 1. *The total capital share of output increases in firm-level risk, that is,*

$$\frac{\partial \Pi}{\partial \sigma} > 0.$$

The argument behind Proposition 1 does not depend on how the level of compensation c to the skilled worker is determined in equilibrium; rather, the essential relation between c and discounted average productivity $\tilde{x}$ stems from the optimality condition that governs the exit decision given in Equation (21).

3.4.2 Rents and the Capital Share

While our focus is on the effect of changes in idiosyncratic volatility on the capital share, other parameters also have effects. In particular, the level of rents ν affects the share of total output that accrues to both the investors and the skilled worker. We can apply a similar argument to that of the previous section to show that the capital share of output is given by

$$\Pi = \underbrace{(1-\nu)\alpha(k,l)}_{\text{Physical Capital Share of Output}} + \nu \underbrace{\left(1 - \left(\frac{\rho-1}{\rho}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\xi}{\xi-1}\right)\right)}_{\text{Capital Share of Rents}}. \tag{30}$$

An increase in ν decreases the physical capital share but does not affect the capital share of rents. Thus, an increase in ν increases the total capital share of output if and only if the decrease in the physical capital share of output is less than the increase in the share of output that investors get from rents.

While an increase in rents ν has an ambiguous effect on the capital share, a joint increase in both ν and σ unambiguously increases the capital share. We summarize this result in the following proposition.

Proposition 2. *The capital share of output increases in the level of rents if and only if the capital share of rents exceeds the elasticity of production with respect to physical capital; that is,*

$$\frac{\partial \Pi}{\partial \nu} > 0$$

if and only if

$$\alpha(k,l) < 1 - \left(\frac{\rho-1}{\rho}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\xi}{\xi-1}\right).$$

Moreover, a joint increase in the level of rents and firm-level risk increases the total capital share *of output i; that is,*

$$\frac{\partial^2 \Pi}{\partial \nu \partial \sigma} > 0.$$

3.4.3 The Capital Share on Average

Although our main focus in this paper is to understand the changes in the aggregate capital share, we also document that the average capital share has decreased over our sample period. As volatility or rents increase, the optimal abandonment point shifts to the left and the mass of less productive firms with low capital shares in the left tail of the stationary distribution increases. These firms can cause the average firm-level capital share to decrease. Recall that the output of a firm with productivity X_t is given by $Y_t = y\frac{X_t}{\hat{X}}$. Thus, we can use similar arguments to those above to calculate that the average capital share is given by

$$\begin{aligned}\text{Average Capital Share} &= 1-(1-\nu)(1-\alpha(k,l))+\frac{c}{y}\int_{\bar{x}}^{\infty}\left(\frac{\hat{X}}{e^x}\right)\phi(x)dx \\ &= \underbrace{(1-\nu)\alpha(k,l)}_{\text{Physical Capital Share of Output}} + \nu\underbrace{\left(1-\left(\frac{\rho}{\rho+1}\right)\left(\frac{\gamma}{\gamma+1}\right)\left(\frac{\xi}{\xi-1}\right)\right)}_{\text{Average Capital Share of Rents}}\end{aligned} \tag{31}$$

As a result, an increase in ν will decrease the average capital share if the average capital share of rents is greater that the elasticity of the production function with respect to physical capital. The comparative static of the average capital share with respect to σ does not have a simple closed form. Intuitively, the increased mass of firms with low productivity—and hence low firm-level capital share—resulting from a delay in the optimal abandonment time have a relatively large effect on the simple average of the capital share across firms. As a result, the average capital share will decrease in response to a comparative static increase in σ, as we show in a calibrated version of our model.

The behavior of the average capital share distinguishes our mechanism from that of Autor et al. (2017). Their mechanism accounts for an increase in the mass of high-productivity, low-labor-share firms by appealing to an increase in concentration, and it does not provide an explanation for the accompanying increase in low-productivity, low-capital-share firms that we document in the data. In our model, an optimal risk-sharing contract implies that low capital share firms will remain active longer in response to increased firm level-risk and that the average capital share can decrease even though the aggregate share increases.

4 Model Calibration and Quantitative Experiments

In this section, we explore the quantitative implications of our model. We calibrate the economy to match the empirical moments of the distribution of the capital share of output across firms in the U.S. Compustat sample. We then consider the effects of changes in the underlying parameters to quantify the effect of our mechanism on the aggregate and average capital shares and the labor share.

Table 1: Benchmark Calibration

The table reports our benchmark calibration. Panel A reports the target moments in the data and the implied moments from our production model. The data moments are computed from the sample Compustat/CRSP Merged Fundamentals Annual from 1960 to 1970. The sample excludes firms that have SIC codes from 6000 to 6799. Panel B reports the calibrated parameters, and Panel C reports the preset parameters. Firm-level value added VA_i is OIBDP plus Extended XLR. To deal with negative values, we identify the minimum operating income (OIBDP) for each year, and we increase the value added of all of the firms by the absolute value of the minimum OIBDP$\times$(1+1%). The average capital share is computed using OIBDP divided by the adjusted value added. The standard deviation and skewness of the capital share are also estimated using the adjusted value added measure. The aggregate capital share is calculated using the unadjusted value added, and the capital share at the exit threshold $\bar{X}$ is measured as the average capital shares three years prior to delisting.

Panel A: Capital Share Moments 1960-1970		
	Data	Model
Average Capital Share	0.208	0.264
Aggregate Capital Share	0.419	0.374
Standard Deviation of Capital Share	0.152	0.096
Capital Share at Exit	0.076	0.040
Power Law Exponent of Firm Size	1.480	1.295
Panel B: Preset Parameters		
r	0.05	Discount Rate
ν	0.2	Share of Rents in GDP
σ	0.2	Idiosyncratic Vol
k/l	1	Capital/Labor Ratio
p	1	Sunk Cost
Panel C: Calibrated Parameters		
μ	0.025	Firm Growth
λ	0.04	Exogenous Exit Rate
ρ	3.5	Entrants' Firm Size Distribution
α	0.3	Aggregate Physical Capital Share of Output

We first calibrate the model to match the aggregate moments from the sample of U.S. publicly traded firms from 1960 to 1970. Panel A in Table 1 reports the moments that we set out to match. One caveat is that we are not able to use value added to compute the firm-level capital share because a large number of firms have negative value added. When estimating the average capital share for calibration, we use the adjusted value added instead of sales at the firm level as the denominator, which allows us to obtain empirical moments of the firm-level capital share that are more consistent with our theoretical counterpart without having to drop the negative value added observations.[11] Further details on the data are provided in Appendix A.

Panel B of Table 1 reports the preset parameters. We choose a discount rate of $r = 5\%$ and note

[11]Although in the empirical exercise, we use income-to-sales ratios to proxy for factor shares, our model does not have the theoretical counterpart of income-to-sales ratios.

that this discount rate also accounts for aggregate growth and risk that we omit from the model. We follow Atkeson and Kehoe (2005) and set $\nu = .2$ and $\alpha = .27$. We set $\sigma = .2$ to match our data on the volatility of returns that we use to generate Figure 5. The capital labor ratio k/l and the cost of starting a new firm do not affect our target moments, so we normalize these parameters to one.

Panel C reports the parameters that we chose to match the moments in Panel A. Although our model is simple, it is able to roughly match the moments of the data given reasonable parameters. We do not directly model the aggregate growth rate of the economy; thus, the drift parameter μ represents idiosyncratic growth beyond aggregate growth prior to firm exit. We calibrate μ, the death rate λ, and the parameter ρ, which governs the entry distribution, to match the cross-sectional standard deviation of the firm-level capital share, the capital share at the exit threshold $\bar{X}$, and the power law exponent of firm size as closely as possible. The aggregate physical capital share of output is then chosen to match the aggregate and average capital share as closely as possible. Our model produces an aggregate capital share that is too low, an average capital share that is too high, and too little dispersion of capital shares, mostly because the entry distribution of new firms is misspecified, which results in a stationary distribution with insufficient dispersion in the left tail and the middle of the distribution. Our calibration of λ and ρ produces an entry rate of firms of 13.4%, while the average IPO rate is only 3.4% in the 1980-2015 sample.[12] Obviously, we cannot match this secular trend in a stationary equilibrium. A more realistic entry distribution would enable the model to match the data moments more closely while producing a lower entry rate, albeit at the cost of reduced tractability. Moreover, introducing some flexibility in the relationship between firm size, expected growth, and volatility would allow for a lower rate of exit—and hence entry—albeit at the cost of reduced tractability.

4.1 Quantitative Experiments

Next, we use the benchmark calibration of the model to conduct the series of experiments reported in Table 2. In Panel A, we report the changes in firm-level capital share distribution when increasing volatility σ, entry parameter ρ, and rent share ν.

As we document in Figure 5, firm-level volatility has increased dramatically over the past five decades. In particular, the volatility of the idiosyncratic component of sales growth has roughly doubled. When we double the volatility from the baseline of $\sigma = 20\%$ to $\sigma = 40\%$, the model predicts a decline in the average capital share of output of 8.7 percentage points and an increase in the aggregate capital share of output of 1.3 percentage points. These numbers mask large changes in the distribution of rents. In the benchmark calibration of our model, the owners only collect 12.2% of total rents at the average firm, but they collect 67.5% of aggregate rents. To translate the change

[12]IPO rates before 1980 are not available. Fama and French (2004) suggests that the IPO rate before 1979 is much lower. The number of publicly traded U.S. firms has been trending down since 1996; half of this decrease is due to the abnormally high delisting rate due to acquisitions, while the rest is due to the abnormally low rate of new listings (Doidge, Karolyi, and Stulz, 2017). It is unclear what is driving these secular trends according to Doidge et al. (2017), but these forces are presumably largely outside of our model.

in the capital share of output to a change in the capital share of rents, we must consider that these rents are a fraction of the output given by ν. Thus, doubling volatility increases the aggregate share of rents collected by owners by 6.4 pps (roughly 1.3 pps divided by $\nu = .2$) and decreases the average share of rents by 43 pps (roughly 8.7 pps divided by $\nu = .2$). Doubling idiosyncratic volatility also leads to a decrease in the power law exponent of firm size from $\gamma = 1.295$ to $\gamma = 1.130$. While this decrease is sizable, the decrease that we document in the data is larger. This could indicate either that σ has more than doubled or that the growth rate μ has increased. In any case, our model would deliver a larger increase in the aggregate capital share if we were to match a larger decline in the power law exponent.

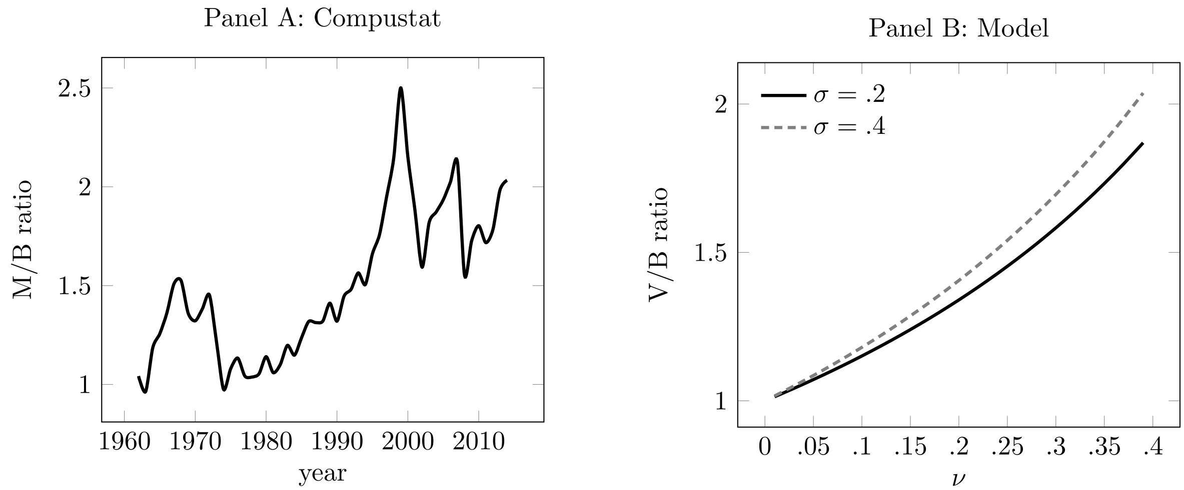
There is increasing evidence that rents, represented in our model by the parameter ν, have increased. In particular, many authors have documented evidence of an increase in intangible capital formed by U.S. corporations (e.g., Hall (2001), Corrado, Hulten, and Sichel (2009), Corrado and Hulten (2010), and Eisfeldt and Papanikolaou (2014)). These empirical measures of intangible capital correspond to the capitalized value of rents that accrue to the owners of the capital stock.[13] To arrive at a reasonable increase in ν to examine in our quantitative experiments, we note that an increase in ν in our model leads to an increase in the firm's valuation. Economic rents are back-loaded in the model, and hence the market value rises relative to its replacement cost as ν rises (See Appendix E). In Figure 8, Panel (1), we document a secular increase in the aggregate market-to-book ratio of the Compustat firms. In Figure 8, Panel (2), we show that doubling rents from the baseline of $\nu = .2$ to $\nu = .4$ corresponds to a similar increase.

The quantitative effects of doubling ν are reported in Panel A of Table 2. If investors did not have to share rents with skilled workers, doubling ν would correspond to a $\nu(1 - \alpha) = 24$ percentage point increase in the capital share. However, because investors only receive a fraction of rents, doubling ν increases the capital share by 7.5 percentage points. While the increase in ν leads to a sizable effect on the aggregate capital share, it has a small effect on the distribution of the capital share across the firm size distribution, and it thus has a relatively small effect on the average capital share, decreasing it by 3.5 percentage points. ν does not affect the power law exponent of the firm size distribution in the model.

The changes in σ and ν alone have limited success in explaining the actual target moments of the data. In particular, doubling σ can explain roughly half of the decrease in the average capital share, but it leads to a small increase in the aggregate capital share. Similarly, doubling ν can explain a third of the increase in the aggregate capital share, but a relatively small part of the increase in the average capital share, and it is not consistent with the change in the size distribution that we see in the data. Either parameter predicts smaller changes in the aggregate and average capital shares than occur in the data. However, the effect of a change in volatility operates through the division of rents: when rents are larger, the effects of a change in volatility on the aggregate and

[13] Falato, Kadyrzhanova, and Sim (2013) estimate that the intangible capital stock relative to total assets increased from 20% at the end of 1970 to 80% in 2010, while Barkai (2016)'s calculations imply that economic rents have increased by more than 200 pps. Finally, De Loecker and Eeckhout (2017) find that U.S. markups have risen from 18% to 64% of marginal costs.

Figure 8: Valuation Ratio



Panel A plots the ratio of the aggregate market value to the aggregate total assets. The market value of an individual firm i is measured as Total Assets + Stock Price $\times$ Common Shares Outstanding $-$ Common/Ordinary Equities. Data Sources: Compustat/CRSP Merged Fundamentals Annual (1960-2014). Parameter values for the value-to-book ratio given in Panel B are taken from our benchmark calibration given in Table 1.

average capital shares of profits will be larger. We therefore next consider a joint increase in both volatility and the size of rents.

