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ABSTRACT

We first show that, at least in theory, open market operations in real assets can be a useful tool for overcoming a liquidity trap because they change the inflation incentives of the government, and thus change private sector expectations from deflationary to inflationary. We argue that this formalizes Ben Bernanke's arbitrage argument for why a central bank can always increase nominal demand, despite the zero lower bound. We illustrate this logic in a calibrated New Keynesian model assuming the government acts under discretion. Numerical experiments suggest, however, that the needed intervention is incredibly high, creating a serious limitation of this solution to the liquidity trap. Our experiments suggest that while asset purchases can be a helpful commitment device in theory, they may need to be combined in practice with fiscal policy coordination to achieve the desired outcome.

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1. Introduction

This paper looks back on the professional consensus about monetary policy at the zero bound prior to the 2008 crisis, and proposes a calibrated model that provides one interpretation that explains why it was somewhat off base. It is easy to forget, but the general consensus in the economics profession in the late 1990's, when Japan was experiencing difficulties due to deflation and the zero bound, was that increasing the money supply in one of a variety of ways was a simple and straightforward answer to stimulating aggregate demand.

One example of this point of view is from Kenneth Rogoff (1998), a leading international macroeconomist, in response to Krugman (1998), an article that launched the modern zero lower bound (ZLB) literature. One of Krugman's key predictions was that increasing the money supply at the ZLB was irrelevant as long as expectations of future money supply were fixed. Rogoff's comment on this summarizes well a commonly held view at the time:

> "No one should seriously believe that the BOJ would face any significant technical problems in inflating if it puts it mind to the matter, liquidity trap or not. For example, one can feel quite confident that if the BOJ were to issue a 25 percent increase in the current supply and use it to buy back 4 percent of government nominal debt, inflationary expectations would rise."

This basic logic was later spelled out more explicitly in a general equilibrium model by Auerbach and Obstfeld (2005) in an article in the American Economic Review, a leading journal of the economics profession, titled "The case for open-market purchases in a liquidity trap." Their argument was that purchasing government debt with money should plausibly lead people to expect a permanent increase in the money supply, in contrast to Krugman's assumption and much as suggested by Rogoff, due to the fact that a permanent increase in the money supply creates seignorage revenues, which reduces tax distortions. In this case increasing the money supply *should* increase prices and output because people should have no reason to expect the money supply to be contracted to its original level once things normalize and the short-term interest rate is positive, as this would imply higher tax distortions.

Since Rogoff's prediction, the Bank of Japan has not only increased the monetary base by 25 percent, but rather it has increased it by about 550 percent. Furthermore, it has accumulated more than 30 percent of outstanding government debt, as well as several types of real assets, such as stocks, foreign exchange and mortgage backed securities. A similar story can be told about many other central banks since 2008. Meanwhile, in Japan, government debt as a fraction of GDP, at 80 percent in 1998, has almost tripled.

The point here is not to single out Kenneth Rogoff for a prediction that in retrospect seems off base as an empirical matter. Instead, it is to illustrate a broad consensus in the profession at the time, a consensus of which the quote from Rogoff is a particularly cogent summary. So as to not seem to be be unfairly singling out any particular author, below we will provide examples in which the former author of this article made statements that had a similar tenor as Rogoff's prediction.

Our suspicion is that the broad consensus at the time had its roots in the classic account of the Great Depression by Friedman and Schwartz (1963), in which the deflation from 1929-1933 was explained by a collapse in the money supply. The Great Inflation of the 1970's also appeared to support Friedman's famous dictum that "Inflation is always and everywhere a monetary phenomenom." It was natural, then, to assume the same applied to Japan, and that all that was needed was increasing the money supply to halt the deflation.

Another indication of the consesus of the time was Lars Svensson's (2001) well known proposal for a "foolproof way" out of a liquidity trap, which, in contrast to Rogoff's proposal, involved printing money to buy up foreign exchange, rather than government debt. The fact that this solution was claimed to be "foolproof" also indicated to some extent the general sense among academic economists at the time, especially in the US, that expansionary monetary policy at the ZLB was only a question of will, rather than posing any technical difficulties for the world's central banks.

From the perspective of the former author of this paper, however, the most pertinent statement about the academic consensus at the turn of the century came up in a conversation with Ben Bernanke, then Chairman, not of the Federal Reserve, but the Princeton economics department, and editor of the American Economic Review. When proposing "the liquidity trap" as a Ph.D. dissertation topic, Bernanke issued the following warning: "I have to warn you. I do not believe in the liquidity trap." While the current understanding of the word "the liquidity trap" is that it reflects some bound on the short-term nominal interest rate (often referred to as zero, albeit recent experience suggests it may be somewhat negative), Bernanke was instead referring more broadly to the fact that he believed in the power of the central banks to do something to stimulate demand, in the tradition of Friedman and Schwartz, zero bound or not. This position seemed to have been very much in line with the thinking of Rogoff, Svensson and Auerbach and Obstfeld, already cited.

Below is a quote from Bernanke (2000), a speech given at the American Economics Association, which later would become very widely known as he assumed the Chairmanship of the Federal Reserve. Some interpreted the speech as a roadmap for the Fed's subsequent policy actions:

> "First, that— despite the apparent liquidity trap— monetary policymakers retain the power to increase nominal aggregate demand and the price level. [..] In my view one can make what amounts to an arbitrage argument —the most convincing type of argument in an economic context—that it must be true. The monetary authorities can issue as much money as they like. Hence, if the price level were truly independent of money issuance, then the monetary authorities could use the money they create to acquire indefinite quantities of goods and assets. This is manifestly impossible in equilibrium. Therefore money issuance *must* ultimately raise the price level, even if nominal interest rates are bounded at zero. This is an *elementary* argument." (italics ours)

In this paper we revisit this elementary argument on the basis of one particular interpretation of Ben Bernanke's logic. We use it to try to illuminate why the pre-2008 consensus about the power of monetary policy may have been a bit too optimistic about the ability of central banks to stimulate demand.[1] In making this case we do not wish to claim that the cental bank – or the government as a whole – is unable to stimulate demand at all. Instead, the point is that doing so may require considerably larger intervention than suggested by the pre-crisis consensus, for example interventions of the size and scope

[1]Paul Krugman has often quipped that he should take Svensson and Bernanke to Japan with him on an apology tour for having made it seem too easy at the time. See e.g. Krugman (2014).

of the radical regime change implemented by Franklin Delano Roosevelt in 1933. This radical regime change is discussed in detail in Eggertsson (2008) and it involved an explicit commitment to inflate the price level by about 30 percent to pre-depression level, the abolishment of the gold standard, and a massive increase in government spending and budget deficits. As an indication of how radical it was at the time, the then director of the budget, Arthur Lewis, declared "this is the end of Western Civilization" and resigned from his post.[2]

To frame the approach of the paper let us first ask a basic question: What is an arbitrage opportunity? An arbitrage opportunity refers to a situation in which an agent can acquire profit without taking on any risk. In the context of Bernanke's argument, he is suggesting that the liquidity trap can be eliminated as a logical possibility because its existence would imply that the government could generate infinite profits. For the argument to make sense – for example in the context of a closed economy – one must have in mind an environment in which the government would care about profits and losses in the first place. At first blush this does not seem obvious, as these profits would necessarily be at the expense of the country's citizens, whose welfare the government should care about in the first place. Nevertheless, we believe that the proposition that the government cares about profits and losses is entirely reasonable, because the government needs to rely on costly and possibly distortionary taxation to pay for its expenditures. Hence if there was truly an arbitrage opportunity for the government, any rational government would wish to take it in order to eliminate taxation costs/distortions altogether (not to mention if it could do so at the cost of foreigners via buying up foreign assets).

