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# EXCESS CAPACITY, MONOPOLISTIC COMPETITION, AND INTERNATIONAL TRANSMISSION OF MONETARY DISTURBANCES

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# Excess Capacity, Monopolistic Competition and International Transmission of Monetary Disturbances

## ABSTRACT

A stochastic two-country neoclassical rational expectations model with sticky prices -- optimally set by monopolistically competitive firms -- and possible excess capacity is developed to examine international spillover effects on output of monetary disturbances. The Mundell-Fleming model predicts that monetary expansion at home leads to recession abroad. In contrast, our main result is that spillover effects of monetary policy may be either positive or negative, depending upon whether the intertemporal elasticity of substitution in consumption exceeds the intratemporal elasticity of substitution. The model in addition is used to determine nominal and real inteest rates, exchange rates, and other asset prices.

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## I. Introduction

The debates on the shift in US monetary policy since late 1979 vividly demonstrate the importance of the international transmission of the effects of monetary policy. The shift to stricter money targets in 1979 has by many been associated with at least the initial rise in the US dollar exchange rate and the worldwide slump that followed upon this shift in monetary operating procedures. However, the theoretical basis for such an assessment of the transmission of monetary policy in the existing literature is not entirely satisfactory in several respects.

Existing international neoclassical monetary models models with flexible prices and continuously clearing markets, like Stockman (1980), Lucas (1982) and Svensson (1985a), while useful as benchmarks, fail to address the sluggishness in price and wage adjustment, and relatively large adjustment in output and employment, that seem a persistent feature of macroeconomic fluctuations. The models mentioned have exogenous output, and hence cannot incorporate any effects of monetary disturbances on output. Also, the price level in these models is as variable as any asset price, like exchange rates and stock prices, and jumps instantaneously when new information arrives. We believe these models exaggerate the variability of the price level. Models that explicitly incorporate the possibility of sticky prices usually suffer, however, from a series of well known shortcomings. Typically, price setting rules are arbitrarily assumed rather than linked to rational behavior of the price setting agents in the economy. Moreover, in spite of the obvious incompatibility of price setting behavior on the one hand, and assumptions of perfect competition on the other, firm behavior is typically based on the assumption of the latter market structure. Often, intertemporal issues concerning expectations, savings and investment behavior are treated in a static context, precluding a meaningful analysis; and so on. While individual papers are emerging dealing with each of these issues,[1] no satisfactory framework has as yet been developed for a full analysis of the international transmission of monetary policy shocks.

Of course, the Mundell-Fleming model,[2] still to a large extent the workhorse of international macroeconomics, can be considered as an early sticky-price model, with Keynesian unemployment and excess commodity supply prevailing throughout the world. It leads to strong conclusions about international transmission of monetary policy. Let us consider, for instance, the two-country version with flexible exchange rates and perfect capital mobility, the straigth-forward flexible-exchange-rate extension of the fixed price IS-LM model with demand determined output.

An expansion of the foreign money supply leads in that model to an incipient capital outflow from the foreign country into the home country, an appreciation of the home currency, and an improvement in the home country's terms of trade. This deteriorates the home country's trade balance, and depresses demand for home output. The world rate of interest falls. Since the home interest rate is equal to the world interest rate by the assumption of perfect capital mobility, the home interest rate falls as well. In the new equilibrium, home money market equilibrium requires that home output has fallen. The home country's trade account has deteriorated. Foreign output has expanded. A monetary expansion is in this sense beggar-thy-neighbor policy.

The IS-LM and hence the Mundell-Fleming model has obvious and well-known weaknesses. The behavioral functions, including the money demand functions, are not derived from intertemporal optimizing behaviour. Expectations are stationary.

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[1] For instance, Aizenman (1985), Blanchard and Kiyotaki (1986) and Svensson (1986) use monopolistic competition; Persson (1982) and van Wijnbergen (1985) introduce savings and investment behavior based on intertemporal optimizing behavior in a fix-price model.

[2] Fleming (1962) and Mundell (1968). For a modern restatement, see Dornbusch (1980).

Saving, investment and the trade balance are treated without regard for the intertemporal considerations that should be inherent in saving and investment decisions. These shortcomings often qualitatively affect the results, as comparision with this paper demonstrates.

Considerable effort has been devoted in the international macro field to remedy these inadequacies, and it would take to long here to survey the major developments. Let us however note one particular development. Recently rigorous two-country monetary general equilibrium models with rational expectations and uncertainty have been developed and used in international finance. These models are essentially monetary and international extensions of Lucas (1978) celebrated model, like Lucas (1982), Stockman (1980), Svensson (1985a) and Stockman and Svensson (1987). These models rely on full employment and flexible prices. More recently, Svensson (1986) has shown one way of incorporating sticky goods prices and demand determined output with maintained rigour in these models, although in a closed economy setting. Here we extend Svensson (1986) to a two-country flexible exchange rate setting, in order to examine the strong results of the Mundell-Fleming model in a more rigorous framework. The new results obtained clearly demonstrate the positive payoff of such effects. We also hope that our model will in general provide a more satisfactory framework for the analysis of international transmission of international disturbances.

The organisation of the paper is as follows. In section II we present the model. In section III we discuss the output effects of monetary disturbances. Section IV discusses the impact of monetary disturbances on interest rates. Section V concludes. Some technical details are presented in an appendix.

## II. The Model

The world consists of two countries, home and foreign. Each country is completely specialized in the production of home and foreign goods, respectively. As further specified below, there is production of differentiated products, but at this stage it is sufficient to consider two aggregate goods only. In period t ($t = \ldots, -1, 0, 1, \ldots$) world per capita production of each good, $Y_t$ and $Y^*_t$, respectively, is costless up to an exogenous stochastic capacity level, $y_t$ and $y^*_t$. Hence

(1) $$Y_t \le y_t \text{ and } Y^*_t \le y^*_t.$$

There is a representative consumer in each country. Hence world population is 2, and world production of the two goods is $2Y_t$ and $2Y^*_t$, respectively.

Goods are perishable and cannot be stored between periods. Hence, whenever output and consumption fall short of capacity there is waste of resources, excess capacity, and underemployment.[3]

There are two currencies, home and foreign. The supply of currencies in period t, $\bar{M}_t$ and $\bar{N}^*_t$, respectively, is stochastic and given by

(2) $$\bar{M}_t = \omega_t \bar{M}_{t-1} \text{ and } \bar{N}^*_t = \omega^*_t \bar{N}^*_{t-1},$$

where $\omega_t$ and $\omega^*_t$ are the (gross rates of) monetary expansion of home and foreign currency. We call the vector $s_t = (y_t, y^*_t, \omega_t, \omega^*_t)$ the state. We assume that the states are serially independently distributed, and that their probability distribution is given by the time-invariant probability distribution function $F(s_t)$.

The home and the foreign consumer have identical preferences. Let us consider the situation for the home consumer. His preferences in period t are given by the expected utility function

(3) $$E_t \Sigma_{\tau=t}^{\infty} \beta^{\tau-t} u(c_{h\tau}, c_{f\tau}), \quad 0 < \beta < 1.$$

Here $E_t$ denotes the expectations operator conditional upon information available in period t; $u(c_{ht}, c_{ft})$ is a standard concave instantaneous utility function of consumption $c_{ht}$ and $c_{ft}$ of home and foreign goods in period t.

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3 With a different interpretation of the model, assuming a linear technology with labor input and goods output, $Y_t$ and $y_t$ can be identified as actual endogenous labor employment and exogenous labor supply, respectively. Then we can interpret excess capacity directly as unemployment.