We report the effect of a joint increase in σ and ν in Panel B of Table 2. In this case, the model predicts an increase of 7.4 pps in the aggregate capital share and a decrease of 15.9 pps in the average capital share. When we increase the discount rate to 10%, the increase in the aggregate capital share is 9.3 pps, while the decrease in the average capital share is 9.2 pps. Increases in ν have minor effects on the aggregate capital share, except when they are augmented by increases in σ. Finally, only increases in σ lower the average capital share.

Finally, we examine the quantitative effects of changes in discounting alone by changing r from 5% to 10%. Such an increase could be the result of an increase in real interest rates or macroeconomic risk. Increasing the discount rate increases the difference between the discounted average productivity and aggregate productivity, and it hence increases the capital share. This increase is similar in magnitude to the increase in the capital share that results from an increase in σ. However, increasing the discount rate leads to an increase in the average capital share because an increase in the discount rate decreases the value of the option to wait and therefore increases the exit threshold. This in turn decreases the mass of small low-capital-share firms and hence raises the average capital share.

Table 2: Changes in the Capital Share and the Firm Size Distributions from 1960-1970 to 1990-2014. This table presents changes in the average and aggregate capital shares and the size distribution of firms in the data and compares them to the changes predicted by the model given a change in the key model parameters. To calculate the change in moments in the data, we subtract the moments measured for the time period of 1960-1970 from the same moments measured in the time period 1990-2014.

	Data	**Model**			
		$\sigma \to 2\sigma$	$\nu \to 2\nu$	$\sigma \to 2\sigma$ $\nu \to 2\nu$	$r \to 2r$
		$r = 0.05$			
Δ Average Capital Share	-0.118	-0.087	-0.035	-0.209	0.030
Δ Aggregate Capital Share	0.138	0.013	0.075	0.101	0.011
Δ Capital Share at $\bar{X}$	-0.574	-0.235	-0.260	-0.732	0.068
Δ Power Law Exponent of Firm Size	-0.346	-0.165	0	-0.164	0
		$\sigma \to 2\sigma$	$\nu \to 2\nu$	$\sigma \to 2\sigma$ $\nu \to 2\nu$	
		$r = 0.10$			
Δ Average Capital Share	-0.118	-0.053	-0.005	-0.111	
Δ Aggregate Capital Share	0.138	0.014	0.086	0.115	
Δ Capital Share at $\bar{X}$	-0.574	-0.150	-0.191	-0.491	
Δ Power Law Exponent of Firm Size	-0.346	-0.165	0	-0.164	

5 Empirical Evidence Linking U.S. Capital Share Dynamics and Idiosyncratic Volatility

In this section, we present empirical evidence on the joint dynamics of compensation, firm size, and the implied capital share dynamics. We show that the findings are largely consistent with the implications of our model.

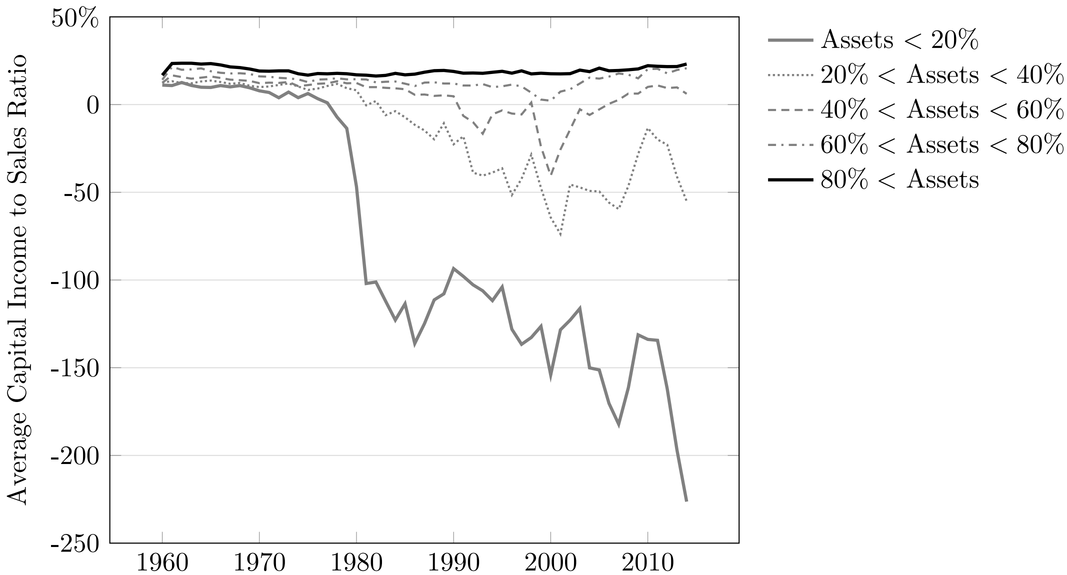
5.1 Cross-Sectional Variation in Capital Share Dynamics

A key prediction of our model is that the distribution of capital shares across firm size becomes more dispersed as idiosyncratic volatility increases. In particular, the capital shares of the smallest firms decreases because small firms with low profitability delay exit when volatility increases. The cross-sectional variation in the capital shares bears this out. Over the period 1960-2014, firm-level volatility doubled, and the capital share of the smallest firms significantly decreased. Figure 9 presents the time series of the average capital-income-to-sales ratio for different size quintiles.[14] All of the firms are sorted into five quintiles based on their total assets, and we compute the average capital-income-to-sales ratio in each quintile. We emphasize two aspects of this figure: first, the capital-income-to-sales ratio is increasing in firm size at each date, which is consistent with the core mechanism of our model: larger and more productive firms have higher capital shares ex post because their shareholders bear more risk than their skilled workers. Second, the average capital-income-to-sales ratio tends to decline more in the smaller size quintiles, while it increases for the large firms (the last quintile). Taken together, these facts indicate that while the aggregate capital share has increased, this increase is driven exclusively by the largest firms. These facts are also consistent with our model. As volatility increases, the dispersion of the size distribution of firms increases, which in turn increases the dispersion in the distribution of capital shares because larger (and more productive) firms have larger capital shares.

The increase in the dispersion of capital shares across the firm size distribution is also present within industries. To demonstrate this, we repeat the exercise carried out in Figure 9 within four industry groups: consumer goods, manufacturing, health products and information, and computers and technology (i.e., high tech industries). Industries are defined by the Fama-French five-industry classification. We omit the industry classification "other" because it contains few firms. We fix the definitions of the industries over time and sort firms into five size quintiles within each industry. Figure 10 plots the results. We find similar cross-sectional patterns within each industry: the dispersion of the capital-income-to-sales ratio across size groups increases over the last five decades, while the more significant decline occurs in the smaller size quintiles. Interestingly, we observe a greater increase in the dispersion of the capital-income-to-sales ratio in the high tech and health products industries, which have relatively larger change in firm-level volatility. In section A of the Separate Online Appendix, we show that these patterns also manifest themselves when we exclude

[14] Recall that because value added can be negative at the firm level, we use the capital-income-to-sales ratio as a measure of the capital share at the firm level.

Figure 9: Capital-Income-to-Sales Ratio by Firm Size



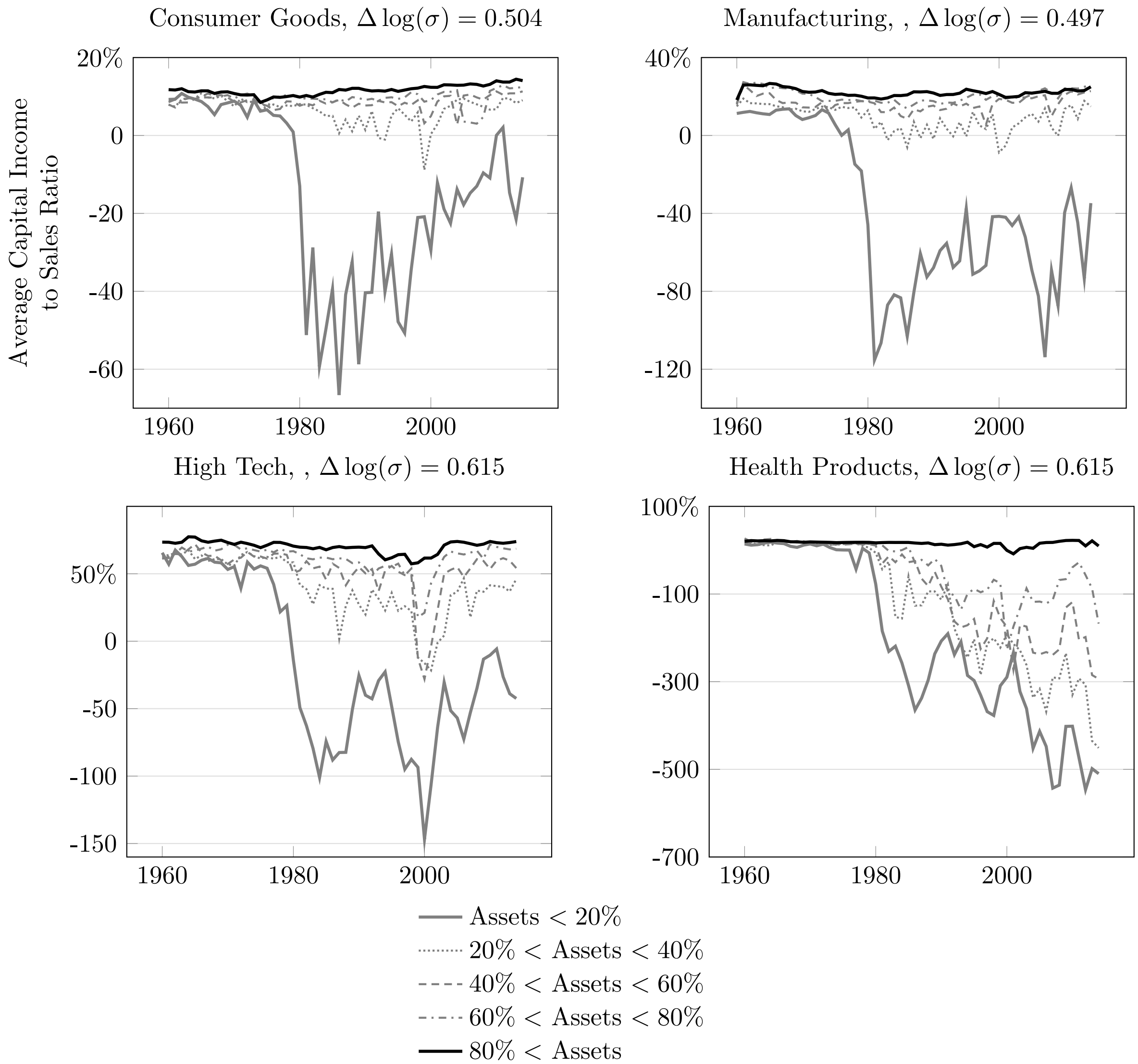
This figure presents the average capital-income-to-sales ratio by size over time. Size is measured by total assets, and the capital-income-to-sales ratio is measured as operating income (OIBDP) divided by sales. For each year, firms are categorized into five groups based on their total assets, and we estimate the average capital-income-to-sales ratio within each group for a given year. Source: Compustat-CRSP merged Fundamentals Annual for 1960-2014.

NASDAQ firms.

To provide more evidence on the link between volatility and the dispersion of the capital share within industries, we regress the industry-level average capital-income-to-sales ratio on the average firm-level volatility within industries at the 2- and 3-digit SIC code level. The decline in the average capital-income-to-sales ratio is mostly a within-industry phenomenon. We include industry and time fixed effects. The regression results are reported in Table 3. As shown in Column (1), a one-standard-deviation (0.21) increase in the firm-level stock return volatility corresponds to a 12-percentage-point decline in the average capital-income-to-sales ratio. In Column (2), a one-standard-deviation (0.186) increase in the moving average of the past idiosyncratic volatility is associated with a 17-percentage-point drop in the industry average capital income-to-sales ratio. The moving average captures the long-run change in the idiosyncratic volatility, and the effect is quantitative larger since the capital share and firm size distribution reflects both the contemporaneous volatility and the cumulative effect of the past volatility. In Column (3), the effect of firm-level sales growth volatility on capital share is less significant, likely because sales growth volatility is estimated using 5-year sales growth data instead of contemporaneous sales growth.[15] However, the effect in Column (2) is economically sizable: a one-standard-deviation (0.204) increase in sales

[15]Since sales growth volatility is estimated using the past 5-years quarterly sales growth data, the sales growth volatility estimates captures, to the large extent, past sales growth information. Hence, we do not include the ten year moving average of past sales growth volatility in the regression.

Figure 10: Average Capital Share of Output: Industries



Industries are defined by the Fama-French five-industry classification. We omit the industry classification "other" because it contains few firms. Within each industry, we sort firms into five groups based on their total assets. The plot shows the average capital share within each size group for four industries. $\Delta \log(\sigma)$ is the difference between the log of the industry level average of idiosyncratic volatility in 1990-2014 and the industry level average of idiosyncratic volatility in 1960-1970. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

growth volatility above that industry's mean volatility leads to a drop of more than 8 pps in the average capital-income-to-sales ratio below that industry's mean ratio.

Table 3: Average Capital Share and Idiosyncratic Volatility: Industry Level 1960 – 2014

The table reports the regression results of industry capital income/sales ratios on the average idiosyncratic volatility (annualized). The industry capital income/sales ratio is calculated as the average of capital income/sales ratios across firms within an industry. *Idio.Vol(ret)* is the average log idiosyncratic stock return volatility within an industry, *MA10.Idio.Vol(ret) is the moving average of industry level idiosyncratic return volatility from year t-10 to year t (including the current year)*, and *Idio.Vol(sales)* is the average log idiosyncratic sales growth volatility within an industry. *Tangibility* is the ratio of the average of gross property, plant, and equipment (PPEGT) to that of total assets (AT) within an industry. *M/B* ratio is the industry average market-to-book ratio within an industry. Columns (1) and (2) define industries using 2-digit SIC codes, and columns (3) and (4) define industries using 3-digit SIC codes. The sample includes all of the firms in the Compustat/CRSP merged database for 1960-2014. t statistics in parentheses, and * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$.

	(1)	(2)	(3)	(4)	(5)	(6)
	Capital Share (SIC2)			Capital Share (SIC3)		
Idio.Vol(ret)	-0.593***			-0.406***		
	(-3.02)			(-6.07)		
MA10.Idio.Vol(ret)		-0.920**			-0.477***	
		(-2.61)			(-3.83)	
Idio.Vol(sales)			-0.389			-0.141*
			(-1.50)			(-1.76)
Tangibility	0.242	0.336*	0.292*	0.161***	0.158**	0.132**
	(1.42)	(1.72)	(1.69)	(2.62)	(2.50)	(2.06)
M/B Ratio	-0.080**	-0.094**	-0.106**	-0.071***	-0.069***	-0.094***
	(-2.42)	(-2.65)	(-2.29)	(-3.81)	(-3.67)	(-4.49)
Constant	0.185	0.254	0.158*	0.155***	0.182***	0.179**
	(1.50)	(1.33)	(1.72)	(2.99)	(3.01)	(2.23)
Year FE	Y	Y	Y	Y	Y	Y
Industry FE	Y	Y	Y	Y	Y	Y
N	3,091	3,106	2,774	11,838	11,896	10,394
N_clust	65	65	63	252	252	249
r2_a	0.161	0.163	0.175	0.084	0.071	0.060

This relation between the average capital share and firm-level volatility is robust to controlling for various concentration measures. (Barkai, 2016; Autor et al., 2017) emphasize that increased concentration drives the secular factor shares. We construct concentration measures based on Compustat-reported sales; we also use census concentration measures and Herfindahl-Hirschman Index measures for manufacturing. The negative effect of firm-level return risk on average capital shares endures at the 2- and 3-digit SIC code levels. These results are reported in Section C of the Separate Online Appendix.