Indeed, framing the question in this way makes clear the tight connection between Bernanke's no-arbitrage argument and Auerbach and Obstfeld (2005). As we already noted, their case for open market operations was made on the basis that open market operations in a liquidity trap *should* imply a permanent increase in the money supply that will last even once the zero bound stops being binding. This was the most reasonable benchmark to them, for contracting the money supply back to its initial level would imply fiscal costs. Hence a permanent increase in the money supply made sense from the perspective of both macroeconomic stabilization ex ante and fiscal solvency ex post. They made their point explicit by numerically computing a comparative statics that showed the beneficial effect of permanently increasing the money supply (which they coined "open market operations"). This argument was made slightly differently in an earlier IMF working paper by one of the authors, Eggertsson (2003), which remains unpublished. That paper explicitly cites Bernanke's no-arbitrage argument as a motivation, using the same quotation as above. In Eggertsson (2003), Bernanke's argument is modeled as a violation of Ricardian Equivalence. Eggertssson (2003) assumes the government cannot collect lump-sum taxes but instead that the government needs to pay tax collection costs as in Barro (1979). In this case the government cares about profits and losses on its balance sheet, as it needs to make up for the losses by costly taxation. By analyzing a Markov Perfect Equilibrium policy game, which presumes that the government cannot make any credible commitment about future policy apart from paying back the nominal value of debt as in Lucas and Stokey (1983), Eggertsson (2003) formally shows that purchasing "real assets" by printing money (or equivalently bonds, since money and bonds are perfect substitutes at the ZLB) implies a *credible* permanent increase in the money supply in the long run due to the fact that the government has no incentive to revert the supply completely back it original level on account of its fiscal consequences (leading

[2]See references in Eggertsson (2008).

to costly taxation). This, in turn, provides direct theoretical foundation for Bernanke's no-arbitrage argument to "eliminate" the liquidity trap.

The interpretation suggested in Eggertsson (2003) is that open market operations in real assets provides a straightforward commitment mechanism to lower future interest rates and higher inflation that mitigates the problem of the ZLB.[3] Indeed, the simulations reported in the paper suggest that open market purchases in real asset seem to allow the government to replicate quite closely the ideal state of affairs in which the government can fully commit to future policy and the problem of the ZLB is trivial in terms of its effect on output and inflation (as in Eggertsson and Woodford (2003) that analyze the full commitment equilibrium in a standard New Keynesian model). In retrospect, however, this interpretation was perhaps a little premature. A careful examination of the numerical results illustrates a disturbing feature. The required intervention in real assets needed to generate this outcome in Eggertsson (2003) corresponds to about 4 times annual GDP. Moreover, the intervention is conducted under the ideal circumstances under which the assets bought are in unlimited supply, their relative returns are not affected by the intervention (but instead equal to the market interest rate in equilibrium), and assuming that the world is deterministic so there are no risks associated with using real asset purchases as a commitment device.

More generally, however, if the government buys real assets corresponding to something like 400 percent of GDP it seems exceedingly likely that all of these assumptions will be violated in one way or the other. First, an operation of this kind is likely to have a substantial distortionary effect on pricing – which is not modeled. Second, it is likely that the government may run into physical constraints such as running out of assets to buy. Third, as the scale of the operations increases and uncertainty is taken into account, the risk to the government's balance sheet may be deemed unacceptable, thus lessening the power of this commitment device. Finally, with an intervention of this scale it is very likely that the central bank will hit some political constraints, either due to public concerns, or concerns from trading partners in the case the assets in question are foreign. Indeed, all the considerations mentioned above have proved to be relevant constraints for banks conducting large asset purchases since 2008. Central banks have faced challenges in finding liquid enough markets to conduct the operation, they have faced strong political backlash for the scale of the operations (e.g. because they are viewed as favoring the financial sector and the richest few), and in some cases both the government and the central banks in question have become exceedingly concerned over balance sheet risks of the central bank. These risks could put central bank independence in question as they could imply that the treasury infuses capital into the central bank to prevent unacceptable high levels of inflation, with the associated budgetary implications.[4]

In this paper we revisit Bernanke's no-arbitrage argument in the prototypical New Keynesian DSGE model, in the tradition of Woodford (2003), using conventional calibra-

[3] In this respect the intervention in Eggertsson (2003) is different from Auerbach and Obstfeld (2005) in that it increases total government liabilities (money + bonds), and thus the overall inflation incentive of the government. Since money and bonds are perfect substitutes at the zero bound, it is not obvious that open market operations themseselves have any effect on future government objectives.

[4] There are several recent papers that try to evaluate the extent to which these risk have become material for current central banks post crisis, such as Hall and Reis (2015) and Del Negro and Sims (2015). Our overall reading of this literature is that these risks are not pertinent for a balance sheet of the size of the Federal Reserve today, however, it seems relatively obvious that they would become relevant in some of the numerical examples we provide later in the paper given how extreme some of the numbers in question are.

tion parameters. This is in contrast to the unpublished work by Eggertsson (2003), who uses a simpler non-conventional modeling approach, which may raise scepticism of the numerical experiments conducted. Inside this model, we ask how large of an intervention the government needs to undertake in real assets to achieve the optimal allocation under discretion, assuming there is no cost of such interventions. As in Eggertsson (2003), we find that the numbers are very large. In fact, in our baseline simulation the corresponding intervention is more than 10 times GDP. This suggests that using the balance sheet of the government as a commitment device may imply asset positions by the central bank that would be difficult to implement in practice. Thus, while we find that Bernanke's no-arbitrage argument can be correct in theory, it may run into constraints in practice. For this reason, following our baseline experiment, which is done in the ideal circumstances in which the asset is in unlimited supply and at no cost for the government, we also consider cases in which the assets purchases are costly. In this case the purchases can lose much of their commitment power.

How does this all relate to recent experience? Consider that on October 31, 2014, the BOJ unexpectedly announced an expansion of its Comprehensive Monetary Easing (CME) program from 50 trillion to 80 trillion yen per year. Along with a change in the size of its balance sheet, the annoucement also included a change in its composition. Beyond long-term government securities, the central bank would purchase additional riskier assets such as exchange traded funds and real estate investment trusts. The expressed goal of the expansion was to meet a 2 percent inflation target within two years. The BoJ Governor, Haruhiko Kuroda, described the program as "monetary easing in an entirely new dimension" and in reference to limits in its size relative to GDP said, "We don't have any particular ceiling." In fact, as of August 2015, the size of the BOJ balance sheet stood at approximately 80 percent of GDP. While this seems like a large number, it is much smaller than what is needed according to our calibrated model. Would the Bank of Japan not hit *some* ceiling if it had to buy assets that are more than 10 times the current size of its balance sheet? In any event, as of this writing, the BoJ is still unable to hit its inflation target, and most projections paint a pessimistic picture of its prospect of hitting it anytime soon.

As another example, the Swiss National Bank bought foreign currency on the order of 90 percent of GDP in order to fight deflation during the crisis, leading to an 800 percent increase in its money supply. They eventually abandoned this policy since the magnitudes involved had become so high that the central bank faced strong political pressures to do so. The effect of this policy on the price level was negligible at best, although for a while the Swiss National Bank did manage to prevent an appreciation of the Swiss franc relative to the Euro.

The bottomline, then, may be that the irrevance result of Wallace (1981) that was later extended by Eggertsson and Woodford (2003) to a model with sticky prices and an explicit zero lower bound, may in the end be stronger than the pre-crisis consensus suggested. The Eggertsson and Woodford (2003) irrelevance result, in turn, is closely related to Krugman's (1998) result that increasing money supply has no effect at the ZLB if people expect it to be contracted again to its original level once interest rates turn positive.[5] Those irrelevance results suggested that absent some restrictions in asset

[5]The difference between Eggertsson and Woodford (2003) and Krugman (1998) is that while Krugman (1998) assumed that the central bank followed a monetary targeting rule, Eggertsson and Woodford (2003) assumed a more conventional Taylor type interest rate reaction function. Moreover, while Krugman (1998) assumes that the money supply is increased via purchases of short-term nominal bonds, Eggertsson and

trade that prevent arbitrage, equilibrium quantities and assets prices are not affected by a change in the relative supplies of various assets owned by the private sector *if the policy rule of the central bank is taken as given.* One way the irrelevance results have been broken in the literature is via changes about expectations about future monetary policy. The results here suggest that at least in a simple calibrated New Keyensian model that imposes a Markov Perfect Equilibrium as an equilibrium selection device, the asset position of the government needed to achieve the desired commitment, and thus break these irrelevance results may be extremely high. To be clear – and this seems worth re-iterating – we do not contend that this implies that nothing can be done at ZLB, nor even that nothing more could have been done in response to the current crisis, as the experience of FDR's radical reflation program, which coordinated monetary, fiscal, industrial, and exchange rate policy, illustrated during the Great Depression. However it does imply that actions by central banks to increase demand may be a bit harder than the pre-crisis consensus suggested and the foolproof ways out of the liquidity trap are hard to come by. One policy, that we do not consider here, but is found in Bhattarai et al. (2015) is the option to shorten the maturity structure of outstanding government debt. Their findings suggest that a policy of that kind may have more potency than the purchases of real assets studied here. Alternatively, if there is a "freeze" in secondary asset markets, e.g. due to a drop in the liquidity of assets, there may also be an important role for asset purchases as shown by Del Negro et al. (2016) in the context of the crisis of 2008. Our model abstracts from different degrees of asset liquidity, and hence this mechanism does not play a role here.

