NBER WORKING PAPER SERIES

EFFICIENT "MYOPIC" ASSET PRICING IN GENERAL EQUILIBRIUM: A POTENTIAL PITFALL IN EXCESS VOLATILITY TESTS

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Working Paper No. 2251

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
May 1987

I would like to thank Kenneth Kletzer and Vittorio Grilli for helpful discussions. The research reported here is part of the NBER's research program in Financial Markets and Monetary Economics. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

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ABSTRACT

Excess volatility tests for financial market efficiency maintain the hypothesis of risk-neutrality. This permits the specification of the benchmark efficient market price as the present discounted value of expected future dividends. By departing from the risk-neutrality assumption in a stripped-down version of Lucas's general equilibrium asset pricing model, I show that asset prices determined in a competitive asset market and efficient by construction can nevertheless violate the variance bounds established under the assumption of risk neutrality. This can occur even without the problems of non-stationarity (including bubbles) and finite samples. Standard excess volatility tests are joint tests of market efficiency and risk neutrality. Failure of an asset price to pass the test may be due to the absence of risk neutrality rather than to market inefficiency.

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I. Introduction

In this short note I point out some problems in conventional excess volatility testing, which may lead to incorrect inferences concerning financial market efficiency. The volatility tests initiated by Shiller [1981 a,b] specify the benchmark efficient market price as the present discounted value of expected future dividends.[1] The maintained hypothesis of risk neutrality that permits the intertemporal first-order condition of the consumer-investor to be purged of unobservable marginal utilities of consumption today and tomorrow, is crucial for the validity of the model and of the excess volatility tests. Using a stripped-down, one good, one asset version of Lucas's [1978] asset pricing model, it is very easy to get tractable solutions for the equilibrium asset price for the constant relative risk aversion family of utility functions.

With logarithmic utility, the price-dividend ratio is a constant in this model, i.e. only the current dividend affects the current asset price, regardless of the nature of the stochastic process governing dividends. In this setting it is easy to come up with examples where the "bubble-free" equilibrium asset price with logarithmic utility violates the variance bound derived under the assumption of linear

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1 Or earnings. My note focuses on problems with the excess volatility tests that are present even when the countless econometric and data problems that complicate empirical tests are absent. The earnings vs. dividends problem is not present in the model used in this note.

utility. Without bubbles and with arbitrarily large sample size, the efficiently determined asset price would, according to the misconceived volatility test, exhibit excessive volatility.

II. The Model and the Equilibrium Behavior of the Asset Price

The model is a baby version of Lucas's [1978] model of asset prices in an exchange economy. The competitive exchange economy is inhabited by a representative infinite-lived household with a time-additive utility function. There is a single non-durable consumption good. The economy receives each period a random "endowment" or "dividend" d_t of this non-durable good. For simplicity, d_t is assumed to be strictly positive for all t, to have bounded support and to follow a Markov process. The household owns a share in this stream of endowments. This share is the only asset and there are no other sources of household income. The share is traded and priced in a competitive market. c_t is consumption of the non-durable good by the representative household in period t. p_t is the price in period t (in terms of period t consumption) of a claim to the entire future stream of endowments i.e. it is the period t "ex-dividend" price of the asset. θ_t is the household's demand in period t for a share in the future endowments.

The representative household faces the following optimization problem.

$$1) \quad \underset{\{c_{t+i},\, \theta_{t+i}\}}{\text{Max}} \; E_t \sum_{i=0}^{\infty} \beta^i u(c_{t+i}) \qquad 0 < \beta < 1,\; u' > 0,\; u'' \leq 0$$

$$2)\quad c_{t+i} \geq 0 \qquad\qquad t \geq 0;\ i \geq 0$$

$$2b)\ c_t + p_t(\theta_t - \theta_{t-1}) \leq d_t\, \theta_{t-1} \qquad\qquad t \geq 1$$

$$2c)\ \theta_0 = 1$$

β is the consumer's subjective discount factor. The single-period utility function, u, is twice continuously differentiable, bounded and increasing with $u(0) = 0$. u is strictly concave except when the case of linear utility (risk-neutrality) is considered, in which case it is concave.

E_t is the expectation operator, conditional on the information set at time t. This information set includes all current and past values of the state variable d_t and the true model, including the transition function of the Markov process governing d_t. The market-clearing asset price function co-determined by the consumer's optimizing behavior is the same as the price function on which consumer decisions are based: the equilibrium is a rational expectations equilibrium.