Finally, using Compustat Global, we ran the same regression for the U.K., Europe, and Japan and found statistically significant negative coefficient estimates for firm-level volatility in all three cases: a one-standard-deviation increase in volatility above that industry's (3-digit SIC-code) average lowers the average capital share by 2.7 pps, 3.5 pps, and .5 pps in the U.K., Europe, and Japan, respectively. This effect is economically significant in the U.K. and Europe (although smaller than that in the U.S.), but not in Japan. Section B.1 of the Separate Online Appendix provides the details.

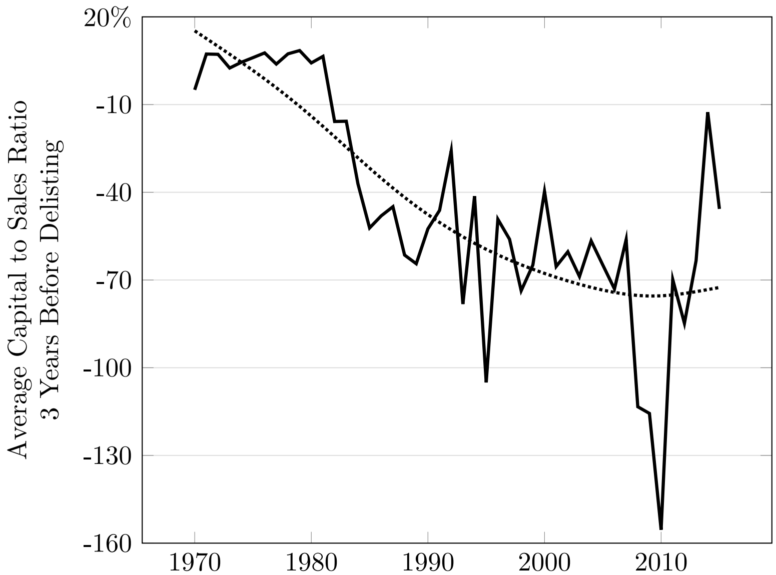
We have also examined the vintage effects of the capital share dynamics. For each year, we identify firms in the top 20 percent of the size distribution (measured by total assets or sales). Among the top 20 (bottom 20) percent of largest (smallest) firms, we identify the entry year of each firm and group them into five entry cohorts (1960s, 1970s, 1980s, 1990s, and 2000s). We then compute the average capital-income-to-sales ratio of each entry cohort. First, the average capital-income-to-sales ratio among the largest firms trends up for all cohorts, not just the most recent ones. Second, there is a strong vintage effect amongst the smallest firms, which is consistent with our model: the capital-income-to-sales ratios are lower for younger vintages that have experienced higher idiosyncratic volatility early on in their lifespans, which is consistent with the higher optional value of waiting. The Autor et al. (2017) mechanism cannot speak to this vintage effect in the left tail. This evidence is reported in section D of the Separate Online Appendix.

To provide further evidence that the patterns that we see in the dynamics of capital shares across the firm size distribution are consistent with firms delaying exit, we directly examine the capital shares of firms close to the exit boundary. Specifically, we investigate firms that exit the public domain due to poor company performance (e.g., liquidation, insolvency, bankruptcy) by obtaining the security delisting information from the CRSP U.S. Stock Event database. Figure 11 plots the capital-to-sales ratio three years before delisting for these firms. Consistent with our model, the average capital share of firms three years before delisting declined by almost 90 pps from 1970 to 2014. This result remains largely unchanged if we consider the capital share five years before delisting.

6 Conclusion

We propose a mechanism by which an increase in firm-level volatility drives a wedge between national income accounting and the ex-ante assessment of firm profitability. A firm's owner insures skilled workers against firm-level productivity shocks and may choose to exit if productivity becomes too low. In our optimal contracting model, the level of the skilled workers' compensation is proportional to the expected value of new firms, which necessarily integrates over paths that end in exiting. In contrast, in the national income accounts, one integrates only over ex-post surviving firms that necessarily feature higher capital shares. This leads to a difference between the aggregate capital share of income, which is calculated ex post, and the capital share of value at the origination of the firm, which is calculated ex ante. When firm-level volatility increases, more firms end up in the right tail of the size distribution. These outcomes are immediately reflected in the stationary distribution of productivity, but they are discounted when the values of new firms are calculated. Thus, they have a larger impact on aggregate productivity, and hence output, than

Figure 11: Exit Threshold: Average Capital-Income-to-Sales Ratio Three Years before Delisting



The plots present the average capital shares of output three years before delisting. We define a firm's exit from the public firm domain by the use of delisting codes 400-490 and 550-591. The dotted line is the HP filtered trend. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014) and CRSP delisting code.

on the payments to skilled workers. As a result, the aggregate capital share increases. We present a calibrated version of our model that replicates the key moments of the data with reasonable parameters via increases in firm level volatility, but only if accompanied by an increase in rents. Finally, we present time series and cross-sectional evidence for Compustat firms that are consistent with our proposed mechanism.

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A Data Appendix

A.1 Data Construction

The Sample. The Compustat/CRSP Merged Fundamental Annual database contains accounting data and stock return data for all publicly traded firms. The sample runs from 1960 to 2014, and it includes all Compustat/CRSP firms (both active and inactive).[16] We exclude financial firms with SIC codes 6000-6799 for our main analysis, and we exclude firms that have negative sales, negative employee numbers, or negative total asset values. Finally, we exclude firms that indicate their currency code for Canadian dollars to focus on U.S. firms only. All of the variables are winsorized at 1% unless indicated otherwise.

Construction of Main Variables. We measure a firm's capital income using operating income before depreciation (OIBDP). The capital share of output is defined as OIBDP/Sales.

Labor income is measured using the labor cost reported by public firms. Because public firms are not required to file Staff Expenses (XLR), we obtain only sparse observations of the labor cost from the Compustat database. Following Donangelo (2016), we construct the *extended labor cost* (extended XLR). First, we estimate the average labor cost per employee (XLR/EMP) within the industry-size group for each year. Industries are classified using the Fama-French 17-industry definition, and firms are sorted into 20 size groups based on their total assets, which yields a total of 340 industry-size groups. Then, the labor cost of a firm with missing XLR equals the number of employees multiplied by the average labor cost per employee of the same industry-size group during that year. We winsorize the extended XLR at 5% to exclude outliers from the approximation. We measure the labor share of output as extended labor cost (XLR)/Sales.

Value added (VA) is defined as OIBDP + extended XLR. We winsorize VA at 5% to exclude outliers from the approximation of extended XLR. We calculate the capital share as OIBDP/VA and calculate the labor share as extended XLR/VA. We also winsorized capital income to value added ratio (OIBDP/VA) and labor share extended XLR/VA at 5% to avoid the influence of outliers. We estimate the *adjusted* value added to deal with negative values. We identify the minimum operating income (OIBDP) for each year, and we increase the value added of all firms by the absolute value of the minimum OIBDP×(1+1%). The adjusted VA is then OIBDP×(1+1%) + extended XLR.

We measure firm-level volatility using both idiosyncratic cash flow volatility and idiosyncratic stock return volatility. Idiosyncratic stock returns are constructed within each year by obtaining the residual of a Fama-French 3-factor model using all of daily stock returns within that year. $r_{it} = \gamma_{0,i} + \gamma_{1,i} MKT_t + \gamma_{2,i} HML_{3,i} + \epsilon_{i,t}$, where t is a daily observation of stock returns within year T. Idiosyncratic stock return volatility for firm i in year T is calculated as the annualized standard deviation of $\epsilon_{i,t}$ within that year.

[16] Using the Compustat/CRSP merged dataset gives us a consistent sample for estimating stock return volatility and the delisting threshold. All of our empirical results for capital shares, capital-income-to-sales ratio, and labor shares remain the same using the Compustat sample.

To obtain the idiosyncratic cash flow volatility, we use the same factor specification as that in the idiosyncratic return volatility. The sales growth is winsorized at 1% in the full sample to exclude the outliers. For each quarter t, we estimate the factor model using quarterly sales growth data in prior 20 quarters up to calender year T. Since there is no predominant factors for cash flow growth, we follow Herskovic et al. (2015) and use the first five major principal components of the prior 20 quarters sales growth. The data requires firms with no missing observations in the 20 quarter window ending in year T. The idiosyncratic cash flow volatility is the annualized standard deviation of the residuals of a sales growth factor specification.

Non-Publicly Traded Firms. We obtain the measure of capital income for non-publicly traded firms by subtracting the aggregate capital income of the Compustat firms from the aggregate capital income of the U.S. economy. The aggregate capital income is measured using the NIPA table *Net Operating Surplus*, which measures the aggregate business income from production after subtracting labor costs, taxes on production and imports (less subsidies), and consumption of fixed capital (economic depreciation) from value added, but before subtracting financing costs (such as net interest) and business transfer payments. Net operating surplus is a profit-like measure that is conceptually closest to earnings before interest and tax (EBIT) in the NIPA tables (See Mead, Moulton, and Petrick (2004)).

A.2 Figures and Tables

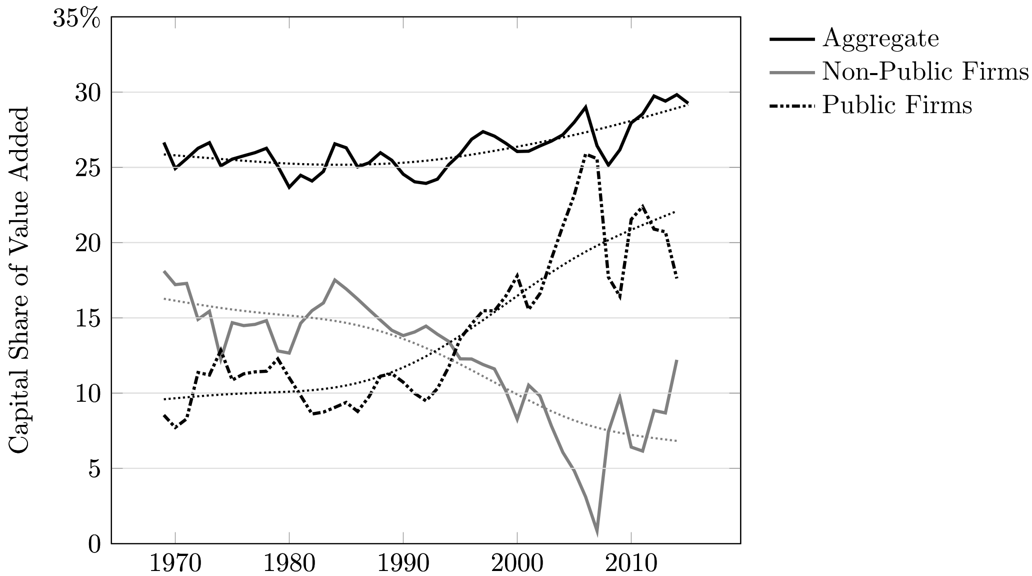
In this section, we provide more details regarding the figures and tables in the paper.

Average and Aggregate Factor Share. Using the Compustat sample, we construct the average and aggregate factor shares as follows: For each year, aggregate factor share $= \frac{\Sigma_i \text{Factor Income}}{\Sigma_i \text{Output}}$, and the average factor share $= \Sigma_i \left(\frac{\text{Factor Income}}{\text{Output}}\right)/N$.

Size Groups. For each year, all of the firms are sorted into five groups based on their total assets. Within each group, we compute the average and aggregate labor share and capital share. Figure 9 and Figure 10 show the time series of the average and aggregate capital shares of each size group.

Delisting Threshold. We obtain the security delisting information from the CRSP U.S. Stock Events database. The *delisting* Code is a 3-digit integer code that (a) indicates that a security is still trading or (b) provides a specific reason for delisting. We consider delisting due to liquidation (delisting codes 400 to 490) and delisting by the current exchange for various reasons due to poor company fundamentals (e.g., insolvency, bankruptcy, insufficient capital (delisting codes 550 to 591)). Then, we calculate the average capital shares either three years or five years before delisting. Figure 11 shows the time series of the pre-delisting performance from 1970 to 2014.

Figure 12: Aggregate Capital Share of U.S.: Public versus Private Decomposition



This figure decomposes the aggregate capital share (black solid line) for all U.S. industries into the share due to private firms (gray solid line) and the share due to public firms (dashed black line). The universe of private firms covers all firms except publicly traded firms in Compustat. The aggregate capital share of U.S. value added is the ratio of NIPA's net operating surplus retrieved from FRED and the total value added from the BEA's GDP-by-industry accounts. Private capital income is obtained by subtracting the aggregate capital income (earnings before interest and tax) reported by Compustat public firms from the NIPA net operating surplus. The private (public) share is the ratio of private (public) aggregate capital income and total value added. The dotted lines are the HP-filtered trends of the three time series of capital shares. Data sources: U.S. Bureau of Economic Analysis, Gross domestic income: Net operating surplus: Private enterprises [W260RC1Q027SBEA] (1969-2015). Bureau of Economic Analysis, GDP by Industry Accounts (1960-2015). Compustat/CRSP Merged Fundamentals Annual (1960-2014).

B Public vs. Private

Our empirical analysis focuses on publicly traded firms. However, Davis, Haltiwanger, Jarmin, and Miranda (2007) find that non-farm firm-level volatility in the private sector has declined in recent decades. According to the logic of our model, the aggregate capital share for private firms in the U.S. should not have increased over the same period of time. Using the NIPA net operating surplus (NOS), which is a profit-like measure that aggregates the overall operating income of the U.S. economy, we infer the operating income of private firms by subtracting the aggregate operating income of publicly traded firms (Compustat/CRSP firms) from the NIPA net operating surplus. Figure 12 decomposes the aggregate capital share of total value added into a private component and a public component. The component due to private firms, the ratio of the aggregate operating income of private firms to total value added, declined from 1969 to 2014 even though the private sector has grown recently: the number of publicly listed firms has decreased by 14% since 1996 (Doidge, Karolyi, and Stulz (2015)).

C Proofs

C.1 Derivation of Equilibrium Compensation

The equilibrium wage is given by the solution to the following equation:

$$\int_{\bar{X}(c)}^{\infty} V(X;c)f(X)dX = P,$$

where $\bar{X}(c)$ is given by Equation (12) and $V(X;c)$ is given by Equation (11). Note that this equation is equivalent to Condition 3 of Definition 1. We have

$$\int_{\bar{X}(c)}^{\infty} V(X;c)f(X)dX = \left(\frac{\xi-1}{\hat{F}\xi}\right)^{-\rho}\left(\frac{\eta}{(r+\lambda)(\rho-1)(\eta+\rho)}\right)c^{-(\rho-1)}.$$

Note that our assumption on the Pareto form $f(X)$ facilitates the computation of the integral shown above because both $V(X;c)$ and $f(X)$ are power functions. This integral represents the expected value of the firm to the shareholder after paying the fixed cost but before realizing the initial productivity of the firm. Because $\rho > 1$, it is monotonically increasing in c, and we can solve to obtain the expression for equilibrium compensation given in (13).