We outline the model in Section 2 and summarize the conditions for a Markov Perfect Equilibrium (MPE) for a coordinated government in Section 3. We present and discuss the calibrated model in Section 4. With costly taxation and coordinated monetary and fiscal policy, deficit spending and real asset purchases both serve as an additional commitment device for solving the credibility problem created by a liquidity trap. They are effective because they act as an additional device through which a discretionary government can commit future governments to a higher money supply, and thus higher inflation and lower real interest rates.

2. The Model

We start by outlining a standard general equilibrium sticky-price closed economy model with output cost of taxation, along the lines of Eggertsson (2006). We assume that monetary and fiscal policy are coordinated to maximize social welfare under discretion. The difference in the model from the literature is the introduction of a real asset in the government budget constraint.

2.1. Private sector

A representative household maximizes expected discounted utility over the infinite horizon:

Woodford (2003) assume that money supply can be increased via purchases of any type of security that is priced in the economy as in Wallace (1981).

$$E_t \sum_{t=0}^{\infty} \beta^t \left[u(C_t) + g(G_t) - v(h_t)\right] \xi_t \tag{1}$$

where β is the discount factor, C_t is a Dixit-Stiglitz aggregate of consumption of each of a continuum of differentiated goods

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\epsilon}{\epsilon-1}} di\right]^{\frac{\epsilon-1}{\epsilon}}$$

with elasticity of substitution equal to $\epsilon > 1$, G_t is a Dixit-Stiglitz aggregate of government consumption defined analagously, h_t is labor supplied, ξ_t is an exogenous shock, and P_t is the Dixit-Stiglitz price index,

$$P_t = \left[\int_0^1 p_t(i)^{(1-\epsilon)} di\right]^{\frac{1}{1-\epsilon}}$$

where $p_t(i)$ is the price of variety i. E_t denotes the mathematical expectation conditional on information available in period t, $u(.)$ is concave and strictly increasing in C_t, $g(.)$ is concave and strictly increasing in G_t, and $v(.)$ is increasing and convex in h_t.[6]

The household is subject to the following sequence of flow budget constraints

$$P_t C_t + B_t + E_t \left\{Q_{t,t+1} D_{t+1}\right\} \leq n_t h_t + (1 + i_{t-1}) B_{t-1} + D_t - P_t T_t + \int_0^1 Z_t(i) di \tag{2}$$

where B_t is a one period risk-free nominal government bond with nominal interest rate i_t, n_t is nominal wage, $Z_t(i)$ is nominal profit of firm i, T_t is government taxes, D_{t+1} is the value of the complete set of state-contingent securities at the beginning of period $t + 1$, and $Q_{t,t+1}$ is the stochastic discount factor.

On the firm side there is a continuum of monopolistically competitive firms indexed by the variety, i, which they produce. Each firm has a production function which is linear in labor $y_t(i) = h_t(i)$ and, as in Rotemberg (1982), faces a cost of changing prices given by $d\left(\frac{p_t(i)}{p_{t-1}(i)}\right)$.[7] The demand function for variety i is given by

$$\frac{y_t(i)}{Y_t} = \left(\frac{p_t(i)}{P_t}\right)^{-\epsilon} \tag{3}$$

where Y_t is total demand for goods. The firm maximizes expected discounted profits

$$E_t \sum_{t=0}^{\infty} Q_{t,t+s} Z_{t+s}(i) \tag{4}$$

where the period profits are given by

$$Z_t(i) = \left[(1+s) Y_t p_t(i)^{1-\epsilon} P_t^{\epsilon} - n_t(i) Y_t p_t(i)^{-\epsilon} P_t^{\epsilon} - d\left(\frac{p_t(i)}{p_{t-1}(i)}\right) P_t\right].$$

[6] We abstract from money by considering the cashless limit of Woodford (1998).

[7] Our results are not sensitive to assuming instead the Calvo model of price setting so long as we do not assume large resource costs of price changes. See Eggertsson and Singh (2015) for a discussion.

We assume that the production subsidy, s, satisfies

$$\frac{\epsilon - 1}{\epsilon}(1+s) = 1$$

in order to eliminate steady-state production inefficiencies from monopolistic competition. The household's optimality conditions are given by

$$\frac{v_h(h_t)}{u_c(C_t)} = \frac{n_t}{P_t} \tag{5}$$

$$\frac{1}{1+i_t} = E_t\left[\beta \frac{u_c(C_{t+1})\xi_{t+1}}{u_c(C_t)\xi_t}\Pi_{t+1}^{-1}\right] \tag{6}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is gross inflation. The firm's optimality condition from price-setting is given by

$$\epsilon Y_t\left[\frac{\epsilon-1}{\epsilon}(1+s)u_c(C_t,\xi_t) - v_y(Y_t,\xi_t)\right] + u_c(C_t,\xi_t)d'(\Pi_t)\Pi_t = E_t\left[\beta u_c(C_{t+1},\xi_{t+1})d'(\Pi_{t+1})\Pi_{t+1}\right] \tag{7}$$

where we have replaced v_h with v_y since we focus on a symmetric equilibrium where all firms charge the same price and produce the same amount.

2.2. Government

We assume that there is an output cost of taxation $s(T_t)$ as in Barro (1979).[8] Real government spending is then given by

$$F_t = G_t + s(T_t).$$

The government can issue one-period nominal bonds B_t and purchase a real asset A_t with rate of return q_t, which we assume satisfies the Fisher no-arbitrage condition in equilibrium. Furthermore, we assume that the government does not internalize the rate of return when optimizing social welfare. That is, the government takes the rate of return on the asset as given when making its policy decision. The consolidated flow budget constraint can be written as

$$B_t + P_tA_t = (1+i_{t-1})B_{t-1} + (1+q_{t-1})P_tA_{t-1} + \psi(A_t) + P_t(F_t - T_t)$$

where $\psi(A_t)$ is a quadratic cost of asset management. We introduce this quadratic cost as a reduced form way to capture two phenomena. First, it captures that managing large amounts of assets will involve some adminstration cost. Second, it is a way to model the relationship that as the scale of the asset purchases increases the real returns of the asset decreases, as this function reflects a loss of real resources. As we noted in the introduction, a key conclusion from the numerical experiment we report shortly is that the intervention done by the central bank is "unreasonably" large. One interesting thought experiment we will consider below is to set this cost high enough so as to rationalize the scale of the balance sheet expansion in some central banks observed post crisis. We can then ask if in this case the intervention has a substantial effect.

[8] The function s(T) is assumed to be twice differentiable with derivates $s'(T) > 0$ and $s''(T) > 0$.

Next, we define the real value of government debt inclusive of interest payments to be paid next period as $b_t = (1+i_t)\frac{B_t}{P_t}$ and the value of the real asset inclusive of returns as $a_t = (1+q_t)A_t$, so that we can then write the budget constraint in real terms as

$$\frac{b_t}{1+i_t} + \frac{a_t}{1+q_t} = b_{t-1}\Pi_t^{-1} + a_{t-1} + \psi\left(\frac{a_t}{1+q_t}\right) + (F_t - T_t) \tag{8}$$

We define fiscal policy as the choice of T_t, F_t, b_t, and a_t. For simplicity, we will abstract from variations in real government spending, so $F_t = F$ in all that follows. Conventional monetary policy is the choice of the nominal interest rate, i_t, which is subject to the zero bound constraint

$$i_t \geq 0. \tag{9}$$

2.3. Private sector equilibrium

The goods market clearing condition implies the overall resource constraint

$$Y_t = C_t + F_t + d(\Pi_t) + \psi\left(\frac{a_t}{1+q_t}\right). \tag{10}$$

We define the private sector equilibrium as a collection of stochastic process $\{Y_{t+s}, C_{t+s}, b_{t+s}, a_{t+s}, \Pi_{t+s}, i_{t+s}, T_{t+s}\}$ for $s \geq 0$ that satisfy equations (5)-(10) for each $s \geq 0$, given a_{t-1}, b_{t-1}, and an exogenous stochastic process for $\{\xi_{t+s}\}$. Policy must now be specified to determine the set of possible equilibria in the model.

3. Markov-perfect Equilibrium

We assume that the government policy is implemented under discretion so that the government cannot commit to future policy. To do so, we solve for a Markov-perfect equilibrium.[9] However, we also assume the government is able to commit to paying back the nominal value of its debt as in Lucas and Stokey (1983). The only way the government can influence future governments, then, is through the endogenous state variables that enter the private sector equilibrium conditions.