The timing of markets and transactions facing the consumer needs to be specified. We shall explain it in some detail for the home consumer. The home consumer enters period t with stocks of six different assets, namely home and foreign currency, $M_{t-1}$ and $N_{t-1}$, shares in home and foreign firms, $z_{ht-1}$ and $z_{ft-1}$, and claims to transfers of home and foreign currency, $x_{Mt-1}$ and $x_{Nt-1}$.[4] Then he learns the current state $s_t$ of current capacities and monetary expansions, and he receives money transfers of home and foreign currency, in proportion to his claims on such transfers. After that, the goods market opens. On the goods market the consumer can buy home and foreign goods. He must pay for home goods with home currency and for foreign goods with foreign currency. Hence he faces the liquidity constraints

(4a) $$P_{ht}c_{ht} \leq M_{t-1} + (\omega_t - 1)\bar{M}_{t-1}x_{Mt-1} \text{ and}$$

(4b) $$P^*_{ft}c_{ft} \leq N_{t-1} + (\omega^*_t - 1)\bar{N}^*_{t-1}x_{Nt-1},$$

where $P_{ht}$ and $P^*_{ft}$ is the home-currency price of home goods and the foreign currency price of foreign goods, respectively, and where the second terms on the right hand sides are the transfers received.

Since nominal goods prices will be sticky and not adjust to the current state of the market, there will be excess demand in some states and excess supply in others. When there is excess demand consumers will be rationed. Therefore the home consumer also faces the rationing constraints

(5) $$c_{ht} \leq Y_t \text{ and } c_{ft} \leq Y^*_t$$

(recall that $Y_t$ and $Y^*_t$ are output per capita). We assume uniform rationing such that home and foreign consumers are treated equally when there is rationing on the markets for home and foreign goods.

After the consumer's transactions on the goods market, that market closes and

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4 Home consumers receive transfers of home currency, and foreign consumers receive foreign currency. Risk averse consumers have an incentive to diversify their portfolio. This gives rise to trade in claims on net transfers of currency (see Lucas (1982)).

the asset market opens. On the asset market, dividends on shares in firms (cash from sales of output) are distributed and the consumer can trade assets according to the budget constraint

(6)
$$M_t + e_tN_t + Q_{ht}z_{ht} + Q_{ft}z_{ft} + R_{Mt}x_{Mt} + R_{Nt}x_{Nt} \leq$$
$$\leq [M_{t-1} + (\omega_t-1)\bar{M}_{t-1}x_{Mt-1} - P_{ht}c_{ht}] +$$
$$+ e_t[N_{t-1} + (\omega^*_t-1)\bar{N}^*_{t-1}x_{Nt-1} - P^*_{ft}c_{ft}] + (Q_{ht} + P_{ht}Y_t)z_{ht-1} +$$
$$+ (Q_{ft} + e_tP^*_{ft}Y^*_t)z_{ft-1} + R_{Mt}x_{Mt-1} + R_{Nt}x_{Nt-1}.$$

Here $e_t$ is the exchange rate, $Q_{ht}$ and $Q_{ft}$ are the home currency prices of shares in home and foreign firms, and $R_{Mt}$ and $R_{Nt}$ are the home currency prices of claims to transfers of home and foreign currency. The inequality (6) simply states that the value of end-of-period asset stocks plus the value of current period consumption cannot exceed the value of asset stocks carried over from the previous period plus current period income. After these transactions the asset market closes, and the home consumer leaves period t and enters period t+1 with new stocks of his six assets: $M_t$, $N_t$, $z_{ht}$, $z_{ft}$, $x_{Mt}$ and $x_{Nt}$.

The home consumer will maximize expected utility (3) subject to the sequence of liquidity, rationing and budget constraints (4)-(6). The foreign consumer will maximize the same utility function, with the same constraints, only his variables are denoted by '*', like $c^*_{ht}$, $c^*_{ft}$, $M^*_t$, $N^*_t$. etc.

Firms' price setting behavior is modelled as in Svensson (1986), by introducing differentiated products and monopolistic competition along the lines of Dixit and Stiglitz (1977).[5] As is further specified in the Appendix, home (foreign) firms produce differentiated home (foreign) goods in monopolistic competition. Firms set prices in their own currency.[6] For some reason they must

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5 See Aizenman (1985), Blanchard and Kiyotaki (1986), Dornbusch (1987) and Giovannini (1985) for alternative price setting stories. Aizenman makes the price setting period endogenous.

6 See Giovannini (1985) for an analysis of what determines whether firms prefer to set prices in home or foreign currency.

set prices one period in advance, that is, before the current state is known. This could be because it takes time to implement a price change, or because the firms cannot observe the current state of the economy except with a lag. For instance, firms may not be able to observe their demand and infere the state of the economy until the end of the week, whereas they must post a price at the beginning of the week. Thus, own-currency goods prices $P_{ht+1}$ and $P^*_{ft+1}$ will be predetermined and prices in period t+1 can only depend on the state variables in period t. These will turn out to be the state $s_t$, the money stocks $\bar{M}_{t-1}$ and $\bar{N}^*_{t-1}$, and the predetermined price levels $P_{ht}$ and $P^*_{ft}$. The pricing functions are the result of the home and foreign firms maximizing their stock market values. However, since the states are serially uncorrelated, the current state conveys no information about the state of tomorrow's markets. Hence, the pricing will be to some extent independent of the current state. Also, there is no money illusion in this world. The outcome is that prices for period t+1 are set proportionally to the stocks of money in period t. It follows that the pricing functions (assumed stationary) have the simple form

(7) $$P_{ht+1} = \bar{M}_t/k = \omega_t\bar{M}_{t-1}/k \text{ and } P^*_{ft+1} = \bar{N}^*_t/k^* = \omega^*_t\bar{N}^*_{t-1}/k^* ,$$

where the constants k and k* are determined by the first-order conditions for maximization of stock market values (see Appendix for details).

We consider a perfectly pooled equilibrium, where the home and foreign consumer hold identical portfolios and consume identical quantities of the goods. That is, they hold the same per capita share of world asset stocks and consume the same per capita share of world output of each good: half of world quantities. Then the market equilibrium conditions for the goods, money and other assets can be written

(8a) $$c_{ht} = c^*_{ht} = Y_t \leq y_t \text{ and } c_{ft} = c^*_{ft} = Y^*_t \leq y^*_t,$$

(8b) $$M_t = M^*_t = \bar{M}_t \text{ and } N_t = N^*_t = \bar{N}^*_t, \text{ and}$$

(8c) $$z_{ht} = z_{ft} = z^*_{ht} = z^*_{ft} = x_{Mt} = x_{Nt} = x^*_{Mt} = x^*_{Nt} = 1.$$

Note that total output is $2Y_t$ and $2Y^*_t$, total money stocks $2\bar{M}_t$ and $2\bar{N}^*_t$, and that total quantity of each other asset is normalized to equal 2. The inequalities in the goods market equilibrium conditions (8a) allow for the possibility of excess supply and underutilization of resources.

The first-order conditions for the consumers and their liquidity, rationing and budget constraints, together with the pricing functions (7) and the market equilibrium conditions (8), determine the equilibrium. The equilibrium is the usual stationary stochastic rational expectations equilibrium, where realizations of the endogenous variables in period t are given as time-independent functions of realizations of the exogenous state, the vector of capacities and monetary expansions, in period t.

At this stage we do not need to discuss the equilibrium conditions in further detail. (The determination of the equilibrium is further discussed in the Appendix.) It is sufficient to note that equilibrium home and foreign output are given by the reduced form <u>output functions</u>

(9) $\quad Y(y,y^*,\omega,\omega^*)$ and $Y^*(y,y^*,\omega,\omega^*)$,

where we have dropped the time subscripts. As further specified below, the precise functional form of (9) will depend on which regime the economy is in, namely whether liquidy constraints or capacity constraints are binding in one, both, or none of the markets for home home and foreign goods.

One can also solve for the other endogenous variables: the exchange rate, the terms of trade, nominal and real interest rates, stock market values, etc. Some of these variables are further discussed below.

## III. Output Effects of Monetary Disturbances

We are interested in the effect on home output of a foreign monetary expansion. Let us first note that the experiment considered is a serially uncorrelated shock to the <u>growth rate</u> of the foreign money stock, which is equivalent to a permanent shock to the <u>level</u> of the foreign monely stock. This is equivalent to the usual experiment in the Mundell-Fleming model.

