NBER WORKING PAPER SERIES

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Working Paper 21355
http://www.nber.org/papers/w21355

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
July 2015

We thank Joe Aldy, Spencer Banzhaf, Geoffrey Heal, Charlie Kolstad, Robert Mendelsohn, and seminar participants at NBER, GSU, RFF, William and Mary, and Yale for helpful comments, and Kalee Burns, Bo Liu, and Drew Moxon for valuable research assistance. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 21355
July 2015
JEL No. C61,H23,Q54,Q58

ABSTRACT

We consider the socially optimal use of solar geoengineering to manage climate change. Solar geoengineering can reduce damages from atmospheric greenhouse gas concentrations, potentially more cheaply than reducing emissions. If so, optimal policy includes less abatement than recommended by models that ignore solar geoengineering, and the price of carbon is lower. Solar geoengineering reduces temperature but does not reduce atmospheric or ocean carbon concentrations, and that carbon may cause damages apart from temperature increases. Finally, uncertainty over climate change and solar geoengineering alters the optimal deployment of solar geoengineering. We explore these issues with an analytical model and a numerical simulation. The price of carbon is 30%-45% lower than the price recommended in a model without geoengineering, depending on the parameterizations of geoengineering costs and benefits. Carbon concentrations are higher but temperature is lower when allowing for solar geoengineering. The optimal amount of solar geoengineering is more sensitive to climate uncertainty than is the optimal amount of abatement.

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I. Introduction

Greenhouse gases (GHGs) like carbon dioxide (CO_2) contribute to climate change and thus create negative externalities. The standard Pigouvian solution to the market failure caused by negative externalities is to price the externality at marginal external damages. Solar geoengineering (SGE, also called albedo modification or solar radiation management (SRM)) is an alternative way to reduce the damages from GHGs: instead of reducing the quantity of GHGs, SGE can, at least in part, reduce the damages that they inflict. A Pigouvian tax on GHG emissions will not create incentives for SGE. If SGE is part of the optimal policy portfolio, then a Pigouvian tax alone cannot bring about the first-best. Furthermore, if the Pigouvian tax is set at the level of marginal external damages without SGE, then the tax will be too high relative to the optimum level (which is equal to marginal external damages with SGE), and abatement (emissions reductions) will be too high, resulting in welfare loss. It has been argued that SGE implementation may be substantially cheaper than abatement, creating a large welfare loss from ignoring SGE. But SGE also introduces new sources of damages and uncertainty, possibly eroding the welfare gains from its implementation, or even yielding welfare losses.

The purpose of this paper is to investigate, theoretically and numerically, how the possibility of SGE affects optimal climate policy. Does SGE substantially reduce the optimal carbon tax? Does ignoring SGE lead to policies that encourage too much abatement at too high a cost? How does uncertainty over climate change and over SGE damages affect optimal policy? How important is the fact the SGE reduces temperature but does not reduce carbon concentrations? We develop a theoretical model that captures these effects and demonstrates the welfare effects of introducing or ignoring SGE. We augment a standard integrated assessment model (IAM) of climate change by adding the possibility of a specific type of SGE to the Dynamic Integrated Climate-Economy (DICE) model (Nordhaus 2008). Costs and benefits of SGE are calibrated from various prior estimates, though we note that there is substantial uncertainty. Therefore, we conduct extensive sensitivity analyses. We caution that the purpose of this paper is not to argue one way or the other about the merits of SGE or to estimate the optimal level of its deployment, but rather to investigate the qualitative point that ignoring SGE in models may lead to biased and incomplete policy recommendations.

Geoengineering is often thought of as an "insurance" against harsh or abrupt climate damages – as something that should be figuratively kept behind glass only to be broken in case of emergency. By contrast, our main approach begins by treating SGE as a policy option, just like abatement, with its own costs and benefits. Optimal use of SGE is evaluated by comparing marginal benefits to marginal costs. When both abatement and SGE are policy tools, both should be used to the point where the marginal benefits of each are equal. In the last section of the paper, we compare this optimal approach to an alternative "insurance" approach in which SGE is not used until a temperature threshold is reached.

A small but growing literature examines the economics of solar geoengineering.[1] One branch of that literature focuses on governance. Barrett (2008) explores the "incredible economics of geoengineering," by which he means the fact that SGE is (potentially) so much cheaper than emissions abatement that it could be undertaken by a single country.[2] This creates a unique set of administrative problems. In fact, if the key problem with administering abatement policy is the inability to achieve international consensus to act, the key problem with SGE might be ensuring that there is not too *much* implemented, since any number of nations may do it independently. A series of studies examine this issue of international governance for SGE. Ricke et al. (2013) look at the incentives behind the formation

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[1] In addition to the small but growing literature in economics on GE, there is a large scientific literature on the subject. Latham et al. (2014) and the associated special journal issue provide a recent introduction.

[2] The prospect of low-cost GE is not universally accepted. For instance, Keller et al. (2014) use an Earth system model to simulate several different types of GE in the presence of high GHG emissions (no abatement), and they find that the effects of GE on warming are limited (less than an 8% reduction) and the side effects are potentially severe.

of coalitions to implement SGE. These incentives are different than those behind coalitions to abate GHGs. With SGE, there are incentives to keep coalitions small so that action can be taken. Victor (2008) argues for norms to govern the deployment of SGE. Weitzman (2012) models SGE as a "free-driver" problem analogous to the "free-rider" problem of abatement. He notes that SGE, depending on the level at which it is undertaken and the nation in question, can be either a public good or a public bad (thus he labels it a "public gob"). Moreno-Cruz (2015) models two countries agreeing on both mitigation and geoengineering, and shows that SGE can lead to inefficiently high levels of mitigation. These papers are primarily concerned with how international agreements can be crafted to implement SGE (or to prevent too much implementation).[3]

A second strand of the literature, which is smaller though perhaps more fundamental, focuses on the optimal use of SGE. Moreno-Cruz and Keith (2013) incorporate SGE into a two-period model of climate change and solve for optimal policy. They find that the uncertainty related to SGE is an important determinant of optimal policy. Including SGE can reduce the overall costs of climate policy by around 2 percentage points of GDP. Other papers have added SGE to integrated assessment models (IAMs) and examined the policy implications. Bickel and Lane (2009) show that SGE promises potentially large net benefits, though there is substantial uncertainty. They model two types of geoengineering: solar radiation management (SRM) and air capture (AC), and they conclude that SRM is cheaper and more cost-effective. They conduct a benefit-cost analysis of various levels of implementation of SGE, and they consider how implementing SGE affects carbon taxes. But, they do not solve for an optimal level of SGE. Goes et al. (2011) make several modifications to the DICE model, including allowing SGE

[3] Barrett (2014) discusses the literature on the governance of geoengineering, what he calls "the fundamental problem posed by geoengineering." Rayner et al. (2013) present the "Oxford Principles," a set of five guidelines for international GE governance. A similar argument is related to moral hazard: deployment of GE may reduce or eliminate the willingness to reduce carbon emissions (Corner and Pidgeon 2014).

and refining the climate dynamics. Their specification imposes an exogenous intermittency in SGE which makes it less effective.[4] They present summaries of policies with an optimal mix of abatement and SGE (subject to the intermittency), but they do not present implications for policy, i.e. carbon taxes with SGE. Bickel and Agrawal (2013) extend the analysis of Goes et al. (2011) by considering alternate conditions under which SGE would be deployed; in contrast to Goes et al. (2011), Bickel and Agrawal (2013) find that under some scenarios a substitution of SGE for abatement can pass a cost-benefit test. Gramstad and Tjøtta (2010) include SGE in DICE and conduct a cost-benefit analysis of GE under various assumptions about the level undertaken and its costs. Under all specifications, SGE passes a cost-benefit analysis, with net benefits ranging from $1.5 trillion to $17.8 trillion. Postponement of SGE by 30-50 years reduces those net benefits by less than 10%. They do not consider carbon taxes or the optimal level of SGE.[5]

Our paper falls under this second strand of the literature that examines optimal SGE policy. Our paper's contribution is threefold. First, we focus on how the inclusion of SGE affects optimal abatement and the optimal carbon tax. Our theoretical model shows that including SGE reduces the price of carbon. Since SGE appears to be so much cheaper than abatement, it is possible that including SGE will drastically reduce the price of carbon. It is theoretically possible that optimal policy will involve a corner solution with no abatement (and hence no carbon taxes), though we show numerically that this does not hold because of SGE's inability to deal with carbon concentrations. To quantify this, we modify the

[4] Jones et al. (2013) also investigate the effect of abrupt suspension of GE (a "termination effect"), using a simulation of 11 different climate models. Also see Ross and Matthews (2009).

[5] Klepper and Rickels (2012 and 2014) provide review articles on the economics of geoengineering. Emmerling and Tavoni (2013) use a different IAM, WITCH, to model SGE and abatement policy. Wigley (2006) uses the Model for the Assessment of Greenhouse Gas-Induced Climate Change (MAGICC) to study how SGE and abatement are related.

DICE model to include the possibility of SGE and use it to solve for optimal policy, where both abatement and SGE are policy options. While solving for the optimal carbon price is a straightforward extension of other papers that have used an IAM with SGE to find optimal policy, we argue that it is nonetheless an important contribution that should not be overlooked. If SGE means that the optimal carbon price is much lower than current estimates of the social cost of carbon, this has very important policy implications. We calculate the welfare loss of ignoring this fact. We also explore how different assumptions about various parameter values affect the time path of optimal policy. For instance, it is well known that small changes to the discount rate have large changes in optimal abatement paths, but little is known about how discounting affects optimal SGE paths, or how discounting affects abatement when SGE is an option.[6]

Our second contribution to the literature is our focus on uncertainty. There are substantial uncertainties about the costs, benefits, and risks of SGE given the present state of scientific understanding.[7] There is also uncertainty in our understanding of the climate, in particular over the equilibrium climate sensitivity, which measures how much temperature changes after doubling CO_2 concentrations from pre-industrial levels. We characterize these uncertainties in our analytical model and derive policy implications. We also use a stochastic version of DICE to model uncertainty in geoengineering and in the climate system.

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[6] Barrett (2014) considers four different options for the time path of SGE, and Keith (2013) recommends starting at a low level of SGE and gradually increasing its use, but neither uses an IAM to generate optimal policy. Likewise, Wigley (2006) notes that more intensive use of SGE can reduce the need for abatement but does not consider optimal SGE levels.

[7] The National Academy of Sciences has recently issued a report providing a technical evaluation of SGE proposals, jointly sponsored by the NOAA, the CIA, NASA, and the Energy Department (National Research Council 2015).

Third, in our models (analytical and numerical) we explicitly distinguish between damages from carbon concentrations and damages from temperature. Unlike abatement, SGE reduces temperatures without reducing carbon concentrations, either atmospheric or oceanic. Both types of carbon stocks may lead to damages, even if temperatures are brought back to preindustrial levels. For instance, ocean acidification may deplete corals and fisheries, and atmospheric carbon may affect precipitation patterns. Other papers have mentioned this issue, but to our knowledge ours is the first to incorporate it into a theoretical model or numerical simulation of geoengineering policy.

We find that SGE unambiguously lowers the optimal level of abatement and the optimal price of carbon in the model. The degree to which it does so is sensitive to parameter values. In our base case specification, the optimal level of abatement is up to 25 percentage points lower than the optimal level without SGE, and the elimination of all carbon emissions is delayed by five decades. Ignoring SGE can increase overall costs of climate change by one-half to one percent of GDP. Optimal abatement levels are less sensitive to parameter values and to uncertainty in equilibrium climate sensitivity than are optimal SGE levels. The degree to which damages from climate change arise from carbon directly, rather than from temperature, substantially affects optimal SGE deployment; if a high fraction of damages are from carbon, then SGE is used less intensively.

We focus on SGE, which is just one type of geoengineering. The recent National Academy of Sciences report (National Research Council 2015) differentiates between two broad categories of geoengineering: albedo modification (which includes SGE) and carbon dioxide removal (CDR). CDR eliminates the damages from carbon and does not have the negative side effects of SGE. However, it is prohibitively costly, so we omit it from our analysis. We model sulfur SGE because it is the most promising of the albedo modification options. We do not explicitly model adaptation, which is a third alternative (along with mitigation/abatement and geoengineering) for dealing with climate change. The notion that geoengineering can be a substitute for mitigation also extends to adaptation.[8] The costs of climate damages in our model can be interpreted as being net of adaptation; we leave it to future work to separately model adaptation from geoengineering.

The next section of the paper introduces our theoretical model, which provides the framework for our inclusion of SGE into the DICE model. Section 3 briefly describes how we include SGE in the DICE model; details are in the appendix. Section 4 presents our simulation results, and section 5 concludes.

II. Theoretical Model

Consider a representative agent who has access to an endowment of a fixed stock of capital k. That capital can be allocated in three ways: towards production k_p, towards abatement k_a, or towards solar geoengineering k_g; $k = k_p + k_a + k_g$. Allocating capital towards production yields a level of potential output $f\left(k_p\right)$, with $f' > 0$ and $f'' < 0$. This is potential output, because actual output is reduced due to damages from pollution x. Actual output (all of which is consumed) is $y = c = f\left(k_p\right)\left(1 - d\left(x; k_g\right)\right)$. The damage function $d \in [0,1]$ represents the fraction of potential output that is lost because of pollution x, and $d_x > 0, d_{xx} > 0$ (damages are increasing and convex in pollution, but must be bounded by 1 at the solution). Solar geoengineering k_g affects damages also, with $d_k < 0$ and $d_{xk} < 0$. That is, solar geoengineering reduces absolute and marginal damages. Pollution x is determined by the total capital endowment and the fraction abated μ, so that $x = (1 - \mu)k$. The fraction abated is a function of abatement capital: $\mu = g(k_a)$, where $g' > 0, g'' < 0$. The model does not explicitly include adaptation, but the damage function can be interpreted as being net of adaptation.

[8] Aldy (2015) argues that both adaptation and geoengineering need to be considered when calculating the social cost of carbon.

The planner's problem is to allocate the capital stock so as to maximize actual output (or equivalently maximize a monotone utility function over actual output). That is,

$$\max_{k_p,k_a,k_g} f(k_p)\left(1-d(x;k_g)\right)$$

such that

$$k = k_p + k_a + k_g$$

$$x = \left(1-g(k_a)\right)k$$

First, consider the constrained solution to this problem that omits SGE, or sets $k_g = 0$. The solution to this constrained problem is analogous to policy recommendations by IAMs that ignore SGE. The first-order condition for the constrained problem is

$$f'(k_p^c)\left(1-d(x^c;0)\right) = f(k_p^c)kg'(k_a^c)d_x(x^c;0), \tag{1}$$

where k_p^c, k_a^c, and x^c indicate solutions to the constrained problem.[9] The left-hand side of equation (1) is the marginal cost of an additional unit of abatement, which is the foregone marginal output that could have been produced by allocating to production k_p instead of abatement k_a. The right-hand side is the marginal benefit of an additional unit of abatement, which is the reduction in damages caused by pollution from the extra unit of abatement. In a decentralized economy, the right-hand side of this equation is the optimal pollution tax when GE is ignored (as it is in many IAMs).

Next, consider the unconstrained problem where SGE is not fixed at zero. This solution is characterized by two first-order conditions:

$$f'(k_p^{opt})\left(1-d(x^{opt};k_g^{opt})\right) = f(k_p^{opt})kg'(k_a^{opt})d_x(x^{opt};k_g^{opt}) \tag{2}$$

$$d_x(x^{opt};k_g^{opt})g'(k_a^{opt})k = -d_k(x^{opt};k_g^{opt}) \tag{3}$$

where $k_p^{opt}, k_a^{opt}, k_g^{opt}$, and x^{opt} indicate solutions to the unconstrained problem (i.e. the optimal levels). Equation (2), as in equation (1) in the constrained case, equates the marginal cost of an

[9] Throughout, we assume interior solutions and assume that the second-order conditions are satisfied.

additional unit of abatement with its marginal benefit. In a decentralized economy, the optimal carbon tax is the right-hand side of equation (2). In equation (3), the left-hand side represents the marginal benefit of an additional unit of abatement (divided through by potential output f), and the right-hand side represents the marginal benefit of an additional unit of GE.

Consider the market for abatement k_a under both the constrained and unconstrained problem, as shown in the top half of Figure 1. The x-axis is the amount of abatement. The line $MC|k_g = 0$ is the marginal cost of abatement conditional on no SGE ($k_g = 0$); it equals $f'(k - k_a)\left(1 - d\left(\left(1 - g(k_a)\right)k, 0\right)\right)$, which is the left-hand side of equation 1 evaluated at arbitrary k_a. The line $MB|k_g = 0$ is the marginal benefit of abatement with no SGE; it equals $f(k - k_a)kg'(k_a)d_x\left(\left(1 - g(k_a)\right)k, 0\right)$. The appendix shows that MC is increasing and MB is decreasing in k_a. Where these are equal, their value is the optimal price of carbon, conditional on no SGE, as indicated by $\tau|k_g = 0$; this is the price determined by equation 1.