Define the expectation variables f_t^E and g_t^E. The necessary and sufficient condition for a private sector equilibrium are now that the variables $\{Y_t, C_t, b_t, a_t, \Pi_t, i_t, T_t\}$ satisfy: (a) the following conditions

$$\frac{b_t}{1+i_t} + \frac{a_t}{1+q_t} = b_{t-1}\Pi_t^{-1} + a_{t-1} + \psi\left(\frac{a_t}{1+q_t}\right) + (F - T_t) \tag{11}$$

$$1 + i_t = \frac{u_C(C_t, \xi_t)}{\beta f_t^E}, i_t \geq 0 \tag{12}$$

$$\beta g_t^E = \epsilon Y_t \left[\frac{\epsilon - 1}{\epsilon}(1+s)u_c(C_t, \xi_t) - v_y(Y_t, \xi_t)\right] + u_c(C_t, \xi_t)d'(\Pi_t)\Pi_t \tag{13}$$

$$Y_t = C_t + F_t + d(\Pi_t) + \psi\left(\frac{a_t}{1+q_t}\right) \tag{14}$$

[9]See Maskin and Tirole (2001) for a formal definition of the Markov-perfect Equilibrium.

given b_{t-1}, a_{t-1} and f_t^E and g_t^E; (b) expectations are rational so that

$$f_t^E = E_t \left[u_c(C_{t+1}, \xi_{t+1}) \Pi_{t+1}^{-1} \right] \tag{15}$$

$$g_t^E = E_t \left[u_c(C_{t+1}, \xi_{t+1}) d'(\Pi_{t+1}) \Pi_{t+1} \right] \tag{16}$$

Since the government cannot commit to future policy apart from its choice of the endogenous state variables a_{t-1} and b_{t-1}, the expectations f_t^E and $g_t^E$are only a function of a_t, b_t, and ξ_t. That is, the expectation functions are defined as

$$f_t^E = \bar{f}^E(a_t, b_t, \xi_t) \tag{17}$$

$$g_t^E = \bar{g}^E(a_t, b_t, \xi_t) \tag{18}$$

and we assume that these functions are continous and differentiable. The discretionary government's dynamic programming problem is

$$V(a_{t-1}, b_{t-1}, \xi_t) = \max \left[U(.) + \beta E_t V(a_t, b_t, \xi_{t+1}) \right] \tag{19}$$

subject to the private sector equilibrium conditions (11)-(14) and the expectation functions (17)-(18), which in equilibrium satisfy the rational expectations restrictions (15)-(16). The period Lagrangian and first-order conditions for this maximization problem are outlined in the appendix, along with their linear approximations.[10] A Markov Perfect Equilbrium can now be defined as a private sector equilbrium that is a solution to the government problem defined by (19).

4. Results

Following Eggertsson (2006) we model a benchmark deflation scenario as a credibility problem. In particular, we assume that the following three conditions are satisfied: the government's only policy instrument is the short-term nominal interest rate; the economy is subject to a large negative demand shock given by the preference shock ξ_t; and the government cannot commit to future policy. We calibrate this benchmark with parameter values from Eggertsson and Singh (2015) that match a 10 percent drop in output and 2 percent drop in inflation.[11]

4.1. Optimal monetary policy under commitment

As shown in Eggertsson and Woodford (2003), to increase inflation expectations in a liquidity trap, the central bank commits to keeping the nominal interest rate at zero after the natural rate of interest becomes positive again. The consequence of the anticipation of this policy is that the benchmark deflation and large output gap scenario are largely avoided. For the particular calibration that we work with here, deflation and the output gap in the first period of the trap are -0.65% and -5.42%, respectively. Figure 3 makes this comparison clear.

With the benchmark deflation scenario and optimal monetary commitment in hand, we are now set to conduct numerical experiments to measure how discretionary fiscal policy with real asset purchases and/or deficit spending compare to the worst and best case scenarios, i.e., limited discretion and full commitment.

[10] Note that we assume that the government and private sector move simulataneously.

[11] They parameterise the model using Bayesian methods as in Denes and Eggertsson (2009) and Denes et al. (2013).

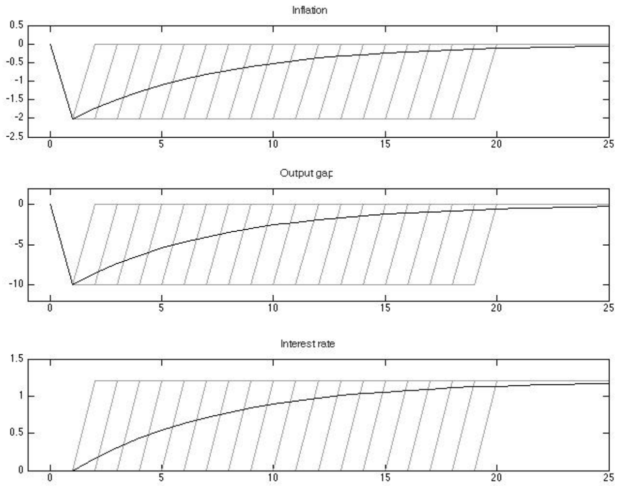

Figure 1: Inflation, the output gap, and the short-term nominal interest rate under discretion when the government's only policy instrument is open market operations

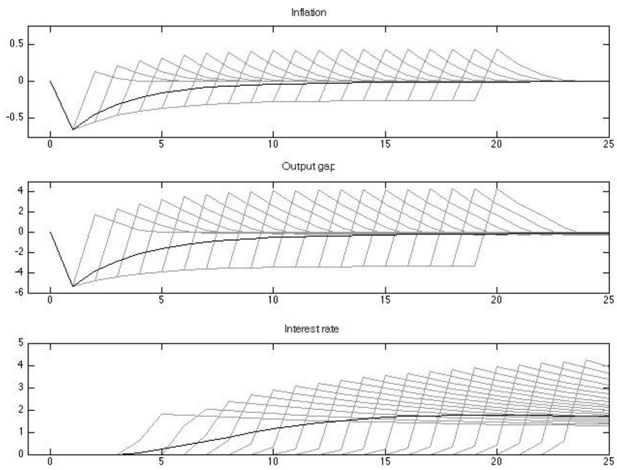

Figure 2: Inflation, the output gap, and the short-term nominal interest rate under commitment when the government can only use conventional monetary policy

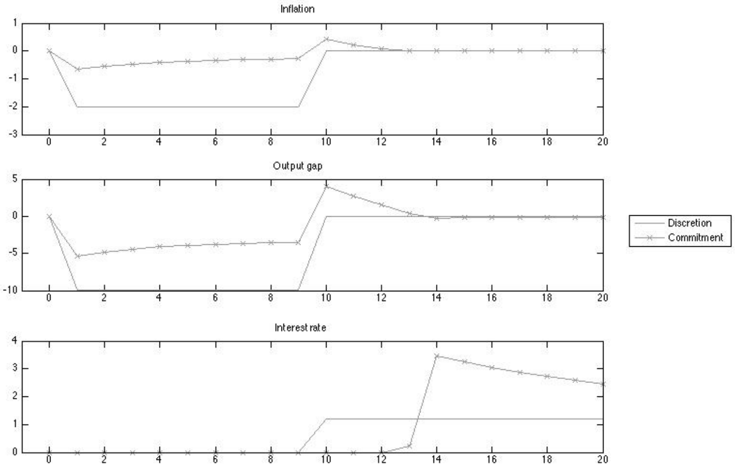

Figure 3: Inflation, the output gap, and the short-term nominal interest rate under the benchmark and commitment when the aggregate demand shock lasts for 10 periods

4.2. Deficit spending as an additional policy instrument

In order to discuss optimal discretion under fiscal policy, we must first calibrate the cost of taxation. We do so by choosing the second derivative of the cost function, s_1, so that 5 percent of government spending goes to tax collection costs. With deficit spending as an additional policy instrument, the government can commit to future inflation and a low nominal interest rate by cutting taxes and issuing nominal debt. Nominal debt commits the government to inflation even if it is discretionary because it creates an incentive for the government to reduce the real value of its debt and future interest payments. Since both inflation and taxes are costly, the government will choose a combination of the two in order to achieve this goal. Figures 4 and 5 summarize this result of Eggertsson (2006) for our parameterization.

