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SHOULD SOCIAL SECURITY  
BENEFITS INCREASE WITH AGE?

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ABSTRACT

This paper shows that the optimal relation between social security benefits and retiree age depends on balancing the advantage of providing an otherwise unavailable actuarially fair annuity against the lower rate of return earned in a pay-as-you-go social security system. The ability of compulsory social security programs to provide an actuarially fair annuity implies that benefits should increase with age while the lower return on social security contributions than on private saving implies that a larger fraction of total benefits should be paid during the early years of retirement. In an economy that contains a mixture of rational life cycle savers and completely myopic individuals who do no saving, it is optimal for benefits to decline during the earlier part of the retirement period and then to begin rising. Numerical calculations based on actual macroeconomic parameters and representative survival probabilities suggest that the optimal age for minimum benefits occurs before age 75.

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## Should Social Security Benefits Increase with Age?

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Although individuals can save while they are working to finance consumption during their retirement years, they cannot purchase actuarially fair annuities with which to spread their accumulated wealth over the uncertain retirement period. Because insurance companies cannot know as much about individuals' health and life expectancy as the individuals themselves, an adverse selection problem leads to the underprovision of annuity insurance.<sup>1</sup> As a result, individuals are forced to leave involuntary bequests and to consume less during their retirement years than an actuarially fair annuity would permit.

In contrast to the limited private annuity market, compulsory public social security retirement systems can provide actuarially fair annuities. This feature is a potentially important justification for mandating such benefits even though the implicit return on social security is less than the return on private investments.<sup>2</sup>

It is perhaps surprising therefore that in practice the social security program provides each retiree with a real benefit that is fixed for life. It seems at least plausible that the social security program should instead provide a lower level of benefits in the early retirement years (when most individuals have savings with which to finance consumption) and a higher level

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of benefits in the later years (when the uncertainty of survival and the absence of actuarially fair private annuities make the availability of social security benefits more important). The present paper examines this conjecture and shows the conditions under which it is true.

It is useful to begin the analysis by considering the simple but extreme case in which individuals are completely myopic: they do no saving and are therefore completely dependent on social security for their retirement consumption. Section 1 discusses the optimal age profile for social security benefits in such an economy. A more appropriate framework for deriving the optimal annuity structure recognizes that some fraction of the population are completely myopic while others save rationally for their old age. Before considering this general case, section 2 goes to the opposite extreme from section 1 and examines the optimal annuity structure when everyone is a rational life-cycle saver. Looking at this case permits separating the effects of imperfections in the private annuity market and individual myopia in the design of the optimal social security annuity structure. Finally, section 3 considers the general problem of a mixed economy with both myopes and lifecyclers who face an imperfect private annuity market. Section 4 presents a numerical example and discusses the application of the analysis. There is a brief concluding section.

#### 1. The Optimal Annuity Structure with Complete Myopia

The economy examined in this paper is an extension of the overlapping generations model first developed by Samuelson (1958).<sup>3</sup> Instead of the Samuelsonian two-period framework, individuals work for two periods and then

retire for either one or two periods. All individuals are alike and each has a probability  $p$  of surviving to the second retirement period. The population grows at rate  $n$  per period and wages grow at rate  $g$ . (For concreteness, I shall later assume that each period is 10 years long.)

If the number of new retirees at time  $t$  is  $R_t$ , the number of older retirees who survived from the previous vintage of retirees is  $pR_{t-1}$ . The social security program pays benefits of  $b_{1t}$  to the younger retirees at time  $t$  and  $b_{2t}$  to the older retirees at time  $t$ .

All workers at each point in time are paid the same wage,  $w_t$ . If the total number of workers at time  $t$  is denoted  $L_t$  and the social security tax rate is  $\theta$ , the social security tax collections are  $T_t = \theta w_t L_t$ . The pay-as-you-go nature of the social security program implies the budget constraint:

$$(1) \quad b_{1t}R_t + b_{2t}pR_{t-1} = \theta w_t L_t.$$

To focus the present analysis on the structure of the annuity, I will assume that the level of the social security tax,  $\theta$ , is fixed. Since individuals are myopic and do no saving, the appropriate welfare criteria in this sector can be written as a function of consumption only.

