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DYNAMIC OPTIMIZATION  
IN TWO-PARTY MODELS

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ABSTRACT

The goal of this paper is to study the problem of optimal dynamic policy formulation with competing political parties. We study a general class of problems, in which the two competing political parties have quadratic intertemporal objective functions, and in which the economy has a linear structure and a multidimensional state space. For the general linear quadratic problem we develop a numerical dynamic programming algorithm to solve for optimal policies of each party taking into account the party's objectives; the structure of the economy ; the probability of future election results; and the objectives of the other political party.

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## Dynamic Optimization in Two-Party Models

### I. Introduction

In recent years, a considerable body of economic analysis has focussed on the positive theory of macroeconomic policymaking. In one strand of this literature, policy choices are assumed to be made by a policy authority with a well defined and stable objective function. Policy choices are then governed by the maximization of the objective function, subject to the structure of the economy. An example of this approach is Barro (1979), who considers the optimal intertemporal choice of budget deficits by a government attempting to minimize the excess burden of taxation. Another approach emphasizes the competition of interest groups on government policymaking. Instead of assuming that policymakers maximize a single, well-defined objective function, governments are assumed to respond to the lobbying or rent-seeking activity of competing interest groups, with policies outcomes a function of the extent of lobbying and the political influence of various rival interests. Examples of this approach may be found in several studies on the formation of tariff policy in the United States <sup>1</sup>.

While these analyses have yielded important insights, both approaches are clearly flawed as models of policy formation. The first approach completely ignores the ongoing competition for political power among rival interests that characterizes most

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<sup>1</sup> See Baldwin (1982) .

economies. Implicitly, the approach assumes that the battle for power has been settled once and for all, and can be summarized in the stable objective function of the government. The second approach treats governments as blank slates upon which particular interest groups operate.

As usually formulated and empirically implemented, both approaches downplay the effects of election outcomes on policy choices. Indeed, in most cases, elections are ignored altogether in the discussion of policy formulation. To the extent that elections are considered, the electoral competition is typically between candidates with identical policy platforms. Elections are then simply a battle for the spoils of office, rather than a competition over the choice of policies. The so-called political business cycle (PBC) approach is of this genre <sup>2</sup>. In the PBC models, elections matter to the extent that politicians manipulate the probability of reelection by the timing of their policies. Elections are not, however, viewed as offering the voters a choice between candidates with distinct policy positions.

A third approach to modelling policy formation, known as partisanship theory, has attracted increasing interest recently. This third approach combines important elements of the first two approaches. Partisanship theory begins by acknowledging the ongoing struggle for political power among competing interest groups, and assumes that in the industrial democracies the

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<sup>2</sup> See Nordhaus (1975) and McRae (1977) for formalizations of the political business cycle approach.

struggle is mediated through the electoral process. In contrast to the PBC approach, competing political parties represent different constituencies, and therefore follow different policies when they win elections. The success of different interest groups in affecting public policies will depend on which political party holds office.

The standard theoretical challenge to the commonsense view that political parties differ is that since political parties seek electoral victories, they are led to pursue policies in the interest of the median voter, and are thus led to a convergence of policies. This challenge has been undermined theoretically by Wittman (1977) and Alesina (1986), who show that convergence to the median voter's preferences is likely to be incomplete, for at least two major reasons. First, assuming that there is randomness in electoral outcomes (e.g. randomness in the preferences of voters), parties will announce electoral strategies that are a compromise between their own preferred policy positions, and those of the median voter. Second, Alesina notes that if parties in fact represent distinct constituencies, then the voters will not fully believe that the party will stick to its promises to pursue the median voters' favored positions after it has won an election. Voters suspect that whatever a party announces before an election, it will at least partly represent its particular constituents' interests after the election. In an equilibrium with rational voters, parties will therefore run election campaigns on distinct platforms. Once again, convergence among parties to the position

of the median voter will be incomplete.

The partisanship viewpoint has now been tested empirically by several authors, starting with the important and influential work of Hibbs (1977). The findings point nearly unanimously to the proposition that different political parties in the United States and Europe indeed pursue distinct macroeconomic policies while in office. In the United States, for example, several writers, including Alesina and Sachs (1986), Beck (1982, 1984), Chapell and Keech (1985) and Hibbs (1977, 1985), have shown that Democratic administrations pursue policies that give more weight than Republican administrations to unemployment relative to inflation, and that favor income redistribution to lower income groups. Put in revealed preference terms, the Democratic Party objective function is revealed to put relatively more weight on unemployment and income redistribution than on inflation, compared with the Republican Party objective function.

The basic insight of partisanship theory, that successive governments are likely to differ in objectives, complicates the positive analysis of government policymaking. Basically, the fact that objectives will change over time adds another dimension to the well-known problem of time consistency<sup>3</sup>. The time consistency problem arises because current governments generally cannot bind the actions of successor governments. Current governments must act

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<sup>3</sup> The issue of the time consistency of government policy was introduced in the macroeconomic literature by Kydland (1977) and Kydland and Prescott (1977). For recent surveys of the large and growing literature on time consistency see Barro (1986), Cukierman (1985), Fischer (1986) and Rogoff (1986).

taking into account that future governments will pursue policies that are optimal from their own future perspective, and not from the current perspective. Current governments may influence the actions of future governments by leaving the economy in a particular state (e.g. with a given structure of public debt, as in Lucas and Stokey (1983)), but in general cannot completely bind the actions of future governments. As is now well known, this problem arises even when the objectives of successive governments are unchanging through time, as long as government policies impinge on a private sector characterized by forward-looking behavior.

The problem of influencing future actions is made more complicated when future governments may not share the objectives of the current government. How should Democrats behave if they know that they may be succeeded by Republicans? How should their optimal behavior change as the probability of a Republican successor government increases? Consider, for example, the problem of fighting inflation. Suppose that Democrats are leery of fighting inflation via recession, because of the effects on unemployment on their working-class constituents. Republicans, meanwhile, don't mind recessions, because their coupon-clipping constituents are unlikely to become unemployed. Should a Democratic administration be more or less inflationary while in office, and thus leave a higher or lower inflation rate to the future, if the probability rises of a Republican successor?

This problem has now been studied in several specific

examples<sup>4</sup>. Alesina and Tabellini (1987) have studied the problem of how competition among political parties affects the choice of budget deficits by each party when it is in power. They show that if the two parties differ by the type of public goods that are preferred by their respective constituencies, then political competition leads the parties to choose larger budget deficits than they would in the absence of political competition (i.e. if the party were certain that it would remain in power in the future). Persson and Svensson (1987) have studied a related problem in which the present government chooses budget policies knowing with certainty that it will be followed by a future government with different fiscal policy objectives.