The intuition is straightfoward. Even with the inability to commit, the government can stimulate aggregate demand in a liquidity trap by increasing inflation expectations. In order to increase inflation expectations, the government can coordinate monetary and fiscal policies in order to run budget deficits. Budget deficits increase nominal debt, which in turn make a higher inflation target credible. Finally, increased inflation expectations lower the real rate of interest and thus stimulate aggregate demand.

Figure 6 makes the comparison between the benchmark scenario, optimal monetary commitment, and discretionary fiscal policy. In the first period following the shock, the inflation rate and the output gap are -0.93% and -6.79% under fiscal discretion, quite close to their levels under optimal monetary commitment. Lastly, Figure 7 shows taxes and the evolution of debt to output when the shocks lasts for 10 periods. Taxes deviate by 60% from steady state, while debt peaks at approximately 35% of output.

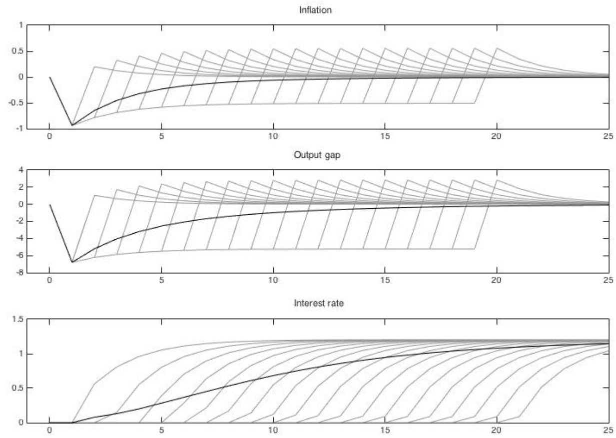

Figure 4: Inflation, the output gap, and the short-term nominal interest rate under discretion when the government can use both monetary and fiscal policies

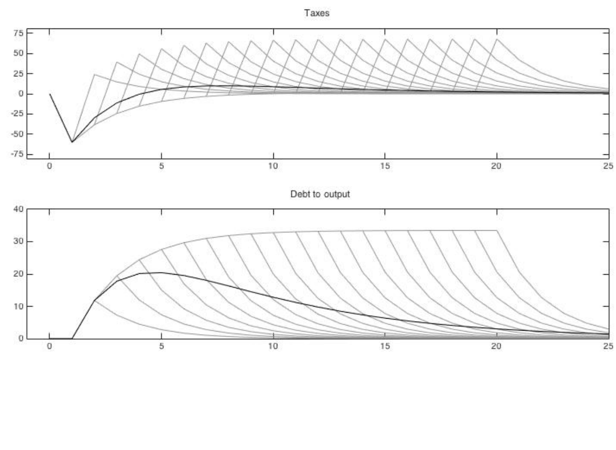

Figure 5: Taxes and debt under discretion when the government can use both monetary and fiscal policies

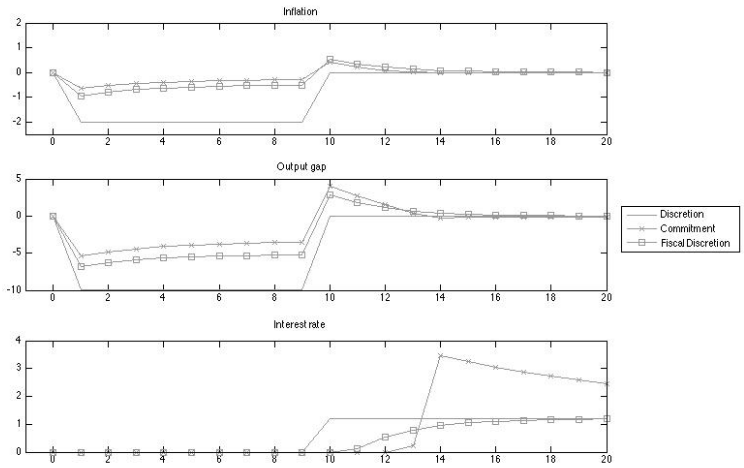

Figure 6: Inflation, the output gap, and the short-term nominal interest rate under the benchmark discretion, monetary commitment, and fiscal discretion when the aggregate demand shock lasts for 10 periods

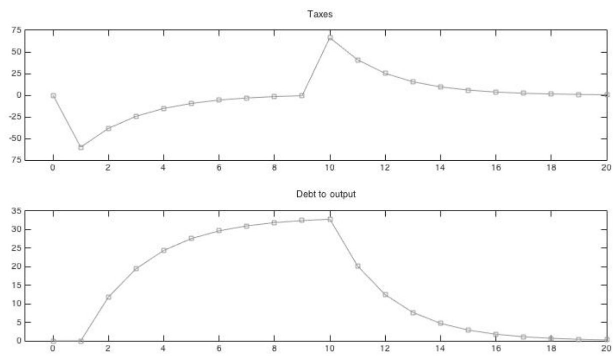

Figure 7: Taxes and debt to output under fiscal discretion when the aggregate demand shock lasts for 10 periods

4.3. Real asset purchases and deficit spending

We now turn to how the optimal policy under discretion changes when real asset purchases are used as an additional policy instrument. Figures 8 and 9 show that when asset management is costless and the output cost of taxation is calibrated to 5% of government spending, the optimal amount of real asset purchases exceeds 2000% of gross domestic product in all contingencies. Although there is a strong inflation incentive and corresponding output boom due to the large increase in nominal debt, it seems clear that the required amount of asset purchases to obtain this response would be infeasible in practice.[12]

Therefore, perhaps a more interesting question is, what does the model predict for inflation and the output gap if we calibrate the asset management cost so that the optimal amount of real asset purchases is 80% of gross domestic product in the first period of the recession? We pick this number as a reference point, as it corresponds approximately to the scale of the Swiss Central Bank in foreign exchange before it abandoned its peg. Figures 10 and 11 show that when we perform this thought experiment, the effectiveness of real asset purchases is much more limited. In fact, inflation and the output gap are only reduced to -1.36% and -8.23%, respectively, which is worse than the case when we have deficit spending only as the policy instrument. It also the case that as the cost of asset management gets very large, asset purchases approach zero, and we converge to the solution under fiscal discretion.[13]

There are two main takeaways from our results: first, although costless real asset purchases perform the best at reducing inflation and the output gap, the required balance-sheet size under this scenario is far too large to be feasible in practice; second, for realistic levels of asset purchases, a combination of deficit spending and asset purchases does not perform much better than the worst-case scenario in the numerical example above. These two points taken together suggest that a combination of fiscal stimulus and central bank balance sheet policies with more weight on the fiscal stimulus may be the most practical. We have abstracted from the ability of the government to increase real government spending in the example above, but the existing literature suggest this is another way in which the discretionary outcome can be improved.

Lastly, figures (12) and (13) make the comparison between all of the policy scenarios that we have considered more precise (i.e. benchmark discretion, commitment, fiscal discretion, costless real asset purchases, and real asset purchases calibrated to match 80% of gross domestic product). We have also confirmed, that as the cost of asset management gets sufficiently high, then the solution converges to the same as we have already analyzed in which case the government only uses deficit spending.

[12]Technically, there still is a negligibly small cost of asset management in this exercise, with $\psi = 1 \times 10^{-7}$. This is the smallest level of ψ which induces stationarity in the equilibrium dynamics. See Schmitt-Grohé and Uribe (2003) for an example of this in closing small open economy models.

[13]This numerical result indicates a non-linearity that is somewhat interesting, in that a discretionary government with intermediate costs of administrating the real assets is better off without the ability to intervene in real assets than with it, as it limits its ability to commit to future inflation. One possible way of getting around this issue, which we do not pursue here, is to impose the constraint that the government cannot have negative asset holdings in which case the government may still be able to commit to inflation in the intermediate asset management cost range. The key point, however, is that in this case commitment arises due to fiscal commitment as opposed to asset purchases.