Each individual's utility during retirement will be written in the separable form  $u(c_3) + v(c_4)$  where  $c_3$  is consumption during the first retirement period (i.e. the third period of life) and  $c_4$  is the consumption during the second period of retirement. I take social welfare in each period to be the sum of the utilities of the individuals alive in that period:

$$(2) \quad w_t = R_t \cdot u(c_{3t}) + pR_{t-1} \cdot v(c_{4t}).$$

Since individuals are completely myopic and therefore do no retirement saving during their working years, all retirement consumption is financed by social security:  $c_{3t} = b_{1t}$  and  $c_{4t} = b_{2t}$ . The optimal design of the social security benefits is then equivalent to maximizing

$$(3) \quad w_t = R_t \cdot u(b_{1t}) + pR_{t-1} \cdot v(b_{2t}),$$

subject to the government budget constraint given by equation (1). It follows immediately that  $u' = v'$  at the optimal levels of benefits. Thus if the two utility functions are identical, the optimal benefits are the same in each period:  $b_{1t}^* = b_{2t}^*$ .<sup>4</sup>

Since real wages are rising at rate  $g$  per period, the common level of benefits is also increasing at rate  $g$  per period. The equality of benefits of the younger and older retirees at each point in time therefore means that the optimal level of each individual's own benefits increases at rate  $g$  between the early retirement period and the late retirement period.

## 2. The Optimal Annuity Structure with Rational Life Cycle Savers

Consider now an economy in which each individual is a rational life-cycle saver but in which no private annuity market exists. Each individual saves during his working years and then chooses an optimal level of consumption from these accumulated assets during the first period of his retirement. The remaining assets plus the interest on them are consumed in the second retirement period if the individual survives. If the individual dies at the

end of the first period, those assets are bequeathed. I shall assume that in deciding his own consumption the individual gives no weight to these bequests. The social welfare function will of course take them into account.

To derive explicit results, I will now follow the log-linear utility specification of Feldstein (1985, 1987) and posit that individuals who will retire in period  $t$  maximize  $\ln c_{1,t-2} + \ln c_{2,t-1} + \ln c_{3,t} + p \ln c_{4,t+1}$  subject to a lifetime budget constraint

$$(4) \quad [(1-\theta)w_{t-2} - c_{1,t-2}](1+r)^3 + [(1-\theta)w_t - c_{2,t-1}](1+r)^2 + (b_{1t} - c_{3t})(1+r) - c_{4,t+1} = 0$$

It follows directly that the individual's optimal spending plan is:

$$(5) \quad c_{s,t+s-3}^* = \frac{p^k(1+r)^{s-1}}{3+p} \{ (1-\theta)w_{t-2} [1 + (1+g)/(1+r)] + \frac{b_{1t}}{(1+r)^2} + \frac{b_{2,t+1}}{(1+r)^3} \}$$

where  $k = 1$  for  $s = 4$  and  $k = 0$  for the previous periods. Note that since wages rise at rate  $g$  per period, benefits per retiree also increase at that rate; thus,  $b_{2,t+1} = b_{2t}(1+g)$ .

Since the number of persons in each cohort rises at rate  $n$ , social welfare at time  $t$  can be written:

$$(6) \quad W_t = R_t [(1+n)^2 \ln c_{1,t} + (1+n) \ln c_{2,t} + \ln c_{3t} + \frac{p}{1+n} \ln c_{4t} + \frac{\beta(1-p)}{1+n} \ln c_{4t}]$$

The last term represents the value of the bequests made by those who do not survive to the second retirement period; the coefficient  $\beta$  reflects the relative weight given to these bequests in the social welfare function.