In terms of the output functions (9), we like to find the partial derivative of home output with respect to foreign monetary expansion, $Y_{\omega*}$.[7] In order to understand how this partial is determined, we need to understand the nature of the equilibrium in more detail. There are two goods markets, the (world) markets for home and for foreign goods. Each of these markets can be in one of three possible regimes, depending upon the realization of capacities and monetary expansions which determines what combination of capacity and liquidity constraints is binding. There are hence nine possible regimes all together. (The regimes are specified in detail in the Appendix.)

It is practical to start with the market for foreign goods. We will need to understand how a foreign monetary expansion affects foreign output, that is, what determines the derivative $Y^*_{\omega*}$. We call the three possible regimes for the foreign goods market the full capacity regime, the liquidity constrained regime, and the underconsumption regime, respectively, and we label them F*, L* and U*. The <u>full capacity regime</u> is the one when consumers are rationed in foreign goods (the rationing constraint (5) binds for foreign goods), and consumption and output of foreign goods is equal to capacity. In this regime foreign output is independent of foreign monetary expansion:

(10a) $\quad Y^* = y^*, \text{ so } Y^*_{\omega*} = 0 \qquad (F^*).$

The full capacity regime is the only one where there is no waste of resources.

The <u>liquidity constrained regime</u> is the one when consumers' liquidity constraint (4b) for foreign currency is binding. Then consumption and output of foreign goods equal real balances in foreign currency. These real balances are $\bar{N}^*/P^*_f$ which by the pricing function (7) equal $k^*\omega^*$ and are proportional to the foreign monetary expansion. Hence, in the liquidity constrained regime, foreign

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7 The output functions are not differentiable on the borderline of the regimes specifed in the Appendix. Hence, derivatives are defined for the interior of the regimes.

output is given by

(10b) $Y^* = k^*\omega^*$, so $Y^*_{\omega^*} = k^* > 0$ $(L^*)$,

and foreign output is proportional to foreign monetary expansion.

In the underconsumption regime, neither the capacity constraint nor the liquidity constraint is binding. There consumption and output of foreign goods is determined by the first-order condition that the marginal utility of consumption of foreign goods, $u_f(c_h, c_f)$, equals the marginal utility of wealth measured in foreign goods, $\lambda^*$ (the Lagrange multiplier of the budget constraint (6) when the latter is deflated by the foreign currency price of foreign goods, $P^*_f$). Hence, in equilibrium

(10c) $u_f(Y, Y^*) = \lambda^*$ $(U^*)$,

where we have substituted home and foreign output for consumption of home and foreign goods. (In general this first-order condition is $u_f(Y, Y^*) = \lambda^* + \mu^*_f + \nu^*_f$, where $\mu^*_f$ and $\nu^*_f$ are the Lagrange multipliers of the liquidity constraint (4b) and the foreign goods rationing constraint in (5).) The marginal utility of wealth measured in foreign currency also fulfills the first-order condition for foreign currency holdings,

(10d) $\lambda^*/P^*_f = \beta E[(\lambda^{*\prime} + \mu^{*\prime}_f)/P^{*\prime}_f]$.

Expectations are conditional upon information available in the current period, and variables with a prime refer to variables next period. Equation (10d) is an asset-pricing equation for foreign currency. The variable $\mu^{*\prime}_f$ is the Lagrange multiplier of the foreign-currency liquidity constraint next period, and can be interpreted as the liquidity services of foreign real balances next period. The equation states that the current value of money reflects its future "resale" value on next period's asset market and the liquidity services the money stock will provide on next period's commodity market. From the pricing function (7) and from the assumption that shocks are serially uncorrelated, we can write (10d) as

(10e) $\lambda^* = A^*/\omega^*$,

where $A^* = \beta E[\lambda^{*\prime} + \mu_f^{*\prime}]$ is constant. The marginal utility of wealth measured in foreign goods is inversely proportional to the foreign monetary expansion. This is so since the current marginal utility of next period's foreign currency real balances, $A^*$, is constant, and the rate of deflation, $P_f^*/P_f^{*\prime}$, for the given pricing function is inversely proportional to the foreign monetary expansion. Hence, the current marginal utility of wealth measured in foreign goods, is completely determined by the asset-pricing equation for foreign currency.

Finally, it follows from (10c) and (10e) that for constant home output, foreign output is increasing in foreign monetary expansion in the underconsumption region. This is so because an increase in foreign monetary expansion decreases the marginal utility of wealth measured in foreign goods. For the marginal utility of consumption of foreign goods to fall, consumption and output of foreign goods must rise. It can be shown (see Appendix A.3) that consumption and output of foreign goods must rise also when home output is allowed to adjust. We conclude that foreign output indeed rises with foreign monetary expansion in the underconsumption region,

(10f) $\quad Y^*_{\omega^*} > 0 \qquad\qquad (U^*).$

This completes our examination of the effect on <u>foreign</u> output of a foreign monetary expansion. Intuitively it is clear what is happening in the different regimes. With full capacity, consumption and output of foreign goods are restricted by the available capacity, and a foreign monetary expansion has no effect. In the liquidity constrained regime, consumption and output is restricted by real balances of foreign currency, and a foreign monetary expansion directly relaxes the foreign-currency liquidity constraint and increases consumption and output of foreign goods. In the underconsumption regime, increased transfers of foreign currency due to the foreign monetary expanision are, partly at least, spent on foreign goods.

A more precise way to to understand the effect on foreign output is with reference to intertemporal substitution. A current foreign monetary expansion implies that next period's foreign currency price of foreign goods increases. The current foreign currency price of foreign goods is predetermined. As we shall see in section IV below, perhaps somewhat surprisingly the foreign nominal interest rate remains unchanged. Then current foreign goods are becoming cheaper relative to next period's foreign goods; the foreign-good real rate of interest falls, and there is substitution in favor of current foreign goods. A second interpretation is to note that increased foreign inflation makes foreign currency real balances less attractive to hold, so consumers spend some of their foreign currency real balances on foreign goods.

Next we need to understand the effect on <u>home</u> output of a foreign monetary expansion. The three possible regimes in the home goods market, the full capacity regime, the liquidity constrained regime, and the underconsumption regime, are denoted by F, L and U. In the full capacity regime, home output is given by home capacity, and there is no effect of foreign monetary expansion,

(11a) $\quad Y = y, \text{ so } Y_{\omega*} = 0 \qquad$ (F).

In the liquidity constrained regime, home output is constraind by home currency real balances. By the same argument as above, these real balances are given by $k\omega$. We thus have,

(11b) $\quad Y = k\omega, \text{ so } Y_{\omega*} = 0 \qquad$ (L),

and again there is no effect of foreign monetary expansion on home output. In the underconsumption regime, finally, consumption and output of foreign goods is implicitly given by the first-order condition

(11c) $\quad u_h(Y, Y*) = \lambda = A/\omega \qquad$ (U).

The marginal utility of consumption of home goods equals the marginal utility of wealth measured in home goods, $\lambda$ (the Lagrange multiplier of the budget constraint (6) when the latter is deflated by the home goods price of home goods). (In general the first-order condition is $u_h(Y,Y*) = \lambda + \mu_h + \nu_h$, where $\mu_h$ and $\nu_h$ are the Lagrange multipliers of the home currency liquidity constraint (4a) and the home goods rationing constraint in (5).) By an argument symmetric to the one preceding (10e), the marginal utility of wealth measured in home goods is inversely proportional to the home monetary expansion (the second equality in (11c)), and independent of foreign monetary expansion. It follows that the effect on home output of foreign monetary expansion is determined exclusively by how changes in consumption of foreign goods affects the marginal utility of consumption of home goods. Differentiating (11c), we can write

(11d) $$Y_{\omega*} = [u_{hf}/(-u_{hh})]Y^*_{\omega*} \qquad (U).$$

Whether home output moves with or against foreign output has only to do with whether the cross derivative $u_{hf}$ is positive or negative, that is, whether home and foreign goods are Edgeworth-Pareto complements or substitutes.