C.2 Derivation of Stationary Distribution

The ODE for $\phi(x)$ has the following general solution:

$$\phi(x) = A_1 e^{\gamma_1 x} + A_2 e^{-\gamma_2 x} + A_3 e^{-\rho x}, \tag{32}$$

where γ_1 and γ_2 are given by

$$\gamma_1 = \frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2\lambda}}{\sigma^2} \tag{33}$$

$$\gamma_2 = \frac{-(\mu - \frac{1}{2}\sigma^2) + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2\lambda}}{\sigma^2}. \tag{34}$$

First note that $\gamma_1 > 0$ implies that $A_1 = 0$. To ease notation, we drop the subscript on γ_2. Next, note that an application of the ODE gives

$$A_3 = -\frac{\rho\psi}{\frac{1}{2}\rho^2\sigma^2 + \rho(\mu - \frac{1}{2}\sigma^2) - \lambda}. \tag{35}$$

Finally, the boundary condition implies that

$$A_2 e^{-\gamma\bar{x}} + A_3 e^{-\rho\bar{x}} = 0,$$

so

$$A_2 = -A_3 e^{(\gamma-\rho)\bar{x}}. \tag{36}$$

The result in Equation (17) directly follows from the solution above and from an application of the market-clearing condition for skilled workers.

C.3 Proof of Proposition 1

We have

$$\begin{aligned}\frac{\partial \Pi}{\partial \sigma} &= -\nu\left(\frac{\rho-1}{\rho}\right)\frac{\partial}{\partial \sigma}\left[\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\xi}{\xi-1}\right)\right] \\ &= -\nu\left(\frac{\rho-1}{\rho}\right)\left[\left(\frac{\xi}{\xi-1}\right)\frac{1}{\gamma^2}\frac{\partial \gamma}{\partial \sigma} - \left(\frac{\gamma-1}{\gamma}\right)\frac{1}{(\xi-1)^2}\frac{\partial \xi}{\partial \sigma}\right]\end{aligned}$$

Now observe that

$$\frac{\xi(\xi-1)\frac{\partial \gamma}{\partial \sigma}}{\gamma(\gamma-1)\frac{\partial \eta}{\partial \sigma}} = \frac{\sqrt{(\mu-\frac{1}{2}\sigma^2)^2 + 2(r+\lambda)\sigma^2}}{\sqrt{(\mu-\frac{1}{2}\sigma^2)^2 + 2\lambda\sigma^2}} > 1,$$

which leads to the desired result.

C.4 Proof or Proposition 2

The first result follows immediately from equation (30). The second result follows from the proof of Proposition 1.

D Pay for Performance

In this appendix, we allow for some exposure in the skilled worker's compensation to firm performance. This exposure could arise for a variety of reasons. For example, there could be a firm-level agency conflict between the skilled worker and investors, or the investor could be risk-averse. In either case, the optimal contract will call for the skilled worker to bear some exposure to firm performance, either for incentive purposes or to improve risk sharing. The precise form of the optimal contract will depend on the nature of the agency problem or the exact preferences of the skilled workers and investors.[17] One possible concern thus far with our results may be that this exposure could mitigate the insurance nature of the relationship between firms' owners and their skilled workers, thus decreasing or reversing the effect of firm-level volatility on the capital share of profits. To ease notation, we assume that $\nu = 1$. Rather than solve directly for an optimal contract

[17] Edmans et al. (2009) derive CEO compensation in a competitive equilibrium with a talent assignment and a moral hazard problem.

for a particular problem, we assume that the skilled worker's contract takes the following simple affine form:

$$c_t = \beta X_t + w. \tag{37}$$

The sensitivity β of the skilled worker's payment c_t to the level of productivity is determined by either the severity of the agency problem or the nature of the risk-sharing problem, and it is exogenous from the standpoint of our model. The fixed wage w is set in equilibrium in the same manner as total wages are set above. This contract has the advantage of being particularly tractable to analysis in the context of our model of equilibrium.

For a given fixed wage w, the investor's problem is

$$\max_{\tau} \left[\int_0^{\tau} e^{-rt}((1-\beta)X_t - w)dt \right]. \tag{38}$$

Again, standard arguments imply that the investor's value function $V(X)$ must satisfy the following ODE:

$$(r+\lambda)V = (1-\beta)X - w + \mu X V' + \frac{1}{2}\sigma^2 X^2 V'', \tag{39}$$

with the boundary conditions

$$V(\bar{X}) = 0, \tag{40}$$

$$V'(\bar{X}) = 0, \tag{41}$$

$$\lim_{X\to\infty} \left| V(X) - \left(\frac{(1-\beta)X}{r+\lambda-\mu} - \frac{w}{r+\lambda} \right) \right| = 0. \tag{42}$$

This problem is essentially the same as the problem given in Equations (7)-(10), up to a scaling of the leading term by a factor of $(1-\beta)$. Thus, the solution to Equations (39)-(42) is

$$\begin{aligned} \bar{X} &= \left(\frac{1}{1-\beta} \right) \left(\frac{\eta}{\eta+1} \right) \frac{w(r+\lambda-\mu)}{r+\lambda} \\ V(X) &= \frac{(1-\beta)X}{r+\lambda-\mu} - \frac{w}{r+\lambda} - \left(\frac{(1-\beta)\bar{X}}{r+\lambda-\mu} - \frac{w}{r+\lambda} \right) \left(\frac{X}{\bar{X}} \right)^{-\eta}, \end{aligned}$$

where η is defined as above.

Given the solution for the investor's value, we can apply the investor's zero ex-ante profit condition to determine the fixed component of the skilled worker's equilibrium contract. This calculation yields

$$w^* = \left(\frac{P(r+\lambda)(\rho-1)(\rho-\eta)}{\eta} \left(\frac{\eta(r+\lambda-\mu)}{(1-\beta)(\eta+1)(r+\lambda)} \right)^{\rho} \right)^{-\frac{1}{\rho-1}}. \tag{43}$$

Comparing Equations (13) and (43) reveals that the fixed component of the equilibrium affine contract is just the equilibrium wage under full insurance scaled by a function of β. Thus, the investor's problem under the affine contract is identical to the problem under full insurance when the firm's productivity is scaled by a factor of $1-\beta$. The stationary distribution of firm productivity is unaffected by our assumption of affine contracts up to a shifting of the optimal abandonment threshold (i.e., the left support of the stationary distribution). Thus, we can again calculate the total capital share of profits in the stationary distribution to obtain

$$\Pi = (1-\beta)\left(1-\left(\frac{r+\lambda}{r+\lambda-\mu}\right)\left(\frac{\rho-1}{\rho}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\eta+1}{\eta}\right)\right). \tag{44}$$

Comparing Equations (30) and (44) shows that the total capital share profits under the affine contract depend on γ and η, and hence also on σ in the same manner as the total capital share of profits under full insurance. In other words, allowing the skilled worker to share in the success of successful firms does not change our main qualitative results.

While allowing skilled workers to share in some of the gains of successful firms does not change the aggregate dynamics of the capital share, it does have important implications for the distribution of the labor share across income levels. Income inequality has been rising, as observed by Piketty and Saez (2003) and Guvenen and Kuruscu (2007). Therefore, the share of output that accrues to the top decile of the income distribution could have increased. That is, the income shares could have become more unequal. Our model is consistent with rising income share inequality when we allow skilled workers to share in the gains of successful firms via the affine contracts that we consider in this section. In this case, the distribution of skilled worker pay essentially inherits the properties of the distribution of firm productivity (or size). As volatility increases, the most successful firms account for a larger share of total output, and because the managers of these firms receive pay in proportion to productivity, their pay is also a larger fraction of total output.

E Valuation Ratio

In this appendix, we solve for the aggregate valuation ratio of our production economy. Given the homogeneity of F, the price of capital is given by solving the first-order condition for the firm's problem and is given by

$$\kappa = (1-\nu)\left(\frac{F_1(k,l)}{F(k,l)}\right)y$$

where F_1 denotes the partial derivative of F with respect to its first argument. Because F, k, and l do not depend on any of the parameters with which we want to take a comparative static, we can just let

$$\alpha = \left(\frac{F_1(k,l)}{F(k,l)}\right)k$$

to get that the aggregate rental payment to physical capital is

$$\kappa k = (1-\nu)\alpha y$$

which makes some intuitive sense. This is basically the capital expenditure share of output obtained from a model with a homogeneous production function. To obtain the book value of physical capital, we can simply capitalize this number to get

$$\text{Book value of physical capital} = \frac{\kappa k}{r} = \frac{(1-\nu)\alpha y}{r}$$

Given the equilibrium $\hat{F}$, c^*, $\bar{X}$ and the firm value $V(X,;c,\hat{F})$ from equation (7), the ratio of the aggregate of market value of rents is

$$\begin{aligned}
\hat{V} &= \int_{\bar{x}}^{\infty} V(X;c^*,\hat{F})\phi(x)dx \\
&= \frac{\hat{F}\hat{X}}{r+\lambda-\mu} - \frac{c^*}{r+\lambda} - \left(\frac{\widehat{F}\bar{X}^{1+\eta}}{r+\lambda-\mu} - \frac{c^*\bar{X}^{\eta}}{r+\lambda}\right)\int_{\bar{x}} e^{-\eta x}\phi(x)dx \\
&= \frac{\nu y}{r+\lambda-\mu} - \frac{c^*}{r+\lambda} - \left(\frac{\widehat{F}\bar{X}^{1+\eta}}{r+\lambda-\mu} - \frac{c^*\bar{X}^{\eta}}{r+\lambda}\right)\int_{\bar{x}} e^{-\eta x}\phi(x)dx
\end{aligned}$$

The ratio of the aggregate value of firms (cumulative of the aggregate value of physical capital) to the book value of physical capital is then

$$\frac{\text{Market}}{\text{Book}} = 1 + \frac{r\hat{V}}{(1-\nu)\alpha y}$$

Capital Share Dynamics When Firms Insure Workers: Separate Online Appendix, Not For Publication.

This Appendix consists of five sections. In Section A, we study compositional changes that drive the factor share dynamics. In Section B, we examine the international evidence on factor share dynamics. In Section C, we examine the evidence on the relation between factor share dynamics and concentration. In Section D, we examine whether the facts we document in Sections 1 are driven by entry cohorts. In Section F, we demonstrate that the findings we present in Sections 1 and 5 are robust to adding R&D expenses to our measure of capital income and to different winsorization criteria.

A Composition Effects and the Aggregate Capital Share

A.1 Exchanges

This section investigates whether our results are driven by the accession of NASDAQ firms to the Compustat database. Figure A.1 plots the aggregate and average capital-income-to-sales ratios for NYSE and NASDAQ firms separately. There is similar divergence between the aggregate and average capital shares in the universe of NYSE firms, though the trends are quantitatively less pronounced.

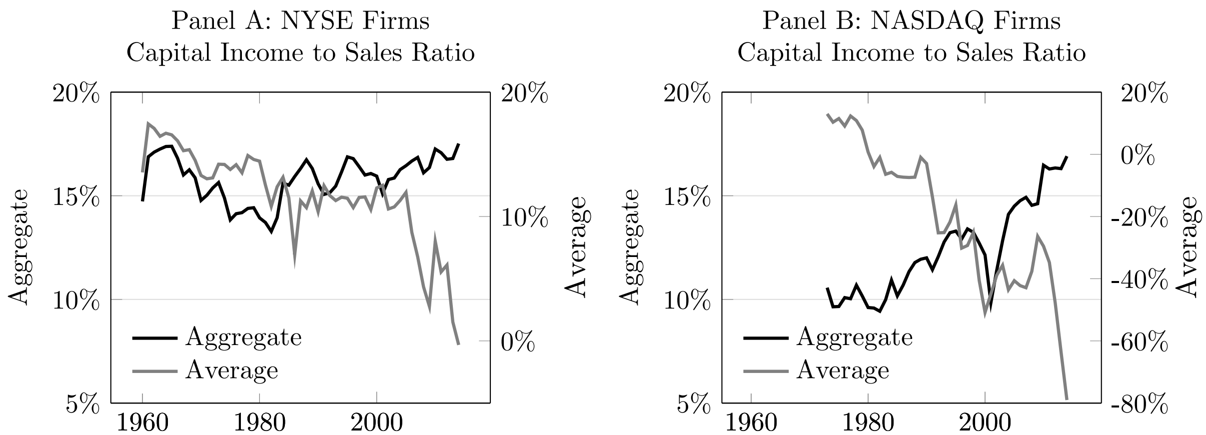
Figure A.2 reports the average capital-income-to-sales ratio by firm size (total assets) for NYSE and NASDAQ separately. We found qualitatively similar patterns in the average capital-income-to-sales ratio in the NYSE and NASDAQ universe separately, as documented in Figure A.2 (a) and (b). Across the two major exchanges, there is a clear ordering of average capital-income-to-sales ratio by firm size and the average capital-income-to-sales ratio trended down dramatically (on average negative) in the smallest firm size group since 1980s. However, the decline is much more pronounced for the smallest NASDAQ firms, as one would expect. Hence, the entry of small NASDAQ firms does contribute to the decline in the capital-income-to-sales ratio that started in the 1980s, but even after excluding the NASDAQ firms, we document a steep decline.