The (interior) intertemporal first-order condition (or "Euler equation") of the representative agent can be written as

$$3)\quad u'(c_t)\ p_t = E_t\left[\beta u'(c_{t+1})(p_{t+1} + d_{t+1})\right]$$

Using the law of iterated projections and imposing the transversality condition given in (4), the current ex-dividend share price can be written as in (5).

$$4)\quad \lim_{\tau\to\infty} \beta^{\tau} E_t \left[u'(c_{t+\tau}) p_{t+\tau}\right] = 0$$

$$5)\quad p_t = \frac{1}{u'(c_t)} \sum_{i=1}^{\infty} \beta^i E_t \left[u'(c_{t+i})\ d_{t+i}\right]$$

Note that by imposing (4) we choose the "bubble-free" solution to (3). The possibility of violating the variance bounds documented below, is therefore not due to the presence of speculative bubbles (see e.g. Flood and Garber [1986]).

Equilibrium in this exchange economy without durable commodities requires that the endowment be consumed each period or (equivalently by Walras' Law) that the representative household willingly holds all shares to the future stream of endowments, i.e.

$$6a)\quad c_t = d_t \qquad t \geq 1$$

or

$$6b)\quad \theta_t = 1 \qquad t \geq 1$$

Substituting (6a) into (5) yields

$$7)\quad p_t = \frac{1}{u'(d_t)} \sum_{i=1}^{\infty} \beta^i E_t (u'(d_{t+i})\ d_{t+i})$$

In the case of linear utility (constant u') this reduces to

$$8)\quad p_t^{(1)} = \sum_{i=1}^{\infty} \beta^i E_t(d_{t+i})$$

Starting with the work of Shiller [1981 a, b], this expression for the current asset price (or share price) as the present discounted value of currently expected future endowments (or dividends), has motivated a number of related tests for the efficiency of financial markets.

Define the ex-post rational price under linear utility, $p_t^{*(1)}$ as in (9):

$$9)\quad p_t^{*(1)} = \sum_{i=1}^{\infty} \beta^i d_{t+i}$$

Clearly,

$$10)\quad p_t^{*(1)} = p_t^{(1)} + u_t$$

where

$$11)\quad u_t \equiv \sum_{i=1}^{\infty} \beta^i (d_{t+i} - E_t d_{t+i})$$

By construction, u_t is orthogonal to $p_t^{(1)}$. The d_t process need not be covariance stationary. If it isn't, its unconditional moments are undefined. The variance bounds arguments will in that case be expressed in terms of the innovations of (possibly non-stationary) stochastic processes, which are well-defined. For any variable x_t, and any finite positive integer n, the innovation based on period t-n information is $x_t - E_{t-n}(x_t)$. The innovation variance is defined by

$$\mathrm{Var}_n(x_t) \equiv E\left\{\left[x_t - E_{t-n}(x_t)\right]^2\right\}$$

E is the unconditional expectation operator.

Except for the trivial case where d_t is strictly deterministic, it then follows that

$$\text{12)}\quad \mathrm{Var}_n\left[p_t^{*(1)}\right] = \mathrm{Var}_n\left[p_t^{(1)}\right] + \mathrm{Var}_n\left[u_t\right] > \mathrm{Var}_n\ p_t^{(1)}$$

If the variance of the observed asset price process were to exceed $\mathrm{var}_n(p_t^{*(1)})$, the variance bound in (12) would have been violated and a rejection of the joint hypothesis of asset market efficiency and risk neutrality would have been established.

Risk neutrality or linear utility is, however, crucial to the correct interpretation of the results from variance bounds tests, as the following simple example makes clear. Let u(c) belong to the class of constant relative risk aversion utility functions:

$$\text{13)}\quad u(c) = \begin{cases} \frac{1}{\gamma}c^{\gamma} & \gamma \leq 1 \quad (\gamma \neq 0) \\ \ln c & (\gamma = 0) \end{cases}$$

The asset pricing formula (7) now becomes

$$14) \quad p_t = d_t^{1-\gamma} \sum_{i=1}^{\infty} \beta^i E_t d_{t+i}^{\gamma}$$

The risk-neutral case of equation (8) is recovered from (14) by setting $\gamma = 1$. Consider instead the logarithmic case ($\gamma = 0$). Asset pricing is now governed by

$$15) \quad p_t^{(0)} = \frac{\beta}{1-\beta} d_t$$ [2]

Thus, with logarithmic utility, regardless of the stochastic or deterministic nature of the process governing d_t, the asset will be priced exclusively with reference to the current realization of the dividend process. $p_t^{(0)}$ behaves like the price of a real consol with a constant coupon of d_t and a constant short-real interest rate $\frac{1-\beta}{\beta}$. The intuition is the following:[3] with logarithmic utility, the marginal utility of consumption is the reciprocal of the level of consumption. The first-order condition (5) therefore becomes:

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[2] Equation (15) gives the ex-dividend price; the price inclusive of the current dividend would be $\frac{1}{1-\beta} d_t$.