It follows from equation (5) that

$$(7) \quad c_{s,t} = \left(\frac{1+r}{1+g}\right)^s \frac{p^k}{3+p} \left\{ (1-\theta)w_t \left[1 + \frac{1+g}{1+r}\right] + b_{1t} \left(\frac{1+g}{1+r}\right)^2 + b_{2t} \left(\frac{1+g}{1+r}\right)^3 \right\}.$$

Combining equations (6) and (7) implies

$$(8) \quad \frac{w_t}{R_t} = \alpha_1 \ln \left\{ (1-\theta)w_t \left[1 + \frac{1+g}{1+r}\right] + b_{1t} \left(\frac{1+g}{1+r}\right)^2 + b_{2t} \left(\frac{1+g}{1+r}\right)^3 \right\} + \alpha_2$$

where  $\alpha_1 = (1+n)^{-1} [(1+n)^3 + (1+n)^2 + (1+n) + p + \beta(1-p)]$  and

$$\alpha_2 = -\alpha_1 \ln(3+p) + (1+n)^{-1} [(1+n)^2 + 2(1+n) + 3p + 3\beta(1-p)] \ln[(1+r)/(1+g)].$$

The important feature of  $\alpha_1$  and  $\alpha_2$  at this point in the analysis is that they are constants, independent of  $\theta$ ,  $b_{1t}$  and  $b_{2t}$ . Thus maximizing social welfare is equivalent to maximizing  $\left\{ (1-\theta)w_t \left[1 + \frac{1+g}{1+r}\right] + b_{1t} \left(\frac{1+g}{1+r}\right)^2 + b_{2t} \left(\frac{1+g}{1+r}\right)^3 \right\}$ .

Moreover, since the current analysis takes the level of the social security tax rate ( $\theta$ ) as given, maximizing social welfare is equivalent to maximizing  $b_{1t} + (1+g)/(1+r)b_{2t}$  subject to the budget constraint is still given by equation (1) that

$$(9) \quad b_{1t} + \frac{p}{1+n} b_{2t} = \frac{\theta w_t L_t}{R_t}.$$

Since  $\theta$  is not a choice variable, the right hand side of (9) is predetermined.

It is immediately clear that the problem of maximizing  $b_{1t} + [(1+g)/(1+r)]b_{2t}$  subject to (9) and to the constraints that  $b_{1t} \geq 0$  and  $b_{2t} \geq 0$  does not have an interior solution. Welfare is an increasing function of  $b_{1t}$  if  $(1+g)/(1+r) < p/(1+n)$ ; in this case the optimal social security program would be a lump sum to new retirees with no benefits to older retirees. Conversely, if  $(1+g)/(1+r) > p/(1+n)$ , the optimal program would provide no

benefit to new retirees and a lump sum to those who survive to the second period.

To understand this result, note that the key inequality can be restated as a comparison of the survival probability  $p$  and the "efficiency of social security,"  $x = (1+n)(1+g)/(1+r)$ . This term can be characterized as the efficiency of social security because it compares the implicit return on social security  $[(1+n)(1+g)]$  to the return earned on private assets  $(1+r)$ . Previous analysis has emphasized that since social security is inefficient in the sense that  $x < 1$ , a social security program is justified only to the extent that the provision of benefits to those myopic individuals who would otherwise save too little outweighs the losses to the rational life cyclers who are forced to sacrifice a return of  $1 + r$  in exchange for a return of  $(1+n)(1+g)$ .

In the present context in which the size of the overall social security program is fixed and in which all individuals are rational life cyclers, the comparison of  $x$  and  $p$  indicates whether the gain from social security's ability to provide a fair annuity outweighs the loss due to its lower rate of return. If  $x < p$ , the return on social security is so low that individuals are better off receiving a lump sum social security payment when they retire with no second period benefit at all. Another way of stating this is to note that  $x < p$  is equivalent to  $(1+g)(1+n) < p(1+r)$ , i.e., the return provided by the social security annuity  $[(1+g)(1+n)]$  is less than the return from private saving reduced by the mortality probability  $[p(1+r)]$ .