Finally, combining (10) and (11d) we can write

(12) $$Y_{\omega*} \gtrless 0 \text{ if and only if } u_{hf} \gtrless 0 \qquad (U;L*,U*).$$

To sum up, when the home market is in the underconsumption regime, and the foreign market is in the liquidity constrained or the underconsumption regime, the effect on home output of a foreign monetary expansion is positive or negative depending upon whether home and foreing goods are Edgeworth-Pareto complements and substitutes. In all other regimes, the effect is zero. In other words, there is an effect of foreign monetary expansion on home output only if there is an effect on foreign output. There is an effect on foreign output only when the foreign market is in the liquidity constrained or in the underconsumption regime. Furthermore, there is an effect on home output only when the home goods market is in the underconsumption regime, that is when neither the liquidity constraint or the capacity constraint is binding.

Thus, the entire transmission of the effect of foreign monetary disturbances on home output seems to take place through the effect of a change in output and consumption of foreign goods on the marginal utility of home goods. Then the standard Mundell-Fleming result is contradicted if home and foreign goods are Edgeworth-Pareto complements.

In order to further clarify this result, we first note that whether goods are Edgeworth-Pareto complements or substitutes depends on the difference between the intertemporal elasticity of substitution in consumption and the intratemporal elasticity of substitution. To see this, we consider the nested CES utility function

(13a) $u(c_h, c_f) = U(c_h, c_f)^{1-1/\sigma}/(1 - 1/\sigma)$, $\sigma > 0$, $\sigma \neq 1$, and

$u(c_h, c_f) = \log U(c_h, c_f)$, $\sigma = 1$, where

(13b) $U(c_h, c_f) = [\gamma c_h^{1-1/s} + (1-\gamma)c_f^{1-1/s}]^{1/(1-1/s)}$, $s > 0$, $s \neq 1$, and

$U(c_h, c_f) = c_h^{\gamma} c_f^{1-\gamma}$, $s = 1$.

That is, $\sigma$ is the intertemporal elasticity of substitution in consumption, and s is the intratemporal elasticity of substitution in consumption between home and foreign goods. Then, as shown in the Appendix, the sign of the cross-derivative $u_{hf}$ is determined by the relative size of the intertemporal and intratemporal elasiticicities of substitution:

(14) $\text{sign } u_{hf} = \text{sign } (\sigma - s)$.

If intertemporal substitution dominates over intratemporal substitution, goods are Edgeworth-Pareto complements, otherwise they are Edgeworth-Pareto substitutes.

Thus equipped, we can interpret our results. To understand the role of intratemporal substitution, we note that the terms of trade $p = eP_f^*/P_h$ fulfill

(15) $p = \lambda^*/\lambda = (A^*/A)(\omega/\omega^*)$,

where we have exploited that the terms of trade equal the ratio of the marginal utility of real wealth measured in foreign goods to the marginal utility of real wealth measured in home goods (the first equality in (15)), the expressions for which we have derived in (10d) and (10e). Hence the terms of trade are proportional to the ratio between home and foreign monetary expansion (the second eqaulity in (15)). Also, it is easy to see that the exchange rate[8] will be proportional to the ratio of the stocks of home and foreign currency and given by[9]

(16) $e = (A^*k/Ak^*)(\bar{M}/\bar{N}^*) = (A^*k/Ak^*)(\omega\bar{M}_{-1}/\omega^*N^*_{-1})$.

From (16) it is clear that a foreign monetary expansion directly decreases the exchange rate (appreciates the home currency). With predetermined goods prices, the home country's terms of trade then improve in proportion to the foreign monetary expansion, and home goods become relatively expensive. Consumers substitute away from home goods, and demand for and output of home goods fall. The amount of such intratemporal substitution away from home goods is measured by the intratemporal elasticity of substitutions s.

However, a foreign monetary expansion also leads to a proportional increase in next period's foreign currency price of foreign goods, because of the pricing behavior of foreign firms. Since, as mentioned before and shown in section IV, the foreign nominal interest rate remains constant, the aggregate of next period's goods becomes more expensive relative to current goods. This induces intertemporal substitution away from next period's goods into current goods. This increases demand for and output of home goods. The amount of such intertemporal substitution into home goods is measured by the intertemporal elasticity of substitution $\sigma$. The

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8 Note that this exchange rate is the one that rules on the asset market after the goods market is closed. The terms of trade (15) are here defined from that exchange rate although goods are actually not traded directly against each other at relative prices equal to those terms of trade. (They are traded against currencies).

One can consider a different exchange rate $\tilde{e}$ on a hypothetical currency market that opens after the current state is known but before goods market opens. A different definition of the terms of trade can then be used, $\tilde{p} = \tilde{e}P^*_f/P_h$. In regimes without rationing of consumers, the terms of trade so defined equal the ratio of marginal utilities, $u_f(c_h,c_f)/u_h(c_h,c_f)$. When both markets are in the underconsumption region, the two exchange rates and terms of trade coincide. See Svensson (1985a, Section 5) and Stockman and Svensson (1987) on the properties of these alternative definitions of term of trade, and exchange rates.

9 Note that with the serially uncorrelated shocks the log of the exchange rate is a random walk.

net effect depends on the difference between $\sigma$ and s.

In summary, a foreign monetary expansion has a first effect by appreciating the home currency and affecting the relative price of home and foreign goods. This induces substitution away from home goods, and a fall in home output. This effect is also present in the Mundell-Fleming model. In addition, a foreign monetary expansion has a second effect by increasing foreign inflation and affecting the intertemporal relative price between current goods and future goods (lowering the foreign-goods real interest rate), which induces substitution in favor of current goods, and an increase in home output. This effect is either missing in the Mundell-Fleming model (when expenditure and saving is assumed independent of the real interest rate) or, if present, always dominated by the the first effect.

The reason the second effect is always dominated in the Mundell-Fleming model is that the decrease in the nominal interest rate caused by the foreign monetary expansion increases demand for home currency. Home money market equilibrium then requires a fall in home output (the transactions variable in the home money demand function). In our case, as both the foreign and home nominal interest rates remains unchanged (which will be demonstrated in section IV), we might expect the money market equilibrium to enforce a constant home output. However, in the underconsumption regime the home currency liquidity constraint is not binding and home currency holdings do not stand in a one-to-one relation to home output.

These results clearly demonstrate the payoff attached to the more careful approach to microeconomic underpinnings of this paper. Explicit attention to price setting and intertemporal decision-making has brought the intertemporal substitution channel effectively ignored in the standard Mundell-Fleming model into the foreground; a careful micro-based analysis of portfolio demand and asset pricing has shown how the usual ad hoc money demand function used in the Mundell-Fleming models arbitrarily restricts the results.

## IV. Nominal and Real Interest Rates

We first consider the effect of a foreign monetary expansion on nominal rates of interest at home and abroad. One over one plus the nominal interest rate equals the present value, measured in money terms, of a sure unit of nominal money paid out next period (after goods markets close; bonds yield no liquidity services):

(17a) $1/(1+i) = \beta E[\lambda'/P_h']/(\lambda/P_h)$ and $1/(1+i*) = \beta E[\lambda*'/P_f*']/(\lambda*/P_f*)$, or

(17b) $1+i = (\lambda/P_h)/\beta E[\lambda'/P_h']$; $1+i* = (\lambda*/P_f*)/\beta E[\lambda*'P_f*']$.

Using the first-order conditions for currency holdings (10d), and its analog for home currency, we can write

(17c) $i = E[\mu_h'/P_h']/E[\lambda'/P_h']$; $i* = E[\mu_f*'/P_f*']/E[\lambda*'/P_f*']$,

where $\mu_h$ ($\mu_f*$) is the liquidity services of home (foreign) real balances (the Lagrange multiplier for the home (foreign) currency liquidity constraint).

The nominal interest rate hence equals the ratio between expected future liquidity services of money, $E[\mu_h'/P_h']$, and the expected utility of future nominal wealth, $E[\lambda'/P_h']$. The interpretation is straightforward: both money and a nominal bond have an end-of-period value equal to one unit of money. During the holding period, money alone yields liquidity services valued at $E[\mu_h'/P_h']$. To offset that advantage, bonds need to pay interest of equal value, so market clearing requires $iE[\lambda'/P_h'] = E[\mu_h'/P_h']$, which immediately yields (17c).