The goal of this paper is to present in a more general setting the problem of optimal dynamic policy formulation with competing political parties. The earlier papers have used restrictive assumptions on the time horizon of the competing parties (e.g. the two-period model of Persson and Svensson (1987)), or on the state space of the economy (e.g. the one-dimensional state space, with further specific restrictions, in Alesina and Tabellini (1987)), to get specific solutions for optimal policies by the parties. In this paper, we study a general class of problems with three main characteristics: the two competing political parties have quadratic intertemporal objective functions; the probability of electoral victory for each party is

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<sup>4</sup> See Alesina (1986) for a seminal analysis of the choice of monetary policy and the inflation rate in a two-party political system.

constant; the economy has a linear structure and a multidimensional state space. For this general linear quadratic problem we develop a numerical dynamic programming algorithm to solve for optimal policies of each party taking into account the party's objectives; the structure of the economy; the probability of future election results; and the objectives of the other political party. We should highlight one key point of the analysis: the constant probability of reelection. By ignoring the links between policy formation and reelection chances, we ignore all the considerations of the PBC literature. In our models, elections count only because they select among parties with different objectives, and not at all because they induce parties to select particular policies in order to improve election chances. While our formulation has the merit of highlighting the difference of the PBC and partisanship models, we recognize that a combination of the two approaches would be more satisfactory.

The plan of the paper is the following. Section II introduces the general policy optimization problem for two political parties that alternate in power according to exogenous reelection probabilities. It is shown that time consistent solutions for this class of problems can be obtained through dynamic programming. This solution technique does not generally lead to closed-form solutions, and thus must be implemented numerically. However, in some simple economic models, discussed in sections III and IV, closed-form analytical solutions can be obtained using the dynamic programming techniques. Applications of the numerical algorithm are

found in section V where the results of simulations on specific models are presented. The numerical algorithm itself is presented in detail in section VI. Concluding remarks follow in section VII.

## II. The General Optimization Problem in a Two-Party Political System

In this section we present the general optimization problem for an economy with two political parties that alternate in power according to exogenous reelection probabilities. The economy can be described in a very general form by the following minimal state-space representation <sup>5</sup>:

$$Z_{t+1} = f_t ( Z_t, U_t ) \quad (2.1)$$

$$r_t = g_t ( Z_t, U_t ) \quad (2.2)$$

where:

$Z$  is a vector of state variables

$U$  is a vector of control variables

$r$  is a vector of target variables

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<sup>5</sup> In (2.1) and (2.2),  $Z_t$  is a vector of state variables that is predetermined at time  $t$ . In the general solution in section VI, we expand the analysis to include a vector of non-predetermined or "jumping variables", as commonly arises in rational expectations models.

Assume that instead of the traditional single social planner there are two political parties labeled party D ("Democrat") and party R ("Republican") characterized by different objective functions. Elections take place at the beginning of every period  $t$  and the elected party chooses the control variables  $U_t$  for period  $t$ . We denote the choice of  $U_t$  when D is in power as  $U_t^D$ , and we define  $U_t^R$  analogously. The probability of reelection of each party is fixed and taken as exogenous: party D is elected with probability  $p$  and party R is elected with probability  $(1-p)$ .

The objective function of the two parties is given by the following welfare functions:

$$W_t^R = - E_t \left( \sum_{j=t}^{\infty} \beta^{j-t} r_j, \Omega^R r_j \right) \quad (2.3)$$

$$W_t^D = - E_t \left( \sum_{j=t}^{\infty} \beta^{j-t} r_j, \Omega^D r_j \right) \quad (2.4)$$

where

$W^i$  is the level of welfare of party  $i$  ( $i = R, D$ )

$\beta$  is  $1/(1+\delta)$  and  $\delta$  is the social rate of time discount <sup>6</sup>

$\Omega^i$  is a matrix of weights on the policy targets ( $i=R, D$ )

$r$  is a vector of target variables

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<sup>6</sup> In principle the rate of time preference could be different for the two parties.

$E_t$  is the expectation operator

The two parties are assumed to differ in the weights that they give to different policy objectives, so that the matrices  $\Omega^R$  and  $\Omega^D$  of target weights will differ for the two parties. Party R chooses  $U_t^R$  when in power to maximize its welfare function (2.3) subject to the dynamic system (2.1) and (2.2) and the knowledge of the the reelection probabilities. Party D solves a similar optimization problem choosing  $U_t^D$  to maximize its welfare function (2.4) subject to the system (2.1) and (2.2) and the reelection probability. We will examine memoryless closed-loop strategies for the two parties of the form:

$$U_t^D = U_t^D(Z_t) \quad (2.5)$$

$$U_t^R = U_t^R(Z_t)$$

in which each party's equilibrium choice is a function only of the current state. The actual  $U_t$  selected in each period will be equal to  $U_t^i$  for party  $i$  in power at time  $t$ <sup>7</sup>.

The above problem for the two parties can be reformulated in terms of value functions of the two parties. In defining these

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<sup>7</sup> This formulation of strategies is restrictive, in that it rules out game solutions in which the parties' moves depend on the past history of the game in addition to the current state of the economy. Reputational equilibria, supported by trigger strategies, are thereby ruled out. See Fershtman (1987) for further discussion of this point.

value functions one should observe the crucial differences between the case of a single social planner and the case of a two party system. Under a single controller a unique value function  $V_t$  can be used to define the optimization problem to be solved by the social planner. In particular the single controller problem can be formulated as:

$$V_t = \text{Max}_{U_t} \{ -r_t' \Omega r_t + \beta V_{t+1}(Z_{t+1}) \} \quad (2.6)$$

subject to (2.1) and (2.2).

The maximization in (2.6) gives rise to a policy rule  $U_t = U_t(Z_t)$ . Note that if (2.1) and (2.2) are time invariant, and the policymaker's horizon is infinite, then the  $V_t$  and  $U_t$  functions will be time invariant functions of the state  $Z_t$ .

When we consider two different parties the problem becomes more complex because we have to define four value functions, two value functions for each party in each period: one for the case in which the party is in power at time  $t$  and one for the case in which the party is not in power. Let  $V_{jt}^i$  signify the value function at time  $t$  of party  $j$  when party  $i$  is in power at time  $t$ . For example,  $V_{Dt}^R$  is the maximum welfare of party  $D$  when party  $R$  is in power in period  $t$ . The equilibrium that we seek is a pair of rules  $U_t^D = U_t^D(Z_t)$  and  $U_t^R = U_t^R(Z_t)$ , and four value functions, such that:

Party D

$$V_{Dt}^D = \underset{U_t^D}{\text{Max}} \{ - r_t^D, \Omega^D r_t + \beta [ p V_{Dt+1}^D + (1-p) V_{Dt+1}^R ] \} \quad (2.7)$$

$$V_{Dt}^R = - r_t^R, \Omega^D r_t + \beta [ p V_{Dt+1}^D + (1-p) V_{Dt+1}^R ] \quad (2.8)$$

$$U_t^D(Z_t) = \underset{U_t}{\text{argmax}} \{ - r_t^D, \Omega^D r_t + \beta [ p V_{Dt+1}^D + (1-p) V_{Dt+1}^R ] \} \quad (2.9)$$