A.2 Across- vs. Within-Industry Effects

Although we document a significant relation between industry level average capital share and the idiosyncratic volatility in Table 3, we do not find a similar relation between industry level aggregate capital share and idiosyncratic volatility. This is because the trend in the aggregate capital share is driven primarily by changes in the cross industry share of aggregate sales, rather than a within industry change in the firm size distribution. Figure A.3 plots the weighted capital-income-to-sales ratio in 1970 where the weights are the sales in year t given by given by:

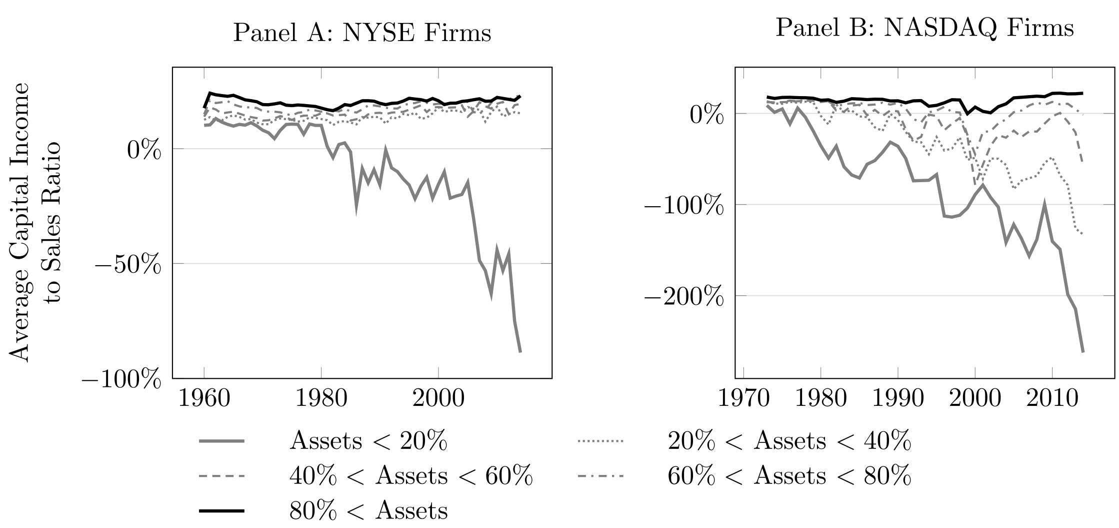
$$\text{Across Industry Effect}_t = \frac{\sum_i \text{Sales}_{it} \frac{\text{Capital Income}_{i1970}}{\text{Sales}_{i1970}}}{\sum_i \text{Sales}_{it}} \tag{45}$$

Figure A.1: Capital Share Dynamics Across the NYSE and NASDAQ



This figure presents the average and aggregate capital income to sales ratios separately for firms listed in the NYSE and NASDAQ Exchanges The aggregate capital-income-to-sales ratio = $\sum_i$ Operating Income$_i$ divided by $\sum_i$ Sales$_i$ for each year. The average capital-income-to-sales ratio = mean (Operating Income divided by Sales) for each year. The sample is winsorized at 1%. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

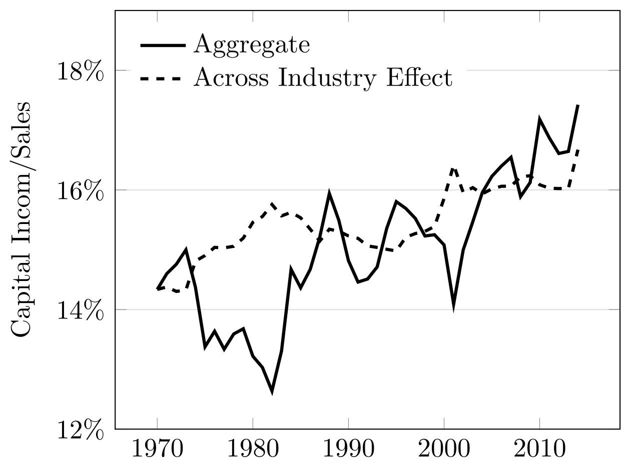
Figure A.2: Capital-Income-to Sales Ratio by Firm Size: Exchanges



This figure presents the average capital-income-to-sales ratio by size over time separately for firms listed in the NYSE and NASDAQ Exchanges. Size is measured by total assets, and the capital-income-to-sales ratio is measured as operating income (OIBDP) divided by sales. For each year, firms are categorized into five groups based on their total assets, and we estimate the average capital-income-to-sales ratio within each group for a given year. The sample is winsorized at 1%. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

This ratio measures the across-industry component of the aggregate capital share. The remainder (i.e., the gap between the aggregate ratio and the across-industry component) measures the within-industry effect. Note that the across-industry effect accounts for most of the increase in the aggregate capital-income-to-sales ratio. Figure A.3 indicates that industries with high within industry aggregate capital income to sales ratios in the 1970s became larger.

Figure A.3: The Across-Industry Effect on the Aggregate Capital Share



Industry classification is according to the Fama-French 48 industry classification. The solid line is the aggregate capital share. The dashed line is the across-industry effect we describe in Equation (45). Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

B International Evidence

To provide further support for our mechanism, we explore the capital share dynamics in Japan, UK and Europe (EU). The EU countries include Austria, Belgium, Switzerland, Germany, Denmark, Spain, Finland, France, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Sweden.[1]

Our main analysis is conducted using Compustat Global. Compustat Global contains the widely available accounting data for us to obtain key firm fundamental variables. The daily stock returns are calculated using Compustat Global Security Daily. We estimated the idiosyncratic return volatility within each year by estimating the Fama/French 3 factor model where we obtain the Fama-French global three factors from Ken French's website. The global three factors start in the year 1990, so our the majority of our international evidence is presented for the sample period from 1990 to 2017. Our sample excludes financial firms (SIC code between 6000 and 6799).

Since the shift in the firm size distribution only gradually reflects the changes in idiosyncratic volatility, we extend the time series of idiosyncratic volatility using stock return data from Datastream. For UK and Japan, we can extend the estimates of idiosyncratic volatility back to 1975 and

[1]The list of EU countries are chosen following the Fama/French European 3 Factors Portfolios excluding the U.K.

1978 respectively by estimating a single factor model. The daily stock return data and interest rate data for most of the European countries are not available before 1990, so we are not able to extend the time series of idiosyncratic volatility for EU.

Our firm-level variables are constructed as follows:

- Idiosyncratic stock return volatility
  - Compustat Global Security Data: The daily stock return of company j in country k is calculated using data from (1990-2017) as follows

    $$R_{j,k,t} = \frac{PRCCD_t/AJEXDI_t \times TRFD_t - PRCCD_{t-1}/AJEXDI_{t-1} \times TRFD_{t-1}}{PRCCD_{t-1}/AJEXDI_{t-1} \times TRFD_{t-1}},$$

    where $PRCCD$ is the closing price at the end of each trading day, $AJEXDI$ is the cumulative adjustment factor (issue) ex-date and $TRFD$ is the total return adjustment factor.

    We then convert the daily stock return in the local currency to US dollars $r_{j,k,t}$, and then we calculate the idiosyncratic volatility within each year τ by estimating a factor model using all daily observations within the year for each country k:

    $$r_{j,k,t} = \delta_{j,k} + \gamma_{\mathbf{j,k}} \mathbf{F_{k,t}} + \epsilon_{j,k,t}$$

    where $\mathbf{F_{k,t}}$ are factors Fama-French global three factors for EU and Japan from Ken-French Data Library (1990-2017). For the U.K., we used European 3 factors for the estimation.

    Idiosyncratic volatility $\sigma_{j,k,\tau}$ is the standard deviation of $\epsilon_{j,k,t}$ within each year τ.
  - Datastream: We obtained individual security return (RI, adjusted and US dollar denominated) for the U.K. (1975-2017) and Japan (1978-2017), and then calculate the idiosyncratic volatility within each year τ by estimating a factor model using all daily observations within the year for each country k:

    $$r_{j,k,t} = \delta_{j,k} + \gamma_{j,k} MKT_{k,t} + \epsilon_{j,k,t}$$

    where $MKT_{k,t}$ are factors excess return of market indexes for the U.K. and Japan over the same period of time. The interest rate for the U.K. is U.K. sterling 1-month deposit rate, and the market index for the U.K. is UK total market index (TOTMKUK). The interest rate for Japan is the 1-month deposit rate, and the market index is the NIKKEI 225 average share index.

    The idiosyncratic volatility of firm j in country k, $\sigma_{j,k,\tau}$ is the standard deviation of $\epsilon_{j,k,t}$ within each year τ. For the U.K. and Japan, we are able to employ longer equity stock return data from Datastream. Given that it is noisy to merge the security daily return data from Datastream to Compustat Fundamental Global, the estimates of idiosyncratic volatility is only used to show the evolution of idiosyncratic volatility over a longer sample period. In Table 1. we show that summary statistics of the idiosyncratic volatility from two databases are very similar.

- Firm fundamental variables (Compustat Global Fundamental Annual)
  - Capital Share: Ratio of the operating income (OIBDP) to sales (SALE).
  - M/B ratio: Ratio of the market value of total assets (AT + PRCCD$\times$ SCHO-CEQ) to the book value of total assets (AT).
  - Tangibility: Ratio of physical assets (PPEGT) to total assets (AT).

Table A.1 reports summary statistics of all variables.

We start by documenting the shifts in the firm size distribution and idiosyncratic volatility in the U.K., the E.U. and Japan. The U.K. experience largely mirrors that of the U.S. Figure A.4 visualizes the change in size distribution and capital share of U.K. firms throughout the sample period (1990-2017). The right tail of the firm size distribution gets fatter as idiosyncratic volatility increases from 1975 to 2017. We also documented the divergence of aggregate and average capital-income-to-sales ratio. Consistent with our model mechanism, the relationship between firm size and capital-income-to-sales ratio has changed dramatically since 1990. In Figure A.4 (d), we see that the capital-income-to-sales ratio was much more strongly increasing in firm size than in 1995. We found qualitatively similar patterns when investigating the major countries in Europe (see Figure A.4).

On the other hand, the Japanese economy behaved differently from the UK and EU economies (Figure A.4), but the evidence is largely consistent with our model mechanism. First, the right tail of the Japanese firm size distribution is not getting fatter over time, and there is no clear trend in the idiosyncratic volatility from 1978-2017. Consistent with our mechanism, there is *no* divergence between the average and the aggregate capital-income-to-sales ratio. The relationship between firm size and capital-income-to-sales ratio has changed since 1990, but 1) the slope of capital-income-to-sales ratio was not much steeper in 2010 than 1995; 2) more than 90% of Japanese firms experienced an increase in capital share over the sample period (not just the tail 10% as seen in US, UK and EU). This implies that there is no strong left tail effect that drives a decline in average capital share.

Figure A.4: International Evidence

This figure presents the time series of idiosyncratic volatility, average and aggregate capital income to sales ratio, the average capital income to sales ratio within each size percentile, the average capital income to sales ratio over each volatility percentile, and the power law coefficient of the top 5-percentile firms for the U.K., the E.U. and Japan.

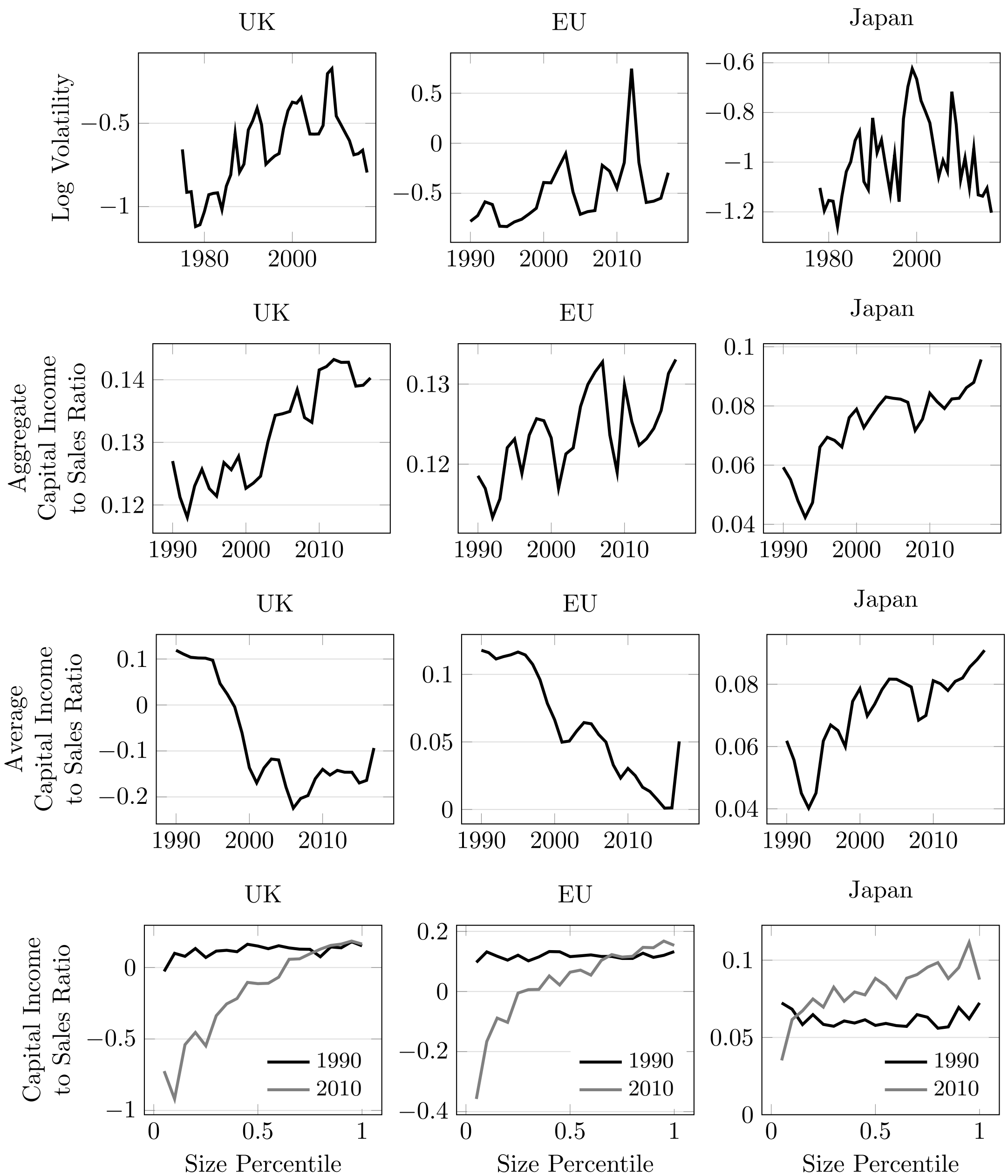


Table A.1: Summary Statistics

This table reports summary statistics of firm-level variables for the international facts. All variables are defined in Section A. Sample period: 1990-2017. All variables are winsorized at 5%. For US data, all variables are winsorized at 1%. Data Source: Compustat Global Fundamental Annual (1990-2017), Compustat Global Security Daily (1990-2017), and Datastream (1975-2017).

Panel A: UK

Variable	Obs	Mean	Std. Dev.	P10	P50	P90
Log(Idio. Vol.) Datastream	92711	-.8944	.7194	-1.8019	-.9711	.1751
Log(Idio. Vol.) Compustat	28310	-.8779	.5588	-1.6142	-.9273	-.0501
MB Ratio	27857	1.9204	1.3886	.7958	1.4263	3.8982
Capital Income/Sales Ratio	32031	-.0955	.6566	-.4915	.0932	.2559
Tangibility	32544	.465	.3862	.0281	.383	1.0494

Panel B: Japan

Variable	Obs	Mean	Std. Dev.	P10	P50	P90
Log(Idio. Vol.) Datastream	110607	-1.0431	.4361	-1.6452	-1.05	-.4232
Log(Idio. Vol.) Compustat	64039	-1.0038	.4341	-1.5926	-1.0217	-.3756
MB Ratio	63594	1.176	.5247	.6999	1.0191	1.8951
Capital Income/Sales Ratio	68930	.0736	.0528	.0111	.0655	.1544
Tangibility	68699	.6497	.4113	.1178	.5963	1.2567

Panel C: EU

Variable	Obs	Mean	Std. Dev.	P10	P50	P90
Log(Idio. Vol.) Compustat	58275	-.8799	.5194	-1.539	-.9465	-.0933
MB Ratio	56417	1.6354	1.0795	.8116	1.2348	3.1694
Capital Income/Sales Ratio	75166	.0518	.2202	-.1331	.0936	.2425
Tangibility	69894	.5568	.4638	.0522	.4444	1.255

Panel D: US

Variable	Obs	Mean	Std. Dev.	P10	P50	P90
Log(Idio. Vol.)	167573	-.7671	.6099	-1.5469	-.7909	.0542
MB Ratio	200611	1.8861	1.6288	.8317	1.332	3.4881
Capital Income/Sales Ratio	204722	-.171	1.6367	-.1686	.1023	.2936
Tangibility	207021	.5581	.3937	.1154	.4764	1.1047

B.1 International Regression Evidence

Table A.2 below replicates our industry level analysis reports industry evidence for UK, EU and Japan using Compustat Global. We confirm the robust and significant negative correlation between average capital share (at SIC 3 digit and SIC 2 digit) and idiosyncratic return volatility. Idiosyncratic stock return volatility is estimated using daily stock return data from Compustat Global Security (1990-2017). Given the relative shorter time series and lower frequency of Compustat Global, we did not use cash flow volatility to proxy idiosyncratic volatility. A one standard deviation increase in vol above that industry's (SIC 3 digit) average lowers the average capital share by 3.8 pps., 2.4 pps. and .5 pps in the U.K., Europe and Japan respectively.[2] This effect is economically significant in the U.K. and Europe., but less so in Japan.