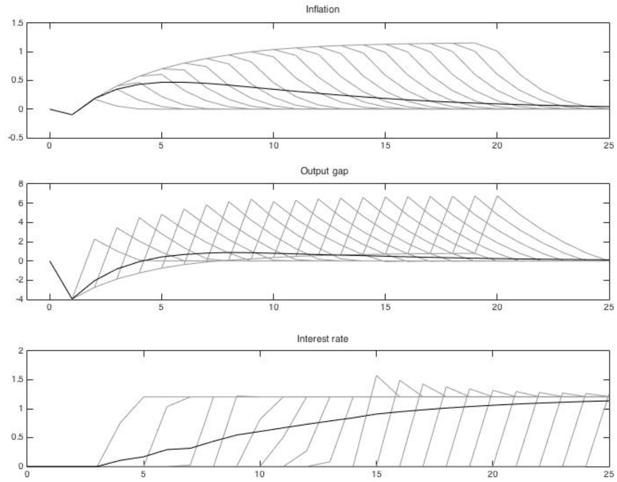

Figure 8: Inflation, the output gap, and the short-term nominal interest rate under discretion when the government can costlessly conduct deficit spending and open market operations in real assets

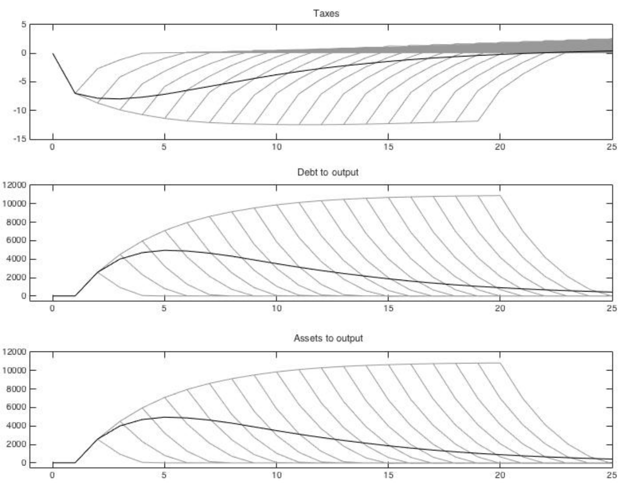

Figure 9: Taxes, debt to output, and asset purchases under discretion when the government can costlessly conduct deficit spending and open market operations in real assets

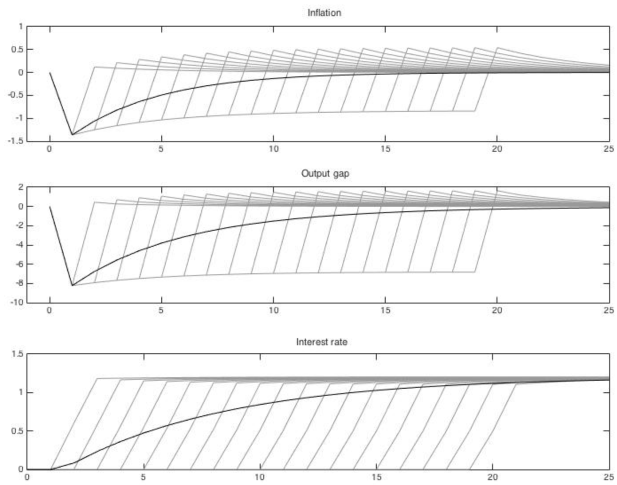

Figure 10: Inflation, the output gap, and the short-term nominal interest rate under discretion when the government can conduct deficit spending and open market operations in real assets, with the cost of asset purchases calibrated to match real asset purchases of 80% of GDP

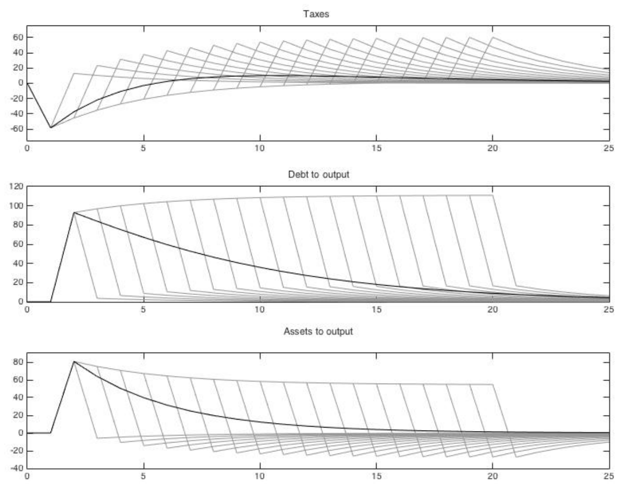

Figure 11: Taxes, debt to output, and asset purchases under discretion when the government can conduct deficit spending and open market operations in real assets, with the cost of asset purchases calibrated to match real asset purchases of 80% of GDP

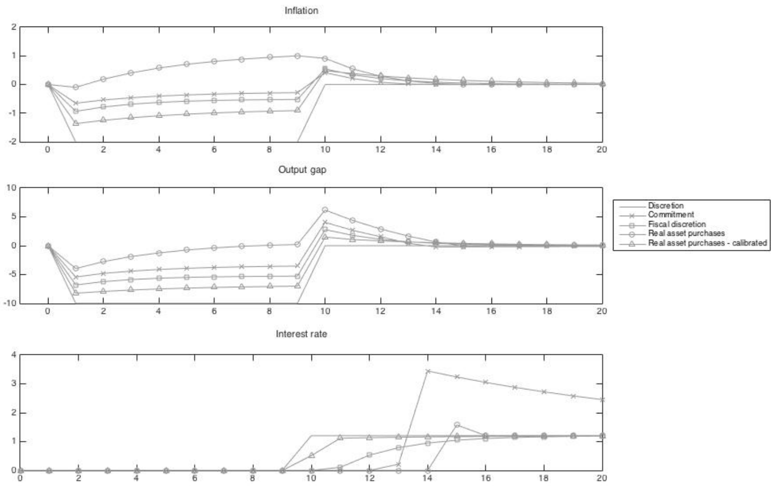

Figure 12: Inflation, the output gap, and the short-term nominal interest rate under the benchmark discretion, monetary commitment, fiscal discretion, and real asset purchases when the aggregate demand shock lasts for 10 periods

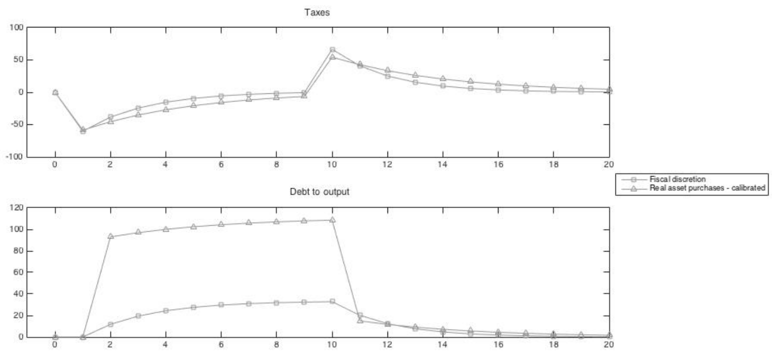

Figure 13: Taxes and debt to output under fiscal discretion and calibrated real asset purchases when the aggregate demand shock lasts for 10 periods

5. Sensitivity Analysis

Table 1: Varying the cost of taxation
as a percentage of government spending

	Fiscal discretion			Discretion with real assets		
Taxation cost (%)	π (%)	y (%)	b/gdp (%)	π (%)	y (%)	b/gdp (%)
0.25	-1.61	-9.00	51.80	-1.78	-9.41	953.00
0.50	-1.47	-8.58	39.44	-1.85	-9.46	2,414.63
1.00	-1.31	-8.10	28.76	-1.87	-9.41	5,790.70
2.50	-1.09	-7.37	17.75	-0.99	-6.21	13,074.97
5.00	-0.94	-6.79	11.81	-0.10	-3.95	2,562.44
7.50	-0.87	-6.52	9.33	-0.05	-3.86	1,390.96
10.00	-0.83	-6.35	7.82	-0.03	-3.85	979.50
15.00	-0.78	-6.13	6.02	-0.02	-3.84	609.61
20.00	-0.75	-6.01	4.93	-0.02	-3.83	461.96

The table above shows the sensitivity of our results to the size of taxation costs. The main takeaway is that for any reasonable value of the taxation cost, very large increases in real purchases are needed under full discretion, suggesting a limitation to this policy once more realistic constraints are added.

6. Conclusion

This paper takes Bernanke's no-arbitrage argument to its logical limit and finds that it implies implausibly large asset purchases in a Markov Perfect Equilbrium. One interpretation of this finding is that open market operations in real assets alone is not sufficient in a liquidity trap, so instead, fiscal policy may be used in one form or another to support a reflation at the zero bound. A key abstraction is that the monetary and fiscal policy objective here corresponds to the utility of the representative household. It may seem more reasonable that the central bank has objectives that are different from social welfare, for example that it cares greatly about its own balance sheet losses, independently of tax distortions. If one takes that perspective, however, there is no guarantee that real asset purchases provide the magic bullet to escape a liquidity trap, for reasons first articulated by Paul Samuleson in the context of the Great Depression. He argued that during the Great Depression the Fed was a "prisoner of its own independence" and paralyzed from taking any action for fear that they may imply balance sheet losses.[14]

An alternative explanation for the relative ineffectiveness of monetary policy post-2008 in guaranteeing inflation at or above target is that central banks never explicitly committed to an inflationary policy. While one reason central banks refrained from doing so was the high perceived cost of inflation, another was that many of them thought a reflationary program by a central bank would not be credible. The pre-crisis consensus was that this objection was not relevant as the central bank had the ability to print an unlimited amount of money and buy whatever assets they wanted. The numerical experiments here suggest governments may in practice face some constraints, due to the scale involved needed to generate that commitment.