Party R

$$V_{Rt}^R = \underset{U_t^R}{\text{Max}} \{ - r_t^R, \Omega^R r_t + \beta [ p V_{Rt+1}^D + (1-p) V_{Rt+1}^R ] \} \quad (2.10)$$

$$V_{Rt}^D = - r_t^D, \Omega^R r_t + \beta [ p V_{Rt+1}^D + (1-p) V_{Rt+1}^R ] \quad (2.11)$$

$$U_t^R(Z_t) = \underset{U_t}{\text{argmax}} \{ - r_t^R, \Omega^R r_t + \beta [ p V_{Rt+1}^D + (1-p) V_{Rt+1}^R ] \} \quad (2.12)$$

subject to:

$$Z_{t+1} = f_t ( Z_t, U_t ) \quad (2.1)$$

$$r_t^i = g_t ( Z_t, U_t^i ) \quad (i=R, D) \quad (2.2')$$

In the infinite horizon case with (2.1) and (2.2') time invariant, all functions  $V_{jt}^i$  and  $U_t^i$  will be time invariant.

A general algorithm for the solution of these optimization problem for the two parties is presented in section VI. The problem is solved through the technique of dynamic programming, using a backward recursion procedure that does not generally lead to closed-form solutions. However, in some simple economic models closed-form analytical solution can be obtained using dynamic programming techniques. In the next two sections we present and derive analytical solutions for two specific models where closed-form solutions are obtainable.

### III. A Simple Closed Economy Two-Party Model

Consider the following "inflation game" of Alesina (1986) which extends the framework of Barro and Gordon (1983) to include two political parties. Let  $q_t$  be output, and  $\pi_t$  be inflation. For each party, the desired level of output is  $\bar{q}$ , and the desired level of inflation is zero. Utility in each period is a quadratic function of the deviations of output and inflation from the target levels. Thus:

$$U^D = - \sum_{t=0}^{\infty} \beta^t \left\{ (q_t - \bar{q})^2 + \phi^D (\pi_t)^2 \right\} \quad (3.1)$$

$$U^R = - \sum_{t=0}^{\infty} \beta^t \left\{ (q_t - \bar{q})^2 + \phi^R (\pi_t)^2 \right\} \quad (3.2)$$

$$q_t = \alpha (\pi_t - {}_{t-1}\pi_t) \quad (3.3)$$

$${}_{t-1}\pi_t = E (\pi_t | I_{t-1}) \quad (3.4)$$

$$\phi^D < \phi^R$$

where  $I_{t-1}$  is the information set available at time  $(t - 1)$  and includes the past inflation rates and the entire history of the game. Note that for concreteness we assume that party R is the less inflationary party, i.e. R has a higher disutility of inflation, since  $\phi^R > \phi^D$ . The party that is in power selects the policy variable  $(\pi)$  to maximize its welfare function subject to the structure of the economy. The state of the economy at time  $t$  is given by the level of inflation expectations at time  $t-1$ ,  ${}_{t-1}\pi_t$ . Each party wants to stabilize output  $q$  at a positive value  $\bar{q}$ . However, according to the "surprise" Phillips curve (3.3), and the assumption of rational expectations (3.4), the average level of output will be zero rather than  $\bar{q}$ . It is assumed that the probability that D will be elected is equal to  $p$  (exogenously given) while the probability that R elected is equal to  $(1-p)$ . Elections occur at the beginning of each period.

Define the value function for the two parties as:

$$V_{Dt}^D = \text{Max}_{\{\pi_t\}} \{-[(q_t - \bar{q})^2 + \phi^D \pi_t^2] + \beta[pV_{Dt+1}^D + (1-p)V_{Dt+1}^R]\} \quad (3.5)$$

$$V_{Rt}^R = \text{Max}_{\{\pi_t\}} \{ -[(q_t - \bar{q})^2 + \phi^R \pi_t^2] + \beta [pV_{Rt+1}^D + (1-p)V_{Rt+1}^R] \} \quad (3.6)$$

subject to (3.3) and (3.4).

To find the time consistent solution to this problem we find first the solution to the finite horizon problem and then take the limit of this problem for the infinite horizon case. Suppose period  $T$  is the final period so that  $V_{T+1}^i = 0$ . Assume, moreover, that party  $i$  ( $i=D,R$ ) is in power in period  $T$ . From the optimization of (3.5) or (3.6), party  $i$  should set:

$$(q_T - \bar{q}) \frac{dq_T}{d\pi_T} + \pi_T \phi^i \frac{d\pi_T}{d\pi_T} = 0 \quad (3.7)$$

Differentiating (3.3) and substituting into (3.7) it can be shown that the policy rule chosen by party  $i$  will be:

$$\pi_T^i = \frac{\alpha^2}{\alpha^2 + \phi^i} ({}_{T-1}\pi_T) + \frac{\alpha}{\alpha^2 + \phi^i} \bar{q} \quad (3.8)$$

The assumption of rational expectations implies that:

$${}_{T-1}\pi_T = E \{ \pi_T \mid I_{T-1} \} = p \pi_T^D + (1-p) \pi_T^R \quad (3.9)$$

Then substituting (3.9) in (3.8) we obtain:

$$\pi_T^D = \frac{\alpha^2}{\alpha^2 + \phi^D} (p \pi_T^D + (1-p) \pi_T^R) + \frac{\alpha}{\alpha^2 + \phi^D} \bar{q} \quad (3.10)$$

$$\pi_T^R = \frac{\alpha^2}{\alpha^2 + \phi^R} (p \pi_T^D + (1-p) \pi_T^R) + \frac{\alpha}{\alpha^2 + \phi^R} \bar{q} \quad (3.11)$$

Solving the system of equations (3.10) and (3.11) for  $\pi_T^D$  and  $\pi_T^R$  we then obtain the equilibrium solutions for the inflation rates chosen by the two parties:

$$\pi_T^D = \frac{\alpha (\alpha^2 + \phi^R)}{\phi^R \alpha^2 (1-p) + \alpha^2 \phi^D p + \phi^D \phi^R} \bar{q} > 0 \quad (3.12)$$

$$\pi_T^R = \frac{\alpha (\alpha^2 + \phi^D)}{\phi^R \alpha^2 (1-p) + \alpha^2 \phi^D p + \phi^D \phi^R} \bar{q} > 0 \quad (3.13)$$

Equations (3.12) and (3.13) represent the reduced form solutions for the inflation rate chosen by the two parties in period T. Before discussing the properties of this equilibrium, one should observe that these solutions will hold not only for the terminal period T but also for all other periods of time as well.

In fact, the original dynamic optimization problem can be reduced to a series of static problems that are identical to the one solved above. The reason for this is that the reduced form solutions for the target variables depend only on an exogenous variable ( $\bar{q}$ ) and not on the state variable. Equations (3.12) and (3.13) show that this is the case for the inflation rate and simple substitutions can prove that the reduced forms for output depend only on  $\bar{q}$  as well.