The effect of a foreign monetary expanions on nominal interest rates is most easily derived by rewriting (17b) by using the pricing functions (7) and the expression for marginal utility of wealth measured in home goods (11c) to substitute for $\lambda$ and $P_h$, which gives

(18a) $$1+i = \frac{1}{\beta E[1/\omega']}.$$

Similarly,

(18b) $$1+i* = \frac{1}{\beta E[1/\omega*']},$$

so we get

(19) $\quad i_{\omega*} = i^*_{\omega*} = 0.$

Both home and foreign nominal interest rates are independent of foreign monetary expansion

The explanation of this maybe somewhat surprizing result is easily seen from equation (17). A temporary (that is, serially uncorrelated) foreign monetary disturbance will affect next period's foreign price level one for one, since $P^{*\prime}_f = \omega^*\bar{N}^*_{-1}/k^*$; this will reduce the liquidity service of foreign (nominal) money next period ($E[\mu^{*\prime}_f/P^{*\prime}_f] = E[\mu^{*\prime}_f]/P^{*\prime}_f$), but in the same proportion as it reduces the marginal utility of nominal wealth measured in foreign currency ($E[\lambda^{*\prime}/P^{*\prime}_f] = E[\lambda^{*\prime}]/P^{*\prime}_f$). Hence the relative attractiveness of foreign nominal bonds and foreign money remains unchanged and the nominal interest rate will not be affected at all. The home price level depends on home monetary expansion only, so it will not be affected by foreign monetary expansion; the argument therefore holds a fortiori for domestic nominal interest rates.

A different story emerges for real interest rates. Real rates can be derived from the present value of a future unit of wealth measured in terms of home goods or foreign goods, to derive home and foreign (own-good) real rates of interest, respectively:

(20) $\quad 1+\rho = \lambda/\beta E[\lambda'] \text{ and } 1+\rho^* = \lambda^*/\beta E[\lambda^{*\prime}].$

Substitution for the marginal utility of wealth measured in home and foreign goods, and using the expressions (18) for the nominal interest rates, we can write

(21) $\quad 1+\rho = \frac{1+i}{\omega}\ ;\ 1+\rho^* = \frac{1+i^*}{\omega^*}.$

Since the rate of inflation is known once current monetary shocks have been observed, the Fisher relation holds exactly, and one plus the real interest rate equals one plus the nominal interest rate divided by the rate of inflation.[10]

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[10] It is well known that when the rate of inflation is stochastic, the Fisher relation does no longer hold exactly. See Svensson (1985b) for an extensive discussion of the Fisher relation in a monetary asset pricing model with random inflation.

Since the nominal interest rates are constant, we can conclude that a foreign monetary expansion decreases the foreign real interest rate since it increases foreign inflation, but that it has no effect on the home real interest rate since it does not affect home inflation.

Let us consider an aggregate real interest rate, the real interest rate that corresponds to a price index.[11] That real interest rate will be a weighted average of the two real interest rates, and it follows that the aggregate real interest rate falls in response to a foreign monetary expansion. We may then interpret the intertemporal substitution discussed above as due to a decrease in the aggregate real interest rate, triggered by the increase in next period's foreign currency prices of foreign goods.

However, these real interest rates do not necessarily equal the expected marginal rates of substitution in consumption (minus one); that is, the real rates of interest on asset markets do not necessarily equal the true consumption rates of interest between goods markets at different points of time. The home and foreign (own-goods) consumption rates of interest, $\tilde{\rho}$ and $\tilde{\rho}*$, are defined from the marginal utility of consumption rather than that of wealth, hence[12]

(22) $$1+\tilde{\rho} = u_h/\beta E[u_h'] \text{ and } 1+\tilde{\rho}* = u_f/\beta E[u_f'].$$

The aggregate consumption interest rate is a weighted average of these two own-goods consumption interest rates. Temporary changes in the tightness of capacity and liquidity constraints (measured by changes in the Lagrange

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11 See Svensson and Razin (1983) for a discussion of terms of trade, consumption interest rates and price indices.

12 The consumption interest rates refer to bonds that are completely liquid, that is, bonds that are traded and mature at the beginning of the period, after the current state is known but before the goods market opens. These bonds are not perfect substitutes to bonds traded at the end of the period, and their interest rates are hence different. See Svensson (1985b) for further discussion of this issue.

Giovannini (1987) discusses the importance of less than perfect liquidity of bonds and stocks for the effect on interest rates and stock prices of increased uncertainty in output and monetary growth.

multipliers) drive a wedge between the consumption rates of interest and the real rates of interest observed on asset markets.

These distinctions between the the interest rates illuminates the different spillover effects in different regimes. In particular it clarifies our surprising result of zero spillover when foreign output is capacity constrained. On first sight, foreign output capacity constrained should block the intratemporal substitution effect, but leave the intertemporal one intact, implying a positive spillover. The reason why that implication is incorrect is related to the distinction between the real interest rates observed on asset markets and the consumption interest rates in the following way. Increased demand for foreign goods left unsatisfied because of a binding capacity constraint leads to a temporary increase in the corrsponding Lagrange multiplier, $\nu^*_f$. This increase is temporary because shocks are serially uncorrelated. Moreover, the increase will exactly offset the decline in $\lambda^*$ (since we have $u_f(Y,Y^*) = \lambda^* + \nu^*_f$ in the full-capacity regime). But then the foreign goods consumption rate of interest $\tilde{\rho}^*$ remaines unchanged, even though the foreign goods real interest rate $\rho^*$ falls. Temporary changes in the Lagrange multipliers of the capacity and liquidity constraints drive a wedge between the consumption interest rates and the real interest rates observed on the asset markets; this is what eliminates the intertemporal channel in the case when the foreign capacity constraint is binding. Feltenstein, Lebow and van Wijnbergen (1987) provide empirical evidence of the importance of the link between changes in the tightness of rationing constraints and the consumption rate of interest.

In the underconsumption regime (U;U*) the Lagrange multipliers on both constraints equal zero; hence equality between consumption rates of interest and the corresponding asset market real rates of interest obtains. Therefore the foreign goods consumption rate of interest $\tilde{\rho}^*$ declines in line with the foreign real rate of interest $\rho^*$. As a consequence a foreign monetary expansion shifts consumption from the next period to the current one under this regime.

This shift in consumption towards the current period also happens when the foreign goods market is liquidity constrained (regime (U;L*)). In that regime, foreign monetary expansion not only lowers the marginal utility of wealth $\lambda^*$, but it also lowers the Lagrange multiplier $\mu^*_f$ associated with the liquidity constraint. Both decrease the marginal utility of consumption of foreign goods. In this case, therefore, the foreign goods consumption interest rate $\tilde{\rho}^*$ defined by (22) declines even more than the foreign real interest rate $\rho^*$ defined by (20).

## V. Conclusions

In this paper we present a new framework for the analysis of international transmission of policy disturbances. We then apply this framework to a study of the effects of monetary policy both in the home country and in the rest of the world.

Our motivation is two-fold: First, a dissatisfaction with the structure of existing international neoclassical business cycle models, with infinitely flexible prices. Second, suspicion on spillover effects of monetary policy derived in Keynesian sticky-price models without proper microeconomic foundation. In this paper we focus specially on the result obtained within Mundell-Fleming models that monetary disturbances have negative spillover effects on output. We show that this result can be rationalized only under specific parameter values.

The framework used is similar to some recent work on open economy macroeconomics in that savings decisions are derived from optimimizing behaviour in a full rational-expectations context. We also derive money demand and other asset choices from maximizing behaviour within a specific transactions technology (cash-in-advance), but without the counterfactual implication of constant velocity characterizing many cash-in-advance models. This is achieved following earlier work of Svensson (1985a).