In the alternative case, the low returns on the social security annuity is nevertheless great enough to exceed the expected return on private saving:

$(1+g)(1+n) > p(1+r)$  or  $x > p$ . In this case, optimal social security benefits should be paid only to those who survive to the second stage of retirement.<sup>5</sup>

These results show that the optimal time structure of retirement benefits for rational life-cyclers depends on balancing the gains from annuity risk protection against the loss from relying on the low yielding social security instead of higher yielding private assets.

The analysis of this case of rational life cyclers is useful in understanding the more general case to which I now turn.

### 3. Optimal Annuity Structure When Some Individuals are Myopic

I now follow the analysis of Feldstein (1985) and assume that a fraction  $\mu$  of the population are complete myopes who never save anything while the remain  $1 - \mu$  are rational life cyclers.

At time  $t$ , the per capita consumption of the working myopes is  $(1-\theta)w_t$  and the per capita consumption of the two cohorts of retired myopes is  $b_{1t}$  and  $b_{2t}$ . The number of newly retired myopes is  $\mu R_t$  and of older myopes is  $\mu p R_{t-1} = \mu p R_t / (1+n)$ . Similarly, the numbers of working myopes in the younger and older cohorts are  $\mu(1+n)^2 R_t$  and  $\mu(1+n) R_t$ . The component of social welfare at time  $t$  attributable to the myopes is thus  $\mu R_t \{ (1+n)^2 \ln(1-\theta)w_t + (1+n) \ln((1-\theta)w_t) + \ln b_{1t} + [p/(1+n)] \ln b_{2t} \}$ . Combining this with the corresponding component of social welfare attributable to the rational life-cyclers as previously derived in equation (8) yields the social welfare value at time  $t$ :

$$(10) \quad w_t = \mu R_t [(1+n)(2+n) \ln \theta w_t + \ln b_{1t} + p(1+n)^{-1} \ln b_{2t}] \\ + (1-\mu)R_t \alpha_1 \ln \left\{ (1-\theta)w_t \left[ 1 + \frac{1+g}{1+r} \right] + b_{1t} \left( \frac{1+g}{1+r} \right)^2 + b_{2t} \left( \frac{1+g}{1+r} \right)^3 \right\} + (1-\mu)R_t \alpha_2.$$

The optimal annuity structure is derived by maximizing 10 subject to the government's budget constraint that  $b_{1t}R_t + b_{2t}pR_{t-1} = \theta w_t L_t$ . After factoring out  $R_t$  and noting that  $\theta$  is a constant, the maximand can be written

$$(11) \quad \hat{w}_t = \mu \ln b_{1t} + \mu p(1+n)^{-1} \ln b_{2t} \\ + (1-\mu)\alpha_1 \ln \left\{ (1-\theta)w_t \left[ 1 + \frac{1+g}{1+r} \right] + b_{1t} \left( \frac{1+g}{1+r} \right)^2 + b_{2t} \left( \frac{1+g}{1+r} \right)^3 \right\} \\ - \lambda [b_1 + p(1+n)^{-1}b_2 - \theta w_t L_t (1+n)^{-1}].$$

The first order conditions are then:

$$(12) \quad \frac{d\hat{w}_t}{db_{1t}} = \frac{\mu}{b_{1t}} + \frac{(1-\mu)\alpha_1 \left( \frac{1+g}{1+r} \right)^2}{(1-\theta)w_t + (1-\theta)w_t \left( \frac{1+g}{1+r} \right) + b_{1t} \left( \frac{1+g}{1+r} \right)^2 + b_{2t} \left( \frac{1+g}{1+r} \right)^3} - \lambda = 0$$

and

$$(13) \quad \frac{d\hat{w}_t}{db_{2t}} = \frac{\mu p}{(1+n)b_{2t}} + \frac{(1-\mu)\alpha_1 \left( \frac{1+g}{1+r} \right)^3}{(1-\theta)w_t + (1-\theta)w_t \left( \frac{1+g}{1+r} \right) + b_{1t} \left( \frac{1+g}{1+r} \right)^2 + b_{2t} \left( \frac{1+g}{1+r} \right)^3} - \frac{\lambda p}{1+n} = 0$$