It then follows that, in each stage of the intertemporal optimization problem (3.5) or (3.6),  $V_{t+1}^i$  is a constant and therefore will not affect the period  $t$  optimization problem. Then the rules (3.8) for the policy variables will be time independent and identical in any period  $t$ , i.e.:

$$\pi_t^D = \frac{\alpha^2}{\alpha^2 + \phi^D} (\pi_{t-1}) + \frac{\alpha}{\alpha^2 + \phi^D} \bar{q} \quad (3.8')$$

$$\pi_t^R = \frac{\alpha^2}{\alpha^2 + \phi^R} (\pi_{t-1}) + \frac{\alpha}{\alpha^2 + \phi^R} \bar{q} \quad (3.8'')$$

for every  $t$ . Similarly, the reduced form solution for inflation and output chosen by the two parties will be the same in each time period.

We can now consider the properties of this time-invariant

equilibrium. The solution is time consistent and the economy has an inflationary bias. In equilibrium, the inflation rate chosen by both parties is positive and will be greater for party D (the party with the lower  $\phi$ ). One can also observe that the inflation rate chosen by each party (when in power) depends not only on the parameters of the loss function of that party but also on the parameters of the other party and on the probability of being reelected.

Consider, in particular, the role of the probability of reelection. It can be shown by differentiating (3.12) and (3.13) that the inflation rate chosen by both parties is positively related to the probability of election of the more inflationary party D, i.e.:

$$\frac{\partial \pi^D}{\partial p} = \frac{\alpha^3 (\alpha^2 + \phi^R) (\phi^R - \phi^D)}{[\phi^R \alpha^2 (1-p) + \alpha^2 \phi^D p + \phi^D \phi^R]^2} > 0 \quad (3.14)$$

$$\frac{\partial \pi^R}{\partial p} = \frac{\alpha^3 (\alpha^2 + \phi^D) (\phi^R - \phi^D)}{[\phi^R \alpha^2 (1-p) + \alpha^2 \phi^D p + \phi^D \phi^R]^2} > 0 \quad (3.15)$$

Note, however, that the ratio between the inflation rates chosen by the two parties is independent of this reelection probability. In fact, if we take the ratio of  $\pi_t^D$  to  $\pi_t^R$  from equations (3.12)

and (3.13), we get:

$$\frac{\pi^D}{\pi^R} = \frac{\alpha^2 + \phi^R}{\alpha^2 + \phi^D} \quad (3.16)$$

which is independent of the probability of reelection. This ratio of inflation rates chosen by the two parties depends only on the divergence of the policy objectives of the two parties as measured by  $\phi^R$  and  $\phi^D$ , and the elasticity of output supply to inflation ( $\alpha$ ). In particular, the more divergent are the policy objectives of the two parties the greater is the divergence of the inflation rates chosen by the two parties. Also, the greater is the output supply elasticity  $\alpha$  the closer are the inflation rates chosen by the two parties.

The two major conclusions of this section are the following:

1) In the two party model analyzed above the policy rules followed by the two parties (3.8) are time invariant and do not depend on the reelection probabilities.

2) The reduced form solutions for the target variables chosen by each party depend on both parties' parameters and the election probabilities. In particular, both parties become more inflationary as the probability of election of D (the more inflationary party) increases.

## IV. A Two Period Model with Backward-Looking Expectations

The model presented in section III was easily solved in a closed form because the intertemporal optimization problem was shown to be reduceable to a series of static maximization problems. In particular the policy rules were shown to be time independent and not dependent on the reelection probabilities. Closed-form solutions for the inflation rates chosen by the two parties were then derived and shown to hold in the same form for any time period. These reduced forms for inflation turned out to be dependent on the reelection probabilities.

More complex models cannot be reduced to a series of static problems because the transition matrix will introduce true dynamics in the problem (i.e. next period's value functions will depend on today's control choices). In this section we present a two-period version of a model similar to the one presented in section III, derive a closed-form time-consistent solution for it, and discuss the issue of policy selection of two parties in this new setting.

Take a two period version of the two-party model introduced in section III:

$$U_1^i = - E_1 \sum_{t=1}^2 \beta^{t-1} \{ (q_t)^2 + \phi^i (\pi_t)^2 \} \quad (i=D,R) \quad (4.1)$$

$$q_t = \alpha (\pi_t - {}_{t-1}\pi_t) \quad (4.2)$$

$${}_0\pi_1 \text{ given at time 1} \quad (4.3)$$

$$\phi^D < \phi^R$$

This problem was solved in section III for the case of rational expectations. Now, assume instead that expectations are formed according to a backward looking mechanism <sup>8</sup>:

$${}_{t-1}\pi_t = \pi_{t-1} \quad (4.4)$$

Then substituting (4.4) in (4.2) the output supply function can be written as:

$$q_t = \alpha (\pi_t - \pi_{t-1}) \quad (4.2')$$

Now define the value functions of the two parties as:

$$V_{D1}^D = \text{Max}_{\pi_1^D} - [ (q_1^D)^2 + \phi^D (\pi_1^D)^2 ] + \beta [ pV_{D2}^D + (1-p)V_{D2}^R ] \quad (4.5)$$

for party D and:

$$V_{R1}^R = \text{Max}_{\pi_1^R} - [ (q_1^R)^2 + \phi^R (\pi_1^R)^2 ] + \beta [ pV_{R2}^D + (1-p)V_{R2}^R ] \quad (4.6)$$

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<sup>8</sup> We use this example not for the realism of the expectation assumption, but to illustrate certain methodological points.

for party R, where:

$$V_{D2}^D = - [(q_2^D)^2 + \phi^D (\pi_2^D)^2] \quad (4.7)$$

$$V_{R2}^R = - [(q_2^R)^2 + \phi^R (\pi_2^R)^2] \quad (4.8)$$

$$V_{D2}^R = - [(q_2^R)^2 + \phi^D (\pi_2^R)^2] \quad (4.9)$$

$$V_{R2}^D = - [(q_2^D)^2 + \phi^R (\pi_2^D)^2] \quad (4.10)$$

One can generally observe that:

$$V_{D2}^D > V_{D2}^R \quad \text{for } \phi^D \neq \phi^R \quad (4.11)$$

$$V_{R2}^R > V_{R2}^D \quad \text{for } \phi^D \neq \phi^R \quad (4.12)$$

i.e. the maximum utility that either party can reach in period 2 is greater if the party can optimize for itself in the period considered. In other words, given different objective functions for the two parties, if party R chooses  $\pi_2$  the welfare that D will obtain must be less than the level that party D would reach if it chooses  $\pi_2$  instead.