Moreover, we depart from much of the recent literature by explicitly incorporating the possibility of sticky prices, and excess demand and supply, which possibility seems to us to be a potentially important aspect of the transmission of macroeconomic disturbances. This puts us in line with the large literature on open economy "disequilibrium" models, of which Mundell-Fleming can be considered as a special but rather dominating case. However, following recent work on sticky prices, in particular Svensson (1986), we base price-setting behaviour in an explicit monopolistic competition structure.

We apply this framework to the analysis of the international transmission of monetary policy, and find results that are markedly different from the standard Mundell-Fleming predictions of negative spillovers on output. More specifically we find that the response to home output of a foreign monetary expansion first of all depends on what regimes the world markets for home and foreign goods are in, more precisely whether they are characterized by full capacity, binding liquidity constraints, or underconsumption. Which regimes the markets are in are determined by the realization of the stochastic money growth rates and capacity levels in the home and foreign countries. The underconsumption regimes are most similar to the Mundell-Fleming setup with excess supply in both countries. In this regime a foreign monetary expansion has basically two effects. First, it appreciates the home currency, increases the price of home goods relative to foreign goods. This induces intratemporal (within period) substitution away from home goods, depresses demand for and output of home goods, and is hence a negative effect on home output. This first effect is also present in in the Mundell-Fleming model. Second, a foreign monetary expansion causes an increase in the future foreign price level, which makes the aggregate of future goods more expensive relative to current goods. Equivalently, increased inflation decreases the aggregate consumption real interest rate. This results in intertemporal (between period) substitution in favor of current goods, and into current home goods, which stimulates demand for and output of home goods, and is hence a positive effect on home output. Which effect dominates is determined by the intertemporal elasticity of substitution in consumption relative to the intratemporal elasticity of substitution. In the Mundell-Fleming model, this second effect is either ignored (when expenditure and saving for simplicity is assumed independent of the real interest rate) or always dominated by the first effect.

Hence, according to this story a monetary contraction in the United States stimulates output in the rest of the world by the appreciation of the US dollar, but it depresses output in the rest of the world by increased real interest rates. With relatively strong intertemporal substitution, the depressing spillover may dominate.

We furthermore show that a permanent increase in the money supply (a serially uncorrelated shock to the rate of growth of money) will lead to a fall in the own real rate of interest of the good produced by the contry expanding its money supply. The own rate of interest on the other good will not be affected. Also, foreign and domestic nominal interest rates are not affected. The latter result follows from the fact that the serially uncorrelated monetary expansion does neither affect the future liquidity services of money in the expanding country nor the future marginal utility of nominal wealth measured in the expanding country's currency; hence the relativ attractiveness of money and nominal bonds is not affected.

The results on interest rates confirm a point made by Lucas and Stokey (1985). It is crucial for the effect on interest rates, and on asset prices in general, what the information content is of current disturbances. Depending upon that information content, for instance whether shocks are positively or negatively serially correlated, asset prices may react one way or another. In our case, shocks are serially uncorrelated, which means that the shocks have no information content relevant to the nominal interest rates.

There are some obvious technical limitations, that our analysis shares with similar international asset-pricing general equilibrium analyses. One is the the assumption of identical home and foreign consumers and the corresponding reliance on a perfectly pooled equilibrium. This means that home and foreign consumers, by holding identical portfolios of all the available assets, including currencies, are identically affected by all disturbances. Hence, wealth and welfare effects are identical across the two countries.[13] Removing the assumption of identical consumers remains an urgent task for future research.[14] Nevertheless, it seems that our result on the importance of intertemporal versus intratemporal substitution should not critically depend on the assumption of perfectly pooled equilibria.

Also, for technical reasons we can so far only solve the model for serially uncorrelated shocks. Whith regard to the comparision with the Mundell-Fleming results this is less of a drawback – a serially uncorrelated shock to the growth rate of money is equivalent to a permanent change in the level of the money stock, which seems to be how the standard Mundell-Fleming experiment should be interpreted. Nevertheless, allowing for serial correlation in the shocks remains a desirable extension.

Overall, we think this paper has demonstrated the positive payoff one can expect from a more serious approach to intertemporal decision-making, asset pricing and market structure. Our choice-theoretic approach to intertemporal decision-making has brought out an intertemporal substitution channel effectively ignored in the literature on international transmission of monetary shocks. This channel, we have shown, can reverse standard results under entirely plausible

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13 This does not exclude capital flows in models with perfectly pooled equilibria. There are indeed capital flows when the value of home and foreign (based) assets change, which causes changes in the net foreign asset position of home and foreign consumers. See Stockman and Svensson (1987) for details.

14 See Lucas (1982) and Svensson (1985a) for further discussion of the perfectly pooled equilibrium, and Svensson (1986) for further discussion of the pricing model used.

parameter values. Also, our rigorous approach to consumers' demand for various assets has shown how money demand functions commonly used in international economics arbitrarily restricts results.

## Appendix

### A1. Derivation of the Equilibrium Equations

Introduce the notation

(A.1) $x = x_t,\ x_{-1} = x_{t-1},\ x' = x_{t+1},\ \pi_M = 1/P_h,\ \pi_N = e/P_h,\ p = eP_f^*/P_h,$

$q_h = Q_h/P_h,\ q_f = Q_f/P_h,\ r_M = R_M/P_h$ and $r_N = R_N/P_h.$

Then the budget, liquidity and rationing constraints can be rewritten

(A.2a) $c_h + pc_f + \pi_M M + \pi_N N + q_h z_h + q_f z_f + r_M x_M + r_N x_N \leq w,$

(A.2b) $w' = \pi_M' M + \pi_N' N + (q_h' + Y')z_h + (q_f' + pY^{*\prime})z_f +$

$+ [r_M' + \pi_M'(\omega'-1)\bar{M}]x_M + [r_N' + \pi_N'(\omega^{*\prime}-1)\bar{N}^*]x_N,$

(A.2c) $c_h \leq \pi_M[M_{-1} + (\omega-1)\bar{M}_{-1}x_{M,-1}],$

(A.2d) $pc_f \leq \pi_N[N_{-1} + (\omega^*-1)\bar{N}^*_{-1}x_{N,-1}],$

(A.2e) $c_h \leq Y$ and

(A.2f) $c_f \leq Y^*.$

In a stationary stochastic rational expectations equilibrium the endogenous variables in period t will be functions of the state variables in period t, $(s,\bar{M}_{-1},\bar{N}^*_{-1},\pi_M,\pi_N^*)$. Then the home consumer's decision problem to maximize (3) subject to (A.2) defines, in the usual way, the value function $v(w,M_{-1},N_{-1},x_{M,-1},x_{N,-1};s,\bar{M}_{-1},\bar{N}^*_{-1},\pi_M,\pi_N^*)$ as the maximum of $u(c_h,c_f) + \beta E[v(w',M,N,x_M,x_N;s',\bar{M},\bar{N}^*,\pi_M',\pi_N^{*\prime})]$ subject to (A.2). The first-order conditions, together with the market equilibrium conditions (8), give

(A.3a) $c_h \leq \pi_M\bar{M} \quad [\mu_h \geq 0],$

(A.4a) $pc_f \leq \pi_N\bar{N}^* \quad [\mu_f \geq 0],$

(A.3b) $c_h \leq y \quad [\nu_h \geq 0],$

(A.4b) $c_f \leq y^* \quad [\nu_f \geq 0],$

(A.3c) $u_h(c_h,c_f) = \lambda + \mu_h + \nu_h,$

(A.4c) $u_f(c_h,c_f) = \lambda p + \mu_f p + \nu_f,$

(A.3d) $\lambda\pi_M = \beta E[(\lambda'+\mu_h')\pi_M'],$

(A.4d) $\lambda\pi_N = \beta E[(\lambda'+\mu_f')\pi_N'],$

(A.3e) $\lambda q_h = \beta E[\lambda'(q_h'+Y')]$,

(A.4e) $\lambda q_f = \beta E[\lambda'(q_f'+p'Y*')]$,

(A.3f) $\lambda r_M = \beta E[\lambda' r_M' + (\lambda'+\mu_h')\pi_M'(\omega'-1)\bar{M}]$ and

(A.4f) $\lambda r_N = \beta E[\lambda' r_N' + (\lambda'+\mu_f')\pi_N'(\omega*'-1)\bar{N}*]$.