These first order conditions together imply

$$(14) \quad \frac{1}{b_2} - \frac{1}{b_1} = -\beta \left[ \frac{(1+g)(1+n)}{1+r} - p \right]$$

where

$$\beta = \frac{(1-\mu)(1+g)^2}{\mu p(1+r)^2 \left[ (1-\theta)w_t + (1-\theta)w_t \left(\frac{1+g}{1+r}\right) + b_{1t} \left(\frac{1+g}{1+r}\right)^2 + b_{2t} \left(\frac{1+g}{1+r}\right)^3 \right]} > 0.$$

Writing  $x = (1+g)(1+n)/(1+r)$  for the efficiency of social security, equation (14) implies that  $b_2 > b_1$  if  $x > p$  and  $b_2 < b_1$  if  $x < p$ .

Two general things should be noted about this result. First, the question of whether older retirees get higher or lower benefits than younger retirees depends only on the comparison of social security efficiency ( $x$ ) and the mortality probability ( $p$ ) and not on the relative frequency of myopes in the population ( $\mu$ ). Second, the effect of the fraction of myopes in the population is to dampen the sensitivity of the benefit difference ( $b_2 - b_1$ ) to the difference between  $x$  and  $p$ . Thus, while the previous section showed that  $x < p$  implies that social security is so inefficient that rational lifecyclelers would prefer to receive all of their benefits at retirement (i.e.,  $b_2 = 0$ ), when there are myopes as well as lifecyclelers, some benefits must be provided in both periods. Nevertheless,  $x < p$  implies that first period benefits are higher than second period benefits ( $b_1 > b_2$ ). Conversely, when social security is efficient enough so that the actuarially fair return at rate  $(1+g)(1+n)$  exceeds  $p(1+r)$ , the expected return on private saving without an annuity, then the optimal structure of social security benefits gives higher benefits to the older retirees.

#### 4. A Numerical Example and a Simple Extension

Although the social security efficiency ( $x$ ) depends only on basic macroeconomic parameters ( $g$ ,  $n$  and  $r$ ), the survival probability relevant for determining the age structure of benefits depends on the age limits of the social security benefit groups. This section examines the relation described by equation (14) with the help of realistic numerical values and shows the implications of dividing the overall retirement period in different ways.

Consider an economy in which individuals who reach age 65 all live for 10 years and may live for another 10 years. No one lives beyond age 85. The survival probability  $p$  will be approximated by the actual probability of surviving from age 70 to age 80. According to The Vital Statistics of the United States (Bureau of the Census, 1986, p. 69), this survival probability for a white male in 1982 was 0.53.

The value of  $x$  depends on the rate of population growth, the rate of wage growth and the real rate of return on private capital. The rate of population growth over the past three decades averaged 1.4 percent a year. Since we are using ten year period,  $1 + n = (1.014)^{10} = 1.15$ . Similarly, real compensation grew at an average annual rate of 2.2 percent, implying  $1 + g = (1.022)^{10} = 1.24$ . Finally, since the real pretax rate of return on nonfinancial corporate capital averaged 10.4 percent from 1955 through 1984 (Feldstein and Jun, 1986),  $1 + r = (1.104)^{10} = 2.69$ . Taken together, these imply that  $x = 0.53$  for a decade period.