Table A.2: Average Capital Share and Idiosyncratic Volatility: International Evidence 1990-2017

The table reports the regression results of industry capital income/sales ratios on the average idiosyncratic volatility for Japan, UK and Europe.

$$CS_{i,t} = \alpha + \beta_1 Idio.Vol._{i,t} + \beta_2 MB_{i,t} + \beta_3 Tang_{i,t} + \gamma_t + \eta_i + \epsilon_{i,t},$$

where i stands for industry. The estimation is done for each country separately. The industry's average capital income/sales ratio is calculated as the equal-weighted average of capital income/sales ratios across firms within industry. *Idio.Vol(ret)* is the average annualized idiosyncratic stock return volatility within industry. *Tangibility* is the average of gross property, plant and equipment (PPEGT) to total assets (AT) ratio within industry. *M/B* ratio is the industry average market-to-book ratio within industry. Column (1) and column (2) define industry using 2-digit SIC code, and column (3) and column (4) define industry using 3-digit SIC code. The sample includes all firms in Compustat Global Daily database, 1990-2017. The sample is winsorized at 5%. t statistics in parentheses, and * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	(1)	(2)	(3)	(4)	(5)	(6)
	U.K.		E.U.		Japan	
	SIC2	SIC3	SIC2	SIC3	SIC2	SIC3
Idio.Vol(ret)	-0.432**	-0.200***	-0.176***	-0.141***	-0.025	-0.041***
	(-2.43)	(-3.85)	(-3.03)	(-6.22)	(-1.60)	(-5.46)
Tangibility	0.311***	0.175***	0.093***	0.023*	0.029*	0.018***
	(2.87)	(3.76)	(2.95)	(1.75)	(1.76)	(2.78)
M/B Ratio	-0.057	-0.067***	-0.013	-0.010**	0.013*	0.014***
	(-1.62)	(-5.21)	(-0.92)	(-2.08)	(1.88)	(4.77)
Constant	0.184	0.192***	0.145***	0.182***	0.031*	0.039***
	(1.26)	(4.43)	(5.57)	(11.00)	(1.74)	(5.30)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Y	Y	Y	Y	Y	Y
N	1,632	5,171	1,665	6,034	1,638	5,960
N_clust	62	232	65	251	64	240
r2_a	0.245	0.129	0.232	0.173	0.352	0.284

[2] A one standard deviation increase in vol above that industry's (SIC 3 digit) average lowers the average capital share by 6.6 pps., 2.6 pps. and .3 pps in the U.K., Europe and Japan respectively.

C Concentration versus Selection

In this section, we run reexamine our industry-level analysis reported in Table 3 linking the average capital share at the industry level to idiosyncratic volatility to determine the extent to which our selection mechanism is robust the concentration explanation advocated by other authors. To conduct this analysis, we use the following measure of industry-level concentration:

- Concentration ratios using the Compustat-CRSP sample
  - $Concentration4$ is the sales share of the top 4 largest firms within industry.
  - $Concentration20$ is the sales share of the top 20 largest firms within industry.
- Census Concentration Ratio (2002 & 2007)
  - $Census4$ is the sales share of the top 4 largest companies within industry (4 digit SIC). Within each 3-digit SIC industry, we compute the average of the concentration ratio to obtain the industry (3-digit SIC) level concentration ratio.
  - $Census20$ is the sales share of the top 20 largest companies within industry (4 digit SIC). Within each 3-digit SIC industry, we compute the average of the concentration ratio to obtain the industry (3-digit SIC) level concentration ratio.
- Herfindahl-Herschmann Index (1982-2007, Manufacturing only): Herfindahl-Herschmann index for 50 largest companies from the Census Bureau. The HH Index is scaled by 10000. The HHIs after 1997 are reported at the NAICS level, but we used the method in Bustamante and Donangelo (2017) to convert NAICS to SIC.[3] The Census provide the HH index every five years starting 1982. We follow Bustamante and Donangelo (2017) and repeat the data from the available year in the following four years after that survey year. For example, we report the data from 92 in the years 93, 94, 95, and 96).

Summary statistics for our concentration measures are reported in Table A.3.

Table A.3: Summary Statistics

Variable	Obs	Mean	Std. Dev.	P10	P50	P90
Concentration4 (SIC2)	3322	.6584	.2408	.3492	.6449	1
Concentration20 (SIC2)	3322	.9201	.1167	.7424	.9831	1
Concentration4 (SIC3)	12665	.8564	.1719	.5986	.9275	1
Concentration20 (SIC3)	12665	.9892	.038	.9769	1	1
Census 4 (SIC3)	727	.3688	.2137	.105	.335	.676
Census 20 (SIC3)	723	.5953	.2505	.23	.619	.932
HH Index (SIC4)	14406	.0697	.0637	.0095	.049	.1615

[3]Thanks to Andres Donangelo for kindly providing the data.

Next, we run the same panel regressions at the industry level of average capital shares on industry-level volatility and some controls, while controlling for variation in concentration. Tables A.4-A.8 report the results. Essentially, the baseline results reported in the paper, which document a sizeable negative effect of firm-level vol. on the industry's average capital share, are robust to controlling for various measures of concentration. Table A.4 uses the Compustat sales-based concentration measures and idiosyncratic return vol as the vol measure. Table A.5 uses the Compustat sales-based concentration measures and idiosyncratic sales vol as the vol measure. Table A.6 uses the Census concentration measures and idiosyncratic sales vol as the vol measure. Table A.7 uses the HHI for manufacturing and the return-based vol measure. Table A.8 uses the HHI for manufacturing and sales-based vol measures. Overall, the evidence is weaker using sales-based vol measures than when using return-based vol measures, but the concentration controls never drive out the vol variables on the right hand side.

Table A.4: Average Capital Share, Idiosyncratic Return Volatility and Concentration: Industry Level 1960 – 2014

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		Capital Share(SIC3)				Capital Share(SIC2)		
	1960-2014	1980-2014	1960-2014	1980-2014	1960-2014	1980-2014	1960-2014	1980-2014
Idio.Vol(ret)	-0.403***	-0.401***	-0.396***	-0.398***	-0.601***	-0.468**	-0.588***	-0.467**
	(-6.08)	(-6.13)	(-6.21)	(-6.14)	(-3.04)	(-2.60)	(-3.00)	(-2.61)
Tangibility	0.157**	0.130*	0.155**	0.123	0.233	0.106	0.230	0.105
	(2.52)	(1.71)	(2.56)	(1.62)	(1.36)	(0.47)	(1.36)	(0.47)
M/B Ratio	-0.071***	-0.123***	-0.071***	-0.123***	-0.083**	-0.103***	-0.084**	-0.103***
	(-3.80)	(-4.70)	(-3.80)	(-4.70)	(-2.50)	(-3.94)	(-2.53)	(-4.01)
Concentration4	0.106	-0.191***			0.061	0.004		
	(1.50)	(-2.62)			(1.20)	(0.06)		
Concentration20			0.977**	0.064			0.478***	0.053
			(2.14)	(0.22)			(3.63)	(0.35)
Constant	0.052	0.437***	-0.812*	0.216	0.152	0.317*	-0.291	0.272
	(0.64)	(4.76)	(-1.77)	(0.72)	(1.20)	(1.78)	(-1.50)	(1.32)
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Industry FE	Y	Y	Y	Y	Y	Y	Y	Y
N	11,836	8,299	11,836	8,299	3,090	2,127	3,090	2,127
N_clust	252	252	252	252	65	65	65	65
r2_a	0.084	0.059	0.087	0.058	0.162	0.077	0.168	0.077

The table reports the regression results of industry capital income/sales ratios on the average idiosyncratic volatility when we control for industry concentration.

$$CS_{i,t} = \alpha + \beta_1 Idio.Vol._{i,t} + \beta_2 MB_{i,t} + \beta_3 Tang_{i,t} + \beta_4 Concentration_{i,t} + \gamma_t + \eta_i + \epsilon_{i,t},$$

where i stands for industry. The estimation is done for each country separately. The industry capital income/sales ratio is calculated as the average of capital income/sales ratios across firms within industry. *Idio.Vol(ret)* is the average idiosyncratic stock return volatility within industry. *Tangibility* is the average of gross property, plant and equipment (PPEGT) to total assets (AT) ratio within industry. M/B ratio is the industry average market-to-book ratio within industry. Industry concentration $Concentration(n)$ is the sales share of the top N largest firms within industry in the Compustat public firm sample. Concentration4 is the sales share of the top 4 largest firms within industry. Concentration20 is the sales share of the top 20 largest firms within industry. Column (1)- (3) define industry using 3-digit SIC code, and column (4) - (6) define industry using 2-digit SIC code. The sample includes all firms in Compustat-CRSP, 1960-2014. The sample is winsorized at 1%. t statistics in parentheses, and * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A.5: Average Capital Share, Idiosyncratic Sales Volatility and Concentration: Industry Level 1960 – 2014

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		Capital Share(SIC3)				Capital Share(SIC2)		
	1960-2014	1980-2014	1960-2014	1980-2014	1960-2014	1980-2014	1960-2014	1980-2014
Idio.Vol(sales)	-0.142*	-0.163*	-0.138*	-0.160*	-0.390	-0.391	-0.386	-0.390
	(-1.75)	(-1.91)	(-1.73)	(-1.87)	(-1.51)	(-1.26)	(-1.49)	(-1.26)
Tangibility	0.127*	0.100	0.127**	0.094	0.288	0.268	0.284	0.260
	(1.95)	(1.39)	(1.99)	(1.30)	(1.66)	(1.38)	(1.64)	(1.33)
M/B Ratio	-0.094***	-0.111***	-0.094***	-0.111***	-0.108**	-0.110***	-0.108**	-0.111***
	(-4.47)	(-4.78)	(-4.47)	(-4.77)	(-2.32)	(-3.20)	(-2.33)	(-3.26)
Concentration4	0.105	-0.180**			0.036	-0.076		
	(1.40)	(-2.50)			(0.75)	(-1.34)		
Concentration20			1.058**	0.206			0.437***	-0.053
			(2.40)	(0.69)			(3.91)	(-0.37)
Constant	0.093	0.286***	-0.893*	-0.067	0.140	0.169	-0.239	0.174
	(0.95)	(3.34)	(-1.96)	(-0.22)	(1.48)	(1.16)	(-1.66)	(0.86)
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Industry FE	Y	Y	Y	Y	Y	Y	Y	Y
N	10,391	8,067	10,391	8,067	2,774	2,102	2,774	2,102
N_clust	249	249	249	249	63	63	63	63
r2_a	0.060	0.039	0.064	0.037	0.176	0.117	0.181	0.116

The table reports the regression results of industry capital income/sales ratios on the average idiosyncratic volatility when we control for industry concentration.

$$CS_{i,t} = \alpha + \beta_1 Idio.Vol._{i,t} + \beta_2 MB_{i,t} + \beta_3 Tang_{i,t} + \beta_4 Concentration_{i,t} + \gamma_t + \eta_i + \epsilon_{i,t},$$

where i stands for industry. The estimation is done for each country separately. The industry capital income/sales ratio is calculated as the average of capital income/sales ratios across firms within industry. *Idio.Vol(sales)* is the average idiosyncratic sales volatility within industry. *Tangibility* is the average of gross property, plant and equipment (PPEGT) to total assets (AT) ratio within industry. M/B ratio is the industry average market-to-book ratio within industry. Industry concentration $Concentration(n)$ is the sales share of the top N largest firms within industry in the Compustat public firm sample. Concentration4 is the sales share of the top 4 largest firms within industry. Concentration20 is the sales share of the top 20 largest firms within industry. Column (1)- (3) define industry using 3-digit SIC code, and column (4) - (6) define industry using 2-digit SIC code. The sample includes all firms in Compustat-CRSP, 1960-2014. The sample is winsorized at 1%. t statistics in parentheses, and * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A.6: Average Capital Share, Idiosyncratic Sales Volatility and Census Concentration: Industry Level 2002 & 2007

	Capital Share (SIC3)			
Idio.Vol(ret)	-0.451**	-0.452**		
	(-2.08)	(-2.06)		
Idio.Vol(sales)			0.089	0.080
			(0.55)	(0.50)
Tangibility	0.364	0.371	0.458	0.476
	(0.93)	(0.94)	(1.11)	(1.14)
M/B Ratio	-0.201	-0.199	-0.172	-0.172
	(-0.89)	(-0.88)	(-1.10)	(-1.10)
Census 4	0.225		0.212	
	(1.17)		(1.17)	
Census 20		0.238		0.401
		(0.66)		(1.16)
Constant	0.207	0.144	-0.160	-0.328
	(0.86)	(0.45)	(-1.20)	(-1.37)
Year FE	Y	Y	Y	Y
Industry FE	Y	Y	Y	Y
N	697	692	642	637
N_clust	437	436	405	404
r2_a	0.032	0.032	0.025	0.027

The table reports the regression results of industry capital income/sales ratios on the average idiosyncratic volatility when we control for industry concentration.

$$CS_{i,t} = \alpha + \beta_1 Idio.Vol._{i,t} + \beta_2 MB_{i,t} + \beta_3 Tang_{i,t} + \beta_4 Census(n)_{i,t} + \gamma_t + \eta_i + \epsilon_{i,t},$$

where i stands for industry. The estimation is done for each country separately. The industry capital income/sales ratio is calculated as the average of capital income/sales ratios across firms within industry. *Idio.Vol(sales)* is the average idiosyncratic sales volatility within industry. *Idio.Vol(ret)* is the average idiosyncratic stock return volatility within industry. *Tangibility* is the average of gross property, plant and equipment (PPEGT) to total assets (AT) ratio within industry. *M/B* ratio is the industry average market-to-book ratio within industry. Industry concentration $Census(n)$ is the sales share of the top N largest firms within industry obtained from the Census Bureau. Census 4 is the sales share of the top 4 largest firms within industry. Census 20 is the sales share of the top 20 largest firms within industry. Industry is defined using 3-digit SIC code. The sample includes all firms in Compustat-CRSP, 1960-2014. The sample is winsorized at 1%. t statistics in parentheses, and * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

D Cohort Effects

First, we show the average and aggregate capital-income-to-sales ratio for firms that entered the public domain at different years and survived throughout the entire sample in Figure (A.5). We keep the identity of firms in each vintage cohort unchanged. Figure A.5 (a) shows that on average firms who entered in the 1960s and survived till the recent decades have the highest capital-income-to-sales ratio, while firms who have entered recently (1990s and 2000s cohorts) on average have

Table A.7: Average Capital Share, Idiosyncratic Return Volatility and Herfindahl-Herschmann Indexes: Manufacturing 1982-2015

	Capital Share(SIC4)		
Idio.Vol(ret)	-0.449***	-0.489***	-0.326***
	(-4.75)	(-4.04)	(-2.78)
Tangibility	0.106	0.071	-0.034
	(0.86)	(0.28)	(-0.40)
M/B Ratio	-0.122**	-0.153**	-0.055**
	(-2.57)	(-2.45)	(-2.14)
HH Index	-0.634	-1.228	-0.169
	(-1.23)	(-1.05)	(-0.13)
Constant	0.326**	0.457*	0.188**
	(2.47)	(1.88)	(2.23)
Year FE	Y	Y	Y
Industry FE	Y	Y	Y
N	3,854	2,228	1,746
N_clust	135	134	126
r2_a	0.044	0.038	0.042