Here $\lambda$, $\mu_h$, $\mu_f$, $\upsilon_h$ and $\upsilon_f$ are the Lagrange multipliers of the constraints (A.2a), (A.2c), (A.2d), (A.2e) and (A.2f), respectively. Equations (A.3c)-(A.3f) are the partials of the Lagrangean with respect to $c_h$, M, $z_h$, and $x_M$, whereas (A.4c)-(A.4f) are the partials with respect to $c_f$, N, $z_f$ and $x_N$. By the definition of the value function it will fulfill

(A.5) $v_w = \lambda$, $v_M = \mu_h\pi_M$, $v_N = \mu_f\pi_N$, $v_{x_M} = \mu_h\pi_M(\omega-1)\bar{M}_{-1}$ and

$v_{x_N} = \mu_f\pi_N(\omega*-1)\bar{N}*_{-1}$,

which has been exploited in (A.3) and (A.4).

Equations (A.3a) - (A.3d), together with the pricing equation (7), give

(A.6a) $c_h \leq k\omega$ $[\mu_h \geq 0]$,

(A.6b) $c_h \leq y$ $[\upsilon_h \geq 0]$,

(A.6c) $u_h(c_h,c_f) = \lambda + \mu_h + \upsilon_h$ and

(A.6d) $\lambda = A/\omega$ where $A = \beta E[\lambda' + \mu_h']$.

Introduce the notation

(A.7) $\lambda* = \lambda p$, $\mu_f^* = \mu_f p$, $\upsilon_f^* = \upsilon_f$ and $\pi_N^* = \pi_N/p$.

Then (A.4a) - (A.4d), together with (A.7) and the pricing function (7), give

(A.8a) $c_f \leq k*\omega*$ $[\mu_f^* \geq 0]$,

(A.8b) $c_f \leq y*$ $[\upsilon_f^* \geq 0]$,

(A.8c) $u_f(c_h,c_f) = \lambda* + \mu_f^* + \upsilon_f^*$ and

(A.8d) $\lambda* = A*/\omega*$ where $A* = \beta E[\lambda*' + \mu_f^{*}{}']$.

The equations/inequalities (A.6) and (A.8) provide the system of functional equations that need to be solved.

The regimes are specified in the following way. Let us consider the market for home goods. We first define the critical levels of home capacity and monetary expansion, $\bar{y}(Y*)$ and $\bar{\omega}(Y*)$, for which both the liquidity and rationing constraints (A.6a) and (A.6b) are just binding, and for which hence the corresponding Lagrange multipliers are zero. This gives

$$\text{(A.9)} \qquad \bar{\omega}(Y*) = A/u_h(\bar{y}(Y*),Y*) \text{ and } \bar{\omega}(Y*) = \bar{y}(Y*)/k,$$

which implicitly determines the point $(\bar{y}(Y*),\bar{\omega}(Y*))$ in $(y,\omega)$-space. Consider the assumption

$$\text{(A.10)} \qquad \left|\frac{\epsilon u_h(c_h,c_f)}{\epsilon c_h}\right| < 1,$$

where $\epsilon y/\epsilon x$ denotes the partial elasticity $(x/y)\partial y/\partial x$. That is, the elasticity of marginal utility of consumption of home goods with respect to home goods is less than unity.

Under assumption (A.10) the underconsumption regime, U, for the home goods market is defined by the set of points $(y,\omega)$ that fulfill the condition $\omega \leq \min(A/u_h(y,Y*),\bar{\omega}(Y*))$. The liquidity constrainted regime, L, is given by the set of $(y,\omega)$ fulfilling the condition $y \geq \bar{y}(Y*)$ and $\bar{\omega}(Y*) \leq \omega \leq y/k$. The full capacity regime, F, is given by the set of $(y,\omega)$ fullfilling the condition $\omega \geq \max(A/u_h(y,Y*),y/k)$. The regimes for the foreign goods market are determined analogously.

Assumption (A.10) determines the relative location of the regimes U, L and F. It is not necessary for any of the results in the paper. See Svensson and van Wijnbergen (1986) for a more detailed analysis of the regimes and the solution, including diagrammatic illustrations. See Svensson (1986) for details of the solution when (A.10) does not hold.

## A2. Pricing Behavior

The solution of (A.6) and (A.8) gives rise to the output functions (9), where the constants k and k* are included among the arguments. Now we shall derive the pricing policy of firms, and, more specificly, explain what determines the constants k and k* in (9). As in Svensson (1986), this is done by introducing differentiated products and monopolistic competition along the lines of Dixit and Stiglitz (1977).

Let there be a continuum of home and foreign firms. The set of home firms is represented by the unit interval, and each home firm is indexed by j, $0 \leq j \leq 1$. Each home firm j produces a unique differentiated home product j, $0 \leq j \leq 1$. The home firms face perfectly correlated economy-wide shocks to their capacity. We hence let y denote the capacity of each home firm as well as the aggregate capacity of the home country (since $\int_{j=0}^{1} y dj = y$). Similarly, there is a continuum of foreign firms, also represented by the unit interval, and foreign firm j*, $0 \leq j^* \leq 1$, produces foreign differentiated product j*, with the (across foreign firms) perfectly correlated foreign country-wide capacity y*.

Let us now look more closely at the home firms. The consumers' preferences for differentiated home goods are given by considering $c_h$ in the utility function (3) to be aggregate real consumption of home goods, an aggregate given by the CES subutility function

(A.11) $$c_h = \left[\int_{j=0}^{1} c_{hj}^{(\sigma_h-1)/\sigma_h} dj\right]^{\sigma_h/(\sigma_h-1)}, \quad \sigma_h > 1,$$

of consumption $c_{hj}$ of differentiated home product j. We identify the home price level $P_h$ with the corresponding CES price index

(A.12a) $$P_h = \left[\int_{j=0}^{1} P_{hj}^{1-\sigma_h} dj\right]^{1/(1-\sigma_h)},$$

where $P_{hj}$ is the home currency price of differentiated home product j. It follows by a standard derivation that per capita demand for home product j is given by

(A.12b) $$c_{hj} = (P_{hj}/P_h)^{-\sigma_h} c_h.$$

Let $Y_j \leq y$ denote actual (per capita) output of home firm j. We specify that it is given by

(A.13) $$Y_j = \min(y, c_{hj}).$$

that is, the minimum of capacity and demand. Furthermore, let $Q_{hj}$ denote the (per capita) home currency stock market value of home firm j. It is given by the asset-pricing equation

(A.14a) $\lambda\pi_M Q_{hj} = \beta E[\lambda'\pi_M'(Q_{hj}' + P_{hj}'Y_j')]$,

which can be solved to give

(A.14b) $Q_{hj} = P_{hj}' E\left[\frac{\lambda'\pi_M'}{\lambda\pi_M} Y_j'\right]/\delta$,

where $\delta$, the rate of time preference, is given by $\beta = 1/(1+\delta)$. The expression for $\lambda'\pi_M'/\lambda\pi_M$, the marginal rate of substitution between nominal wealth in period t and t+1, can be simplified using (7) and (A.6d) which yields:

(A.15) $\frac{\lambda'\pi_M'}{\lambda\pi_M} = \frac{1}{\omega'}$.

Then the stock market value of home firm j can be written

(A.16) $Q_{hj} = P_{hj}' E[Y_j'/\omega']/\delta$.