Allowing for SGE affects the marginal benefit of abatement, and intuition suggests that the marginal benefit curve allowing for SGE will be lower than the marginal benefit curve with no SGE. SGE reduces the damages from a unit of emissions, and therefore it reduces the marginal benefits of abatement. This intuition is verified in the appendix, and thus the curve $MB|k_g^{opt}$, which is the marginal benefit of abatement conditional on the optimal level of SGE, is drawn in Figure 1 lower than $MB|k_g = 0$. Assuming that introducing SGE does not change the marginal cost of abatement, then the optimal price of carbon conditional on optimal SGE is lower than the optimal price of carbon ignoring

SGE. Figure 1 demonstrates the deadweight loss in the abatement market (the triangle labeled DWL) from setting a carbon tax that ignores SGE.[10]

The bottom half of Figure 1 shows that in equilibrium, the marginal benefit of an additional unit of abatement capital will equal the marginal benefit of an additional unit of solar geoengineering. The left-hand side is the market for abatement, and the right-hand side is the market for solar geoengineering. The curves in blue represent the marginal costs and benefits of solar geoengineering, at a constant level of abatement (equal to k_a^c). As k_g increases and solar geoengineering is implemented, the optimal level of abatement k_a decreases, so emissions increase. Thus, the marginal benefit of each unit of solar geoengineering increases, since more pollution is allowed and temperatures are warmer without solar geoengineering. This is represented by an upward shift in the marginal benefit curve to the red curve. In equilibrium, the marginal benefits of solar geoengineering (on the right half) will increase just enough and the marginal benefits of abatement (on the left half) will decrease just enough so that the optimal quantity of each choice variable is such that the marginal benefits are equal across the two

[10] Allowing for SGE also affects the marginal cost curve, though in Figure 1 we have ignored the fact that $MC\big|k_g = 0 \neq MC\big|k_g^{opt}$. The appendix shows that $MC\big|k_g = 0 < MC\big|k_g^{opt}$. This is because, for any abatement level, allowing SGE makes the damages from pollution lower, and thus potential output higher, and therefore the marginal cost of abatement (foregone output) higher. But, the appendix also argues that the difference between the two marginal cost curves is likely to be small, unlike the difference between the two marginal benefit curves, which is why we have ignored the change in MC in Figure 1. Since the marginal cost under optimal SGE is (slightly) higher than under no SGE, the optimal price of carbon under optimal SGE will be closer to the optimal price of carbon ignoring GE than shown in Figure 1 ignoring the change in marginal costs. But, the deadweight loss from ignoring SGE could be higher or lower than that shown in Figure 1 (although the quantity of abatement will be lower, the cost of each unit over the optimal is higher).

markets. Extending the new optimal carbon price across the graphs will intersect the equilibrium in the solar geoengineering market.

II.A. Uncertainty

We amend the model to include uncertainty. Suppose that there are two random variables that affect the damage function *d*: call them θ_x and θ_g. θ_x represents uncertainty about the relationship between pollution and damages, for instance, uncertainty about equilibrium climate sensitivity, or uncertainty over how temperatures affect the economy. The other shock, θ_g, represents uncertainty over solar geoengineering – either its implementation costs, its efficacy in controlling temperatures, or its negative side effects. Realistically, the third of these is the major source of uncertainty regarding SGE.

We assume that the variance of either of these shocks affects marginal damages in the following way: $\frac{\partial E[d_x]}{\partial Var(\theta_x)} > 0$ and $\frac{\partial E[d_k]}{\partial Var(\theta_g)} > 0$. A higher variance in θ_x means that the expected marginal damages from pollution are higher. This could arise from the damage function itself, or it could reflect risk aversion in preferences, where the damage function d incorporates that risk aversion. A higher variance in θ_g increases d_k, that is, it makes it less negative – so it reduces the marginal benefits from solar geoengineering. Again, this could arise from the form of the damage function itself, or it could reflect risk aversion. We also assume that $\frac{\partial E[d]}{\partial Var(\theta_x)} > 0$ and $\frac{\partial E[d]}{\partial Var(\theta_g)} > 0$: both of these shocks affect the expected level of damages, not just the derivative. We make no assumptions over the "cross" effect of the shocks: $\frac{\partial E[d_x]}{\partial Var(\theta_g)}$ or $\frac{\partial E[d_k]}{\partial Var(\theta_x)}$.

Given these uncertainties, the planner allocates production, abatement, and solar geoengineering to maximize expected net output. The implicit function theorem can be used on the two first-order conditions to present comparative static results on how uncertainty in either pollution damages or in solar geoengineering effectiveness affects optimal policy. The details of the derivation are presented in the appendix; here we present the results.

The appendix shows that

$$\frac{\partial k_a}{\partial Var(\theta_x)} = \frac{1}{Det}\left\{\left[f'(k_p)\left(g'(k_a)k\frac{\partial E[d_x]}{\partial k_g} + \frac{\partial E[d_k]}{\partial k_g}\right)\right]\frac{\partial E[d]}{\partial Var(\theta_x)} + A\frac{\partial E[d_x]}{\partial Var(\theta_x)} + B\frac{\partial E[d_k]}{\partial Var(\theta_x)}\right\} \tag{4}$$

Here Det is the determinant of the Jacobian matrix of the first-order conditions and is positive, and A and B are positive terms defined in the appendix. The second and third terms in the brackets are signed and easily interpretable. Since uncertainty over climate damages increases the expected marginal damages from pollution ($\frac{\partial E[d_x]}{\partial Var(\theta_x)} > 0$), it increases optimal abatement. The increase in the uncertainty of pollution damages makes abatement more attractive. If uncertainty over climate damages also increases d_k – i.e. reduces the marginal benefits from SGE – then through this channel it also increases optimal abatement. The first term in equation 4, which is multiplied by $\frac{\partial E[d]}{\partial Var(\theta_x)}$, has ambiguous sign. Its first component, $g'(k_a)k\frac{\partial E[d_x]}{\partial k_g}$, is negative. The extent to which solar geoengineering reduces the marginal damages from pollution – d_{kx} – means that climate uncertainty's effect on total expected damages reduces optimal abatement. This is because more solar geoengineering will be employed, and the more that SGE reduces marginal damages from pollution, the less abatement is needed. The second component of the coefficient on $\frac{\partial E[d]}{\partial Var(\theta_x)}$, $\frac{\partial E[d_k]}{\partial k_g}$, is positive so long as $d_{kk} > 0$ – that is, the marginal benefits of solar geoengineering are decreasing. Since uncertainty over climate increases expected damages, increased use of SGE will be less effective and so more abatement will need to be used to compensate, hence this effect makes $\frac{\partial k_a}{\partial Var(\theta_x)}$ positive.

Also, the appendix shows that

$$\frac{\partial k_g}{\partial Var(\theta_x)} = \frac{1}{Det}\left\{\left[f'(k_p)\left(-g''(k_a)kE[d_x] - g'(k_a)k\frac{\partial E[d_x]}{\partial k_a} - \frac{\partial E[d_k]}{\partial k_a}\right)\right]\frac{\partial E[d]}{\partial Var(\theta_x)} - C\frac{\partial E[d_x]}{\partial Var(\theta_x)} - D\frac{\partial E[d_k]}{\partial Var(\theta_x)}\right\} \quad (5)$$

Here the terms C and D are both positive and defined in the appendix. As with equation 4, here the second and third terms are unambiguous. Since uncertainty over climate increases marginal damages from pollution, this effect decreases optimal SGE – more abatement is used instead of SGE. If uncertainty over climate increases d_k (reduces marginal benefits from SGE), then this reduces optimal SGE because it is less effective. The first term in equation 5, multiplied by $\frac{\partial E[d]}{\partial Var(\theta_x)}$, has ambiguous sign. The first two components, $-g''(k_a)kE[d_x] - g'(k_a)k\frac{\partial E[d_x]}{\partial k_a}$, are positive. As expected damages are higher with more climate uncertainty, this will lead to more optimal SGE, since marginal damages are positive ($E[d_x]$) and increasing ($\frac{\partial E[d_x]}{\partial k_a}$). The remaining component multiplying $\frac{\partial E[d]}{\partial Var(\theta_x)}$, $-\frac{\partial E[d_k]}{\partial k_a}$, is negative since $d_{kx} < 0$. Because increased use of SGE will decrease marginal damages from pollution, there is an effect making optimal use of SGE lower – less is needed since marginal damages are lower.

Comparing the two sets of ambiguous terms multiplying $\frac{\partial E[d]}{\partial Var(\theta_x)}$ in equations 4 and 5 yields the following conclusion: if the cross-partial derivative d_{kx} is not too large, then higher uncertainty in climate damages will increase use of both abatement and SGE. That is, there is a scale effect since expected damages are larger, so it is optimal to use more of both tools available. The negative cross-partial derivative d_{xk} means that each of the two policy tools (abatement and SGE) makes the other less effective, and thus this effect serves to reduce the use of each.

The equations for the effect of uncertainty in solar geoengineering on optimal policy are identical to the equations above, except with partial derivatives with respect to $Var(\theta_g)$:

$$\frac{\partial k_a}{\partial Var(\theta_g)} = \frac{1}{Det}\left\{\left[f'(k_p)\left(g'(k_a)k\frac{\partial E[d_x]}{\partial k_g} + \frac{\partial E[d_k]}{\partial k_g}\right)\right]\frac{\partial E[d]}{\partial Var(\theta_g)} + A\frac{\partial E[d_x]}{\partial Var(\theta_g)} + B\frac{\partial E[d_k]}{\partial Var(\theta_g)}\right\} \quad (6)$$

$$\frac{\partial k_g}{\partial Var(\theta_g)} = \frac{1}{Det}\left\{\left[f'(k_p)\left(-g''(k_a)kE[d_x] - g'(k_a)k\frac{\partial E[d_x]}{\partial k_a} - \frac{\partial E[d_k]}{\partial k_a}\right)\right]\frac{\partial E[d]}{\partial Var(\theta_g)} - C\frac{\partial E[d_x]}{\partial Var(\theta_g)} - D\frac{\partial E[d_k]}{\partial Var(\theta_g)}\right\} \quad (7)$$

As before, there are unambiguous effects from how uncertainty affects the first derivatives. Since $\frac{\partial E[d_k]}{\partial Var(\theta_g)} > 0$ (uncertainty about SGE reduces expected marginal benefits of SGE), more uncertainty about SGE leads to less SGE and more abatement; SGE is a less-attractive option, and abatement is a substitute for it. Note that $Var(\theta_g)$ could also be taken to represent uncertainty about damages from SGE; for example, the possibility that SGE will damage the ozone layer. In fact, it is this aspect of d_k that motivates most of the uncertainty around SGE technology. However, the ambiguous first term (multiplying $\frac{\partial E[d]}{\partial Var(\theta_g)}$) indicates that uncertainty over SGE could increase SGE use.

In the numerical simulations below we incorporate uncertainty over both climate damages and SGE.

II.B. Decomposition of Climate Damages

We amend the model to consider that damages from climate change may occur both from temperature changes, which SGE addresses, and from carbon concentrations, which SGE does not address. To model this simply, we separate the damage function into two components, only one of which is affected by solar geoengineering: $d(x; k_g) = \lambda_d d_1(x) + d_2(x; k_g)$. Damages that occur from temperature change, which SGE can remedy, are modeled by d_2, and damages from carbon concentrations, which SGE cannot remedy, by d_1. When $\lambda_d = 0$, this becomes the original model. But when $\lambda_d > 0$, there is a separate component of damages that cannot be alleviated with SGE, and so each unit of SGE is less effective at reducing damages from pollution. We conduct comparative statics on how this value affects optimal abatement and SGE policy. When the fraction of climate damages from carbon, rather than from temperature, increases, λ_d will increase.

The appendix shows that

$$\frac{\partial k_a}{\partial \lambda_d} = C \cdot [g'(k_a)kd_{2xk} + d_{2kk}] \tag{8}$$

$$\frac{\partial k_g}{\partial \lambda_d} = C \cdot [(-g''(k_a)kd_{2x} + g'(k_a)^2k^2d_{2xx} + kg'(k_a)d_{2xk})] \tag{9}$$

The constant C is defined in the appendix and is positive.

In equation 8, the first term in brackets is negative, and the second term is positive. As more climate damages come from carbon rather than temperature (higher λ_d), the second (positive) term reflects the fact that more abatement is warranted, since it is the only approach that addresses carbon. In equation 9, the first two terms in brackets are positive, and the third term is negative. As more abatement is used because of a higher λ_d, there is less need for SGE to alleviate d_2 because the cross-partial derivative is negative. This is captured in the third (negative) term. However, in each of the above equations there is a term or terms of opposite sign to the above intuition. The negative term in equation 8 reflects the fact that, as more abatement is employed to counter increased damages from carbon (d_1), the damages from temperature (d_2) are less intensive and therefore somewhat less abatement may be needed. The positive terms in equation 9 reflect the fact that an increase in λ_d increases the magnitude of climate change damages overall, and some of that can be alleviated with increased SGE. This effect will be larger as d_{2x} and d_{2xx} are larger; that is, as marginal damages from temperature are greater and increasing.

The model in this section provides intuition but omits many important details. For instance, it is static, though climate change is inherently dynamic. Thus, in the following section we incorporate SGE into a dynamic IAM. Though the output of the IAM is difficult to interpret intuitively (it is a "black box"), the intuition developed in this section will be carried over to the results from the IAM simulations.

III. Solar Geoengineering and DICE

The dynamic integrated climate-economy (DICE) model by William Nordhaus is an IAM designed to solve for optimal GHG abatement policy and calculate the social cost of carbon. It includes a representative-agent economic model with an endogenous capital stock and an exogenous level of technological growth. Carbon emissions are a byproduct of production but can be reduced through expenditure on abatement. The climate component of the model includes several equations describing the dynamic interaction between carbon concentrations in several layers: the atmosphere and upper and lower oceans. The atmospheric carbon concentration affects the atmosphere's radiative forcing; that is, the difference between the amount of heat energy absorbed by Earth and that radiated back into space. The human-caused change in radiative forcing is ultimately what affects atmospheric temperatures. Finally, the climate and economy sections of the model are integrated together since increases in temperature cause reductions in total economic output. The model can be run to solve for optimal (welfare-maximizing) carbon abatement trajectories. Given marginal abatement costs, the optimal price of carbon is a byproduct of the model's output. A time period in the 2007 version of the model is one decade, and the model is typically run over several dozen periods (hundreds of years).

The DICE model and its results have been refined over the years, and summaries of the model's equations and results are available in Nordhaus (2008) as well as Nordhaus's personal webpage.[11] A key takeaway from the model runs are that the social cost of carbon in the present is positive (typically around $30 per ton of CO_2), and it is gradually increasing over time to reflect the increase in carbon concentrations and thus in marginal damages per ton of carbon.

IAMs like DICE have been criticized. Pindyck (2013) argues that they tell us "very little" and are "close to useless" because so many of the calibrated parameter values are ad hoc with little empirical foundation. This is demonstrated by the fact that the policy recommendations can be so sensitive to arbitrarily chosen parameter values, for instance the discount rate. Because our numerical analysis relies

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[11] http://www.econ.yale.edu/~nordhaus/homepage/

on DICE, it is subject to these criticisms. However, even if one accepts these critiques and is skeptical of IAMs, we argue that our analysis has merit. Though the point estimates of optimal policy paths should be interpreted with caution, how they vary with parameter values (i.e. the sensitivity analysis) still provides insight. Further, the simulations demonstrate that it is important to consider SGE in optimal policy design, with or without IAMs. In these respects, the use of DICE can be seen as another argument in favor of Pindyck's (2013) and others' critiques of IAMs.

III.A. Modifications to DICE

Here we briefly describe our modifications to DICE to include SGE; details are available in the appendix. The appendix also details our calibration of the model. We have modified DICE in the following five ways. First, while the only choice variable in the original DICE model is carbon abatement, we add a second choice variable to reflect the choice of the intensity of solar geoengineering. Abatement intensity a can take values between zero and one: when $a = 0$, there is no abatement, and $a = 1$ means all carbon emissions are abated (zero emissions). The choice variable for the intensity of SGE is g. When g equals zero, this represents no SGE. When $g = 1$, this represents "full" SGE, i.e. fully offsetting the warming effects from increased carbon concentrations. However, unlike abatement intensity a, SGE intensity g could take a value larger than 1, representing more than fully offsetting temperature increases from climate change.

Second, there is a cost of implementing solar geoengineering, analogous to the cost of abatement. This cost, expressed as a fraction of gross output that is lost to SGE implementation, is a quadratic function of SGE intensity g. To completely offset global warming ($g = 1$) costs 0.27% of global GDP in our base case.

Third, we add damages from solar geoengineering, analogous to the original model's specification of damages from climate change. For instance, sulfates are expected to exacerbate ozone depletion (Heckendorn et al. 2009). Precipitation could be drastically reduced (Ferraro et al. 2014, Robock et al. 2008). The sulfates injected into the stratosphere may condense and fall back to the atmosphere, contributing to acid rain[12]. Like the damages from climate change, these are expressed as a fraction of gross output that is lost, and they are a quadratic function of SGE intensity g. We are very conservative (i.e. biased *against* SGE) in our base-case calibration: SGE at full intensity ($g = 1$) leads to damages of 3% of global GDP.

Fourth, the benefits of solar geoengineering are modeled as directly modifying the radiative forcing equation (see the appendix for details). This implies that SGE can reduce temperatures much more quickly than abatement can. Fifth and finally, we decompose the damages from climate change so that they depend directly on temperature and also on atmospheric and ocean carbon concentrations. In the base case, 80% of climate change damages are from temperature increase, 10% are from atmospheric carbon concentrations, and 10% are from ocean carbon concentrations. By contrast, in the original DICE model and all of the previous studies that have modified DICE to include geoengineering, 100% of damages are from temperature.

A unique contribution of this paper is to treat uncertainty and stochastic processes by adopting a stochastic optimization technique, rather than only using conventional sensitivity analyses or Monte Carlo (MC) simulation. The advantage of our approach over sensitivity analysis is that it incorporates the prior knowledge about probability distributions of uncertain parameters into the solution method. Although our approach features a similar random sampling as MC simulation, its advantage over MC simulation is that the numerical results are used to develop the optimal strategy rather than merely demonstrating the range of possible outcomes. The optimal strategy then is used to produce the prediction for any new realization of the uncertain parameter. This is the key advantage of stochastic optimization techniques over conventional MC simulation: in stochastic optimization (or "reinforcement

[12] Though Kravitz et al. (2009) find that this effect will be insubstantial.

learning" in computer science language) the agent updates its optimal decision in the face of uncertainty based on a large (finite) number of previous observations of random realizations of uncertain parameter and outcome but in MC simulation, the collection of the agent's responses to realizations of random variable form a probability distribution that defines the range of optimal policy.