The table reports the regression results of industry capital income/sales ratios on the average idiosyncratic volatility when we control for industry concentration.

$$CS_{i,t} = \alpha + \beta_1 Idio.Vol._{i,t} + \beta_2 MB_{i,t} + \beta_3 Tang_{i,t} + \beta_4 HHI_{i,t} + \gamma_t + \eta_i + \epsilon_{i,t},$$

where i stands for industry. The estimation is done for each country separately. The industry capital income/sales ratio is calculated as the average of capital income/sales ratios across firms within industry. *Idio.Vol(sales)* is the average idiosyncratic sales volatility within industry. *Idio.Vol(ret)* is the average idiosyncratic stock return volatility within industry. *Tangibility* is the average of gross property, plant and equipment (PPEGT) to total assets (AT) ratio within industry. *M/B* ratio is the industry average market-to-book ratio within industry. Industry concentration is measured using Herfindahl-Herschmann Indexes (HHI) obtained from the Census Bureau. Industry is defined using 4-digit SIC code. The sample includes all firms in Compustat-CRSP, 1960-2014. The sample is winsorized at 1%. t statistics in parentheses, and * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A.8: Average Capital Share, Idiosyncratic Sales Volatility and Hirschmann-Herfindahl Indexes: Manufacturing 1982-2015

	Capital Share(SIC4)		
Idio.Vol(sales)	-0.205*	-0.189	-0.016
	(-1.79)	(-1.11)	(-0.29)
Tangibility	0.053	0.066	-0.041
	(0.61)	(0.41)	(-0.44)
M/B Ratio	-0.126***	-0.155***	-0.082***
	(-2.92)	(-2.75)	(-2.89)
HH Index	-0.517	-0.833	-0.613
	(-1.00)	(-0.76)	(-0.50)
Constant	0.182*	0.245	-0.025
	(1.89)	(1.46)	(-0.33)
Year FE	Y	Y	Y
Industry FE	Y	Y	Y
N	3,733	2,150	1,583
N_clust	133	133	121
r2_a	0.031	0.025	0.030

The table reports the regression results of industry capital income/sales ratios on the average idiosyncratic volatility when we control for industry concentration.

$$CS_{i,t} = \alpha + \beta_1 Idio.Vol._{i,t} + \beta_2 MB_{i,t} + \beta_3 Tang_{i,t} + \beta_4 HHI_{i,t} + \gamma_t + \eta_i + \epsilon_{i,t},$$

where i stands for industry. The estimation is done for each country separately. The industry capital income/sales ratio is calculated as the average of capital income/sales ratios across firms within industry. *Idio.Vol(sales)* is the average idiosyncratic sales volatility within industry. *Idio.Vol(ret)* is the average idiosyncratic stock return volatility within industry. *Tangibility* is the average of gross property, plant and equipment (PPEGT) to total assets (AT) ratio within industry. *M/B* ratio is the industry average market-to-book ratio within industry. Industry concentration is measured using Herfindahl-Herschmann Indexes (HHI) obtained from the Census Bureau. Industry is defined using 4-digit SIC code. The sample includes all firms in Compustat-CRSP, 1960-2014. The sample is winsorized at 1%. t statistics in parentheses, and * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

the lowest highest capital-income-to-sales ratio, consistent with selection. Furthermore, the steeper upward drift over time in average ratios for the more recent 1990 and 2000 vintages is the hallmark of our selection mechanism. This selection mechanism is stronger for recent vintages because they have been exposed to higher vol early on in their life. However, from the aggregate capital shares plotted in Figure A.5 (b), we see that there is no clear pattern across vintages in aggregate capital shares. It is not the case that only firms who entered recently (Google, Apple, or Facebook which are in the 90s and 2000s cohorts) have become more profitable. The largest firms in the right tail of firm size distribution, who are the driving force behind the aggregate ratio, can be either firms from older vintages that have survived long enough or firms that have entered recently with good draws of productivity.

Second, we explore the cohort effects among the largest and smallests firms in Figure A.6. Each year, we identify firms in the top (bottom) 20 percentile of size (measured by total assets or sales) distribution. Among the top 20 (bottom 20) percentile largest (smallest) firms, we identify the entry year of each firms and group them into five entry cohorts (1960s, 1970s, 1980s, 1990s, 2000s). We then compute the average capital-income-to-sales ratio of each entry cohort.The average capital-income-to-sales ratio among the largest firms has been trending up for all cohorts, not just the most recent ones. There is some evidence to suggest that the largest firms from recents cohorts are somewhat more profitable than those from older cohorts, but the evidence is not overwhelming.

Third, as shown in Figure A.6, there is a very strong vintage effect amongst the smallest firms, consistent with our model: lower capital-income-to-sales ratios are more likely to prevail in the recent sample period for the young vintages who have experienced higher idiosyncratic volatility early on in their lifespan, when they are more likely to generate negative profits. They are willing to wait because the option value of waiting is so high. The Autor et al. (2017) mechanism cannot speak to this vintage effect in the left tail. One important difference between our mechanism and the "super-star" firm mechanism (or other prevailing explanations of declining costs of capital goods, etc.) is that our real option mechanism predicts the increase in the left tail of firm size distribution.

E Additional Cross Sectional Evidence

Since the firm size distribution may reflect not just the change in contemporaneous volatility but also, to some extent, reflect the cumulative changes in the past volatility. In this section, we show our industry-level regression of industry average capital income to sales ratio on past idiosyncratic volatility to provide further evidence on the past volatility and the dispersion of capital share. We consider two regression models: 1) use lagged industry level idiosyncratic volatility directly as a control; 2) use a moving average of current and past idiosyncratic volatility to capture the cumulative effect of past volatility.

Figure A.5: Capital-Income-to-Sales Ratio by Entry Cohorts

This figure presents the average and the aggregate capital-income-to-sales ratio by different entry cohorts. Vintage 1960s represents the set of firms who went public between 1960-1970 and survived throughout the entire sample period (till 2014), and we keep the composition of firms in this group fixed and plot the average and aggregate capital-income-to-sales ratio within each cohorts. The vintage here represents the survival period of firms. The sample is winsorized at 1%. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

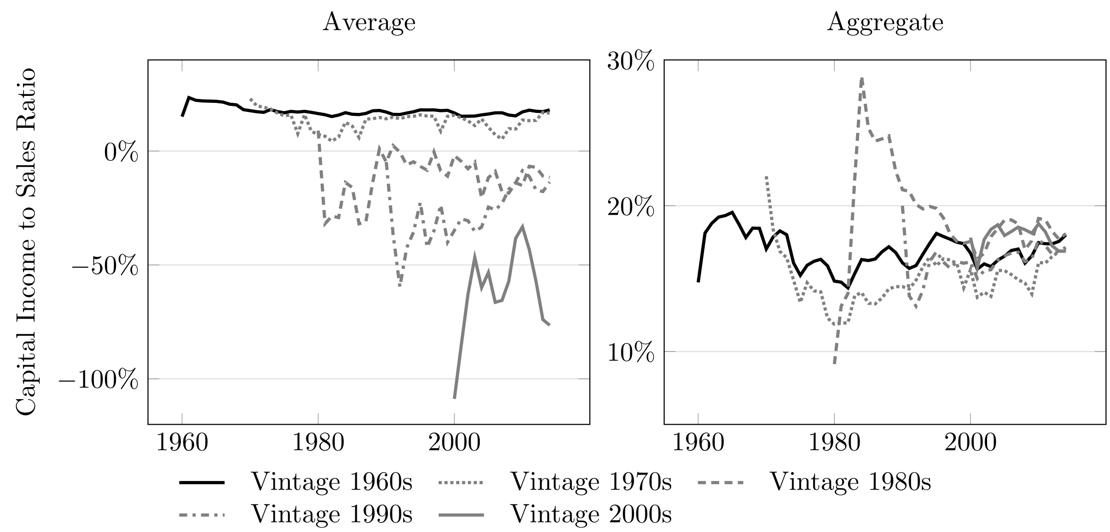


Figure A.6: Capital-Income-to-Sales Ratio by Entry Cohorts among Largest and Smallest Firms

This figure presents the average and the aggregate capital-income-to-sales ratio by different entry cohorts among the largest and smallest firms. Each year, we identify firms in the top and bottome 20 percentile of size distribution. Within each size group, we then identify the entry year of each firms and group them in to five entry cohorts. We then compute the average capital-income-to-sales ratio of each entry cohort. Source: Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

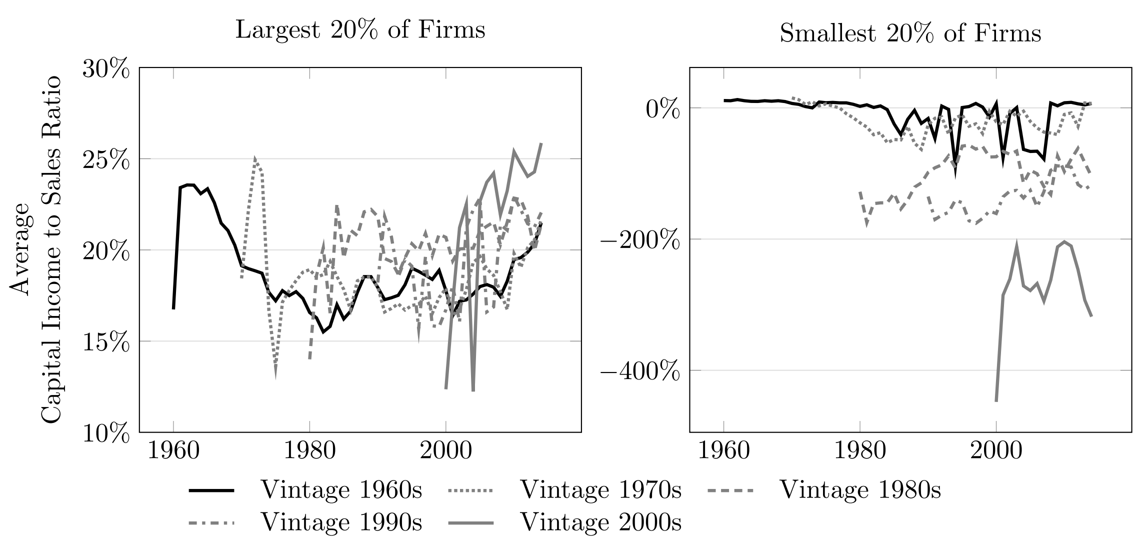


Table A.9: Average Capital Share and Past Idiosyncratic Volatility: Industry Level 1960 – 2014

	(1)	(2)	(3)	(4)	(5)	(6)
	Capital Share(SIC2)			Capital Share(SIC3)		
Idio.Vol(ret)	-0.593***			-0.406***		
	(-3.02)			(-6.07)		
idio_ret_vol_MA5		-0.800***			-0.449***	
		(-2.78)			(-4.61)	
idio_ret_vol_MA10			-0.920**			-0.477***
			(-2.61)			(-3.83)
Tangibility	0.242	0.287	0.336*	0.161***	0.160**	0.157**
	(1.42)	(1.58)	(1.72)	(2.62)	(2.57)	(2.50)
M/B Ratio	-0.080**	-0.089**	-0.094**	-0.071***	-0.069***	-0.069***
	(-2.42)	(-2.57)	(-2.65)	(-3.81)	(-3.69)	(-3.67)
Constant	0.185	0.238	0.254	0.155***	0.171***	0.182***
	(1.50)	(1.53)	(1.33)	(2.99)	(3.16)	(3.01)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Y	Y	Y	Y	Y	Y
N	3,091	3,100	3,106	11,838	11,888	11,896
N_clust	65	65	65	252	252	252
r2_a	0.161	0.160	0.163	0.084	0.075	0.071

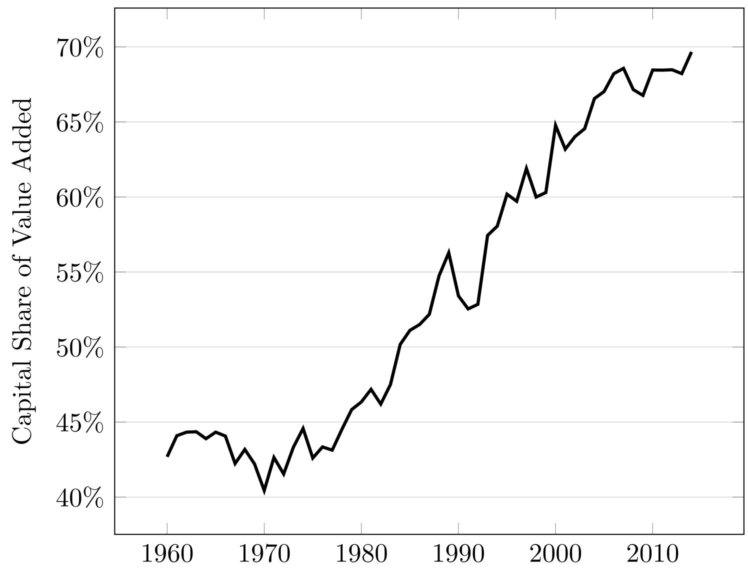
The table reports the regression results of industry capital income/sales ratios on the the past average idiosyncratic volatility. The industry capital income/sales ratio is calculated as the average of capital income/sales ratios across firms within industry. *Idio.Vol(ret)* is the average annualized idiosyncratic stock return volatility within industry. *MA(n).Idio.Vol(ret)* is the moving average of industry level idiosyncratic return volatility over from year t-n to year t (including the current year). *Tangibility* is the average of gross property, plant and equipment (PPEGT) to total assets (AT) ratio within industry. *M/B* ratio is the industry average market-to-book ratio within industry. Column (1)- (3) define industry using 3-digit SIC code, and column (4) - (6) define industry using 2-digit SIC code. The sample includes all firms in Compustat-CRSP, 1960-2014. The sample is winsorized at 1%. t statistics in parentheses, and * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

F Robustness

F.1 Adjusting For R&D

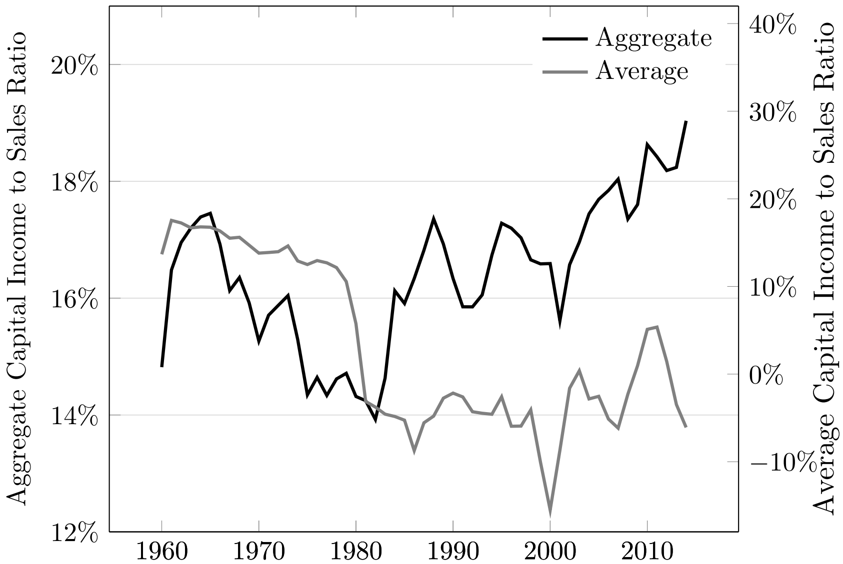
In this section, we reconstruct our main results from 1 and 5 adjusting our measure of capital income by adding R&D expenses.