Home firm j chooses $P_{hj}'$ to maximize (A.16).[15] In order to see what this implies, let us define expected discounted sales $g_{hj}(P_{hj}/P_h, k, k*)$ by

(A.17) $g_{hj}(\frac{P_{hj}'}{P_h}, k, k*) = E[\frac{Y_j'}{\omega'}] =$

$$= \int_{\upsilon_h'>0} \frac{y'}{\omega'} dF(s') + \int_{\upsilon_h'=0} (\frac{P_{hj}'}{P_h'})^{-\sigma_h} \frac{Y(s', k, k*)}{\omega'} dF(s'),$$

where we have used (A.13) and (A.12b). We now realize that maximization of (A.16) implies that $P_{hj}'$ is chosen such that the elasticity of expected discounted sales (A.17) with respect to the relativce price $p_{hj} = P_{hj}/P_h$ is set equal to minus unity. (This corresponds to choosing the marginal revenue equal to zero - recall that marginal cost is zero.) Furthermore, all home firms have identical demand. Hence they will all choose the same home currency price $P_{hj}$.[16] Then we have

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[15] It does not matter whether the firm maximizes its nominal or real stock market value since it perceives the price level in any currency as independent of its actions.

[16] They will also choose the same output level, $Y_j = Y$, $0 \leq j \leq 1$, and in

(A.18a) $P_{hj} = P_h,\ 0 \leq j \leq 1$, and

(A.18b) $$-\frac{g_{hp}(1,k,k*)}{g_h(1,k,k*)} = 1,$$

where we have dropped the index j since the expected-discounted-sales function is identical for all home firms, and where $g_{hp}$ denotes the partial with respect to $P_{hj}/P_h$. The first-order condition (A.18b) can by (A.17) be written as

(A.19a) $$\sigma_h \frac{\int_{\upsilon_h=0} \frac{Y(s,k,k*)}{\omega} dF(s)}{g_h(1,k,k*)} = 1.$$

By a completely symmetric argument, the first-order condition for foreign firms can be written

(A.19b) $$\sigma_f \frac{\int_{\upsilon^*_f=0} \frac{Y*(s,k,k*)}{\omega*} dF(s)}{g_f(1,k,k*)} = 1.$$

with obvious notation.

The first-order conditions (A.19) have an intuitive interpretation. The home firm faces an ex post price elasticity of sales equal to $\sigma_h$ when there is excess supply, that is when $\upsilon_h = 0$. It faces an ex post price elasticity of sales equal to zero when there is excess demand and full capacity, that is when $\upsilon_h > 0$. Since it must settle on a price for its output ex ante, it chooses the price so that the price elasticity of expected discounted sales equals unity. The latter is a weighted average of the ex post price elasticity when there is excess supply ($\sigma_h$) and the ex post price elasticity when there is excess demand (zero). Thus, the weight multiplying $\sigma_h$ on the left hand side of (A.19a) is the probability-weighted share of expected discount sales with excess supply.

Now the full equilibrium is determined. The system (A.6) and (A.8) determines, for given k and k*, the output functions Y(s,k,k*) and Y*(s,k,k*), and

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equilibrium consumers will consume the same quantity of home differentiated products, $c_{hj} = c_h = Y_h$.

these together with the firms' first-order conditions (A.19) determine the constants k and k*.

A3. Derivation of $Y^*_{\omega^*}$ in region (U,U*)

In region (U;U*) the output functions are given by the system

(A.20a) $u_h(Y,Y^*) = A/\omega$ and

(A.20b) $u_f(Y,Y^*) = A^*/\omega^*$.

It is straight forward to derive the derivative

(A.21) $Y^*_{\omega^*} = [-u_{hh}A^*/(\omega^*)^2]/\Delta$,

where the determinant $\Delta$ equals $u_{hh}u_{ff} - u_{hf}u_{hf}$. By concavity of the utility function the determinant is positive and $u_{hh}$ is negative, and it is clear that the derivative $Y^*_{\omega^*}$ is positive.

A4. Derivation of (14)

From (13) we have

(A.22) $u_h = U^{-1/\sigma}U_h$ and $u_f = U^{-1/\sigma}U_f$. Furthermore,

(A.23) $U_h = U^{1/s}\gamma c_h^{-1/s}$ and $U_f = U^{1/s}(1-\gamma)c_f^{-1/s}$. Then

(A.24a) $\epsilon u_h/\epsilon c_h = -(\epsilon U/\epsilon c_h)/\sigma + \epsilon U_h/\epsilon c_h$ and

(A.24b) $\epsilon u_h/\epsilon c_f = -(\epsilon U/\epsilon c_f)/\sigma + \epsilon U_h/\epsilon c_f$, where

(A.25) $\epsilon U/\epsilon c_h = c_h U_h/U = \alpha_h$ and $\epsilon U/\epsilon c_f = c_f U_f/U = \alpha_f$, $\alpha_h + \alpha_f = 1$. Also

(A.26a) $\epsilon U_h/\epsilon c_h = (\epsilon U/\epsilon c_h)/s - 1/s = (\alpha_h - 1)/s = -\alpha_f/s$ and

(A.27b) $\epsilon U_h/\epsilon c_f = (\epsilon U/\epsilon c_f)/s = \alpha_f/s$.

Combining this gives

(A.28a) $\epsilon u_h/\epsilon c_h = -(\sigma\alpha_f + s\alpha_h)/\sigma s$.

(A.29b) $\epsilon u_h/\epsilon c_f = \alpha_f(\sigma - s)/\sigma s$.

which implies (14).

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## References


Aizenman, J., "Monopolistic Competition, Relative Prices and Output Adjustment in the Open Economy," mimeo., 1985.

Blanchard, O. J., and N. Kiyotaki, "Monopolistic Competition, Aggregate Demand Externalities, and Real Effects of Nominal Money," NBER Working Paper No. 1770, 1986.

Dixit, A. K., and J. E. Stiglitz, "Monopolistic Competition and Optimal Product Diversity," American Economic Review, LXVII (1977), 297-308.

Dornbusch, R., Open Economy Macroeconomics, (New York: Basic Books, 1980).

———, "Exchange Rates and Prices," American Economic Review LXXVII (1987), 93-106.

Feltenstein, A., D. Lebow and S. van Wijnbergen, "Savings, Commodity Market Rationing and the Real Rate of Interest in China," DRD Discussion Paper No. 000, World Bank, 1987.

Fleming, M., "Domestic Financial Policies Under Fixed and Under Floating Exchange Rates," IMF Staff Papers, IX (1962), 369-379.

Giovannini, A., "Exchange Rates and Traded Goods Prices," mimeo., Columbia University, 1985.

———, "Uncertainty and Liquidity," mimeo., Columbia University, 1987.

Lucas, R. E., Jr., "Asset Prices in an Exchange Economy," Econometrica, XLVI (1978), 1429-1445.

———, "Interest Rates and Currency Prices in a Two-Country World," Journal of Monetary Economics, X (1982), 335-360.

———, and Nancy L. Stokey, "Money and Interest in a Cash-In-Advance Economy," NBER Working Paper No. 1618, 1985.

Mundell, R. A., International Economics, (New York: Macmillan, 1968).

Persson, T., "Global Effects of National Stbilization Policy under Fixed Exchange Rates," Scandinavian Journal of Economics, VXXXIV (1982), 165-192.

Stockman, A. C., "A Theory of Exchange Rate Determination," Journal of Political Economy, LXXXVIII (1980), 673-698.

———, and L. E. O. Svensson, "Capital Flows, Investment, and Exchange Raes," Journal of Monetary Economics, XIX (1987), 171-201.

Svensson, L. E. O., "Currency Prices, Terms of Trade and Interest Rates: A General-Equilibrium Asset-Pricing Cash-in-Advance Approach," Journal of International Economics XVIII (1985a), 17-41.

———, "Money and Asset Prices in a Cash-in-Advance Economy," Journal of Political Economy, XCIII (1985b), 919-944.

———, "Sticky Goods Prices, Flexible Asset Prices, Monopolistic Competition, and Monetary Policy," Review of Economic Studies LIII (1986), 385-405.

———, and A. Razin, "The Terms of Trade and the Current Account: The Harberger-Laursen-Metzler Effect," Journal of Political Economy, XCI (1983), 97-125.

———, and S. van Wijnbergen, "International Transmission of Monetary Policy," IIES Seminar Paper No. 362, 1986.

van Wijnbergen, S., "Oil Price Shocks, Investment, Employment and the Current Account: an Intertemporal Disequilibrium Analysis," Review of Economic Studies, LII (1985), 627-648.