The other SGE DICE papers conduct sensitivity analyses, but they do not model uncertain or stochastic parameters. Several papers, including Baker and Solak (2011) and Kolstad (1996), modify DICE to include uncertainty, but without SGE.[13] In this paper we consider continuous state and probability spaces and adopt a unique stochastic approximation technique for finding the optimal strategy in the face of uncertainty. We do not "invent" a new approximation function, and we use the already calculated functions within the model as building blocks of our value function approximation. This reduces the number of tunable parameters and substantially limits the subsequent optimal search domain. Furthermore, this algorithm is intuitive in the sense that it forecasts a limited number of steps ahead given the current realization of the uncertain parameter and uses this as an insight to predict values of future states. Details of the solution method are available in the appendix. Finally, the appendix compares our modifications to DICE to the modifications made by the small number of other papers that have considered geoengineering (see appendix table 1).

IV. Simulation Results

IV.A Baseline Simulations

[13] Kelly and Kolstad (1999) use neural network approximations to obtain flexible functional form of the value function. Oppenheimer et al. (2008) discretize the uncertainty over equilibrium climate sensitivity and solve the stochastic DICE model in discrete deterministic stages. Webster et al. (2012) use two parametric and non-parametric methods to approximate the value function. Cai et al. (2013) apply Chebyshev polynomial approximation for value function estimation.

In order to understand the way solar geoengineering affects optimal climate policy, we start by analyzing the deterministic case. We compare the outcomes of the baseline scenario to the case of no solar geoengineering; that is, a model that does not allow for solar geoengineering. The results are presented in Figure 2. The first panel shows how abatement is affected when solar geoengineering is introduced as a viable policy instrument. The introduction of solar geoengineering lowers the level of abatement and delays the time when we transition to a clean economy. Once abatement reaches it maximum level, geoengineering begins to decline. However, it stays positive for some time because of the lag in the effect of emissions on temperature. The optimal SGE deployment is a "ramping-up" policy, starting out at low levels and gradually increasing as the damages from climate change increase. Although SGE is allowed to take a value greater than 1, its maximum value is just about one-half (i.e. offsetting half of the increase in radiative forcing from carbon concentrations). This is because the benefits from SGE are traded off against the (substantial) damages. Eventually, SGE use declines towards zero, since carbon concentrations are reduced. SGE is a substitute for abatement in the short- and medium-run, but eventually abatement dominates.

In the next two panels we look at carbon dioxide concentrations and temperature changes. Because of the lower level of abatement, carbon dioxide concentrations peak at a higher level and later in the presence of solar geoengineering. Concentrations peak at 1600ppm, relative to the case of No SGE where concentrations peak at 1400ppm. But with solar geoengineering, temperature peaks much earlier and it is kept at check below the 2 degrees mark. This is the buying-time effect, often cited in the literature, where solar geoengineering keeps the system below deleterious levels of climate change while the abatement technology improves enough to eliminate emissions (Keith 2013, Moreno-Cruz and Smulders 2007). This is done at the cost of allowing for higher concentrations. Thus, there is a tradeoff between carbon damages and temperature damages, as well as solar geoengineering costs and abatement costs.

The fourth panel in Figure 2 shows that the carbon price is lower when solar geoengineering is introduced; it peaks at a lower level before it starts to decline at the rate of learning by which the costs of the backstop technology decline. As the analytical model shows, ignoring solar geoengineering leads to an optimal carbon price that is too high. After 100 years, the optimal carbon price is about 30% lower than the price from the model ignoring solar geoengineering; after 200 years it is about 45% lower.[14]

What is remarkable about all these results is that they do not arise because solar geoengineering is very cheap, since we are very conservative about the costs and damages of SGE. These results are due to the use of solar geoengineering directly on the radiative forcing equation. This reduces the inertia of the climate system, reducing the amount of abatement needed today to reduce concentrations in the future. Thus, by postponing costly abatement to future periods, solar geoengineering helps to reduce the aggregate costs of climate change. This is demonstrated in the last panel of Figure 2, which plots the costs in proportional GDP loss of ignoring solar geoengineering. For instance, at year 200, this value is 1.52%, indicating that net GDP (after accounting for climate damages, SGE damages, abatement costs, and SGE costs) is 1.52% lower in the "No SGE" simulation than it is in the baseline simulation. This corresponds to the area of deadweight loss from the analytical model in Figure 1.

These deterministic simulation results verify what we find in the analytical model – allowing for solar geoengineering reduces the optimal level of abatement, reduces the optimal carbon price, and reduces total policy costs.

[14] This carbon price equals marginal external damages along the optimal policy path as solved through DICE. In contrast, the term "social cost of carbon" often refers to marginal external damages along the baseline path, as a way of valuing reductions in carbon emissions. Aldy (2015) notes that this price (the social cost of carbon) can also be affected by the availability of geoengineering (and adaptation).

IV.B Variation in the Composition of Damages – Temperature vs. Carbon

Because SGE reduces temperatures without reducing atmospheric or ocean carbon concentrations, it cannot completely offset all damages from climate change. In the baseline specification, we assume that 80% of climate damages are directly from temperature, 10% are from atmospheric carbon concentrations, and 10% are from ocean carbon concentrations (see the appendix for details). In Figure 3, we present simulation results where we vary this decomposition of climate damages. In addition to the baseline case, we simulate three other damage decompositions, in each of which damages from temperature only account for 50% of climate damages. The remaining 50% of damages are split between atmospheric and ocean carbon 25/25, 40/10, or 10/40.

Comparing the baseline case to any of the three alternate decompositions shows that there is more solar geoengineering and less abatement when temperature accounts for a higher fraction of climate damages. SGE is less effective relative to abatement when temperature accounts for less damages, and so less of it is deployed. This corresponds to the results from the analytical model that $\frac{\partial k_a}{\partial \lambda_d} < 0$ and $\frac{\partial k_g}{\partial \lambda_d} > 0$. Comparing the three alternative decompositions to each other shows that there is more solar geoengineering and less abatement when ocean carbon concentrations account for a higher fraction of damages than do atmospheric carbon concentrations. Abatement more directly affects atmospheric rather than ocean carbon, since the absorption of emitted carbon by the ocean is gradual and slow. If atmospheric carbon is more damaging than ocean carbon, more abatement and less SGE are needed.

The actual composition of damages between ocean carbon, atmospheric carbon, and temperature is unknown. In fact, atmospheric carbon may yield benefits from increased agricultural productivity. The purpose of this analysis is not to provide policy recommendations but rather to demonstrate the importance of research on measuring these distinct damages from climate change and incorporating them into assessment models. For mitigation policy, the distinction is unimportant. But because SGE severs the direct link between carbon and temperature, the distinction matters.

IV.C Uncertainty and Stochasticity

We now allow DICE to be solved allowing for uncertainty or stochasticity in certain parameters as described above. For a distribution of policy paths, we present the mean of the policy outcome variables and their 5th and 95th percentile paths. We also compare the solutions under uncertainty with the solutions in the deterministic case. We allow two different variables to be random: equilibrium climate sensitivity and SGE damages. Of course, in the real world there is uncertainty over many more parameters in the model (perhaps all of them) and over the model specification itself. We focus on the uncertainty in these two parameters because they allow us to compare more general uncertainty in the climate system with uncertainty that is specific to SGE.

First, we allow equilibrium climate sensitivity to be uncertain. This parameter describes the equilibrium temperature change that results from a doubling of atmospheric carbon. In our deterministic case this is set to 3. It takes on a truncated log-normal distribution, calibrated based on the IPCC report (IPCC 2013). The lower and upper bounds are 0.1 and 20, respectively; the mean and standard deviation are 1.1 and 0.55, respectively. This parameterization is used in Shayegh and Thomas (2015).

Figure 4 presents the policy simulation results. For each of the simulated outcomes (abatement, solar geoengineering, temperature, atmospheric carbon, and the price of carbon), we present the mean value (in green) across the 1000 simulations, the 5th and 95th percentiles, and the value from the deterministic case (in red). The green and red curves are very close to each other, indicating that the average policy outcome is not much different than the deterministic case (this is because the distribution of the equilibrium climate sensitivity is set so that its average is the deterministic value).

However, both abatement and SGE are slightly higher in the deterministic case than in the mean value across the random simulations. This corresponds to our prediction from the theoretical model that $\frac{\partial k_a}{\partial Var(\theta_x)}$ and $\frac{\partial k_g}{\partial Var(\theta_x)}$ can be positive. The 5th and 95th percentile values demonstrate that uncertainty over equilibrium climate sensitivity affects optimal solar geoengineering policy much more so than it affects optimal abatement policy. The 5th to 95th percentile bands for abatement, carbon concentrations, and the carbon price are very narrow. This is surprising, since the only uncertain variable in this simulation is equilibrium climate sensitivity, which is not directly related to solar geoengineering. This reflects the flexibility enabled by the use of SGE directly on the radiative forcing equation, making the SGE response more sensitive to small variations.[15]

Although these simulations allow for uncertainty in climate sensitivity, they all use an identical probability distribution of the uncertain parameter (equilibrium climate sensitivity). In fact, this distribution itself is unknown, and many different estimated distributions arise from various models (see, for example, Figure 1 in Millner et al. 2013). Thus, we re-run these simulations under alternative distributions for the uncertain climate sensitivity parameter, but with all other parameters kept at base case values. The results, which are presented in Appendix Figure 1, demonstrate that the optimal policy values are largely invariant to this distribution.

Second, we allow the damages from SGE, measured by the parameter ν_G, to be stochastic (see the appendix for details). These damages represent the primary source of uncertainty over SGE.[16] The distribution of this parameter is assumed to be lognormal, with a mean value of 0.03, identical to the

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[15] The confidence bands here are smaller than those in other simulations of optimal abatement policy (without SGE). We also run these simulations without the possibility of SGE, and we find confidence bands that are much larger and in line with the magnitudes from other studies. Allowing for SGE thus implies that optimal abatement paths are much less uncertain – the uncertainty is instead "transferred" to the optimal SGE path.

[16] National Research Council (2015).

value on the deterministic case, and a scale parameter equal to 1. Figure 5 presents these simulations. Here, SGE use is lower and abatement is higher in the deterministic simulations, compared to the mean of the random simulations (the green curves vs. the red curves). The theoretical model (equations 6 and 7) were ambiguous about this comparison. This result appears counterintuitive but it follows from the way geoengineering is defined in the model. SGE can be implemented to quickly reduce temperatures (that is, it can be greater than 1), and it enters directly in the forcing equation, eliminating the inertia associated with the carbon cycle. This implies that SGE is always useful, irrespective of how damaging it can be, but when its damages are low it is used substantially more relative to the deterministic case. [17] As in the case of uncertainty over equilibrium climate sensitivity, here with stochasticity over SGE damages, we find that stochasticity affects the distribution of optimal SGE policy by a much greater amount than it affects the distribution of optimal abatement policy. Optimal SGE can peak at anywhere between 10% and 150% intensity. As a result, temperatures can peak between 0.5 and 2.5 degrees above preindustrial levels.

IV.D SGE as Insurance

As discussed earlier, SGE is often thought of as an "insurance" policy that should only be used as an emergency response to unprecedented climate change. For instance, policymakers may want to prohibit SGE unless global average temperature increases beyond 2°C. While this is not optimal in our model (assuming that all costs and benefits of SGE are captured in our model), it may be more politically viable. In Figure 6, we present simulations in which SGE can only be deployed after temperature increases by 2°C; afterwards it is free to take any value. These simulations allow for uncertainty over

[17] We confirm this result by trying multiple values for the variance, also by changing the distribution form lognormal to normal and finally by treating SGE damages as uncertain and not stochastic. The result holds in all cases.

climate sensitivity, as in Figure 4. The red curve (representing the deterministic case) shows an abrupt increase in SGE once it is allowed, and a corresponding decrease in abatement activity and temperature. Before that point is reached, there is a slight inflation in the values of optimal abatement in the case of SGE as an insurance policy (Figure 6 relative to Figure 4). Since the timing of deploying SGE is uncertain and correlated with uncertainty in equilibrium climate sensitivity, more abatement is the only available option to hedge against such uncertainty. Lastly, the lower confidence interval band for optimal SGE deployment is flat at zero, since for those simulations temperature never reaches the threshold 2°C increase and SGE is never allowed.[18]

IV.E Sensitivity analysis

Lastly we consider how variation in certain parameters affects optimal policy. In these simulations, presented in Figure 7, we conduct deterministic simulations for several different values of certain parameters, along with the No SGE scenario. Figure 7 presents the optimal SGE deployment path under each parameter value; the Appendix Figures 3 through 6 present the other policy outcomes (including abatement and the price of carbon). We vary the costs of solar geoengineering (G_{coef}, Panel A), its effectiveness at counterbalancing radiative forcing (ϕ, Panel B), the damages associated with its implementation (ν_g, Panel C), and the social discount rate (ρ, Panel D). See the appendix for details on these parameters and their baseline calibrations.

As the implementation costs of solar geoengineering increase, less solar geoengineering is deployed. Because these costs are so low in the base case, an order-of-magnitude change in the coefficient in front of these costs has only a modest effect on SGE deployment; the maximum level of SGE intensity varies from 20% to 50%. Appendix Figure 3 shows that abatement, carbon concentrations,

[18] Simulations that allow for SGE and insurance under stochasticity of SGE damages, as in Figure 5, are presented in Appendix Figure 2, and they are qualitatively similar to the simulations in Figure 6.

and the carbon price are not very sensitive to this parameter. But even if we make solar geoengineering 10 times more costly than the base case, there is a substantial amount of warming that is still compensated by SGE. This reflects the fact that by eliminating the inertia of the carbon cycle and therefore allowing for postponing abatement, solar geoengineering decreases the total costs of climate change and increases welfare.

Panel B of Figure shows that solar geoengineering effectiveness affects its deployment – as SGE is more effective, it is used more intensively. When its effectiveness is very low ($\phi = 0.01$), it is barely used. When it is very effective ($\phi = 5.0$), it is immediately ramped up to 25% intensity, after which it gradually increases. Even after 500 years we see no decline in its intensity. Appendix Figure 4 shows that more effective SGE results in lower abatement and higher carbon concentrations, but lower temperatures. With a high effectiveness of $\phi = 2$, temperatures are brought back to pre-industrial levels after just 200 years.

Next, variation in the damages from SGE cause a very wide range of optimal SGE deployment, as seen in Panel C of Figure . When damages are an order-of-magnitude lower than the base case ($\nu_G = 0.003$), SGE eventually reaches greater than 100% intensity. The variation from damages, in Panel C, is so much larger than the variation from costs, in Panel A, because costs are so small and damages (at least in our conservative calibration) are quite large. Appendix Figure 5 demonstrates that the variation in optimal abatement and carbon concentrations is smaller than the variation in SGE deployment. With the lowest level of GE damages, mean temperatures are brought back to within 0.5 degrees of preindustrial levels by 200 years.

Finally, increasing the discount rate (Panel D of Figure and Appendix Figure 6) decreases the amounts of both abatement and solar geoengineering, as well as the carbon price. Abatement and solar geoengineering are postponed to a later stage, but also less solar geoengineering is implemented overall and at its peak. This suggests that more patient societies would tend to favor abatement over solar geoengineering.[19]

V. Conclusion

Solar geoengineering has the potential to lower the costs of dealing with climate change and reduce the need for abatement and a high carbon price. Three points are crucial. First, models that ignore solar geoengineering may prescribe policies that abate too much, cost too much, and have a carbon price that is too high. Second, uncertainty over both climate damages and solar geoengineering costs and damages can substantially affect optimal policy. Third, because solar geoengineering reduces temperatures but not carbon concentrations, it is merely an imperfect substitute for abatement. We explore these issues through both an analytical theoretical model and a numerical integrated assessment model of climate change. Our modification of the DICE model provides quantitative insights as to how solar geoengineering can affect optimal abatement policy. The level of abatement can be about 25% lower when allowing for solar geoengineering, and the optimal atmospheric carbon concentrations can be more than 20% higher. Despite that, temperature changes can be kept about one-and-a-half degrees Celsius lower because of the use of solar geoengineering, and total GDP losses can be lower by up to one-and-a-half percentage points of GDP. These base-case results are of course sensitive to the parameter values, which are very uncertain. Still, under a wide (two orders of magnitude) range in parameters describing the costs and damages of solar geoengineering, the optimal carbon price and level of abatement do not vary substantially, although the optimal level of solar geoengineering does vary substantially (ranging from nearly no solar geoengineering to more than 100%

[19] In the appendix, we also consider sensitivity analyses over a variable that is not directly related to SGE: the cost of abatement. See Appendix Figure 7 and the corresponding discussion.

solar geoengineering). As with all climate models, more precise parameter values are essential for pinning down specific policy recommendations.

We caution that these results should not be interpreted as a policy prescription for immediate deployment of solar geoengineering. The uncertainties surrounding the calibration of the model, in particular the damages associated with solar geoengineering, are too great to be able to do so. Instead, the main contribution of this paper is in its qualitative and quantitative exploration of how including SGE in climate models affects the optimal deployment of abatement and the price of carbon, of how uncertainty affects optimal policy, and of how important it is that solar geoengineering reduces temperatures but not carbon.