Now, consider the problem faced by the two parties in the terminal period 2. At time 2 party  $i$  ( $i = D$  or  $R$  according to which party is in power in period 2) has the following problem:

$$\text{Max}_{\pi_2^i} U_2^i = - [(q_2^i)^2 + \phi^i (\pi_2^i)^2] \quad (4.13)$$

subject to :

$$q_2^i = \alpha (\pi_2^i - \pi_1) \quad (4.2'')$$

The first order condition for this problem is:

$$\alpha [\alpha (\pi_2^i - \pi_1)] + \phi^i \pi_1^i = 0 \quad (i=D,R) \quad (4.14)$$

Solving the system (14) for the values of  $\pi_2^i$  we get:

$$\pi_2^i = \Psi^i \pi_1 \quad (i=D,R) \quad \text{where} \quad \Psi^i = \frac{\alpha^2}{\alpha^2 + \phi^i} \quad (4.15)$$

Substituting the solutions (4.15) in the equation for output (4.2'') we obtain the solution for the output level chosen by the two parties in period 2 as:

$$q_2^i = \alpha (\Psi^i - 1) \pi_1 \quad (i=R,D) \quad (4.16)$$

The four value functions for period 2 are therefore:

$$V_{D2}^D = - \{ \alpha^2 [(\alpha \Psi^D - 1)^2 + \phi^D (\Psi^D)^2] \} (\pi_1)^2 = - \Sigma_{D2}^D (\pi_1)^2 \quad (4.17)$$

$$V_{R2}^R = - \{ \alpha^2 [(\alpha \Psi^R - 1)^2 + \phi^R (\Psi^R)^2] \} (\pi_1)^2 = - \Sigma_{R2}^R (\pi_1)^2 \quad (4.18)$$

$$V_{D2}^R = - \{ \alpha^2 [(\alpha \Psi^R - 1)^2 + \phi^D (\Psi^R)^2] \} (\pi_1)^2 = - \Sigma_{D2}^R (\pi_1)^2 \quad (4.19)$$

$$V_{R2}^D = - \{ \alpha^2 [(\alpha \Psi^D - 1)^2 + \phi^R (\Psi^D)^2] \} (\pi_1)^2 = - \Sigma_{R2}^D (\pi_1)^2 \quad (4.20)$$

Note that these value functions are quadratic in the first period inflation rate,  $\pi_1$ . Moreover, simple computations can show that:

$$\Sigma_{D2}^R > \Sigma_{D2}^D \quad (4.21)$$

$$\Sigma_{R2}^D > \Sigma_{R2}^R \quad (4.22)$$

so that the results stated in (4.11) and (4.12) are confirmed.

Now let us go back to period 1. Consider first the case in which party D is in power in period 1 (the case in which party R is in power at 1 will be solved similarly). Party D's problem at time 1 can be expressed as:

$$V_{D1}^D = \text{Max}_{\pi_1^D} \quad -[(q_1^D)^2 + \phi^D (\pi_1^D)^2] + \beta [p V_{D2}^D + (1-p) V_{D2}^R] \quad (4.5)$$

subject to (4.17), (4.19) and

$$q_1^D = \alpha (\pi_1^D - \pi_0) \quad (4.2''')$$

Taking the first order conditions for this problem we obtain after several steps:

$$(\alpha \phi + \phi_1) \pi_1^D = \alpha^2 \pi_0 \quad (4.23)$$

where :

$$\Phi = (\alpha + \beta p \alpha (\Psi^D - 1)^2 + \beta p \phi^D (\Psi^D)^2 + \beta (1-p) \alpha (\Psi^R - 1)^2 - \beta \phi^D (\Psi^R)^2)$$

Solving (4.23) for  $\pi_1^D$  we get:

$$\pi_1^D = \frac{\alpha^2}{\alpha \Phi + \phi^D} \pi_0 \quad (4.24)$$

i.e. the inflation rate chosen by party D is a function of inherited inflation and the output objective.

The question now becomes: what is the effect of a change in the probability of reelection of party D on its choice of the inflation rate in period 1 ? Will party D become more or less expansionary as its probability of reelection increases ? More formally we want to know what is the sign of  $(\partial \pi_1^D / \partial p)$ . Since  $p$  affects  $\pi_1^D$  only through  $\Phi$  we get:

$$\frac{\partial \pi_1^D}{\partial p} = \frac{\partial \pi_1^D}{\partial \Phi} \frac{\partial \Phi}{\partial p} =$$

$$= \left( \frac{-\alpha^3}{(\alpha \Phi + \phi^D)^2} \pi_0 \right) \left( \frac{\beta \alpha^3 (\phi^D - \phi^R) (\phi^R - \phi^D)}{(\alpha_2 + \phi^D) (\alpha^2 + \phi^R)^2} \right) > 0 \quad (4.25)$$

The above derivative is always positive because the terms in the two brackets are both negative. The positive sign means that an increase in  $p$ , the probability of election of party D, makes this party more expansionary (it will choose a higher level of inflation at time 1 for any  $\pi_0$ ). It can be proved that the opposite result holds for party R, i.e. a rise in  $p$  makes party R more contractionary. Put another way, for either party an increase in the election probability of that party makes that party more expansionary. This result differs substantially from the one obtained in section III for the case of forward looking wage setters, where a rise in the election probability for R made party R more contractionary.

The intuition behind the result on the election probability is straightforward. Consider the problem from the point of view of D. The expected value for D in the second period is  $p V_{D2}^D + (1-p) V_{D2}^R$ . We have shown that both  $V_{D2}^D$  and  $V_{D2}^R$  can be written as quadratic functions of  $\pi_1$ , the amount of inflation in period 1:

$$V_{D2}^D = - \Sigma_{D2}^D (\pi_1)^2 \quad (4.17)$$

$$V_{D2}^R = - \Sigma_{D2}^R (\pi_1)^2 \quad (4.19)$$

Moreover, as pointed out earlier, it must be the case that  $\Sigma_{D2}^D < \Sigma_{D2}^R$ , since party D always benefits from itself being in power in period 2. Thus, from party D's perspective, second period

expected utility is:

$$\begin{aligned}
 & p v_{D2}^D + (1-p) v_{D2}^R - v_{D2}^R + p (v_{D2}^D - v_{D2}^R) - \\
 & - \Sigma_{D2}^R (\pi_1)^2 + p (\Sigma_{D2}^R - \Sigma_D^D) (\pi_1)^2 \qquad (4.26)
 \end{aligned}$$

Clearly, as party D's probability of election rises, i.e. as  $p$  increases, the second period cost to D of high  $\pi_1$  diminishes. Thus the more likely it is that D will be in power in period 2, the less costly is it for D to have a high inflation rate in period 1. D therefore becomes more inflationary in period 1 as D's probability of election increases.

Now consider the problem from R's perspective. Second-period expected utility for R can be written as:

$$\begin{aligned}
 & p v_{R2}^D + (1-p) v_{R2}^R - v_{R2}^D + (1-p)(v_{R2}^R - v_{R2}^D) - \\
 & - \Sigma_{R2}^D (\pi_1)^2 + (1-p)(\Sigma_{R2}^D - \Sigma_R^R) (\pi_1)^2 \qquad (4.27)
 \end{aligned}$$

Once again,  $\Sigma_{R2}^D$  exceeds  $\Sigma_{R2}^R$ . Thus, for R, the second-period cost of  $\pi_1$  diminishes as R's probability of reelection,  $1-p$ , rises. Thus,  $\pi_1^R$  will rise with  $1-p$ , and fall with  $p$ .