[3] I would like to thank Vittorio Grilli for guiding my intuition.

$$5')\quad \frac{p_t}{c_t} = \sum_{i=1}^{\infty} \beta^i E_t \left[\frac{d_{t+i}}{c_{t+i}}\right]$$

With a perishable commodity $c_t = d_t$. The marginal benefit from postponing consumption in period t (the r.h.s. of (5')) is therefore independent of d_t and of current expectations of future d_{t+i}. The marginal cost of postponing consumption in period t should therefore also be independent of d_t and of current expectations of future d_{t+i}. This requires that the current asset price moves proportionally with the current dividend.

Note that with $\gamma = 0$, $p_t^{(0)}$ in (15) is the bubble-free asset price; the asset market is efficient by construction, in spite of the myopic appearance of the equilibrium asset price equation. Yet $p_t^{(0)}$ may easily fail to pass the conventional variance bounds test based on linear utility. Consider e.g. the case where d_t is a constant plus a zero mean, constant variance, serially independent error term η_t.

$$16)\quad d_t = \bar{d} + \eta_t \qquad ;\eta_t > -\bar{d};\ E\eta = 0;\ E\eta_t\eta_s = \begin{cases} 0, & s \neq t \\ \sigma_\eta^2, & t = s \end{cases}$$

From (9), (15) and (16), $p_t^{*(1)}$ and $p_t^{(0)}$ are given by:

$$17)\quad p_t^{*(1)} = \frac{\beta}{1-\beta}\bar{d} + \sum_{i=1}^{\infty} \beta^i \eta_{t+i}$$

$$18)\quad p_t^{(0)} = \frac{\beta}{1-\beta}\bar{d} + \frac{\beta}{1-\beta}\eta_t$$

Comparing the variances of $p_t^{(0)}$ and $p_t^{*(1)}$ [1] we obtain

$$19) \quad \mathrm{Var}\; p_t^{(0)} = \sigma_\eta^2 \frac{\beta^2}{(1-\beta)^2} > \mathrm{Var}_t^{*(1)} = \sigma_\eta^2 \frac{\beta^2}{(1+\beta)(1-\beta)}$$

Thus the actual (and efficiently determined) asset price $p_t^{(0)}$ with logarithmic utility is more volatile than the ex-post rational price would be under linear utility, $p_t^{*(1)}$.

If instead of (16), d_t is governed by the stationary AR1 process given in (20), a similar result is obtained.

$$20) \quad d_t = \alpha d_{t-1} + \epsilon_t \qquad |\alpha| < 1;\; 0 < \underline{\epsilon} \le \epsilon_t < \bar{\epsilon} < \infty$$

$$E\epsilon = 0;\; E\epsilon_t\epsilon_s = \begin{cases} 0 & s \neq t \\ \sigma_\epsilon^2 & t = s \end{cases}$$

Given (20), $p_t^{*(1)}$ and $p_t^{(0)}$ are given by:

$$21) \quad p_t^{*(1)} = \frac{\alpha\beta}{1-\alpha\beta} d_t + \sum_{i=1}^{\infty} \beta^i \sum_{j=0}^{i-1} \alpha^j \epsilon_{t+i-j} = \frac{\alpha\beta}{1-\alpha\beta} d_t + \frac{1}{1-\alpha\beta} \sum_{i=1}^{\infty} \beta^i \epsilon_{t+i}$$

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1 As the two series are covariance-stationary, the unconditional comments are well-defined.

$$22)\ p_t^0 = \frac{\beta}{1-\beta} d_t$$

Since, given (20), Var $d_t = \frac{1}{1-\alpha^2} \sigma_\epsilon^2$, it follows that

$$23)\ \text{Var } p_t^{*(1)} = \frac{(1+\alpha\beta)\ \beta^2}{(1-\alpha^2)(1-\beta^2)(1-\alpha\beta)} \sigma_\epsilon^2$$

$$24)\ \text{Var } p_t^{(0)} = \frac{\beta^2}{(1-\beta)^2} \frac{1}{(1-\alpha^2)} \sigma_\epsilon^2$$

It is easily checked that, provided $\alpha < 1$ (as was assumed), Var $p_t^{(0)} >$ Var $p_t^{*(1)}$. An excess volatility test based on the linear utility model incorrectly rejects market efficiency when utility is logarithmic.

Conclusion

Conventional volatility tests are joint tests of rational expectations, competitive market clearing and risk-neutrality. Relaxing the latter assumption may (and in the examples solved for in this note) does make it impossible to interpret violation of variance bounds as evidence against market efficiency.

The optimal "myopic" pricing function in the case of logarithmic utility permits the construction of easy examples of an efficiently determined asset price exhibiting apparent excess volatility. Finally, in the perishable commodity universe of this note, a degree of risk aversion in excess of that given by the logarithmic case ($\gamma < 0$), generates a negative effect of future expected dividends on the efficiently determined asset price while current dividends have a more than proportional positive effect! It's hard indeed to test for efficiency.

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