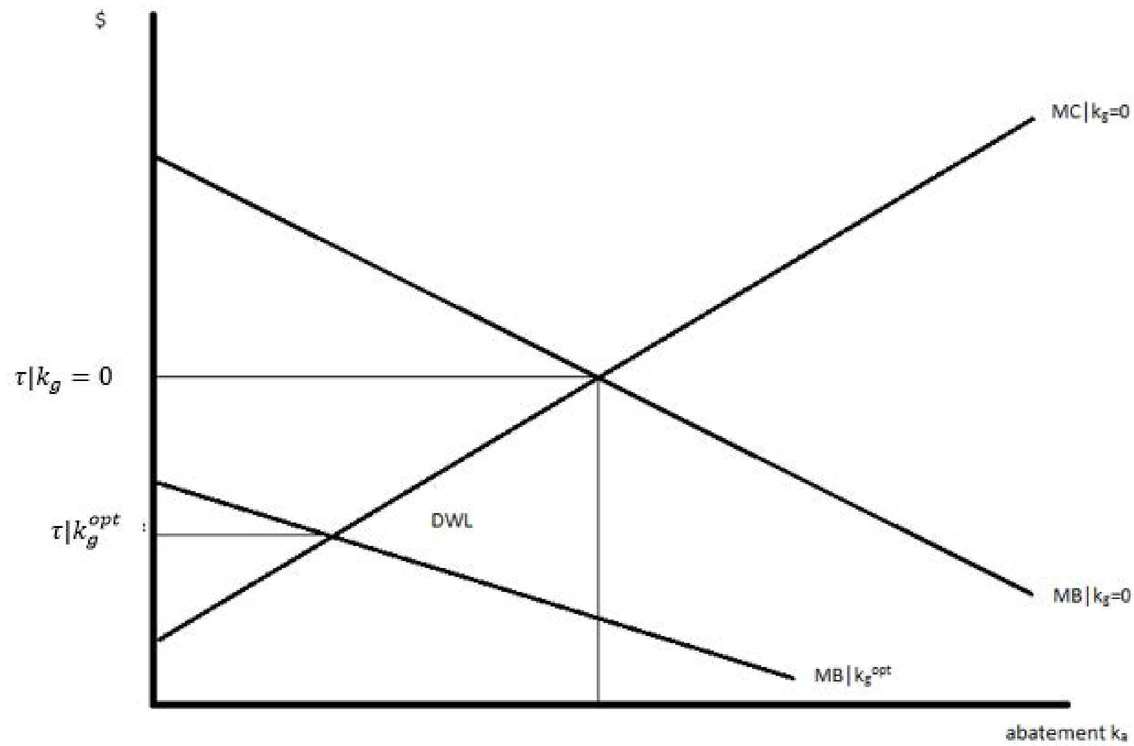
Still, the fundamental contribution made by this study has important policy implications. It is not efficient to merely estimate the marginal external damages of a ton of carbon and institute that carbon tax, if the external damages are estimated in a model without the possibility of solar geoengineering. Our results suggest that this may in fact be the case, and that for this reason the carbon price currently being used by policymakers may be too high.[20] Of course, there are many other potential reasons why the carbon price currently used may be too low – estimates may omit many benefits from carbon reductions.

Our research emphasizes the need for more information on costs and benefits of solar geoengineering. Extensions to our analysis may yield valuable policy lessons. Further research could expand the set of parameters modeled as uncertain variables, or add refinements to either the climate model in DICE or its treatment of economic costs or growth. The damages from SGE represent probably the most "unknown" of all of the features of this model. Research in progress is examining how solar geoengineering can address the issue of tipping points, or irreversibilities and discontinues in climate

[20] The SCC used by the EPA and other federal agencies is described here: http://www.epa.gov/climatechange/EPAactivities/economics/scc.html.

damages. Endogenous learning about climate or SGE should be considered, for instance by adopting the Bayesian learning framework of Kelly and Kolstad (1999). This would allow for a calculation of the value of information about SGE, and the benefits of research and development or field experiments. Explicitly modeling how adaptation affects optimal abatement and geoengineering is a fruitful extension that could yield additional insights. Because the model is dynamic, it can be used to examine intergenerational justice. The model could be disaggregated by region – potentially important if the effects of SGE are not uniform across the globe.[21] Finally, there are many issues related to SGE that we do not or cannot address using an IAM – including a fat-tailed distribution of risks, distributional effects, and ethical issues related to the question of abatement versus SGE.

[21] The RICE model is a regionally disaggregated extension of the DICE model. Kravitz et al. (2014) studies the regional disparities arising from SGE deployment. Moreno-Cruz et al. (2012) account for regional inequalities in SRM effectiveness.