In sum, the more certain each party is of its second-period tenure in office, the more inflationary it will be in the first period. Since  $p$  is the probability of D in period 2,  $\pi_1^D$  is a rising function of  $p$ , and  $\pi_1^R$  is a falling function of  $p$ .

## V. Simulation Results

This section presents the results of numerical simulations obtained using the dynamic programming algorithm that solves the linear-quadratic version of the two-party problems introduced in section II <sup>9</sup>. The model used in this section to exemplify the algorithm is a version of the model presented in section IV. There the model was solved analytically for the two-period case; here it is assumed that the policy-makers' programming horizon is infinite and the numerical solution to this dynamic programming problem is obtained. We are interested to find policy rules chosen by the two parties and solutions for the target variables.

In order to use the solution algorithm we need to parametrize the model; in particular, we assume that <sup>10</sup> :

$$\phi^D = 1 \quad \phi^R = 4 \quad \alpha = 2 \quad \beta = 10/11$$

Table 1 presents the inflation rule and the equilibrium values of the policy targets (inflation and output) chosen by the two parties for different values of the reelection probabilities <sup>11</sup>.

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<sup>9</sup> A detailed derivation of this algorithm is presented in section VI.

<sup>10</sup> It should be noted that the qualitative results obtained below do not depend on the particular parameter choice but, as seen in section IV, are a structural feature of the inflation model used in these sections.

<sup>11</sup> Note that the inflation rate is the control variable and a target variable at the same time.

The results for the inflation rules of the two parties are consistent with the parameter choice of R as a party more concerned about inflation than party D ( $\phi^R > \phi^D$ ). In setting the inflation rate, when in power, party R accommodates past inflation less than party D does (in table 1 the coefficient on  $\pi_{t-1}$  in the inflation equation is always smaller for party R relative to party D). However, the policy rules followed by the two parties depend on the reelection probabilities. Party R becomes more contractionary as its reelection probability falls; in fact, R accommodates 0.39 of past inflation when it is certain of reelection ( $(1-p)=1$ ) while it accommodates 0.36 of  $\pi_{t-1}$  when its election probability is zero. Similarly, party D becomes more contractionary (expansionary) when its election probability falls (increases): D accommodates 0.63 of past inflation when it is certain of reelection while it accommodates only 0.59 of past inflation when its election probability is zero. These results for the infinite horizon case confirm those obtained in section IV for the two-period case.

It can also be seen from Table 1 that the results for the other target variable, output, mirror those obtained for the inflation rate: whenever the inflation policy of one party becomes more contractionary, the outcome for output becomes more contractionary as well.

The model chosen in this section to exemplify the solution algorithm is very simple (one state variable and two targets). It should, however, be observed that the algorithm (derived in

TABLE 1. BACKWARD LOOKING PRICE EXPECTATIONS

INFLATION RATE AND OUTPUT CHOSEN BY THE TWO PARTIES

P = Probability of election of party D

(1 - P) = Probability of election of party R

P = 0

$$\pi_t^R = 0.39 \pi_{t-1}$$

$$q_t^R = -1.21 \pi_{t-1}$$

$$\pi_t^D = 0.59 \pi_{t-1}$$

$$q_t^D = -0.81 \pi_{t-1}$$

P = 0.5

$$\pi_t^R = 0.38 \pi_{t-1}$$

$$q_t^R = -1.23 \pi_{t-1}$$

$$\pi_t^D = 0.60 \pi_{t-1}$$

$$q_t^D = -0.78 \pi_{t-1}$$

P = 1

$$\pi_t^R = 0.36 \pi_{t-1}$$

$$q_t^R = -1.27 \pi_{t-1}$$

$$\pi_t^D = 0.63 \pi_{t-1}$$

$$q_t^D = -0.73 \pi_{t-1}$$

section VI) is very general in that it can handle models with any number of state, jumping, control and target variables where closed-form solutions could not be otherwise obtained.

## VI. The General Dynamic Programming Solution of the Linear-Quadratic Problem Under a Two-Party Political System

This section describes the general solution technique to solve the two-party optimization problem introduced in section II. We will show that time consistent solutions for this two-party infinite horizon dynamic game can be obtained through a technique of dynamic programming for the general class of problems where the objective function of the two parties is quadratic and the transition matrix is linear <sup>12</sup>.

Consider the linear formulation of the state-space model presented in section II. Partition the state vector  $Z$  between between a vector of predetermined state variables ( $X$ ) and a vector of jumping variables ( $e$ ). The model can be written as :

$$X_{t+1} = A X_t + B e_t + C U_t \quad (6.1)$$

$$e_{t+1} = D X_t + F e_t + G U_t \quad (6.2)$$

$$r_t = M X_t + L e_t + N U_t \quad (6.3)$$

<sup>12</sup> In the single controller case this class of linear-quadratic problems leads to the optimal linear regulator problem of dynamic programming. See Sargent (1987) for details.

where:

$X$  is a vector of state variables (predetermined)

$e$  is a vector of jumping variables

$U$  is a vector of control variables

$r$  is a vector of target variables <sup>13</sup>

$${}_t e_{t+1} = E \{ e_{t+1} \mid I_t \}$$

There are two political parties labeled party D and party R characterized by the different objective functions (2.3) and (2.4). Elections take place at the beginning of every period  $t$  and the elected party chooses the control variables for period  $t$ . The probability of election of each party is given and taken as exogenous: party D is elected with probability  $p$  and party R is elected with probability  $(1-p)$ . The four value functions for D and R are given by (2.7), (2.8), (2.10) and (2.11).

In order to solve this optimization problem with two parties we need to find time-invariant party-specific matrices  $\Gamma^i$  ( $i=D,R$ ) for the linear policy rules of the two parties:

$$U_t^i = \Gamma^i X_t \quad (i=D,R) \quad (6.4)$$

and time-invariant matrices  $S^{DD}, S^{DR}, S^{RD}, S^{RR}$  such that:

---

<sup>13</sup> The model can be easily extended to consider the case in which exogenous variables and expected state variables appear in the model representation. The numerical algorithm written by the authors explicitly considers these additional types of variables and an analytical derivation is available upon request.

$$V_{Dt}^D (X_t) = - X_t' S_D^D X_t \quad (6.5)$$

$$V_{Rt}^R (X_t) = - X_t' S_R^R X_t \quad (6.6)$$

$$V_{Dt}^R (X_t) = - X_t' S_D^R X_t \quad (6.7)$$

$$V_{Rt}^D (X_t) = - X_t' S_R^D X_t \quad (6.8)$$

Note that (6.5) is defined by the problem (2.7) subject to (6.1) to (6.3) and (6.6) is defined by the problem (2.10) subject to (6.1) to (6.3).