[14] See Mayer, Thomas (1993) p.6.

We do not wish to interpret this as suggesting that monetary policy is impotent at the zero bound, however. Instead, our interpretation is that the central banks need to more explicitly inflate, and second, they may need some fiscal backing to achieve their objective, via, for example, direct government spending, fiscal transfers, and debt accumulation, together with or perhaps in addition to some additional institutional reforms which coordinate monetary and fiscal policy. Exploring how this coordination may take place in practice is likely to be a fertile ground for future research (see e.g. Turner (2015)).

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7. Appendix

7.1. Functional forms

We make the following functional form assumptions

$$u(C,\xi) = \xi \bar{C}^{\frac{1}{\sigma}} \frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$
$$v(h(i),\xi) = \xi \lambda \frac{h(i)^{1+\phi}}{1+\phi}$$
$$g(G,\xi) = \xi \bar{G}^{\frac{1}{\sigma}} \frac{G^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$
$$y(i) = h(i)$$
$$d(\Pi) = d(\Pi - 1)^2$$
$$s(T) = \frac{s_1}{2} T^2$$
$$\psi(a) = \frac{\psi_1}{2} a^2$$

Note that the discount factor shock, ξ, equals 1 in steady-state, and we scale hours such that Y = 1 in steady-state, too. This implies

$$\tilde{v}(Y,\xi) = \frac{1}{1+\phi} \lambda \xi Y^{\frac{1+\phi}{\kappa}}.$$

7.2. Calibration

Table 2: Model parameters

Parameter	Value
α	0.7871
β	0.9970
σ	$\frac{1}{1.29}$
ϵ	13.6012
ϕ	1.7415
d''	5776.7
κ	0.0072
$\bar{F}$	0.30
$\bar{T}$	0.30
$\bar{G}$	0.25
s_1	0.3333
ψ_1	0.0000362

Table 3: ZLB experiment

Parameter	Value
r_L^e	-0.0136
γ	0.1369

7.3. Non-linear markov-perfect equilibrium

Formulate the Lagrangian

$$
\begin{aligned}
L_t = {} & u(C_t, \xi_t) + g(F - s(T_t)) - \tilde{v}(Y_t) + \beta E_t V(a_t, b_t, \xi_{t+1}) \\
& + \phi_{1t}\left(\frac{b_t}{1+i_t} + \frac{a_t}{1+q_t} - b_{t-1}\pi_t^{-1} - a_{t-1} - \psi\left(\frac{a_t}{1+q_t}\right) - (F - T_t)\right) \\
& + \phi_{2t}\left(\beta f_t^E - \frac{u_c(C_t,\xi_t)}{1+i_t}\right) \\
& + \phi_{3t}\left(\beta g_t^E - \epsilon Y_t\left[\frac{\epsilon-1}{\epsilon}(1+s)u_c(C_t,\xi_t) - \tilde{v}_y(Y_t,\xi_t)\right] - u_c(C_t,\xi_t)d'(\pi_t)\pi_t\right) \\
& + \phi_{4t}\left(Y_t - C_t - F - \psi\left(\frac{a_t}{1+q_t}\right) - d(\pi_t)\right) \\
& + \eta_{1t}(f_t^E - \bar{f}^E(a_t, b_t, \xi_t)) \\
& + \eta_{2t}(g_t^E - \bar{g}^E(a_t, b_t, \xi_t)) \\
& + \gamma_{1t}(i_t - 0)
\end{aligned}
$$

First-order conditions

$$
\begin{aligned}
\pi_t &: \phi_{1t}[b_{t-1}\pi_t^{-2}] - \phi_{3t}[u_c d''\pi_t + u_c d'] - \phi_{4t}d' \\
Y_t &: -\tilde{v}_y - \phi_{3t}\left[\epsilon\left(\frac{\epsilon-1}{\epsilon}(1+s)u_c\right) - \epsilon Y_t\tilde{v}_{yy} - \epsilon\tilde{v}_y\right] - \phi_{4t} \\
i_t &: -\phi_{1t}[b_t(1+i_t)^{-2}] + \phi_{2t}[u_c(1+i_t)^{-2}] + \gamma_{1t} \\
C_t &: u_c - \phi_{2t}[u_{cc}(1+i_t)^{-1}] - \phi_{3t}\left[\epsilon Y_t\frac{\epsilon-1}{\epsilon}(1+s)u_{cc} + u_{cc}d'\pi_t\right] - \phi_{4t} \\
T_t &: g_G(-s'(T_t)) + \phi_{1t} \\
a_t &: \beta E_t V_a(a_t, b_t, \xi_{t+1}) + \phi_{1t}\left[(1+q_t)^{-1} - \psi'\left(\frac{a_t}{1+q_t}\right)\right] - \phi_{4t}\psi'\left(\frac{a_t}{1+q_t}\right) - \eta_{1t}\bar{f}_a^E - \eta_{2t}\bar{g}_a^E \\
b_t &: \beta E_t V_b(a_t, b_t, \xi_{t+1}) + \phi_{1t}[(1+i_t)^{-1}] - \eta_{1t}\bar{f}_b^E - \eta_{2t}\bar{g}_b^E \\
f_t^E &: \beta\phi_{2t} + \eta_{1t} \\
g_t^E &: \beta\phi_{3t} + \eta_{2t}
\end{aligned}
$$

Complementary slackness condition

$$\gamma_{1t} \le 0,\ i_t \ge 0,\ \gamma_{1t}i_t = 0$$

Benveniste-Scheinkman conditions

$$
\begin{aligned}
V_a(a_{t-1}, b_{t-1}, \xi_t) &= -\phi_{1t} \\
V_b(a_{t-1}, b_{t-1}, \xi_t) &= -\phi_{1t}\pi_t^{-1}
\end{aligned}
$$

7.4. Steady-state

We linearize around an inefficient steady-state with positive output cost of taxation so that

$$\bar{\phi}_1 = g_G(s'(\bar{T})).$$

Although we linearize around an inefficient steady-state, to simplify we still assume an appropriate production subsidy, as well as no resource loss from price adjustments, which requires

$$d(\Pi) = 0$$

so that

$$Y = C + F.$$

This requires that we linearize around a zero inflation steady-state

$$\Pi = 1$$

which implies

$$d'(\Pi) = 0.$$

Furthermore, we assume that $\bar{a} = \bar{b} = 0$ in steady-state, so that from the first order condition with respect to π_t

$$\bar{\phi}_3 = 0.$$

We assume that the production subsidy satisfies

$$\frac{\epsilon - 1}{\epsilon}(1 + s) = 1$$

so that

$$u_c = \tilde{v}_y.$$

Also, we linearize around a steady-state with positive interest rates so

$$1 + i = \frac{1}{\beta}$$

which implies

$$\bar{\gamma}_1 = 0$$

and from the first order condition for i_t

$$\bar{\phi}_2 = 0.$$

Using $d'(\Pi) = 0$ and the first order condition for π_t

$$\bar{\phi}_4 = 0$$

which implies from the first order conditions for Y_t and C_t

$$\bar{\phi}_5 = \tilde{v}_y = u_c.$$

The first order conditions with respect to the expectation variables imply

$$\bar{\eta}_1 = \bar{\eta}_2 = 0$$

so that we do not need to know the derivatives of the unknown functions.

7.5. Linear approximation

Private sector equilibrium conditions We approximate the equilibrium conditions around an inefficient non-stochastic steady-state with zero inflation, $1 + \mathrm{i} = 1 + \mathrm{q} = \beta^{-1}$, and $\bar{a} = \bar{b} = 0$. We also normalize steady-state output to $\bar{Y} = 1$.

Linearizing the resource constraint $Y_t = C_t + F + d(\Pi_t) + \psi\left(\frac{a_t}{1+q_t}\right)$ gives

$$\hat{Y}_t = \bar{C}\hat{C}_t \tag{20}$$

where $\hat{C}_t = \frac{C_t - \bar{C}}{\bar{C}}$.