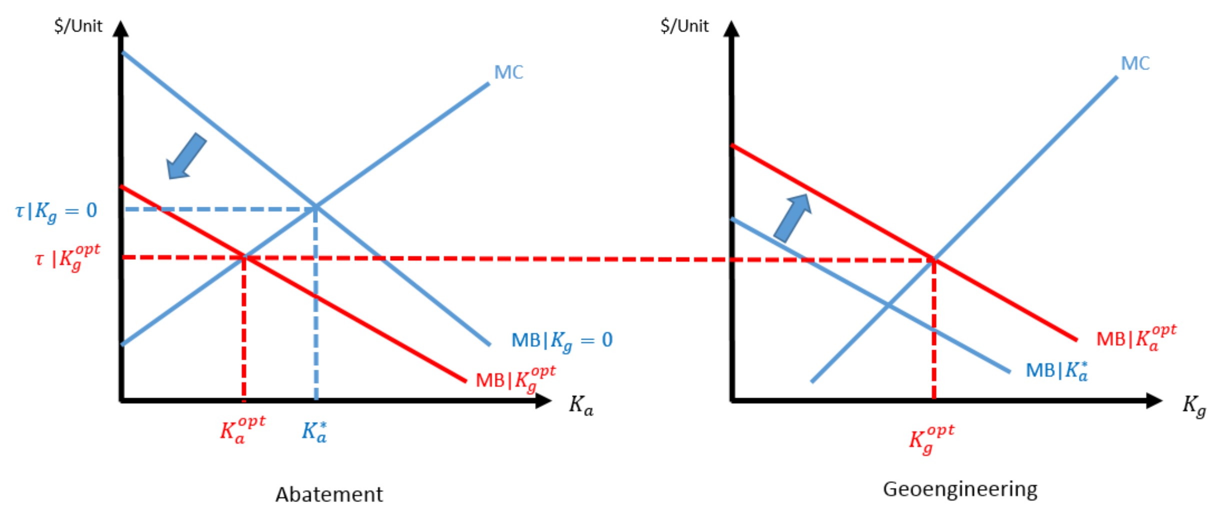


Figure 1 – Static Model

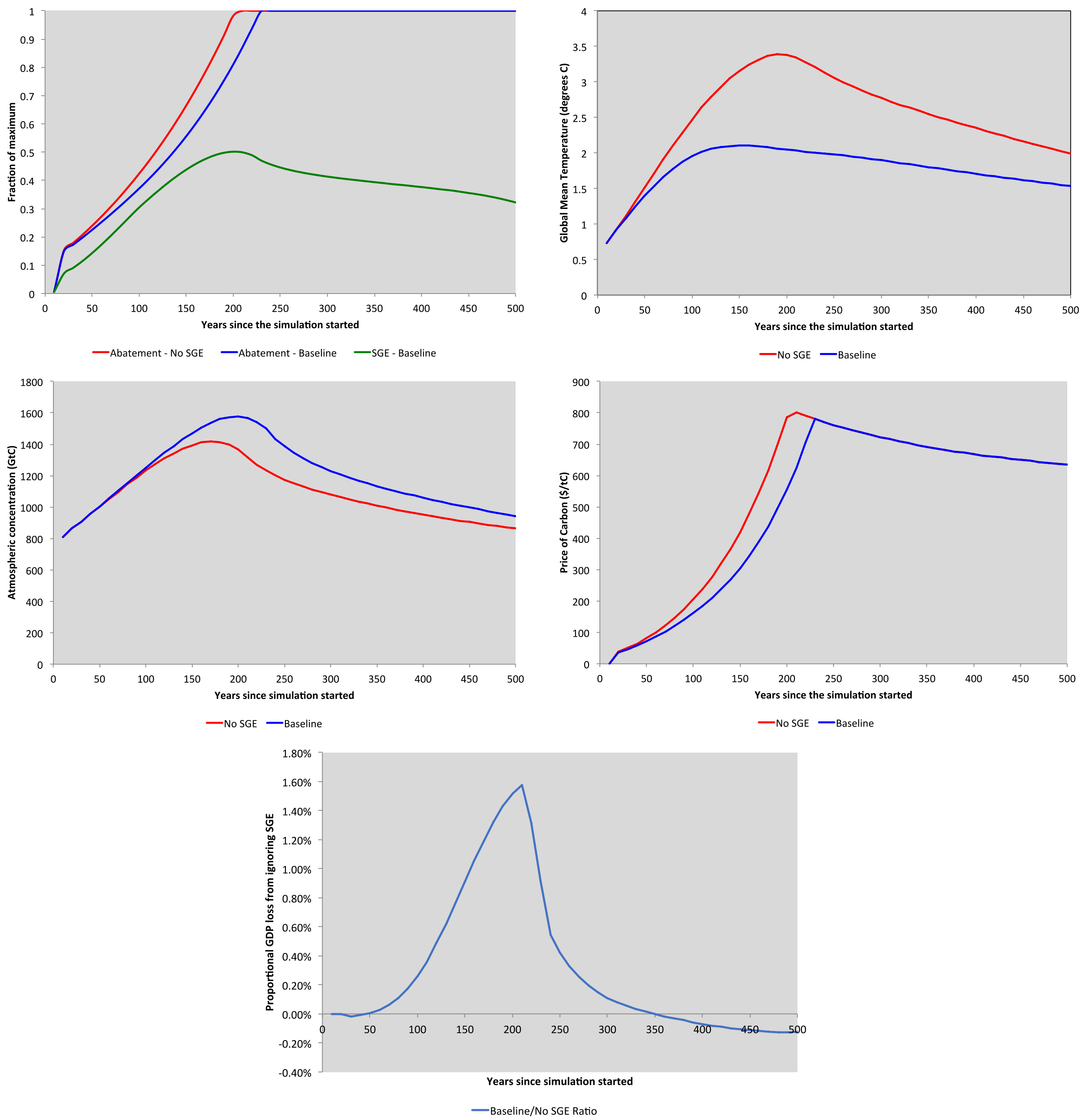


Figure 2 – Baseline simulations

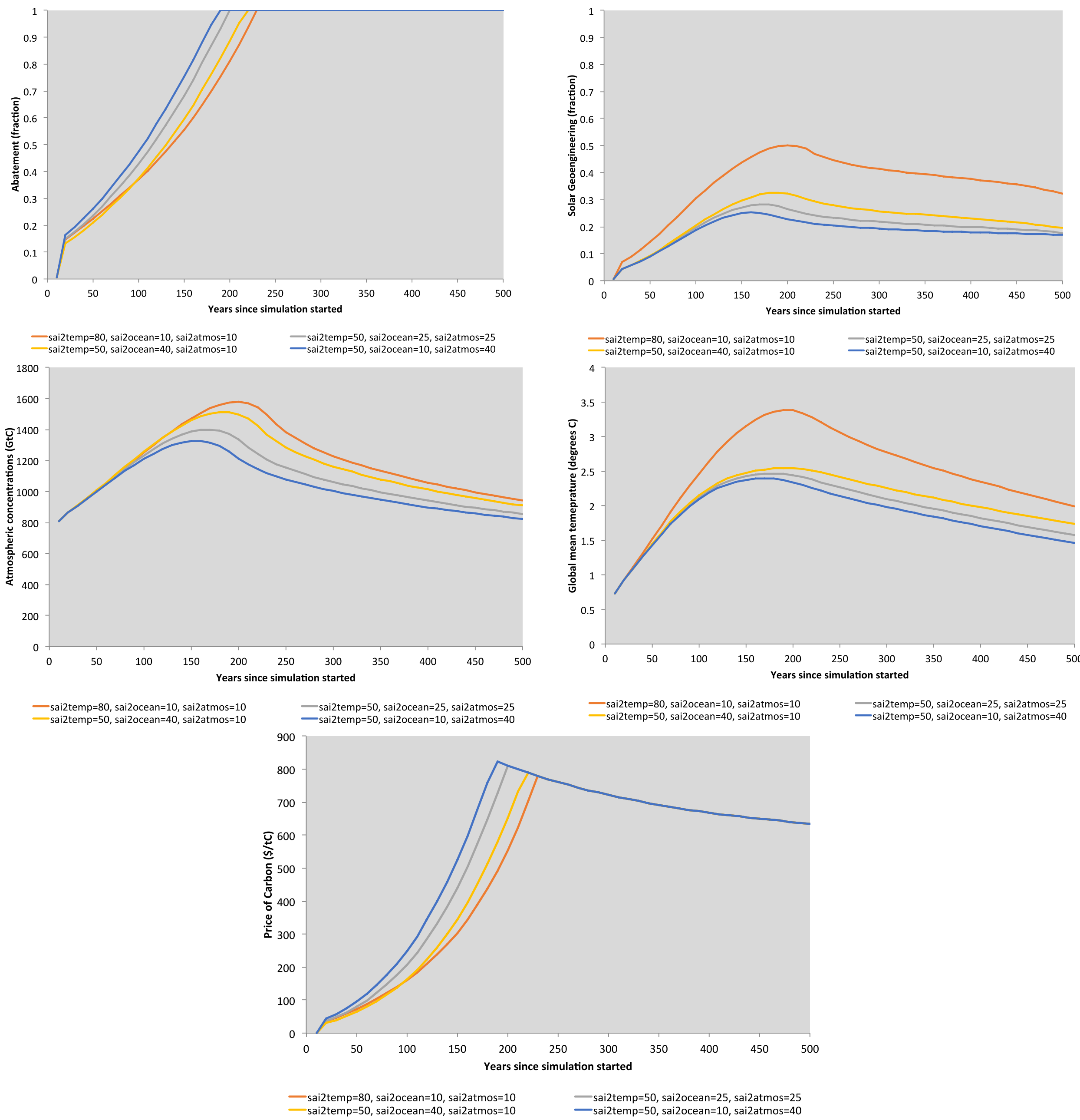


Figure 3 – Variation in the Composition of Damages

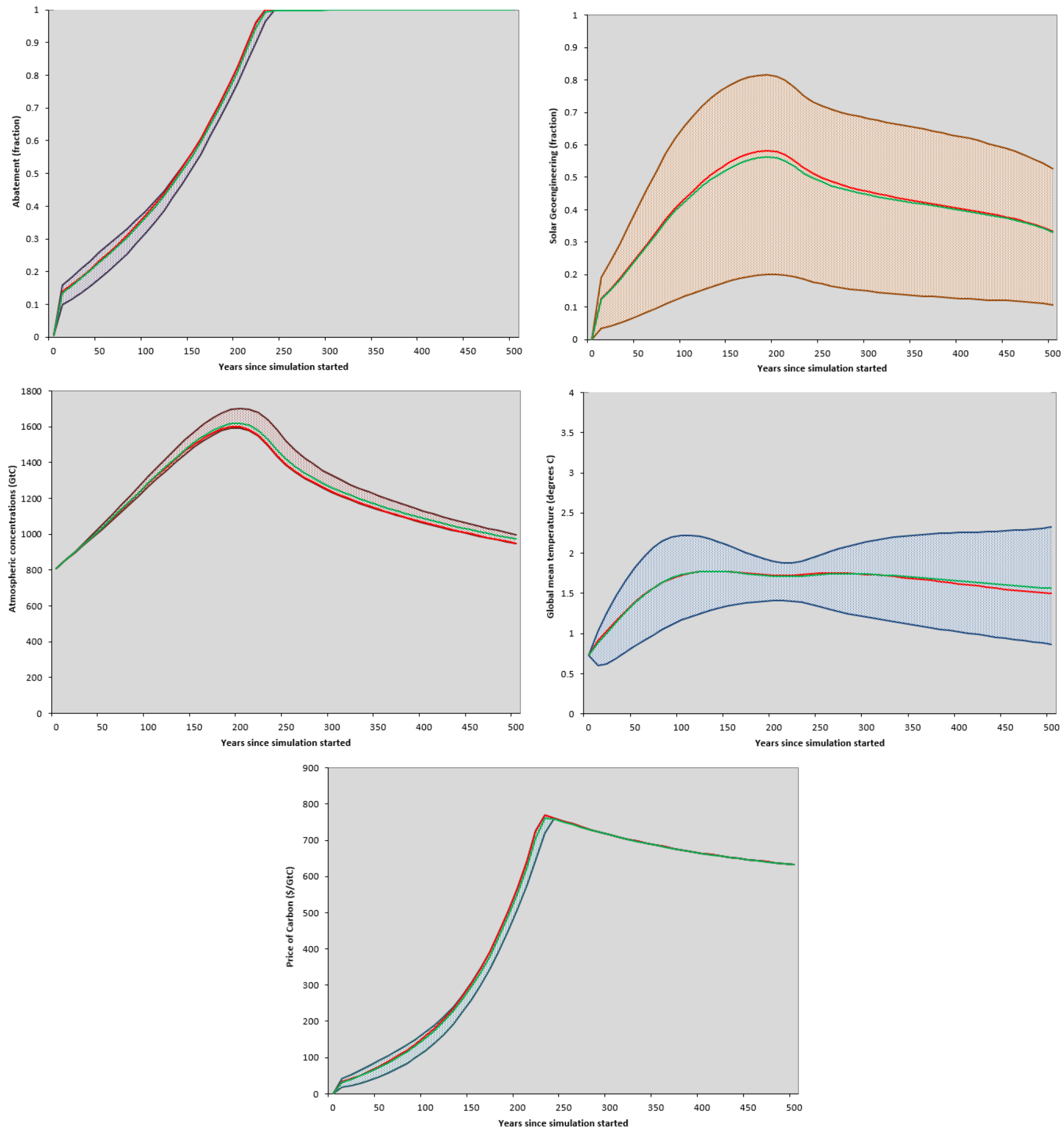


Figure 4 – Uncertainty in Equilibrium Climate Sensitivity

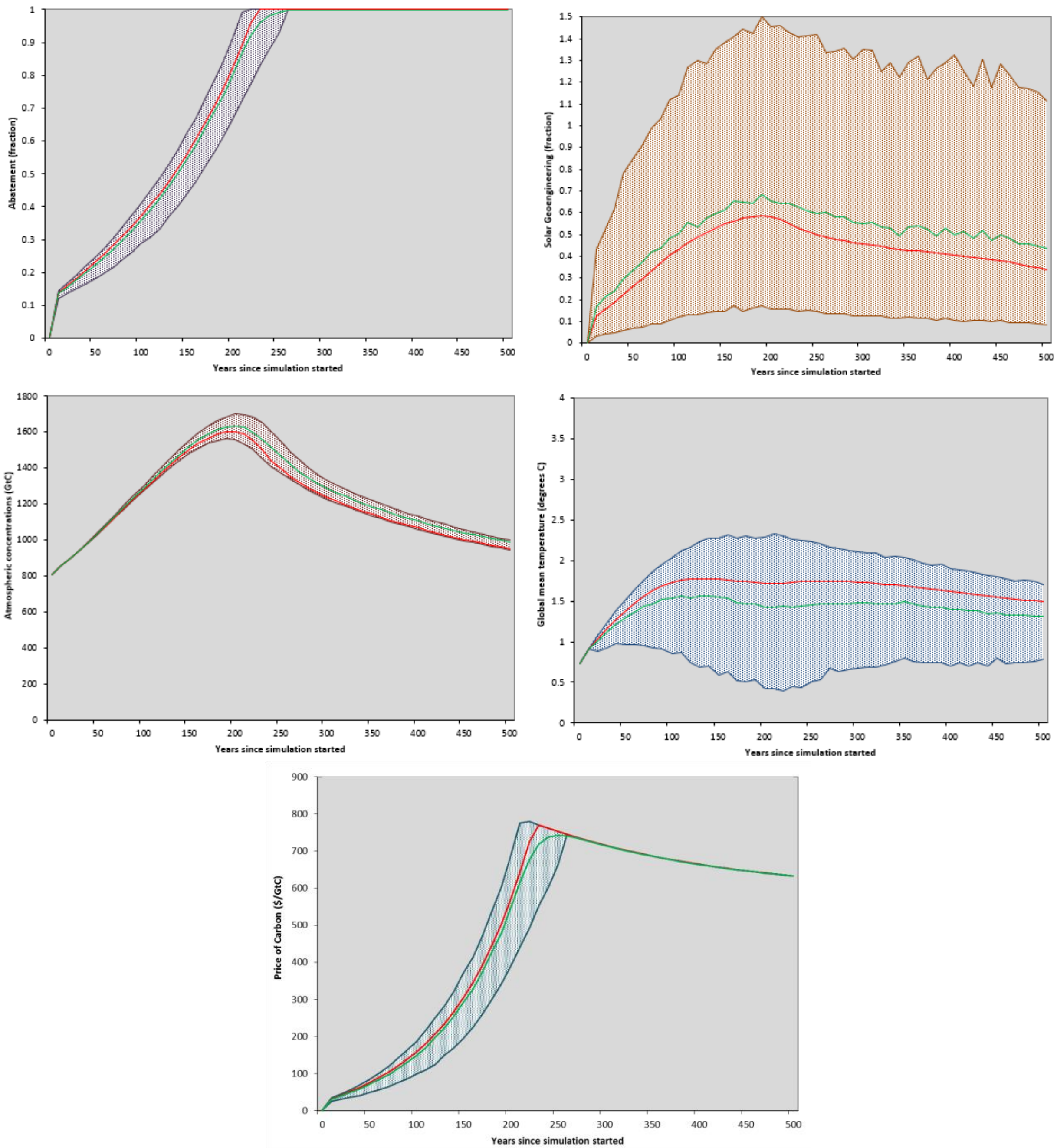


Figure 5 –Stochasticity in Geoengineering Damages

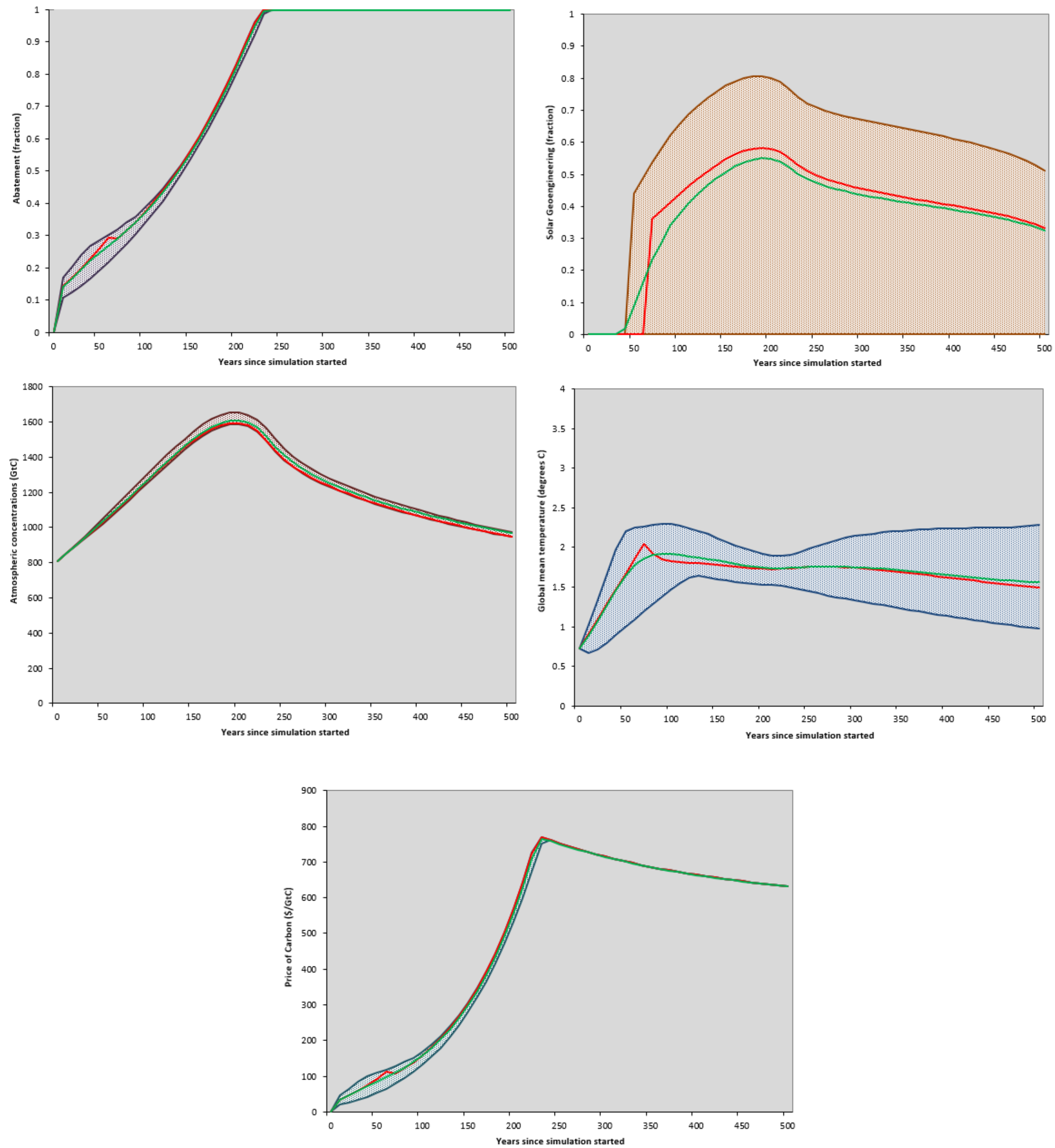


Figure 6 –SGE as Insurance

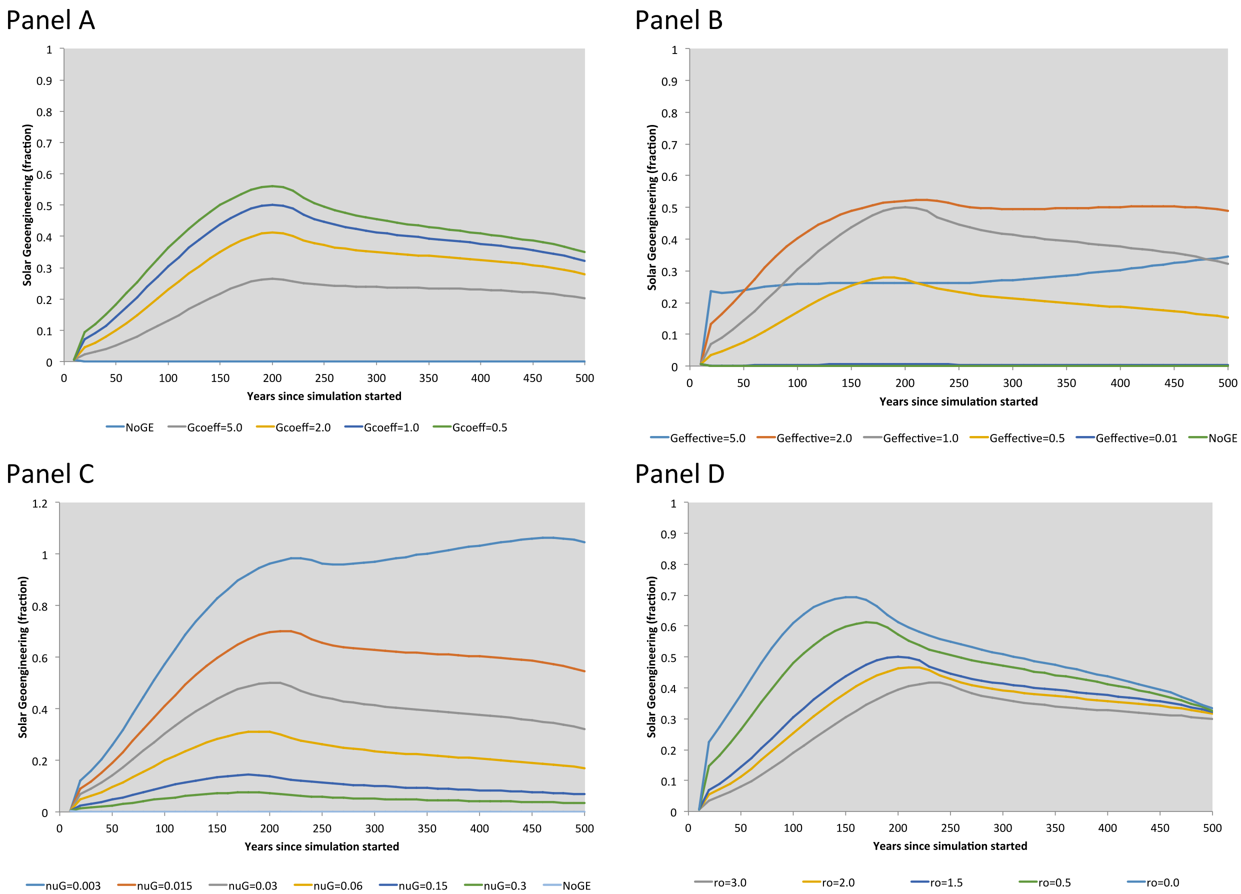


Figure 7 – Sensitivity analysis

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Solar Geoengineering, Uncertainty, and the Price of Carbon – Appendix

A.I: Details of the Theoretical Model

We verify that 1) the marginal cost of abatement is increasing in k_a, 2) the marginal benefit of abatement is decreasing in k_a, 3) the marginal cost of abatement is higher for a positive value of SGE k_g than it is for $k_g = 0$, and 4) the marginal benefit of abatement is lower for a positive value of SGE k_g than it is for $k_g = 0$. We also argue that the difference in marginal costs (3) is likely to be smaller in magnitude than the difference in marginal benefits (4).

The marginal cost of abatement as a function of abatement k_a is $f'(k - k_a - k_g)\left(1 - d\left((1 - g(k_a))k, k_g\right)\right)$, for some fixed k_g. The first half is increasing in k_a since f is concave. The second half is increasing in k_a since g is increasing and d is decreasing in x. Thus, marginal cost is monotone increasing.

The marginal benefit of abatement is $f(k - k_a - k_g)kg'(k_a)d_x\left((1 - g(k_a))k, k_g\right)$. Because f is increasing the first part is decreasing in k_a. Assuming that g is concave, the middle part is decreasing in k_a. Lastly, because the cross-partial derivative $d_{xk} < 0$, the third part of this expression is decreasing in k_a, and so the entire expression for marginal benefit is monotone decreasing.

The marginal cost of abatement at zero SGE is $f'(k - k_a)\left(1 - d\left((1 - g(k_a))k, 0\right)\right)$, and for an arbitrary level of SGE (for instance, k_g^{opt}) it is $f'(k - k_a - k_g)\left(1 - d\left((1 - g(k_a))k, k_g\right)\right)$. Because f is concave, the first part of the expression is higher for $k_g > 0$. Because $d_k < 0$, the second part of the expression is higher for $k_g > 0$. Thus, the marginal cost of abatement is higher for a positive value of SGE k_g than it is for $k_g = 0$.

The marginal benefit of abatement at zero SGE is $f(k-k_a)kg'(k_a)d_x\left(\left(1-g(k_a)\right)k,0\right)$, and for an arbitrary level of SGE it is $f\left(k-k_a-k_g\right)kg'(k_a)d_x\left(\left(1-g(k_a)\right)k,k_g\right)$. Because f is increasing, the first part of this expression is lower for $k_g>0$. Because $d_{xk}<0$, the second part of the expression is lower for $k_g>0$. Thus, the marginal benefit of abatement is lower for a positive value of SGE k_g than it is for $k_g=0$.

Lastly, we argue that the magnitude of the difference in the two marginal benefit curves is likely to be large, while the magnitude of the difference in the two marginal cost curves is likely to be small. This cannot be mathematically demonstrated like the rest of the claims in the appendix. Rather, it follows from our intuition of the application of the model. Consider first the difference in marginal costs. The first half of the expression is the difference between $f'(k-k_a)$ and $f'\left(k-k_a-k_g\right)$. This is likely to be small, because k_g is likely to be very small relative to k (i.e. only a small fraction of total capital will be spent on GE). The second difference between the two expressions is $1-d\left(\left(1-g(k_a)\right)k,0\right)$ versus $1-d\left(\left(1-g(k_a)\right)k,k_g\right)$. This is also likely to be small because the damages from climate change as a proportion of total potential output (d) is likely to be only a few percentage points. Thus, even if the optimal level of k_g completely eliminated climate change damages ($d=0$), the value of $1-d$ would change only from, say, 98% to 100%.

Consider instead the differences in marginal damages. The first difference is the difference between $f(k-k_a)$ and $f\left(k-k_a-k_g\right)$, which from the argument in the previous paragraph is likely to be small since k_g is small relative to k. The other difference is the difference between $d_x\left(\left(1-g(k_a)\right)k,0\right)$ and $d_x\left(\left(1-g(k_a)\right)k,k_g\right)$. This difference is likely to be large (first-order). Even though damages d may be small (a few percentage points), the difference in the marginal damages d_x may be large depending on the presence of SGE. At the extreme, if k_g is sufficiently high to eliminate any damages from climate change, then $d_x(x, k_g)$ will be zero though $d_x(x, k_g)$ is positive.

Uncertainty

We now derive the expressions in section II.A in which pollution damages and solar geoengineering benefits are uncertain. The two first-order conditions for the planner's problem can be written as:

$$F \equiv f'(k - k_a - k_g) \cdot (1 - E[d(x, k_g, \theta_x, \theta_g)]) - f(k - k_a - k_g) \cdot k \cdot g'(k_a) \cdot E[d_x(x, k_g, \theta_x, \theta_g)] = 0$$

$$G \equiv g'(k_a) \cdot k \cdot E[d_x(x, k_g, \theta_x, \theta_g)] + E[d_k(x, k_g, \theta_x, \theta_g)] = 0$$

The variance of the shocks, $Var(\theta_x)$ and $Var(\theta_g)$, are treated as exogenous parameters that affect the expected values of the damage function and its partial derivatives, as defined in the text. Therefore, the implicit function theorem can be used to find the following derivatives:

$$\begin{pmatrix} \dfrac{\partial k_a}{\partial Var(\theta_x)} \\ \dfrac{\partial k_g}{\partial Var(\theta_x)} \end{pmatrix} = -\begin{pmatrix} \dfrac{\partial F}{\partial k_a} & \dfrac{\partial F}{\partial k_g} \\ \dfrac{\partial G}{\partial k_a} & \dfrac{\partial G}{\partial k_g} \end{pmatrix}^{-1} \begin{pmatrix} \dfrac{\partial F}{\partial Var(\theta_x)} \\ \dfrac{\partial G}{\partial Var(\theta_x)} \end{pmatrix}$$

$$\begin{pmatrix} \dfrac{\partial k_a}{\partial Var(\theta_g)} \\ \dfrac{\partial k_g}{\partial Var(\theta_g)} \end{pmatrix} = -\begin{pmatrix} \dfrac{\partial F}{\partial k_a} & \dfrac{\partial F}{\partial k_g} \\ \dfrac{\partial G}{\partial k_a} & \dfrac{\partial G}{\partial k_g} \end{pmatrix}^{-1} \begin{pmatrix} \dfrac{\partial F}{\partial Var(\theta_g)} \\ \dfrac{\partial G}{\partial Var(\theta_g)} \end{pmatrix}$$

The inverse of the 4-by-4 matrix in these expressions (the Jacobian matrix) is

$$\frac{1}{Det}\begin{pmatrix} \dfrac{\partial G}{\partial k_g} & -\dfrac{\partial F}{\partial k_g} \\ -\dfrac{\partial G}{\partial k_a} & \dfrac{\partial F}{\partial k_a} \end{pmatrix}$$

The determinant of the Jacobian, Det, is positive from the second-order condition of the planner's maximization problem. The elements of the Jacobian matrix are:

$$\frac{\partial F}{\partial k_a} = -f''(k_p)(1 - E[d]) + f'(k_p)\left(-\frac{\partial E[d]}{\partial k_a}\right) + f'(k_p)kg'(k_a)E[d_x] - f(k_p)kg''(k_a)E[d_x]$$
$$- f(k_p)kg'(k_a)\frac{\partial E[d_x]}{\partial k_a} > 0$$

$$\frac{\partial F}{\partial k_g} = -f''(k_p)(1 - E[d]) + f'(k_p)\left(-\frac{\partial E[d]}{\partial k_g}\right) + f'(k_p)kg'(k_a)E[d_x] - f(k_p)kg'(k_a)\frac{\partial E[d_x]}{\partial k_g} > 0$$

$$\frac{\partial G}{\partial k_a} = g''(k_a)kE[d_x] + g'(k_a)k\frac{\partial E[d_x]}{\partial k_a} + \frac{\partial E[d_k]}{\partial k_a}$$

$$\frac{\partial G}{\partial k_g} = g'(k_a)k\frac{\partial E[d_x]}{\partial k_g} + \frac{\partial E[d_k]}{\partial k_g}$$

All terms in $\frac{\partial F}{\partial k_a}$ and $\frac{\partial F}{\partial k_g}$ are positive. However, $\frac{\partial G}{\partial k_a}$ and $\frac{\partial G}{\partial k_g}$ have ambiguous sign. In $\frac{\partial G}{\partial k_a}$, the first two terms are negative, and the third term is positive. In $\frac{\partial G}{\partial k_g}$, the first term is negative, and the second term is positive. Since the final term in each expression is a multiple of d_{kk}, which we assume is negative, $\frac{\partial G}{\partial k_a}$ is negative and $\frac{\partial G}{\partial k_g}$ is positive so long as d_{xk} is not too negative.