We also need to find party-specific matrices  $H_1^i$  that ensure that the jumping variables adjust to keep the model on the stable manifold:

$$e_t^i = H_1^i X_t \quad (i=D,R) \quad (6.9)$$

where  $i$  is the party in power at time  $t$ .

The iterative technique which solves this problem is similar to the dynamic programming procedure used for the case of unique social planner. We begin by converting the infinite period problem into a finite period problem where the terminal period is some arbitrary period  $T$ . Then we can solve the problem for the terminal period  $T$  twice: in one case we assume that party  $D$  is in power at  $T$  and in the other case we assume that party  $R$  is in power at time  $T$ . With these results, we solve for period  $T-1$ , and

by induction, for all periods until the first period.

Assume that in period T+1 the jumping variables and the expected state variables have stabilized and that the value functions for T+1 are equal to zero. This implies:

$${}^T e_{T+1} = e_T \quad (6.10)$$

Using (6.10) in (6.2) and we get:

$$e_T = \beta_1 X_T + \beta_2 U_T \quad (6.11)$$

where:  $\beta_1 = (I - F)^{-1} D$

$$\beta_2 = (I - F)^{-1} G$$

Now, take (6.11) and substitute it into (6.3) to obtain an expression for the targets at time T as a function of the states and the control variables at time T:

$$r_T = \gamma_1 X_T + \gamma_2 U_T \quad (6.12)$$

where:  $\gamma_1 = (M + L\beta_1)$

$$\gamma_2 = (N + L\beta_2)$$

Suppose now that party i is in power in period T. i will be equal to D or R according to which party is in power in the terminal

period  $T$ . Since  $T$  is the terminal period  $V_{T+1} = 0$  and the problem faced by party  $i$  will be:

$$\text{Max}_{U_T^i} - \beta^T r_T' \Omega^i r_T \quad (6.13)$$

subject to (6.12) or:

$$\text{Max}_{U_T^i} - \beta^T (\gamma_1 X_T + \gamma_2 U_T^i)' \Omega^i (\gamma_1 X_T + \gamma_2 U_T^i) \quad (6.13')$$

The first order conditions for this problem will be:

$$\gamma_2' \Omega^i \gamma_1 X_T + \gamma_2' \Omega^i \gamma_2 U_T^i = 0 \quad (6.14)$$

that can be written in compact form as:

$$MM_T^i U_T^i = - NN_T^i X_T \quad (6.15)$$

$$\text{where } MM_T^i = \gamma_2' \Omega^i \gamma_2$$

$$NN_T^i = \gamma_2' \Omega^i \gamma_1$$

Solving for  $U_T^i$  we get:

$$U_T^i = \Gamma_T^i X_T \quad (6.16)$$

$$\text{where: } \Gamma_T^i = - (MM_T^i)^{-1} NN_T^i$$

The next step is to take the policy rule (6.16), substitute it in (6.11) to obtain the values of the jumping variables when party  $i$

is in power at time T. It is obvious that the values assumed by jumping variables will differ depending on which party is in power at time T. We will then get:

$$e^i_T = H^i_T X_T \quad (6.17)$$

$$\text{where: } H^i_T = \beta_1 + \beta_2 \Gamma^i_T$$

At this point it is possible to take the above rules for the policy variables and the jumping variables and substitute them back in the equations for their target variables (block (6.3)) for period T. Doing so we obtain the values of the targets when party i is in power at time T as a function of the state variables at time T:

$$r^i_T = [ M + L H^i_T + N \Gamma^i_T ] X_T \quad (6.18)$$

Substituting these solutions for the targets in the value functions of their two parties for time T (6.13) and equating these solutions to the guess solution for these value functions (equations (6.5) to (6.8)) we can obtain the initial starting guesses for the S's matrices for time T:

$$S^D_{DT} = (M+L H^D_{T+N} \Gamma^D_T)' \Omega^D (M+L H^D_{T+N} \Gamma^D_T)$$

$$S^R_{RT} = (M+L H^R_{T+N} \Gamma^R_T)' \Omega^R (M+L H^R_{T+N} \Gamma^R_T)$$

$$S_{RT}^D = (M+L H_{T+N}^D \Gamma_T^D)' \Omega^R (M+L H_{T+N}^D \Gamma_T^D)$$

$$S_{DT}^R = (M+L H_{T+N}^R \Gamma_T^R)' \Omega^D (M+L H_{T+N}^R \Gamma_T^R)$$

Given the value function in each period , we can solve the problem in any period  $t$ . Consider then the problem for period  $t$ . At time  $t$  the state of the system is described by:

$$X_{t+1} = A X_t + B e_t + C U_t \quad (6.1)$$

$${}^t e_{t+1} = D X_t + F e_t + G U_t \quad (6.2)$$

We therefore need an expression for  ${}^t e_{t+1}$ ; from the solution for the problem for period  $t+1$  we know that:

$$e_{t+1} = H_{t+1}^D X_{t+1} \quad (6.19)$$

if party D is in power at  $t+1$  and:

$$e_{t+1} = H_{t+1}^R X_{t+1} \quad (6.19')$$

if party R is in power.

It must then be the case that as of time  $t$ :

$$\begin{aligned} {}^t e_{t+1} &= E ( e_{t+1} \mid I_t ) = \\ &= p(H_{t+1}^D X_{t+1}) + (1-p)(H_{t+1}^R X_{t+1}) \\ &= H_{t+1}^* X_{t+1} \end{aligned} \quad (6.20)$$

where:

$$H^*_{t+1} = p H^D_{t+1} + (1-p) H^R_{t+1} \quad (6.21)$$

Then, if we substitute (6.20) in (6.2) we can obtain an expression for the jumping variables at  $t$  as a function of state and control variables:

$$e_t = \beta_{1t} X_t + \beta_{2t} U_t \quad (6.22)$$

$$\text{where: } \beta_{1t} = (H^*_{t+1} B - F)^{-1} (D - H^*_{t+1} A)$$

$$\beta_{2t} = (H^*_{t+1} B - F)^{-1} (G - H^*_{t+1} C)$$

Substitution of (6.22) in the equations for the state variables at  $t+1$  (6.1) and the target variables (6.3) allow us to express the values of these variables at  $t$  as a function of the state and control variables at time  $t$ :

$$X_{t+1} = \theta_{1t} X_t + \theta_{2t} U_t \quad (6.23)$$

$$r_t = \gamma_{1t} X_t + \gamma_{2t} U_t \quad (6.24)$$

$$\text{where: } \theta_{1t} = A + B\beta_{1t} \quad \theta_{2t} = C + B\beta_{2t}$$

$$\gamma_{1t} = M + L\beta_{1t} \quad \gamma_{2t} = N + L\beta_{2t}$$

We can now consider the optimization problem faced by party D assuming that this is the party in power at time  $t$  (a similar optimization problem can be obtained by assuming that R is in power at  $t$ ). As seen above the value function of party D at  $t$  will be:

$$V_{Dt}^D = \text{Max}_{U_t^D} \{ -r_t' \Omega^D r_t + \beta [ p V_{Dt+1}^D + (1-p) V_{Dt+1}^R ] \} \quad (2.7)$$