Figure A.7: The Aggregate Capital Share of Value Added



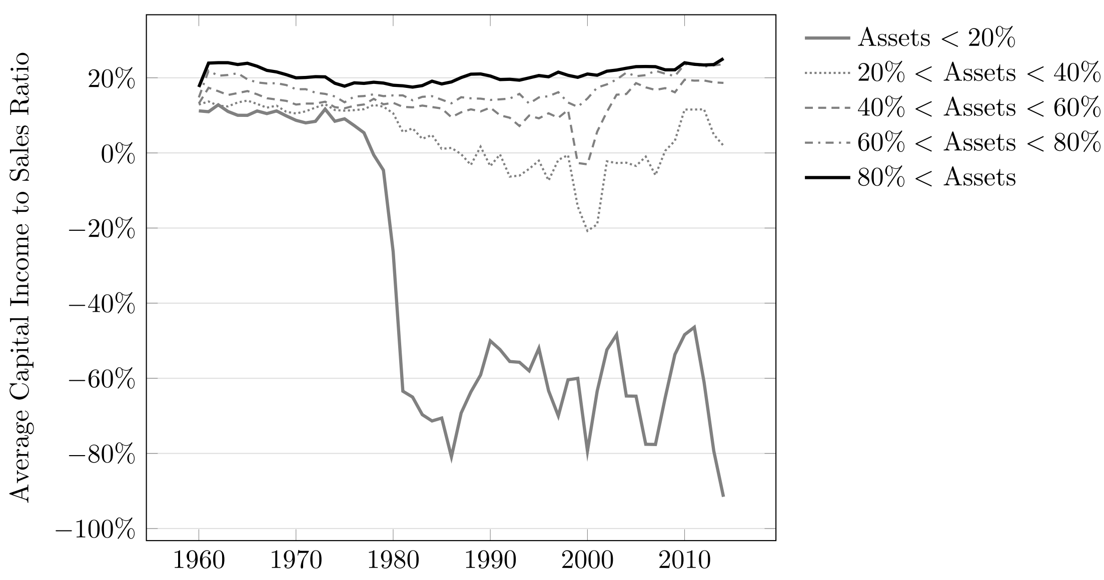
This figure plots the aggregate capital share.The aggregate capital share is $\sum_i$ Operating Income$_i$ + R&D Expenses$_i$ divided by $\sum_i$ Imputed XLR$_i$ + Operating Income$_i$ + R&D Expenses$_i$. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

Figure A.8: Aggregate and Average Capital Share



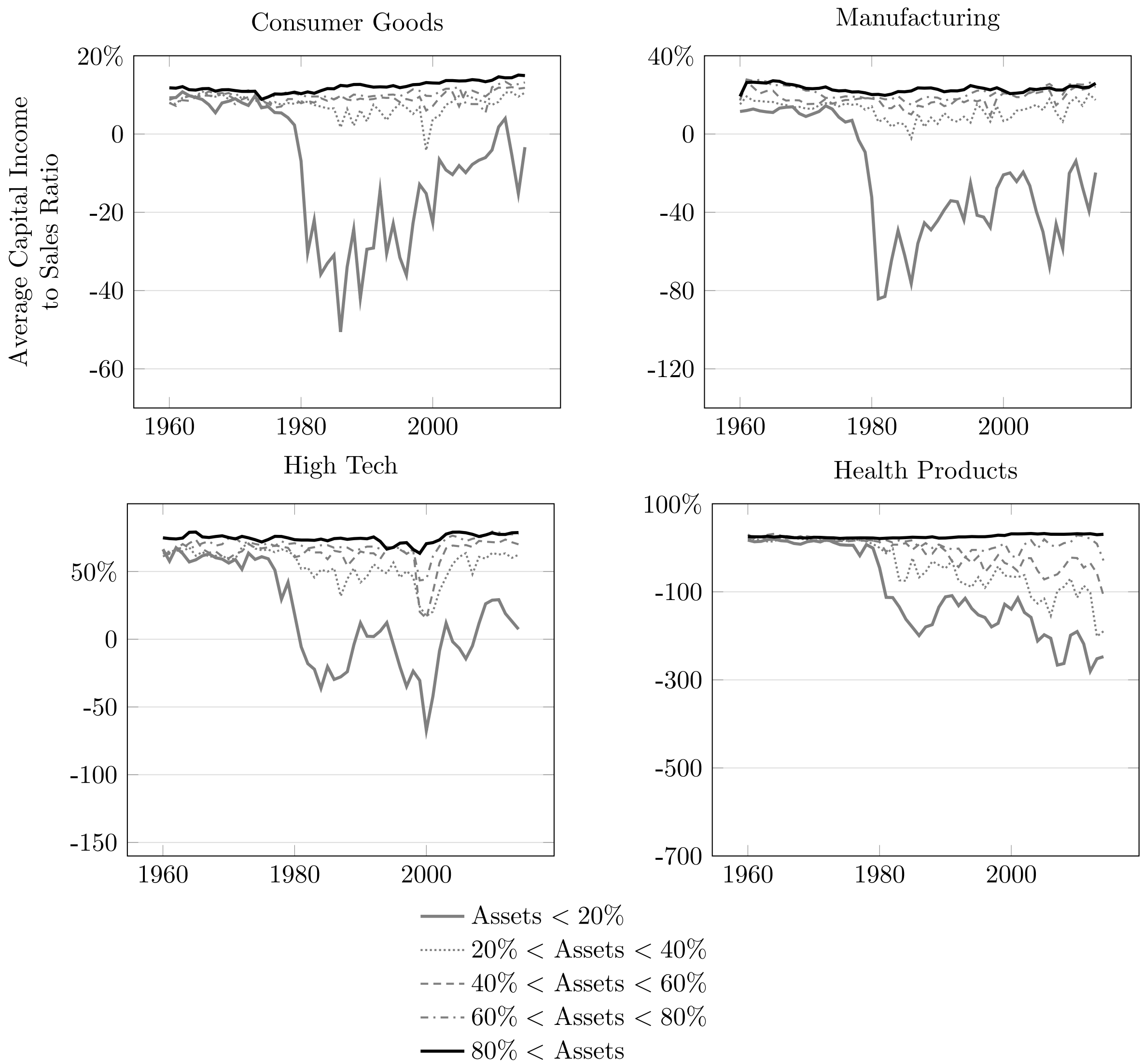
This figure plots the aggregate and average capital income to sales ratio. We define capital income as operating income + R&D expenses. The aggregate capital income to sales ratio is $\sum_i$ Operating Income$_i$ + R&D Expenses$_i$ divided by $\sum_i$ Sales$_i$ for each year. The average capital-income-to-sales ratio is the simple average of the firm level capital capital-income-to-sales ratio for each year. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

Figure A.9: Average Capital Share in Size Groups



This figure presents the average capital-income-to-sales ratio by size over time. We measure capital income as operating income + R&D expenses. Size is measured by total assets, and the capital-income-to-sales ratio is measured as capital income (OIBDP+XRD) divided by sales. For each year, firms are categorized into five groups based on their total assets, and we estimate the average capital-income-to-sales ratio within each group for a given year. The sample is winsorized at 1%. The sample includes all firms in Compustat-CRSP merged Fundamentals Annual for 1960-2014.

Figure A.10: Average Capital Income to Sales Ratio in Size Groups (FF 5 Industry)

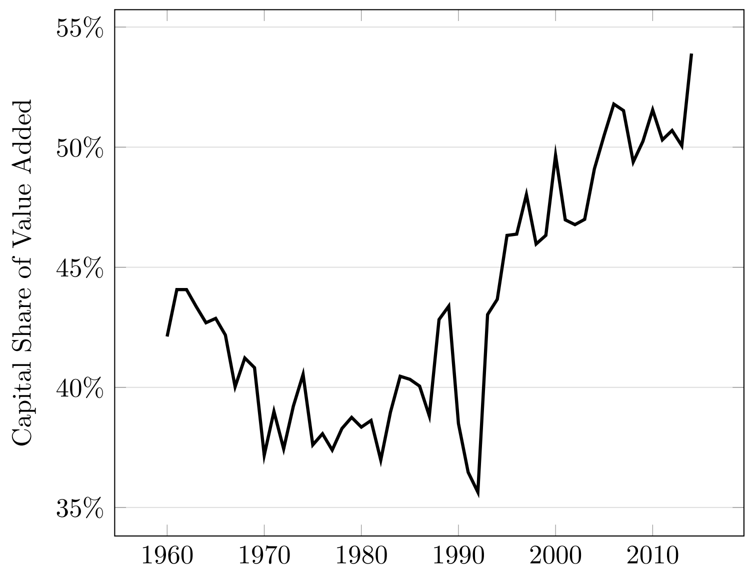


Industries are defined Fama-French five-industry classification. We omit the industry classification ”other” because it contains few firms after excluding financial firms. Within each industry, we sort firms into five groups based on their total assets. The plot shows the average capital income (OIBDP+XRD) to sales ratio within each size group for four industries. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

F.2 Winsorization

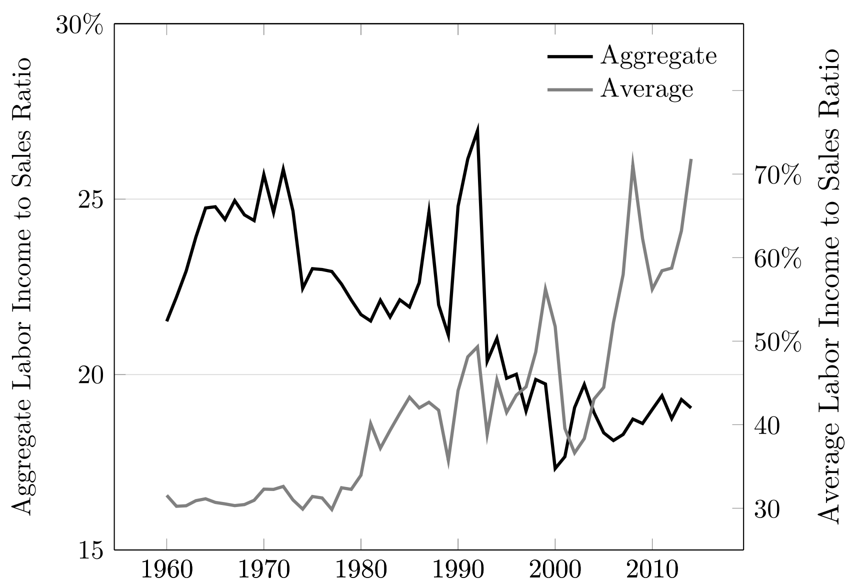
This subsection presents our results with different winsorization procedures.

Figure A.11: The Aggregate Capital Share of Value Added



This figure plots the aggregate capital share. The aggregate capital share is $\sum_i \text{Operating Income}_i$ divided by $\sum_i \text{Imputed XLR}_i + \text{Operating Income}_i$. The sample is winsorized at **1%**. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

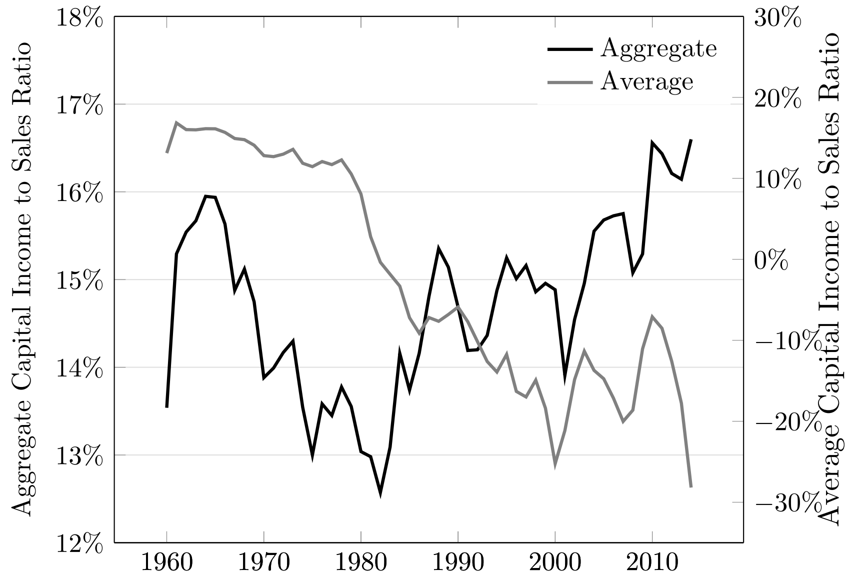
Figure A.12: Aggregate and Average Labor Income to Sales Ratio



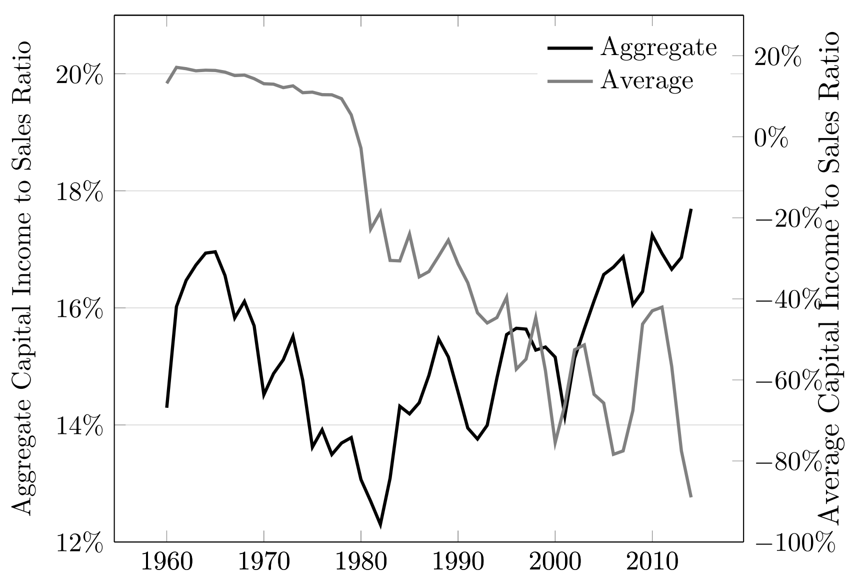
The aggregate labor income to sales ratio is $\sum_i \text{Extended XLR}_i$ divided by $\sum_i \text{Sales}_i$ for each year. The average labor income to sales ratio is the simple average of the within firm labor income to sales ratio for each year. The sample is winsorized at **1%**. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).

Figure A.13: Aggregate and Average Capital Share

(a)



(b)



This figure reports the capital income to sales ratio. The aggregate capital income to sales ratio is $\sum_i$ Operating Income$_i$ divided by $\sum_i$ Sales$_i$ for each year. The average capital-income-to-sales ratio is the simple average of the firm level capital capital-income-to-sales ratio for each year. Part (a) use the sample with winsorization at 2.5%. Part(b) use the sample with winsorization at .5%. Source: Compustat/CRSP Merged Fundamentals Annual (1960-2014).