Linearizing the price-setting optimality condition gives

$$\bar{u}_c d'' \pi_t + \epsilon \bar{u}_{cc} \bar{C} \hat{C}_t - \epsilon \bar{v}_{yy} \hat{Y}_t - \epsilon \bar{v}_{y\xi} \hat{\xi}_t + \epsilon \bar{u}_{c\xi} \hat{\xi}_t = \beta \bar{u}_c d'' E_t \pi_{t+1}$$

which can be simplified by making use of the linearized resource constraint

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{Y}_t \tag{21}$$

where $\kappa = \frac{\epsilon(\phi + \sigma^{-1})}{d''}$.

Linearizing the Euler equation gives

$$\bar{u}_{cc} \bar{C} \hat{C}_t + \bar{u}_{c\xi} \hat{\xi}_t = \bar{u}_c \hat{i}_t + \bar{u}_{cc} \bar{C} E_t \hat{C}_{t+1} + \bar{u}_{c\xi} E_t \hat{\xi}_{t+1} - \bar{u}_c E_t \pi_{t+1}$$

which can be simplified by making use of the linearized resource constraint

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \sigma(\hat{i}_t - E_t \pi_{t+1} - \hat{r}^e_t) \tag{22}$$

where $\hat{r}^e_t = -\frac{\bar{u}_{c\xi}}{\bar{u}_c}\left[E_t \hat{\xi}_{t+1} - \hat{\xi}_t\right]$ and $\sigma = -\frac{\bar{u}_c}{\bar{u}_{cc}} = \tilde{\sigma}\bar{C}$.

Imposing the Fisher arbitrage relation as an equilibrium condition and linearizing gives

$$\hat{q}_t = \hat{i}_t - E_t \pi_{t+1} \tag{23}$$

Linearizing the government budget constraint

$$b_t + a_t = \beta^{-1} b_{t-1} + \beta^{-1} a_{t-1} - \beta^{-1} \bar{T} \hat{T}_t \tag{24}$$

where $\hat{T}_t = \frac{T_t - \bar{T}}{\bar{T}}$.

Lastly, linearizing the expectation functions gives

$$\hat{f}^E_t = -\sigma^{-1} E_t \hat{Y}_{t+1} + E_t \hat{\xi}_{t+1} - E_t \pi_{t+1} \tag{25}$$

$$\hat{g}^E_t = d'' E_t \pi_{t+1} \tag{26}$$

Markov-perfect FOCs Note that in steady-state all Lagrange multipliers besides $\bar{\phi}_1 = g_G s'$ and $\bar{\phi}_3 = \tilde{v}_y = u_c$ are equal to zero. Linearizing each FOC in the order given above and using appropriate functional form assumptions:

$$\pi_t : g_G s' b_{t-1} - d''\hat{\phi}_{3t} - d''\pi_t$$
$$Y_t : \phi\hat{Y}_t - \epsilon\phi\hat{\phi}_{3t} - \hat{\phi}_{4t}$$
$$i_t : -\beta^2 g_G s' b_t + \beta^2\hat{\phi}_{2t} + \hat{\gamma}_{1t}$$
$$C_t : \hat{Y}_t - \sigma\hat{\xi}_t - \beta\hat{\phi}_{2t} - \epsilon\hat{\phi}_{3t} + \sigma\hat{\phi}_{4t}$$
$$T_t : -\left(\frac{s'\bar{T}\bar{C}}{\bar{G}\sigma} + \frac{s''\bar{T}}{s'}\right)\hat{T}_t + \hat{\phi}_{1t}$$
$$a_t : -E_t\hat{\phi}_{1t+1} + \hat{\phi}_{1t} - \hat{q}_t - \psi\beta\frac{g_G s' + 1}{g_G s'}a_t + \frac{\bar{f}_a^E}{g_G s'}\hat{\phi}_{2t} + \frac{\bar{g}_a^E}{g_G s'}\hat{\phi}_{3t}$$
$$b_t : -E_t\hat{\phi}_{1t+1} + \hat{\phi}_{1t} - \hat{i}_t + E_t\pi_{t+1} + \frac{\bar{f}_b^E}{g_G s'}\hat{\phi}_{2t} + \frac{\bar{g}_b^E}{g_G s'}\hat{\phi}_{3t}$$

Guess solutions for all variables at positive interest rates as a linear function of a_{t-1}, b_{t-1}, and $\hat{r}_t^e$. Expectations will take the form

$$\hat{f}_t^E = -\sigma^{-1}E_t\hat{Y}_{t+1} + E_t\hat{\xi}_{t+1} - E_t\pi_{t+1} = \bar{f}_a^E a_t + \bar{f}_b^E b_t + \bar{f}_r^E\hat{r}_t^e$$
$$\hat{g}_t^E = d''E_t\pi_{t+1} = \bar{g}_a^E a_t + \bar{g}_b^E b_t + \bar{g}_r^E\hat{r}_t^e$$

Under the assumptions about the shock process, $\hat{\xi}_t$, we have

$$E_t\hat{\xi}_{t+1} = (1-\gamma)\hat{\xi}_t$$

and

$$\hat{r}_t^e = \gamma\hat{\xi}_t$$

where γ is the probability of remaining at the ZLB. Note that when the ZLB no longer binds, $\hat{r}_t^e = 0$.

7.6. Optimal Policy Commitment

Formulate the Lagrangian

$$\begin{aligned}
L_t = E_t\sum_{t=0}^{\infty}\beta^t &\left\{u(C_t,\xi_t) + g(F - s(T_t)) - \tilde{v}(Y_t)\right\} \\
&+ \phi_{1t}\left(\beta u_c(C_{t+1},\xi_{t+1})\pi_{t+1}^{-1} - \frac{u_c(C_t,\xi_t)}{1+i_t}\right) \\
&+ \phi_{2t}\left(\beta u_c(C_{t+1},\xi_{t+1})d'(\pi_{t+1})\pi_{t+1} - \epsilon Y_t\left[\frac{\epsilon-1}{\epsilon}(1+s)u_c(C_t,\xi_t) - \tilde{v}_y(Y_t,\xi_t)\right] - u_c(C_t,\xi_t)d'(\pi_t)\pi_t\right) \\
&+ \phi_{3t}(Y_t - C_t - F - d(\pi_t)) \\
&+ \gamma_{1t}(i_t - 0)
\end{aligned}$$

First order conditions

$$\pi_t : -\phi_{1t-1}[u_c\pi_t^{-2}] - \phi_{2t}[u_c d''\pi_t + u_c d'] - \phi_{2t-1}[u_c d''\pi_t + u_c d'] - \phi_{3t}d'$$
$$Y_t : -\tilde{v}_y - \phi_{2t}\left[\epsilon\left(\frac{\epsilon-1}{\epsilon}(1+s)u_c\right) - \epsilon Y_t\tilde{v}_{yy} - \epsilon\tilde{v}_y\right] + \phi_{3t}$$
$$i_t : \phi_{1t}[u_c(1+i_t)^{-2}] + \gamma_{1t}$$
$$C_t : u_c - \phi_{1t}[u_{cc}(1+i_t)^{-1}] + \phi_{1t-1}[u_{cc}\pi_t] - \phi_{2t}\left[\epsilon Y_t\frac{\epsilon-1}{\epsilon}(1+s)u_{cc} + u_{cc}d'\pi_t\right] + \phi_{2t-1}[u_{cc}d'\pi_t] - \phi_{3t}$$

Complementary slackness condition

$$\gamma_{1t} \leq 0,\ i_t \geq 0,\ \gamma_{1t} i_t = 0$$

7.7. Linear approximation

Note that in steady-state all Lagrange multipliers besides $\bar{\phi}_3 = \tilde{v}_y = u_c$ are equal to zero. Linearizing each FOC in the order given above and using appropriate functional form assumptions:

$$\begin{aligned}
\pi_t &: -\hat{\phi}_{1t-1} - d''\hat{\phi}_{2t} - d''\hat{\phi}_{2t-1} - d''\pi_t \\
Y_t &: \phi\hat{Y}_t - \epsilon\phi\hat{\phi}_{2t} - \hat{\phi}_{3t} \\
i_t &: \beta^2\hat{\phi}_{1t} + \hat{\gamma}_{1t} \\
C_t &: \hat{Y}_t - \sigma\hat{\xi}_t - \beta\hat{\phi}_{1t} + \hat{\phi}_{1t-1} - \epsilon\hat{\phi}_{2t} + \sigma\hat{\phi}_{3t}
\end{aligned}$$