Furthermore,

$$\frac{\partial F}{\partial Var(\theta_x)} = -f'(k_p)\frac{\partial E[d]}{\partial Var(\theta_x)} - f(k_p)kg'(k_a)\frac{\partial E[d_x]}{\partial Var(\theta_x)}$$

$$\frac{\partial G}{\partial Var(\theta_x)} = g'(k_a)k\frac{\partial E[d_x]}{\partial Var(\theta_x)} + \frac{\partial E[d_k]}{\partial Var(\theta_x)}$$

Substituting these expressions into the matrix equation above, simplifying, and collecting terms yields

$$\frac{\partial k_a}{\partial Var(\theta_x)} = \frac{1}{Det}\left\{\left[f'(k_p)\left(g'(k_a)k\frac{\partial E[d_x]}{\partial k_g} + \frac{\partial E[d_k]}{\partial k_g}\right)\right]\frac{\partial E[d]}{\partial Var(\theta_x)}\right.$$

$$+\left[f(k_p)kg'(k_a)\frac{\partial E[d_k]}{\partial k_g}\right.$$

$$\left. + g'(k_a)k\left(-f''(k_p)(1-E[d]) - f'(k_p)\frac{\partial E[d]}{\partial k_g} + f'(k_p)kg'(k_a)E[d_x]\right)\right]\frac{\partial E[d_x]}{\partial Var(\theta_x)}$$

$$\left. + \left[\frac{\partial F}{\partial k_g}\right]\frac{\partial E[d_k]}{\partial Var(\theta_x)}\right\}$$

Define

$A \equiv f(k_p)kg'(k_a)\frac{\partial E[d_k]}{\partial k_g} + g'(k_a)k\left(-f''(k_p)(1-E[d]) - f'(k_p)\frac{\partial E[d]}{\partial k_g} + f'(k_p)kg'(k_a)E[d_x]\right) > 0$ and $B \equiv \frac{\partial F}{\partial k_g} > 0$, and the expression is as appears in the text.

Next,

$$\frac{\partial k_g}{\partial Var(\theta_x)} = \frac{1}{Det}\left\{\left[f'(k_p)\left(-g''(k_a)kE[d_x] - g'(k_a)k\frac{\partial E[d_x]}{\partial k_a} - \frac{\partial E[d_k]}{\partial k_a}\right)\right]\frac{\partial E[d]}{\partial Var(\theta_x)}\right.$$

$$+\left[-f'(k_p)kg'(k_a)\left(\frac{\partial E[d_k]}{\partial k_a}\right)\right.$$

$$\left. - g'(k_a)\left(-f''(k_p)(1-E[d]) - f'(k_p)\frac{\partial E[d]}{\partial k_a} + f'(k_p)kg'(k_a)E[d_x]\right)\right]\frac{\partial E[d_x]}{\partial Var(\theta_x)}$$

$$\left. + \left[-\frac{\partial F}{\partial k_a}\right]\frac{\partial E[d_k]}{\partial Var(\theta_x)}\right\}$$

Define

$C \equiv f'(k_p)kg'(k_a)\left(\frac{\partial E[d_k]}{\partial k_a}\right) + g'(k_a)\left(-f''(k_p)(1-E[d]) - f'(k_p)\frac{\partial E[d]}{\partial k_a} + f'(k_p)kg'(k_a)E[d_x]\right) > 0$ and $D \equiv \frac{\partial F}{\partial k_a} > 0$, and the expression is as appears in the text.

The solutions for $\frac{\partial k_a}{\partial Var(\theta_g)}$ and $\frac{\partial k_g}{\partial Var(\theta_g)}$ are identical to those for $\frac{\partial k_a}{\partial Var(\theta_x)}$ and $\frac{\partial k_g}{\partial Var(\theta_x)}$, respectively, except for replacing all partials with respect to $Var(\theta_x)$ with partials with respect to $Var(\theta_g)$.

<u>Decomposition of Climate Damages</u>

We now derive the expressions in section II.B where damages occur from both temperature and from carbon. The first-order conditions in for the planner's problem are identical as in the original model, except that the damage function is now $d(x;k_g) = \lambda_d d_1(x) + d_2(x;k_g)$.

$$F \equiv f'(k-k_a-k_g)\cdot\left(1-d(x;k_g)\right) - f(k-k_a-k_g)\cdot k\cdot g'(k_a)\cdot d_x(x;k_g) = 0$$

$$G \equiv g'(k_a)\cdot k\cdot d_x(x;k_g) + d_k(x;k_g) = 0$$

As with the last model, the implicit function theorem can be used to conduct comparative statics:

$$\begin{pmatrix} \frac{\partial k_a}{\partial \lambda_d} \\ \frac{\partial k_g}{\partial \lambda_d} \end{pmatrix} = -\begin{pmatrix} \frac{\partial F}{\partial k_a} & \frac{\partial F}{\partial k_g} \\ \frac{\partial G}{\partial k_a} & \frac{\partial G}{\partial k_g} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial F}{\partial \lambda_d} \\ \frac{\partial G}{\partial \lambda_d} \end{pmatrix}$$

Again, it can be shown that $\frac{\partial F}{\partial k_a} > 0$, $\frac{\partial F}{\partial k_g} > 0$, and the determinant of the Jacobian is positive. The partial derivatives $\frac{\partial G}{\partial k_a}$ and $\frac{\partial G}{\partial k_g}$ have ambiguous sign:

$$\frac{\partial G}{\partial k_a} = g''(k_a)kd_{2x} + g'(k_a)k\frac{\partial Ed_{2x}}{\partial k_a} + \frac{\partial Ed_{2k}}{\partial k_a}$$

$$\frac{\partial G}{\partial k_g} = g'(k_a)k\frac{\partial Ed_{2x}}{\partial k_g} + \frac{\partial Ed_{2k}}{\partial k_g}$$

As in the model in the prior section, the first two terms in $\frac{\partial G}{\partial k_a}$ are negative, and the last is positive. The first term in $\frac{\partial G}{\partial k_g}$ is negative, and the last is positive. Also,

$$\frac{\partial F}{\partial \lambda_d} = -f'(k_p)d_1 - f(k_p)d_{1x}kg'(k_a) < 0$$

$$\frac{\partial G}{\partial \lambda_d} = 0$$

After substituting in for each of these partial derivatives and simplifying, we get

$$\frac{\partial k_a}{\partial \lambda_d} = \frac{1}{Det} \cdot \left[f'(k_p)d_1 + f(k_p)d_{1x}kg'(k_a)\right]\left[g'(k_a)kd_{2xk} + d_{2kk}\right]$$

$$\frac{\partial k_g}{\partial \lambda_d} = \frac{1}{Det} \cdot \left[f'(k_p)d_1 + f(k_p)d_{1x}kg'(k_a)\right]\left[(-g''(k_a)kd_{2x} + g'(k_a)^2k^2d_{2xx} - kg'(k_a)d_{2xk})\right]$$

The first set of terms in front of each expression, $\frac{1}{Det} \cdot \left[f'(k_p)d_1 + f(k_p)d_{1x}kg'(k_a)\right]$, is positive and is defined as the constant C in the text.

A.II. Details of the Numerical Simulation Model

Modification of DICE and calibration

In the original DICE model, the cost of abatement is modeled as a power function of a: $AbatementCost = \theta_1(t)a^{\theta_2}$. The exponent θ_2 is 2.8 in the base case, indicating convex costs. The coefficient $\theta_1(t)$ decreases with time, halving after about 100 years (10 periods) to reflect technological advancement in abatement. The outcome variable $AbatementCost$ is the fraction of gross output that is sacrificed for abatement. For instance, in period 1 where $\theta_1(1) = 0.0561$, the cost of abating 10 percent of gross emissions would be 0.009% of gross output ($0.00009 = 0.0561 \times 0.10^{2.8}$). We define solar geoengineering costs analogously as a fraction of gross output: $GeoengCost = G_{coef}\theta_{GE}(t)g^{\theta_3}$. To calibrate this cost function of aerosol-sulfate-based SGE, we use two sources.[22] First, doing well-informed back-of-the-envelope calculations, Crutzen (2006) estimates the amount of sulfur needed to reduce the radiative forcing equivalent to doubling CO_2 to be equal to 5.3 Mt of sulfur. The second piece of information is related to the costs of delivering sulfur at the distance required to have a global impact. Crutzen (2006) has estimated something in the order of $25 Billion for 1 Mt S. Recent estimates, using new aircraft designs, estimated the costs at $3 Billion for 1 Mt S or $8 Billion to deliver 5 Mt S (McClellan et al. 2012). These two pieces of data imply that reducing the radiative forcing equivalent to

[22] There are alternatives solar radiation management technologies other than sulfate aerosols, though sulfate aerosols are likely the most cost-effective and dependable technology. Marine cloud brightening (MCB) would increase reflectivity by injecting seawater particles into clouds (Latham et al. 2014). Cirrus cloud seeding would increase outgoing radiation by reducing cirrus cloud cover (Storelvmo et al. 2014).

a doubling of CO_2 costs between \$8 Billion and \$125 Billion. Furthermore, we assume that particles are required at an increasing rate (Rasch et al. 2008), and for simplicity the costs are quadratic (less convex than mitigation, but linear costs are unrealistic due to coagulation of particles and other such processes). GDP in 2005 was \$46 Trillion, so the lowest estimate of \$8 Billion is only 0.02% of global GDP and the highest estimate is 0.27% of global GDP. (Compare this with the 3% in terms of mitigation costs associated with the optimal policy in DICE.) Because solar geoengineering is a fraction in our model, reducing a doubling of CO_2 to nothing is equivalent to setting $g = 1$. Thus our solar geoengineering cost estimate is $\theta_{GE}(t) = 0.0027$ and $\theta_3 = 2$. We set $\theta_{GE}(t)$ constant over time, so that unlike abatement technology there is no learning or improvement in solar geoengineering technology. We also include the coefficient G_{coef} to represent a scaling of solar geoengineering costs. In the base case we set $G_{coef} = 1$, and we will vary this in sensitivity analysis. By using the high cost estimate and not allowing technological growth in SGE technology, this base case value for SGE costs is very conservative, that is, biased against deployment of solar geoengineering.

In addition to these implementation costs of solar geoengineering, there may also be damages from solar geoengineering. We model these damages in the same way that the original DICE model models damages from climate change – as a factor of total potential output that is unrealized due to these damages (and of course we keep damages from climate change as well). In the original DICE, the damage function is $\Omega(t)$, where output $Q_{net} = \frac{1}{1+\Omega(t)} Q_{gross}$ and $\Omega(t) = \psi_1 T_{AT}(t) + \psi_2 T_{AT}(t)^2$. The damage function is a function of atmospheric temperature at time t, $T_{AT}(t)$, and the ψs are calibrated coefficients. We amend this by also allowing for solar geoengineering g to directly reduce total output. In addition to the $\Omega(t)$ term representing damages from climate change, we also include damages from geoengineering: $Q_{net} = \frac{1}{1+\Omega(t)} \times \frac{1}{1+\nu_G g^2} \times Q_{gross}$. The coefficient ν_g represents how damages from solar geoengineering scale net output. In our base case, we set $\nu_g = 0.03$, which implies that solar geoengineering at full intensity ($g = 1$) leads to damages that amount to 3% of gross output. This is slightly higher than SGE damages in Goes et al. (2011), where damages range from 0% to 2%, or in Gramstad and Tjotta (2010), where damages range from 0.1% to 2.4%. SGE damages are similar in scale to climate change damages in DICE associated with about 6 degrees Celsius of warming, thus this damage estimate (like the cost estimate) is very conservative (i.e. biased *against* geoengineering).

The purpose of engaging in solar geoengineering is to alter the radiative forcing of the Earth's atmosphere. Radiative forcing is the difference in net heat loss due to anthropogenic GHG emissions relative to preindustrial levels. In the DICE model, the radiative forcing equation is

$$F(t) = \eta\{\log_2\left[\frac{M_{AT}(t)}{M_{AT}(1750)}\right]\} + F_{EX}(t)$$

It is a function of the ratio of the current atmospheric carbon stock ($M_{AT}(t)$) to the pre-industrial atmospheric carbon stock ($M_{AT}(1750)$ = 596.4 Gt C, equivalent to about 280 ppm CO_2), exogenous forcing $F_{EX}(t)$ due to anthropogenic emissions of GHGs other than CO_2 (assumed exogenous in DICE), and a calibrated radiative forcing parameter η. Atmospheric temperature $T_{AT}(t)$ is affected by radiative forcing through the following equation:

$$T_{AT}(t) = T_{AT}(t-1) + \xi_1\{F(t) - \xi_2 T_{AT}(t-1) - \xi_3[T_{AT}(t-1) - T_{LO}(t-1)]\}.$$

This function also depends on the lower ocean temperature in the previous period $T_{LO}(t-1)$. A higher value of radiative forcing $F(t)$ (which is caused by higher atmospheric carbon $M_{AT}(t)$) leads to higher atmospheric temperatures $T_{AT}(t)$ all else equal. Our modification is to the radiative forcing equation:

$$F(t) = \left(\eta\{\log_2\left[\frac{M_{AT}(t)}{M_{AT}(1750)}\right]\} + F_{EX}(t)\right)\left(1 - \phi g(t)\right)$$

The variable $g(t)$ is the amount of solar geoengineering in period t, and ϕ is a positive parameter that captures the leverage of solar geoengineering. Higher ϕ means less solar geoengineering needs to be implemented to achieve a given level of radiative forcing reduction. At a value of 1, radiative forcing F is reduced to zero (regardless of the carbon stock), completely eliminating anthropogenic climate change effects on temperature. When $g > 1$, radiative forcing is negative, and global temperatures will reduce faster than they would even with no anthropogenic GHGs in the atmosphere.

Our final change to the DICE model is a modification of the damage function from climate change (not from solar geoengineering). In DICE, climate change damages are a function of temperature only. However, climate change damages are expected to come from more than just temperature changes. Absent its effect on temperature, atmospheric carbon is expected to affect precipitation patterns, which may cause damages (Allen 2002, Bala et al 2008). Bony et al. (2013) find that atmospheric carbon increases account for about half of predicted tropical circulation change and a large fraction of precipitation changes from climate change. The acidification of oceans cause damages too. Brander et al. (2012) suggest that find that damages from ocean acidification alone can account for 0.14% to 0.18% of gross GDP. There is also the possibility that increased atmospheric carbon concentrations may have benefits to agricultural productivity, holding temperature and precipitation constant (Pongratz et al. 2012, Matthews et al. 2005). Because solar geoengineering decreases temperature but does not change the carbon stock in either the atmosphere or the oceans, it is crucial to decompose the damages from climate change into damages directly from temperature and damages from carbon stocks. Thus, we modify DICE's damage function $\Omega(t)$ to be a function of temperature $T_{AT}(t)$ as well as atmospheric carbon $M_{AT}(t)$ and upper ocean carbon $M_{UP}(t)$:

$$\Omega(t) = \psi_T[T_{AT}(t)]^2 + \psi_O[M_{UP}(t) - M_{UP}(1750)]^2 + \psi_{AT}[M_{AT}(t) - M_{AT}(1750)]^2$$

The temperature T_{AT} is already defined as degrees Celsius relative to preindustrial (1750) average temperature, but the other two components of damages are not, so we calculate damages by subtracting the preindustrial levels, and we allow damages to be a quadratic function of the deviation from preindustrial levels. We calibrate the new damage coefficients ψ in the following way. We impose that the original DICE model's total climate change damages in the initial period is correct, but we allocate some of those damages to be directly from temperature, some from ocean carbon concentrations, and some from atmospheric carbon concentrations. Direct damages from atmospheric or ocean carbon concentrations are difficult to calibrate. Therefore, we begin by assuming that, of the total damages from climate change in the initial period calibrated in DICE, 80% is directly a function of atmospheric temperature, 10% is a function of atmospheric carbon, and 10% is a function of upper ocean carbon. By imposing that the total damages from climate in the initial period are identical to those in DICE, this allows us to calibrate ψ_T, ψ_O, and ψ_{AT}. Specifically, in the initial period in DICE, climate change damages amount to 0.15157% of gross output ($\Omega(t) = \psi T^2 = 2.8388 \times 10^{-3} \times 0.7307^2$). We set 80% of this total a function of temperature, yielding $\psi_T = 0.00227$ ($\psi_T \times 0.7307^2 = 0.80 \times 0.0015157$). And similarly we calibrate the other damage parameters.