This coincidental equality of the macroeconomic parameter  $x = 0.53$  and the decade survival probability  $p = 0.53$  implies that  $b_1^* = b_2^*$ , i.e., that the benefits of retirees aged 65 to 75 should be the same as the benefits of concurrent retirees aged 75 to 85. Note that since the overall benefit level

rises over time at the rate of growth of wages, this equality of the benefits of younger and older retirees means that each individual's benefit increases as he ages. More specifically, real wages growing at 2.2 percent a year implies that  $1 + g = (1.022)^{10} = 1.24$  and therefore that the benefits of 75 to 85 year olds should be 24 percent higher than the benefits that those same individuals received when they were 65 to 75 years old.

More generally, this analysis suggests that if social security benefits can vary with each year of age rather than by decade, the benefits of retirees should decline at early ages while the annual value of  $x = (1.014)(1.022)/(1.104) = 0.94$  is less than the annual survival probability and should then begin increasing with age. For example, in 1982 the annual survival probability of white males fell to 0.94 at age 75. Thus, optimal benefits would decline among retirees aged 65 to 75 and would then begin to increase.

Since this is a cross-sectional relation, each individual's benefits would begin to increase at an earlier age. The age at which an individual's benefits begin to increase corresponds to the point at which the 2.2 percent annual rise in the general level of benefits outweighs the cross-sectional decline in benefits. Thus at some point between the ages of 65 and 75 optimal benefits stop declining and begin to increase.

## 5. Conclusion

This paper shows that the optimal relation between social security benefits and retiree age depends on balancing the advantage of providing an otherwise unavailable actuarially fair annuity against the lower rate of

return earned in a pay-as-you-go social security system. If the tax rate that finances social security benefits is fixed, the ability of compulsory social security programs to provide an actuarially fair annuity implies that benefits should increase with age while the lower return on social security contributions than on private saving implies that a larger fraction of total benefits should be paid during the early years of retirement. In an economy that contains a mixture of rational life cycle savers and completely myopic individuals who do no saving, it is optimal for benefits to decline during the earlier part of the retirement period and then to begin rising. Numerical calculations based on actual macroeconomic parameters and survival probabilities suggest that the optimal age for minimum benefits occurs before age 75.

The present analysis has assumed a complete absence of private annuity markets. If private annuity markets do exist but provide a return that is less than actuarially fair, the value of the survival probability ( $p$ ) in the present analysis must be replaced by  $p/p^*$  when  $p^* > p$  is the survival probability implicit in the private annuity. Thus benefit rise with age if and only if  $x > p/p^*$ . In the limiting case of an actuarially fair private annuity ( $p^* = p$ ), social security benefits should decline with age as long as  $x < 1$ .

This analysis suggests that the ability of social security to provide an actuarially fair annuity that cannot be provided by the market implies that the optimal level of social security taxes is higher than the level derived in Feldstein (1985) for an economy in which the retirement period was certain. Similarly, the annuity aspect of compulsory social security may alter the conditions (derived in Feldstein, 1987) in which a means tested program is

preferable to a universal program. These issues will be examined in subsequent research.

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Footnotes

1. On the absence of actuarially fair annuities, see Friedman and Warshawsky, (1985a, 1985b).
2. See Feldstein (1985) for an analysis that shows that it may be optimal to have no social security in an economy in which the implicit return on social security is low even though individuals save too little privately for their own old age.
3. I have extended that model to deal with related issues in Feldstein (1985, 1987).
4. Although it is of course possible to argue that differences in the utility function imply a different benefit structure, it is not clear in which direction this difference points. Younger retirees may have a higher marginal utility of consumption at each level of spending because they are healthier and therefore able to engage in a broader range of activities. Alternatively, the older retirees may have a higher marginal utility of consumption at each level of spending because they have higher fixed costs for medical care and other personal services. While recognizing both possibilities, the present analysis proceeds on the assumption that both utility functions are the same.
5. Since borrowing secured by future social security benefits is illegal, the optimal level of  $b_2$  may be constrained to be not greater than the level of final period consumption that the individual would choose conditional on that value of  $b_2$ . This type of consideration is clearly important if we recognize a large number of retirement subperiods.

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