Given our definitions of the guesses for the value functions of the 2 parties in (6.5) to (6.8) we can rewrite (2.7) as:

$$\begin{aligned} V_{Dt}^D = \text{Max}_{U_t^D} \{ & -r_t' \Omega^D r_t + \beta [ p [X_{t+1}' S_{Dt+1}^D X_{t+1}] + \\ & + [(1-p) [X_{t+1}' S_{Dt+1}^R X_{t+1}] - \\ & - \text{Max}_{U_t^D} \{ -r_t' \Omega^D r_t + \beta [ X_{t+1}' S_{t+1}^D X_{t+1} ] \} \} \end{aligned} \quad (2.7')$$

$$\text{where: } S_{t+1}^D = p S_{Dt+1}^D + (1-p) S_{Dt+1}^R \quad (6.25)$$

We can then substitute the equations (6.23) and (6.24) in (2.7') and compute the following first order conditions:

$$\begin{aligned} & [\gamma_{2t}' \Omega^D \gamma_{1t} + \beta \theta_{2t}' S_{t+1}^D \theta_{1t}] X_t + \\ & [\gamma_{2t}' \Omega^D \gamma_{2t} + \beta \theta_{2t}' S_{t+1}^D \theta_{2t}] U_t^D = 0 \end{aligned} \quad (6.26)$$

that can be rewritten as:

$$NN^D_t X_t + MM^D_t U^D_t = 0 \quad (6.27)$$

Then solving for  $U^D_t$  we get:

$$U^D_t = \Gamma^D_t X_t \quad (6.28)$$

$$\text{where: } \Gamma^D_t = -(MM^D_t)^{-1} NN^D_t$$

that represents the rule for the control variables followed by party D if it is in power at time  $t$ .

We can similarly find the policy rule followed by party R if we assume that this is the party in power at  $t$ . We would then maximize the value function (2.10) (instead of (2.7)) that is the one relative to party R. Then, repeating the procedure for R we would get the following policy rule:

$$U^R_t = \Gamma^R_t X_t \quad (6.28')$$

$$\text{where: } \Gamma^R_t = -(MM^R_t)^{-1} NN^R_t$$

Substituting these rules in (6.22) (equations for  $e_t$ ) we then get the stable manifold for the jumping variables assuming alternatively that D or R is in power at  $t$ :

$$e_t = H_t^i X_t \quad (i=D,R) \quad (6.29)$$

$$\text{where:} \quad \beta_{1t} + \beta_{2t} \Gamma_t^i \quad (i=D,R)$$

Then, through substitutions of the policy rules (6.28) and of the jumping variable rules (6.34) in the equations (6.23) and (6.24), we can finally express the state vector  $X_{t+1}$  and the targets  $r_t$  as functions of the states  $X_t$  when party  $i$  is in power at time  $t$ :

$$X_{t+1}^i = \Sigma_t^i X_t \quad (i=D,R) \quad (6.30)$$

$$r_t^i = \Psi_t^i X_t \quad (i=D,R) \quad (6.31)$$

$$\text{where:} \quad \Sigma_t^i = \theta_{1t} + \theta_{2t} \Gamma_t^i \quad (i=D,R)$$

$$\Psi_t^i = \gamma_{1t} + \gamma_{2t} \Gamma_t^i \quad (i=D,R)$$

Finally we can go back to the value functions (2.7'), (2.8) (2.10) and (2.11) for the two parties, substitute back the solutions for states and targets ((6.30)-(6.31)) and obtain general recursion rules for the  $S$  matrices:

$$S_{Dt}^D = (\Psi_t^D, \Omega^D \Psi_t^D + \beta \Sigma_t^D, S_{t+1}^D \Sigma_t^D) \quad (6.32)$$

$$S_{Rt}^R = (\Psi_t^R, \Omega^R \Psi_t^R + \beta \Sigma_t^R, S_{t+1}^R \Sigma_t^R) \quad (6.33)$$

$$S_{Dt}^R = (\Psi_t^R, \Omega^D \Psi_t^R + \beta \Sigma_t^R, S_{t+1}^D \Sigma_t^R) \quad (6.34)$$

$$S_{Rt}^D = (\Psi_t^D, \Omega^R \Psi_t^D + \beta \Sigma_t^D, S_{t+1}^R \Sigma_t^D) \quad (6.35)$$

We have therefore derived recursion rules and starting values for the policy rules of the two parties (equations (6.28) and (6.28')), for the jumping variables (equations (6.29)) and for the matrices defining the value functions (6.5) to (6.8). Then the time consistent solution is the stationary solution to which the system converges for  $t=0$  as  $T$  goes to infinity. The backward recursion procedure is repeated until the rule matrices converge to a stable time-independent value. We do not know of a general proof of convergence in the presence of jumping variables, but have in practice experienced no difficulties in achieving convergence.

## VII. Conclusions

In this paper we have modelled the effects of electoral competition on the formulation of policies by rival political parties. In a sense, we look at the flip side of the political business cycle approach. In PBC models, competing politicians do not have intrinsic preferences over alternative policies; they choose policies only to improve their chances for election or

reelection. In the partisanship models discussed in this paper, the parties differ in their intrinsic preferences, presumably because the parties represent distinct constituencies of voters. The difference in approach is highlighted by our assumption of a constant, exogenous probability of reelection of each party.

We demonstrate that each political party adjusts the optimal rule that it would follow in the absence of political competition from the other party. In fighting inflation, for example, the optimal rate of disinflation for each party depends on the chance of the other party coming to power. In general, both parties opt for a more rapid rate of disinflation the higher is the chance that the other party will come to power in a future election.

Using dynamic programming techniques, we provide an algorithm for solving for the policy equilibrium of the two parties, given quadratic objectives, a linear dynamic structure, and exogenous election probabilities.

There are two major extensions to the analysis which we should like to mention briefly. First, it would be useful to extend the model to make the electoral probabilities a function of the policies pursued by the two parties and/or the performance of the economy during their tenure in office. Alesina (1986) has taken an important step in this direction. Second, it would be useful to expand the types of strategic interactions that are allowed for between the two parties. In particular, if the parties can base the current policies not merely on the current state of the economy, but also on the past history of the economy, then far

richer kinds of equilibria can be found. For example, the parties may find it to be in their common interest to converge to a common set of policies, with such convergence sustained by the mutual threat that if either party diverges from the common policies, then the other party will revert to party-specific policies to the detriment of the other party. Such an equilibrium of course requires a history-dependent strategy for each party, which has been so far ruled out in this paper. Once again, see Alesina (1986) for some results in this area.

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