These are the five areas in which we modify DICE to include geoengineering and its costs, benefits, and damages. We have described the base case parameterizations that we use in our model, but we will also conduct a very broad range of sensitivity analyses, since many of the parameters are difficult to quantify.

Solution Method and Uncertainty

The model is solved using the two-step-ahead approximation method described in Shayegh and Thomas (2015). This algorithm was originally developed to find the optimal solution for the stochastic case of uncertainty in equilibrium climate sensitivity in DICE. The approximation technique was tested and tuned in the deterministic case and then applied to the stochastic model.

We consider two different specifications of uncertain parameters, and the algorithm is slightly different in the two cases. First, we model uncertainty in the climate system by allowing equilibrium climate sensitivity to be uncertain. In this case, we are dealing with a physical constant with a true value that is unknown to the planner, and therefore we can formulate this problem as decision-making under uncertainty. To solve this problem we generate 1000 samples from the distribution of equilibrium climate sensitivity. We solve the model for each realization and find the best set of tunable parameters for the value function approximation so that it generates the optimal climate change abatement and SGE action under any realization of this uncertain parameter. In other words, we find the optimal decision rule by simulating and solving finitely many deterministic problems.

The second case considers uncertainty in the damages from SGE (the parameter ν_G). In this case, we treat uncertainty as random shocks in the system, i.e., a stochastic process. We assume that the parameter values at each period are independently drawn randomly from a probability distribution. Under this characterization of uncertainty, for each simulation we generate a "path" of realizations of the random parameter instead of assuming a constant value throughout the modeling horizon.

Thus, the distinction highlights the difference between *uncertainty* and *stochasticity* in the system. With equilibrium climate sensitivity, there is a true value, but we (and the planner) do not know what it is. Uncertainty is subject to change over time with more observations.[23] Damages from SGE are described by stochasticity: randomness in the system that cannot be resolved or reduced over time by observation.

The algorithm works as follows: at each period t, the decision-maker estimates the value of the current state by projecting the values of the subsequent two states. The values of the two projected states are calculated under a deterministic forecast of the stochastic parameter and brought back to the present using an artificial and tunable discount rate. In the case of uncertainty in the climate system, the deterministic forecast of the uncertain parameter remains fixed over time within an iteration but varies from iteration to iteration. In the case of uncertainty in GE deployment, the forecast varies both over time and within an iteration. These values reflect the social utility under the deterministic assumption and are used to construct the value function of the current state. The optimal action (abatement or

[23] However, in our study we assume that uncertainty about equilibrium climate sensitivity remains unresolved and do not incorporate Bayesian updating. Recent studies incorporating Bayesian updating about equilibrium climate sensitivity in IAMs include Kelly and Tan (2013), Hwang (2014), and Shayegh and Thomas (2015).

geoengineering) is found by maximizing this value function. The algorithm starts at time $t = 1$ and progress until the final period. To update the optimal decision rule, the algorithm finds the best set of parameters for the value function approximation. At each iteration, assuming a constant set of parameters, the value functions are used to approximate future values and derive optimal actions. Once an iteration is complete, it moves backwards and calculate the “actual” values of future states given such actions. The difference between these actual and the estimated values (generated from the value function approximation) constitutes an error margin for that iteration. For the next iteration, the algorithm updates the value function approximation parameters to close the gap between actual and estimated values. The algorithm iterates until it “learns” the decision rule (i.e. finds the best value function approximation). In theory, the iteration ends when the error (difference between old and new values) converges to zero. Our experiments here are, instead, run for a finite set of iterations and the validity of error margin is confirmed at the end of 1000 iterations.

Although the algorithm is designed to deal with random variables in stochastic problems, it can be used in a deterministic model to approximate the optimal solution or in the model with uncertainty to generate approximate solution for each realization of an uncertain parameter. The algorithm is developed in MATLAB and is available upon request.

Other DICE models with geoengineering

Other studies have also modified DICE to include geoengineering, and Appendix Table 1 compares our modifications to these other papers’. All of the papers allow SGE to directly modify the radiative forcing equation; our paper is the only one to allow SGE to enter as a multiplicative factor rather than a linear additive term. This is not a major difference, since in either case the substantive effect is to reduce the value of $F(t)$. We choose a multiplicative factor for ease of interpretation: the policy variable represents the fraction of total anthropogenic forcing that is eliminated via SGE. The next column notes that all of the previous studies except one allow for damages from SGE (apart from their implementation costs). We, like Gramstad and Tjotta (2010), allow for these damages to be a quadratic function of SGE intensity and to be a multiplier on gross output; in this way they are modeled analogously to damages from climate change. Next, only this paper and Goes et al. (2011) and Bickel and Agrawal (2013) modify DICE's damages from climate change function. The other two papers allow for damages to be a function both of temperature and of the rate of temperature change, based on the fact that SGE can lead to rapid temperature changes (Matthews and Caldeira 2007). Their damage function is taken from Lempert et al. (2000). We are more direct in that we allow for damages to be a function of more than just temperature; this innovation is unique to this paper.

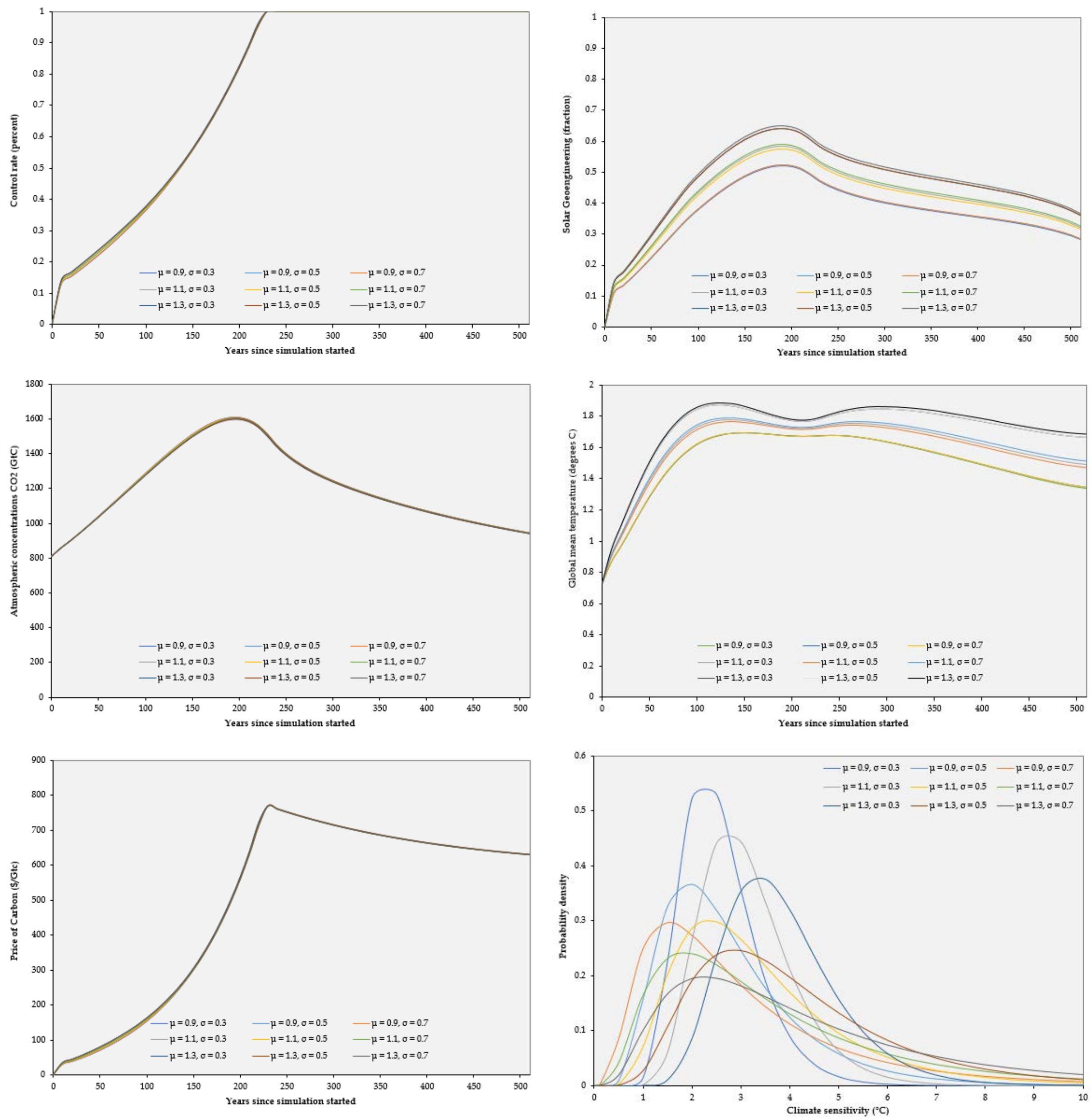
There are other modifications as well. Bickel and Lane (2009) is the only paper that also considers carbon capture, and Goes et al. (2011) and Bickel and Agrawal (2013) make several other modifications, including using a different climate model altogether.

Additional sensitivity analyses

We consider sensitivity analysis over parameters unrelated to SGE. Because SGE is a substitute for abatement, any change in the relative price of abatement affects optimal SGE deployment. Appendix Figure 7 shows sensitivity analysis over the value of the main parameter of the abatement cost function, θ_2. In DICE, θ_2 is the exponent of the abatement cost function: $Abatement\ Cost = \theta_1 {a_t}^{\theta_2}$, where this cost is in terms of the fraction of gross output sacrificed to abate a fraction a_t of emissions in period t. The base case value is $\theta_2 = 2.8$, and in Appendix Figure 7 we also consider three other values for θ_2. Since the abatement rate a_t is a fraction between zero and one, a larger θ_2 means cheaper abatement. Therefore, larger θ_2 yields a higher optimal level of abatement and a lower optimal level of SGE.

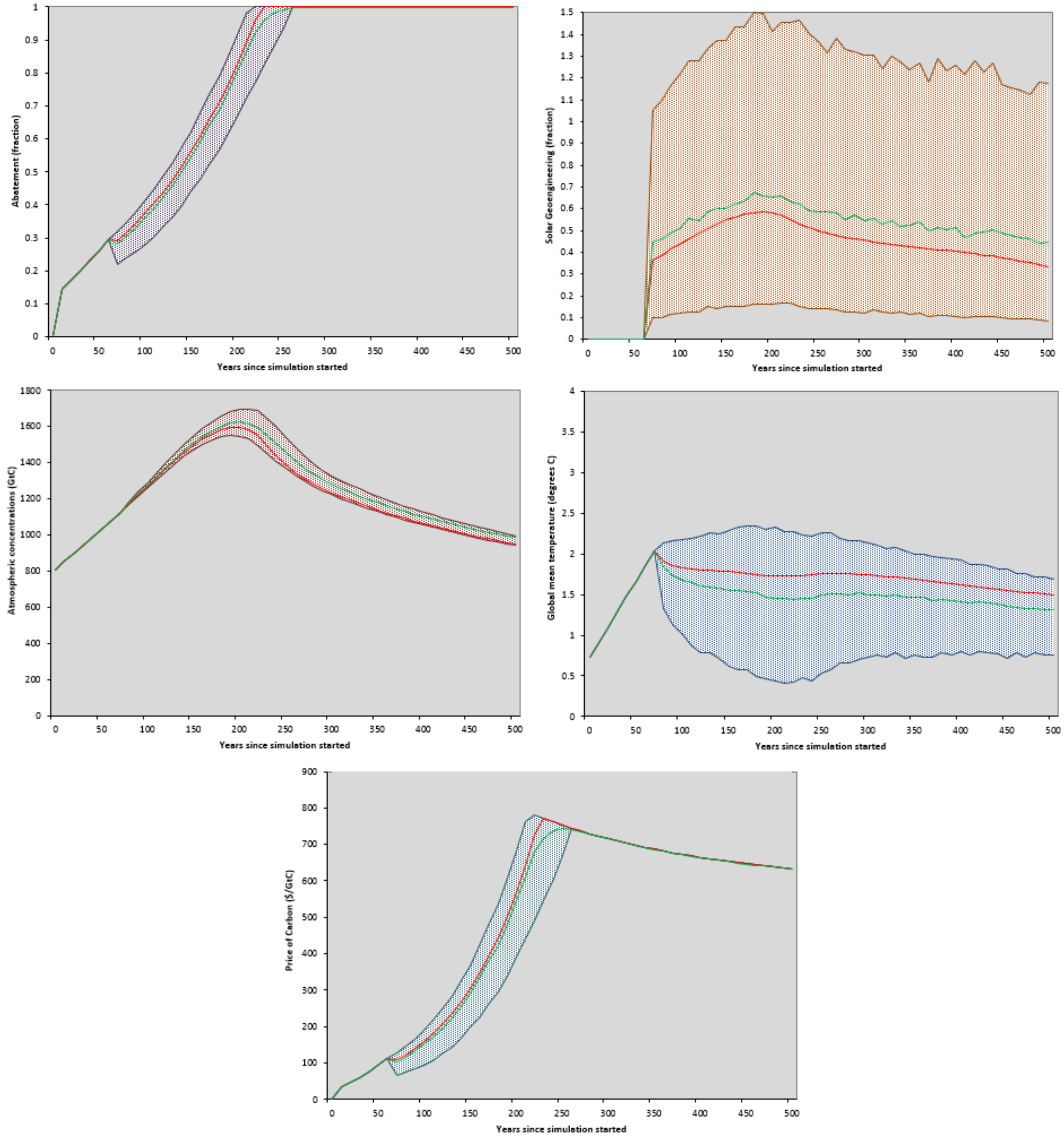
Appendix Table 1 – Summary of modifications to DICE

	Radiative Forcing	Damages from SGE	Climate Change Damages	Other modifications	Outcomes
Bickel and Lane (2009)	Linear term in forcing equation	None	No Modifications	Also model carbon capture geoengineering	Cost-benefit analysis for fixed levels of SGE; carbon price
Gramstad and Tjotta (2010)	Linear term in forcing equation	Quadratic multiplier on gross output	No Modifications	None	Cost-benefit analysis for fixed levels of SGE
Goes et al. (2011) and Bickel and Agrawal (2013)	Linear term in forcing equation	Linear function of aerosols deployed	Damages a function of temperature and rate of temperature change	Alter discounting formula; change climate model to DOECLIM; intermittency in GE	Cost-benefit analysis for fixed levels of SGE and for optimal SGE/abatement mix; Bickel and Agrawal (2013) considers sensitivity analysis of Goes et al. (2011)
This paper	Multiplicative factor in forcing equation	Quadratic multiplier on gross output	Damages a function of temperature, atmospheric carbon, and ocean carbon	Uncertainty analysis of equilibrium climate sensitivity	Optimal levels of SGE and abatement; carbon price; sensitivity analyses

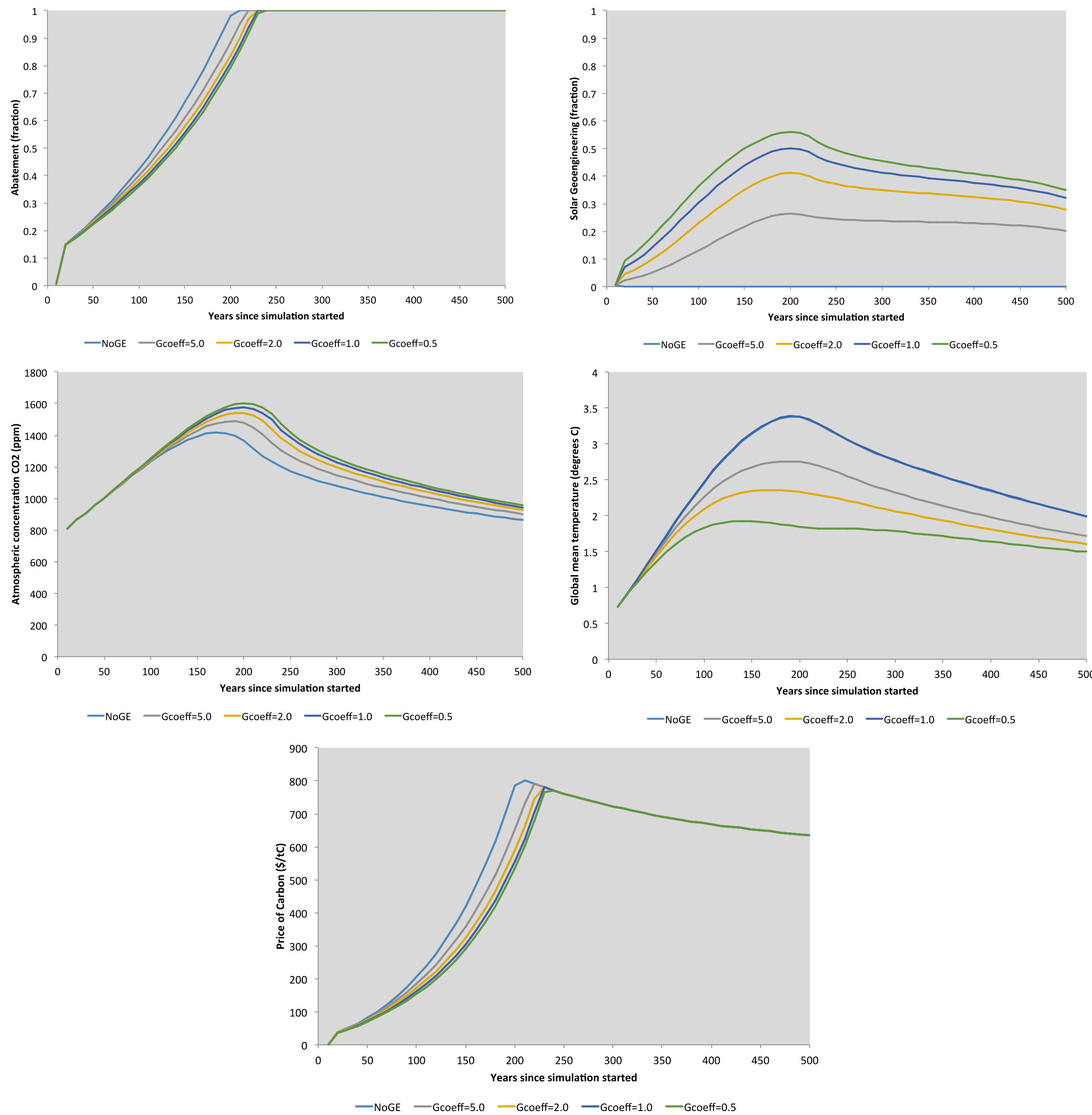


Appendix Figure 1 – Sensitivity Analysis over Equilibrium Climate Sensitivity Distributions

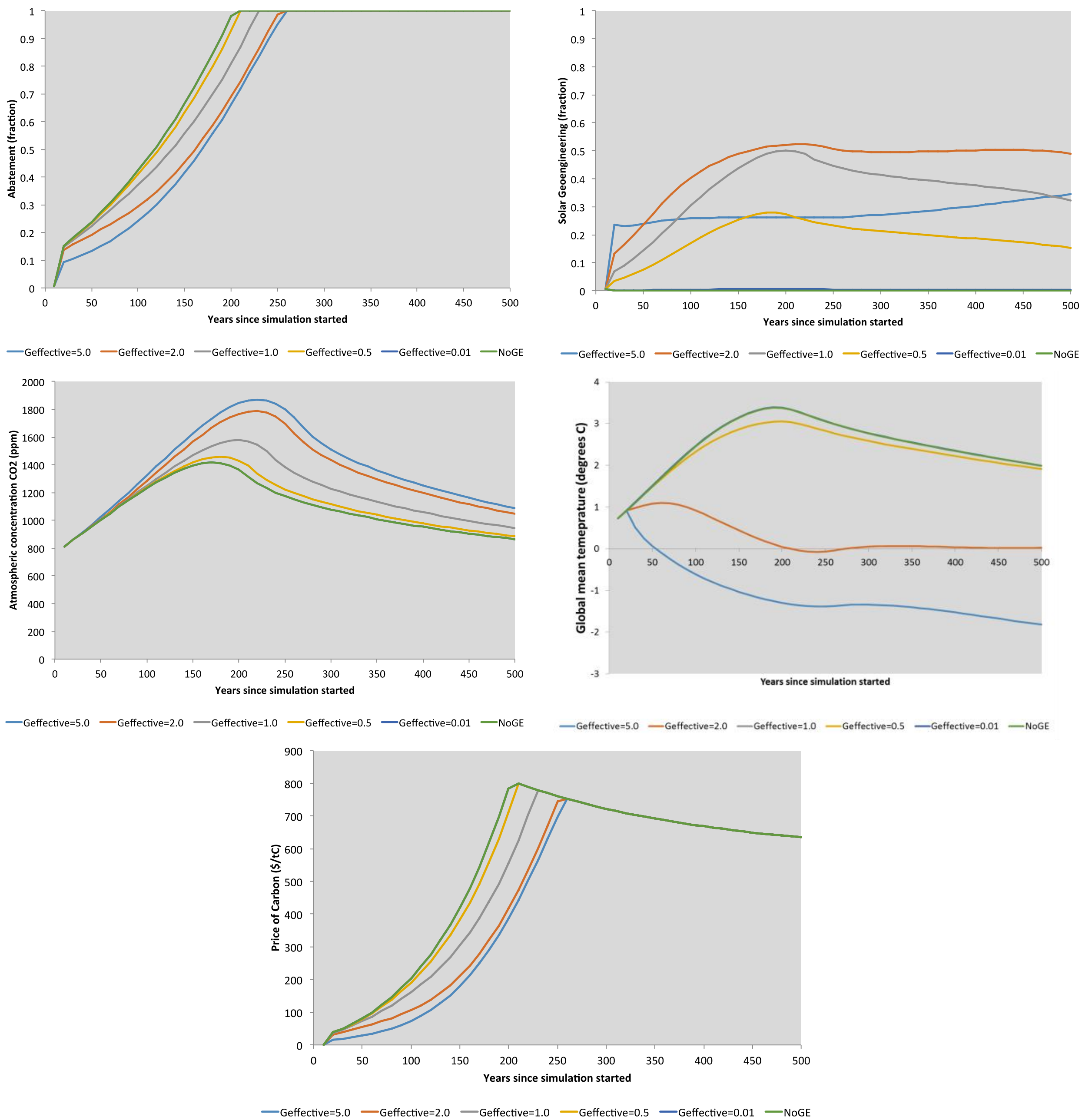
Note: Appendix Figure 1 shows a set of 9 lognormal distributions with different location (μ) and scale (σ) parameters. Each pair of these parameters define a unique distribution that is shown in the bottom right corner of Appendix Figure 1, roughly calibrated to match Millner et al. (2013) Figure 1. For each distribution, we run the uncertainty model from 1000 draws and calculate the median value for the state variables and optimal policies. The first five panels of Appendix Figure 1 Appendix Figure represent the median values of policy variables for different distributions.



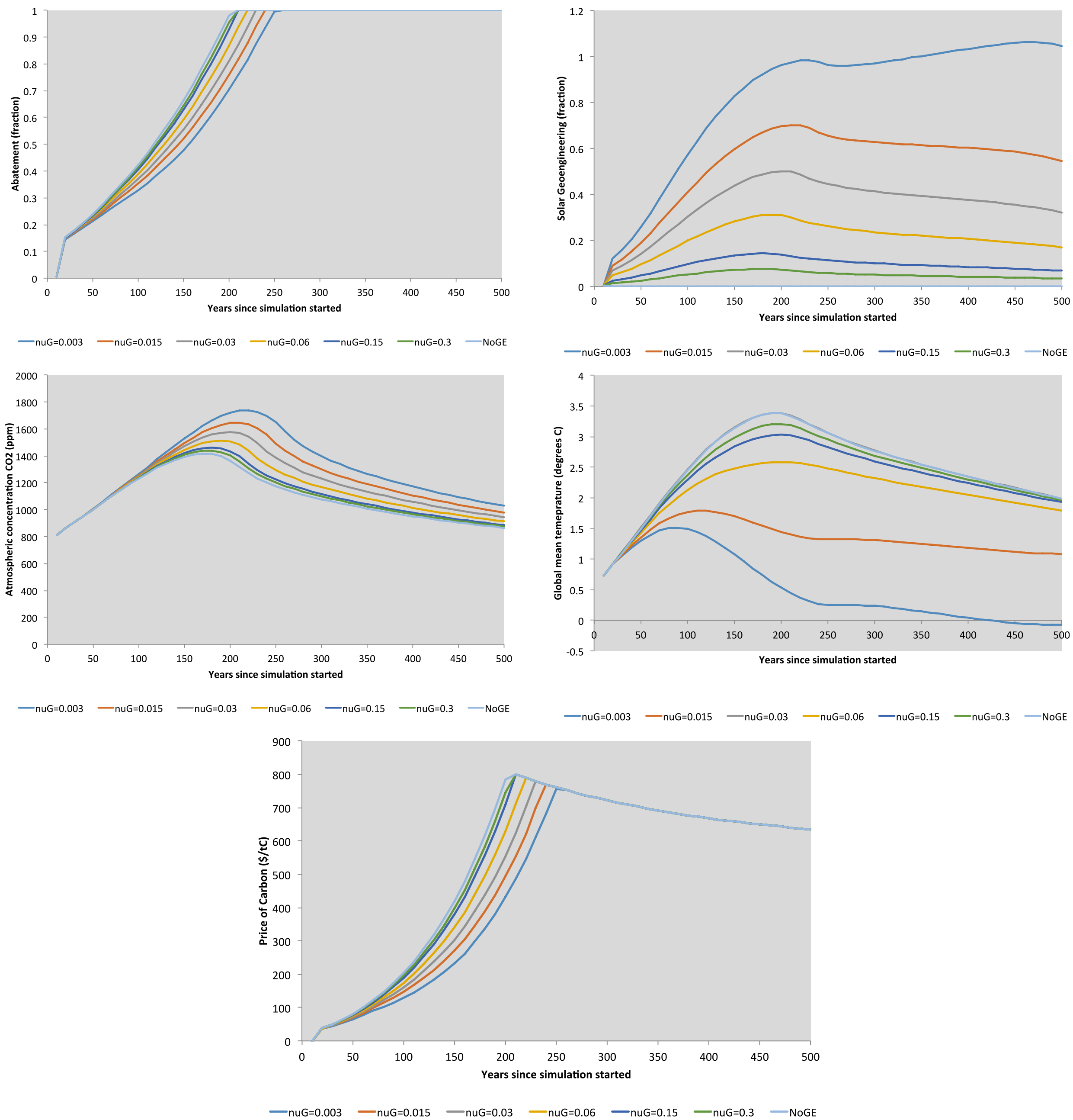
Appendix Figure 2 – SGE as Insurance – Stochasticity in Geoengineering Damages



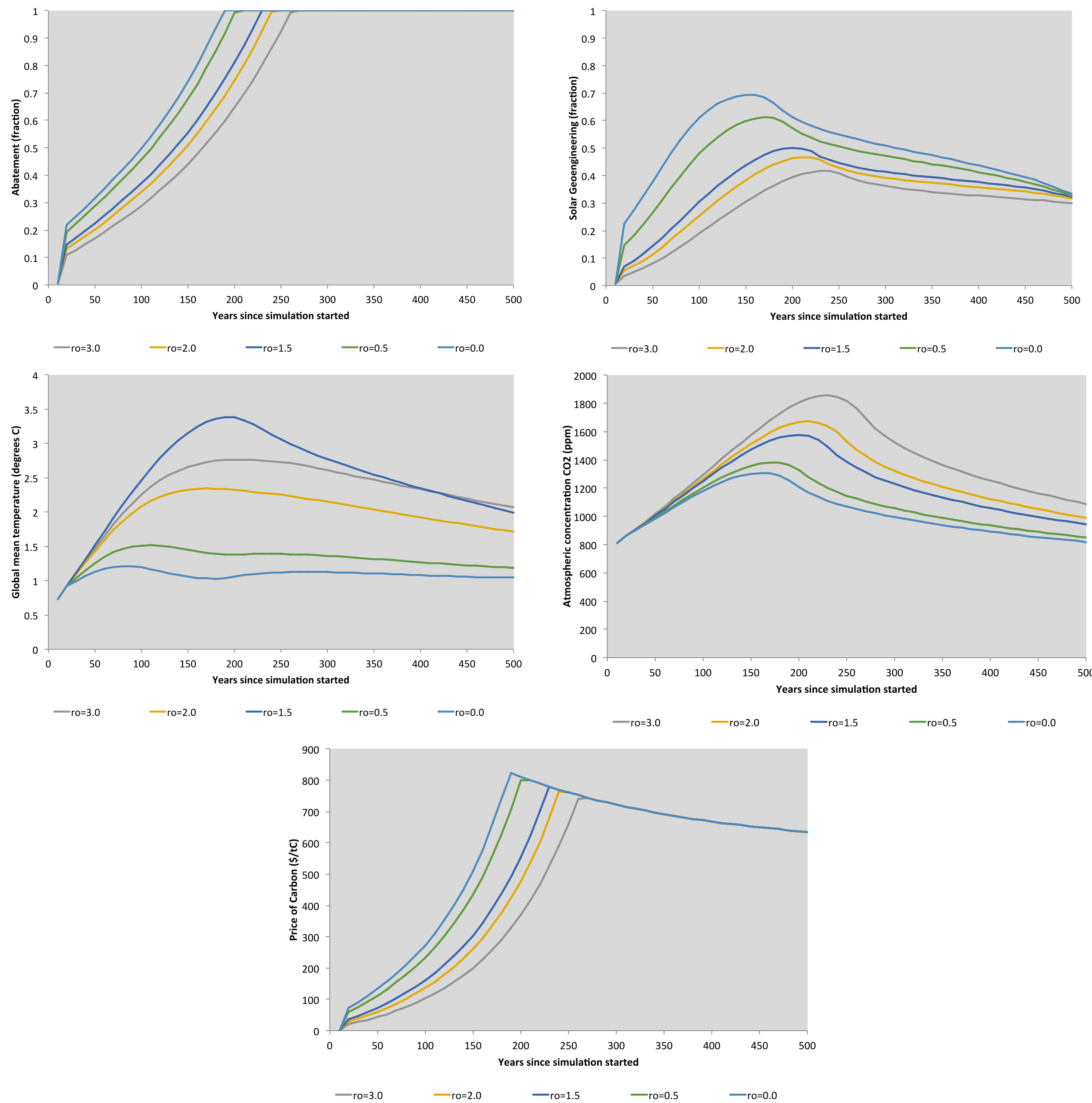
Appendix Figure 3 – Sensitivity analysis – Cost of SGE



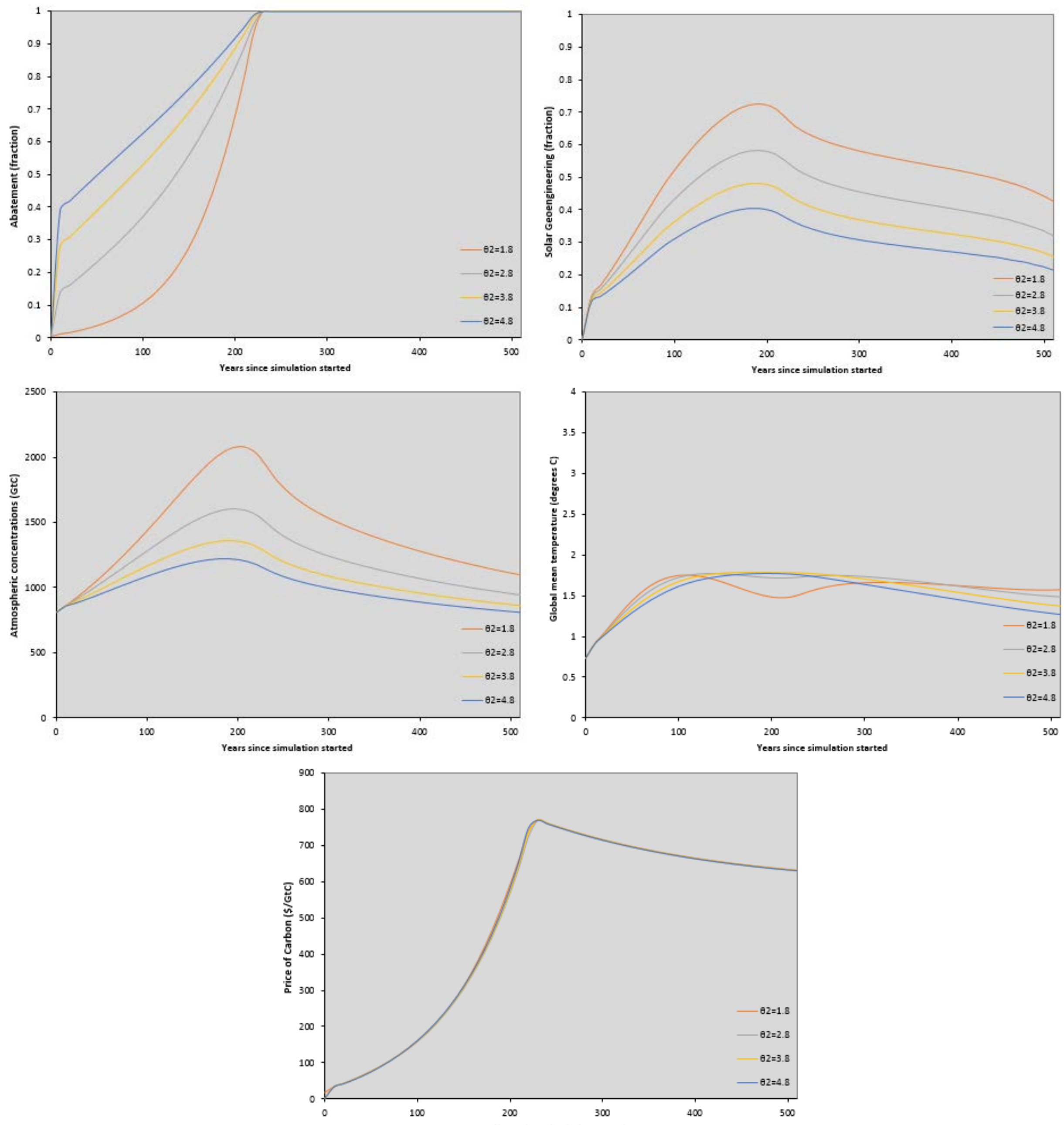
Appendix Figure 4 – Sensitivity analysis – Effectiveness of SGE



Appendix Figure 5 – Sensitivity analysis – Damages from SGE



Appendix Figure 6 – Sensitivity analysis – Discount Rate



Appendix Figure 7 – Sensitivity analysis over Abatement Cost Function Parameter

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